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Centralized filtering and smoothing algorithms from outputs with random parameter matrices transmitted through uncertain communication channels

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Abstract

The least-squares linear centralized estimation problem is addressed for discrete-time signals from measured outputs whose disturbances are modeled by random parameter matrices and correlated noises. These measurements, coming from different sensors, are sent to a processing center to obtain the estimators and, due to random transmission failures, some of the data packet processed for the estimation may either contain only noise (uncertain observations), be delayed (sensor delays) or even be definitely lost (packet dropouts). Different sequences of Bernoulli random variables with known probabilities are employed to describe the multiple random transmission uncertainties of the different sensors. Using the last observation that successfully arrived when a packet is lost, the optimal linear centralized fusion estimators, including filter, multi-step predictors and fixed-point smoothers, are obtained via an innovation approach; this approach is a general and useful tool to find easily implementable recursive algorithms for the optimal linear estimators under the least-squares optimality criterion. The proposed algorithms are obtained without requiring the evolution model of the signal process, but using only the first and second-order moments of the processes involved in the measurement model.

1. Introduction

In recent years, the use of sensor networks has been widely encouraged in many different fields of application, due to the fact that they usually provide more information than traditional single-sensor communication systems. For this reason, much thought has been given to the multi-sensor fusion estimation problem in many significant research fields of engineering, computing, and mathematics, mainly because of its wide variety of applications, including target tracking, habitat monitoring, animal tracking or communications, among others. Depending on the way the multi-sensor measurements are combined and processed, there are two fundamental fusion techniques: (1) centralized fusion approach, where the measurements of all the sensors are sent directly to the processing center and fused for signal estimation, and (2) distributed fusion approach, where the measurements of each sensor are processed independently to obtain local estimators before they are sent to the processing center for fusion (see e.g. [1]-[3]). Centralized algorithms provide estimators by jointly processing the measurements of all the sensors at each instant of time; hence, when all the sensors work correctly, and the connections are perfect, they have the optimal estimation accuracy.

The aforementioned literature deals with different fusion estimation algorithms for conventional network systems with additive noises, when there are no error in the sensors (except those described by the additive noises) and the measured data packets are transmitted to the processing center over perfect connections. However, uncertainties in the sensor output measurements, such as stochastic sensor gain degradation, multiplicative noises, missing or fading measurements (see e.g. [4]-[8]), and failures during the transmissions, as for example random delays and packet dropouts or uncertain observations (see e.g. [9]-[12]), commonly occur and can spoil dramatically the quality of the fusion estimators designed without considering these drawbacks.

A unified framework to model the random disturbances in the output measurements is provided by the use of random measurement matrices and, for this reason, the estimation problem in this type of systems has become an issue of great concern in the last years (see e.g. [13]-[19] and references therein). Also, it is well known that the correlation of sensor measurement noises is inevitable in many network systems, due to the internal structure of the sensor and the influence of the practical environment. Systems with correlated measurement noises usually arise in situations where all the sensors operate in the same noisy environment, or when augmented systems are used to describe random delays and measurement losses. So, the assumption of uncorrelated sensor noises is commonly weakened and, in the past years, a great deal of efforts have been devoted to the research of the signal estimation problem in systems with correlated noises (see e.g. [5], [7], [11], [17] and [18]).

As already indicated, some fusion estimation algorithms consider conventional systems, where the sensors transmit their output measurements to the processing center over perfect connections. However, usually the communication channels may not be completely reliable and some anomalies (e.g. uncertain observations -measurements that contain only noise-, random delays and/or packet dropouts) may arise when the sensor measurements are sent to the processing center. Hence, the design of new fusion estimation algorithms for systems featuring one of these uncertainties (see e.g. [11], [14] and references therein), or even several of them simultaneously (see e.g. [10], [15], [20], [21] and references therein), has become an active research topic.

In some practical systems, these three random transmission failures can coexist with uncertainty in the measured outputs and correlation in the sensor noises. Up to now, to the best of the authors' knowledge, the signal estimation problem in sensor networks with noise correlation and simultaneous transmission uncertainties of sensor delays, packet dropouts and uncertain observations, has not been fully investigated in the framework of random measurement matrices modelling the random disturbances in the measured outputs, so it is still a challenging research topic.

Motivated by the above discussion, in the current paper, we aim to investigate the centralized fusion linear signal estimation problem from measurement outputs, whose disturbances are modeled by random parameter matrices and correlated noises, coming from multiple sensors subject to mixed uncertainties of random sensor delays, packet dropouts and uncertain observations. The major contributions and novelties of this paper are highlighted as follows: (1) Random matrices are considered in the measurement outputs of the sensors to provide a unified framework to address some random uncertainties, such as missing and fading measurements or sensor gain degradation, and, simultaneously, random delays, packet dropouts and uncertain observations are considered in the data transmissions from each sensor. (2) The proposed recursive prediction, filtering and fixed-point smoothing algorithms, based on covariance information, do not require the signal evolution model and they are computationally simple and suitable for online applications. Compared to [21], the main contributions of the current paper are: (a) The consideration of measurement outputs with random parameter matrices, which provides a unified framework to model some random phenomena as stochastic sensor gain degradation, missing or fading measurements, which cannot be described only by the usual additive disturbances. (b) The design of optimal linear centralized fusion estimation algorithms, including, not only filtering as in [21], but also multi-step prediction and fixed-point smoothing.

The rest of the paper is organized as follows. In Section 2, we present the measurement model of the different sensors and the assumptions under which the estimation problem is addressed. In Section 3, the original model is rewritten in a stacked form to carry out the centralized fusion estimation and the necessary statistical properties of the stacked observations are displayed. In Section 4, the LS linear centralized fusion estimation algorithms, obtained by the innovation approach, are presented. The performance of the proposed estimators is illustrated by numerical simulations in Section 5 and the paper concludes with some final comments in Section 6.

Notation. The notation used throughout the paper is standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. For a matrix A, A^T and A^{-1} denote its transpose and inverse, respectively. $\mathbf{1}_n = (1, \ldots, 1)^T$ denotes the all-ones $n \times 1$-vector and I_n the $n \times n$ identity matrix. If a matrix dimension is not specified, it is assumed to be compatible with algebraic operations. The Kronecker and Hadamard product of matrices will be denoted by $\otimes$ and $\circ$, respectively. $\delta_{k,s}$ denotes the Kronecker delta function. For any $a, b \in \mathbb{R}$, $a \wedge b$ is used to mean the minimum of a and b. Finally, for any function $G_{k,s}$, depending on the sampling times k and s, for simplicity we will write $G_k = G_{k,k}$; analogously, $K^{(i)} = K^{(ii)}$ will be written for any function $K^{(ij)}$ depending on the sensors i and j.

2. Observation model and hypotheses

The aim of this section is to design a model for the observations to be processed in the least-squares (LS) linear estimation problem of discrete-time random signals from multi-sensor noisy measurements transmitted through imperfect communication channels, when three types of random uncertainties may arise in the transmission process. More specifically, it is assumed that the measured outputs of each sensor, perturbed by random parameter matrices, are transmitted to a processing center (PC), and the observations processed for the estimation may randomly be *one-step delayed*, contain only noise (*uncertain observations*), or be *lost* during transmission. Different sequences of Bernoulli random variables with known probabilities are introduced to depict these different uncertainties in the transmission and, in case of loss, the last observation that successfully arrived is used for the estimation.

In this context, our goal is to find recursive algorithms for the LS linear prediction, filtering and fixed-point smoothing problems using the centralized fusion method. These algorithms will be obtained under the assumption that the evolution model of the signal to be estimated is unknown and only information about its mean and covariance functions is available; this information is specified in the following hypothesis:

Hypothesis 1: *The n_x-dimensional signal process $\{x_k\}_{k\geq 1}$ has zero mean and its autocovariance function is expressed in a separable form, $E[x_k x_s^T] =$*

$A_k B_s^T$, $s \le k$, *where* $A_k, B_s \in \mathbb{R}^{n_x \times M}$ *are known matrices.*

2.1. Multi-sensor measured outputs with random parameter matrices

Usually, the signal measured outputs are subject to uncertainties which cannot be described only by the usual additive disturbances. For example, stochastic sensor gain degradation [4], multiplicative noises [5], missing [6] or fading [7] measurements, or even both multiplicative noises and missing measurements [8]. A unified framework to model these random phenomena is provided by the use of random measurement matrices.

In this paper, we consider measured outputs with random parameter matrices. So, we assume that there are m sensors which provide measurements of the signal process according to the following model:

$$z_k^{(i)} = H_k^{(i)} x_k + v_k^{(i)}, \quad k \ge 1, \quad i = 1, \dots, m, \tag{1}$$

where $z_k^{(i)} \in \mathbb{R}^{n_z}$ is the signal measured output from the i-th sensor at time k, which will be transmitted to the PC by an unreliable network, $H_k^{(i)}$ is a random parameter matrix and $v_k^{(i)}$ is the measurement noise vector. The following hypotheses are assumed:

Hypothesis 2: $\{H_k^{(i)}\}_{k \ge 1}$, $i = 1, \dots, m$, *are independent sequences of independent random parameter matrices with known means,* $E[H_k^{(i)}] = \overline{H}_k^{(i)}$. *Moreover, by denoting* $h_{pq}^{(i)}(k)$ *the* (p, q)*-th entry of* $H_k^{(i)}$*, the expectations* $E[h_{pq}^{(i)}(k) h_{p'q'}^{(i)}(k)]$ *are also assumed to be known, for* $p, p' = 1, \dots, n_z$ *and* $q, q' = 1, \dots, n_x$.

Hypothesis 3: *The measurement noises* $\{v_k^{(i)}\}_{k \ge 1}$, $i = 1, \dots, m$, *are zero-mean second-order processes with*

$$E[v_k^{(i)} v_s^{(j)T}] = R_k^{(ij)} \delta_{k,s} + R_{k,k-1}^{(ij)} \delta_{k-1,s}, \quad s \le k.$$

2.2. Observation transmission model with mixed uncertainties

As already indicated, due to eventual imperfections of the communication channels, one-step delays, uncertain observations or packet dropouts may occur randomly in data transmissions from the individual sensors to the PC, with different rates. Specifically, the following model is considered for the processed measurements, $y_k^{(i)}$, coming from the i-th sensor:

$$\begin{aligned} y_k^{(i)} &= \gamma_{0,k}^{(i)} z_k^{(i)} + \gamma_{1,k}^{(i)} z_{k-1}^{(i)} + \gamma_{2,k}^{(i)} v_k^{(i)} + \gamma_{3,k}^{(i)} y_{k-1}^{(i)}, \ k \geq 2; \\ y_1^{(i)} &= \gamma_{0,1}^{(i)} z_1^{(i)} + \gamma_{2,1}^{(i)} v_1^{(i)}, \quad i = 1, \dots, m, \end{aligned} \tag{2}$$

where $\gamma_{d,k}^{(i)}$, $d = 0, 1, 2, 3$, are Bernoulli random variables such that, for $k \geq 2$, $\sum_{d=0}^{3} \gamma_{d,k}^{(i)} = 1$, and $\gamma_{0,1}^{(i)} + \gamma_{2,1}^{(i)} = 1$. The following hypothesis on these variables is assumed:

Hypothesis 4: *$\left\{\left(\gamma_{0,k}^{(i)}, \gamma_{1,k}^{(i)}, \gamma_{2,k}^{(i)}\right)^T\right\}_{k\geq1}$, with $\gamma_{1,1}^{(i)} = 0$, $i = 1, \dots, m$, are independent sequences of independent random vectors, whose components are Bernoulli random variables with known probabilities, $\overline{\gamma}_{d,k}^{(i)} \equiv P[\gamma_{d,k}^{(i)} = 1]$, for $i = 1, \dots, m$ and $d = 0, 1, 2$.*

From this assumption, it is clear that $\overline{\gamma}_{3,k}^{(i)} \equiv P[\gamma_{3,k}^{(i)} = 1] = 1 - \sum_{d=0}^{2} \overline{\gamma}_{d,k}^{(i)}$ and, also, for $i, j = 1, \dots, m$, and $d, d' = 0, 1, 2, 3$, the correlation of the variables $\gamma_{d,k}^{(i)}$ and $\gamma_{d',s}^{(j)}$ is known, and it is given by:

$$E\left[\gamma_{d,k}^{(i)} \gamma_{d',s}^{(j)}\right] = \begin{cases} \overline{\gamma}_{d,k}^{(i)} \delta_{d,d'}, & i = j \text{ and } \ k = s \\ \overline{\gamma}_{d,k}^{(i)} \overline{\gamma}_{d',s}^{(j)}, & i \neq j \text{ or } k \neq s. \end{cases} \tag{3}$$

Finally, the following independence hypothesis is also required:

Hypothesis 5: *For each $i = 1, \dots, m$, the signal, $\{x_k\}_{k\geq1}$, and the processes $\{H_k^{(i)}\}_{k\geq1}$, $\{v_k^{(i)}\}_{k\geq1}$ and $\left\{\left(\gamma_{0,k}^{(i)}, \gamma_{1,k}^{(i)}, \gamma_{2,k}^{(i)}\right)^T\right\}_{k\geq1}$ are mutually independent.*

3. Stacked observation model

To address the estimation problem by the centralized fusion method, the observations from the different sensors are gathered and jointly processed at each sampling time to yield the optimal signal estimator. Therefore, our aim is to obtain a recursive algorithm for the LS linear estimator of x_k based on $\left\{y_1^{(i)}, \dots, y_L^{(i)}, \ i = 1, \dots, m\right\}$, which will be denoted by $\widehat{x}_{k/L}$, and the problem will be addressed considering, at each time $k \geq 1$, the vector constituted by the measurements of all sensors. For this purpose, the equations (1) and (2) are combined to yield the following stacked observation model:

$$
\begin{aligned}
z_k &= H_k x_k + v_k, \quad k \geq 1. \\
y_k &= \Gamma_{0,k} z_k + \Gamma_{1,k} z_{k-1} + \Gamma_{2,k} v_k + \Gamma_{3,k} y_{k-1}, \quad k \geq 2; \\
y_1 &= \Gamma_{0,1} z_1 + \Gamma_{2,1} v_1,
\end{aligned}
\tag{4}
$$

where:
$z_k = \left(z_k^{(1)T}, \dots, z_k^{(m)T}\right)^T$, $H_k = \left(H_k^{(1)T}, \dots, H_k^{(m)T}\right)^T$, $v_k = \left(v_k^{(1)T}, \dots, v_k^{(m)T}\right)^T$, and $\Gamma_{d,k} = diag\left(\gamma_{d,k}^{(1)}, \dots, \gamma_{d,k}^{(m)}\right) \otimes I_{n_z}, \quad d = 0, 1, 2, 3.$

Therefore, the problem is reformulated as that of obtaining the LS linear estimator of the signal, x_k, based on the observations $\{y_1, \dots, y_L\}$, given in (4). Next, the statistical properties of the processes involved in the observation model (4), necessary to address the LS linear estimation problem are specified:

- $\{H_k\}_{k \geq 1}$ is a sequence of independent random matrices with known means, $\overline{H}_k \equiv E[H_k] = \left(\overline{H}_k^{(1)T}, \dots, \overline{H}_k^{(m)T}\right)^T$, and it satisfies
$$E[H_k x_k x_k^T H_k^T] = E[H_k A_k B_k^T H_k^T] = \left(E[H_k^{(i)} A_k B_k^T H_k^{(j)T}]\right)_{i,j=1,\dots,m}$$
where the (p, q)-th entries, $p, q = 1, \dots, n_z$, of these matrices are given by:
$$\left(E[H_k^{(i)} A_k B_k^T H_k^{(j)T}]\right)_{pq} = \sum_{a=1}^{n_x} \sum_{b=1}^{n_x} E[h_{pa}^{(i)}(k) h_{qb}^{(j)}(k)] \left(A_k B_k^T\right)_{ab}.$$
Also, for $s \neq k$, we have $E[H_k A_k B_s^T H_s^T] = \overline{H}_k A_k B_s^T \overline{H}_s^T$.

- $\{v_k\}_{k \geq 1}$ is a zero-mean noise process with $E[v_k v_s^T] = R_k \delta_{k,s} + R_{k,k-1} \delta_{k-1,s}$, for $s \leq k$, where $R_{k,s} = \left(R_{k,s}^{(ij)}\right)_{i,j=1,\dots,m}$.

- $\{\Gamma_{d,k}\}_{k \geq 1}$, $d = 0, 1, 2, 3$, are sequences of independent random matrices with means $\overline{\Gamma}_{d,k} \equiv E[\Gamma_{d,k}] = diag\left(\overline{\gamma}_{d,k}^{(1)}, \dots, \overline{\gamma}_{d,k}^{(m)}\right) \otimes I_{n_z}$. If we denote $\gamma_{d,k} = \left(\gamma_{d,k}^{(1)}, \dots, \gamma_{d,k}^{(m)}\right)^T \otimes 1_{n_z}$, the Hadamard product properties guarantee that, for any deterministic matrix S, $E[\Gamma_{d,k} S \Gamma_{d',k}] = K_{d,d'}^{\gamma_k} \circ S$, being $K_{d,d'}^{\gamma_k} \equiv E[\gamma_{d,k} \gamma_{d',k}^T]$, for $d, d' = 0, 1, 2, 3$, the correlation matrices whose entries are given in (3).

- The processes $\{x_k\}_{k\geq 1}$, $\{H_k\}_{k\geq 1}$, $\{v_k\}_{k\geq 1}$ and $\{(\Gamma_{0,k}, \Gamma_{1,k}, \Gamma_{2,k}, \Gamma_{3,k})\}_{k\geq 1}$ are mutually independent.

Remark 1. In order to simplify the algorithm derivations, the observation model (4) is rewritten in an equivalent way as follows:

$$\begin{aligned} y_k &= \Gamma_{0,k} H_k x_k + \Gamma_{1,k}\overline{H}_{k-1}x_{k-1} + \Gamma_{3,k}y_{k-1} + W_k, \;\; k\geq 2; \\ y_1 &= \Gamma_{0,1}H_1x_1 + v_1. \\ W_k &= \Gamma_{1,k}\widetilde{H}_{k-1}x_{k-1} + \big(\Gamma_{0,k}+\Gamma_{2,k}\big)v_k + \Gamma_{1,k}v_{k-1}, \;\; k\geq 2, \end{aligned} \tag{5}$$

with $\widetilde{H}_k = H_k - \overline{H}_k$, for $k \geq 1$.

In the following lemmas we present the expressions of the covariance matrices of the processes involved in the observation model. From the previous properties, the proof of these lemmas is straightforward, so the details are omitted.

Lemma 1. *$\{z_k\}_{k\geq 1}$ is a zero-mean mn_z-dimensional process whose autocovariance function, $\Sigma^z_{k,s} \equiv E\big[z_k z_s^T\big]$, is given by:*

$$\Sigma^z_{k,s} = E\big[H_k A_k B_s^T H_s^T\big] + R_k\delta_{k,s} + R_{k,k-1}\delta_{k-1,s}, \quad 1\leq s\leq k.$$

Lemma 2. *$\{y_k\}_{k\geq 1}$ is a zero-mean mn_z-dimensional process and the covariance matrices $\Sigma^y_{k,s} \equiv E[y_k y_s^T]$, for $s = k, k-1$, and $\Sigma^y_{k-1,k} = \Sigma^{yT}_{k,k-1}$ are obtained by the following expressions:*

$$\begin{aligned} \Sigma^y_k = {} & K^{\gamma_k}_{0,0}\circ\Sigma^z_k + K^{\gamma_k}_{1,1}\circ\Sigma^z_{k-1} + (K^{\gamma_k}_{0,2} + K^{\gamma_k}_{2,0} + K^{\gamma_k}_{2,2})\circ R_k + K^{\gamma_k}_{3,3}\circ\Sigma^y_{k-1} \\ & + K^{\gamma_k}_{0,1}\circ\Sigma^z_{k,k-1} + K^{\gamma_k}_{1,0}\circ\Sigma^z_{k-1,k} + K^{\gamma_k}_{0,3}\circ\Sigma^{zy}_{k,k-1} + K^{\gamma_k}_{3,0}\circ\Sigma^{yz}_{k-1,k} \\ & + K^{\gamma_k}_{1,3}\circ\Sigma^{zy}_{k-1} + K^{\gamma_k}_{3,1}\circ\Sigma^{yz}_{k-1} + K^{\gamma_k}_{1,2}\circ R_{k-1,k} + K^{\gamma_k}_{2,1}\circ R_{k,k-1} \\ & + K^{\gamma_k}_{3,2}\circ\big((\overline{\Gamma}_{0,k-1}+\overline{\Gamma}_{2,k-1})R_{k-1,k}\big) \\ & + K^{\gamma_k}_{2,3}\circ\big(R_{k,k-1}(\overline{\Gamma}_{0,k-1}+\overline{\Gamma}_{2,k-1})\big), \quad k\geq 2, \\ \Sigma^y_1 = {} & K^{\gamma_1}_{0,0}\circ\Sigma^z_1 + (K^{\gamma_1}_{0,2} + K^{\gamma_1}_{2,0} + K^{\gamma_1}_{2,2})\circ R_1, \end{aligned}$$

$$\begin{aligned} \Sigma^y_{k,k-1} = {} & \overline{\Gamma}_{0,k}\Sigma^{zy}_{k,k-1} + \overline{\Gamma}_{1,k}\Sigma^{zy}_{k-1} + \overline{\Gamma}_{2,k}R_{k,k-1}\left(\overline{\Gamma}_{0,k-1}+\overline{\Gamma}_{2,k-1}\right) \\ & + \overline{\Gamma}_{3,k}\Sigma^y_{k-1}, \quad k\geq 2, \end{aligned}$$

where $\Sigma^{zy}_{k,s} \equiv E\big[z_k y_s^T\big]$ *are given by:*

$$\begin{aligned}\Sigma^{zy}_{k,s} &= \Sigma^{z}_{k,s}\overline{\Gamma}_{0,s} + \Sigma^{z}_{k,s-1}\overline{\Gamma}_{1,s} + R_k\overline{\Gamma}_{2,k}\delta_{k,s} + R_{k,k-1}\overline{\Gamma}_{2,k-1}\delta_{k-1,s} \\ &\quad + \Sigma^{zy}_{k,s-1}\overline{\Gamma}_{3,s}, \quad 2 \le s \le k, \\ \Sigma^{zy}_{k,1} &= \Sigma^{z}_{k,1}\overline{\Gamma}_{0,1} + \big(R_1\delta_{k,1} + R_{2,1}\delta_{k-1,1}\big)\overline{\Gamma}_{2,1}, \quad k \ge 1.\end{aligned}$$

Lemma 3. *The* mn_z*-dimensional process* $\{W_k\}_{k\ge 2}$*, defined in (5), has zero mean and the covariance matrices* $\Sigma^{W}_{k,k-1} \equiv E[W_k W_{k-1}^T]$ *are obtained by:*

$$\begin{aligned}\Sigma^{W}_{k,k-1} &= \big(\big(\overline{\Gamma}_{0,k} + \overline{\Gamma}_{2,k}\big)R_{k,k-1} + \overline{\Gamma}_{1,k}R_{k-1}\big)\big(\overline{\Gamma}_{0,k-1} + \overline{\Gamma}_{2,k-1}\big) \\ &\quad + \overline{\Gamma}_{1,k}R_{k-1,k-2}\overline{\Gamma}_{1,k-1}, \ k \ge 3; \\ \Sigma^{W}_{2,1} &= \big(\overline{\Gamma}_{0,2} + \overline{\Gamma}_{2,2}\big)R_{2,1} + \overline{\Gamma}_{1,2}R_1.\end{aligned}$$

4. Centralized fusion estimators

Our aim in this section is to obtain recursive algorithms for the LS linear centralized prediction, filtering and fixed-point smoothing problems. For this purpose, an innovation approach will be used.

4.1. Innovation approach to the LS linear estimation problem

The innovation approach consists of transforming the observation process $\{y_k\}_{k\ge 1}$ into an equivalent one of orthogonal vectors, the *innovation process*, $\{\mu_k\}_{k\ge 1}$, defined as $\mu_k = y_k - \widehat{y}_{k/k-1}$, where $\widehat{y}_{k/k-1}$, the one-stage observation predictor, is the orthogonal projection of y_k onto the linear space generated by $\{\mu_1, \dots, \mu_{k-1}\}$. So, denoting $\Pi_h = E[\mu_h \mu_h^T]$, the following general expression for the LS linear estimator of a vector ξ_k based on the observations $\{y_1, \dots, y_L\}$ is obtained

$$\widehat{\xi}_{k/L} = \sum_{h=1}^{L} E\big[\xi_k \mu_h^T\big]\Pi_h^{-1}\mu_h. \tag{6}$$

From this expression, the first step to obtain the signal estimators is to find an explicit formula for the innovation or, equivalently, for the one-stage linear predictor of the observation. Using (5) and applying orthogonal projections, we have:

$$\widehat{y}_{k/k-1} = \overline{\Gamma}_{0,k}\overline{H}_k\widehat{x}_{k/k-1} + \overline{\Gamma}_{1,k}\overline{H}_{k-1}\widehat{x}_{k-1/k-1} + \overline{\Gamma}_{3,k}y_{k-1} + \widehat{W}_{k/k-1}, \quad k \ge 2.$$

Now, from (6), denoting $\mathcal{W}_{k,h} \equiv E\big[W_k \mu_h^T\big], \; h \leq k-1$, and taking into account that $\mathcal{W}_{k,h} = 0$ for $h \leq k-3$, we obtain:

$$\widehat{W}_{k/k-1} = \mathcal{W}_{k,k-2}\Pi_{k-2}^{-1}\mu_{k-2} + \mathcal{W}_{k,k-1}\Pi_{k-1}^{-1}\mu_{k-1}, \quad k \geq 3; \quad \widehat{W}_{2/1} = \mathcal{W}_{2,1}\Pi_1^{-1}\mu_1,$$

and, hence, the observation predictor is given by

$$\begin{aligned} \widehat{y}_{k/k-1} = {} & \overline{\Gamma}_{0,k}\overline{H}_k\widehat{x}_{k/k-1} + \overline{\Gamma}_{1,k}\overline{H}_{k-1}\widehat{x}_{k-1/k-1} + \overline{\Gamma}_{3,k}y_{k-1} \\ & + \sum_{h=1}^{(k-1)\wedge 2} \mathcal{W}_{k,k-h}\Pi_{k-h}^{-1}\mu_{k-h}, \quad k \geq 2. \end{aligned} \tag{7}$$

This expression for the one-stage observation predictor, along with the general expression (6) for the LS linear estimators as linear combination of the innovations, are the starting points to derive the centralized prediction and filtering recursive algorithm presented in Theorem 1.

4.2. Centralized prediction and filtering recursive algorithm

In the following theorem, a recursive algorithm is given for the optimal LS linear centralized fusion estimators $\widehat{x}_{k/L}$, $L \leq k$, of the signal x_k based on the observations $\{y_1, \ldots, y_L\}$ given in (4) or, equivalently, in (5). The theorem includes a recursive expression for the error covariance matrices, which are a measure of the estimator performance.

Theorem 1. *The centralized predictors and filter, $\widehat{x}_{k/L}, \;\; L \leq k$, and their corresponding error covariance matrices, $\widehat{\Sigma}_{k/L} \equiv E[(x_k - \widehat{x}_{k/L})(x_k - \widehat{x}_{k/L})^T]$, are obtained by*

$$\widehat{x}_{k/L} = A_k O_L, \qquad \widehat{\Sigma}_{k/L} = A_k\left(B_k - A_k r_L\right)^T, \quad L \leq k, \tag{8}$$

where the vectors O_L and the matrices $r_L \equiv E\big[O_L O_L^T\big]$ are recursively obtained from

$$O_L = O_{L-1} + J_L \Pi_L^{-1}\mu_L, \quad L \geq 1; \quad O_0 = 0, \tag{9}$$

$$r_L = r_{L-1} + J_L \Pi_L^{-1} J_L^T, \quad L \geq 1; \quad r_0 = 0, \tag{10}$$

and the matrices $J_L \equiv E\big[O_L \mu_L^T\big]$ satisfy

$$J_L = \overline{\mathcal{H}}_{B_L}^T - r_{L-1}\overline{\mathcal{H}}_{A_L}^T - \sum_{h=1}^{(L-1)\wedge 2} J_{L-h}\Pi_{L-h}^{-1}\mathcal{W}_{L,L-h}^T, \quad L \geq 2; \quad J_1 = \overline{\mathcal{H}}_{B_1}^T. \tag{11}$$

The innovations, μ_L, and their covariance matrices, Π_L, are given by

$$\begin{aligned}\mu_L &= y_L - \overline{\Gamma}_{3,L}y_{L-1} - \overline{\mathcal{H}}_{A_L}O_{L-1} - \sum_{h=1}^{(L-1)\wedge 2}\mathcal{W}_{L,L-h}\Pi_{L-h}^{-1}\mu_{L-h}, \quad L\geq 2;\\ \mu_1 &= y_1,\end{aligned} \tag{12}$$

and

$$\begin{aligned}\Pi_L &= \Sigma_L^y - \overline{\Gamma}_{3,L}\Sigma_{L-1,L}^y - \Sigma_{L,L-1}^y\overline{\Gamma}_{3,L} + \overline{\Gamma}_{3,L}\Sigma_{L-1}^y\overline{\Gamma}_{3,L} - \overline{\mathcal{H}}_{A_L}\big(\overline{\mathcal{H}}_{B_L}^T - J_L\big)\\ &\quad - \sum_{h=1}^{(L-1)\wedge 2}\mathcal{W}_{L,L-h}\Pi_{L-h}^{-1}\big(\overline{\mathcal{H}}_{A_L}J_{L-h} + \mathcal{W}_{L,L-h}\big)^T, \quad L\geq 2;\\ \Pi_1 &= \Sigma_1^y.\end{aligned} \tag{13}$$

The coefficients $\mathcal{W}_{L,L-h} = E\big[W_L\mu_{L-h}^T\big]$, for $h=1,2$, satisfy

$$\begin{aligned}\mathcal{W}_{L,L-2} &= \overline{\Gamma}_{1,L}R_{L-1,L-2}\left(\overline{\Gamma}_{0,L-2} + \overline{\Gamma}_{2,L-2}\right), \quad L\geq 3.\\ \mathcal{W}_{L,L-1} &= \overline{\Gamma}_{1,k}E[\widetilde{H}_{k-1}A_{k-1}B_{k-1}^T\widetilde{H}_{k-1}^T]\overline{\Gamma}_{0,k-1} + \Sigma_{L,L-1}^W\\ &\quad -\mathcal{W}_{L,L-2}\Pi_{L-2}^{-1}\big(\overline{\mathcal{H}}_{A_{L-1}}J_{L-2} + \mathcal{W}_{L-1,L-2}\big)^T, \quad L\geq 3;\\ \mathcal{W}_{2,1} &= \overline{\Gamma}_{1,2}E[\widetilde{H}_1A_1B_1^T\widetilde{H}_1^T]\overline{\Gamma}_{0,1} + \Sigma_{2,1}^W.\end{aligned} \tag{14}$$

Finally, the matrices Σ_L^y, $\Sigma_{L,L-1}^y$ and $\Sigma_{L,L-1}^W$ are given in lemmas 2 and 3, respectively, and the matrices $\overline{\mathcal{H}}_{\Psi_L}$ with $\Psi_L = A_L, B_L$, are defined by

$$\overline{\mathcal{H}}_{\Psi_L} = \overline{\Gamma}_{0,L}\overline{H}_L\Psi_L + \overline{\Gamma}_{1,L}\overline{H}_{L-1}\Psi_{L-1}, \quad L\geq 2; \quad \overline{\mathcal{H}}_{\Psi_1} = \overline{\Gamma}_{0,1}\overline{H}_1\Psi_1. \tag{15}$$

Proof. From the general expression (6), to obtain the LS linear estimators $\widehat{x}_{k/L}$, $L\leq k$, it is necessary to calculate the coefficients

$$\mathcal{X}_{k,h} \equiv E\big[x_k\mu_h^T\big] = E\big[x_ky_h^T\big] - E\big[x_k\widehat{y}_{h/h-1}^T\big], \quad h\leq k.$$

Using the separable form of the signal covariance (*Hypothesis 1*) and the independence hypotheses on the model, it is clear that $E[x_ky_h^T] = A_k\overline{\mathcal{H}}_{B_h}^T$, with $\overline{\mathcal{H}}_{B_h}$ given in (15). Now, from expression (7) for $\widehat{y}_{h/h-1}$, together with (6) for $\widehat{x}_{h/h-1}$ and $\widehat{x}_{h-1/h-1}$, it is immediately deduced that the coefficients $\mathcal{X}_{k,h}$ can be expressed as $\mathcal{X}_{k,h} = A_kJ_h,\ 1\leq h\leq k$, with J_h given by

$$J_h = \overline{\mathcal{H}}_{B_h}^T - \sum_{j=1}^{h-1}J_j\Pi_j^{-1}J_j\overline{\mathcal{H}}_{A_h}^T - \sum_{j=1}^{(h-1)\wedge 2}J_{h-j}\Pi_{h-j}^{-1}\mathcal{W}_{h,h-j}^T, \ \ h\geq 2; \quad J_1 = \overline{\mathcal{H}}_{B_1}^T. \tag{16}$$

Then, by denoting $O_L = \sum_{h=1}^{L} J_h \Pi_h^{-1} \mu_h$ for $L \geq 1$, which clearly satisfies (9), expression (8) for $\widehat{x}_{k/L}$ is easily obtained from (6).

Now, denoting $r_L = \sum_{h=1}^{L} J_h \Pi_h^{-1} J_h^T$, for $L \geq 1$, which obviously satisfies (10), expression (11) for J_L is easily derived just making $h = L$ in (16). Next, by substituting $\widehat{x}_{L/L} = A_L O_L$ and $\widehat{x}_{L/L-1} = A_L O_{L-1}$ in (7) for $k = L$ and using (15), expression (12) for the innovation is obtained.

To prove expression (13) for $\Pi_L = E[\mu_L \mu_L^T]$, we apply the Orthogonal Projection Lemma (OPL) to write $\Pi_L = \Sigma_L^y - E\big[\widehat{y}_{L/L-1} \widehat{y}_{L/L-1}^T\big]$, with $\Sigma_L^y = E\big[y_L y_L^T\big]$ given in Lemma 2, and applying again the OPL, we have

$$
\begin{aligned}
E\big[\widehat{y}_{L/L-1} \widehat{y}_{L/L-1}^T\big] &= E\big[\widehat{y}_{L/L-1} y_L^T\big] \\
&= E\big[\widehat{y}_{L/L-1} \big(y_L - \Gamma_{3,L} y_{L-1}\big)^T\big] + \Sigma_{L,L-1}^y, \overline{\Gamma}_{3,L},
\end{aligned}
$$

where $\Sigma_{L,L-1}^y = E\big[y_L y_{L-1}^T\big]$ is also given in Lemma 2. Now, using that $\widehat{y}_{L/L-1} = \overline{\Gamma}_{3,L} y_{L-1} + \overline{\mathcal{H}}_{A_L} O_{L-1} + \sum_{h=1}^{(L-1)\wedge 2} \mathcal{W}_{L,L-h} \Pi_{L-h}^{-1} \mu_{L-h}$ in the expectation $E\big[\widehat{y}_{L/L-1}\big(y_L - \Gamma_{3,L} y_{L-1}\big)^T\big]$ and taking into account that

- $E\big[y_{L-1}\big(y_L - \Gamma_{3,L} y_{L-1}\big)^T\big] = \Sigma_{L-1,L}^y - \Sigma_{L-1}^y \overline{\Gamma}_{3,L}$,
- $E\big[O_{L-1}\big(y_L - \Gamma_{3,L} y_{L-1}\big)^T\big] = E\big[O_{L-1}\big(\widehat{y}_{L/L-1} - \overline{\Gamma}_{3,L} y_{L-1}\big)^T\big] = \overline{\mathcal{H}}_{B_L}^T - J_L$,
- $E\big[\mu_{L-h}\big(y_L - \Gamma_{3,L} y_{L-1}\big)^T\big] = \big(\overline{\mathcal{H}}_{A_L} J_{L-h} + \mathcal{W}_{L,L-h}\big)^T, \ h = 1, 2$,

expression (13) for Π_L is easily obtained.

Next, expression (14) for $\mathcal{W}_{L,L-h} = E\big[W_L \mu_{L-h}^T\big], \ h = 1, 2$, with W_L given in (5), is derived. Taking into account that W_L is uncorrelated with $y_h, \ h \leq L-3$, we have that, for $L \geq 3$,

$$
\mathcal{W}_{L,L-2} = E\big[W_L y_{L-2}^T\big] = E\big[W_L W_{L-2}^T\big] = \overline{\Gamma}_{1,L} R_{L-1,L-2} \left(\overline{\Gamma}_{0,L-2} + \overline{\Gamma}_{2,L-2}\right).
$$

Now, using (6) for $\widehat{y}_{L-1/L-2}$ in $\mathcal{W}_{L,L-1} = E\big[W_L y_{L-1}^T\big] - E\big[W_L \widehat{y}_{L-1/L-2}^T\big]$, we have $\mathcal{W}_{L,L-1} = E\big[W_L y_{L-1}^T\big] - \mathcal{W}_{L,L-2} \Pi_{L-2}^{-1} E[\mu_{L-2} y_{L-1}^T]$. From (5) and the independence between the signal and the observation noise, the first expectation involved in the previous formula is given by

$$E[W_L y_{L-1}^T] = E[W_L x_{L-1}^T H_{L-1}^T]\overline{\Gamma}_{0,L-1} + \mathcal{W}_{L,L-2}\overline{\Gamma}_{3,L-1} + \Sigma_{L,L-1}^W,$$

and using again (5) for W_L, we have

$$E[W_L x_{L-1}^T H_{L-1}^T] = \overline{\Gamma}_{1,L} E[\widetilde{H}_{L-1} A_{L-1} B_{L-1}^T \widetilde{H}_{L-1}^T].$$

Finally, using that $E\big[\mu_{L-2} y_{L-1}^T\big] = \big(\overline{\mathcal{H}}_{A_{L-1}} J_{L-2} + \mathcal{W}_{L-1,L-2}\big)^T + \Pi_{L-2}\overline{\Gamma}_{3,L-1}$, after some manipulations, expression (14) is proven and the proof of Theorem 1 is complete. □

4.3. Centralized fixed-point smoothing recursive algorithm

Starting with the filter, $\widehat{x}_{k/k}$, the fixed-point smoothers $\widehat{x}_{k/k+N}$, $N > 0$, $k \geq 1$, and their error covariance matrices are calculated in the following theorem by a recursive algorithm.

Theorem 2. *The fixed-point smoothers of the signal, $\widehat{x}_{k/k+N}$, $N > 0$, and their error covariance matrices, $\widehat{\Sigma}_{k/k+N} \equiv E[(x_k - \widehat{x}_{k/k+N})(x_k - \widehat{x}_{k/k+N})^T]$, are calculated as*

$$\widehat{x}_{k/k+N} = \widehat{x}_{k/k+N-1} + \mathcal{X}_{k,k+N}\Pi_{k+N}^{-1}\mu_{k+N}, \quad N \geq 1, \quad k \geq 1 \tag{17}$$

and

$$\widehat{\Sigma}_{k/k+N} = \widehat{\Sigma}_{k/k+N-1} - \mathcal{X}_{k,k+N}\Pi_{k+N}^{-1}\mathcal{X}_{k,k+N}^T, \quad N \geq 1, \quad k \geq 1. \tag{18}$$

The matrices $\mathcal{X}_{k,k+N} \equiv E[x_k \mu_{k+N}^T]$ satisfy

$$\mathcal{X}_{k,k+N} = \big(B_k - M_{k,k+N-1}\big)\overline{\mathcal{H}}_{A_{k+N}}^T - \sum_{h=1}^{(k+N-1)\wedge 2} \mathcal{X}_{k,k+N-h}\Pi_{k+N-h}^{-1}\mathcal{W}_{k+N,k+N-h}^T, \tag{19}$$

where $M_{k,k+N} \equiv E[x_k O_{k+N}^T]$ are obtained from the recursive formula

$$M_{k,k+N} = M_{k,k+N-1} + \mathcal{X}_{k,k+N}\Pi_{k+N}^{-1}J_{k+N}^T, \quad k \geq 1; \quad M_{k,k} = A_k r_k. \tag{20}$$

Proof. From (6), the signal smoothers are written as

$$\widehat{x}_{k/k+N} = \sum_{h=1}^{k+N} \mathcal{X}_{k,h}\Pi_h^{-1}\mu_h, \;\; N \geq 1;$$

then, by starting with the filter, $\widehat{x}_{k/k}$, it is immediately clear that such estimators are recursively obtained by (17), and from it, the recursive formula (18) for the error covariance matrices, $\widehat{\Sigma}_{k/k+N}$, is easily deduced.

Since $\mathcal{X}_{k,k+N} = E\big[x_k y_{k+N}^T\big] - E\big[x_k \widehat{y}_{k+N/k+N-1}^T\big], \; N \geq 1$, expression (19) is derived, calculating each of these expectations, as follows:

- *Hypothesis 1* together with (15), lead us to

$$E\big[x_k y_{k+N}^T\big] = B_k \overline{\mathcal{H}}_{A_{k+N}}^T + E\big[x_k y_{k+N-1}^T\big]\overline{\Gamma}_{3,k+N}, \quad N \geq 1.$$

- Using the expression of $\widehat{y}_{k+N/k+N-1}$ obtained from (12) is clear that

$$\begin{aligned} E\big[x_k \widehat{y}_{k+N/k+N-1}^T\big] &= E\big[x_k O_{k+N-1}^T\big]\overline{\mathcal{H}}_{A_{k+N}} + E\big[x_k y_{k+N-1}^T\big]\overline{\Gamma}_{3,k+N} \\ &+ \sum_{h=1}^{(k+N-1)\wedge 2} \mathcal{X}_{k,k+N-h}\Pi_{k+N-h}^{-1}\mathcal{W}_{k+N,k+N-h}^T, \quad N \geq 1. \end{aligned}$$

From the above items, we conclude that expression (19) holds true simply by denoting $M_{k,k+N} = E\big[x_k O_{k+N}^T\big]$. Using (9) for O_{k+N}, the recursive expression (20) for the matrices $M_{k,k+N}$ is also clear. $\square$

5. Numerical simulation example

In this section, a numerical example is shown to illustrate the application of the proposed centralized filtering and fixed-point smoothing algorithms and how the estimation accuracy is influenced by the probabilities of occurrence of missing measurements, random delays and packet dropouts during transmission. This example also illustrates some of the sensor uncertainties which are particular cases of the current measurement model (1) with random parameter matrices.

Let us consider that the system signal to be estimated is a zero-mean scalar process, $\{x_k\}_{k\geq 1}$, with autocovariance function $E[x_k x_j] = 1.025641 \times 0.95^{k-j}$,

$j \leq k$, which is factorizable according to *Hypothesis 1* just taking, for example, $A_k = 1.025641 \times 0.95^k$ and $B_k = 0.95^{-k}$.

The measured outputs of this signal, which are provided by four different sensors, are described in a unified way as in the proposed model (1) with random measurement matrices and correlated noises. Specifically,

$$z_k^{(i)} = H_k^{(i)} x_k + v_k^{(i)}, \quad k \geq 1, \quad i = 1, 2, 3, 4,$$

where the additive noises are defined as $v_k^{(i)} = c_i(\eta_k + \eta_{k+1}),\ i = 1, 2, 3, 4,$ with $c_1 = c_3 = 0.25$, $c_2 = 0.75$, $c_4 = 0.5$, and $\{\eta_k\}_{k\geq 1}$ is a standard Gaussian white process. These noises are clearly correlated, with $R_k^{(ij)} = 2c_i c_j,\ R_{k,k-1}^{(ij)} = c_i c_j,\ i, j = 1, 2, 3, 4$. The random measurement matrices are defined by $H_k^{(i)} = \theta_k^{(i)} C_k^{(i)}$, for $i = 1, 2, 3$, where $C_k^{(1)} = 0.82$, $C_k^{(2)} = 0.75$, $C_k^{(3)} = 0.74$, and $H_k^{(4)} = \theta_k^{(4)}\big(0.75 + 0.95\varphi_k\big)$, with $\{\varphi_k\}_{k\geq 1}$ a standard Gaussian white process, and $\{\theta_k^{(i)}\}_{k\geq 1},\ i = 1, 2, 3, 4$, white processes with the following time-invariant probability distributions:

- $\{\theta_k^{(1)}\}_{k\geq 1}$ are uniformly distributed over $[0.2, 0.7]$.
- $P\big(\theta_k^{(2)} = 0\big) = 0.3, \quad P\big(\theta_k^{(2)} = 0.5\big) = 0.3, \quad P\big(\theta_k^{(2)} = 1\big) = 0.4, \quad k \geq 1.$
- For $i = 3, 4$, $\{\theta_k^{(i)}\}_{k\geq 1}$ are Bernoulli variables with $P\big(\theta_k^{(i)} = 1\big) = \overline{\theta}, \quad k \geq 1.$

According to the theoretical observation model, suppose that random one-step delays, packet dropouts and uncertain observations with different rates happen in the data transmissions. More precisely, the possibility that uncertain observations, delays and packet dropouts simultaneously occur in the transmissions from sensor 4 is considered, while the measurements transmitted by the other sensors are only subject to one random failure: uncertain observations in sensor 1, one-step delays in sensor 2 and packet dropouts in sensor 3. Specifically, let us consider the observation model (2):

$$\begin{aligned} y_k^{(i)} &= \gamma_{0,k}^{(i)} z_k^{(i)} + \gamma_{1,k}^{(i)} z_{k-1}^{(i)} + \gamma_{2,k}^{(i)} v_k^{(i)} + \gamma_{3,k}^{(i)} y_{k-1}^{(i)},\ k \geq 2; \\ y_1^{(i)} &= \gamma_{0,1}^{(i)} z_1^{(i)} + \gamma_{2,1}^{(i)} v_1^{(i)}, \end{aligned}$$

where, for $i = 1, 2, 3, 4$, and $d = 0, 1, 2, 3$, $\{\gamma_{d,k}^{(i)}\}_{k\geq 1}$ are sequences of independent Bernoulli variables with $\overline{\gamma}_{0,1}^{(1)} = \overline{\gamma}_{0,1}^{(4)} = 0.9$, $\overline{\gamma}_{0,1}^{(2)} = \overline{\gamma}_{0,1}^{(3)} = 1$, and for $k \geq 2$, $\overline{\gamma}_{d,k}^{(i)} = \overline{\gamma}_{d}^{(i)}$.

To illustrate the feasibility and effectiveness of the proposed algorithms, they were implemented in MATLAB and fifty iterations of the centralized filtering and fixed-point smoothing algorithms were run.

For $\overline{\theta} = 0.5$, $\overline{\gamma}_0^{(i)} = 0.5$, $i = 1, 2, 3$, $\overline{\gamma}_2^{(1)} = \overline{\gamma}_1^{(2)} = \overline{\gamma}_3^{(3)} = 0.5$ and $\overline{\gamma}_d^{(4)} = 0.25$, $d = 0, 1, 2, 3$, Figure 1 displays the error variances of the centralized prediction, filtering and smoothing estimators. On the one hand, this figure shows that, as expected, the centralized fusion filtering estimators outperform the prediction ones and the error variances corresponding to the smoothers are less than those of the filter, thus confirming that the estimation accuracy of the smoothers is superior to that of the filters which, in turn, are more accurate than the predictors. On the other, it is also gathered that the accuracy of the predictors and fixed-point smoothers is better as the number of available observations increases. Similar results are obtained for other values of the probabilities $\overline{\theta}$ and $\overline{\gamma}_d^{(i)}$, as we show below in Figure 2.

Next, for $\overline{\gamma}_0^{(i)} = 0.9$, $i = 1, 2, 3$, $\overline{\gamma}_2^{(1)} = \overline{\gamma}_1^{(2)} = \overline{\gamma}_3^{(3)} = 0.1$ and $\overline{\gamma}_d^{(4)} = 0.25$, $d = 0, 1, 2, 3$, in order to show how the estimation accuracy is influenced by the probability $\overline{\theta}$ that the signal is present in the measured outputs of sensors 3 and 4, the centralized filtering and smoothing error variances are displayed in Figure 2 for different values of these probabilities. From this figure, it is observed that the performance of the centralized fusion estimators is indeed influenced by the probability $\overline{\theta}$ and, as expected, the filtering and smoothing error variances become smaller as $\overline{\theta}$ increases, which means that the performance of the centralized fusion estimators improves as the probability $1 - \overline{\theta}$ of only-noise measured outputs decreases, although this improvement is practically imperceptible for small values of $\overline{\theta}$ (see e.g. $\overline{\theta} = 0.2$ and 0.4).

Next, in order to show how the estimation accuracy is influenced by the effect

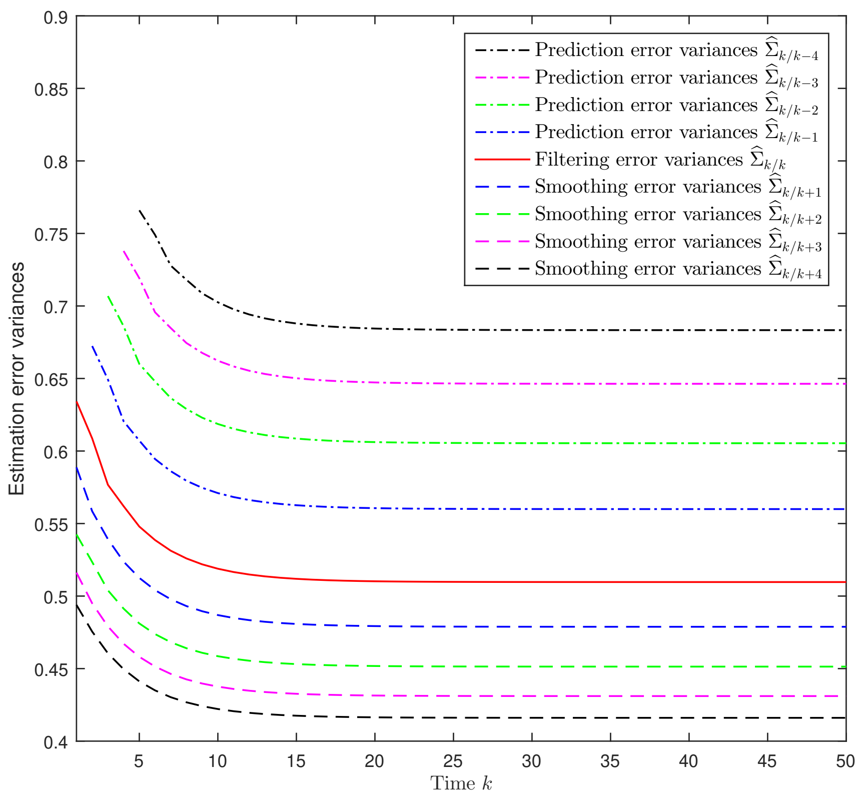

Figure 1: Error variance comparison of the centralized fusion filter and smoothers.

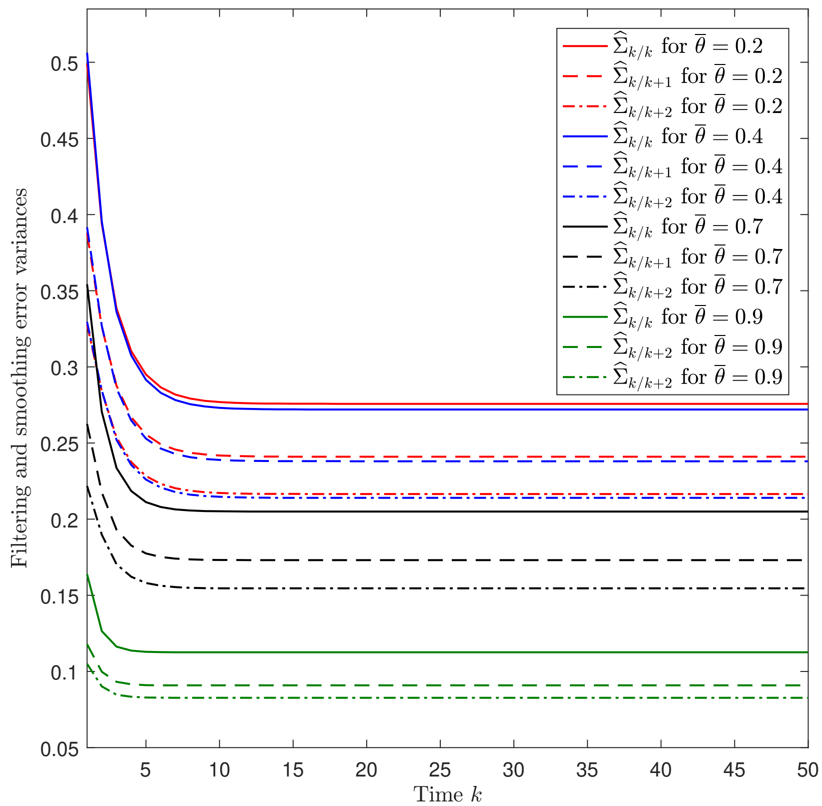

Figure 2: Centralized fusion filtering, $\widehat{\Sigma}_{k/k}$, and smoothing, $\widehat{\Sigma}_{k/k+N}$, $N = 1, 2$, error variances for different values of $\overline{\theta}$.

of missing measurements, random delays and packet dropouts in the transmissions from the sensors 1, 2 and 3, the centralized filtering error variances are displayed in Figure 3 for different probabilities, $\overline{\gamma}_0^{(i)}$, for $i = 1,2,3$, considering, as in Figure 1, the value $\overline{\theta} = 0.5$, and $\overline{\gamma}_d^{(4)} = 0.25, \quad d = 0,1,2,3$. From this figure, it is deduced that the performance of the filters is indeed influenced by these uncertainties and, as expected, the centralized error variances become smaller as some of the probabilities $\overline{\gamma}_0^{(i)}$ increase, which means that the performance of the centralized filter improves when $1-\overline{\gamma}_0^{(i)}$, for $i = 1,2,3$ (missing measurement probability in sensor 1, delay probability in sensor 2 and packet dropout probability in sensor 3) decrease. For example:

- If $\overline{\gamma}_0^{(1)} = \overline{\gamma}_0^{(2)} = \overline{\gamma}_0^{(3)}$, better estimators are obtained as $\overline{\gamma}_0^{(i)}$ increases (see $\overline{\gamma}_0^{(i)} = 0.7,\ 0.8,\ 0.9,\ i = 1,2,3$).
- If $\overline{\gamma}_0^{(2)} = \overline{\gamma}_0^{(3)}$, the error variances become lower as $\overline{\gamma}_0^{(1)}$ is higher (see $\overline{\gamma}_0^{(i)} = 0.9,\ i = 2,3$ and $\overline{\gamma}_0^{(1)} = 0.3,\ 0.9$).
- If $\overline{\gamma}_0^{(1)} = \overline{\gamma}_0^{(3)}$, better estimators are obtained as $\overline{\gamma}_0^{(2)}$ is higher (see $\overline{\gamma}_0^{(i)} = 0.9,\ i = 1,3$ and $\overline{\gamma}_0^{(2)} = 0.4,\ 0.9$).
- If $\overline{\gamma}_0^{(1)} = 0.9$ and $\overline{\gamma}_0^{(3)} = 0.2$, also lower error variances are obtained as $\overline{\gamma}_0^{(2)}$ increases (see $\overline{\gamma}_0^{(2)} = 0.5,\ 0.9$).

Finally, in order to analyze the performance of the proposed centralized filter in comparison with the centralized filter in [21] and the centralized Kalman filter, the different estimates are compared using the filtering mean-squared error at each time instant k (MSE_k), calculated as $\text{MSE}_k = \dfrac{1}{1000}\sum_{s=1}^{1000}\left(x_k^{(s)} - \widehat{x}_{k/k}^{(s)}\right)^2$, where $\left\{x_k^{(s)};\ 1 \leq k \leq 150\right\}$ denote the s-th set of artificially simulated data and $\widehat{x}_{k/k}^{(s)}$ is the filter at the sampling time k in the s-th simulation run. The results, assuming the same probabilities $\overline{\theta}$ and $\overline{\gamma}_{d,k}^{(i)}$ as in Figure 1, are displayed in Figure 4, which, as expected, shows that the MSE_k for the proposed filtering estimates are less than those of the other two filtering estimates. Indeed, the performance of the proposed filter was expected to be better than that of the centralized

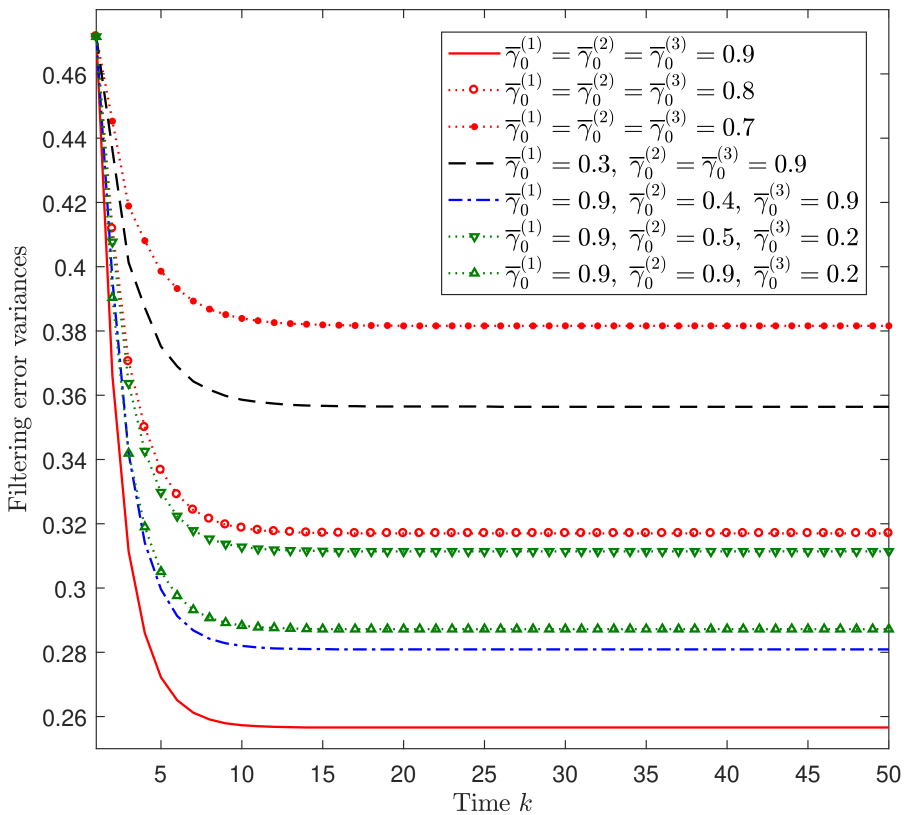

Figure 3: Centralized fusion filtering error variances for different values $\overline{\gamma}_0^{(i)}$, for $i = 1, 2, 3$.

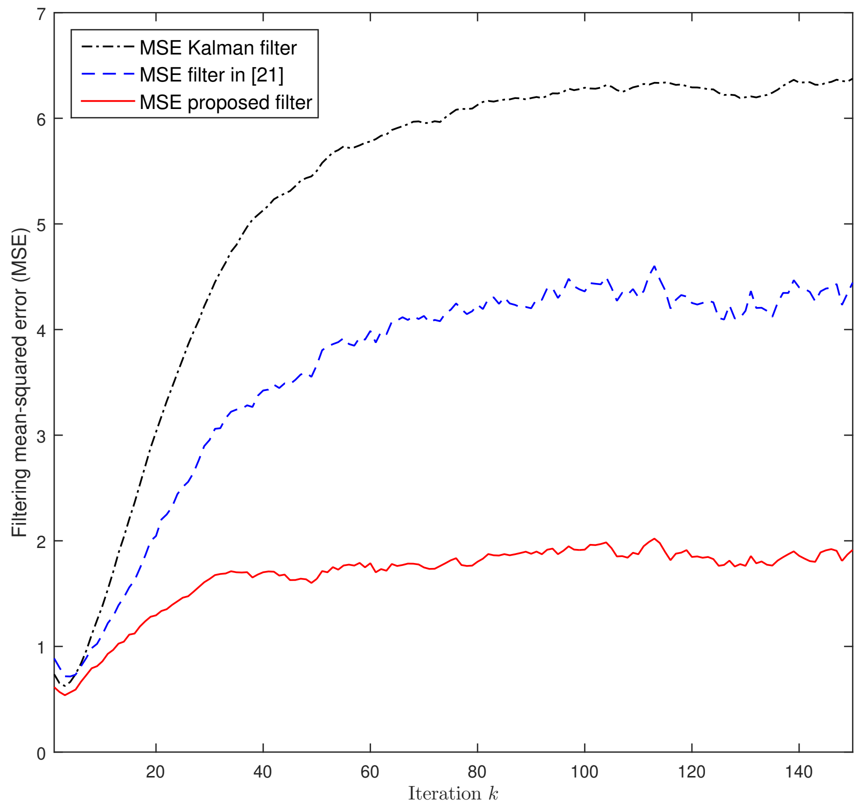

Figure 4: Comparison of mean-squared errors for the Kalman filter, filter in [21] and proposed filter.

filter [21], since the latter does not take into account the uncertainties in the sensor measurements, and the centralized Kalman filter was also expected to provide the worst estimations as it ignores the uncertainties in both the sensor measurements and the transmissions.

6. Conclusion

Centralized fusion prediction, filtering and fixed-point smoothing algorithms have been designed in sensor networks with measured outputs perturbed by random parameter matrices and correlated noises, using an innovation approach. Owing to the unreliable network characteristics, the information transmission through the network communication channels is assumed to be subject to mixed random failures. The current measured output model with random parameter matrices and correlated noises provides a general framework to deal with a great variety of networked systems featuring different network-induced stochastic uncertainties.

The proposed recursive estimation algorithms are easily implementable and do not require the evolution model generating the signal process, since it is based on covariance information. The estimation accuracy is measured by the estimation error covariances, which can be calculated offline as they do not depend on the current set of observed data.

A more general model, suggested by the anonymous reviewer, would be obtained by considering that, for each sensor, the transmission noise is not equal to the measurement noise. For this new model, estimation algorithms with a similar structure to the proposed ones would be obtained, but essential differences would arise in the expressions of the covariance matrices given in Lemma 2 and Lemma 3, as well as in expression (14) of Theorem 1. Consequently, the algorithms in the current paper would not be directly applicable in this situation.

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References

[1] E. Song, Y. Zhua, J. Zhou, Z. You, Optimal Kalman filtering fusion with cross-correlated sensor noises, Automatica 43 (2007) 1450–1456.

[2] J. Feng, M, Zeng, Optimal distributed Kalman filtering fusion for a linear dynamic system with cross-correlated noises, Int. J. Syst. Sci. 43 (2012) 385–398.

[3] L. Yan, X. Rong Li, Y. Xia, M. Fu, Optimal sequential and distributed fusion for state estimation in cross-correlated noise, Automatica 49 (2013) 3607–3612.

[4] Y. Liu, Z. Wang, X. He, D.H. Zhou, Minimum-variance recursive filtering over sensor networks with stochastic sensor gain degradation: Algorithms and performance analysis, IEEE Trans. Control Netw. Syst. 3 (2016) 265–274.

[5] X. Wang, S. Sun, Optimal recursive estimation for networked descriptor systems with packet dropouts, multiplicative noises and correlated noises, Aerosp. Sci. Technol. 63 (2017) 41–53.

[6] J. Hu, Z. Wang, F.E. Alsaadi, T. Hayat, Event-based filtering for time-varying nonlinear systems subject to multiple missing measurements with uncertain missing probabilities, Inf. Fusion 38 (2017) 74–83.

[7] W. Li, Y. Jia, J. Du, Distributed filtering for discrete-time linear systems with fading measurements and time-correlated noise, Digit. Signal Process. 60 (2017) 211–219.

[8] R. Caballero-Águila, A. Hermoso-Carazo, J. Linares-Pérez, Fusion estimation from multisensor observations with multiplicative noises and correlated random delays in transmission, Mathematics 5 (2017) 45.

[9] M.J. García-Ligero, A. Hermoso-Carazo, J. Linares-Pérez, Least-squares linear estimation of signals from observations with Markovian delays, J. Comput. Appl. Math. 236 (2011) 234–242.

[10] J. Ma, S. Sun, Centralized fusion estimators for multisensor systems with random sensor delays, multiple packet dropouts and uncertain observations, IEEE Sensors J. 13 (2013) 1228–1235.

[11] S. Sun, T. Tian, H. Lin, Optimal linear estimators for systems with finite-step correlated noises and packet dropout compensations, IEEE Trans. Signal Process. 64(21) (2016) 5672–5681.

[12] Y. Guo, Switched filtering for networked systems with multiple packet dropouts, J. Franklin Inst. 354 (2017) 3134–315.

[13] J. Hu, Z. Wang, H. Gao, Recursive filtering with random parameter matrices, multiple fading measurements and correlated noises, Automatica 49 (2013) 3440–3448.

[14] R. Caballero-Águila, A. Hermoso-Carazo, J. Linares-Pérez, Optimal state estimation for networked systems with random parameter matrices, correlated noises and delayed measurements, Int. J. Gen. Syst. 44 (2) (2015) 142–154.

[15] R. Caballero-Águila, A. Hermoso-Carazo, J. Linares-Pérez, Fusion estimation using measured outputs with random parameter matrices subject to random delays and packet dropouts, Signal Process. 127 (2016) 12–23.

[16] Y. Yang, Y. Liang, Q. Pan, Y. Qin, F. Yang, Distributed fusion estimation with square-root array implementation for Markovian jump linear systems with random parameter matrices and cross-correlated noises, Inf. Sci. 370–371 (2016) 446–462.

[17] R. Caballero-Águila, A. Hermoso-Carazo, J. Linares-Pérez, Covariance-based fusion filtering for networked systems with random transmission delays and non-consecutive losses, Int. J. Gen. Syst. 46 (2017) 752–771.

[18] S. Sun, T. Tian, H. Lin, State estimators for systems with random parameter matrices, stochastic nonlinearities, fading measurements and correlated noises, Inf. Sci. 397–398 (2017) 118–136.

[19] F. Han, H. Dong, Z. Wang, G. Li, F.E. Alsaadi, Improved Tobit Kalman filtering for systems with random parameters via conditional expectation, Signal Process. 147 (2018) 35–45.

[20] J. Ma, S. Sun, Distributed fusion filter for networked stochastic uncertain systems with transmission delays and packet dropouts, Signal Process. 130 (2017) 268–278.

[21] R. Caballero-Águila, A. Hermoso-Carazo, J. Linares-Pérez, Least-squares estimation in sensor networks with noise correlation and multiple random failures in transmission, Math. Probl. Eng. 2017 (2017) Article ID 1570719.