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Model-Based Damage Evaluation of Layered CFRP Structures

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Abstract. An ultrasonic evaluation technique for damage identification of layered CFRP structures is presented. This approach relies on a model-based estimation procedure that combines experimental data and simulation of ultrasonic damage-propagation interactions. The CFPR structure, a $[0/90]_{4s}$ lay-up, has been tested in an immersion through transmission experiment, where a scan has been performed on a damaged specimen. Most ultrasonic techniques in industrial practice consider only a few features of the received signals, namely, time of flight, amplitude, attenuation, frequency contents, and so forth. In this case, once signals are captured, an algorithm is used to reconstruct the complete signal waveform and extract the unknown damage parameters by means of modeling procedures. A linear version of the data processing has been performed, where only Young modulus has been monitored and, in a second nonlinear version, the first order nonlinear coefficient β was incorporated to test the possibility of detection of early damage. The aforementioned physical simulation models are solved by the Transfer Matrix formalism, which has been extended from linear to nonlinear harmonic generation technique. The damage parameter search strategy is based on minimizing the mismatch between the captured and simulated signals in the time domain in an automated way using Genetic Algorithms. Processing all scanned locations, a C-scan of the parameter of each layer can be reconstructed, obtaining the information describing the state of each layer and each interface. Damage can be located and quantified in terms of changes in the selected parameter with a measurable extension. In the case of the nonlinear coefficient of first order, evidence of higher sensitivity to damage than imaging the linearly estimated Young Modulus is provided.

INTRODUCTION

Industry can improve performance when a condition based approach to maintenance can be adopted and intervention planned. For such an approach reliable information is needed to be guide such decisions. One of the sources of information is the results that come from prognosis algorithms, which predict the evolution of the remaining life during which a component is expected to continue performing the required function (1). These algorithms utilize the NDE history of the component, and need several previous nondestructive testing cycles to begin to provide reliable predictions (2,3). An early detection of damage is able to provide information to these algorithms, in such a way that, reliable prognostics are given which can be used to inform and guide maintenance strategies.

Particular challenges are faced in managing carbon fiber-reinforced polymers (CFRP) which are complex materials, both, in structure (inhomogeneous, anisotropic), and damage modes. If, additionally, the aim is an early detection of damage, it is necessary to move NDE techniques to their limits. This complexity is expressed in terms of weak damage signatures inside the received signals, which are particularly difficult to detect. One efficient way to cope with this situation is the use of a model-based evaluation, consisting of solving an inverse problem (IP) (4) in

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order to calculate the effects of damage on the mechanical properties of the specimen. This paper reports work which is focused on testing the capabilities of the model-based IP to evaluate different kind of damage in a cross-ply CFRP specimen for an ultrasonic immersion transmission nondestructive testing approach.

Although a through transmission technique is widely used in CFRP health monitoring, there are few studies in literature which consider the reconstruction of the material mechanical properties using model-based inverse methods. The first inverse problem methods aimed to provide thickness reconstruction and phase velocity for a thin coating on a thick substrate, using a Newton-Raphson method to obtain convergence between simulated and experimental transfer functions (5). This approach was extended to a three layered material (6) minimizing the difference between experimental and theoretical results in the frequency domain, by considering amplitude and phase. Further research allowed the estimation of one of the following four parameters given the other three; thickness, wave speed, density or attenuation (7). In the field of FRP and immersion through transmission technique, Balasubramanian and Whitney (8) presented an inverse method to calculate the elastic constants of thick glass composites. After this work, they applied genetic algorithms (9) to improve the convergence of the procedure with unidirectional composites. Fahim et al. (10) also obtained stiffness and attenuation effects on data using genetic algorithms with composites damaged by impact, and simultaneously reconstructing three parameters. It has been found that there are a small number of studies, reconstructing just a very low number of parameters, and that there are even fewer which are based in experimental data.

This paper presents a model based approach to evaluation of damage in layered CFRP. It includes a methodology for the evaluation process to quantify the degree of damaged in a CFRP structure using a model-based IP with experimental signals. Additionally, a reconstruction process is described which supplies a large number of parameters (nine), in such a way that several damage modes can be interpreted at the same time, namely cross-ply micro-cracking and delamination between crossed layers. It appears that there is no previous study which sought to reconstruct the nonlinear behavior of the material. This manuscript describes the specimens, the damage process and the experimental system. It then provides a description of the inverse problem procedure using genetic algorithms (GA’s). This is followed by a description of the used models and finally the results are presented and discussed.

SPECIMEN AND EXPERIMENTAL SETUP

The test specimen was a CFRP plate, 250x35x2 mm^3 size, manufactured from Cycom 977-2-35-12k HTS pre-pregs mounted on a $[0/90]_{4S}$ stacking sequence, compacted every four layers by applying vacuum for 15 minutes. A 117 ºC, 7 bar, 3 hour curing process was performed. The manufacturer supplied mechanical and geometric features of the layers in the thickness direction, shown in Table 1.

TABLE 1. Mechanical and geometrical properties of the carbon layers and resin interface, reported by the manufacturer.

	Young M. (GPa)	Poisson’s R.	Density (kg/m^3)	Attenuation (Np/m)	Thickness(mm)
0º & 90º layers	11.1616	0.3007	1,589.5	293.0230	0.1215
Interface-matrix	05.2728	0.3500	1,310.0	361.1595	0.0100

This specimen was damaged by impact and fatigue processes. First, impact damage was generated with a drop weight tower with an anti-rebound system (3.8 Joule). Following impact, a fatigue tension-tension loading was applied in an Instron/Schenk load frame to 100kN, under a stress ratio of R = 0.1 and 100,000 cycles.

The damage is expected to be in the form of delamination between the last two back-wall layers, at the opposite side to the impact, together with other delaminations between the 0º and 90º layers, a significant stiffness decrease in the 90º layers, especially affected by fatigue load, and intense matrix cracking.

The setup is a typical ultrasonic immersion through transmission technique, where an automated scan parallel to the specimen is carried out around the impacted area with an area of 40 x 20 mm^2 and data taken at 1 mm steps. The sample is placed in degassed water and experiments performed at room temperature. The specimen was located at the focal distance in front of the transmitting transducer; 30 mm. In reception, the signal passed to a pre-amplified with 40 dB gain and the analogue signal sampled at 12bit-200MHz obtaining a data record of 2000 samples. Averaging 500 signals per scanned location, signal to noise ratio was improved by about 27 dB.

There are some differences between the linear and nonlinear tests. The former uses a transducer in reception, with the specimen located at the mid-point between the transducers. The excitation is accomplished with an Agilent 33220 wave generator producing a one cycle of sinusoidal shape 5 MHz signal with 5 Vpp amplitude and 2μs duration. The non-linear tests are performed with a hydrophone at reception located close to the specimen, which has a nearly flat

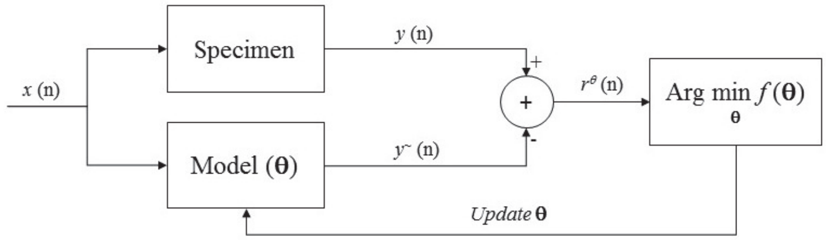

FIGURE 1. Iteration scheme for the inverse problem resolution.

frequency response up to 20 MHz. This wide band receiver enables reception of the second harmonic of the incident signals. In this case, excitation is a sine-burst, 10 cycles long, 5 MHz central frequency and 1.6 V amplitude.

MODEL-BASED INVERSE PROBLEM

The direct problem is commonly used to simulate the received signals with a theoretical model. Complex models use a larger number of parameters and incorporate more complete physical descriptions. The presence of damage affects the local mechanical properties, and if the model parameters properly include these, the simulation will be able to reconstruct the damage. While the direct problem results in simulated signals from a pre-determined set of model parameters, one can think of solving the inverse problem by obtaining the model parameters that simulate the experimental signals. Damage is interpreted by comparing the calculated mechanical properties with the undamaged values.

The assumed result is the set of mechanical properties ($\boldsymbol{\theta}$) used in the model that generated the simulated signals ($\mathbf{y}^{(\boldsymbol{\theta})}$) which are in best agreement with the experimental data ($\mathbf{y}$), or at least agree to within a certain error. It is not usually to deduce analytical expressions for the parameters used in the model equations. The solution has to be achieved by solving an optimization problem (see Figure 1) of an N-dimensional function, with N being the number of parameters in the model. It is a non-obvious problem when the number of parameters is large. The function $f^{(\boldsymbol{\theta})}$, which must be minimized, is a measure of the error and a function of the model parameter set ($\boldsymbol{\theta}$). In this case, the L_2-norm of the residue $\mathbf{r}^{(\boldsymbol{\theta})} = \mathbf{y} - \mathbf{y}^{(\boldsymbol{\theta})}$ is selected. In global search algorithms a different version of the error function is commonly used to speed up convergence, $\hat{f}^{(\boldsymbol{\theta})} = \log (f^{(\boldsymbol{\theta})} + \varepsilon)$, where ε is a fixed small quantity (11).

Different strategies are used to accomplish this convergence process. This present work uses GA's. The main reason is that they are a global search method, therefore, they explore the whole solution space and the risk of selecting a local minima is avoided, which is a common problem with gradient minimization methods. Figure 2 shows a scheme of the model-based IP using GA's, where a population of $N_p = 800$ parameter sets is tested by simulation, selected and completed with operations of mutation, tournament and cross-over, to fulfill one iteration out of 3,200.

MODELS

The models that are selected for this study are based on the Transfer Matrix Formalism (TMF) (12). The TMF is a one-dimensional model that is able to simulate the ultrasonic propagation of plane waves in layered materials whose interfaces are perpendicular to the propagation direction. Each layer is supposed to be homogeneous. Under a pure sinusoidal excitation, the amplitude and phase of the particle displacements are computed in the interfaces applying continuity conditions across the interface for displacements and stress. Both, the direct insonification and the back-propagated disturbance coming from other interfaces are accounted for. The wave field in the next interface (forward- and back-propagated components) is computed by a multiplication of a transfer matrix of the layer by the field in the previous interface. Building the aggregation of all layers, a Transfer Matrix for the complete material is obtained and therefore the signals in the face behind can be directly calculated from the signals in the face ahead. The model is easily expanded to broadband excitations, sampling the temporal frequency space, by applying the Transfer Matrix to each frequency, and then performing an inverse Fourier transform, which calculates the solution in the time domain.

The model uses five parameters per layer; Young modulus in the propagation direction (E), Poisson's ratio due to stresses in the propagation direction (ν), attenuation for absorbing layers (α), density (ρ) and thickness (a).

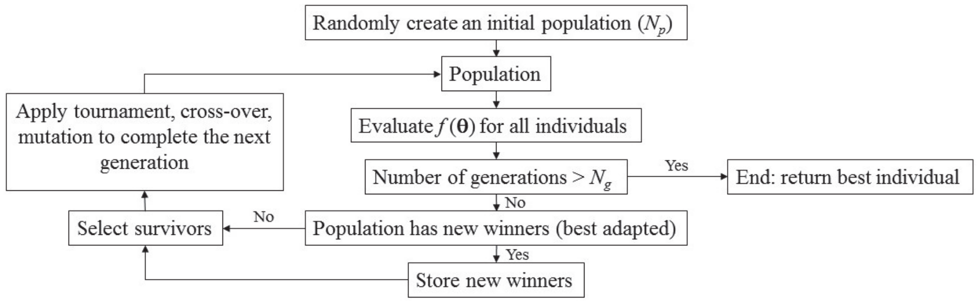

FIGURE 2. Iteration process governed by the GA.

The nonlinear version of the model, Nonlinear Transfer Matrix Formalism (NMTF) (13), is deduced in a similar way, but has to additionally compute the second harmonic, for both the forward- and back-propagated waves, besides the aforementioned fundamental. Given that the second harmonic is calculated, the first order nonlinear coefficient (β) for each layer is added as a parameter in the model, giving then, six parameters per layer.

INVERSE PROBLEM IMPLEMENTATION

The model-based evaluation process must be applied to the experimental data. In that vein, 33 layers are accounted for in the model; 16 are the carbon fiber layers of the specimen, 15 are the matrix interfaces in between the fiber layers with different mechanical properties (those of the matrix), and 2 additional layers for the water either side of the specimen. This situation then implies that the model uses 165 parameters (33 layers x 5 parameter/layer) for the linear case and 198 parameters (33 x 6) for the nonlinear one.

This large number of dimensions in the parameter space is not amenable to fast computation for the inverse problem. Therefore, the most significant parameters are selected by adding some hypothesis based rules regarding parameter behavior as it is related to the expected damage (cross ply cracking and interface delamination), which is based on a prior knowledge (4,14). With this information, parameters can be divided into two groups; those expected to be affected in a straightforward way by damage (and will be calculated), and the rest of the parameters that are taken as constant values. In order to fix these constant values and to know the expected undamaged values of the selected parameters, properties supplied by the manufacturer could be used, but there is usually a high probability that they will differ from their real values. Therefore, a calibration process is performed, where twelve locations are chosen, far from the impacted area, and the inverse problem with the genetic algorithm is solved on each location with two selected parameters; the Young modulus and attenuation coefficient of the interface between fiber layers. All interfaces in between are assumed to have the same values for these parameters. The rest of the parameters are assigned the manufacturer's values (see Table 1). The new calculated values are the average of the 12 locations, summarized in Table 2. Once this calculation has been performed, the Young modulus and attenuation coefficient of the carbon plies is deduced applying mixing rules considering the known fiber/epoxy volume ratio of the plies.

TABLE 2. Calibrated values of Young modulus and attenuation of interfaces in between and fiber layers.

	Young modulus		Attenuation	
	Manufacturer's	Calibrated	Manufacturer's	Calibrated
0º and 90º layers	11.1616	11.0346	293.023	291.8043
Interface	5.2728	5.1921	361.1595	354.0515

Once the model has been calibrated, the IP to detect damage can be addressed. In order to select the significant parameters and to make the inverse problem computationally affordable, the following hypothesis is assumed. Given the damage process, it is expected that a significant delamination will occur at in the interface between the last two fiber layers opposite from the impact, and that this will affect the Young modulus E_i^* of the interface. The first interface between the two first carbon fiber layers at the impact side, will also be assumed to be delaminated and its Young modulus E_i altered in the damage area. The fatigue process will damage 90º layers, and their Young moduli, E_{90}^j, with j = 2, 4, 6, 8-9, 11, 13, 15, will decrease. Under these expectations, all the Young moduli discussed are selected as parameters to be calculated by the inverse problem algorithm, so as to give as a meaningful description of damage for the linear case (see Table 3).

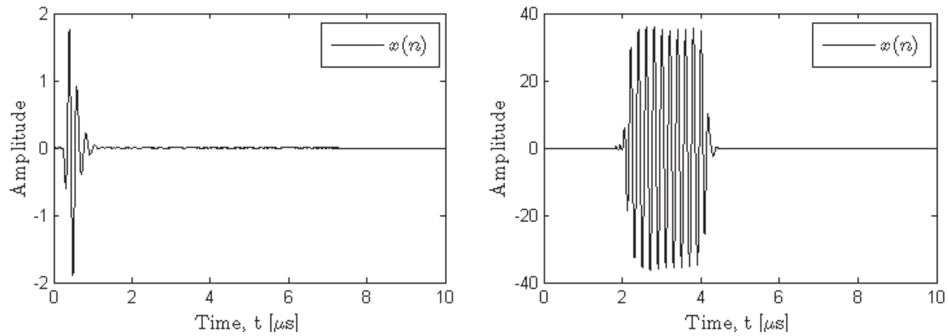

FIGURE 3. Input signals for the linear case (left plot) and nonlinear case (right plot).

TABLE 3. Selected parameters for the linear and nonlinear cases.

Linear	E_i	E_{90}^{2}	E_{90}^{4}	E_{90}^{6}	$E_{90}^{8\text{-}9}$	E_{90}^{11}	E_{90}^{13}	E_{90}^{15}	E_i^*
Nonlinear	E_i	$E_{90}^{2\text{-}15}$	$E_{90}^{4\text{-}13}$	$E_{90}^{6\text{-}11}$	$E_{90}^{8\text{-}9}$	E_i^*	β_{90}	β_i	β_i^*

In the case of the nonlinear model, the aim is different; it is addressed early detection of damage. Therefore, the process can be applied to individual locations that are far from the impact zone, with none or slight damage. In this case, the hypothesis should be different. In order to maintain the nine parameters, the number of previously selected Young moduli must be reduced. As these are slightly damaged areas, the supposition for early damage is assumed to be nearly symmetrically distributed through the specimen's thickness. This will reduce the number of Young moduli parameters to six and provides the opportunity to include three new parameters, the first order nonlinear coefficient from the first and last interfaces, β_i^*, β_i, and a common parameter β_{90} for all layers with 90º orientation having the same value.

Input signals used in the models are shown in Figure 3 (linear left, nonlinear case right) and these are the calculated signals, as seen in water, which are used to enable the effect of the transducer transfer function on the signal to be considered.

RESULTS

Results of the Linear Experience

For the simulated signal shown in Fig. 4 the model data (black) that best fits the experimental one (red) for two different locations are given. The set of parameters that generates these best-adapted simulated signals are shown above each graphic. Damage is inferred by comparing these values with the expected undamaged data (e.g. 11.0 for the Young modulus of the cross layers and 5.1 for the Young modulus of the interfaces).

The signals show good agreement, especially in the part with larger amplitude. Looking at the values of the mechanical properties, the plot on the left (Fig 4a) shows a point with a stiffness reduction along all the layers, clearly stronger in the 15th layer (E_{90}^{15} = 2.1) and the last interface between the layers 15 and 16 (E_i^* = 0.6), that has almost lost its strength. It represents a large stiffness reduction from the expected undamaged values (11.0 and 5.1 respectively). This last interface was expected to have a strong delamination, as the result shows, and it is an observed pattern along most scanned signals. The plot on the right (Fig 4b) depicts the signal corresponding to another point of the scan, with a higher level of damage. Although the Young modulus for the commented layers have some recovery, they continue to be damaged, since the values are E_{90}^{15} = 5.2 is below the undamaged value of 11.0, and E_i^* = 3.9 as compared with 5.1. Central layers are more affected, especially in layers 11 (E_{90}^{11} = 0.8) and 13 (E_{90}^{13} = 1.2).

The reconstruction of the values of the mechanical properties for each layer, allows the possibility to produce C-scan plots for every parameter. Figure 5 shows the C-scan of the reconstructed parameter for the double cross layer numbers 8 and 9 in the center of the thickness, $E_{90}^{8\text{-}9}$. This kind of graphic allows evaluation of the damage state of this individual layer, in terms of damage detection, location and sizing. This detailed information for one layer cannot be detected with conventional C-scan imaging techniques, because they show the amplitude of the received signal or

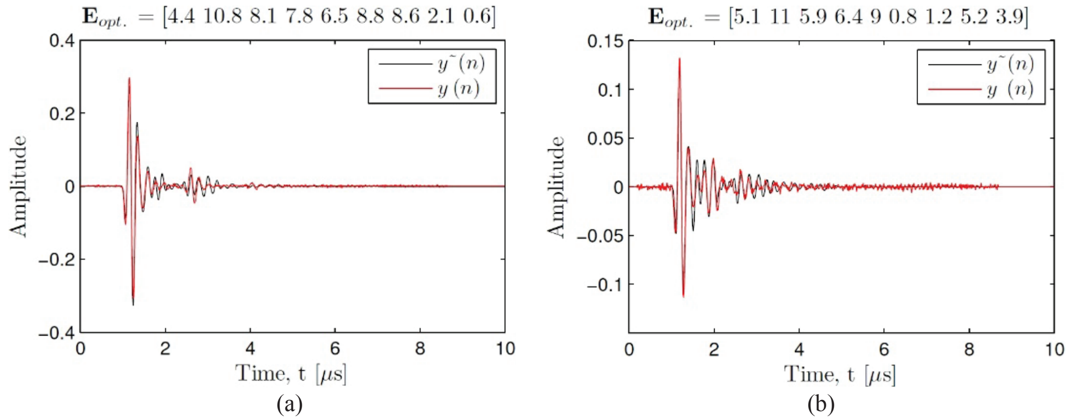

FIGURE 4. Results of linear experience showing a location with moderate damage (a) and some more intense damage (b). The black signal is the best-adapted simulated signal for the location, and the red one is the experimental signal for this location. Above, the best-adapted set of parameters that generates the simulated signal.

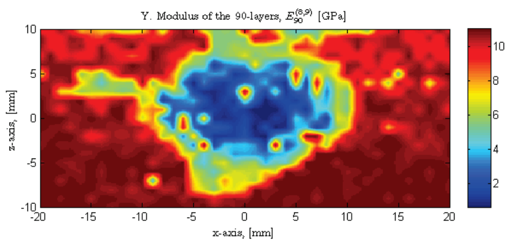

FIGURE 5. Results of the linear case. C-scan of the Young modulus of the middle layer (double 90 degree 8-9 layer).

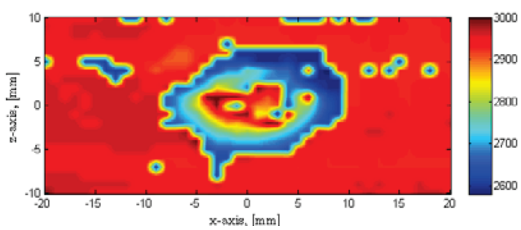

FIGURE 6. Conventional C-scan of the signal propagation velocity (m/s) through the thickness.

the average propagation velocity across the thickness, as depicted in Figure 6, which are averaged magnitudes for the interaction with the whole thickness. The difference between Figure 5 and Figure 6, is mainly in the central part of the impact, where the layer 8-9 is more affected than could be determined from the data in a C-scan or data averaged plot. It can be also observed that the lower part of the layer mantains better stiffness than shown in the C-scan or data averaged plot.

Results of the Nonlinear Experience

Although it is recognized that using a highly damaged sample is probably not the best way to validate an approach to early detection of damage capability using nonlinear ultrasonics, these results are used to provide initial early data and to demonstrate potential to detect such damage.

The technique has been applied over areas with low levels of damage. Figure 7 shows the signals from a location that is far from the impacted area. The agreement between the model and experimental data is good the in time domain. The magnitude spectrum has been included in the figures to enable differences to be seen. The NMTF is able to

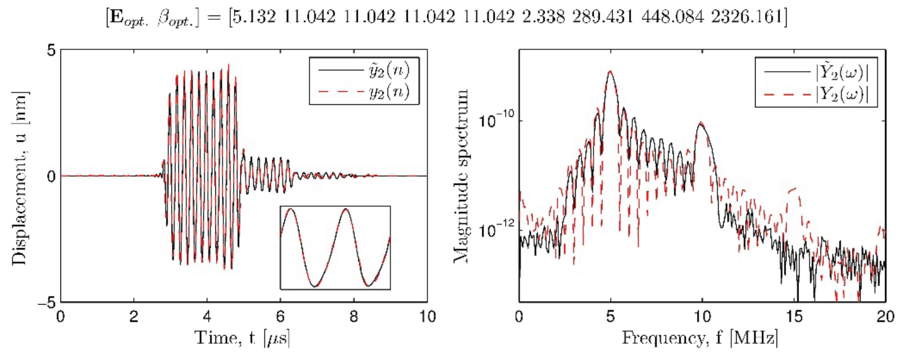

FIGURE 7. Results of nonlinear experience. Signal from a slightly damaged location.

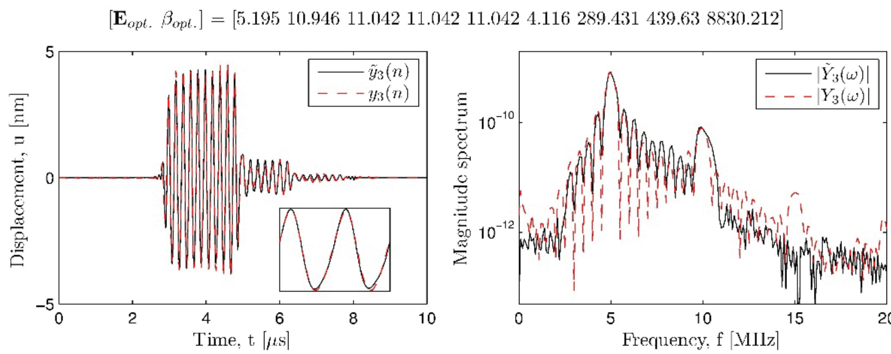

FIGURE 8. Results of the nonlinear model and measurements at a slightly damaged location. (a) time domain signals (b) magnitude spectra.

simulate the second harmonic generation and is not currently able to simulate higher harmonics. This is the main difference seen between the simulated and experimental signals, where third and fourth harmonics are experimentally evident, but they are not included in the simulated signal. The adjusted mechanical properties at the interfaces seemed to be slightly affected, while the stiffness of the 90º layers is not altered. No data appear to be available for the β parameter for epoxy materials or a combination of epoxy/fiber for the 90º, therefore no reference is available for those values, other that the calibration process resulting in a value of about 173.2. The calculated values for the interfaces are higher than the calibrated value, especially the back-wall interface parameter $\beta_i^* = 2{,}326$. Additionally, this value corresponds to a layer where the linear experience has shown to be more severely delaminated than the first interface, which is showing a lower nonlinear coefficient of first order $\beta_i^* = 448$. Figure 8 shows another low damaged point. The Young modulus of the cross plies is maintained at the same value and their β_{90} parameter is similar to the last case. In the case of the interfaces, the back-wall interface parameter $\beta_i^* = 8{,}830$ is again higher than the value of the first interface $\beta_i = 439$. As this pattern is repeated in other similar points, it seems that there could be some correlation between the nonlinear coefficient of first order and the damage state of the layers, with higher absolute sensitivity than the Young modulus.

CONCLUSION

A model-based inverse problem has been presented for damage evaluation of layered CFRP structures inspected under a through-transmission ultrasound configuration, which can be applied to any other layered material. Although, the current models require layers to be homogeneous and isotropic, they can be used as a good approximation in materials with other kind of layers if their thickness is small or there is a small correlation between effects in different directions which can be assumed. The technique is able to supply detailed information regarding the damage state layer by layer instead of averaged values for the whole thickness. The possibility to compute 9 parameters, enables the capability to evaluate different kinds of damage at the same time; considering the interfaces between carbon fiber plies as another layer in the model, with its own mechanical properties, giving the technique the capability to easily distinguish between matrix cracking and delamination when damage is interpreted.

The selection of the model parameters to be calculated is a crucial issue, since simulation processes with too large number of parameters affect the convergence of the solution, in both computation time and the possibility to execute ill-posed processes. Damage hypotheses based on prior knowledge regarding experience and specimen history are necessary to enable the selection of suitable parameters.

Concerning the nonlinear experience, it seems that it could be a good procedure to detect early damage. Although this methodology should be compared with data from other techniques such as micrograph, or crack counting, etc., these preliminary results are promising and open the possibility to design a specific method of estimating damage and properties. Further work must be performed to validate this hypothesis and determine early damage thresholds using the through transmission technique. This idea should consider two important points, 1) the use of specimens with low levels of damage rather than higher degrees of damage and 2) the design of an experimental procedure allowing the isolation of the nonlinear response of the specimen from the rest of nonlinear sources, mainly that of the emission electronics and transducer. This issue would approach obtaining a technique, which is only specimen-dependent and would enable reproduction of similar results by different users.

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