Received: 9 October 2020 | Revised: 22 March 2021 | Accepted: 4 May 2021

DOI: 10.1002/int.22467

RESEARCH ARTICLE

WILEY

A novel intelligent system for securing cash levels using Markov random fields

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Abstract

The maintenance of cash levels under certain security thresholds is key for the health of the banking sector. In this paper, the monitoring process of branch network cash levels is performed using a single intelligent system which should provide an alert when there are cash shortages at any point of the network. Such an integral solution would provide a unified insight that guarantees that branches with similar cash features are secured as a whole. That is to say, a triggered alarm at a specific branch would indicate that attention must also be paid to similar (in-cash-feature) branches. The system also incorporates a (complementary) specific treatment for individual branches. The Early Warning System for securing cash levels presented in this paper (cash level EWS) is deliberately free of local demographic specifications, thereby overcoming the current lack of worldwide definitions for local demographics. This aspect would be particularly valuable for banking institutions with branch networks all over the world. A further benefit is the cost reductions that are a result of replacing several approaches with a single global one. Instead of local demographic parameters, a solid theoretical model based on Markov random fields (MRFs) has been developed. The use of MRFs means a reduction in the amount of information required. This would mean a higher processing speed as well as a significant reduction in the amount of storage capacity required. To the best of the author's knowledge, this is the first time that MRFs have been applied to cash monitoring.

KEYWORDS
cash level EWS, intelligent system, local demographics free, markov random fields, securing cash levels

1 | INTRODUCTION

Alert systems and warning tools are essential for preventing disasters of any type. In the financial context, Early Warning Systems (EWSs) detect vulnerabilities and malperformance thus preventing institutions from potential bankruptcies derived from liquidity risks. Although there are many definitions, liquidity risks are associated with the the ability of banking institutions to obtain cash (with immediacy) from other products when necessary, without any loss of capital.[1]

Liquidity risks may arise due to several factors, which have intricate and interdependent relationships. Despite the complexity of the subject, most authors agree that accurate cash management of liquid resources at both bank and branch level is determinant in reducing risk exposure as well as guaranteeing financial solvency.[2-4] In particular, some authors[5,6] think that precise cash management at a bank level is largely supported by careful cash management at a branch level, see Reference [7], for further details.

This paper provides a tool for enhancing the cash management capacity at a branch level. To achieve this, the maintenance of cash levels under certain security thresholds is considered as a surveillance process of the whole branch network. A cash level EWS is presented here as an individual tool for (network) cash monitoring which should provide alerts when there are cash shortages at any point of the network. Among other benefits, this type of integral solution would provide a unified insight that guarantees that branches with similar cash features are subjected to similar treatments. This would mean that, for instance, a triggered alarm at a specific branch would indicate that attention must also be paid to similar (in-cash-feature) branches. Besides this treatment of similar (in-cash-feature) branches as a whole, the system also incorporates a (complementary) specific treatment for individual branches.

The cash level EWS presented in this paper has been deliberately designed to be free of local demographic specifications. Instead, a new framework based on "internationally accepted" notions (as opposed to local ones) has been developed. In this sense, this cash level EWS is sometimes referred to as *universal*. In this regard, it should be noted that the main purpose of bypassing local demographic specifications is to avoid the confusion surrounding local demographics derived from the lack of common definitions for local parameters (In fact, the differences concerning the local demographic parameters that exist, complicate efforts in both academic and corporate fields), see Reference [8]. In short, one of the major advantages of a cash level EWS that is free of local specifications is that it would boost the use (and the export) of global technologies for banking institutions that expand their branch networks all over the world. It would also offer cost reductions when replacing several proposals with a single global one.

Our proposal is specifically based on the notion of Markov random fields (MRFs) which was first mentioned in 1980,[9] as a particular kind of graphical model, see Reference [10]. The cash level EWS has been built around two alarm functionalities. First of all, a spatial alarm devoted to covering the whole branch network, for which MRFs have been used as the theoretical groundwork. By means of the corresponding Gibbs distribution associated to any given MRF, the entire network is monitored in such a way that an alarm is generated once a given threshold has been exceeded. This type of joint probability distribution can be expressed as a function of only a few nodes of the network (the cliques) thereby reducing the amount of information required. This would mean a higher processing speed as well as a significant reduction in the amount of storage capacity required. The secondary alarm notification provides temporal coverage thereby recording information as to whether the frequency of cash shortages is acceptable or not. To the best of our knowledge this is the first time in literature that MRFs have been applied in designing an intelligent system for securing cash levels (cash level EWS).

The paper is organized as follows. In Section 2, a review of related previous works is provided. Section 3 contains the preliminaries on MRFs. Section 4 is devoted to developing the EWS structural model through its primary and secondary alarm notifications. In Section 5, the cash level EWS for the whole network is derived as well as complementary alert services for individual branches. Moreover, an example of the monitoring of cash holdings is provided to illustrate how our EWS works. Besides, model assessment is performed by using some experiments on the risk behavior of unsafe cash levels. Finally, Section 6 presents the concluding remarks.

2 | RELATED REVIEWS

The literature offers an exhaustive range of mathematical trends for addressing the maintenance of cash levels and risk control in the financial context. Some approaches integrate the information monitoring in the different possible scenarios by means of stochastic techniques: see Reference [11], where an EWS is discussed using a statistical tool (the risk measurement) which serves as a rational index for the definition of warning thresholds. Other perspectives rely on Fuzzy Neural Networks due to their ability to derive the relationships between the selected inputs and outputs. For instance, see Reference [12], where the authors propose the use of a neural fuzzy system (GenSoFNN) for predicting banking failure. A comparison between neural network methods and standard statistical methods is given in Reference [13], where the authors examine the effect of such models to detect financially troubled life insurers. The paper [14] promote the use of stochastic optimization to improve cash management at a branch level, focusing on two items: the cash forecast in branch ATMs and in credit card transactions. And more recently, the cash forecast for branches have been approached by coupling machine learning and robust optimization in the paper [15]. Other approaches design EWSs for financial risk detection, as in Reference [16], where an EWS model based on data mining (and developed by means of χ^2 Automatic Interaction Detector algorithm) is presented. Bayesian methods are also widely used. In this regard, in the paper [17], the authors show how Bayesian networks may be used for the assessment of credit concentration risk by modeling the destructive power of credit concentrations using the identification of Bayesian graphical models correlations and correlations among borrowers, which are very difficult for the banking industry to calibrate. Another example is the paper [18] which represents a novel approach on banking crisis EWSs by applying dynamic Bayesian networks to systemic banking crisis since literature on these has been mainly based on the signal extraction and the logit model methods

(Reviews on EWSs for predicting banking crisis are to be found in both [18] and [16]). A further example is the paper [19], where the author develops an algorithm for diffusion across networks of economic agents provided that the networks are weighted Bayesian graphs. The algorithm is then employed to transfer the cash flow when operating with interbank liquidity networks.

Bayesian networks differ from MRFs in the underlying domain: Bayesian networks rely on a direct graph while the underlying domain is undirected for MRFs. Our proposal on MRF is intended for bypassing some problems of the previously mentioned methods. On one hand, Bayesian-based methods need large amounts of data: actually, one of their major drawbacks is that they are data-intensive which may make them difficult to use in data-poor contexts. On the other hand, neural networks and related statistical methods rely on a given data distribution. The use of MRFs avoids these problems: since the joint distribution depends only on a small number of nodes, the amount of information needed is much smaller than for Bayesian-based methods. Furthermore, the use of MRF means that there is no fixed distribution, which is highly beneficial for all the dynamic situations that require frequent updates, as happens in financial scenarios.

3 | PRELIMINARIES AND NOTATION

A graph $G = (V, E)$ is a pair consisting of a set of vertices V (also named nodes) and a set consisting of edges E that connect the nodes in pairs. $u \sim v$ means adjacent vertices, that is, vertices u and v such that $(u, v) \in E$. An *undirected graph* is a graph with bidirectional edges. Otherwise, the graph is called directed (the edges are pointing in one direction). For a node $u \in V$, its *neighborhood* is the set of nodes such that one edge exists that connects them, $N(u) = \{v \in V \mid u \sim v\}$, in such a way that to define neighborhood of a node is equivalent to defining the set of edges. A subset of V is called a clique if it is formed either by a single node or by nodes that are fully connected to each other. In a condensed form, this is usually expressed as: $C \subseteq V$ is a clique if and only if $C \subseteq \{u, N(u)\}, \forall\, u \in C$. The graphs (or networks) whose nodes can be identified with values of a random variable, $\{X_v \mid v \in V\}$, and whose edges stand for the statistical connections between the variables are called graphical models. Other names for them are spatial stochastic processes or random fields. Depending on whether the underlying graph is directed or not, graphical models are commonly categorized into Bayesian or Markov networks, also known as MRFs. With the aim of reducing the amount of required data, we will focus on MRFs.

Consider a random variable X which takes values in a finite set V, $X = \{X_v \mid v \in V\}$ and let $P[X]$ denote the corresponding joint distribution, as follows:

$$P[X] = P[\{X_v = x_v | v \in V\}] = \{P[X_v = x_v] | v \in V\}.$$

The joint distribution of X is called a Gibbs distribution if it can be factorized as a product of clique potentials in G. That is to say, $P[X]$ may be written in terms of functions ϕ_C ($C \in \mathcal{C} \subset V$ is a clique) such as $P[X] = \frac{1}{Z}\prod_{C\in\mathcal{C}} \phi_C(X_C)$. These functions $\phi_C(X_C)$ are called Cth clique potentials since they take values only in cliques C and they are required to be positive. $Z = \sum\prod_{C\in\mathcal{C}} \phi_C(X_C)$ is a normalization constant to form a valid probability distribution. Although they can take many forms, clique potentials are usually of the form $\phi_C(X_C) = \exp(-f(C))$, where $f(C)$ is called an energy function. Thus, the probability distribution $P[X]$ can be rewritten as follows:

$$P[X] = \frac{1}{Z}\exp\left[-\sum_{C\in\mathcal{C}} f(C)\right] \tag{1}$$

Generally speaking, a graphical model $X = \{X_v \mid v \in V\}$ is said to be a MRF (with the underlying domain $G = (V, E)$) if its distribution $P[X]$ depends only on the *nearest nodes*, when the term *nearest* must be specified. The underlying idea is the same as for Markov chains: actually, a Markov chain is a random process X_n in which the full conditional distribution of each state X_n, $P[X_n = x_n \mid X_k = x_k, \forall\, k \neq n]$, depends solely on the immediately previous state X_{n-1}:

$$P[X_n = x_n | X_k = x_k, \forall\, k \neq n] = P[X_n = x_n | X_{n-1} = x_{n-1}].$$

However, the natural time-direction for Markov chains (displayed in Figure 1) disappears when moving to MRFs (as shown in Figure 2).

In fact, since MRFs move across two-dimensional surfaces, *nearest* would be defined once the notions of direction and proximity were specified. Both notions shall be detailed by using the notion of neighborhood of a node.

The notion of neighborhood of a node v, $N(v)$, allows us to broaden the Markov property from Markov chains to MRFs:

$$P[X_v = x_v | X_{V-\{v\}} = x_{V-\{v\}}] = P[X_v = x_v | X_{N(v)} = x_{N(v)}].$$

The notion of clique may be derived from that of neighborhood: a *clique*C is a set of nodes such that any pair of elements $c_i, c_j \in C$ satisfies that $c_i \in N(c_j)$ and $c_j \in N(c_i)$. Finally, the joint distribution of a MRF $X \geq 0$ is a Gibbs distribution, according to the Hammersley–Clifford theorem.

4 | THE EWS STRUCTURAL MODEL: A PROPOSAL

This section contains the theoretical framework for the EWS structural model. This will consist of two alarm functionalities, the first one aimed at covering the whole spatial network. This shall be achieved by using a probability function which shall trigger an alarm whenever a given threshold is exceeded (Theorem 4.9 and Corollary 4.11). This function works with very

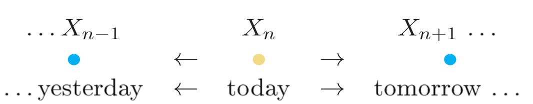

FIGURE 1 Natural time direction [Color figure can be viewed at wileyonlinelibrary.com]

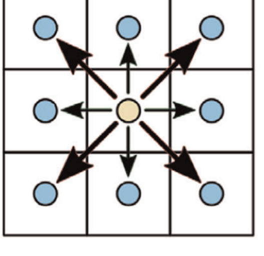

FIGURE 2 Spread time duration [Color figure can be viewed at wileyonlinelibrary.com]

little information (from a few nodes, the cliques). The secondary alarm notification provides temporal coverage by recording information as to whether the frequency of cash shortages is acceptable or not. In the design of the EWS structure, variables which can be freely selected/enlarged, shall be highlighted.

4.1 | Spatial coverage: Primary alarm notification

We begin by defining a theoretical network. For this, nodes v are identified with the random variable to be monitored, X_v. To cover all the aspects, X_v shall be extensively detailed by means of a collection of features $x_v^k, k = 1, \ldots, n$: that is, $X_v \simeq (x_v^1, x_v^2, \ldots, x_v^n)^t$, where both the vector components (i.e., $x_v^k, k = 1, \ldots, n$) and number of them may be selected as needed. As a result, every node can be identified with a collection of *of random variables* $x_v^k, k = 1, \ldots, n$ which can be taken as weighted ones if required. Hence,

$$v \sim X_v = \left(x_v^1, x_v^2, \ldots, x_v^n\right)^t, \quad \text{for each} \quad v \in V.$$

The notion of explanatory variable is conducted through the notion of *feature vector* as an n-dimensional vector of *numerical* scores which represent (label) an item. In this regard, the following considerations should be taken:

- the passage from the random variable $X_v \sim (x_v^1, x_v^2, \ldots, x_v^n)^t$ to a realization (of the random variable) is made by rating the feature coordinates $x_v^k, k = 1, \ldots, n$. To this end, several methodologies could be considered. Once the rating/scoring process is over, each node v can be identified with a feature vector $(score(x^1), score(x^2), \ldots, score(x^n))^t$ which represents the numerical evaluation of X_v.
- Depending on whether X_v can be described qualitatively or quantitatively, the corresponding data sets—where each feature coordinate $x_v^k, k = 1, \ldots, n$ takes values- can be either numerical or wording ones. Once the random variable X_v and its coordinates $x_v^k, k = 1, \ldots, n$ have been selected, what differentiates some nodes from others are the lower and upper bounds of such variables.

Remark 4.1 (Open-door 1). The random variable $X = \{X_v, v \in V\}$ can be selected as required by the context as well as the X_v features $x_v^k, k = 1, \ldots, n$, both the number and the features themselves.

A graphical model should be defined from the set of random variables $X = \{X_v, v \in V\}$, once the links between nodes have been described. In fact, once a neighborhood system N has been defined—which is referred to below- this shall determine the links between the nodes according to the neighboring relationship, namely: • $v \notin N(v)$• $v_j \in N(v_i) \Leftrightarrow v_i \in N(v_j)$. The graphical model thus defined will be denoted by $(X_v, N_v)_{v \in V}$.

Therefore, the distance between nodes could be specified. One possibility (even when the feature coordinates $x_v^k, k = 1, \ldots, n$ are values in wording data sets) is to consider the Euclidean distance between the corresponding mean feature vectors:

Definition 4.2. Consider two nodes $X_{v_i} = (x^1_{v_i}, ..., x^n_{v_i})^t$, $X_{v_j} = (x^1_{v_j}, ..., x^n_{v_j})^t$, $i \neq j$, and let $\overline{X_{v_i}}$ denote the mean value of the random variable over some interval of time,

$$\overline{X_v} = \left(\overline{x^1_v}, \overline{x^2_v}, ..., \overline{x^n_v}\right)^t, \quad \text{for each} \quad v \in V,$$

(i.e., $\overline{X_v}$ is a specific realization of the random variable). Thus, the distance between nodes X_{v_i}, X_{v_j}, $i \neq j$, written d_{ij}, is the Euclidean distance between $\overline{X_{v_i}}, \overline{X_{v_j}}$:

$$d_{ij} = d(\overline{X_{v_i}}, \overline{X_{v_j}}) = \sqrt{\sum_{k=1}^{n} \left(\overline{x^k}_{v_i} - \overline{x^k}_{v_j}\right)^2}. \tag{2}$$

Remark 4.3 (Open-door 2). In some contexts, Euclidian distance may not work well because the components are taken on different scales. In such situations, the components should be rescaled (normalized or standarized).

In fact, the definition of distance between two nodes is of free choice. For instance, Mahalanobis distance is a multivariate measure that determines the distance between nodes and distributions. In general, any distance function on the given set V could be considered provided that it meets the requirements of the features of the context. That is to say, a set V can be viewed as a metric space if there is a function $d : V \times V \to \mathbb{R}$ that satisfies the conditions (for $x, y, z \in V$) (i) $d(x, y) \geq 0$, and $d(x, y) = 0 \Leftrightarrow x = y$, (ii) $d(x, y) = d(y, x)$ (symmetry) and (iii) $d(x, z) \leq d(x, y) + d(y, z)$.

Any metric d on a set induces a topology on the set. In particular, a neighborhood system $N = \{N(v) \mid v \in V\}$ can be defined, where $N(v)$ denotes the neighborhood of $v \in V$. Note that any definition of the neighborhood on a node is allowed as long as the neighboring relationship is satisfied.

In our context, where an order on V need not exist, the neighborhood on a node $v \in V$*is* defined as follows:

Definition 4.4. The neighborhood of a node $v_i \in V$ is $N(v_i) = \{v_j \in V \mid d(v_i, v_j) \leq \epsilon, k \in \mathbb{R}, \epsilon \neq 0\}$, where the degree of similarity materializes through a benchmark ϵ which should be specified for each particular case. Hence, for $v_j \in N(v_i)$ an edge (v_i, v_j) will join them if $d(v_i, v_j) \leq \epsilon \neq 0$.

A full description of the notion of neighborhood follows:

Proposition 4.5 (Code neighborhoods). *The neighborhood of any node $v \in V$ consists of all nearby nodes where the term "nearby" means that, regarding X_v, the nodes are very similar with a degree of similarity ϵ.*

Proof. From Definition 4.4,

$$\begin{aligned} N(v_i) &= \{v_j \in V \mid (v_i, v_j) \in E\} = \\ &= \{v_j \in V \mid d(v_i, v_j) \leq \epsilon, \quad \epsilon \in \mathbb{R}, \epsilon \neq 0\}. \end{aligned}$$

It is clear that both v_i and v_j are very similar with regard to X_v since the distance between v_i, v_j is as short as ϵ. □

Using the equivalence between edges and neighborhood systems, the edges in the network $(X_v, N_v)_{v \in V}$ can be fully described as follows: an edge (v_i, v_j) will join two vertices v_i, v_j if and only if $d(v_i, v_j) \leq k \neq 0 (\Leftrightarrow v_j \in N(v_i))$.

From Proposition 4.5, the next result can also be derived:

Corollary 4.6. *Consider X_{v_i}, X_{v_j} two random variables. If they belong to the same neighborhood, then they have equal conditional distributions. If X_{v_i}, X_{v_j} do not belong to the same neighborhood, they are independent variables.*

Proof. Let X_{v_i}, X_{v_j} be two random variables. Let us suppose that they are in the same neighborhood. Thus, their distance in similarity to the selected features is as small as desired, as is the benchmark ϵ. Hence, their marginal distributions are equal, $P[X_{v_i}] = P[X_{v_j}]$. By applying the Bayes's theorem:

$$P[X_{v_i} | X_{v_j}] = \frac{P[X_{v_j} \mid X_{v_i}] \cdot P[X_{v_i}]}{P[X_{v_j}]} = P[X_{v_j} | X_{v_i}],$$

the result follows. Let it be noticed that, for ϵ small enough, the fact that a node $v_i \notin N(v_j)$ is equivalent to that X_{v_i} and X_{v_j} have no features in common. Thus, the result follows. □

Finally, the definition of *cliques* can be given as maximally connected subgraphs.

More specifically, the set of cliques consists of either a single-node $C_1 = \{c_i\}$, a pair of neighboring nodes $C_2 = \{c_i, c_j\}$, a triple of neighboring nodes $C_3 = \{c_i, c_j, c_k\}$ in such a way that the collection of all cliques is $C = C_1 \cup C_2 \cup C_3 ...$ where C_i are possible sets of larger cliques. Note that the nodes in a clique are ordered and $\{c_i, c_j\}$ is not the same clique as $\{c_i, c_j\}$. For instance, the maximal cliques in Figure 3 are $C_2 = \{6, 7\}$, $C_3 = \{4, 5, 6\}$ and $C_4 = \{1, 2, 3, 4\}$.

The following result identifies the cliques:

Proposition 4.7 (Code cliques). *A clique C consists of the nodes with a high degree of similarity, where the term "high" depends on the benchmark ϵ_i, $\epsilon_i \neq \epsilon$. It should be noted*

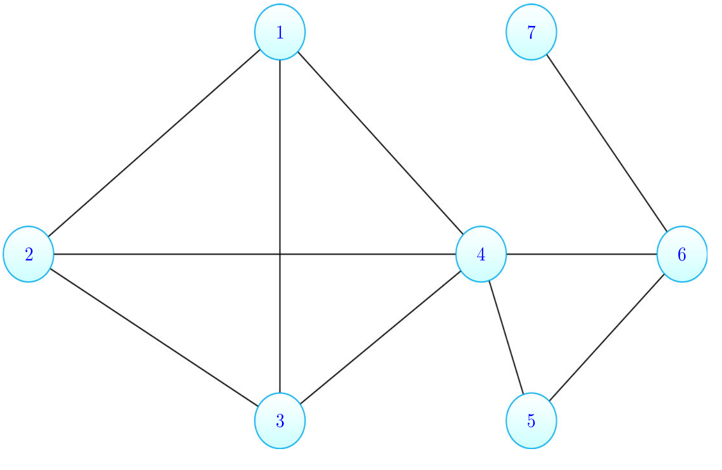

FIGURE 3 Example of cliques [Color figure can be viewed at wileyonlinelibrary.com]

that the index of similarity shall be different for each C_i*, that is, nodes* $c_1^i, c_2^i \in C_i$ *may be similar with a degree* k_i *while* $c_1^j, c_2^j \in C_j$ *may be similar with different degree of similarity* $\epsilon_i \neq \epsilon_j$ *whenever* $i \neq j$*.*

Proof. A clique C_i is a collection of nodes such that each pair are connected by an edge. In consequence, both $c_1^i, c_2^i \in C_i$ belong to the corresponding neighborhood from definition 4.4. Therefore, $d(c_1^i, c_2^i) \leq \epsilon_i \leq \epsilon$. □

Remark 4.1 has significant implications.

Corollary 4.8. *In general, cliques are fully connected (sub)groups. Thus, each choice of the random variable* X_v *as well as of the feature coordinates* $x_v^k, k = 1, \ldots, n$ *generates different ways of grouping the nodes.*

The aim at this stage is to derive a Gibbs distribution for the previously defined graphical model. To this end, the following result is crucial:

Theorem 4.9. *The graphical model* $(X_v, N_v)_{v \in V}$ *is a MRF.*

Proof. To prove that $P[X_v = x_v \mid X_{V-\{v\}} = x_{V-\{v\}}] = P[X_v = x_v \mid X_{N(v)} = x_{N(v)}]$, we consider a partition of the set of nodes: specifically, for each node $v \in V$, $V = N(v) \cup \overline{N(v)}$, where $\overline{N(v)}$ denotes the complementary set (over V) of $N(u)$. According to the neighboring relationship, since $v \notin N(v)$, it is assumed that $v \in \overline{N(v)}$. Hence, $V - \{v\} = N(v) \cup (\overline{N(v)} - \{v\})$. Therefore, random variables $X_{V-\{v\}}$ are either $X_{N(v)}$ or $X_{\overline{N(v)-\{v\}}}$. As a result, the above statement follows from the Corollary 4.6, since variables X_v and $X_{\overline{N(v)}-\{v\}}$ are independent variables. □

Remark 4.10. It should be noted that the graphical model $(X_v, N_v)_{v \in V}$ could be referred to as *universal* in the sense that neither geographical constraints nor local specifications have been considered in its definition.

Corollary 4.11. *The joint distribution* $P[X]$ *of the graphical model* $(X_v, N_v)_{v \in V}$ *can be written in terms of clique potentials* $\phi_c(X_C)$*, where* T *is known as temperature and* $V_c(X_C)$ *are the energy functions:*

$$P[X] = \frac{1}{Z} \prod_{c \in C} \phi_c(X_C) = \frac{1}{Z} e^{\frac{-1}{T} \sum_{c \in C} V_c(X_C)} \tag{3}$$

Proof. Since $(X_v, N_v)_{v \in V}$ is a MRF from the Theorem 4.9, it is associated to a Gibbs distribution, according to the Hammersley–Clifford theorem. In consequence, the joint distribution of X is written either as a product of clique potentials $P[X] = \frac{1}{Z} \prod_{c \in C} \phi_c(X_C)$, or in terms of energy functions $V_c(X_C)$ due to $\phi_c(X_C) = e^{\frac{-1}{T} V_c(X_C)}$. Thus $P[X]$ is

$$P[X] = \frac{1}{Z} \prod_{c \in C} \phi_c(X_C) = \frac{1}{Z} \prod_{c \in C} e^{\frac{-1}{T} V_c(X_C)} = \frac{1}{Z} e^{\frac{-1}{T} \sum_{c \in C} V_c(X_C)}.$$

□

Both the Theorem 4.9 and the Corollary 4.11 state that the designed graphical model $(X_v, N_v)_{v \in V}$ works as an intelligent system which can notify whether misalignments have occurred, depending on whether the values of the joint distribution function $P[X]$ exceed a given threshold or not. In consequence, hereinafter we will refer to $(X_v, N_v)_{v \in V}$ as an EWS. The practical value of these results is that they give access to a joint probability distribution (with a known expression) which should provide an alert when certain security thresholds are exceeded. Thus, a spatial monitoring system based on the information from cliques can be derived for network type situations.

Efficient EWSs should be provided with a simple visual system to identify the different alarm levels. In most cases this is done through a color system in such a way that the colors are both linked to the alarm levels as well as to the corresponding probability of occurrence. To this end, different procedures can be devised to relate probabilities to colors.

A sound, nonexpensive procedure is LATEX (A document preparation system mainly employed for scientific documents), which shall identify each different color with a range of probabilities: taking into account the fact that any probability $P[-]$ Should be between 0 and 1, $0 \leq P[\text{-}] \leq 1$, a partition $\mathcal{P}$ of $[0, 1]$ should be taken: that is, an ordered n-tuple of real numbers $\mathcal{P} = (x_0, x_1, \dots, x_n)$ such that $0 = x_0 < x_1 < \cdots < x_n = 1$, $[0, 1] = [0 = x_0, x_1] \cup [x_1, x_2] \cup \cdots \cup [x_{n-2}, x_{n-1}] \cup [x_{n-1}, x_n = 1]$.

An example of this type of partition of the interval $[0, 1]$ is as follows:

$$[0,1] = \underbrace{\left[0, \frac{1}{4}\right]}_{\text{green}} \cup \underbrace{\left[\frac{1}{4}, \frac{1}{2}\right]}_{\text{yellow}} \cup \underbrace{\left[\frac{1}{2}, \frac{3}{4}\right]}_{\text{orange}} \cup \underbrace{\left[\frac{3}{4}, 1\right]}_{\text{red}}$$

such that the programme matches values with colors by simply assigning warm colors to probability occurrences which either approach or exceed a given limit. The result is a colored area like the one shown one in Figures 4 and 5.

4.2 | Time coverage: Secondary alarm notification

The main alarm functionality is to cover the spatial network. Nevertheless, this does not register the frequency of some potentially troubling events. This is the reason why the proposed EWS has to be endowed with a temporal follow-up system. To visualize the state of alarm, a color-coding system would trigger the corresponding color in the scale of alert as a response to a certain probability of occurrence at an instant of time t. To assess whether it is just a random event or if there is a real possibility of liquid shortages, a temporary system is needed to record incidents. To this end,

- the system should register the time moments t_i in which the joint probability function $P[X]$ is represented by warm colors.
- Time instants associated to such probability distribution values over time meet a temporal sequence $\{t_i\}_i$. These temporal data sets have to be preprocessed into a time series data frame which includes converting information into dates, indexing the dates and assigning a frequency as well as accounting for missing values.
- Once the corresponding thresholds have been fixed, the analysis (e.g., whether the corresponding probability values increase or decrease) of the time series $\{t_i\}_i$ shall indicate the evolution, frequency, and duration of the risk of cash shortages.

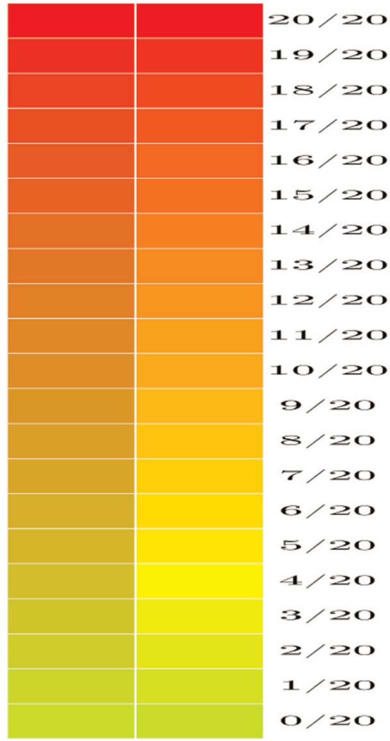

FIGURE 4 The color-coding system [Color figure can be viewed at wileyonlinelibrary.com]

FIGURE 5 Surveillance of cash flows [Color figure can be viewed at wileyonlinelibrary.com]

Let it be noted that, so far, the EWS acts as integral protection for *the whole network* due to a *joint* probability distribution. However, it could be useful to secure some individual node(s) as well. This shall be carried out when deriving the cash level EWS, as discussed in next section.

5 | CASH LEVEL EWS

5.1 | Securing cash levels in the branch network

In this section an EWS for securing cash levels (a cash level EWS) is derived from the previously developed structural model. To contextualize this task, we refer to the framework to which central banks must use To achieve financial stability. The appropriate tools are the banking liquidity stress tests which should be performed by banking entities to adhere to those policies which set the *safety liquidity levels* that banks must attain (Basel III rules with two minimum ratios, “Liquidity Coverage Ratio” [LCR] a sort of stress test, and “Net Stable Funding Ratio” [NSFR] are particularly important as they ensure that the assets of a bank are well supported by stable funding sources.).

This section is specifically related to the LCR since it is intended for controlling whether the bank branches have sufficiently safe liquidity levels. In fact, the LCR was devised to be able to check if banks hold a sufficient reserve of high-quality liquid assets that allow them to survive in conditions of significant liquidity stress lasting for 30 calendar days, see Reference [20]. Such tests are periodically performed not only to meet the external regulatory standards but also for internal security reasons to avoid dormant money at branches, since liquidity comes at a cost: keeping liquid assets in cash involves the corresponding opportunity costs of not investing in other alternative products which bring clear benefits, see Reference [7].

The reasons that make the proposed cash level EWS specially suitable for the banking context are: on one hand, as an EWS free of local constraints, it may be employed as an exportable methodology to be used for banking institutions which expand their branch networks all over the world. Moreover, this would lead to a reduction in costs when replacing several approaches with a single global (universal) one. On the other hand, since the joint probability function is not required to follow a fixed data distribution, it is highly beneficial in contexts with dynamic data which require frequent updates, as in financial scenarios. Additionally, our cash level EWS only requires data sets from cliques thereby leading to a significant reduction in the reporting burden on the remaining nodes (branches).

Let us formulate the cash level EWS. Consider a financial institution and let BN be its Branch Network. Following References [6,17], branches are the nodes and the business relationships between them are the edges, distinguishing between headquarters and branch offices in a directed graph mode. For simplicity, we consider here that the Bank Branch Network BN is an *undirected* graph where each branch $b \in BN$ is a grid node which is characterized by its *cash holdings*, that is, the total amount of cash allowed in the branch b, denoted by CH_b. That is to say, here the general random variable X shall be the branch *cash holdings* CH so that

$$b \sim X_b = CH_b.$$

Potential threats for BN are the *liquidity risks* that comprise either the risk of not reaching (*underperforming* branches) or the risk of exceeding (branches with *dormant money*) the corresponding security cash levels. The usual categorization of branches by branch practitioners divides branches into different categories: city center, rural or business center, depending on the total amount of cash holdings that they are allowed to maintain. As will be detailed shortly, the main differences between nodes b are lower and upper bounds for branch cash holdings corresponding to different kind of branches (note that, in previous sections, it was highlighted that "once a random variable X_v as well as the n-tuple of random variables x_v^k, $k = 1, \ldots, n$ have been selected, what differentiates some nodes from others are the lower and upper bounds of such variables"). The cash level EWS gives a warning about branches whose liquidity levels are both *under* and *over* the required security standards (see next section for further details).

The main determinants of branch cash holdings must be selected as the associated collection of features $x_b^k, k = 1, \ldots, n$ of CH_b. For simplicity, two features $x^k, k = 1, 2$ shall be considered as determinants of CH_b: n_b stands for the maximum number of transactions while v_b denotes the upper volume of cash transactions authorized by the central offices. As mentioned before, however, the number of features can be freely enlarged as needed. The current selection of features results from the fact that both n_b, v_b are the most commonly used features when branch managers specify branch cash holdings (which can alternatively be specified by means of the notion of *size* of a branch by practitioners). Hence,

Proposition 5.1. *Each branch b in BN can be viewed as* $b \sim CH_b = \begin{pmatrix} n_b \\ v_b \end{pmatrix}$.

Following the EWS model foundations, BN is viewed as an undirected graph whose nodes are the random variables $CH = \{CH_b, b \in BN\}$ while the (undirected) edges provide financial similarities between branches. Specifically, using Proposition 4.5, the neighborhood of a branch may be described as follows.

Proposition 5.2. *For a branch $b_i = (n_{b_i}, v_{b_i})$, its neighborhood $N(b_i)$ is the set of all branches b_j with similar features to that of b_i in relation to their cash holdings: $N(b_i) = \{b_j \in BN \mid d(b_i, b_j) \leq k\}$.*

Branch *size* is an open concept which can be defined by basing it on several parameters. Practitioners consider that both the number and volume of cash transactions are the values which act as determinants of the branch size. According to this, the neighborhood of a branch thus contains all the branches of a very similar size. Cliques are also described in this scenario:

Proposition 5.3 (Code cliques are in the Branch Network). *Cliques C in the Branch Network BN consist of branches of the same (or almost the same) size. It is important to note that the size varies for each clique, that is, C_i contains all the branches of size s_i while the branches of size s_j are in C_j, with $s_i \neq s_j$.*

Proof. The result comes from Proposition 4.7. □

However, the main results are the following ones, which can be directly derived from Theorem 4.9 and Corollary 4.11.

Theorem 5.4. *The graphical model $\{CH_b, b \in BN\}$ formed by the branch cash holdings is a MRF.*

Theorem 5.5. *A measurement system for branch cash levels in the branch network BN is given by $P[CH] = \frac{1}{Z} e^{-\sum_{c \in C} V_c(Ch_C)}$, where $V_c(Ch_C)$ are functions which solely depend on groups of branches with the same size (e.g., cliques in BN).*

5.2 | Variations on the cash level EWS: Monitoring individual branches

The previous section contains detailed information about how to secure the network as a whole thanks to a *joint* probability distribution which provides coverage for the entire network. Nevertheless, in some specific situations it is useful to know about a single node (e.g., a single bank branch). This section aims to detail other surveillance options for the cash level EWS proposed, like complementary alert services for individual nodes of the network. Both the surveillance of the whole network and that of an individual branch could be executed in parallel, thus providing comprehensive security. From a computational point of view, these preferences (both procedures are interchangeable) can be arranged as part of the setting options.

Securing the cash levels of individual branches can be performed as a particular application of the EWS structural model, based on the following result.

Theorem 5.6. *The neighborhood of any branch considered as a subnetwork of BN is a MRF itself. Moreover, for any branch* b^**, the network* $N(b^*) \cup \{b^*\}$ *also is a MRF. As a consequence, a Gibbs joint probability distribution for* $N(b^*) \cup \{b^*\}$ *can be obtained.*

Proof. As mentioned before, the neighborhood of a branch $b^* N(b^*) = \{b \in NB \mid b^* \sim b\}$ is the set of branches with similar cash holding features according to Proposition 5.2. It should be noted that the general Markov property $P[CH_b = ch_b \mid CH_{NB-\{b\}} = ch_{NB-\{b\}}] = P[CH_b = ch_b \mid CH_{N(b)} = ch_{N(b)}]$ is satisfied in the particular case of $NB = N(b^*)$,

$$P\left[CH_{b^*} = ch_{b^*} \mid CH_{N(b^*)-\{b^*\}} = ch_{N(b^*)-\{b^*\}}\right] = P\left[CH_{b^*} = ch_{b^*} \mid CH_{N(b^*)} = ch_{N(b^*)}\right]$$

since no node belongs to its neighborhood and hence $N(b^*) - \{b^*\} = N(b^*)$. As for the second statement, $NB = N(b^*) \cup \{b^*\}$, the corresponding Markov property is also satisfied since

$$\begin{aligned} P\left[CH_{b^*} = ch_{b^*} \mid CH_{N(b^*)\cup\{b^*\}-\{b^*\}} = ch_{N(b^*)\cup\{b^*\}-\{b^*\}}\right] &= P\left[CH_{b^*} = ch_{b^*} \mid CH_{N(b^*)}\right. \\ &\left.= ch_{N(b^*)}\right]. \end{aligned} \tag{4}$$

□

Corollary 5.7. *Let* b^* *be a single branch in BN whose cash levels must be separately monitored. The distribution function (which gives the likelihood of branch cash levels reaching a given threshold) takes the form* $P[CH_{b^*}] = \frac{1}{Z} e^{-\sum_{c \in C} V_c(CH_C)}$*, for* C *the set of branches with the* same *size as* b^*.

Proof. The result comes from applying both Theorem 5.6 and the description of cliques in BN (Proposition 5.3) to the specific case of the branch network $NB = N(b^*) \cup \{b^*\}$. □

The next remark thus condenses how to secure the cash levels of individual branches.

Remark 5.8. To secure the cash levels of an individual branch b^*,

- adjust the original set of features n_b (the maximum number of transactions) and v_b (the upper volume of cash transactions authorized) so that this best suits the specific characteristics of b^*.
- Consider (the subnetwork $N(b^*) \cup \{b^*\}$ which contains) both the branch b^* and all the branches of same size as b^*.
- Select appropriate potential functions $V_c(CH_C)$ ($c \in C$ means branches in a clique and hence, "branches of the same size as b^*"), according to the b^* features.
- Compute the corresponding likelihood of having safe/unsafe cash levels using $P[CH_{b^*}] = \frac{1}{Z} e^{-\sum_{c \in C} V_c(CH_C)}$.

Other variations of the cash level EWS would include the surveillance of liquid resources of the whole banking institution. Then, instead of the LCR, the bank as an entity is related to NSFR as long as it is intended for assessing the long-term funding structure for long-term assets.

5.3 | An example of the monitoring of cash holdings

This section is devoted to developing an example of the monitoring of cash holdings to illustrate how our EWS works. In the general model, the branch network BN is formed by a cloud of branches $b \in NB$ that can be grouped into n cliques, $C_1, C_2, ..., C_n$ according to their size: that is, since cliques are groups of branches with the same branch size, each clique C_i has a particular size s_i attached. Moreover, the function $P[CH] = \{P[CH_b] | \, b \in NB\}$ denotes the joint probability distribution of all cash holdings in BN whose output is, according to Theorem 4.5, the probability of the branch cash holdings taking a specific value by aggregating the corresponding information on cliques. $P[CH]$ will be completely determined as soon as the energy functions are selected.

Energy functions are determinant in a Gibbs probability distribution. These can be chosen arbitrarily as long as they meet the "energy constraint" which states that they must be decreasing functions as for the matching of the features of cliques with regard to a given template. That is to say, if the features in a clique match the features in a given template, the energy function should decrease, otherwise it should increase. Moreover, they must be nonnegative functions. Thus, from the general expression given in Theorem 5.5, each choice of clique energy functions shall provide different distributions of BN cash holdings, as shown in Table 1.

We shall consider the case of Gaussian energy functions for the ith clique, $V_c(Ch_{C_i}) = \frac{(CH_{C_i} - \mu_i)^2}{\sigma_i^2}$, where μ_i is the mean and σ_i^2 is the variance of the cash holdings at

TABLE 1 Different distributions of BN cash holdings

Name	Clique energy functions	Joint probability distribution
Gaussian	$\frac{(CH_{C_i} - \mu_i)^2}{\sigma_i^2}$	$P[CH] = \frac{1}{Z} e^{-\sum_{i=1}^{n} \frac{(CH_{C_i} - \mu_i)^2}{\sigma_i^2}}$
Log linear	$\exp(c + \sum_i w_i f_i(CH_{C_i}))$	$P[CH] = \frac{1}{Z} e^{-\sum_{i=1}^{n} \exp\left(c + \sum_i w_i f_i(CH_{C_i})\right)}$
Quadratic form	$CH_{C_i}^t A CH_{C_i}$	$P[CH] = \frac{1}{Z} e^{-\sum_{i=1}^{n} CH_{C_i}^t A CH_{C_i}}$

ith-clique CH_{C_i}. Let us suppose that r represents a certain value for branch cash holdings (r is the initial of *ready money* since choosing c—the initial of the word cash—can cause confusion with the notation of cliques). Let us set out the safe levels for branch cash holdings which would depend on branch size: for underperforming branches of size s_i, let r_i^{min} be the lower threshold while r_i^{Max} denotes the corresponding upper threshold which indicates the possible presence of dormant money at branches of size s_i. Then, for each C_i whose cash holdings behave within the established parameters it should be $r_i^{min} \leq r \leq r_i^{Max}$. To reach a common domain, let $\underline{r} = inf\{r_i^{min} | i = 1, ..., n\}$ and $\bar{r} = sup\{r_i^{Max} | i = 1, ..., n\}$ be the infimum and supremum, respectively so that $\underline{r} \leq r \leq \bar{r}$.

From the general expression of the joint probability distribution which measures the state of the BN cash holdings, the probability of BN cash holdings taking the value r is given by

$$P[CH = r] = \frac{1}{Z} e^{-\sum_{i=1}^{n} \frac{(r-\mu_i)^2}{\sigma_i^2}}$$

For a clique C_i, the probability of having cash holdings for quantities exceeding its upper threshold, $r \in [\bar{r} - r_i^{Max}, \bar{r}]$ should be very small, as happens when this is computed using Gaussian energy functions: indeed, the further the amount r moves away from the established maximum, the greater the difference in the exponent and, therefore, the lower the total probability.

The "constant" Z in 5.3 is known as the "partition function" and its primary role is to ensure that the sum of the probabilities is equal to 1. By using the fact that probabilities must add up to 1, Z can be isolated:

$$1 = \sum_{r_k \in \mathbb{R}} P[CH = r_k] = \sum_{r_k \in \mathbb{R}} \frac{1}{Z} e^{-\sum_{i=1}^{n} \frac{(r_k-\mu_i)^2}{\sigma_i^2}} = \frac{1}{Z} \sum_{r_k \in \mathbb{R}} e^{-\sum_{i=1}^{n} \frac{(r_k-\mu_i)^2}{\sigma_i^2}} \Rightarrow$$
$$\Rightarrow Z = \sum_{r_k \in \mathbb{R}} e^{-\sum_{i=1}^{n} \frac{(r_k-\mu_i)^2}{\sigma_i^2}}$$

In reality, the determination of Z entails at an early stage the delimitation of its domain since, in general, the sum over r_k is understood to be a sum over all possible values that the variable "cash holdings" may take ($r_k \in \mathbb{R}$). As this example has the purpose of illustrating how our model works, we will not enter into the process of computing Z. Instead, we will use the reproductive property to conclude that Z, under certain conditions, can be taken as equal to 1. We assume for our example that the conditions that make Z equal 1 are met.

The reproductive property (see Reference [21]) states that:

Proposition 5.9 (Reproductive property [21, p. 659]). *If two (or more) independent random variables with a certain distribution are added together, the resulting random variable has a distribution of the same type as that of the summands.*

This property is applied twice: firstly, to the exponent in $Z = \sum_{r_k \in \mathbb{R}} e^{-\sum_{i=1}^{n} \frac{(r_k-\mu_i)^2}{\sigma_i^2}}$, viewed as a random variable $\sum_{i=1}^{n} \frac{(r_k - \mu_i)^2}{\sigma_i^2}$. Hence, since the variable $\sum_{i=1}^{n} \frac{(r_k - \mu_i)^2}{\sigma_i^2}$ has a distribution of the same type as that of the summands, it must be $\frac{\left(r_k - \sum_{i=1}^{n} \mu_i\right)^2}{\sum_{i=1}^{n} \sigma_i^2}$ by the properties of Gaussian distribution. In consequence,

$$Z = \sum_{r_k \in \mathbb{R}} e^{-\frac{(r_k-\mu)^2}{\sigma^2}}, \quad \mu = \sum_{i=1}^{n} \mu_i, \sigma^2 = \sum_{i=1}^{n} \sigma_i^2 \tag{5}$$

Second, we apply the reproductive property to the random variable Z itself. Accordingly, Z has an exponential distribution, $Z = e^{-\lambda}$ for some parameter λ (a Gamma$(\lambda, 1)$ distribution), see References [21] and [22]. As a result, Z can be taken as equal to 1 whenever $\lambda = 0$.

Expression (4) allows the corresponding cumulative distribution for a quantity $r \in [\underline{r}, \bar{r}]$ to be fixed:

$$P[CH \leq r] = \sum_{r_k \leq r} P[CH = r_k] = \sum_{r_k \leq r} \frac{1}{Z} e^{-\sum_{i=1}^{n} \frac{(r_k-\mu_i)^2}{\sigma_i^2}} = \frac{1}{Z} \sum_{r_k \leq r} e^{-\sum_{i=1}^{n} \frac{(r_k-\mu_i)^2}{\sigma_i^2}},$$

which is finally equal to

$$P[CH \leq r] = \sum_{r_k \leq \bar{r}} e^{-\frac{(r_k-\mu)^2}{\sigma^2}}, \quad \mu = \sum_{i=1}^{n} \mu_i, \sigma^2 = \sum_{i=1}^{n} \sigma_i^2$$

by applying the reproductive property and assuming that $Z = 1$.

In particular, the likelihood that the cash holdings of BN stay in a safe band is given by

$$\begin{aligned} P[CH \leq \bar{r}] &= \sum_{r_k \leq \bar{r}} P[CH = r_k] = \sum_{r_k \leq \bar{r}} \frac{1}{Z} e^{-\sum_{i=1}^{n} \frac{(r_k-\mu_i)^2}{\sigma_i^2}} = \sum_{r_k \leq \bar{r}} e^{-\frac{(r_k-\mu)^2}{\sigma^2}} \\ \mu &= \sum_{i=1}^{n} \mu_i, \sigma^2 = \sum_{i=1}^{n} \sigma_i^2, Z = 1, \\ P[CH \geq \underline{r}] &= 1 - P[CH < \underline{r}]. \end{aligned}$$

Note that the first formula computes the probability of performing under the permitted threshold and hence, of not having dormant money (since once this threshold has been exceeded, there is a risk that liquid resources are not being invested or used). That means that the probability of having dormant money can be computed as $1 - P[CH \leq \bar{r}] = P[CH \geq \bar{r}]$. In parallel, the second formula provides the probability of performing over the corresponding threshold and hence, of having enough liquidity so that there is no risk of underperformance. Accordingly, $1 - P[CH \geq \underline{r}] = 1 - 1 + P[CH \leq \underline{r}] = P[CH \leq \underline{r}]$ reflects the probability of a risk of liquidity underperformance existing. Table 2 thus collects the basic formulas for computing the probability of BN cash holdings suffering liquidity risks (Table 3).

Let us consider the dormant money detection for a specific branch network BN in a particular example, to show how our model works. We assume that the branch network BN consists of three cliques, gathering branches of small (clique C_1 SS), medium (clique C_2 MS), and large (clique C_3 LG) sizes, respectively.

The following information on lower and upper thresholds of the cash holdings r_i^{min}, r_i^{Max} in scale 1 to 10^n for a positive integer n, is obtained from the cliques, from which $\underline{r} = 1$ and $\bar{r} = 4$:

We consider the sample $r_k = \{0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 7\}$. Thus, the probability of not having dormant money is given by

TABLE 2 Detection of unsafe levels of branch cash holdings

Under/over thresholds	Probability	How to compute the probability
Dormant money detection	$P[CH \leq \bar{r}]$	$\sum_{r_k \leq \bar{r}} P[CH = r_k] =$ $=\sum_{r_k \leq \bar{r}} e^{-\frac{(r_k-\mu)^2}{\sigma^2}}$, $\mu = \sum_{i=1}^{n} \mu_i$, $\sigma^2 = \sum_{i=1}^{n} \sigma_i^2$, $Z = 1$
Underperforming branches detection	$P[CH \geq \underline{r}]$	$1 - P[CH < \underline{r}] =$ $= 1 - \sum_{r_k < \underline{r}} P[CH = r_k] =$ $= 1 - \sum_{r_k < \underline{r}} e^{-\frac{(r_k-\mu)^2}{\sigma^2}}$, $\mu = \sum_{i=1}^{n} \mu_i$, $\sigma^2 = \sum_{i=1}^{n} \sigma_i^2$, $Z = 1$

TABLE 3 Information on thresholds, mean and variance of the cash holdings from cliques

	r_i^{min}	r_i^{Max}	μ_i	σ_i
Clique C_1 SS	1	2	$\mu_1 = 1.5$	$\sigma_1^2 = 0.1$
Clique C_2 MS	2	3	$\mu_2 = 2.5$	$\sigma_2^2 = 0.1$
Clique C_3 LS	3	4	$\mu_3 = 3.5$	$\sigma_3^2 = 0.1$
			$\sum_{i=1}^{3} \mu_i$	$\sigma^2 = \sum_{i=1}^{3} \sigma_i^2$
			$\mu = 7.5$	$\sigma^2 = 0.3$

$$P[CH \leq \bar{r}] = \sum_{r_k \leq \bar{r}} e^{-\sum_{i=1}^{3} \frac{(r_k-\mu_i)^2}{\sigma_i^2}} = \sum_{r_k \leq \bar{r}} e^{-\frac{(r_k-7.5)^2}{0.3}} =$$
$$= e^{-\frac{(0.5-7.5)^2}{0.3}} + e^{-\frac{(1-7.5)^2}{0.3}} + e^{-\frac{(1.5-7.5)^2}{0.3}} + e^{-\frac{(2-7.5)^2}{0.3}} + e^{-\frac{(2.5-7.5)^2}{0.3}} +$$
$$+ e^{-\frac{(3-7.5)^2}{0.3}} + e^{-\frac{(3.5-7.5)^2}{0.3}} + e^{-\frac{(4-7.5)^2}{0.3}} + e^{-\frac{(4.5-7.5)^2}{0.3}} + e^{-\frac{(5-7.5)^2}{0.3}} +$$
$$e^{-\frac{(5.5-7.5)^2}{0.3}} + e^{-\frac{(6-7.5)^2}{0.3}} + e^{-\frac{(6.5-7.5)^2}{0.3}} + e^{-\frac{(7-7.5)^2}{0.3}} = 0.470826907,$$

computed through the information given by Table 4.

5.4 | Risk behavior depending on branch type

The analysis of risk behavior when unsafe cash levels have occurred will shed light on the cash level variations thereby allowing their moves to be anticipated. This paragraph is devoted to this type of study depending on the type of branches.

TABLE 4 Computation of $P[CH \leq \bar{r}]$ for a given sample

r_k	$(r_k - 7.5)$	$(r_k - 7.5)^2$	$\frac{(r_k-7.5)^2}{0.3}$	$-\frac{(r_k-7.5)^2}{0.3}$	$e^{-\frac{(r_k-7.5)^2}{0.3}}$
0.5	−7	49	163.3333333	−163.3333333	1.16208E−71
1	−6.5	42.25	140.8333333	−140.8333333	6.86848E−62
1.5	−6	36	120	−120	7.66765E−53
2	−5.5	30.25	100.8333333	−100.8333333	1.61674E−44
2.5	−5	25	83.33333333	−83.33333333	6.43863E−37
3	−4.5	20.25	67.5	−67.5	4.84309E−30
3.5	−4	16	53.33333333	−53.33333333	6.88062E−24
4	−3.5	12.25	40.83333333	−40.83333333	1.84633E−18
4.5	−3	9	30	−30	9.35762E−14
5	−2.5	6.25	20.83333333	−20.83333333	8.95774E−10
5.5	−2	4	13.33333333	−13.33333333	1.6196E−06
6	−1.5	2.25	7.5	−7.5	0.000553084
6.5	−1	1	3.333333333	−3.333333333	0.035673993
7	−0.5	0.25	0.833333333	−0.833333333	0.434598209
				$P[CH \leq \bar{r}]$	0.470826907

As mentioned, the usual categorization of branches by practitioners divides branches into different types: city center, rural or business center, depending on the authorized upper and lower thresholds for their cash holdings. For this reason, the main differences between different types of branches are the lower and the upper bounds for branch cash holdings.

Under this categorization, the dynamics of the risk of having unsafe cash holding levels is shown in Figures 6–9, depending on the kind of branch.

To reach conclusions, we have considered both probabilities (dormant money detection $P[CH \leq r_i^{Max}]$ and underperforming branch detection, $P[CH \geq r_i^{min}]$) as a function of the variance σ_i^2 for each clique C_i. As far as σ_i^2 is concerned, differences between metropolitan and rural branches should be considered at this stage.

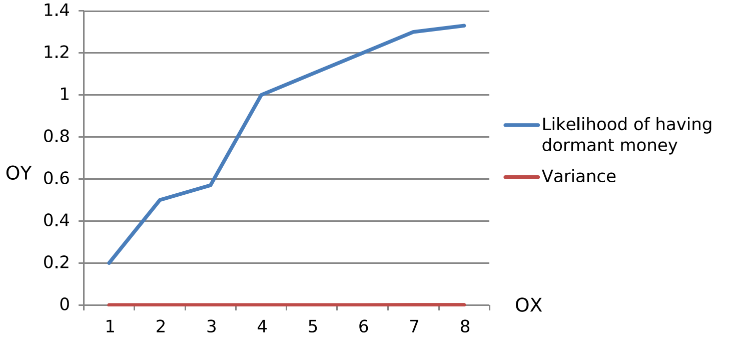

FIGURE 6 Likelihood of having dormant money for metropolitan branches (σ_i^2 taking high values) [Color figure can be viewed at wileyonlinelibrary.com]

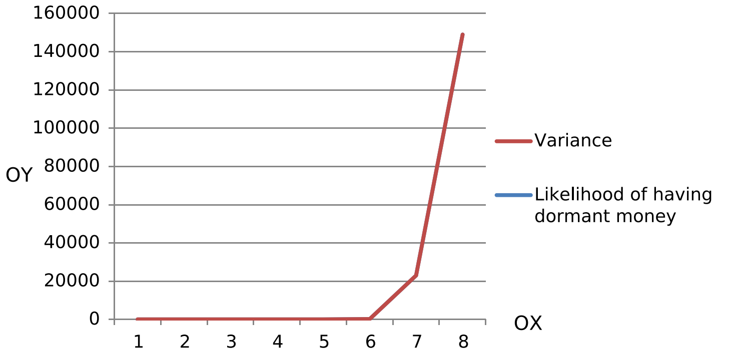

FIGURE 7 Likelihood of underperforming for metropolitan branches (σ_i^2 taking high values) [Color figure can be viewed at wileyonlinelibrary.com]

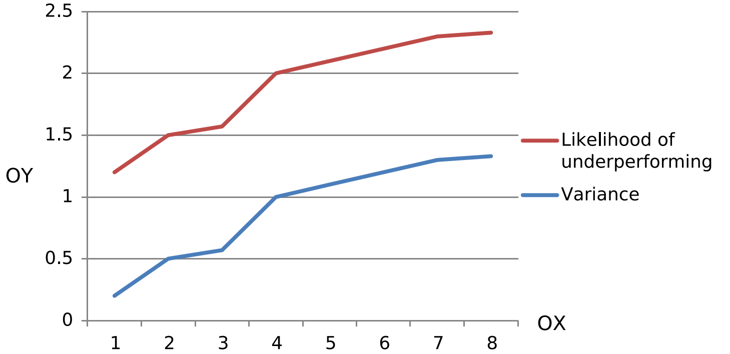

FIGURE 8 Likelihood of underperforming for rural branches (σ_i^2 taking small values) [Color figure can be viewed at wileyonlinelibrary.com]

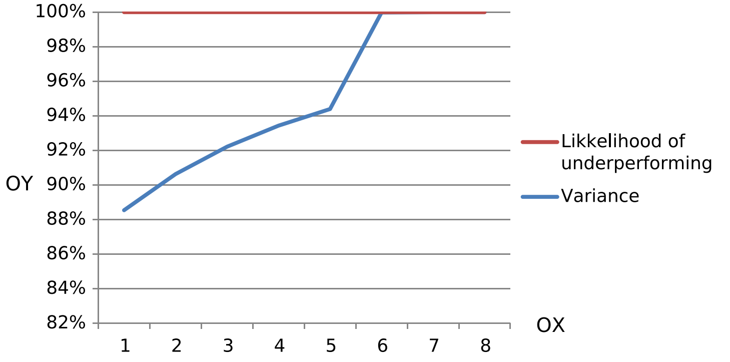

FIGURE 9 Likelihood of underperforming for metropolitan branches (σ_i^2 taking high values) [Color figure can be viewed at wileyonlinelibrary.com]

According to bank managers' expertize, the main feature of *metropolitan bank branches* is a high client flow, where over 50% of users are not habitual ones, and no complete information on the habits of these type of branch consumers. Hence, cash entries and withdrawals produce a *heterogeneous* cash flow. In consequence, for these kind of branches, the variance σ_i^2, can take high values as a result of a higher dispersion. On the contrary, the main features of *bank* *branches located in rural areas* have a constant medium/low client flow, where less than 20% are not regular users. In consequence, branch consumer habits are well known by branch staff. The entry/withdrawal cash flow is *homogeneous* with medium/low level on cash quantities. Therefore, the variance σ_i^2 is low.

As indicated above, Figures 6–9 display the risk behavior of unsafe levels of cash holdings when measured using the corresponding probability. It becomes apparent that this type of risk is an increasing function of σ_i^2 in all cases. Furthermore, the greater the dispersion, the higher the probability of having unsafe cash levels.

6 | CONCLUSIONS

This paper presents a cash level EWS which provides alerts when unsafe liquidity levels are reached at a branch level by working as an integral surveillance system for securing the cash levels of the whole bank network. Among other benefits, this type of integral solution would provide a unified insight that guarantees that branches with similar cash features are subjected to similar treatment. Besides its integral coverage, it provides other options that go beyond the network as a whole, like securing single branches. The implementation of these options may be executed in parallel. From a computational point of view, these preferences can be arranged as part of the setting options.

The design of the structural model is based on a MRF framework thereby providing a solid theoretical groundwork on which the distribution function must no longer be subject to any fixed distribution. This feature makes it very suitable for scenarios with dynamic data which require frequent updates as is the case of financial contexts. Additionally, the distribution function relies only on a few nodes (the cliques) thus reducing the amount of data required.

The cash level EWS has been designed free of local demographic constraints. This last feature provides export facilities, which is particularly valuable for banking institutions with branch networks all over the world. Another advantage of this type of integrated solution is the cost reductions achieved when replacing several proposals with a single global one.

In the cash level EWS presented here, the random variable considered is the discrete variable cash holdings, CH, which is a hard variable as opposed to a fuzzy one. In contexts where the analysis of cash holdings would require a fuzzy variable, see Reference [23], MRFs should be replaced by Fuzzy Markov random fields (FMRFs), which are just MRFs in a fuzzy space. The main difference between MRFs and FMRFs is the energy functions which turn out to be membership functions for FMRFs. Future applications of this proposal also include network-shaped situations such as social networks, e-commerce or e-governance.

ACKNOWLEDGMENTS

Financial support from the Spanish Ministry of Universities "Disruptive group decision making systems in fuzzy context: Applications in smart energy and people analytics" (PID2019-103880RB-I00), and Junta de Andalucía (SEJ340) is gratefully acknowledged.

CONFLICT OF INTERESTS

The authors declare that there are no conflict of interests.

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How to cite this article: García Cabello J. A novel intelligent system for securing cash levels using Markov random fields. *Int J Intell Syst*. 2021;1-23.
https://doi.org/10.1002/int.22467