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Using argumentation in expert's debate to analyze multi-criteria group decision making method results ☆

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ABSTRACT

Recent multi-criteria group decision making methods focus their analysis on the experts preferences. They do not take into account the reasons why each expert has provided a specific set of preferences. In this paper, a method that introduces novel measures capable of explaining the reasons behind experts decisions is presented. A novel concept, the arguments are presented. They represent the experts have for maintaining a certain position in the debate. Several measures related to the arguments are proposed. These new argumentation measures, along with consensus measures, help us to get a clear idea about how and why a specific resolution has been reached. They help us to determine which is the most influential expert, that is, the expert whose contributions to the debate have inspired the rest. Also, the proposed method allows us to determine which are the arguments that most of the experts have followed. A clear overview about how the debate is evolving in terms of arguments is also provided. The novel presented analysis indicate how the experts change their opinions in every round and what was the reason for it, which changes have occurred between rounds and they also provide global analysis results.

1. Introduction

Multi-criteria group decision making methods are an interesting tool for aiding experts to make decisions. Based on a set of criteria values, experts must sort a set of alternatives. The system takes into account all the experts' preferences and generate a final ranking of alternatives.

Multi-criteria group decision making is a field that is quite present in the recent literature. For instance, in [13], probabilistic hesitant fuzzy sets are used for representing the information in a multi-criteria group decision making method. In [17], single-valued trapezoidal neutrosophic information is used in order to define a novel multi-criteria group decision mak-

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ing process. In [23], authors present a novel multi-criteria group decision making method that works in environments that have a high number of alternatives. Finally, in [18], an interval-valued ORESTE method is applied in order to solve a multi-criteria group decision making problem.

Most of the multi-criteria group decision making methods that are present in the recent literature focus on modelling three main issues. The first one is establishing the method itself, that is, establishing an algorithm for carrying out the required process. They focus on how to combine experts' preferences and how to deal with the criteria values [27,33,39]. The second one is determining the way that the experts provide information to the system and how this information is dealt with. There is a need of allowing the experts to be comfortable with the information that they are providing. Therefore, different solutions are given [2,13,22,26]. The third issue is about consensus [4,8,11,21,31]. Methods for calculating consensus among experts are presented.

As it can be seen, all multi-criteria group decision making methods always work over the experts' preferences. They do not take into account the debate that has been carried out or the reasons that make experts to have a certain opinion. Because of that, the information that is provided about the process is hindered and the decision process cannot be completely understood. That is, it is possible to know which experts agree and have similar opinions, but it is not possible to know why they agree or what are the reasons that make them to be on the same side of the discussion. This information is critical since it allows us to get a full understanding of the process. Nevertheless, most of the recent multi-criteria group decision making methods leave aside this information and only focus on results and consensus measures. Although consensus measures can determine if experts agree on a solution, they do not provide specific information about why does that occur. The goal of the proposed measures is to provide solutions to all these problems.

In this paper, a novel multi-criteria group decision making method that implements an analysis based on the discussion arguments that the experts are using in their debates are presented. The goal of the presented measures is to obtain a better overview of the decision process that the experts are participating in. Also, to understand why a certain alternatives ranking is reached. Concretely, the presented method focuses on measuring two main issues:

- **Key arguments**: When carrying out a debate, a lot of ideas and arguments arise. Nevertheless, only a small subset of them are sharp enough to have a critical impact over the decision process and in the way that experts tackle the dealt problem. Identifying those key arguments are critical if the way that experts behaviour want to be understood. In the presented methodology, experts are asked to pinpoint which arguments are the ones supporting their preferences. By analysing which arguments are influencing the multi-criteria group decision process, it is possible to comprehend why a specific set of results is supported by each of the experts.
- **Changes in the mindset of the experts**: In order to understand what are making the experts to change their opinions and provide a different set of preferences, it is important to measure when are they having a change of mind. A multi-criteria group decision making process has a set of rounds where, in each round, a preference providing step is performed. Understanding when and why an expert changes his mind provides us with a lot of information that explains why the experts are behaving as they do.

By using all these measures, it is expected that the decisions result become more reliable due to the fact that they are supported by a clear comprehension of the process that have led to them.

The paper is organized as follows. In Section 2, basis needed to understand the designed measures is presented. In Section 3, the developed measures and multi-criteria group decision making process used are described in detail. In Section 4, an application example is shown. In Section 5, advantages, future work and novelty of the presented method are discussed. Finally, some conclusions are pointed out.

2. Preliminaries

In this section, the basis needed to comprehend the method are presented. In subSection 2.1, multi-criteria group decision making processes structure is described. In subSection 2.2, the basis of consensus measures are exposed.

2.1. Multi-criteria group decision making

Multi-criteria group decision making is the field that is currently being more studied inside the group decision making area [1,24,32,37]. In a traditional group decision making method, a set of experts need to carry out a thorough debate in order to rank a set of alternatives. The final ranking is built using the preferences values that the experts provide to the system. Multi-criteria group decision making methods include a new element: the criteria. The criteria values can be defined as a set of characteristics that the alternatives must fulfil in order to be considered as adequate. Criteria values are a good tool for allowing the experts to carry out a more objective analysis over the tackled problem. Formally, a multi-criteria group decision making problem can be set out as follows:

Definition 2.1. Let define a set of experts $E = \{e_1, \ldots, e_n\}$, a set of alternatives, $X = \{x_1, \ldots, x_m\}$, and a set of criteria values $C = \{c_1, \ldots c_l\}$. A multi-criteria group decision making method consists in defining the process in which the set of experts ranks the set of alternatives using as guidance the set of criteria values. For this purpose, each expert e_i provides a set of preferences, $P_i = \{p_{ik}\}, k = 1 \ldots l$, that are used by the system to calculate the final ranking of alternatives.

An overall scheme of the process can be seen in Fig. 1. It should be noticed that the final ranking of the alternatives totally depends on the preferences provided by the experts to the system.

It should be noticed that it is possible to define different importance levels for the experts. For this purpose, a weighting value, $W = \{w_1, \ldots, w_n\}$ can be assigned to each of them.

2.2. Consensus measures

In multi-criteria group decision making methods, it is quite important that the final reached alternatives ranking is supported by as many people as possible. In other words, it is not recommendable that the experts make a rush resolution. On the contrary, a thorough final decision through an in-depth debate is preferable. For this purpose, it is important to promote debate among the experts until a common resolution of the problem is reached. In order to measure if the experts have reached or not an agreement, it is possible to use consensus measures over the experts' preferences. Generally, three different consensus levels can be distinguished:

- **Consensus over two pair of alternatives**: These consensus measures calculate the agreement level of experts when comparing two specific alternatives.
- **Consensus over alternatives**: These consensus measures calculate the consensus that the experts have on each of the alternatives. If experts agree on a specific alternative, it means that they have similar opinions about the importance of that alternative on the group decision process.
- **Consensus in the decision**: The overall consensus reached by the experts in the decision process. Generally, it can be calculated by aggregating the consensus reached on each alternative. If this value is high enough, it means that the experts' preferences are similar and, therefore, they have reached an agreement. If it is low, then the experts should carry out more debate in order to bring opinions closer.

Consensus in the group decision making area is a quite popular topic in the recent literature. For instance, in [21], different consensus measures that are used in group decision making environments are compared. In [34], a consensus model for large-scale group decision making that uses hesitant fuzzy sets is presented. Finally, in [7], challenges and research paradigms in the consensus calculation process for group decision making methods that are carried out on social networks are presented.

3. Measuring argumentation in multi-criteria group decision making processes

In this section, the developed multi-criteria group decision making method and measures are described in detail. In subSection 3.1, the multi-criteria group decision making method steps are described in detail. In subSection 3.2, the novel proposed measures are exposed.

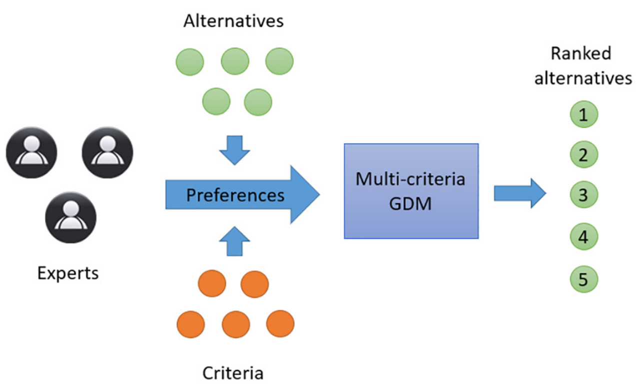

Fig. 1. Multi-criteria group decision making scheme.

3.1. Multi-criteria group decision making scheme

In this subsection, the steps followed by the proposed multi-criteria group decision making method are described. A summary of the notation can be found on Table 1. The proposed method follows the next steps:

1. **Defining the multi-criteria group decision making parameters**: Initial parameters of the multi-criteria group decision making process are defined.
2. **Carrying out the discussion and providing arguments**: Experts start carrying out the discussion. Each of their arguments and reasons about why alternatives should be chosen are listed.
3. **Providing preferences to the system**: Experts provide their preferences to the system. They also indicate which arguments support them.
4. **Calculating the collective preference matrix**: Preferences provided by the experts are aggregated into a single collective preference matrix that represents the overall opinion of all of them.
5. **Calculating consensus measures**: The preferences provided by the experts are used in order to determine the overall consensus of the decision process. If the consensus is low, experts are asked to carry out more discussion (step 2). On the contrary, if the reached consensus is high, final ranking results are calculated.
6. **Calculating argumentation measures**: Using all the argumentation and preferences information provided by the experts in all the group decision making rounds, the key arguments, the different lines of arguing and changes of opinions that the experts have experienced during the process can be retrieved. As a result, it is possible to obtain a summary about how the process has been performed.
7. **Calculating final ranking results**: The final alternative ranking is calculated. Preferences of the last round are used for this purpose.

In Fig. 2, an overall scheme of the presented method is shown. The exposed steps are described in more detail in the following subsections.

On the following subsections, these steps are described in detail.

3.1.1. Defining initial parameters

First of all, the initial parameters of the multi-criteria group decision making process must be defined:

- **Set of experts,** $E = \{e_1, \ldots, e_n\}$: The set of experts participating in the multi-criteria group decision process is defined.
- **Set of alternatives,** $X = \{x_1, \ldots, x_m\}$: This set contains all the alternatives that the experts have to discuss about.
- **Set of criteria values,** $C = \{c_1, \ldots, c_l\}$: A set of criteria values that the experts have to take into account when providing their preferences is defined.
- **Consensus threshold,** CTH: It is the minimum consensus value that, when reached, implies that it can be considered that the experts have reached an agreement. If the consensus value is lower than CTH, then a new discussion round is initiated. Otherwise, the process ends.

Table 1
Notation summary.

Notation	Description
E	Set of experts.
X	Set of alternatives.
C	Set of criteria.
W_e	Weight of the experts in the decision.
D_i	Set of arguments provided by e_i.
PS_i^j	Set of preferences provided by e_i in the round j.
p^{ikj}	Preference matrix provided by e_i in the round j about criterion k.
LA^{ij}	Arguments defended by e_i in the round j.
pc^{ij}	Aggregation of the preferences matrices provided by e_i in round j for all the criteria.
W_c^i	Weight that e_i applies to each criterion.
C^j	Collective matrix that aggregates all the experts' preferences for round j.
rd_k^j	Times that an d_k is mentioned in round j
DLA_i^k	Common arguments between experts e_i and e_k.
$imp_{d_k}^j$	Offset of d_k in the argument raking.
$SimJ_{ij}^{j-1}$	Similarity of preferences provided by e_i compared with the ones provided on the previous round.
$SimLAJ_{ij}^{j-1}$	Similarity of arguments provided by e_i with the ones provided on the previous round.
grd_k	global position of d_k in the arguments' ranking for the whole decision making process.

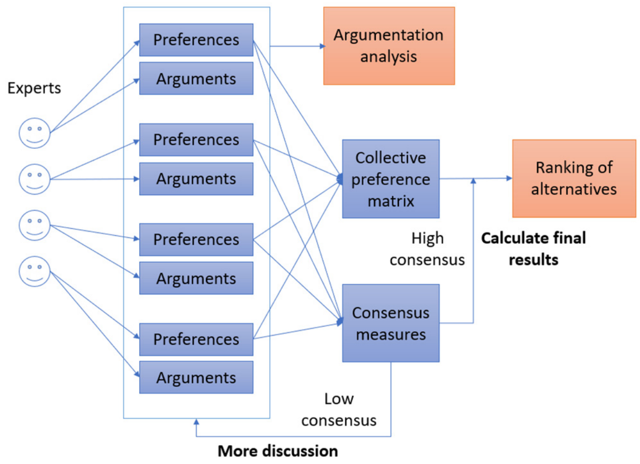

Fig. 2. Developed multi-criteria group decision making method scheme.

- **Maximum number of rounds,** *MR*: Since there are cases where the experts are not capable of reaching an agreement, it is important to delimit the time that the experts are asked to tackle the decision problem. By defining a maximum number of decision rounds, the decision process can end after a certain number of rounds, even if the obtained consensus value is not high enough. This is useful in cases where no matter how many time the experts debate, they do not reach an agreement. Since the decision should end at a certain point or could last forever, the *MR* value delimits the maximum time. This assures that the GDM process ends after some predefined time.
- **Providing preferences procedure**: The procedure that the experts will use to provide their preferences is set. Generally, it is interesting to use a mean that the experts find themselves comfortable with. It should be taken int account that in a group decision making method, the experts are continuously in contact with the system. Therefore, there is a need for a comfortable expert-system communication. For instance, linguistic label sets can be used. They allow experts to express themselves using words instead of numbers. If multi-granular fuzzy linguistic modelling methods are used [30,36,38], each expert can select the linguistic label set that he/she feels more comfortable with.

Once that the initial parameters of the multi-criteria group decision making process are defined, the process can start.

3.1.2. Providing arguments and preferences to the system

Once that the multi-criteria group decision making parameters are set, experts can start the discussion process. When discussing, experts debate about the pros and cons of all the alternatives according to the established criteria. Each of the arguments employed to defend their positions are defined and stored in the system along with the expert that has provided it. Therefore, for each expert e_i in $E = \{e_1, \ldots, e_n\}$, a set $D_i = \{d_1, \ldots, d_h\}$ is generated. h is the total number of arguments that the expert has provided to the discussion.

Once that the arguments have been highlighted, experts provide their preferences to the system. Afterwards, they must indicate which are the arguments that support the values that they provide. They can use their own provided arguments or arguments that other experts have shared.

In order to provide the preference values, experts carry out a pairwise comparison of each pair of alternatives according to a single criterion. Therefore, each expert provides a set of preference relation matrices, one matrix per each criterion. Formally, the provided information is represented by the following set:

$$PS_i^j = \{P^{ikj}\} \quad k = 1, \ldots, l \tag{1}$$

where each P^{ikj} is a preference relation matrix that has been provided by expert e_i in the round j taking into account the criteria value c_k. Each preference relation matrix, P^{ikj}, is defined as follows:

$$P^{ikj} = \begin{pmatrix} - & p_{12}^{ikj} & \cdots & p_{1m}^{ikj} \\ p_{21}^{ikj} & \cdots & \cdots & p_{2m}^{ikj} \\ \cdots & \cdots & \cdots & \cdots \\ p_{m1}^{ikj} & \cdots & p_{m(m-1)}^{ikj} & - \end{pmatrix} \tag{2}$$

where k refers to the criterion c_k. Therefore, each value p_{uv}^{ikj} is the preference value provided by e_i that compares alternative x_u with alternative x_v for the criterion c_k in the decision round j.

Once that experts provide their preferences, they share a list of arguments that support their opinions. Each expert e_i provides the following list of arguments:

$$LA^{ij} = \{la_1^{ij} \ldots, la_r^{ij}\} \tag{3}$$

where elements from LA^{ij} are values of D_i that are generated after the debate.

In Fig. 3, an example of how the preference information is provided is shown. In round 1 experts $\{e_1, \ldots e_4\}$ provide the preferences values p_a, p_b, p_c and p_d respectively. e_1, e_2 and e_3 indicate that argument d_a is the one sustaining their provided values while e_4 indicates that his/her preferences are motivated by argument d_d. On the second round, a new argument d_e is provided by some expert. This new argument motivates a change of mind on experts e_2, e_3 and e_4 that provide new preferences values motivated by that argument. The new argument does not convince e_1 which maintains the provided preferences. He/She is still motivated by d_a.

3.1.3. Calculating the collective preference matrix

Once that the experts have provided their preferences, a collective preference matrix containing the overall opinion of all the experts is generated. This matrix can be calculated by following the next two steps:

1. **Aggregating preference relation matrices per each criteria value**: P^{ikj} matrices for the same e_i can be aggregated into a single preference relation matrix that represents the overall opinion of one expert. For this purpose, the following expression can be applied:

$$pc^{ij} = WA(p^{i1j}, \ldots p^{ikj}) = \begin{pmatrix} - & pc_{12}^{ij} & \ldots & pc_{1m}^{ij} \\ pc_{21}^{ij} & \ldots & \ldots & pc_{2m}^{ij} \\ \ldots & \ldots & \ldots & \ldots \\ pc_{m1}^{ij} & \ldots & pc_{m(m-1)}^{ij} & - \end{pmatrix} \tag{4}$$

   where WA is the weighted aggregation operator. If criteria values have different importance degrees, the preferences of the expert are aggregated using a weighting vector associated to the criteria values. That is, the following weighting vector is used:

$$W_c^i = \{w_1^i, \ldots, w_k^i\} \tag{5}$$

   where $\sum_{u=1}^{k} W_u^i = 1$. w_1^i indicates the importance given by the e_i to c_1.
2. **Aggregating experts' preference relation matrices**: Once that one preference relation matrix per each expert has been generated, the information is aggregated into a single collective preference value. This piece of information contains a summary of all the preferences provided by the experts participating in the process. It can be calculated as follows:

$$C^j = WA(PC^{1j}, \ldots, PC^{nj}) = \begin{pmatrix} - & c_{12}^j & \ldots & c_{1m}^j \\ c_{21}^j & \ldots & \ldots & c_{2m}^j \\ \ldots & \ldots & \ldots & \ldots \\ c_{m1}^j & \ldots & c_{m(m-1)}^j & - \end{pmatrix} \tag{6}$$

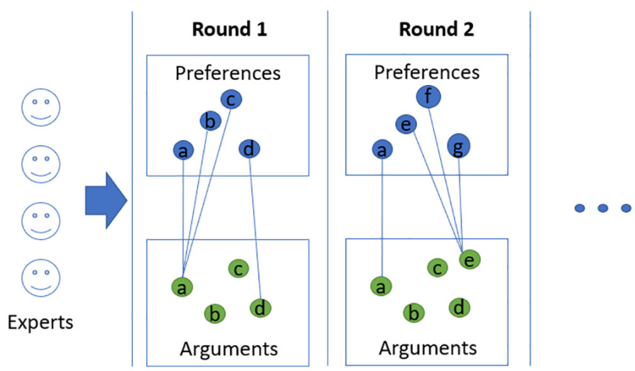

Fig. 3. Providing arguments and preferences example.

where PC^{ij} is the aggregated preference matrix for expert e_i in the round j. If experts have different importance degrees in the decision, a weighting vector can be used in order to carry out a weighted aggregation. For this purpose, the following weighting vector can be used:

$$W_e = \{w_1, \ldots, w_n\} \tag{7}$$

3.1.4. Calculating consensus measures

Before calculating the final ranking results, it is important to determine if the experts have reached an agreement. In case that the obtained consensus degree is high, final raking results can be calculated. Otherwise, more debate is recommended.

First of all, it is necessary to calculate the similarity between two specific preference relation matrices. In order to calculate the similarity between preference aggregations from experts e_i and e_o, the following expression can be used [3,10]:

$$Sim_{PC^i}^{PC^o} = 1 - \phi_1\left(abs\left(\begin{pmatrix} - & pc_{12}^{oj} & \ldots & pc_{1m}^{oj} \\ pc_{21}^{oj} & \ldots & \ldots & pc_{2m}^{oj} \\ \ldots & \ldots & \ldots & \ldots \\ pc_{m1}^{oj} & \ldots & pc_{m(m-1)}^{oj} & - \end{pmatrix} - \begin{pmatrix} - & pc_{12}^{ij} & \ldots & pc_{1m}^{ij} \\ pc_{21}^{ij} & \ldots & \ldots & pc_{2m}^{ij} \\ \ldots & \ldots & \ldots & \ldots \\ pc_{m1}^{ij} & \ldots & pc_{m(m-1)}^{ij} & - \end{pmatrix}\right)\right) \tag{8}$$

where ϕ_1 is a definition of the arithmetic mean operator. ϕ_1 aggregates all the values of a matrix A and return a single scalar value with the result as follows:

$$\phi_1(A) = \frac{\sum_{i=0,j=0}^{i=m,j=m,i\neq j} A_{ij}}{m^2 - m} \tag{9}$$

abs refers to the absolute value operator and it is used to eliminate the negative values and work in terms of distances. Using expression (8), several interesting values can be calculated:

- **Consensus between experts**: If the consensus degree between two different experts in a specific round wants to be calculated, expression (8) can be applied.
- **Global consensus value**: By aggregating all the possible combinations of *SimE*, it is possible to calculate the global consensus value in a discussion round. This calculation can be done as follows:

$$GCV = \phi(SimE_{e_i}^{e_o}) \quad i, o = 1 \ldots l, \; i < o, i \neq o \tag{10}$$

where ϕ is the arithmetic mean applied over scalar values.
- **Consensus on a criteria value**: It is possible to measure the consensus reached on a specific criteria value by obtaining the similarity measure among the preference values provided for that criteria. In order to calculate the similarity value of c_k the following expression can be applied:

$$SimC_{c_k} = \phi_1(\phi_2(abs(P_i^k - P_j^k)), \quad i, j = 1 \ldots l, \; i < j, i \neq j) \tag{11}$$

where ϕ_2 is applied over a set of matrices and returns a matrix that have in the (i, j) position the mean of the values located in the same position on the input matrices. This is done following the expression below:

$$\phi_2(A, B) = \frac{A_{ij} + B_{ij}}{2} \quad i, j \in 1, \ldots, m \tag{12}$$

where A and B are matrices, P_i^k is the preference relation matrix provided by e_i for the criterion c_k.
- **Consensus over the alternatives:** Finally, it is possible to calculate consensus over the alternatives by aggregating the similarity values that are related to them. First of all, the global consensus matrix has to be calculated. This matrix contains the consensus among each pair of alternatives. It can be computed as follows:

$$GSM = \phi_1(\phi_2(abs(PC^i - PC^o))i, \quad o \in 1 \ldots n, \; i < o, i \neq o) \tag{13}$$

where PC^i refers to the aggregation of the preferences provided by the expert e_i. Using *GSM*, the consensus over alternative x_h is calculated as follows:

$$C_{x_h} = \phi(gsm_{vh}), \quad v \in 1 \ldots m \tag{14}$$

In order to obtain even more information about how the multi-criteria group decision making process is going on, argumentation measures are calculated. They are exposed in subSection 3.2.

3.1.5. Calculating final ranking results

Final ranking results are calculated when one of the following two conditions are met:

- The *GSV* consensus degree is over the defined threshold, *CTH*.
- The maximum number of rounds, *MR* has been reached.

In order to calculate the alternative ranking, the guided dominance degree (GDD) operator [3] is applied over the collective preference matrix, *C*. This operator quantifies the importance that alternative x_i has over all other alternatives. For this purpose, it makes a row-wise aggregation of the values in the matrix. Row *i* indicates the importance that x_i has over the rest of the alternatives. For obtaining the GDD value associated to x_i, the following expression is applied:

$$GDD_i = \phi_2(c_{i1}, c_{i2}, \ldots, c_{i(i-1)}, c_{i(i+1)}, \ldots, c_{im}) \tag{15}$$

The final ranking of alternatives is obtained by sorting the alternatives using the obtained GDD value.

3.2. Argumentation measures

Understanding how experts came out with a certain resolution and how the debate has been evolving in order to reach the final conclusions is critical if the alternatives ranking wants to be trusted. In this section, a novel set of measures that clarify what has been happening in the debate process are presented. Summary of the presented notation can be found on Table 1. First of all, we focus on measuring what has happened in a single round. For this purpose, the following novel measures are proposed:

- **Determining the key arguments of the round**: For round *j*, it is possible to establish which are the arguments that have had more impact in the debate round by ranking them using the number of times that the experts have referenced them. Knowing which the most voted arguments are is useful since it allows us to determine the reasons that most of the experts have for providing their preferences. In order to calculate how many times one argument, d_k, has appeared in the *LA* sets in the round *j*, the following expression can be used:

  $$rd_k^j = count(d_k \in \{LA_{ij}\}) \quad i \in [1, n] \tag{16}$$

  where *count* calculates how many times the condition inside is fulfilled. By sorting the arguments based on their rd_k^j values, an argument ranking, RD^j, is obtained. This process makes it possible to identify which are the most popular arguments on a specific round. It also allows us to understand why experts have provided a specific set of preferences.
- **Establishing groups of experts based on the arguments that they rely on:** It is possible to know which experts have the same way of thinking by analyzing the differences between the *LA* sets that they have provided. This measure allows us to determine which experts think similarly. As a result, it is possible to have a clear overview of the different postures on the debate. Let e_i and e_k be two experts that in the round *j* have relied on arguments described in the sets LA^{ij} and LA^{kj} respectively. A similarity measure between the two experts can be defined as follows:

  $$DLA_i^k = |\bigcap(LA^{ij}, LA^{kj})| \tag{17}$$

  where $|\bigcap(A, B)|$function calculates the cardinal of the intersection of sets *A* and *B*. Once that the similarity between all the experts have been calculated, it is possible to cluster them in several groups based on the *DLA* obtained values. By using the similarity values as distances, any clustering algorithm [12] can be applied in order to generate the groups.
- **Establishing groups of experts based on the preferences that they have provided**: It should be noticed that, using the sets of preferences relation matrices and the expression (8), it is possible to measure the similarity between two different experts based on their preferences. The same grouping process as carried out in the previous point can be performed in order to generate the groups. Argumentation measures are especially useful when determining similar experts in a group decision making process. Since consensus measures classify them by only relying on the preferences provided, it is possible that experts who think differently are classified together. This is due to the fact that they can vote the same but for different reasons. For instance, imagine that a set of experts are deciding where to invest a certain amount of money. One of the companies is considered as a good choice due to the fact that the technology that they develop is highly innovative and also because they got benefits in the last month. Expert e_1 supports it because of the first argument and e_2 because of the second. If experts similarity is calculated using consensus measures, then the similarity between e_1 and e_2 is high. On the contrary, if the similarity is calculated using the argumentation measures, then e_1 and e_2 are not similar since they vote using different arguments. In other words, they care about different aspects and think differently. As a result, argumentation, since it classifies experts based on the arguments, provides more accurate results when identifying experts that think similarly. It relies on what they think, not on the preferences that they provide to the system.

- **Determining the most influential expert of the round**: The expert who has provided the argument or arguments that are in the lead can be considered as the most influential expert of the round. This is an interesting measure since it allows us to know which is the expert that provides the arguments that influence the biggest quantity of experts. Therefore, it is possible to classify the experts according to their influence. If several experts draw in the top of the ranking, then it is considered that there is a set of influential experts that lead the decision process. In Fig. 4, an example of a decision round is shown. The fourth expert is the one providing the argument that has obtained more support. Therefore, he/she is the most influential expert of the round and his/her argument the key argument of the round.

The presented measures allow us to understand what has happened in a single round. Nevertheless, it is also interesting to measure how a new discussion round has made experts to modify the results provided in the previous one. For this purpose, the following measures are proposed:

- **Measuring experts' changes of opinion**: A debate round, j, can be considered as critical, or influential, if it makes the experts to change the opinions that they have provided in the round $j-1$. The strength value of the change of opinion that an expert e_i has made from one round to the next can be calculated using the following expression:

$$SimJ_{ij}^{j-1} = \phi_1(PC^{ij} - PC^{i(j-1)}) \tag{18}$$

By analyzing these values, it is possible to determine which experts change their minds the most. Also, it indicates if they totally change their view or if they only provide small changes in their preferences. Experts that totally change their minds from time to time are usually the ones that do not have a clear opinion on the dealt topic.

In order to know which are the arguments that have promoted that change of opinion, the following expression can be applied:

$$SimLAJ_{ij}^{j-1} = LA^{ij} - LA^{i(j-1)} \tag{19}$$

where $SimLAJ_{ij}^{j-1}$ ends up containing all the arguments that e_i did not include in the last round. That is, the arguments that support their change of opinion.

The $SimJ_{ij}^{j-1}$ values can be aggregated in order to calculate a unique value determining the overall change of opinions of the experts from round $j-1$ to j. In order to calculate this value, the following expression can be applied:

$$SimJ_j^{j-1} = \phi_1(SimJ_{ij}^{j-1}), \quad i \in [1, n] \tag{20}$$

- **Determining the key argument changing**: By comparing sets RD^j and RD^{j-1}, it is possible to measure how the ranking of arguments have evolved from one debate round to the next. For each d_k in the ranking, it is possible to know if the popularity of the argument has increased or decreased. As a result, this measure allows us to know if the arguments have lost strength and which are the ones whose importance arise. By keeping track of this information, it is possible to get an overview about how the discussion is evolving. For this purpose, the following expression can be applied:

$$imp_{d_k}^j = pos(d_k, RD^j) - pos(d_k, RD^{j-1}) \tag{21}$$

It should be noticed that negative values indicate that the argument has lost popularity while a positive value means that the popularity of that argument has increased. A 0 value indicates that the argument popularity is maintained. In Fig. 5, an example about arguments ranking in two rounds is shown. The values indicating the key argument changing are shown on the right side of the Figure.

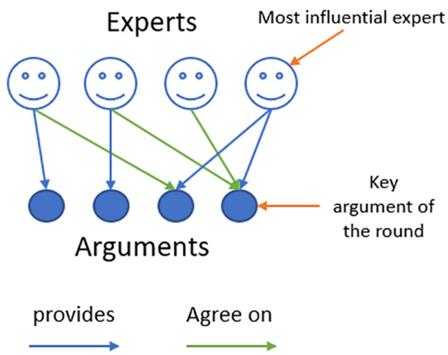

Fig. 4. Argument providing step example for one round.

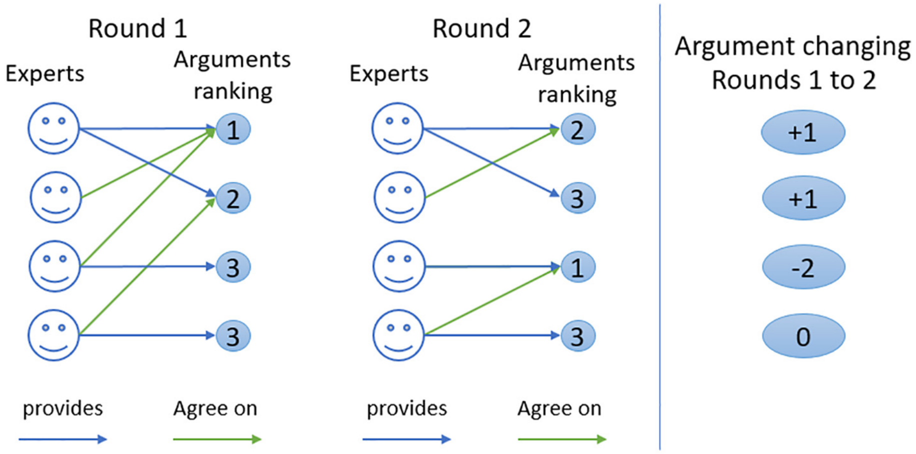

Fig. 5. Argument providing step example for two rounds.

Finally, measures whose main purpose is to provide an overview of the whole multi-criteria group decision making process are presented. They make it possible to detect the main reasons and elements that have influenced the final decision results. These measures are shown below:

- **Key arguments ranking**: It is possible to calculate a ranking by sorting the arguments according to the importance that they have had along the decision process. This way, it is possible to know which are the arguments that have had more influence along the process on the experts. This can be done by ranking the arguments using the mean of the positions that they have had in the argument rankings of all the debate rounds. Formally, the following expression can be applied in order to calculate the position mean of the argument d_k:

$$grd_k = \phi_1(pos(d_k, RD_j)), \quad j \in [1, r] \tag{22}$$

where r is the total number of rounds that have been carried out. $pos(d_k, RD_j)$ is a function that returns the position that d_k has in the ranking RD_j. Using the grd_k values, the arguments can be sorted according to its popularity in the whole decision making process. It is important to notice that there is a semantic difference between the arguments that have supported the final decision with the most popular arguments along the multi-criteria group decision making process. The second case refers to the ranking generated by rd_k^r values. That is, the arguments of the ranking generated in the last round.
- **Rounds with more preferences modification**: It is interesting to determine which are the rounds where experts have applied more modifications to the provided preferences. Identifying these rounds is interesting since it allows us to know where new ideas have arisen or where a subset of experts has reached agreements. As a result, it is possible to determine which rounds have been the most decisive during the decision process. For this purpose, it is possible to rank the multi-criteria group decision making rounds by their associated $SimJ_j^{j-1}$ values. The round that has the first place is the ranking is the one where experts have had more critical changes of opinion. This way, this measure can pinpoint critical rounds where the arguments provided have changed the way that the experts were approaching the dealt problem.
- **Experts' similarity along the process**: Using the $SimJ_{ij}^{j-1}$ values and aggregating them by the round j, it is possible to calculate the overall preferences changing degree that a specific expert e_i has had along the whole multi-criteria group decision making process. In order to obtain this value, the following expression can be used:

$$SimJ_i = \phi_1(SimJ_{ij}^{j-1}) \quad j \in [2, r] \tag{23}$$

where r is the total number of rounds.
- **Most influential expert of the process**: The expert whose argument is in the first position of the global key argument ranking can be considered as the most influential expert in the multi-criteria group decision making process. It is important to notice that his/her ideas have been the most valued. Consequently, this is the expert that has led the discussion. This result can pinpoint an expert or a set of them.

4. Illustrative example

In order to enhance the comprehension of the method, an application example is shown in this section. In order to solve it, the procedure described in subSection 3.1 will be used. Imagine that a set of four experts, $E = \{e_1, e_2, e_3, e_4\}$ has to rank a

Table 2
Descriptions of the alternatives and criteria.

x_l/c_k	Description
x_1	Buy new computers.
x_2	Create an exchange program.
x_3	Introduce a new optional subject.
x_4	Improve sport facilities.
c_1	Durability.
c_2	Impact.
c_3	Viability.

set of four alternatives $X = \{x_1, x_2, x_3, x_4\}$ taking into account a set of three criteria values $C = \{c_1, c_2, c_3\}$. Specifically, experts are part of a high school team and they need to decide where to invest some recent acquired money. In Table 2, a description of the alternatives and criteria values is exposed. Values *CTH* and *MR* are set to 0.75 and 5 respectively. Experts will provide their preferences by using the linguistic label set $S = \{s_1, \ldots, s_5\}$.

First of all, experts carry out a debate process. In that debate process, the arguments shown in Table 3 are pinpointed. The Table indicates which expert has provided each argument and the content of it. Once that the debate process has been carried out, experts must provide their preferences to the system. Each expert generates a preference matrix for each criteria value. In the first round, the experts provide the following preferences matrices:

$$P^{111} = \begin{pmatrix} - & s^5_4 & s^5_4 & s^5_2 \\ s^5_2 & - & s^5_2 & s^5_1 \\ s^5_2 & s^5_4 & - & s^5_1 \\ s^5_4 & s^5_5 & s^5_5 & - \end{pmatrix} \quad P^{112} = \begin{pmatrix} - & s^5_4 & s^5_5 & s^5_3 \\ s^5_1 & - & s^5_2 & s^5_2 \\ s^5_2 & s^5_1 & - & s^5_1 \\ s^5_5 & s^5_4 & s^5_5 & - \end{pmatrix}$$

$$P^{113} = \begin{pmatrix} - & s^5_5 & s^5_5 & s^5_4 \\ s^5_2 & - & s^5_1 & s^5_2 \\ s^5_2 & s^5_2 & - & s^5_1 \\ s^5_5 & s^5_5 & s^5_5 & - \end{pmatrix} \quad P^{211} = \begin{pmatrix} - & s^5_1 & s^5_2 & s^5_1 \\ s^5_5 & - & s^5_4 & s^5_5 \\ s^5_5 & s^5_4 & - & s^5_4 \\ s^5_1 & s^5_1 & s^5_2 & - \end{pmatrix}$$

$$P^{212} = \begin{pmatrix} - & s^5_2 & s^5_2 & s^5_3 \\ s^5_5 & - & s^5_4 & s^5_5 \\ s^5_5 & s^5_4 & - & s^5_5 \\ s^5_2 & s^5_3 & s^5_1 & - \end{pmatrix} \quad P^{213} = \begin{pmatrix} - & s^5_2 & s^5_2 & s^5_1 \\ s^5_5 & - & s^5_4 & s^5_5 \\ s^5_4 & s^5_4 & - & s^5_4 \\ s^5_2 & s^5_1 & s^5_1 & - \end{pmatrix}$$

$$P^{311} = \begin{pmatrix} - & s^5_5 & s^5_4 & s^5_5 \\ s^5_2 & - & s^5_3 & s^5_2 \\ s^5_1 & s^5_2 & - & s^5_1 \\ s^5_5 & s^5_5 & s^5_3 & - \end{pmatrix} \quad P^{312} = \begin{pmatrix} - & s^5_5 & s^5_4 & s^5_5 \\ s^5_3 & - & s^5_2 & s^5_3 \\ s^5_1 & s^5_2 & - & s^5_1 \\ s^5_4 & s^5_5 & s^5_5 & - \end{pmatrix}$$

$$P^{313} = \begin{pmatrix} - & s^5_4 & s^5_5 & s^5_5 \\ s^5_3 & - & s^5_3 & s^5_3 \\ s^5_1 & s^5_2 & - & s^5_1 \\ s^5_5 & s^5_5 & s^5_4 & - \end{pmatrix} \quad P^{411} = \begin{pmatrix} - & s^5_2 & s^5_1 & s^5_2 \\ s^5_5 & - & s^5_5 & s^5_4 \\ s^5_5 & s^5_5 & - & s^5_4 \\ s^5_2 & s^5_2 & s^5_1 & - \end{pmatrix}$$

$$P^{412} = \begin{pmatrix} - & s^5_2 & s^5_1 & s^5_2 \\ s^5_5 & - & s^5_4 & s^5_5 \\ s^5_5 & s^5_5 & - & s^5_4 \\ s^5_1 & s^5_2 & s^5_2 & - \end{pmatrix} \quad P^{413} = \begin{pmatrix} - & s^5_2 & s^5_1 & s^5_2 \\ s^5_5 & - & s^5_5 & s^5_4 \\ s^5_5 & s^5_4 & - & s^5_5 \\ s^5_3 & s^5_2 & s^5_1 & - \end{pmatrix}$$

Also, experts provide the arguments that support their preferences. This information is shown below:

$LA^{11} = \{d_1, d_4\}$
$LA^{21} = \{d_2, d_3\}$
$LA^{31} = \{d_1\}$
$LA^{41} = \{d_2, d_3\}$

Table 3
Arguments of round 1.

e_i	d_j	Description
e_1	d_1	Buying supplies is more necessary than including new educational programs because the educational program of the high school is complete enough.
e_2	d_2	In terms of durability, it is better to create new educational plans. Also, supplies are not needed right now.
e_2	d_3	Introducing an exchange program is more useful for the students than a new subject.
e_1	d_4	Sports facilities are the most needed option, it is the main lack of our high school.

In order to generate the global preference matrices for the experts, the matrices are aggregated using the weighting vectors provided by the experts. Each expert can select the importance that he/she wants to give to each criteria value. Weighting vectors chosen by the experts are shown below:

$$W_c^1 = \{0.5, 0.25, 0.25\}$$
$$W_c^2 = \{0.33, 0.34, 0.33\}$$
$$W_c^3 = \{0.25, 0.25, 0.5\}$$
$$W_c^4 = \{0.2, 0.6, 0.2\}$$

After aggregating the preferences matrices, the following experts' collective matrices are obtained:

$$PC^{11} = \begin{pmatrix} - & 1.67 & 2 & 1.68 \\ 1.75 & - & 1.75 & 1.5 \\ 2 & 2.75 & - & 1 \\ 4.5 & 4.75 & 5 & - \end{pmatrix} \quad PC^{21} = \begin{pmatrix} - & 1.67 & 2 & 1.68 \\ 5 & - & 4 & 5 \\ 4.67 & 4 & - & 4.34 \\ 1.67 & 1.68 & 1.33 & - \end{pmatrix}$$
$$PC^{31} = \begin{pmatrix} - & 4.5 & 4.5 & 5 \\ 2.75 & - & 2.75 & 2.75 \\ 1 & 2 & - & 1 \\ 4.75 & 4.75 & 4 & - \end{pmatrix} \quad PC^{41} = \begin{pmatrix} - & 2 & 1 & 2 \\ 5 & - & 4.4 & 4.6 \\ 5 & 4.8 & - & 4.2 \\ 1.6 & 2 & 1.6 & - \end{pmatrix}$$

The aggregation process is performed over the indexes of the labels. Therefore, the result is a numerical value matrix. By aggregating the experts' collective matrices, it is possible to obtain the global collective matrix that resumes the preferences provided by all the experts. The resulting matrix for round 1 is shown below:

$$C = \begin{pmatrix} - & 3.105 & 3 & 2.8575 \\ 3.625 & - & 3.225 & 3.4625 \\ 3.1675 & 3.3875 & - & 2.635 \\ 3.13 & 3.295 & 2.9825 & - \end{pmatrix}$$

Using the PC^{i1} matrices, consensus among experts can be measured. Results are shown on Table 4. By aggregating the obtained values, the global consensus value is calculated. The result for the first round is 0.6271. Finally, the consensus reached on each alternative is shown on Table 5.

According to the information about the arguments that the experts provided to the system, the ranking of arguments for round 1 is as follows:

$$RD^1 = \{\{d_1, d_2, d_3\}, d_4\} \tag{24}$$

where d_1, d_2 and d_3 have had the same importance in this round. Therefore, e_1 and e_2 can be considered as the most influential experts of the round. Similarity values among experts based on the arguments that they provide, DLA_i^k, are shown on Table 6. It should be noticed that two groups could be created: $\{e_1, e_3\}$ and $\{e_2, e_4\}$.

Table 4
Consensus among the experts in round 1.

	e_1	e_2	e_3
e_2	0.50031		
e_3	0.86328	0.49641	
e_4	0.4875	0.93157	0.483594

Table 5
Consensus reached on each alternative in round 1.

x_1	x_2	x_3	x_4
0.18854	0.32694	0.25812	0.24416

Table 6
Similarity among the experts based on the arguments, that is, DLA_i^k values, in round 1.

	e_1	e_2	e_3
e_2	0		
e_3	1	0	
e_4	0	2	0

Since the consensus is not high enough, another debate round is carried out. In this round, two new arguments are provided by e_3. They are specified in Table 7.

Experts can now modify their preferences according to the new information that has arisen on the debate. e_1, e_2 and e_3 maintain their preferences values. Nevertheless, e_4 provide the following preference information:

$$P^{421} = \begin{pmatrix} - & s_4^5 & s_5^5 & s_4^5 \\ s_2^5 & - & s_2^5 & s_1^5 \\ s_1^5 & s_2^5 & - & s_2^5 \\ s_5^5 & s_4^5 & s_4^5 & - \end{pmatrix} \quad P^{422} = \begin{pmatrix} - & s_5^5 & s_5^5 & s_4^5 \\ s_2^5 & - & s_1^5 & s_1^5 \\ s_2^5 & s_2^5 & - & s_1^5 \\ s_5^5 & s_4^5 & s_5^5 & - \end{pmatrix}$$

$$P^{423} = \begin{pmatrix} - & s_4^5 & s_4^5 & s_4^5 \\ s_1^5 & - & s_2^5 & s_2^5 \\ s_1^5 & s_2^5 & - & s_2^5 \\ s_5^5 & s_4^5 & s_4^5 & - \end{pmatrix}$$

After aggregating the information, the PC^{42} obtained matrix is shown below:

$$PC^{42} = \begin{pmatrix} - & 4.6 & 4.8 & 4 \\ 1.8 & - & 1.4 & 1.2 \\ 1.6 & 2 & - & 1.4 \\ 5 & 4 & 4.6 & - \end{pmatrix} \tag{25}$$

The collective matrix is, therefore, recalculated. Results are shown below:

$$C = \begin{pmatrix} - & 3.755 & 3.95 & 3.3575 \\ 2.825 & - & 2.475 & 2.6125 \\ 2.3175 & 2.6875 & - & 1.935 \\ 3.98 & 3.795 & 3.7325 & - \end{pmatrix}$$

For round 2, the arguments that each expert uses to base their decisions are the following:

$$\begin{aligned} LA^{12} &= \{d_1, d_4, d_5\} \\ LA^{22} &= \{d_2, d_3\} \\ LA^{32} &= \{d_1, d_5, d_6\} \\ LA^{42} &= \{d_4, d_5, d_6\} \end{aligned}$$

Therefore, the new ranking of arguments is as follows:

$$RD^2 = \{d_5, \{d_1, d_4, d_6\}, \{d_2, d_3\}\}$$

Since d_5 has been provided by e_3, he/she is the most influential expert of the round. After applying consensus measures over the experts and preferences on the alternatives, results are shown in Tables 8 and 9 respectively. Since experts e_1, e_2 and e_3 have the same preference values, the same values are obtained when comparing them. Only e_4 related consensus values are modified in round 2. The global consensus value reached on this round is 0.6844. It should be noticed that, since the consensus degree is higher in this round, experts are bringing their opinions closer.

Table 7
Arguments of round 2.

e_i	d_j	Description
e_3	d_5	A new exchange program proposal cannot be submitted until 2 years have passed. That is a long time.
e_3	d_6	It is better to choose an alternative whose effects can be seen soon. Another money fund can be used for far away plans.

Table 8
Consensus among the experts in round 2.

	e_1	e_2	e_3
e_2	0.5		
e_3	0.86328	0.4964	
e_4	0.90937	0.45969	0.87734

Table 9
Consensus reached on each alternative in round 2.

x_1	x_2	x_3	x_4
0.6739583	0.5364583	0.6002083	0.7141667

Table 10
Similarity among the experts based on the arguments, that is, DLA_i^k values, in round 2.

	e_1	e_2	e_3
e_2	0		
e_3	2	0	
e_4	2	0	2

Finally, similarity among the experts based on the arguments that they provide are calculated and shown on Table 10. Two groups of experts, $\{e_1, e_3, e_4\}$ and $\{e_2\}$ can be differentiated in this round.

In order to have an overview of the changes that have happened from the first round to the second, the following measures are applied:

- **Experts' change of opinion**: Experts e_1, e_2 and e_3 have provided the same preference matrices to the system. Therefore, $SimJ_{i2}^1 = 0$ for all of them. On the other hand, e_4 has modified his/her preferences. By applying expression (18), it is possible to measure the strength of the change of opinion. In this case, the result is 0.73 which is a quite high value. This entails a quite harsh change of mind for e_4 since the change of mind value is located in the interval $[0, 1]$. In order to determine, which information have promoted this change of mind, the set $SimLAJ_{42}^1$ can be obtained by applying expression (19). Results are shown below:

  $$SimLAJ_{42}^1 = \{d_4, d_5, d_6\}$$

  In this case, it can be stated that the change of mind of the expert is due to the new arguments that he/she has embraced in this round. Although the rest of the experts have not changed their opinions, they can add new arguments to their sets. In this case, this means that new arguments that have helped them to reinforce their position have appeared. Their $SimLAJ_{i2}^1$ sets are shown below:

  $$SimLAJ_{12}^1 = \{d_5\}$$
  $$SimLAJ_{22}^1 = \{\}$$
  $$SimLAJ_{32}^1 = \{d_5, d_6\}$$

  It should be noticed that e_2 has not changed his/her opinion and he/she did not find any new argument. This situation refers to cases where the expert did not find any new argument that he/she agrees with and he/she maintains his/her previous position.
- **Key arguments changing**: By calculating the $imp_{d_k}^j$ values, it is possible to analyze how the importance of the arguments have changed from the first round to the second. In Table 11, the $imp_{d_k}^2$ values are shown. From the results, it can be stated that the arguments from the first round have lost popularity. Since d_5 and d_6 were not present in the first round, their

$imp^{j}_{d_k}$ value cannot be calculated until the third round is reached.

Since the consensus is still low, another decision round begins. Therefore, experts initiate another debate round. Once that the debate is over, experts determine that no new arguments were generated. e_1, e_3 and e_4 decided to maintain their arguments and preferences. But, e_2 modifies his/her preferences. His/Her new preferences are shown below:

$$P^{231} = \begin{pmatrix} - & s_1^5 & s_2^5 & s_1^5 \\ s_4^5 & - & s_3^5 & s_2^5 \\ s_5^5 & s_4^5 & - & s_4^5 \\ s_4^5 & s_3^5 & s_2^5 & - \end{pmatrix} \quad P^{232} = \begin{pmatrix} - & s_2^5 & s_2^5 & s_3^5 \\ s_4^5 & - & s_2^5 & s_2^5 \\ s_5^5 & s_4^5 & - & s_5^5 \\ s_5^5 & s_3^5 & s_4^5 & - \end{pmatrix}$$

$$P^{233} = \begin{pmatrix} - & s_2^5 & s_2^5 & s_1^5 \\ s_3^5 & - & s_2^5 & s_1^5 \\ s_4^5 & s_4^5 & - & s_4^5 \\ s_3^5 & s_1^5 & s_4^5 & - \end{pmatrix}$$

The new provided argument set for e_2 is as:

$$LA^{23} = \{d_4, d_5, d_6\}$$

After aggregating the information, the preference matrix PC^{23} is as:

$$PC^{23} = \begin{pmatrix} - & 1.67 & 2 & 1.68 \\ 3.67 & - & 2.33 & 1.67 \\ 4.67 & 4 & - & 4.34 \\ 4.01 & 2.34 & 3.34 & - \end{pmatrix}$$

The collective preference matrix that contains the overall opinion of all the experts in round 3 is shown below:

$$C = \begin{pmatrix} - & 3.755 & 3.95 & 3.3575 \\ 2.4925 & - & 2.0575 & 1.78 \\ 2.3175 & 2.6875 & - & 1.935 \\ 4.565 & 3.96 & 4.235 & - \end{pmatrix}$$

For round 3, consensus among the experts and on the alternatives are shown on Tables 12 and 13 respectively. It can be noticed that since e_2 has brought opinions closer with the rest of the experts, all the consensus values have improved. The global consensus value for this round is 0.76528. Since it is higher than 0.75, the ranking results can be calculated and this is the last round of the group decision making process.

The ranking of the arguments in this round is shown below:

$$RD^3 = \{d_5, \{d_4, d_6\}, d_1, \{d_2, d_3\}\}$$

Therefore, e_3 continues to be the most influential expert of the round. For round 3, similarity among the experts according to the results of the arguments are shown in Table 14. As it can be seen, the similarity among the experts has increased from the last round. This is another symptom that they are reaching an agreement.

Next, it is possible to measure the changes that have happened from the second round to the third:

- **Experts' change of opinion**: Since experts e_1, e_3 and e_4 have provided the same preferences matrices, $SimJ^2_{i3} = 0$ for them. Nevertheless, e_2 has provided new preferences. By applying expression (18), the change strength can be calculated. The resulting value is 0.23625. As it can be seen, the change of opinion is smoother than the one performed by e_4 in the previous round. The $SimLAJ^2_{23}$ set that determines the new arguments that e_2 has assumed is shown below:

Table 11
Changes in the arguments importance ranking from round 1 to round 2.

Argument	Ranking change
d_1	−1
d_2	−2
d_3	−2
d_4	0
d_5	–
d_6	–

Table 12
Consensus among the experts in round 3.

	e_1	e_2	e_3
e_2	0.6775		
e_3	0.86328	0.62671	
e_4	0.90937	0.63687	0.87734

Table 13
Consensus reached on each alternative in round 3.

x_1	x_2	x_3	x_4
0.55541	0.5048	0.61229	0.44895

Table 14
Similarity among the experts based on the arguments, that is, DLA_i^k values, in round 3.

	e_1	e_2	e_3
e_2	2		
e_3	2	2	
e_4	2	3	2

$$SimLAJ_{23}^{2} = \{d_4, d_5, d_6\}$$

Therefore, these are the arguments that have promoted the change of opinion that e_2 has had in this round.

- **Key arguments changing**: As it has be done in the previous step, the $imp_{d_k}^{3}$ values are calculated and shown on Table 15. As it can be seen, there are no strong changes. These entails that the importance of the arguments have not significantly changed from round 2 to round 3.

In order to calculate the alternatives' final ranking, the GDD operator shown in expression (15) can be applied. Results are shown below:

$$GDD = \{0.3767953, 0.8016158, 0.7468582, 0.2244165\}$$

Therefore, the alternatives ranking is as:

$$Rank = \{x_4, x_1, x_3, x_2\}$$

Finally, argument measures referring to the whole multi-criteria group decision making process can be calculated. First of all, arguments can be ranked according to their position in the argument ranking along the process. For this purpose, expression (22) is used. Table 16 shows the mean of the positions that each argument has had during the process. The overall ranking of arguments is shown below:

$$GRD = \{d_5, \{d_1, d_4, d_6\}, \{d_2, d_3\}\}$$

As it can be seen, the most critical argument in the multi-criteria group decision making process has been d_5 while d_1 and d_2 are the arguments that have had less impact in the process. In fact, experts stopped using them in the third round of the process. Since d_5 belongs to e_3, he/she can be considered as the most influential expert of the multi-criteria group decision making process.

The round with more preference modification has been round 2. Only e_2 and e_4 have modified preferences in round 3 and 2 respectively. e_4 was the expert whose preferences modifications were harsher.

Table 15
Changes in the arguments importance ranking from round 2 to round 3.

Argument	Ranking change
d_1	+1
d_2	−1
d_3	−1
d_4	0
d_5	0
d_6	0

Table 16
Position of the arguments taking into account the whole multi-criteria group decision making process.

Argument	Position
d_1	2
d_2	2.66
d_3	2.66
d_4	2
d_5	1
d_6	2

Finally, it is possible to determine the level of preferences changing that the experts have maintained along the process. By applying expression (23), the following results are obtained:

$$Sim\,J_1 = \frac{0+0}{2} = 0$$
$$Sim\,J_2 = \frac{0+0.23625}{2} = 0.118125$$
$$Sim\,J_3 = \frac{0+0}{2} = 0$$
$$Sim\,J_4 = \frac{0.73+0}{2} = 0.365$$

e_4 is the expert who has changed most of opinion.

5. Discussion

In this paper, a novel multi-criteria group decision making method that includes measures for describing what is happening in the debate process is presented. A new concept, the argument, is included. It provides the following advantages to the presented multi-criteria group decision making method:

- **Understanding what is happening in the debate**: Traditional multi-criteria group decision making methods focus on analysing only the preferences provided by the experts. Consequently, all the information that could be extracted from the debate process is lost. This is quite an important lack since it is in the debate where the experts carry out the reasoning process and expose their arguments. The presented multi-criteria group decision making method avoids this loss of information by the design of a novel structure, the argument, that includes the facts and reasoning logic that have made experts to come up with the provided preferences. It should be noticed that this is quite important information that helps us to understand how the experts reason and why a specific result set is achieved.
- **Verifying the results**: Using arguments to understand how a specific result is achieved is important since it allows us to verify that the alternative ranking reached is right and concur with the arguments. They make it possible to use the arguments and the ranking results in order to verify that the obtained results are logical and no error has been committed by the system.
- **Identifying critical rounds and arguments**: The presented method identifies which are the arguments that have lead to a certain decision and which are the rounds where the most critical parts of the debate have been carried out. As a result, it is possible to have an overview about what has happened on the experts' discussion. Most of the multi-criteria group decision making methods focus on observing and analysing the preferences provided by the experts. Nevertheless, it is very important to also understand what has happened on the debate. As a result of the inclusion of the arguments' conceptualization, our method makes this possible.

As future work, it would be interesting to introduce text analysing methods to automatically extract the inner meaning of the arguments. In their current state, the arguments are sentences that are easy to understand for the experts but not for the computational system. When each round or the multi-criteria group decision making process is over, the arguments sentences are shown to the experts and they interpret the results. It would be interesting, as future work, to allow the computational system to analyze and extract conclusions from the generated arguments. This can be done, for instance, by applying sentiment analysis [9,25,28] and text mining procedures [15,20,29].

Argumentation is a quite important part in all group decision making processes. Although this is stated since the beginning of the field [5,16], there are few group decision making methods in the recent literature that includes that in their designs. Mainly, argumentation has been employed to create decision making methods that make decisions on their own taking into account some information. For instance, in [40], an argumentation framework is employed for explaining why the system has reached a specific decision. They use a linguistic transcription in order to make results easy to understand for humans. In [14], argumentation is used to aid a set of experts in making a decision. This method focuses on modelling the arguments that have appeared in the debate. It does not model a complete group decision making process with several decision rounds where experts provide their preferences to the system. Also, it does not implement any consensus measures to actually measure how the group decision making process is going on. In [19], a collaborative medical-centered argumentation process similar to the one exposed in [14] is presented. Medical experts expose their ideas in a whiteboard and, by using argumentation procedures, a decision is made. In [6], a group decision making method that define arguments in a similar way as in our method is presented. Nevertheless, in [6], authors focus on using the arguments to make the decision while our method goal is to analyze what is happening round by round in order to get a clear overview about how the multi-criteria group decision making process has evolved from the first rounds to the last ones. In other words, their method focuses on obtaining results while our presented method tries also to elucidate how the decision results have been obtained. For this purpose, our method includes a set of novel argumentation and consensus measures. Also, measures for determining when the experts change their opinion and why are included. In conclusion, there is no method in the literature that have included the argumentation conceptualization in a multi-criteria group decision making environment. Also, no method in the literature focuses on defining argumentation in a decision scheme that includes consensus and several rounds to help the experts come to an agreement. The presented paper covers all these options which are of great help to understand, not only the results of the process, but also how it has developed and why results have been obtained.

Since the main goal of the presented method is to provide the experts with a thorough analysis of the whole group decision making process, the method includes consensus measures that cover all the possible elements involved on the decision. That is, alternatives, criteria and experts. Most of the recent consensus methods, such as, [3,31,34,35] only focus on measuring some of the decision aspects since their main goal is to calculate a global consensus value for the round. Nevertheless, our method provides an overall view of how the group decision making process has been evolving. Furthermore, the main contribution of our paper, the argumentation measures, improves these results as it provides information related to the motivation of the experts. While actual consensus measures just focus on establishing similarities among the experts' preferences, argumentation measures provide an overview of the motivation that experts have when providing a certain set of preferences. All this information is vital for the decision making analyzers since it allows them to understand what is going on in the debate and why experts have reacted and voted in a certain way.

The presented method can be easily applied to any real world group decision making process. All the presented measures and computations are conformed by direct and fast $O(1)$ operations that barely consume any computer time. The method is designed this way to make it competitive in real world scenarios. As a result, it can provide real time responses to the experts. The total computational effort employed is totally related with the amount of information that the system has to deal with. For instance, if there are ne experts, nc criteria values and nx alternatives, the system ends up having to deal with $ne \cdot nc \cdot (nx \cdot nx - nx)$values. This also entails that each expert has to provide $nc \cdot nx \cdot nx$ values. In cases where there is not a high number of alternatives and criteria available, computations are not problematic. For instance, imagine a group decision making process with 7 experts, 5 alternatives and 3 criteria values. In this case, each expert needs to provide $3 \cdot (5 \cdot 5 - 5) = 60$ values. The system will operate with $60 \cdot 7 = 420$ values. Therefore, the amount of information is fair and easy to manage for the computational system. Now, imagine a real world case with 10 experts, 10 alternatives and 5 criteria values. In this example, the number of values that the system has to deal with is $10 \cdot 5 \cdot (10 \cdot 10 - 10) = 4500$ which is, affordable, but higher. Consequently, it can be stated that the method works fine for real world multi-criteria group decision making processes that deal with a low-moderate amount of information. Preference relation matrices are a reliable way of retrieving information from the experts since they allow them to make a pairwise comparison among each pair of alternatives. Nevertheless, they require more experts' effort since one value needs to be provided for each pair of alternatives and criteria. One way of drastically reducing the high amount of information that experts need to provide is asking them to provide only one preference matrix. In this approach, experts directly provide P^i matrices to the system. It should be noticed that the novel presented method and analysis can work fine since they do not depend on the criteria provided information. By providing one preference matrix instead of nc, the amount of information that the system has to deal with is drastically reduced. In the examples provided, the amount of information stored on the system is reduced to 140 and 900 respectively, instead of 420 and 4500. The amount of data that each expert has to provide is also reduced to 20 and 90 values. The main drawback of this approach is that the quality of the provided information is reduced. This occurs because experts provide their preferences by taking into account all the criteria values at once instead of evaluating the alternatives for one criterion at a time. Nevertheless, the amount of information that experts have to provide and the quantity of the values that the presented group decision making method can deal with exponentially grows.

6. Conclusions

Understanding how the experts have reached a specific conclusion is critical in order to verify the multi-criteria group decision making process results. In this paper, a novel multi-criteria group decision making method that provides novel measures for understanding how the process has been performed is presented. Moreover, a novel conceptualization is introduced: the arguments. Apart from providing preferences, experts are asked to indicate the argumentation that support their debate positions. Using these arguments, it is possible to know which are the main reasons that support the obtained multi-criteria group decision making results.

The presented argumentation measures focus on measuring the multi-criteria group decision making process in three perspectives. First of all, the situation occurred in each round is analysed. Afterwards, how the situation has changed from one round to the next is studied. Finally, the whole group decision making process is described. Arguments play an important role in this analysis due to the fact that they gave us the key to understand what are the reasons that makes the expert provide their preferences. As a result, it is possible to understand why a specific alternatives ranking has been obtained. Getting an overview about how the multi-criteria group decision making problem has been solved is important since it allows us to verify the results and understand the key points that have influenced the final decision results.

CRediT authorship contribution statement

J.A. Morente-Molinera: Conceptualization, Methodology, Formal analysis, Software, Validation, Writing - original draft, Investigation, Data curation, Visualization. **G. Kou:** Visualization, Funding acquisition, Supervision. **K. Samuylov:** Funding acquisition, Writing - review & editing, Supervision. **F.J. Cabrerizo:** Validation, Methodology, Visualization. **E. Herrera-Viedma:** Writing - review & editing, Supervision, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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