

# Cosmology of an Axion-Like Majoron

A.J. Cuesta,<sup>(1)</sup> M.E. Gómez,<sup>(2)</sup> J.I. Illana,<sup>(3)</sup> M. Masip<sup>(3)</sup>

<sup>(1)</sup>*Departamento de Física  
Universidad de Córdoba, E-14071 Córdoba, Spain*

<sup>(2)</sup>*Departamento de Ciencias Integradas  
Universidad de Huelva, E-21071 Huelva, Spain*

<sup>(3)</sup>*CAFPE and Departamento de Física Teórica y del Cosmos  
Universidad de Granada, E-18071 Granada, Spain*

ajcuesta@uco.es, mario.gomez@dfa.uhu.es, jillana,masip@ugr.es

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## Abstract

We propose a singlet majoron model that defines an inverse seesaw mechanism in the  $\nu$  sector. The majoron  $\phi$  has a mass  $m_\phi \approx 0.5$  eV and a coupling to the  $\tau$  lepton similar to the one to neutrinos. In the early universe it is initially in thermal equilibrium, then it decouples at  $T \approx 500$  GeV and contributes with just  $\Delta N_{\text{eff}} = 0.026$  during BBN. At  $T = 26$  keV (final stages of BBN) a primordial magnetic field induces resonant  $\gamma \leftrightarrow \phi$  oscillations that transfer 6% of the photon energy into majorons, implying  $\Delta N_{\text{eff}} = 0.55$  and a 4.7% increase in the baryon to photon ratio. At  $T \approx m_\phi$  the majoron enters in thermal contact with the heaviest neutrino and it finally decays into  $\nu\bar{\nu}$  pairs near recombination, setting  $\Delta N_{\text{eff}} = 0.85$ . The boost in the expansion rate at later times may relax the Hubble tension (we obtain  $H_0 = (71.4 \pm 0.5)$  km/s/Mpc), while the processes  $\nu\bar{\nu} \leftrightarrow \phi$  suppress the free streaming of these particles and make the model consistent with large scale structure observations. Its lifetime and the fact that it decays into neutrinos instead of photons lets this axion-like majoron avoid the strong bounds that affect other axion-like particles of similar mass and coupling to photons.

# 1 Introduction

Neutrinos are a crucial ingredient of the  $\Lambda$ CDM cosmological model. Their mass, their interactions, or the presence of additional (sterile) modes may have implications on primordial nucleosynthesis (BBN), the cosmic microwave background (CMB) or the formation of large scale structures (LSSs) [1]. In astrophysics they play an equally important role, for example, in the dynamics of stars like the Sun or in proto-neutron stars, where the neutrino sector is probed up to energies around 100 MeV [2]. On the other hand, some fundamental questions about their nature remain unanswered. Are they Dirac particles that get massive at the electroweak (EW) scale, just like the rest of standard fermions, or is their mass revealing a new scale in particle physics? It is apparent that the search for answers can benefit from a large variety of different observations.

From a model-building point of view, the so called seesaw mechanism [3, 4] provides a very simple and minimal completion of the sector. It explains that neutrinos are light because of a large hierarchy between the EW scale ( $v$ ) and the scale ( $M_R$ ) where their right-handed partners, unprotected by chirality, get their mass. The parameters (neutrino masses and mixings) that define this setup are able to explain well the data [5], despite some persistent anomalies in baseline [6, 7] and flavor [8] experiments or possible difficulties in the modelling of supernova explosions [9, 10]. One should keep in mind, however, that the neutrino sector is basically *dark* and that the data leaves still plenty of room for departures from minimality. In this context, cosmology is becoming a powerful guide, signaling the new physics that could be favored: CMB [11] or BBN [12] observables have been measured at precision levels, whereas the volume of data on LSS [13, 14] will increase substantially in upcoming experiments like EUCLID [15] or J-PAS [16]. Indeed, the Lithium problem (BBN implies 2–3 times more primordial Li than observed [17]) or the  $H_0$  tension (type-Ia supernovae calibrated on Cepheids [18] and strong lensing [19] suggest a 4–6  $\sigma$  larger expansion rate than predicted by CMB and BAO data) could be related to the physics of neutrinos.

Here we will explore a different completion of the neutrino sector that does not imply minimality [20, 21]. We propose a singlet majoron model where two different scales appear in a single phase transition: a scale  $v_X \approx 1$  TeV that breaks a global  $U(1)_X$  symmetry, plus a much smaller scale  $\epsilon v_X$  that breaks a  $Z_3 \subset U(1)_X$  subgroup of discrete symmetries. The value of  $m_\nu$  is then explained by the spontaneous breaking of the discrete symmetry,  $m_\nu \propto \epsilon v_X$ , like in inverse seesaw models [22–26], whereas all

$U(1)_X$	$(\nu e)$	$e^c$	$N$	$N^c$	$n$	$(h^+ h^0)$	$s_1, s_2, \dots$
$Q_X$	+1	-1	-2	-1	0	0	1, 2, ...

Table 1: Charges under the global symmetry.

the extra particles except for the majoron  $\phi$  get masses of order  $v_X$ .

Our scenario is a variation of the one proposed in [27,28] (see also [29]); in particular, it implies the same type of majoron coupling to neutrinos,  $\lambda_\nu \approx m_\nu/v_X$ . However, our proposal can also accommodate tiny couplings to the charged leptons that induce the dimension-5 operator  $\phi \tilde{F}_{\mu\nu} F^{\mu\nu}$ : this is an axion-like majoron (ALM).

## 2 The model

Consider an extension of the SM that includes three fermion singlets\*  $\{N, N^c, n\}$  plus several complex scalar singlets  $\{s_1, s_2, \dots\}$ . Let us assume that the model is valid up to a cutoff scale  $\Lambda \approx 10$  TeV and that, up to gravitational effects [30], it is invariant under the global  $U(1)_X$  symmetry

$$\psi \rightarrow e^{-iQ_X \theta} \psi, \quad (2.1)$$

with the charges  $Q_X$  given in Table 1. Then a linear combination  $s$  of the singlets  $s_i$  gets a VEV  $v_X$  and breaks the global symmetry:

$$s = \frac{1}{\sqrt{2}} (v_X + r) e^{i\phi/v_X}, \quad (2.2)$$

with  $\phi$  the Goldstone boson. We will suppose that the scalar potential may define  $s$  as *any* combination of singlet flavors, including combinations with a tiny component along a given flavor. For example, for two scalars  $s_1$  and  $s_2$  the potential

$$V = -m_1^2 s_1^\dagger s_1 + \alpha \left( s_1^\dagger s_1 \right)^2 + |\beta s_1^2 - m_2 s_2|^2 \quad (2.3)$$

is invariant under  $U(1)_X$ . If we take  $\beta \ll \alpha$  and  $m_1 \approx m_2$  this potential implies a VEV for the flavor combination  $s \approx s_1 + \epsilon s_2$ , with  $\epsilon = \beta m_1 / (\sqrt{2\alpha} m_2) \ll 1$ . This means VEVs  $\langle s_1 \rangle = O(m)$  and  $\langle s_2 \rangle = O(\epsilon m)$ , a majoron  $\phi$  with components along both flavors,

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\*We use a 2-component spinor notation with all the fields of left-handed chirality; at the end of the section we will translate it to the usual 4-spinor notation.

$\phi \approx \phi_1 + \epsilon\phi_2$ , and masses of  $O(m)$  for the rest of degrees of freedom in  $s_1$  and  $s_2$ . More complicated effective potentials could break the global symmetry along any combination of singlet flavors and leave the majoron as the only scalar with a mass much smaller than  $v_X$  ( $m_\phi$  will not be zero if the global symmetry is not exact).

Let us then take the main component in  $s$  along  $s_3$ , with charge  $Q_X = 3$ . The terms relevant for neutrino masses and majoron couplings allowed by the symmetry are

$$-\mathcal{L} \supset y_i h^0 \nu_i N^c + y' s_3 N N^c + \frac{\Lambda_n}{2} n n + \text{h.c.}, \quad (2.4)$$

whereas the Yukawas  $h^0 \nu N$ ,  $h^0 \nu n$ ,  $s_3 N N$ ,  $s_3 N^c N^c$ ,  $s_3 n N$ ,  $s_3 n N^c$  and  $s_3 n n$  are all forbidden. The VEV  $\langle s_3 \rangle$  will obviously break  $U(1)_X$ , but we notice that a global transformation of parameter  $\theta = -2\pi/3$  leaves the vacuum invariant:

$$\langle s_3 \rangle \rightarrow e^{-3i(-2\pi/3)} \langle s_3 \rangle = \langle s_3 \rangle. \quad (2.5)$$

The spinors in Table 1 change non-trivially under this transformation:

$$\nu \rightarrow \alpha \nu \quad N \rightarrow \alpha N \quad N^c \rightarrow \alpha^* N^c \quad n \rightarrow +1 n, \quad (2.6)$$

with  $\alpha = e^{i2\pi/3}$ . Therefore, the effective Lagrangian must respect to all order this unbroken  $Z_3$  symmetry. In particular, Higgs ( $v/\sqrt{2}$ ) and singlet ( $v_X/\sqrt{2}$ ) VEVs will not generate bilinears of type  $\nu N$ ,  $\nu n$ ,  $NN$ ,  $N^c N^c$ ,  $nN$  or  $nN^c$  through operators of any dimension (again, up to gravitational effects), as these terms would break the discrete symmetry. The neutrino mass matrix reads then

$$-\mathcal{L} \supset \frac{1}{2} \begin{pmatrix} \nu_1 & \nu_2 & \nu_3 & N & N^c & n \end{pmatrix} \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & m_1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & m_2 & \cdot \\ \cdot & \cdot & \cdot & \cdot & m_3 & \cdot \\ \cdot & \cdot & \cdot & \cdot & M & \cdot \\ m_1 & m_2 & m_3 & M & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \Lambda_n \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ N \\ N^c \\ n \end{pmatrix} + \text{h.c.}, \quad (2.7)$$

where  $m_i \equiv y_i v/\sqrt{2}$  and  $M \equiv y' v_X/\sqrt{2}$ . This is a rank 3 matrix that implies 3 massless neutrinos, two neutrinos defining a Dirac field of mass  $M' = \sqrt{M^2 + m_1^2 + m_2^2 + m_3^2}$ , heavy-light mixings  $\theta_{N\nu} \approx m_i/M$ , and a Majorana neutrino of mass  $\Lambda_n$ . The discrete symmetry protects the light neutrinos from a mass  $m_\nu \propto v^2/v_X$ , which is the ratio of scales setting  $m_\nu$  in seesaw scenarios. In addition, since under the  $Z_3$  symmetry  $(\nu_i, N) \rightarrow \alpha(\nu_i, N)$  and  $\phi_3 \rightarrow \phi_3$ , a coupling  $i\lambda_\nu \phi \nu \nu$  is also forbidden.

To generate  $\nu$  masses we need that the singlet VEV includes a small component breaking that symmetry. A minimal possibility is

$$s = s_3 + \epsilon s_4 \implies \langle s_4 \rangle = \epsilon \langle s_3 \rangle; \quad \phi = \phi_3 + \epsilon \phi_4, \quad (2.8)$$

with  $\epsilon \ll 1$ . The VEV  $\langle s_4 \rangle$  may then induce entries everywhere in the matrix  $\mathcal{M}$  in (2.7). In particular, we obtain masses for the three light neutrinos using one dimension 4 and two dimension 6 operators:

$$-\mathcal{L} \supset y_{IS} s_4 N N + \tilde{y}_i \frac{s_3^\dagger s_4}{\Lambda^2} \begin{pmatrix} h^+ \\ h^0 \end{pmatrix} \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} N + \tilde{y}'_i \frac{s_3 s_4^\dagger}{\Lambda^2} \begin{pmatrix} h^+ \\ h^0 \end{pmatrix} \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} n. \quad (2.9)$$

The first term implies an entry  $\mu = y_{IS} \sqrt{2} \epsilon v_X$  in  $\mathcal{M}_{44}$  that gives mass to one of the neutrinos ( $\nu'_3$ ; we use a prime to indicate mass eigenstates) through an inverse seesaw mechanism,

$$m_{\nu'_3} \approx \mu \left( \frac{m}{M} \right)^2 \quad (2.10)$$

with  $m = \sqrt{m_1^2 + m_2^2 + m_3^2}$ . In addition,  $\nu'_3$  has now components along  $N$  and also  $N^c$ ,

$$\theta_{N\nu} \approx \frac{m}{M}; \quad \theta_{N^c\nu} \approx \frac{\mu m}{M^2}, \quad (2.11)$$

that induce couplings of the majoron to  $\nu'_3$  via two different operators:

$$y_{IS} s_4 N N \rightarrow i \frac{m_{\nu'_3}}{2v_X} \phi \nu'_3 \nu'_3, \quad y' s_3 N N^c \rightarrow i \frac{m_{\nu'_3}}{v_X} \phi \nu'_3 \nu'_3. \quad (2.12)$$

Including the mass contributions from all the operators in (2.9) we obtain

$$-\mathcal{L} \supset \frac{1}{2} \begin{pmatrix} \nu_1 & \nu_2 & \nu_3 & N & N^c & n \end{pmatrix} \begin{pmatrix} \cdot & \cdot & \cdot & 0 & 0 & \tilde{\mu}' \\ \cdot & \cdot & \cdot & \tilde{\mu} & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot & m & \cdot \\ 0 & \tilde{\mu} & \cdot & \mu & M & \cdot \\ 0 & 0 & m & M & \cdot & \cdot \\ \tilde{\mu}' & \cdot & \cdot & \cdot & \cdot & \Lambda_n \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ N \\ N^c \\ n \end{pmatrix}, \quad (2.13)$$

where we have redefined the three active neutrinos  $\nu_i$  so that  $m_{1,2} = 0$  and  $\tilde{\mu}_1 = 0$ . The second light neutrino ( $\nu'_2$ ) gets a mass  $m_{\nu'_2} \approx \tilde{\mu}^2/\mu$ , while the mass  $m_{\nu'_1} \approx \tilde{\mu}^2/\Lambda_n$  of the lightest one is generated through a type I seesaw mechanism. It is easy to deduce that the couplings of  $\nu'_{1,2}$  to the majoron are  $\lambda_{\nu'_1} = m_{\nu'_1}/(2v_X)$  and  $\lambda_{\nu'_2} = m_{\nu'_2}/v_X$ . The rest of entries (dots in the matrix above) are also suppressed by  $\epsilon$  and inverse powers of the cutoff; they are not necessary to define the model, although they may introduce  $O(1)$  corrections to the  $\nu$  masses and mixings.

Another aspect of our setup concerns the possible coupling of the majoron to charged leptons. Let us consider  $\ell = \mu, \tau$  (the coupling to electrons is severely constrained [31]); we assign  $Q_X(\ell) = +1$  and  $Q_X(\ell^c) = -1$  so that the usual Yukawa term  $h^0 \ell \ell^c$  is allowed by the symmetry. In turn, this implies that higher dimensional operators of type  $s_3^\dagger s_3 h^0 \ell \ell^c$  or  $s_4^\dagger s_4 h^0 \ell \ell^c$  will not introduce couplings to the majoron, just to the (massive) radial component in  $s$ . Unlike neutrinos, chiral charged leptons do not couple to the majoron. We can, however, add a pair of heavy (vectorlike) lepton singlets  $(E_1, E_{-1}^c)$  and  $(E_4, E_{-4}^c)$  with charges  $Q_X = \pm 1$  and  $Q_X = \pm 4$ , respectively, and masses  $m_{E_{1,4}} > v_X$ . These leptons may couple to the majoron through Yukawas like  $y_E s_3 E_1 E_{-4}^c$ . In that case  $\ell$  could also couple to the majoron via mixing with the heavy sector, *e.g.*,  $s_3 h^0 \ell E_{-4}^c$  induces a bilinear  $m' \ell E_{-4}^c$  and a term

$$- \mathcal{L} \supset i \lambda_\ell \phi \ell' \ell^{c'} + \text{h.c.} \quad (2.14)$$

of order  $\lambda_\ell \approx m'/v_X (m_\ell/m_E)^2$ .

It may be useful to translate this 2-spinor notation to the usual one with 4-spinors in the chiral representation. All the neutrinos discussed here can be arranged as the left-handed component of a self-conjugate (Majorana) field,

$$\nu_{i=1,2,3} \equiv \begin{pmatrix} \nu_i \\ \bar{\nu}_i \end{pmatrix}; \quad \nu_4 \equiv \begin{pmatrix} N \\ \bar{N} \end{pmatrix}; \quad \nu_5 \equiv \begin{pmatrix} N^c \\ \bar{N}^c \end{pmatrix}; \quad \nu_6 \equiv \begin{pmatrix} n \\ \bar{n} \end{pmatrix}, \quad (2.15)$$

where bi-spinor indexes ( $\psi_\alpha$  or  $\bar{\psi}^{\dot{\alpha}}$ ) are omitted. For the charged leptons we have

$$\ell \equiv \begin{pmatrix} \ell \\ \bar{\ell}^c \end{pmatrix}. \quad (2.16)$$

Dropping the prime, the majoron couplings in (2.12) and (2.14) would read

$$\mathcal{L} \supset i \lambda_{\nu_i} \phi \bar{\nu}_i \gamma_5 \nu_i + i \lambda_\ell \phi \bar{\ell} \gamma_5 \ell. \quad (2.17)$$

Actually, in our model the neutrino basis with diagonal majoron couplings used above does not need to coincide with the basis of mass eigenstates, as some of the operators contributing to the mass have no axial dependence. For simplicity, however, we will take diagonal couplings and  $\lambda_{\nu_i} \approx m_{\nu_i}/v_X$ .

A final observation concerns the axion-like character of the majoron in this model. The coupling  $\lambda_\ell$  with a charged lepton discussed above induces at one loop (see Fig. 1a) the operator

$$\mathcal{L} \supset -\frac{1}{4} g_{\phi\gamma\gamma} \phi \tilde{F}_{\mu\nu} F^{\mu\nu}, \quad (2.18)$$

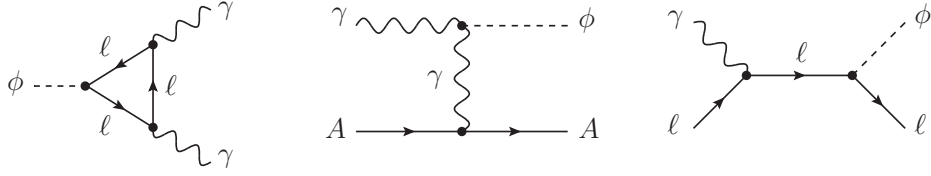


Figure 1: One-loop contribution to  $g_{\phi\gamma\gamma}$  (a); ALM production through Primakoff (b) and Compton (c) processes (a  $u$ -channel diagram in Compton is omitted).

with

$$g_{\phi\gamma\gamma} = -\frac{\alpha\lambda_\ell}{4\pi\tau m_\ell} \left( \text{Li}_2\left(\frac{2}{1+\sqrt{1-\tau^{-1}}}\right) + \text{Li}_2\left(\frac{2}{1-\sqrt{1-\tau^{-1}}}\right) \right) \approx -\frac{\alpha\lambda_\ell}{2\pi m_\ell}, \quad (2.19)$$

an approximation that is valid when  $\tau = m_\phi^2/(4m_\ell^2) \ll 1$  (*i.e.*, once we integrate out the charged lepton). Notice that there may be other contributions to  $g_{\phi\gamma\gamma}$  from, for example, the heavy charged lepton singlets introduced above. Therefore, in this model a given value of  $g_{\phi\gamma\gamma}$  just implies  $\lambda_\ell \leq -(2\pi m_\ell g_{\phi\gamma\gamma})/\alpha$ , where the equality holds when the dominant contribution comes from the diagram in Fig. 1. In any case, the axion-like coupling  $g_{\phi\gamma\gamma}$  will be determinant in the cosmological evolution of the ALM.

### 3 Cosmological evolution: $T > 10$ eV

Let us summarize the model under study and choose definite values for all its parameters. As discussed in the previous section, the model implies the presence of three heavy neutrinos; we will take a quasi-Dirac pair with  $M_{4,5} \approx 500$  GeV and  $M_6 \approx 1$  TeV for the Majorana field. These neutrinos have unsuppressed Yukawa interactions [see Eq. (2.4)], so they will be in thermal equilibrium with the cosmic plasma at  $T \geq 500$  GeV. For the majoron, also in equilibrium at these temperatures, we take

$$m_\phi = 0.5 \text{ eV}; \quad g_{\phi\gamma\gamma} = 1.46 \times 10^{-11} \text{ GeV}^{-1} \quad (3.1)$$

and

$$\lambda_{\nu_3} = 6.8 \times 10^{-14}; \quad \lambda_{\nu_2} = 1.3 \times 10^{-14}; \quad \lambda_{\nu_1} = 0.49 \times 10^{-14}. \quad (3.2)$$

The chosen value of  $g_{\phi\gamma\gamma}$  avoids astrophysical bounds [32–34] and fixes  $\lambda_\mu \leq 1.3 \times 10^{-9}$  and  $\lambda_\tau \leq 2.2 \times 10^{-8}$ . Because of its tiny mass, the majoron cannot decay into these charged leptons, just into photon or neutrino pairs:

$$\Gamma(\phi \rightarrow \nu_i \bar{\nu}_i) = \frac{\lambda_{\nu_i}^2}{4\pi} m_\phi \sqrt{1 - \frac{4m_{\nu_i}^2}{m_\phi^2}}; \quad \Gamma(\phi \rightarrow \gamma\gamma) = \frac{g_{\phi\gamma\gamma}^2}{64\pi} m_\phi^3. \quad (3.3)$$

However, for the couplings that we have assumed the branching ratio into gammas is negligible (around  $10^{-15}$ ): the majoron decays always into neutrinos with a lifetime

$$\tau_\phi = 3.5 \times 10^{12} \text{ s}. \quad (3.4)$$

The branching ratios  $\text{BR}_{\nu_i}$  of  $\phi \rightarrow \nu_i \bar{\nu}_i$  to the three flavors are

$$\text{BR}_{\nu_3} = 0.96, \quad \text{BR}_{\nu_2} = 0.035, \quad \text{BR}_{\nu_1} = 0.005. \quad (3.5)$$

To reconstruct the cosmological history of the ALM we need the Hubble parameter  $H$  at each temperature:

$$H^2 = \frac{8\pi}{3M_P^2} \rho. \quad (3.6)$$

Let us focus on  $T > 10$  eV, when the universe is still radiation-dominated. The energy density can be expressed in terms of the total number of effectively massless degrees of freedom ( $g_*$ ):

$$\rho_R = \frac{\pi^2}{30} g_* T^4. \quad (3.7)$$

We will take the standard values of  $g_*$  in [35]. At  $T \approx 500$  GeV all the SM degrees of freedom plus the heavy neutrinos and the majoron are in thermal equilibrium and contribute to  $g_*$ . Then the temperature drops below  $M_{4,5}$  and the heavy neutrinos disappear, transferring their entropy to the thermal bath. Our initial universe consists of a plasma with all the standard particles ( $g_* = 106.75$ ) and the ALM ( $\Delta g_* = 1$ ), all of them with the same temperature.

Once the heavy neutrinos have disappeared the majoron goes out of equilibrium and its abundance freezes out. Primakoff collisions  $\gamma A \rightarrow \phi A$  with  $A$  any charged particle in the plasma (see Fig. 1b) were in equilibrium at  $T > 1$  PeV [32] but are negligible at  $T < 500$  GeV, whereas Compton-like processes (in Fig. 1c) are also ineffective [36]. In particular, we estimate  $\Delta\rho_\phi \leq 5 \times 10^{-4} \rho_\phi^{\text{eq}}$  from Compton collisions at  $T \approx m_\mu$  if the dominant contribution in Eq. (2.19) came from the muon loop or  $\Delta\rho_\phi \leq 8 \times 10^{-3} \rho_\phi^{\text{eq}}$  if it came from  $\ell = \tau$ . All the interactions of the majoron with the neutrinos are negligible as well. Therefore, as  $T$  decreases the entropy in the heavy degrees of freedom of the SM goes into the lighter ones, but not into majorons. This produces a difference in the temperature in both components of the plasma. At  $T \approx 1$  MeV, right before  $\nu$  decoupling,  $e^+e^-$  annihilation and BBN, we have

$$T_\phi = \left( \frac{g_{*s}(1 \text{ MeV})}{g_{*s}(200 \text{ GeV})} \right)^{1/3} T = 0.463 T, \quad (3.8)$$



where  $T$  is still the temperature of both photons and neutrinos and  $g_{*s} \approx g_*$  [35] does not include the majoron. We may give the contribution of the majoron to  $\rho_R$  in terms of  $N_{\text{eff}}$ ,

$$\rho_\nu + \rho_\phi = \frac{7\pi^2}{120} N_{\text{eff}} T_\nu^4, \quad (3.9)$$

with

$$\Delta N_{\text{eff}} = \frac{4}{7} \left( \frac{T_\phi}{T_\nu} \right)^4 = 0.026. \quad (3.10)$$

At  $T < 1$  MeV the majoron-neutrino sector is decoupled from photons and electrons, so this  $\Delta N_{\text{eff}}$  already present at the beginning of BBN (plus a later 0.044 contribution from  $e^+e^-$  annihilation and other subleading effects, see [37,38]) could evolve unchanged down to low temperatures. The ALM model, however, admits a very different possibility: the resonant conversion of a fraction of CMB photons into majorons [39,40] at the end of BBN. This possibility rests on two basic observations. First, photons in a medium get a mass that can be expressed in terms of the index of refraction describing their propagation. In the cosmic plasma the main contribution to this mass comes from their interaction with free (not bounded in atoms) electrons. Second, the presence of a background magnetic field mixes the photon with the axion-like particle; this separates mass and interaction eigenstates and may produce oscillations that become resonant when the two masses coincide.

Let us then assume the presence of a primordial magnetic field  $\mathbf{B}$  with cosmological coherence length ( $\lambda_0 \geq 1$  Mpc) and field lines frozen in the plasma, so that  $B \approx B_0 (T/T_0)^2$ . We will take<sup>†</sup>  $B_0 = 3$  nG [41] and assume that the strength of the magnetic field is similar everywhere, although its direction may change in different domains. We will follow the density matrix formalism developed by Ejlli and Dolgov (ED) in [42], that accounts for the breaking of coherence (photon interactions that interrupt the oscillations) and provides a very simple estimate when the dominant contribution is at the resonant temperature.

At  $T < m_e$  the effective operators involved in the photon-ALM oscillations are

$$\begin{aligned} \mathcal{L} \supset & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{g_{\phi\gamma\gamma}}{4} \phi \tilde{F}_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (\partial_\mu \phi \partial^\mu \phi - m_\phi^2 \phi^2) \\ & + \frac{\alpha^2}{90m_e^4} \left( (F_{\mu\nu} F^{\mu\nu})^2 + \frac{7}{4} (\tilde{F}_{\mu\nu} F^{\mu\nu})^2 \right). \end{aligned} \quad (3.11)$$

Notice that the external field  $\mathbf{B}$  will also give an effective mass  $m_{\text{QED}}$  to the photon through the dimension-8 Euler-Heisenberg operator. In the WKB approximation the

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<sup>†</sup>The effect to be discussed is proportional to  $g_{\phi\gamma\gamma} B_0 \approx 4 \times 10^{-11}$  GeV<sup>-1</sup> nG.

coupled equations of motion reduce to [39]

$$\left( (\omega + i\partial_{\mathbf{x}}) \mathbb{1} + \begin{pmatrix} m_+ & 0 & 0 \\ 0 & m_{\times} & m_{\phi\gamma} \\ 0 & m_{\phi\gamma} & m_a \end{pmatrix} \right) \begin{pmatrix} A_+ \\ A_{\times} \\ \phi \end{pmatrix} = 0, \quad (3.12)$$

where  $\omega$  is the photon energy,  $\mathbf{x}$  its direction of propagation,  $A_+$  and  $A_{\times}$  correspond, respectively, to the polarizations perpendicular and parallel to  $\mathbf{B}$ ,  $m_{+, \times} = \omega(n-1)_{+, \times}$  with  $n$  the total refractive index,  $m_{\phi\gamma} = g_{\phi\gamma\gamma} B_T/2$  with  $B_T$  the component of  $\mathbf{B}$  orthogonal to  $\mathbf{x}$ , and  $m_a = -m_{\phi}^2/(2\omega)$ . At a temperature  $T$ , for a photon of energy  $xT$  these mass parameters are [42]

$$\begin{aligned} m_{\phi\gamma}(T) &= 1.10 \times 10^{-13} \left( \frac{T}{\text{GeV}} \right)^2 \text{ GeV}; \\ m_{\times}(T) &= \left( 193 x \left( \frac{T}{\text{GeV}} \right)^5 - 75.9 x^{-1} \left( \frac{n_e(T)}{T \text{ GeV}^2} \right) \right) \text{ GeV}; \\ m_a(T) &= -1.07 \times 10^{-19} x^{-1} \left( \frac{\text{GeV}}{T} \right) \text{ GeV}, \end{aligned} \quad (3.13)$$

where  $m_{\times} = (m_{\text{QED}} + m_{\text{pla}})_{\times}$  and

$$n_e(T) \approx 4 \left( \frac{m_e T}{2\pi} \right)^{3/2} \exp\left(-\frac{m_e}{T}\right) + 0.88 \eta_B n_{\gamma}(T), \quad (3.14)$$

with  $\eta_B = 6 \times 10^{-10}$  and  $n_{\gamma} = (2\zeta(3)/\pi^2) T^3$ . ED obtain an analytic solution for the transition probability when the universe crosses the resonant temperature  $\bar{T}$  where  $\Delta m \equiv m_{\times} - m_a = 0$  (see Fig. 2). The approximation is valid if (i)  $\rho_{\phi} \ll \rho_{\gamma} \approx \rho_{\gamma}^{\text{eq}}$  and (ii) the ALM interaction rate is  $\Gamma_{\phi} \approx 0$ . They express the result in terms of the occupation number  $n_{\phi}(x, T)$  of the energy level  $xT$  relative to the occupation number of photons in equilibrium:

$$P_{\phi}(x, T) \equiv \frac{\Delta n_{\phi}}{n_{\gamma}^{\text{eq}}} \approx - \left( \frac{2\pi}{3H} \right) \frac{m_{\phi\gamma}^2}{m_a + m_{\text{QED}}} \Big|_{T=\bar{T}}. \quad (3.15)$$

For our choice of parameters we have  $m_{\text{QED}} \ll |m_{\text{pla}}|$  and then a  $\bar{T}$  that is independent from  $x$ , *i.e.*, all photons oscillate at the same temperature:

$$\bar{T} = 26 \text{ keV}. \quad (3.16)$$

The probability of oscillation, however, is proportional to the photon energy. We obtain that 4.4% of the photons, carrying a 6.3% of the photon energy, convert into majorons

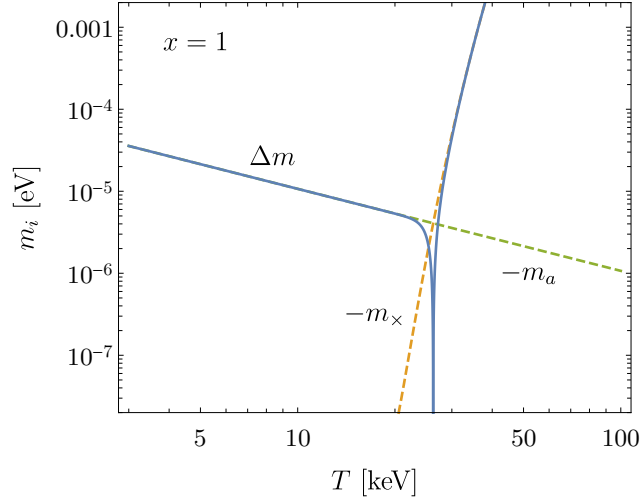


Figure 2: Masses for a photon/axion of  $\omega = xT$  at temperatures near the resonance.

at  $T \approx \bar{T}$ . After this resonant conversion the effective mass of the photon becomes fast much smaller than  $m_\phi$  and the number of ALMs freezes again. Notice that afterwards all photons rethermalize and the spectral distortion is completely erased.

In the simplified scenario just outlined the net effect is a sudden 1.6% drop in the photon temperature and a 4.7% increase in the baryon to photon ratio respect to the BBN value. Since neutrinos are not affected, at  $T < \bar{T}$  the ratio  $T_\gamma/T_\nu$  becomes a 1.6% smaller than in the SM:

$$\left. \frac{T_\gamma}{T_\nu} \right|_{SM} = \left( \frac{11}{4} \right)^{1/3} \implies \frac{T_\gamma}{T_\nu} = (1 - r_\gamma)^{1/4} \left( \frac{11}{4} \right)^{1/3}. \quad (3.17)$$

where  $r_\gamma = 0.063$  is the fraction of photon energy transferred to majorons. In addition to the effective 3.044 neutrino species, now the cosmic radiation also includes these majorons,

$$\rho_\phi^{(1)} = r_\gamma \frac{\pi^2}{15} \frac{T_\gamma^4}{1 - r_\gamma}; \quad n_\phi^{(1)} = 0.041 \frac{2\zeta(3)}{\pi^2} \frac{T_\gamma^3}{(1 - r_\gamma)^{3/4}}, \quad (3.18)$$

plus the initial thermal population of majorons at

$$T_\phi^{(2)} = \frac{0.463}{\left(\frac{11}{4}\right)^{1/3} (1 - r_\gamma)^{1/4}} \implies \rho_\phi^{(2)} = \frac{\pi^2}{30} T_\phi^{(2)4}. \quad (3.19)$$

We may express  $\rho_R = \rho_\gamma + \rho_\nu + \rho_\phi$  at  $T_\gamma < 26$  keV as

$$\rho_R = \frac{\pi^2}{15} T_\gamma^4 + \frac{7}{8} \frac{\pi^2}{15} \left( \frac{T_\gamma}{\left(\frac{11}{4}\right)^{1/3}} \right)^4 (3 + \Delta N_{\text{eff}}), \quad (3.20)$$

with

$$\Delta N_{\text{eff}} = 0.577. \quad (3.21)$$

This radiation will evolve down to  $T_\gamma < 10$  eV until neutrinos and majorons enter in thermal contact at  $T \approx m_\phi$  through decays and inverse decay collisions [21, 27, 28]:

$$\nu\bar{\nu} \leftrightarrow \phi. \quad (3.22)$$

## 4 Cosmological evolution: $T < 10$ eV

Let us analyze the final stage of the ALM evolution: from 10 eV to past recombination at  $T_\gamma \approx 0.26$  eV, when all majorons have already decayed.

As described in the previous section, the population of majorons that results from photon oscillations does not have a thermal distribution. We may, however, find a temperature  $T_\phi$  and a chemical potential  $\mu_\phi$  defining a distribution that reproduces the actual values of both  $\rho_\phi$  and  $n_\phi$ . In particular, we evolve  $\rho_\phi = \rho_\phi^{(1)} + \rho_\phi^{(2)}$  and  $n_\phi = n_\phi^{(1)} + n_\phi^{(2)}$  down to  $T_\gamma = 10$  eV and obtain

$$T_\phi = 0.72 T_\gamma; \quad \mu_\phi = -0.41 T_\gamma. \quad (4.1)$$

In a similar way, we evolve the number and energy density of the three neutrino species; including the small (non-thermal) component from  $e^+e^-$  annihilations we get

$$T_\nu = 0.72 T_\gamma; \quad \mu_\nu = 0.13 T_\gamma. \quad (4.2)$$

For the initial population of neutrinos we will take the same proportion of the three mass eigenstates<sup>‡</sup>.

We will follow the procedure described by Escudero in [37]<sup>§</sup> adapted to a majoron with stronger coupling to heavier neutrinos. The thermodynamic evolution of a  $\phi$ - $\nu$  fluid in thermal contact through  $\nu\bar{\nu} \leftrightarrow \phi$  can be expressed in terms of the decay width

$$\Gamma_\phi \approx \frac{\sum_i \lambda_{\nu_i}^2}{4\pi} m_\phi = 3.7 \times 10^{-28} m_\phi \quad (4.3)$$

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<sup>‡</sup>This is an example of scenario where the Quantum Mechanics approach fails [43] and we must work with neutrino mass eigenstates.

<sup>§</sup>The code NUDEC\_BSM [37, 44] is publicly available.

and the three branching ratios in Eq. (3.5)<sup>¶</sup>. In particular, for the majoron we have

$$\partial_t n_\phi = \frac{\Gamma_\phi m_\phi^2}{2\pi^2} \left[ \sum_i \text{BR}_{\nu_i} T_{\nu_i} e^{2\frac{\mu_{\nu_i}}{T_{\nu_i}}} K_1\left(\frac{m_\phi}{T_{\nu_i}}\right) - T_\phi e^{\frac{\mu_\phi}{T_\phi}} K_1\left(\frac{m_\phi}{T_\phi}\right) \right]; \quad (4.4)$$

$$\partial_t \rho_\phi = \frac{\Gamma_\phi m_\phi^3}{2\pi^2} \left[ \sum_i \text{BR}_{\nu_i} T_{\nu_i} e^{2\frac{\mu_{\nu_i}}{T_{\nu_i}}} K_2\left(\frac{m_\phi}{T_{\nu_i}}\right) - T_\phi e^{\frac{\mu_\phi}{T_\phi}} K_2\left(\frac{m_\phi}{T_\phi}\right) \right], \quad (4.5)$$

whereas for the 3 neutrinos

$$\partial_t n_{\nu_i} = \frac{\Gamma_\phi m_\phi^2}{2\pi^2} \text{BR}_{\nu_i} \left[ 2T_\phi e^{\frac{\mu_\phi}{T_\phi}} K_1\left(\frac{m_\phi}{T_\phi}\right) - 2T_{\nu_i} e^{2\frac{\mu_{\nu_i}}{T_{\nu_i}}} K_1\left(\frac{m_\phi}{T_{\nu_i}}\right) \right]; \quad (4.6)$$

$$\partial_t \rho_{\nu_i} = \frac{\Gamma_\phi m_\phi^3}{2\pi^2} \text{BR}_{\nu_i} \left[ T_\phi e^{\frac{\mu_\phi}{T_\phi}} K_2\left(\frac{m_\phi}{T_\phi}\right) - T_{\nu_i} e^{2\frac{\mu_{\nu_i}}{T_{\nu_i}}} K_2\left(\frac{m_\phi}{T_{\nu_i}}\right) \right], \quad (4.7)$$

being  $K_n(x)$  modified Bessel functions of the second kind. The equations describing the evolution of  $T$  and  $\mu$  of a generic species read

$$\frac{dT}{dt} = \frac{1}{\partial_\mu n \partial_T \rho - \partial_T n \partial_\mu \rho} [-3H((p + \rho) \partial_\mu n - n \partial_\mu \rho) + \partial_\mu n \partial_t \rho - \partial_\mu \rho \partial_t n], \quad (4.8)$$

$$\frac{d\mu}{dt} = \frac{-1}{\partial_\mu n \partial_T \rho - \partial_T n \partial_\mu \rho} [-3H((p + \rho) \partial_T n - n \partial_T \rho) + \partial_T n \partial_t \rho - \partial_T \rho \partial_t n]. \quad (4.9)$$

In Fig. 3 we provide the energy densities of the majoron and the three neutrino flavors relative to  $\rho_\gamma$  (left) together with  $N_{\text{eff}}$ ,

$$N_{\text{eff}} = \frac{8}{7} \left( \frac{11}{4} \right)^{4/3} \frac{\rho_\nu + \rho_\phi}{\rho_\gamma}. \quad (4.10)$$

The plots show that inverse decays  $\nu_3 \bar{\nu}_3 \rightarrow \phi$  keep the energy density in the  $\phi$  component of the fluid close to the equilibrium one up to  $t \approx 10 \tau_\phi$ . Then, at temperatures below  $m_\phi$  majorons disappear and transfer all their entropy to (mostly)  $\nu_3$ . In Fig. 3-right we plot the evolution of  $N_{\text{eff}}$ ; notice that the transfer of energy between  $\nu_3$  and the majorons results in a significant net increase in  $N_{\text{eff}}$ : it goes from 3.58 at  $T > 10$  eV to 3.85 at  $T < 0.1$  eV. This effect, discussed in [37], appears because the majoron decays into neutrinos at temperatures below its mass, when it is mildly relativistic. In our scenario the majoron-neutrino interaction rate [in Fig. 3-bottom, see also Eq. (4.11)] is slightly below the one required for thermal equilibrium (it corresponds to  $\Gamma_{\text{eff}} = 0.8$  in the notation of [28]), which enhances the effect.

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<sup>¶</sup>Notice that our definition of  $\lambda_\nu$  in (2.17) differs by a factor of 2 from the one in [37] and that  $\Gamma_\phi$  refers there to the partial decay width.

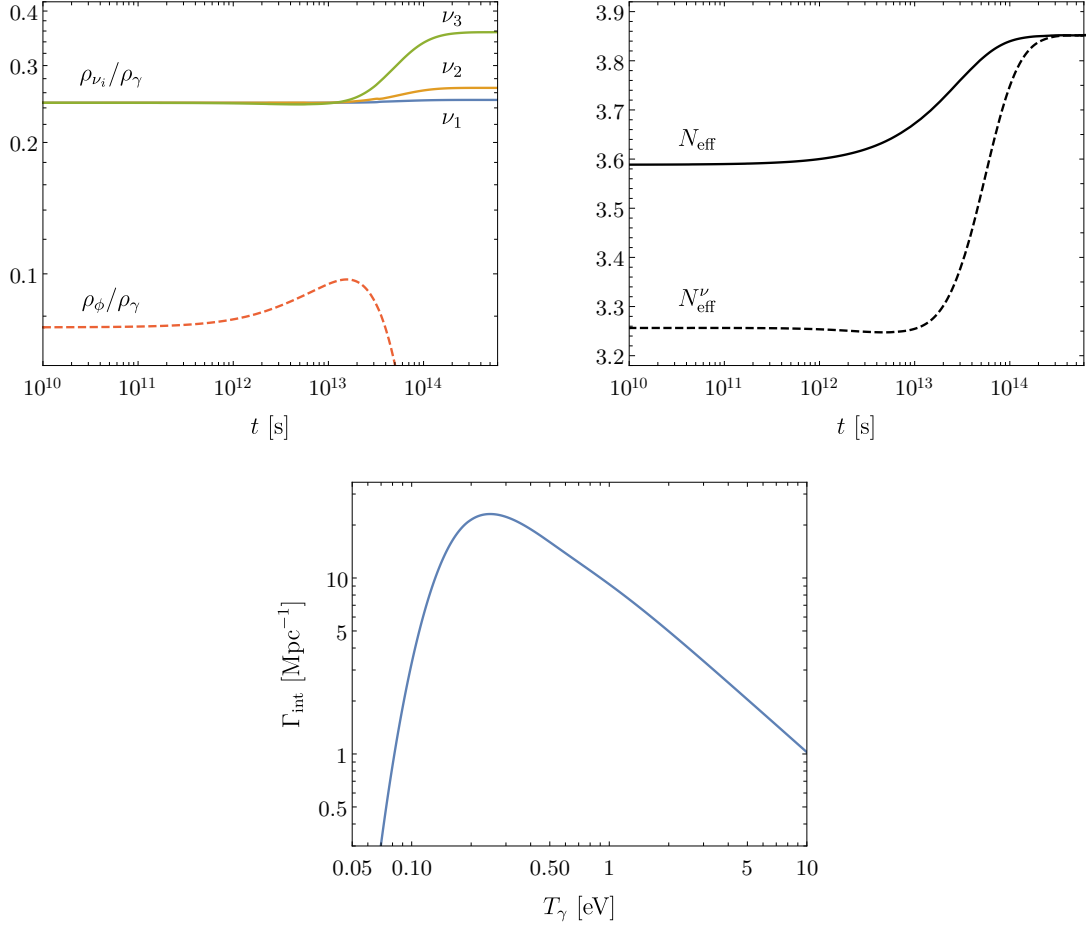


Figure 3: **Left:** Ratios  $\rho_\phi/\rho_\gamma$  and  $\rho_{\nu_i}/\rho_\gamma$  for  $i = 1, 2, 3$  for  $t$  between  $10^{10}$  s ( $T_\gamma = 11$  eV) and  $6 \times 10^{14}$  s ( $T_\gamma = 0.045$  eV). **Right:**  $N_{\text{eff}}$  (in dashes, the contribution from neutrinos). **Bottom:** Interaction rate of neutrinos at temperatures  $T_\gamma \approx m_\phi$ .

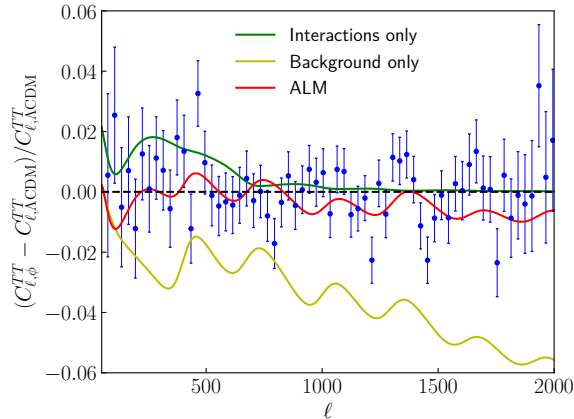


Figure 4: Effect (relative to that of  $\Lambda$ CDM) on the TT power spectrum produced by (i) the  $T$ -dependent increase of  $N_{\text{eff}}$  in the ALM model; (ii) the damping of the  $\nu$  free streaming due to interactions; and (iii) the total effect after redefining the cosmological parameters (see Table 2) in the ALM model.

In addition, the processes  $\nu_3 \bar{\nu}_3 \leftrightarrow \phi$  will damp the number of free streaming neutrinos, something that is necessary to achieve isotropy and preserve the position of the peaks of the CMB acoustic oscillations [21]. The net effect is a time-dependent modification to the growth of potential wells; in particular, these interactions are effective between  $10 m_\phi \geq T_\gamma \geq m_\phi/10$ , implying that they will only alter CMB multipoles  $l \leq 1000$ .

Following the method and approximations<sup>||</sup> described in [37], we have obtained the interaction rate (in Fig. 3)

$$\Gamma_{\text{int}} = \frac{1}{\rho_\nu} \frac{\delta \rho_\nu}{\delta t} = \frac{\Gamma_\phi}{\rho_\nu} \sum_i \text{BR}_{\nu_i} e^{\frac{\mu_{\nu_i}}{T_{\nu_i}}} \left( \frac{m_\phi}{T_{\nu_i}} \right)^3 K_2 \left( \frac{m_\phi}{T_{\nu_i}} \right), \quad (4.11)$$

we have modified the equations for the time evolution of the density, velocity, shear and higher anisotropic moments of the phase space distribution in the synchronous gauge [46] and we have included them in CLASS [47, 48], using MontePython [49, 55] to deduce the cosmological implications of this scenario.

Our results are summarized in Fig. 4 and Table 2. Throughout our analysis we use the definite ALM model with  $m_\phi = 0.5$  eV and  $\tau_\phi = 3.5 \times 10^{12}$  s defined in previous sections. We do not explore the probability density distributions of cosmological parameters for

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<sup>||</sup>These include neglecting neutrino masses, assuming that neutrinos and majorons form a single coupled fluid, and using the relaxation time approximation for their collision term [45].

Parameter	$\Lambda$ CDM	ALM $m_\phi = 0.5$ eV $\tau_\phi = 3.5 \times 10^{12}$ s
$100 \Omega_b h^2$	$2.242 \pm 0.015$	$2.295 \pm 0.014$
$\Omega_{\text{cdm}} h^2$	$0.119 \pm 0.001$	$0.129 \pm 0.001$
$100 \theta_s$	$1.0420 \pm 0.0003$	$1.0407 \pm 0.0003$
$\ln(10^{10} A_s)$	$3.046 \pm 0.015$	$3.062 \pm 0.016$
$n_s$	$0.967 \pm 0.004$	$0.991 \pm 0.004$
$\tau_{\text{reio}}$	$0.055 \pm 0.008$	$0.056 \pm 0.008$
$H_0$ [km/s/Mpc]	$67.71 \pm 0.44$	$71.4 \pm 0.5$

Table 2: Cosmological parameters in  $\Lambda$ CDM and the ALM model from a combined analysis of Planck2018 (low and high multipoles of the CMB temperature and polarization power spectra, as well as the CMB lensing likelihood, [51]) + BAO (BOSS, SDSS MGS [52–54]) data. The quoted  $\Lambda$ CDM values can be found in [50].

other input values, which would correspond to a different thermal history. Fig. 4 shows the effect (relative to that of  $\Lambda$ CDM) on the  $T$  angular power spectrum (TT) of (i) the  $T$ -dependent increase of  $N_{\text{eff}}$  in the ALM model; (ii) the damping of the free streaming due to the neutrino interactions; and (iii) the net effect after redefining the cosmological parameters as given in the table. The first two lines are produced fixing  $\Omega_b H_0^2$ ,  $z_{\text{mr}}$  and  $\theta_s$  to their  $\Lambda$ CDM values, whereas the red line is obtained by varying the 6 cosmological parameters to the values given in Table 2. The baryon to photon ratio in our model ( $100 \Omega_b h^2 = 2.295 \pm 0.014$  versus  $2.242 \pm 0.015$  in  $\Lambda$ CDM) may seem inconsistent with the recent values around  $2.20 \pm 0.05$  deduced from data on the abundance of primordial Deuterium (see [56] and references therein). However, in the ALM model the baryon to photon ratio at the beginning of BBN (before a fraction of photons convert into majorons) is a 4.7% smaller,

$$100 \Omega_b h^2 = 2.186 \pm 0.014, \quad (4.12)$$

which is in perfect agreement with that data.

A comment about our choice of parameters in the ALM model is here in order. We took  $m_\phi = 0.5$  eV to damp the free streaming of neutrinos and majorons at the *right* cosmological time (near recombination). Then we used  $g_{\phi\gamma\gamma}$  to convert 6% of CMB photons into ALMs, so that  $N_{\text{eff}} = 3.85$  after all majorons have decayed. This value of  $N_{\text{eff}}$  is larger than the ones considered in previous literature [27, 28], and thus it implies an also larger expansion rate  $H_0$ . Finally, the majoron-neutrino coupling



$\lambda_{\nu_3} = 6.8 \times 10^{-14}$  was chosen so that  $v_X \approx 900$  GeV and collider bounds on the heavy neutrinos are easily avoided [25]. Notice that  $\lambda_{\nu_3}$  fixes both the lifetime of the ALM and the frequency of neutrino-majoron interactions: its value must let the majoron decay before recombination (a critical process to reduce the free streaming of the extra radiation during that period) and imply an acceptable fit to the CMB multipoles.

## 5 Summary and discussion

Our understanding of the neutrino sector is still work in progress. The SM admits a minimal completion of the sector with just the addition of the dimension-5 Weinberg operator and no new degrees of freedom at EW energies. Indeed, the large scale  $\Lambda \approx 10^{10}$  GeV suppressing that operator could be explained naturally by the usual (type I) seesaw mechanism. However, there are other possibilities that are also well motivated and imply the presence of light exotic particles. In these other scenarios the tiny value of the neutrino masses is justified not by a large scale but by the breaking of a global symmetry. Ultimately, only the data may decide the right venue.

Here we have proposed one of such models and have studied its cosmological implications. The model is an explicit realization of the generic scenario discussed in [21]. A scalar VEV breaks an approximate global symmetry and implies the presence of a majoron  $\phi$  as the only new particle below the EW scale; a small component of the VEV also breaks a discrete  $Z_3$  symmetry, generating neutrino masses and couplings to the majoron. Mixing of the SM charged leptons with heavy fields may then generate a small coupling of  $\phi$  with the  $\tau$  lepton and axion-like couplings of type  $\phi \tilde{F}_{\mu\nu} F^{\mu\nu}$ . This is a *special* axion-like particle: an ALM that decays with an almost 100% branching ratio into neutrinos, not into gammas. For majoron models with axion couplings to gluons see [57, 58].

We show that in the early universe, right after BBN, the presence of a cosmological magnetic field may drive a fraction of photons into ALMs. Other mechanisms to reduce  $T_\gamma - T_\nu$  after BBN but before recombination include the photon cooling by gravitational interactions with a Bose condensate of axions in [59] or by kinetic mixing with hidden photons in [60].

At lower temperatures,  $T_\gamma \approx m_\phi = 0.5$  eV, the ALMs enter in thermal contact with the heaviest neutrino, transfer all their entropy and define a scenario with  $N_{\text{eff}} \approx 3.85$ , with the final value of  $\rho_{\nu_3}$  50% larger than that of  $\rho_{\nu_1}$ . This is basically the mechanism

proposed in [27], although we do not change significantly the standard value of  $N_{\text{eff}}$  during BBN and thus favor larger values of  $N_{\text{eff}}$  (and thus of  $H_0$ ) at later times.

The model proposed in this work constitutes then another variation of the  $\Lambda$ CDM model that may help to relax the  $H_0$  tension if it persists (we obtain  $H_0 = (71.4 \pm 0.5)$  km/s/Mpc) and provides an acceptable fit for the CMB multipoles and the rest of cosmological observables. Although it has been argued [61] that the most likely solutions to this tension are those that modify the expansion history of the universe just before recombination, there actually exists a large number of plausible interpretations (see [62] for a recent analysis of the redshift dependence of supernova data and [63, 64] for a compilation of alternative solutions). The data on LSS from upcoming experiments will probe the  $\Lambda$ CDM model to new limits; we think that the results described here are an example of the effects that could be expected in scenarios that are well motivated by particle physics.

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