Soft Computing
https://doi.org/10.1007/s00500-018-3362-z

METHODOLOGIES AND APPLICATION

A model towards global demographics: an application—a universal bank branch geolocator based on branch size

Julia García Cabello[1]

Abstract

Branch size strongly depends on branch cash holdings. However, while any exhaustive study into branch cash holdings must include demographics around branches, there are major variations when defining demographics according to "local" parameters, as opposed to "internationally accepted" ones. This wide fluctuation in definitions makes cross-border comparisons very difficult. The present paper intends to overcome these difficulties by developing a *global* spatial model that uses cash holdings as a major determinant of branch size and where geographical concepts are replaced by "internationally accepted" notions. Specifically, the contributions of this paper are twofold: firstly, it presents a theoretical model (based on Markov and Gibbs random fields) to analyse the branch cash holdings from a *global* spatial standpoint. Secondly, it introduces a universal branch geolocator based around a decision model that redesigns the bank branch network according to branch size. Importantly, the model variables (including branch size as the main criterion) can be replaced/expanded as needed through the use of a *highly versatile* decision-making tool that can be applied to a wide range of contexts, even non-banking ones as long as they are influenced by demographics.

Keywords Universal geolocator · Branch size · Branch cash holdings · Spatial stochastic processes · Demographic parameters

Mathematics Subject Classification C61 · C63 · G17 · G21

1 Introduction

Demographics are an important determinant in the research of several fields. However, there are major variations when it comes to defining demographics using "local" rather than "internationally accepted" parameters. Actually, the distinction between urban and rural areas is growing *fuzzy*. While the main criteria used to define these areas commonly include population size/density and availability of certain support services such as secondary schools and hospitals, the combination of criteria applied can vary greatly: even different population thresholds can be used. This lack of a precise definition of demographics at a global level complicates both research in academic and scientific fields and the possibility of international comparisons.

The fuzziness of local demographics also adversely affects studies into the banking sector. One example is the branch-site selection problem. This issue consists of finding the best location for branches. In the present scenario of a highly competitive banking industry, demographic branch-site selection is one of the key factors in maximising bank' profitability and increasing its market share. Unfortunately, local demographics fuzziness prevents—from a classical point of view—the design of procedures that are intended to validate all possible (international) scenarios. Actually, most bank branch analyses that consider local demographics as key determinants cannot be extrapolated to more general contexts. Nevertheless, this example lets us introduce the more general problem of reordering the bank branch network according to some fixed criteria.

This paper deals with bank branch network' restructuring based on branch size. Particularly, it formulates a method that redesigns the bank branch network according to branch

Communicated by V. Loia.

✉ Julia García Cabello
cabello@ugr.es

[1] Department of Applied Mathematics, Faculty of Economics and Business Sciences, University of Granada, Campus de Cartuja, 18071 Granada, Spain

size. To this end, *branch cash holdings were selected as the main determinant of branch size*.

Here (branch cash holdings), we encounter the difficulty of local demographics' fuzziness because demographics play a central role in the analysis of branch cash holdings. As a matter of fact, any exhaustive study into branch cash holdings has to include the dependence on the demographics around each branch since their cash transactions will depend on their customers' cash needs.[1] However, as mentioned previously, demographic parameters have to be managed carefully due to the significant variations when defining them as either local or internationally accepted parameters. This means that cross-border comparisons are difficult (see André et al. 2014), "Scientists from different disciplines diverge when defining these zones (rural/urban) or their limits; they even often mention the zones without any definition. This practice excludes comparison between studies", [sic]). The present paper intends to overcome these difficulties by developing a *global* spatial model with regard to branch cash holdings where classical geographical concepts are replaced by "internationally accepted" notions. Once the aforementioned fuzziness of local parameters has been exceeded, apart from allowing cross-border comparisons, a global approach will also be beneficial to many cases: for instance, in order to decrease costs when replacing several local approaches with a universal one.

Another positive point is the fact that the proposed spatial framework would still be valid if the selected variable[2] "branch cash holdings" was replaced by another one (number of users, brick-and-mortar branch dimensions, etc.). The cash holdings approach is just one instance of the spatial model' functionality in the banking sector.

Importantly, this new methodology can also be applied in other contexts outside a banking scenario. As a matter of fact, replacing classical standards with "internationally accepted" notions would also be fruitful in any contexts where demographics play a role, thus enabling other disciplines to fully enjoy the benefits of the globalisation of demographic parameters.

Specifically, the contributions of this paper are twofold. The first is a theoretical setting (based on the notions of Markov and Gibbs random field) designed to analyse the branch cash holdings from a *global* spatial standpoint. Its main objective is to move towards a global unified vision of the classical geographical standards, thereby surpassing local demographics. The second is a universal branch geolocator in function of branch size, developed from the given theoretical setting. The geolocator is a decision model that could help managers select the best locations for branches depending on their size, working on the premise that cash holdings are a major determinant of branch size. In fact, the geolocator is designed as a decision-making tool to be used as required when redesigning the bank branch network. As far as the author knows, this is the first time that Markov random fields, which are mainly employed for vision and image processing (Wainwright and Jordan 2008), have been applied to the banking sector.

With respect to work published in the literature, I could not find any studies that deal with the problem of globalising demographic parameters into a unified approach. Conversely, there are several studies that address the selection of the best locations for branches. Actually, branch-site selection is one of the most important decision-making processes for banks because, if done correctly, it can provide access to the best customers and the greatest market potential. This issue is approached from different perspectives in the current literature.

One perspective treats site selection according to certain pre-established criteria. In this case, in addition to the large variety of factors presented in the literature, a full range of mathematical techniques is used. In Abbasi (2003), a decision support system was developed for locating bank branches using a database of local demographics. In Boufounou (1995), a model was designed for planning new branch locations using regression analysis. In Cinar (2009) having previously identified five main criteria (local demographics, socioeconomic factors, banking indicators, recruitment in accordance and trade potential), a decision support model for bank branch location selection was devised using the fuzzy analytic hierarchy process (AHP). More recently, in Zainab et al. (2014), the hybrid method of AHP and Monte Carlo simulation was used in order to prioritise locations and select the best. In Allahi et al. (2015), a more sophisticated model for selecting optimal site location was proposed by integrating available data sources and decision models such as the AHP, a geographic information system (GIS) and the maximal covering location problem (MCLP).

The problem of selecting the best site for a new branch can also be viewed as part of the more general problem of restructuring the bank branch network, (Cerutti et al. 2007). In Ioannou and Mavri (2007), authors presented a decision support system for reconfiguring branch networks, based specifically on information about the bank' local demographics. In Miliotis et al. (2002), mathematical programming was used to present a method for reorganising the bank service network by combining geographical information systems (GIS) representing local geographical/social attributes—with demand-covering models. Other authors, see Ruiz-Hernandez et al. (2015), presented a branch restructuring model by using integer 0–1 programming and aimed specifically at restructuring the branch networks after mergers and acquisitions, where banks frequently have to face the

[1] A heavy retail area will require much more cash than a predominantly industrial area where firms do not deal with much cash.

[2] As major determinant of branch size.

Journal: 500 MS: 3362 ☐ TYPESET ☐ DISK ☐ LE ☐ CP Disp.:2018/6/30 Pages: 13 Layout: Large

problem of redundant branches competing in the same market. Other approaches on geographically modelling financial organisations are based on fuzzy cognitive maps (FCMs), see Glykas and Xirogiannis (2005) where authors generate a hierarchical and dynamic network of interconnected financial knowledge concepts by using FCMs.

The remainder of the paper is organised as follows. Since the theoretical framework is based on two special kinds of graphical models (Markov and Gibbs random fields), Sect. 2 of this paper sets out the background on graphs and graphical models, while Sect. 3 provides an overview of Markov and Gibbs random fields as well as laying down the relationship between them. Section 4 focuses on developing the global spatial model for bank branches. In Sect. 5, the universal geolocator (the decision model) is derived from the previous theoretical framework. Section 6 discusses the versatility of the proposed geolocator. Finally, Sect. 7 presents the conclusions of the paper.

2 Background: graphs and graphical models

2.1 A short glossary of terms: directed and undirected graphs

Recall that a graph in discrete mathematics is a set of vertices (or nodes) and a collection of edges, each connecting a pair of vertices. They are represented by $G = (V, E)$, where V represents the set of vertices and E the set of edges. An *undirected graph* (also called undirected network) is a graph where all the edges are bidirectional. In contrast, a graph where the edges point in a single direction is called a directed graph (see Fig. 1).

When drawing an undirected graph, the edges are typically drawn as lines between pairs of nodes instead of arrows, which are reserved for directed graphs, that is, in directed graphs edges have a specific direction while in undirected graphs they do not (edges are two ways). Hence, we can formally define an undirected graph as $G = (V, E)$ consisting of the set V of vertices and the set E of edges such that an *edge* is an *unordered* pair of elements of V. A *path* is a list of a graph' vertices where there is an edge between each vertex and the next vertex. An undirected graph that has a path between every pair of vertices is called a *connected graph*. Two vertices u and v are *adjacent*, $u \sim v$, if $(u, v) \in E$. By convention, we assume that there are no edges from a vertex to itself. If the cardinality of V is n, thus the cardinality of E is at most $\binom{n}{2}$. To end this short "glossary" of terms, a subgraph $S = (V', E')$ of a given graph $G = (V, E)$ (i.e. S is a graph whose vertices and edges are subsets of V and E) is called a *maximally connected subgraph* if S is connected, and if for all vertices u such that $u \in V, u \notin V'$, there is no vertex $v \in V'$ for which $(u, v) \in E$. That is, a *maximally connected subgraph* is a connected subgraph of a graph to which no vertex can be added and it still be connected. A *clique* C in a graph $G = (V, E)$ is a subset C of the set of vertices V, $C \subset V$, such that

- C consists of a single node or
- for every par of vertices $u, v \in C$ must be that $u \sim v$.

That is, cliques in a graph are maximally connected subgraphs.

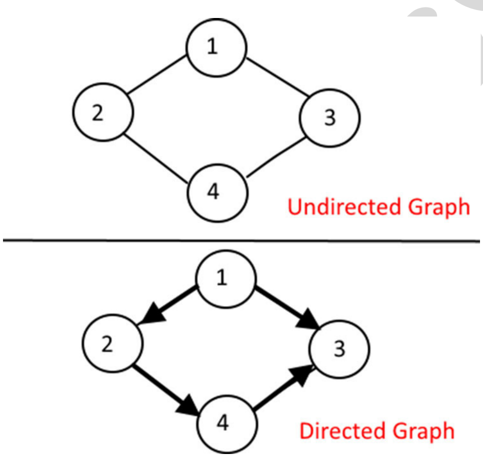

Fig. 1 Differences between directed and undirected graphs

2.2 More than graphs: graphical models

Graphical models are a powerful approach that provides a joint representation of knowledge about random variables and their interrelationships. In other words, graphical models bring together graph and probability theory: while graphs are an intuitive way of representing and visualising relationships amongst random variables, graphical models also allow us to express the conditional dependence structure between them (see Fig. 2).

Besides being a language for formulating models, graphical models inherit the good computational properties of graphs. For instance, while the running time of an algorithm or the magnitude of an error bound can be characterised in

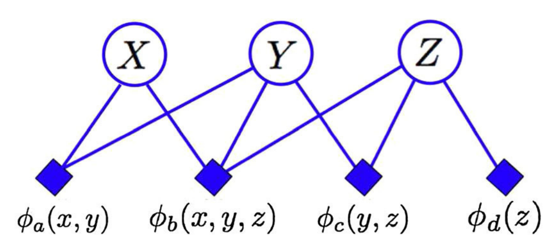

Fig. 2 Graphical models

Journal: 500 MS: 3362 TYPESET DISK LE CP Disp.:2018/6/30 Pages: 13 Layout: Large

terms of structural properties of graphs, this also holds true for graphical models.

Since graphical models represent knowledge about random variables and their interrelationships, they can be viewed as graphs where nodes correspond to random variables and edges represent statistical dependencies between the variables. Thus, graphical models can also be considered as *spatial stochastic processes* understood as those collections of random variables $\{X_v, v \in V\}$ which take a value X_v for each location v over the region of interest V. Such spatial stochastic processes are also called *random fields on* V. Commonly, the region V is a discrete lattice. For simplicity, the non-negative finite two-dimensional grid in the plane $\{1, 2, \ldots, n\} \times \{1, 2, \ldots, n\}$ will be taken as V.

The two most common types of graphical models are Bayesian and Markov networks (also called Markov random fields). The main difference between them is the underlying graph, Bayesian networks are based on a directed graph, whereas Markov random fields use an undirected graph. For the purposes of modelling branch cash holdings based on demographics, we will focus on Markov random fields. In fact, a Markov random field (MRF) is an undirected graphical model that explicitly expresses the conditional independence relationships between nodes in such a way that two nodes are conditionally independent if all paths between them are blocked by given nodes. Both MRFs and their connections with branch cash holdings will be described in more detail in the following section.

3 Gibbs random fields and Markov random fields

This section summarises the structures known as Gibbs random fields (Gibbs distributions) and Markov random fields as well as the connection between them. Roughly speaking, both Gibbs (GRF) and Markov random fields (MRF) are representations of a set of random variables and their relationships that can be depicted as an *undirected* graph. More specifically, a set of random variables is said to be a Gibbs random field if and only if its configurations obey a Gibbs distribution while a Markov random field is characterised by the Markov property. This "Markovianity" is a local property, whereas the Gibbs distribution that characterises a GRF is a global property. However, the Hammersley–Clifford theorem (Hammersley and Clifford 1971) establishes the equivalence of these two types of properties.

Let us examine this in greater detail. Consider a finite collection of random variables $X = \{X_v\}$ taking values in a finite set V, $X = \{X_v, v \in V\}$ (as mentioned, V is the finite two-dimensional grid in the plane $V = \{1, 2, \ldots, n\} \times \{1, 2, \ldots, n\}$). We denote $P[X]$ to the joint distribution of this finite collection of variables. That is, $P[X]$ is the corresponding set of each variables' distribution, as follows:

$$P[X] = P[\{X_v = x_v/v \in V\}] = \{P[X_v = x_v]/v \in V\}.$$

The set V can be viewed as the set of vertices of some graph $G = (V, E)$.

3.1 Gibbs random fields

For a collection of random variables $X = \{X_v, v \in V\}$, we say that the joint distribution of X is a Gibbs distribution relative to the graph $G = (V, E)$ if it can be expressed as a product of clique potentials in G. That is, if we denote $X_C = \{X_v, v \in C\}$, where $C \in \mathcal{C} \subset V$ is a clique in $G = (V, E)$, then functions ϕ_C are required so that the joint distribution of X, $P[X]$, takes the form

$$P[X] = \frac{1}{Z} \prod_{C \in \mathcal{C}} \phi_C(X_C), \tag{1}$$

where $\phi_C(X_C)$ is the Cth clique potential (function), a function that only considers the values of the clique members in C. Each potential function ϕ_C must be positive, but unlike probability distribution functions, they do not need to total a value of 1. A normalisation constant Z is required to create a valid probability distribution $Z = \sum \prod_{C \in \mathcal{C}} \phi_C(X_C)$. Usually, these potentials are only taken to be functions over the *maximal cliques*, that is, cliques which are not proper subsets of any other clique.

Moreover, clique potentials usually take the form $\phi_C(X_C) = \exp(-f(C))$ where $f(C)$ is an energy function over values of C. The energy assigned by the function $f(C)$ is an indicator of the likelihood of the corresponding relationships within the clique, with a higher-energy configuration having a lower probability and vice versa. If this is the case, Eq. (1) can be rewritten as

$$P[X] = \frac{1}{Z} \exp\left[-\sum_{C \in \mathcal{C}} f(C)\right]. \tag{2}$$

If energy functions $f(C)$, $C \in \mathcal{C}$ are quadratic functions, the Gibbs field is known as a Gaussian Gibbs field.

3.2 Markov random fields

For a collection of random variables $X = \{X_v/v \in V\}$, we say that X is a Markov random field relative to $G = (V, E)$ so long as the full conditional distribution of X depends only on the neighbours, according to the previous definition of "neighbourhood". This local property is known as Markov property ("Markovianity"), and it has a well-understood significance for Markov chains. A Markov chain is a random process X_n in which the full conditional distribution of X_n, $P[X_n = x_n | X_k = x_k, \forall k \neq n]$, depends only on the past neighbours X_{n-1}. In other words,

Journal: 500 MS: 3362 TYPESET DISK LE CP Disp.:2018/6/30 Pages: 13 Layout: Large

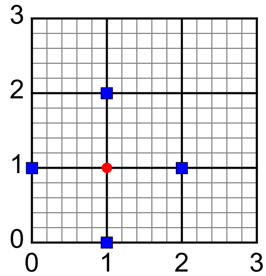

Fig. 3 Neighbourhood of site (1, 1)

$$P[X_n = x_n | X_k = x_k, \forall k \neq n] = P[X_n = x_n | X_{n-1} = x_{n-1}].$$

In order to move from Markov chains to MRFs, let us say that, although both are stochastic processes, the main difference between them is the underlying domain: discrete Markov chains move across a one-dimensional surface ($\{1, 2, \ldots, n\}$), while MRFs go across a two-dimensional surface (for simplicity, the finite two-dimensional grid in the plane, $V = \{1, 2, \ldots, n\} \times \{1, 2, \ldots, n\}$). In fact, the key feature that differentiates the two underlying domains is the universally accepted direction present in the real number line (particularly in the discrete subset $\{1, 2, \ldots, n\}$) due to its linear nature,

$$\begin{array}{ccccc} X_{n-1} & \leftarrow & X_n & \rightarrow & X_{n+1} \\ \text{past location} & \leftarrow & \text{present location} & \rightarrow & \text{future location} \end{array}$$

which means that "proximity" can be defined by the distance and/or the degree of proximity employed. However, for moves in V the primary concept that must be defined is their *direction* move because if it is not specified then we would not know which direction to move? Once this has been done, the notion of distance/proximity should be defined.

In summary, in order to transfer from Markov chains to MRFs, both concepts of *direction and proximity* need to be specified. In the literature, this is usually done through the neighbourhood of a site concept. Let $V = \{1, 2, \ldots, n\} \times \{1, 2, \ldots, n\}$ be the set of nodes of a finite gride. Each $v \in V$ may be also called *site*. The *neighbourhood of a site* (i, j), written $N(i, j)$, is commonly defined as follows:

$$N(i, j) = \{(i-1, j), (i+1, j), (i, j-1), (i, j+1)\},$$

where one may take $(0, j) = (n, j), (n+1, j) = (1, j), (i, 0) = (i, n), (i, n+1)(i, 1)$. For instance, the neighbourhood of (1, 1), $N(1, 1)$, is shown in Fig. 3.

Note that, in the above definition, both direction and proximity are implicitly and explicitly specified. Based on the neighbourhood of a site concept, the Markov property can now be extended from chains to MRFs as follows:

$$P[X_v = x_v | X_{V-\{v\}} = x_{V-\{v\}}] = P[X_v = x_v | X_{N(v)} = x_{N(v)}].$$

Also, the concept of neighbourhood of a site allows to consider the notion of *clique*: A *clique* c is a set of sites such that any pair of elements $c_i, c_j \in c$ hold that $c_i \in N(c_j)$ and $c_j \in N(c_i)$. In the current context of bank branches, the neighbourhood of a branch shall not be defined in terms of geographical coordinates but rather in terms of the distinctive features of each kind of branch (for instance, urban, rural or business centres). Also the notion of clique will be translated into banking practice terms. These points are developed in Sect. 4.

3.3 Relationship between the Gibbs and Markov random fields

Given a Markov random field and its associated conditional dependence relationships, what is the form of the joint probability distribution $P[X]$? Indeed, can we even show that such a distribution exists? The Hammersley–Clifford theorem proves that a Markov random field and Gibbs field are equivalent with regard to the same graph as long as $P[X] \geq 0$. This requirement is known as the "positivity condition".

Theorem 1 (Hammersley–Clifford theorem) *According to the positivity condition, X is a Gibbs random field relative to an undirected graph G is and only if X is a Markov random field relative to G.*

Summarising the above results, the following statements are true:

1. Given any Markov random field, all joint probability distributions that satisfy the conditional independencies can be written as clique potentials over the maximal cliques of the corresponding Gibbs field.
2. Given any Gibbs field, all of its joint probability distributions satisfy the conditional independence relationships specified by the corresponding Markov random field.

In particular, the first condition will be central to our purposes of monitoring branch cash holdings in function of local demographics.

4 Bank branch network' cash holdings as a Markov random field

This section demonstrates that the bank branch network is a Markov random field for which branch cash holdings will be the variable with dependence on "internationally accepted" demographics. On one hand, as mentioned previously, branch cash holdings have been selected since they are major determinants of branch size. On the other hand, recall that Markov random fields (all graphical models indeed) can be considered as graphs with nodes that correspond to random

Journal: 500 MS: 3362 TYPESET DISK LE CP Disp.:2018/6/30 Pages: 13 Layout: Large

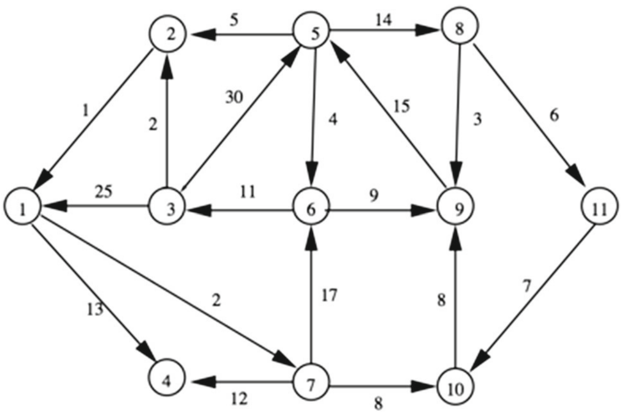

Fig. 4 The branch network as directed graph

variables and edges representing statistical dependencies between the variables or, equivalently, as spatial stochastic processes understood as those collections of random variables $\{X_v, v \in V\}$ which take a value X_v for each location v over the region of interest V. Thus, the task of presenting the bank branch network over the branch cash holdings as a Markov random field would produce a spatial model of the bank branch network in function of branch size.

In order to present the branch cash holdings as a spatial process, let *CH* stand for the cash holdings, while *BN* denotes the bank branch network, which could resemble the non-negative finite two-dimensional grid in the plane $V = \{1, 2, \ldots, n\} \times \{1, 2, \ldots, n\}$. So there are two stochastic processes associated with the random variable CH. Firstly, $\{CH^n, n \in \mathbb{N}\}$ represents the *temporal* stochastic process of cash holdings' movements through time, where n is the time unit.[3] And secondly, $\{CH_b, b \in BN\}$ denotes the *spatial* stochastic process (or random field on BN) where b represents a branch belonging to BN.

In the literature, see Zhou (2016) for example, the bank branch network is usually represented as a directed graph, weighted or unweighted, where the nodes are the branches, the directed edges are the branch ties with arrows pointing from head offices to branch offices and the edges' weights (when applicable) represent ownership as well as business relationships (see Fig. 4). In order to express the bank branch network as a Markov random field, we shall extend conventional knowledge by moving from graphs to graphical models. First, we shall replace branches with their corresponding cash holdings. To do this, we shall identify each branch b with the mean value of its corresponding cash holdings over some (fixed from now on) interval of time, CH_b. Second, the set of random variables $CH = \{CH_b, b \in BN\}$ is considered as an *undirected* graph where the nodes are the corresponding cash holdings' mean value over some interval of time, CH_b for each branch b, and the (undirected) edges indicate financial similarity between branches (to be specified later).

Note that there is a parallel between the temporal stochastic process $\{CH^n, n \in \mathbb{N}\}$ and the spatial one $\{CH_b, b \in BN\}$, such that the main difference between them is the dimension of the underlying domain. While the temporal process has been analysed by the author in previous publications, see García Cabello (2017) or García Cabello and Lobillo (2017), here we focus on the spatial process $\{CH_b, b \in BN\}$.

There is a notion which should be underlined whenever describing the background for graphs and graphical models: *the clique*. The importance of cliques in our context relies on the fact that there is a complete connection within cliques (remember that cliques in a graph are maximally connected subgraphs) which simulates the banking practice of forming highly connected networks through multi-location operations in order to diversify their business and hedge against risks, see, for example, Zhou (2016).

Therefore, the following steps should be taken in order to model branch networks as spatial processes. Firstly, the set of nodes (sites) must be defined for a banking scenario: we shall consider $V = \{1, 2, \ldots, n\} \times \{1, 2, \ldots, n\}$ as the Bank Branch Network *BN*. Next, the *neighbourhood of a branch (a site)* b, written $N(b)$, needs to be defined. As mentioned earlier, there may be major variations in the definitions of demographics according to "local" (as opposed to "internationally accepted") parameters, thus complicating international comparisons, see García Cabello and Lobillo (2017) for further details. Hence, the traditional standpoint based on common geographical distance/directions should be replaced by wider concepts.

For this purpose, let us recap on the notion of *feature vector*. In pattern recognition and machine learning, a feature vector is an n-dimensional vector of numerical features representing an object. We shall use $n = 2$, as shown by the following definition:

Definition 1 Each branch $b \in BN$ is represented by the feature vector (n_b, v_b), defined as follows:

n_b stands for the number of branch transactions at branch b,
v_b stands for the maximum volume of branch transactions allowed at b,

where these definitions of n_b, v_b are considered as the corresponding mean values over some interval of time since time is not the representative variable.[4]

[3] A week could be as an instance of time unit.

[4] That means that the current study is not focused on the temporal stochastic process of cash holdings' movements through time but on the *spatial* stochastic process, $\{CH_b, b \in BN\}$.

Journal: 500 MS: 3362 TYPESET DISK LE CP Disp.:2018/6/30 Pages: 13 Layout: Large

Remark 1 Branch cash holdings have been selected since they are major determinants of branch size.[5] Then, *if the size is the criterion used*, it is apparent that the previous two variables in Definition 1 (number of branch transactions and maximum volume of transactions allowed) have been chosen because they are key factors in both branch size and branch location. In fact, there is a close link between branch size and branch location: while branch size depends on branch cash transactions—number and amounts involved—branch cash transactions depend on customers' needs for cash, which are strongly related to demographics around branches.

However, many other branch features would have been taken into account either in addition to or instead of the ones considered: that is, former n_b and v_b could be replaced by other geographical determinants of branch location/size such as unemployment, population density, foreigner population percentage, per capita income.... Moreover, although feature vector is presented with $n = 2$, the number of vector coordinates can also be extended as required. In other words, both which variables and the number of variables can be freely selected.[6]

Now, distance between branches may be defined as follows:

Definition 2 The distance between two branches $b_i = (n_{b_i}, v_{b_i}), b_j = (n_{b_j}, v_{b_j}), i \neq j$, is the Euclidean distance between their corresponding feature vectors:

$$d(b_i, b_j) = +\sqrt{(n_{b_i} - n_{b_j})^2 + (v_{b_i} - v_{n_j})^2}.$$

We shall simply denote this by $b_{ij} = d(b_i, b_j)$.

Any notion of distance can be used to define the neighbourhood of a branch $b = (n_b, v_b)$, denoted by $N(b)$. Our particular choice of distance based on feature vectors produces a notion of branch neighbourhood which simulates the branch managers' practice of grouping branches according to their common features in terms of their cash holdings. That is,

Definition 3 The neighbourhood of a branch $b_i = (n_{b_i}, v_{b_i})$, $N(b_i)$, consists of all *nearby* branches b_j in the sense that their features with regard to their cash holdings are *very* similar to that of b_i's:

$$N(b_i) = \{b_j \in \text{BN such that } b_{ij} \leq k\},$$

where the degree of similarity (i.e. the benchmark k) should be specified by branch managers. If we accept that branch *size* mainly depends on branch cash transactions-number and amounts involved-then Definition 2 establishes that a branch neighbourhood is formed by those branches with the same or a very similar size.

Finally, the neighbourhood of a branch concept can be used to consider the notion of *cliques* (which are maximally connected subgraphs of the graph): a clique C is a set of branches such that any pair of elements $c_i, c_j \in C$ hold that $c_i \in N(c_j)$ and $c_j \in N(c_i)$.

Remark 2 The choice of Definition 2 makes cliques appear as different groups of branches, simulating different branch managers' practices. While merging branches according to similar sizes is one practice, there are others such as configuring the branch network through multi-location operations in order to diversify business and hedge against risks. Thus, it is noticeable that each choice of Definition 2 leads to a re-configuration of the branch network.

We shall now consider the branch cash holdings as a spatial process $\{\text{CH}_b, b \in \text{BN}\}$. The author proved in García Cabello (2017) that the branch cash holdings constitute a discrete-time Markov chain $\{\text{CH}^n\}_{n\in\mathbb{N}}$, where n denotes the time unit (a week in that case). Now, the main result of the current work is as follows:

Theorem 2 *With the above definition, branch network' cash holdings* $\{CH_b, b \in BN\}$ *are a Markov random field.*

Proof In order to evidence that the branch network BN is a Markov random field, then the Markov property must be shown to hold true, that is,

$$P\left[\text{CH}_b = c_b | \text{CH}_{\text{BN}-\{b\}} = c_{\text{BN}-\{b\}}\right]$$
$$= P\left[\text{CH}_b = c_b | \text{CH}_{N(B)} = c_{N(b)}\right].$$

Remember that the features considered in Definition 1 as determinant of the branch cash holdings are the number of branch transactions and the maximum volume of branch transactions allowed. Then, from Definition 3, the neighbourhood of a branch $b_i = (n_{b_i}, v_{b_i})$, $N(b_i)$, consists of all nearby branches b_j in the sense that their features concerning their cash holdings are very similar to those of b_i's. Then, the result follows. □

Remark 3 As mentioned before, the neighbourhood of a branch $b_i = (n_{b_i}, v_{b_i})$, $N(b_i)$, consists of all branches b_j such that their cash holdings features (their cash holdings' key determinants) are very similar to that of b_i's. In consequence, the neighbourhood is comprised of those branches that have similar mean values for their cash holdings over

[5] We must bear in mind that there are many criteria to quantify the size of a branch amongst bank managers: maximum cash holdings allowed, volume of deposits, volume of credits, number of business/private clients, number of staff etc.

[6] Size may be also replaced as the criterion used, as will be evidenced throughout the paper. This will increase the *versatility* of the proposed model so that it can be applied to a wider range of scenarios. Actually, the *generality* of the current approach is one of its best features, since it can be adjusted as required (see Sect. 6 for further details).

some interval of time (where the degree of similarity must be specified by managers). Equivalently, the neighbourhood of a branch is formed by those branches with the same/very similar sizes, provided that we assume that branch size mainly depends on branch cash transactions-number and amounts involved.

It should be noticed that the concepts of neighbourhood and clique depend on the definition of distance and the degree of similarity. In this particular case, both concepts (neighbourhood and clique) are essentially the same due to the symmetry of the Euclidean distance.

Once the main theorem has been proved, the application of the background work from the previous Sect. 3 leads us to state the following result:

Theorem 3 *The joint distribution of the branch network' cash holdings $CH = \{CH_b, b \in BN\}$ can be expressed as a product of clique potentials in BN, say ϕ_C. That is, if we denote $CH_C = \{CH(c)/c \in C\}$, where C is a clique in BN, we require functions ϕ_C such that the joint distribution of CH, $P[CH]$, takes the form*

$$P[CH] = \frac{1}{Z}\prod_{c\in C}\phi_c(CH_C). \tag{3}$$

Usually, each $\phi_c(CH_C)$ takes the form $\phi_c(CH_C) = e^{\frac{-1}{T}V_c(CH_C)}$ where T is called the temperature and often it is equal to 1 whereas $V_c(CH_C)$ are usually referred as clique potentials. Thus, the joint distribution of CH, $P[CH]$, has the alternate form

$$\begin{aligned} P[CH] &= \frac{1}{Z}\prod_{c\in C}\phi_c(CH_C) \\ &= \frac{1}{Z}\prod_{c\in C} e^{\frac{-1}{T}V_c(CH_C)} \\ &= \frac{1}{Z} e^{\frac{-1}{T}\sum_{c\in C} V_c(CH_C)}. \end{aligned}$$

Hence,

$$\boxed{P[CH] = \frac{1}{Z} e^{\frac{-1}{T}\sum_{c\in C} V_c(CH_C)}}, \tag{4}$$

or $P[CH] = \frac{1}{Z} e^{\frac{-1}{T}U(CH)}$, where $U(CH) = \sum_{c\in C} V_c(CH_C)$ is called the energy.

Proof We can conclude from Theorem 2 that the branch network' cash holdings $\{\mathrm{CH}_b, b \in \mathrm{BN}\}$ are a Markov random field. Since the Hammersley–Clifford theorem establishes the equivalence between Gibbs distributions and Markov random fields, all properties of Gibbs random fields hold true for $\{\mathrm{CH}_b, b \in \mathrm{BN}\}$.

Particularly, a set of random variables is said to be a Gibbs random field if and only if its configurations obey a Gibbs distribution. Hence, the results follows. □

5 A universal geolocator of branches depending on the size

The application of the previous findings provides a decision-making tool that can help identify the best location(s) for branches according to the criterion of branch size (see Fig. 5). This is a decision model that will be used to redesign the bank branch network when required. This section of the paper describes this decision model.

Before proceeding, it should be noted that the joint distribution of the branch network' cash holdings $\mathrm{CH} = \{\mathrm{CH}_b, b \in \mathrm{BN}\}$ will be at the heart of the decision model. In fact, it will act as a numerical score that can be assigned to each scenario of a new branch joining the existing branches, provided that the branch network has previously been divided into subnets. Once the joint distribution has been used to assign a numerical score to each possible juncture, then comparisons can be made between different possible alternatives, thus helping identify the best location(s) for the new branch. Specifically:

The objective is to find the best location(s) for a new branch $b^* \in \mathrm{BN}$ with specific cash holdings corresponding to some specific entity' needs.[7] The following steps could be considered in order to identify the best site for a new branch in function of its cash holdings:

Step 1 *The branch network* BN *is divided into subnets* $S^i\mathrm{BN}$ in such a way that this partition offers different scenarios for locating a new branch b^*:

$$\mathrm{BN} = \bigcup_{i=1} S^i\mathrm{BN}.$$

The criterion used to divide the network should be specified by bank managers. For instance, a branch network consisting of n branches is grouped into two subnets, $S^1\mathrm{BN}$ and $S^2\mathrm{BN}$, one containing the branches with low/medium cash holdings and the other comprised of branches with medium/high cash holdings. The branch network can be subdivided

[7] For example, the entity may need to reinforce their current set of branches with a given volume of cash holdings.

Journal: 500 MS: 3362 TYPESET DISK LE CP Disp.:2018/6/30 Pages: 13 Layout: Large

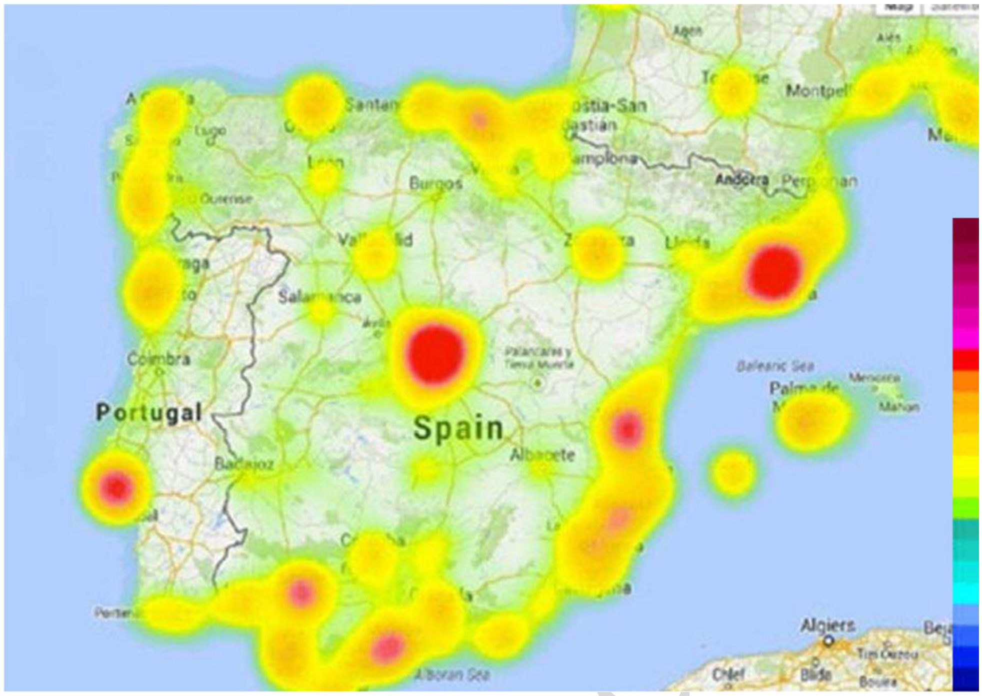

Fig. 5 Spatial areas of highest interest in Spain

according to other criteria depending on the entity's needs.

Step 2 *Each subnet considers the corresponding cash holdings' spatial stochastic process*, provided that the new branch b^* subsequently belongs to each subnet:

$$\mathrm{CH}^i = \{\mathrm{CH}_b, b \in S^i\mathrm{BN} \cup b^*\}.$$

Step 3 *The joint distribution of the subnetwork' cash holdings,* $\{\mathbf{P}_{\mathrm{CH}^i}\}_i$, *is computed*:

$$\left\{P_{\mathrm{CH}^i}\right\}_i \quad \text{where } P_{\mathrm{CH}_{b_1},\ldots,\mathrm{CH}_{b_n}}(c_1,\ldots,c_n) = P[\mathrm{CH}_{b_1} = c_1,\ldots,\mathrm{CH}_{b_n} = c_n).$$

These numerical scores $\{P_{\mathrm{CH}^i}\}_i$ are compared and used to make decisions about the most suitable locations according to the banking entity' needs. The comparison amongst numerical scores $\{P_{\mathrm{CH}^i}\}_i$ makes visible the bank branches with highest cash holdings as well as those with lowest ones. Such comparison may be best revealed through a geostatistical mapping in which the spatial areas of highest interest are explicitly shown on solid colours making possible to identify broader areas where there is a high probability of having high volumes of branch cash holdings, see Fig. 5:

The general steps presented above can be complemented with further fine-tunings:

Step 4 *The computation of the joint probability distributions can only be carried out through the clique potentials* (see Theorem 3), where the cliques of a branch are all those branches with similar mean values for their cash holdings over some interval of time. Remember that in our case, the concepts of neighbourhood and clique are essentially the same given the symmetry of the Euclidean distance (see Remark 3).

Step 5 *From all the outputs, select the most convenient one according to pre-established criteria*. Such criteria may take many forms including minimising costs (total set-up costs, fixed cost, total annual operating cost, etc.), minimising the distances between the existing facilities (average time/distance travelled, maximum time/distance travelled, etc.) and maximising service, amongst others.

It should be noticed that, when numerically valued examples are attempted, a huge quantity of output data has to be managed. In such cases (when a huge quantity of factors is managed), implementing the procedure into an algorithm should provide an easy-to-handle system which are useful for conducting the selection procedure. Such computational version[8] could still be carried out throughout the banking institutions' own computer services at a minimal cost, thus providing a low-cost decision-making tool for financial entities.

[8] This is a future research project within a foreseeable period of time.

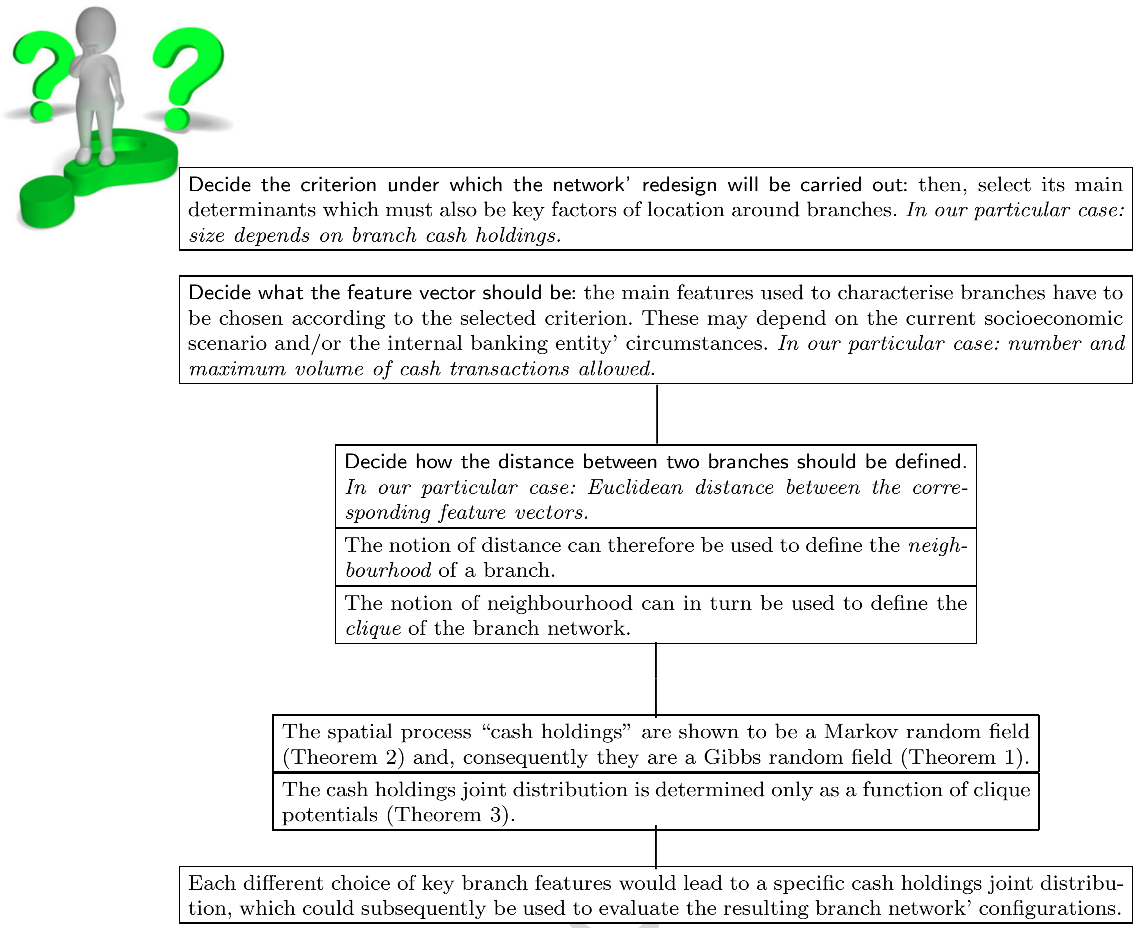

Fig. 6 A guiding thread for this paper' findings

6 The versatility of the geolocator

As mentioned, the branch geolocator can be translated into a computational procedure to provide a low-cost decision-making tool for financial entities and companies. However, while the tool's low cost is very attractive, its generality is actually one of its better features. In fact, it can be adjusted as required and custom-made to suit the specific requirements of each banking institution (or each kind of branch). This section aims to demonstrate the high versatility of the proposed approach by highlighting that many of its variables can be freely selected and expanded as required.

On the one hand, *let us suppose that the criterion used to identify the best location(s) for branches is their size*. As pointed out in Remark 1, bank managers use many criteria to quantify branch size: maximum cash holdings allowed, volume of deposits/credits, number of business/private clients or staff and the brick-and-mortar branch dimensions amongst others. Here we selected the branch cash holdings because it is a major determinant of branch size whereas the two variables considered in Definition 1 to configure the feature vector (i.e. number of branch transactions and their maximum volume allowed) were chosen because they are key factors in branch size and branch location. However, many other branch features would have been taken into account with regard to the selections made: actually, both the variables and their number are freely selectable.

On the other hand, *another criterion can be used instead of size*. Then, in order to generalise the proposed framework, we shall first select the criterion according to which the network' redesign will be carried out. We shall then choose the main determinants for the criterion selected. Recall that demographics around branches are a primary concern when undertaking a branch network redesign. This is why these determinants must also be key factors of location around branches. The next step consists of configuring the feature vector by selecting the main features that characterise branches according to the main determinants of the selected criterion. These may depend on the current socioeconomic scenario and/or the internal banking entity' circumstances and can be freely selected (both the type of variables and their quantity).

There is one more choice to be made in this model: to decide how to define the distance between two branches. In

Journal: **500** MS: **3362** TYPESET DISK LE CP Disp.:**2018/6/30** Pages: **13** Layout: **Large**

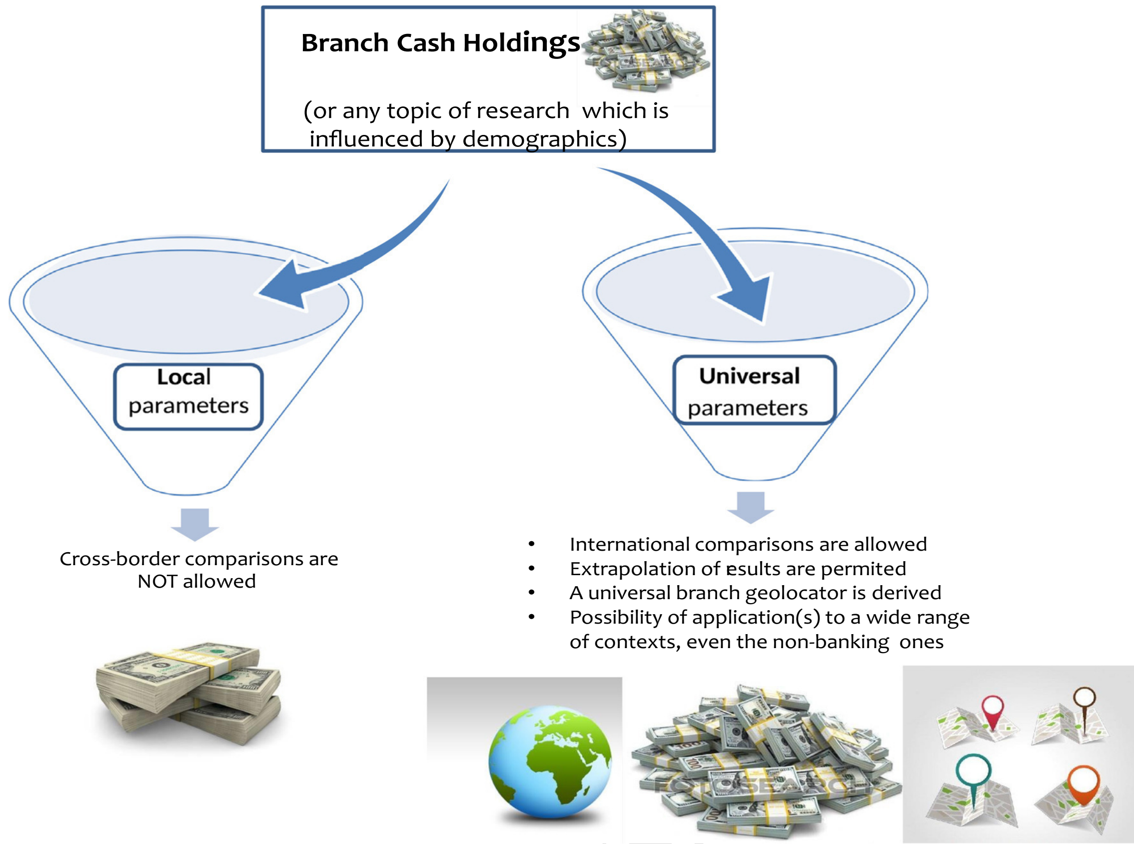

Fig. 7 Contributions of the paper

our particular case, we have selected the Euclidean distance between the corresponding feature vectors, but there are other options that may be more relevant to the entity' needs in each individual case.

Once the notion of distance has been established, the branch *neighbourhood* and the branch *clique* can be defined. In our case, the neighbourhood $N(b_i)$ of a branch b_i is not defined in terms of geographical coordinates but rather in terms of the distinctive features of each kind of branch,[9] reflecting bank managers' practices of categorising and grouping the branches according to their features. The notion of clique will also have a translation in terms of banking practices. As a matter of fact, the choice of distance between branches makes cliques appear as different groups of branches, simulating different branch managers' practices, of which merging branches according to similar sizes is just one.[10] Anyhow, it is remarkable that each choice of distance between branches (which determines the concepts of both neighbourhood and clique) leads to a re-configuration of the branch network. 3

Importantly, it should be noticed that, for the same branch network, the choice of feature vector (i.e. each choice of the key branch features) would also lead to a specific cash holdings joint distribution. This will allow banking entities to better evaluate the results of reconfiguring their branch networks under the different possibilities available. Let us briefly bring together these reflections as well as the previous findings in the following guiding thread scheme (Fig. 6).

As evidenced, the proposed model is highly versatile and could be applied to a wide range of scenarios. Actually, the *generality* of the proposed approach is one of its best features, as it can be adjusted as required.

7 Conclusions

This paper has designed and presented a theoretical model for analysing branch cash holdings from a *global* spatial point of view. This is a potential solution to the fuzziness that exists when defining demographics according to "local"—as opposed to "internationally accepted"—parameters. Moreover, a universal decision model (branch geolocator) aimed

[9] Recall that the neighbourhood of a branch is formed by those branches with the same/a very similar size, where the degree of similarity is defined by managers.

[10] There are other ways of grouping branches such as configuring the branch network through multi-location operations in order to diversify business and hedge against risks.

Journal: 500 MS: 3362 TYPESET DISK LE CP Disp.:2018/6/30 Pages: 13 Layout: Large

at redesigning the bank branch network based on the criterion of branch size is derived from the previous spatial model. As mentioned throughout the paper, the variables considered for the model—including size as the main criterion used—can be replaced/expanded as needed resulting in a *highly versatile* decision-making tool that can be applied to a wide range of contexts. As an instance of adding as many variables as desired,[11] *an extra-variable representing branch' geographical coordinates* could be added. This possibility would take into consideration the fact that *distance matters in banking* and that companies' geographical locations are still shaping corporate behaviour: so spatial proximity remains a factor in branch network formation despite ongoing advances in communication technology (Fig. 7).

Both contributions rely on Markov random fields in order to obtain an explicit joint probability function, while most approaches in literature draw on Bayesian random fields. However, the main disadvantage of Bayesian methods is that they are data intensive, requiring sufficient input in order to derive the probabilistic relationships used in their predictions. This can make their application in data-poor environments challenging. Other approaches are based on neural networks and/or statistical tools, which strongly rely on the assumption of the underlying data distribution.

These disadvantages are avoided by the proposed approach based on Markov random fields, where the likelihood of the entire network (the joint distribution of the collection of variables located at nodes) depends only on cliques, thereby reducing the required amount of data. They must therefore no longer be subject to any default distribution. This is highly favourable especially in those scenarios where data are ever changing, and requires frequent update (financial contexts for instance).

The theoretical structure designed in this paper can be translated into computational terms by means of algorithms because, besides being a language for formulating models, graphical models inherit the excellent computational properties of graphs. This could be a solution when numerically valued examples are attempted, since a huge quantity of output data has to be managed. In such cases (when a huge quantity of determinants is managed), implementing the procedure into an algorithm should provide an easy-to-handle system which are useful for conducting the selection procedure. Such computational version of the proposed method is a future research project within a foreseeable period of time although it could still be carried out throughout the banking institutions' own computer services providing a low-cost decision-making tool that can be adjusted as required and custom-made to suit the specific requirements of each banking institution or each kind of branch.

[11] Standing for all required branch features.

Additionally, this new methodology can also be applied in other contexts besides the banking industry. In fact, the generality of the proposed method would also allow it to be applied—with minor changes according to the specific needs in any given context—to supermarkets, petrol stations or other businesses with networks. Taking into account the versatility of the proposed methodology, such a global approach is beneficial in several ways, for example, decreasing costs by replacing several local approaches with a universal one. Going beyond that, the insight of replacing classical geographical concepts with other "internationally accepted" notions presented in this paper would also be fruitful in any context where demographics play a role.

Once the proposed methodology has been decoupled from geographical premises, it may also apply to non-physical networks such as social ones. Importantly, the case of groups decision-making when viewed as non-physical networks where nodes may be identified with opinions. In this type of participatory processes, where multiple individuals act collectively and/or analyse problems or situations, the proposed methodology may be useful as long as it could evaluate the impact of consider the entry of a new node (viewed as an alternative course of action). For these scenarios, linguistic fuzzy variables are required in order to detail the different meanings of each person when he/she elicits linguistic information (see Cabrerizo et al. 2017; Li et al. 2017).

These ideas will form part of a further research project to be conducted in the near future.

Acknowledgements Financial support from the Spanish Ministry of Science and Innovation "Regulación Financiera y Sector Bancario en Tiempos de Inestabilidad: Mecanismos de Prevención y Resolución de la Crisis" (ECO2014-59584-P), Junta de Andalucía "Excellence Groups" (P12.SEJ.2463) and Junta de Andalucía (SEJ340) is gratefully acknowledged.

Compliance with ethical standards

Conflict of interest The author declares that she has no conflict of interest.

Human and animal rights This article does not contain any studies with human participants or animals performed by the author.

References

Abbasi GY (2003) A decision support system for bank location selection. Int J Comput Appl Technol 16:202–210. https://doi.org/10.1504/IJCAT.2003.000326

Allahi S, Mobin M, Vafadarnikjoo A, Salmon C (2015) An integrated AHP-GIS-MCLP method to locate bank branches. In: Proceedings of the 2015 industrial and systems engineering research conference. ISBN: 978-098376244-7

André M, Mahy G, Lejeune P, Bogaert J (2014) Toward a synthesis of the concept and a definition of the zones in the urban-rural gradient. Biotechnol Agron Soc 18(1):61–74

Journal: 500 MS: 3362 TYPESET DISK LE CP Disp.:2018/6/30 Pages: 13 Layout: Large

Boufounou PV (1995) Theory and methodology, evaluating bank branch location and performance: a case study. Eur J Oper Res 87:389–402. https://doi.org/10.1016/0377-2217(92)00080-5

Cabrerizo FJ, Al-Hmouz R, Morfeq A, Balamash AS, Martínez MA, Herrera-Viedma E (2017) Soft consensus measures in group decision making using unbalanced fuzzy linguistic information. Soft Comput 21(11):3037–3050. https://doi.org/10.1007/s00500-015-1989-6

Cerutti E, Dell' Ariccia G, Martínez Pería MS (2007) How banks go abroad: branches or subsidiaries? J Bank Financ 31:1669–1692. https://doi.org/10.1596/1813-9450-3753

Cinar N (2009) A decision support model for bank branch location selection. World Acad Sci Eng Technol 60:126–131

García Cabello J (2017) The future of branch cash holdings management is here: new Markov chains. Eur J Oper Res 259(2):789–799. https://doi.org/10.1016/j.ejor.2016.11.012

García Cabello J, Lobillo J (2017) Sound branch cash management for less: a low-cost forecasting algorithm under uncertain demand. Omega Int J Manag Sci 70(C):118–134. https://doi.org/10.1016/j.omega.2016.09.005

Ioannou G, Mavri M (2007) Performance-net: a decision support system for reconfiguring a bank's branch network. Omega Int J Manag Sci 35:190–201. https://doi.org/10.1016/j.omega.2005.05.007

Glykas M, Xirogiannis AG (2005) A soft knowledge modeling approach for geographically dispersed financial organizations. Soft Comput 9:579–593. https://doi.org/10.1007/s00500-004-0401-8

Li C-C, Dong Y, Herrera F, Herrera-Viedma E, Martínez L (2017) Personalized individual semantics in computing with words for supporting linguistic group decision making. An application on consensus reaching. Inf Fusion 33(1):29–40. https://doi.org/10.1016/j.inffus.2016.04.005

Miliotis P, Dimopoulou M, Giannikos I (2002) A hierarchical location model for locating bank branches in a competitive environment. Int T Oper Res 9(5):549–65. https://doi.org/10.1111/1475-3995.00373

Ruiz-Hernandez D, Delgado-Gomez D, Lopez-Pascual J (2015) Restructuring bank networks after mergers and acquisitions: a capacitated delocation model for closing and resizing branches. Comput Oper Res 62:316–324. https://doi.org/10.1016/j.cor.2014.04.011

Wainwright MJ, Jordan MI (2008) Graphical models, exponential families, and variational inference. Found Trends® Mach Learn 1(1–2):1–305 ISBN: 978-1-60198-184-4

Zainab L, Zahra NA, Mostafa K (2014) Locating the bank branches using a hybrid method. Tech J Eng Appl Sci 4(3):124–134. https://doi.org/10.5121/ijfcst.2015.560215

Zhou B (2016) Applying the Clique Percolation Method to analyzing crossmarket branch banking network structure: the case of Illinois. Soc Netw Anal Min 6(11):1–14. https://doi.org/10.1007/s13278-016-0318-0

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Journal: 500
Article: 3362

Author Query Form

Please ensure you fill out your response to the queries raised below and return this form along with your corrections

Dear Author

During the process of typesetting your article, the following queries have arisen. Please check your typeset proof carefully against the queries listed below and mark the necessary changes either directly on the proof/online grid or in the 'Author's response' area provided below

Query	Details required	Author's response
1.	Please check and confirm the edit made in article title.	
2.	Please confirm if the author names are presented accurately and in the correct sequence (given name, middle name/initial, family name). Author 1 Given name: [J.] Last name: [García Cabello]. Also, kindly confirm the details in the metadata are correct.	
3.	Figures 6 and 7 were received; however, no citations were provided in the manuscript. Please check and confirm the inserted citation of Figure is correct. If not, please suggest an alternative citation. Please note that figures should be cited in ascending numerical order in the main body of the text.	
4.	Kindly provide valid DOI for Ref. Cinar (2009).	