Economic Research-Ekonomska Istraživanja

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/rero20

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To cite this article: Xunjie Gou , Zeshui Xu , Wei Zhou & Enrique Herrera-Viedma (2021): The risk assessment of construction project investment based on prospect theory with linguistic preference orderings, Economic Research-Ekonomska Istraživanja, DOI: 10.1080/1331677X.2020.1868324

To link to this article: https://doi.org/10.1080/1331677X.2020.1868324

Published online: 17 Jan 2021.

ECONOMIC RESEARCH-EKONOMSKA ISTRAŽIVANJA
https://doi.org/10.1080/1331677X.2020.1868324

The risk assessment of construction project investment based on prospect theory with linguistic preference orderings

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ABSTRACT
Multiple experts decision-making (MEDM) can be regarded as a situation where a group of experts are invited to provide their opinions by evaluating the given alternatives, and then select the optimal alternative(s). As a useful linguistic expression model, linguistic preference orderings (LPOs) were established in which the order of alternatives and the relationships between two adjacent alternatives are fused well. Considering that prospect theory has the superiority in depicting risk attitudes (risk seeking for losses and risk aversion for gains) during the uncertain decision-making process, this paper develops a consensus model based on prospect theory to deal with MEDM problems with LPOs. Firstly, each LPO provided by expert is transformed into the responding DHLPR with complete consistency. Then, the reference point of expert is determined and the prospect preference matrix is established. Moreover, we can obtain the overall prospect consensus degree for a MEDM problem by calculating the similarity degree between individual and collective prospect preference matrix. Furthermore, a consensus improvement method is developed to complete the consensus reaching process. Finally, we apply the proposed method to deal with a practical MEDM problem involving the construction project investment, and make some comparative analyses with existing methods.

ARTICLE HISTORY
Received 1 October 2020
Accepted 19 December 2020

KEYWORDS
Risk assessment; construction project investment; prospect theory; linguistic preference orderings; consensus

SUBJECT CLASSIFICATION CODES
C44; C49; D81

1. Introduction

Multiple experts decision-making (MEDM) can be regarded as a situation in which a group of experts are invited to provide their individual opinions by evaluating the given alternatives, and then select the optimal alternative(s) by aggregating their opinions or using some decision-making methods (Gou et al., 2020c). Obviously, the first and most important step of MEDM is to acquire the assessment information of

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experts, and the preference ordering (PO) structures are usually used by experts to express their opinions considering that people prefer to rank the alternatives according to their own ideas or common sense (Chiclana et al., 1998; Hervés-Beloso & Cruces, 2018; He & Xu, 2018; Tanino, 1984; Zhang et al., 2018). Meanwhile, there are lots of PO structures such as POs (Chiclana et al., 1998; Zhang et al., 2018), hesitant PO sets (He & Xu, 2018), continuous POs (Hervés-Beloso & Cruces, 2018), fuzzy POs (Tanino, 1984), etc.

However, there exist two critical problems when using the existing POs. Firstly, the existing POs can only reflect the orderings of alternatives, but lack the research on the precise relationship between any two adjacent alternatives in the POs. In fact, some experts may prefer a more detailed sentence to express their opinions, such as "*A_2 is very faster than A_1, and A_1 is slightly faster than A_3*", instead of only using an order $A_2 \succ A_1 \succ A_3$. In addition, the existing methods are more inclined to aggregate the POs and obtain the final ordering of alternatives directly (He & Xu, 2018), but ignore the unbalanced relationship between any two adjacent alternatives. To overcome the first shortcoming, primarily, we can utilize the double hierarchy linguistic terms (DHLTs, the basic elements of double hierarchy linguistic term set (DHLTS) (Gou et al., 2017)) to express the relationship between any two adjacent alternatives. The reason is that the DHLTS can be used to handle complex linguistic information well by dividing them into two simple linguistic hierarchies, where the first hierarchy linguistic term set (LTS) is the main linguistic hierarchy and the second hierarchy LTS is the linguistic feature or detailed supplementary of each linguistic term in the first hierarchy LTS (Gou et al., 2017). Then, Gou et al. (2020c) developed a new concept of linguistic preference ordering (LPO) which is composed of PO and DHLTs, and they also focused on dealing with two kinds of LPOs, respectively. To overcome the second shortcoming, Gou et al. (2020c) transformed the LPOs into the corresponding double hierarchy linguistic preference relations (DHLPRs) with complete consistency (2020 b), and then obtained the decision-making result by handling these DHLPRs.

In the past decision-making processes, some existing methods are only used to deal with the experts' assessments (Fan et al., 2006; Liu et al., 2019) and do not consider the experts' psychological behaviors. What is more, lots of empirical evidences (Camerer, 1998; Kahneman & Tversky, 1979; Tversky & Kahneman, 1992) have shown that the experts' psychological behaviors would play an important role in decision analysis (Dong et al., 2015). As we know, Kahneman and Tversky (1979) proposed the concept of prospect theory, and it has the superiority in depicting risk attitudes (risk seeking for losses and risk aversion for gains) during the uncertain decision-making process (Tversky & Kahneman, 1992). Meanwhile, a lot of scholars have devoted to this aspect research and demonstrated the usefulness of behavioral decision-making revealed by the prospect theory (Abdellaoui, 2000; Birnbaum, 2005; Bleichrodt et al., 2009; Gonzalez & Wu, 1999; He & Zhou, 2011; Lu et al., 2020; Wu & Gonzalez, 1999; Zhou et al., 2019). In recent years, prospect theory has been applied to deal with some decision-making problems such as behavioral decision-making (Fan et al., 2013; Wang et al., 2015), multi-attribute decision-making (Fan et al., 2013; Liu et al., 2014; Tian et al., 2018, 2020) and MEDM (Yan & Liu, 2014), etc.

However, as far as we know, there is no research about the extension of prospect theory in consensus reaching process (CRP) of MEDM problems with LPOs. Therefore, the purpose of this paper is to develop a consensus model based on prospect theory to deal with MEDM problems with LPOs. Firstly, the LPOs provided by experts can be transformed into DHLPRs with complete consistency, which can ensure the integrity of the original assessment information (Gou et al., 2020c). Then, we can determine the reference point $s_{0<o_0>}$ of expert considering that it is the demarcation point of the positive and negative DHLTs, and establish the prospect preference matrix by calculating the gains and losses with respect to alternatives for each expert based on the equivalent transformation function (Gou et al., 2017) and the prospect value function introduced by Kahneman and Tversky (1979). Moreover, by calculating the similarity degree between individual prospect preference matrix and the collective prospect preference matrix, we can obtain the overall prospect consensus degree for a MEDM problem, and check whether the consensus is reached or not. If not, we develop a consensus improvement method which consists of identifying the experts and the pairs of alternatives that need to improve their consensus degrees, and feeding the suggestions back to the corresponding experts and telling them how to adjust their preferences. Finally, a model is set up to obtain the priority vector of each expert, and then the rank of alternatives is obtained based on the collective priority vector which is got by aggregating these individual priority vectors.

The main innovation points of this paper are highlighted as follows:

1. By fusing LPOs and prospect theory, we cannot only obtain more comprehensive assessment information of expert, but also have the superiority of prospect theory in depicting risk attitudes (risk seeking for losses and risk aversion for gains) during the decision-making process.
2. The prospect preference matrix of each expert is established by calculating the gains and losses with respect to alternatives.
3. A consensus improvement method is developed, which consists of the identification rules and the direction rules.
4. The final rank of alternatives can be obtained by establishing and solving a model which can be used to calculate the priority vector of each expert and the collective priority vector.

The remainder of the paper is organized as follows: Section 2 reviews some related concepts of DHLPR, LPOs and prospect theory. Section 3 develops a MEDM resolution framework with LPOs based on prospect theory. Section 4 applies the proposed method to deal with a practical MEDM problem involving the construction project investment, and makes comparative analysis with the existing methods. Some concluding remarks are summarized in Section 5.

2. Preliminaries

As the basis of this paper, some related concepts are reviewed in this section including DHLPR, LPOs and prospect theory.

2.1. Double hierarchy linguistic preference relation

For dealing with natural languages, Zadeh (2012) provided the concept of Computing with Words (CW). Based on CW, lots of linguistic representation models were developed (Juang & Chen, 2013; Pang et al., 2016; Rodríguez et al., 2012; Wang et al., 2018; Wei et al., 2020; Wei & Gao, 2020; Xu & Wang, 2017). However, the above linguistic representation models usually have some gaps when expressing some more complex and detailed linguistic information such as "*only a little fast*" and "*slightly high*". By splitting complex linguistic term into two parts with the form of "adverb + adjective" and expressing them by different kinds of linguistic terms respectively, Gou et al. (2017) defined the concept of DHLTS. Let $S = \{s_t | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ and $O^t = \{o_k^t | k = -\varsigma, \ldots, -1, 0, 1, \ldots, \varsigma\}$ be the first hierarchy LTS and the second hierarchy LTS of linguistic term s_t in S, respectively. A DHLTS, S_O, can be expressed by

$$S_O = \left\{s_{t<o_k^t>} | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau; k = -\varsigma, \ldots, -1, 0, 1, \ldots, \varsigma\right\} \quad (1)$$

where the basic element $s_{t<o_k^t>}$ is called DHLT, and o_k^t expresses the second hierarchy linguistic term of the linguistic term s_t in S. For convenience, Eq. (1) can be rewritten by a unified form $S_O = \{s_{t<o_k>} | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau; k = -\varsigma, \ldots, -1, 0, 1, \ldots, \varsigma\}$.

In recent years, many scholars began to pay attention to the research of double hierarchy linguistic information and developed a lot of research results including preference relations (Gou et al., 2018a, 2019, 2020a, 2020b, 2020c, 2020d), measure methodologies (Fu & Liao, 2019; Gou et al., 2018b) and decision methodologies (Fu & Liao, 2019; Gou et al., 2017, 2018a, 2018b, 2019, 2020a, 2020b, 2020c; Krishankumar et al., 2019; Liu et al., 2019a, 2019b; Wang et al., 2020), etc.

Let $\bar{S}_O = \{s_{t<o_k>} | t \in [-\tau, \tau]; k \in [-\varsigma, \varsigma]\}$ be a continuous DHLTS, then the numerical scale γ and the subscript (t, k) of the DHLT $s_{t<o_k>}$ which expresses the equivalent information to the membership degree γ can be transformed to each other by the following functions f and f^{-1} :

$$f : [-\tau, \tau] \times [-\varsigma, \varsigma] \to [0, 1], f(s_{t<o_k>}) = \frac{t + (\tau + k)\varsigma}{2\varsigma\tau} = \gamma \quad (2)$$

$$\begin{gathered} f^{-1} : [0, 1] \to [-\tau, \tau] \times [-\varsigma, \varsigma], \\ f^{-1}(\gamma) = [2\tau\gamma - \tau]<o_{\varsigma(2\tau\gamma - \tau - [2\tau\gamma - \tau])}> = [2\tau\gamma - \tau] + 1<o_{\varsigma((2\tau\gamma - \tau - [2\tau\gamma - \tau]) - 1)}> \end{gathered} \quad (3)$$

To get the concept of DHLPR, some operational laws of DHLTs were developed (Gou et al., 2017). Suppose that $s_{t<o_k>}$, $s_{t^1<o_{k^1}>}$ and $s_{t^2<o_{k^2}>}$ are three different DHLTs, and $\lambda (0 \le \lambda \le 1)$ is a real number. Then,

1. $s_{t^1<o_{k^1}>} \oplus s_{t^2<o_{k^2}>} = s_{t^1+t^2<o_{k^1+k^2}>}$, if $t^1 + t^2 \le \tau, k^1 + k^2 \le \varsigma$;
2. $\lambda s_{t<o_k>} = s_{\lambda t<o_{\lambda k}>}$.

Definition 1 (Gou et al., 2020b). Let S_O be a DHLTS. A DHLPR $\mathbb{R}$ is presented by a matrix $\mathbb{R} = (r_{ij})_{m\times m} \subset A \times A$, where $r_{ij} \in S_O(i,j = 1,2, \ldots, m)$ is a DHLT, indicating the degree of the alternative A_i over A_j. For all $i,j = 1,2, \ldots, m$, $r_{ij}(i < j)$ satisfies the conditions $r_{ij} + r_{ji} = s_{0<o_0>}$ and $r_{ii} = s_{0<o_0>}$.

In addition, a DHLPR $\mathbb{R}$ can be called an additively consistent DHLPR (Gou et al., 2020c) if it satisfies

$$f(r_{ij}) = f(r_{i\rho}) + f(r_{\rho j}) - 0.5 \quad (i,j,\rho = 1,2, \ldots, m, i \neq j) \tag{4}$$

To obtain the additively consistent DHLPR conveniently, Gou et al. (Gou et al., 2020c) proposed the following theorem:

Theorem 1 (Gou et al., 2020c). *Let* $\mathbb{R} = (r_{ij})_{m\times m} \subset A \times A$ *be a DHLPR. If* $f(\bar{r}_{ij}) = \frac{1}{m}\left(\oplus_{\rho=1}^{m}(f(r_{i\rho}) + f(r_{\rho j}) - 0.5)\right)$ *for all* $i,j,\rho = 1,2, \ldots, m, i \neq j$, *then* $\bar{\mathbb{R}} = (\bar{r}_{ij})_{m\times m} \subset A \times A$ *is an additively consistent DHLPR.*

2.2. Linguistic preference orderings

In MEGM processes, when experts evaluate alternatives and provide their POs, two forms of LPOs are very familiar. One is to rank all alternatives using a LPO in continuous form directly, and the other one is to give the relationship between any two alternatives and then all these relations make up a set of POs (2020c).

1. **The LPO in continuous form**

Let S_O and A be a DHLTS and a set of alternatives, respectively. Suppose that an expert e^a provides his/her linguistic preference information by a LPO denoted by:

$$LPO'^{a} = \left\{ A^a_{\sigma(1)} \overset{s^{(\sigma(1),\sigma(2))}_{t<o_k>}}{\succ} A^a_{\sigma(2)} \overset{s^{(\sigma(2),\sigma(3))}_{t<o_k>}}{\succ} \cdots \overset{s^{(\sigma(m\bar{1}),\sigma(m))}_{t<o_k>}}{\succ} A^a_{\sigma(m)} \right\} \tag{5}$$

where $\oplus_{i=1}^{m-1} s^{(\sigma(i),\sigma(i+1))}_{t<o_k>} \leq s_{\tau<o_\varsigma>}$, the $A^a_{\sigma(i)}(i = 1,2, \ldots, m)$ denotes the $i-$th largest alternative, and the linguistic preference, denoted as a DHLT $s^{(\sigma(i),\sigma(i+1))}_{t<o_k>}(i = 1,2, \ldots, m-1)$, means that the degree of the $i-$th largest alternative is better than the $i+1-$th largest alternative.

1. **The LPO in decentralized form**

Considering that the complexity of things and the fuzziness of people's cognition, sometimes some experts prefer to give some pairwise comparisons between any two alternatives rather than provide a complete PO. Therefore, in this case, the preference information provided by an expert e^a on A can be called a LPO in decentralized form, which is a set of PO pairs and can be shown as follows:

$$LPO''^{a} = \left\{ A_i^a \overset{s_{t<o_k>}^{ij}}{\succ} A_j^a \middle| s_{t<o_k>}^{ij} \in S_O, \quad i,j = 1, 2, \ldots, m; i \neq j \right\} \tag{6}$$

where $s_{t<o_k>}^{ij}$ expresses the relationship between A_i^a and A_j^a $(i,j = 1,\ 2,\ \ldots, m; i \neq j)$.

Next, two examples are given to show these two LPOs. Let $A = \{A_1, A_2, A_3, A_4\}$ be a set of alternatives, then $LPO' = \left\{A_3 \succ^{verymuchhigh} A_2 \succ^{alittlehigh} A_1 \succ^{onlyalittleveryhigh} A_4\right\}$ is a LPO in continuous form, and $LPO'' = \left\{A_2 \succ^{alittlehigh} A_3,\ \ A_2 \succ^{verymuchhigh} A_1,\ A_4 \succ^{onlyalittleveryhigh} A_3\right\}$ is a LPO in decentralized form.

Remark 1. For a LPO in continuous form, considering that the evaluation $s_{t<o_k>}^{(\sigma(1),\sigma(m))}$ between the alternative ranked in the first position A_1 and that in the final position A_m must be less than $s_{\tau<o_\varsigma>}$, and the sum of all evaluations in the LPO should be equal to $s_{t<o_k>}^{(\sigma(1),\sigma(m))}$. Therefore, we have $s_{t<o_k>}^{(\sigma(1),\sigma(m))} = \oplus_{i=1}^{m-1} s_{t<o_k>}^{(\sigma(i),\sigma(i+1))} \leq s_{\tau<o_\varsigma>}$.

For a LPO in decentralized form, to obtain the preference information more completely and transform the LPO into DHLPR successfully and exactly, the original preference pairs should contain all alternatives and should also have some relations among alternatives directly or indirectly. Therefore, the number of the PO pairs should not be less than $m-1$.

As we know, the LPOs can express linguistic information more completely and correctly, but it is very difficult to make calculations among them. Therefore, Gou et al. (2020c) developed a transformation model to transform the LPOs into a unified form, namely, a completely consistent DHLPR. The model consists of $(m-1)(m-2)/2$ equations, which can be used to obtain the remaining elements of the upper triangular matrix of DHLPR $\mathbb{R} = (r_{ij})_{m\times m} \subset A \times A$:

$$\begin{cases} r_{\sigma(1),\sigma(3)} = \dfrac{1}{m}\left(\overset{m}{\underset{\rho=1}{\oplus}}(r_{\sigma(1),\rho} + r_{\rho,\sigma(3)})\right); \cdots; r_{\sigma(1),\sigma(m)} = \dfrac{1}{m}\left(\overset{m}{\underset{\rho=1}{\oplus}}(r_{\sigma(1),\rho} + r_{\rho,\sigma(m)})\right) \\ r_{\sigma(2),\sigma(4)} = \dfrac{1}{m}\left(\overset{m}{\underset{\rho=1}{\oplus}}(r_{\sigma(2),\rho} + r_{\rho,\sigma(4)})\right); \cdots; r_{\sigma(2),\sigma(m)} = \dfrac{1}{m}\left(\overset{m}{\underset{\rho=1}{\oplus}}(r_{\sigma(2),\rho} + r_{\rho,\sigma(m)})\right) \\ \quad\vdots \\ r_{\sigma(m-2),\sigma(m)} = \dfrac{1}{m}\left(\overset{m}{\underset{\rho=1}{\oplus}}(r_{\sigma(m-2),\rho} + r_{\rho,\sigma(m)})\right) \end{cases} \tag{7}$$

By deleting some repeating the elements $r_{ii} (i = 1, 2, \ldots, m)$, Eq. (7) is changed to the following form:

$$\begin{cases} r_{\sigma(1)\sigma(3)} = \frac{1}{m-2}\left(\bigoplus_{\rho=1,\,\rho\neq\sigma(1),\,\rho\neq\sigma(3)}^{m}(r_{\sigma(1)\rho}+r_{\rho\sigma(3)})\right);\cdots;r_{\sigma(1)\sigma(m)} = \frac{1}{m-2}\left(\bigoplus_{\rho=1,\,\rho\neq\sigma(1),\,\rho\neq\sigma(m)}^{m}(r_{\sigma(1)\rho}+r_{\rho\sigma(m)})\right) \\ r_{\sigma(2)\sigma(4)} = \frac{1}{m-2}\left(\bigoplus_{\rho=1,\,\rho\neq\sigma(2),\,\rho\neq\sigma(4)}^{m}(r_{\sigma(2)\rho}+r_{\rho\sigma(4)})\right);\cdots;r_{\sigma(2)\sigma(m)} = \frac{1}{m-2}\left(\bigoplus_{\rho=1,\,\rho\neq\sigma(2),\,\rho\neq\sigma(m)}^{m}(r_{\sigma(2)\rho}+r_{\rho\sigma(m)})\right) \\ \vdots \\ r_{\sigma(m-2)\sigma(m)} = \frac{1}{m-2}\left(\bigoplus_{\rho=1,\,\rho\neq\sigma(m-2),\,\rho\neq\sigma(m)}^{m}(r_{\sigma(m-2)\rho}+r_{\rho\sigma(m)})\right) \end{cases} \tag{8}$$

2.3. Prospect theory

Tversky and Kahneman (1992) proposed the prospect theory, which can be used to describe the bounded rational behavior of experts when dealing with decision-making problems. The key point of prospect theory is that the risk seeking for losses and risk aversion for gains are unsymmetrical (Tian et al., 2018). Based on the prospect theory, the prospect value function can be obtained:

$$v(x) = \begin{cases} (x-\bar{x})^{\alpha} & x-\bar{x}>0 \\ 0 & x-\bar{x}=0 \\ -\lambda(\bar{x}-x)^{\beta} & x-\bar{x}<0 \end{cases} \tag{9}$$

In Eq. (9), the x is the assessment value of a project or an alternative. Specially, the $\bar{x}$ is called as reference point and it can be used to determine the assessment value belongs to the gain or the loss. If $x_l-\bar{x}>0,$ then the possible assessment value x means a gain; Otherwise, the possible assessment value x represents a loss. The parameters α and β are the risk attitudes of expert for gains and losses respectively, and λ is a loss aversion coefficient. Generally, $\lambda \geq 1$ and it means that the graph of the value function for losses is steeper than gains. Based on (Tversky & Kahneman, 1992), we obtain $\alpha = \beta = 0.88$ and $\lambda = 2.25$. Suppose that the reference point $\bar{x} = 0$, for any an assessment value x_0, the prospect value function is shown in Figure 1.

3. Prospect consensus with LPOs

In MEDM processes, it is common that experts usually have different education background, cognition and experience of practical decision-making problem. Therefore, it is necessary to reach group consensus before obtaining the final decision-making result. Considering that prospect theory has been demonstrated as a common phenomenon in decision-making because it can express the behavior of risk aversion for gains and risk seeking for losses. Thus, In this section, we develop a MEDM resolution framework and propose a consensus model based on prospect theory with LPOs.

A MEDM problem can be described as: Let $E = \{e^1, e^2, \ldots, e^n\}$ be a set of experts, and $A = \{A_1, A_2, \ldots, A_m\}$ be a set of alternatives. Each expert e^a evaluates all alternatives and provides his/her individual LPO. Based on Eq. (7) or (8), each LPO can be transformed into the corresponding completely consistent DHLPR $\mathbb{R}^a = (r^a_{ij})_{m\times m}$.

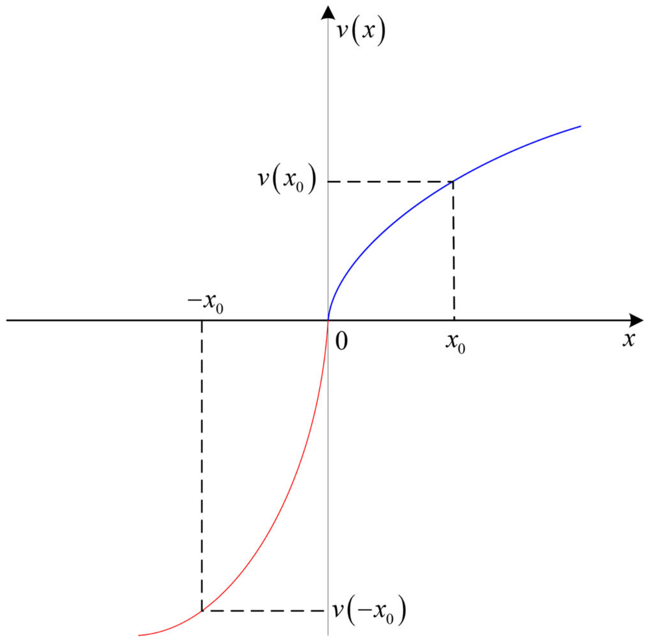

Figure 1. The prospect value function.
Source: The Authors.

3.1. The selection process

Based on the prospect theory and the DHLPR $\mathbb{R}^a = (r_{ij}^a)_{m\times m} \subset A \times A$ transformed from the LPO provided by the expert e^a, the prospect preference matrix P^a is established:

$$P^a = \left(p_{ij}^a\right)_{m\times m} \tag{10}$$

where p_{ij}^a is a real number and determined by the following formula:

$$p_{ij}^a = \begin{cases} \left(f\left(r_{ij}^a\right) - \bar{r}\right)^{\alpha} & f\left(r_{ij}^a\right) - \bar{r} > 0 \\ 0 & f\left(r_{ij}^a\right) - \bar{r} = 0 \\ -\lambda\left(\bar{r} - f\left(r_{ij}^a\right)\right)^{\beta} & f\left(r_{ij}^a\right) - \bar{r} < 0 \end{cases} \tag{11}$$

where the function f is the equivalent transformation function, $\bar{r}$ is the reference point. In this paper, considering that the element is positive when $f(r_{ij}^a) > 0.5$, and the element is negative when $f(r_{ij}^a) < 0.5$, so we can let $\bar{r} = 0.5$. Therefore, the p_{ij}^a is the prospect preference element of the alternative A_i to the alternative A_j from the expert e^a.

For each prospect preference matrix P^a, we can obtain the individual priority vector $w^a = (w_1^a, w_2^a, \ldots, w_m^a)^T$ of the expert e^a by establishing and solving the following model:

Model 1.
$$\min z^a = \sum_{i=1}^{m}\sum_{j=1}^{m}\left(\frac{w_i^a}{w_i^a + w_j^a} - p_{ij}^a\right)^2$$
$$s.t.\begin{cases}\sum_{i=1}^{m} w_i^a = 1\\ w_i^a \geq 0; i,j = 1,2,\ldots,m\end{cases}$$

Then, the collective priority vector $w^c = (w_1^c, w_2^c, \ldots, w_m^c)^T$ can be obtained by aggregating all experts' priority vectors, where

$$w_i^c = \frac{1}{n}\sum_{a=1}^{n} w_i^a, \quad i = 1,2,\ldots,m \tag{12}$$

Obviously, the bigger value of w_i^c is, the higher ranking order of the alternative A_i will be. Therefore, the rank of all alternatives and the optimal alternative can be obtained.

3.2. The consensus processes

In MEDM processes, the other important step is that all experts should reach group consensus before making decision. Therefore, this subsection researches the consensus process in the MEDM framework with LPOs based on prospect theory, which consists of consensus measure and feedback adjustment method.

Generally, we can utilize the distance or similarity degree between individual preferences and the collective preference to express the consensus degree for the decision-making problem (Dong et al., 2010). Similarly, next we will propose a prospect consensus degree (*PCD*) for the MEDM problem with LPOs based on prospect theory.

Firstly, by aggregating all prospect preference matrices $P^a(a = 1,2,\ldots,n)$, the collective prospect preference matrix $P^c = (p_{ij}^c)_{m\times m}$ is obtained, where $p_{ij}^c = \frac{1}{n}\sum_{a=1}^{n} p_{ij}^a$.

Then, the *PCD* of the expert e^a is obtained by measuring the similarity degree between the prospect preference matrix P^a and collective prospect preference matrix P^c.

Definition 2. Let P^a and P^c be the prospect preference matrix of e^a and the collective prospect preference matrix, respectively. The *PCD* of the expert e^a is defined by

$$PCD(e^a) = 1 - \sqrt{\frac{1}{m(m-1)}\sum_{i=1}^{m}\sum_{j=1}^{m}\left(p_{ij}^a - p_{ij}^c\right)^2}, \quad a = 1,2,\ldots,n \tag{13}$$

Then, the overall prospect consensus degree (*OPCD*) among experts is obtained by

$$OPCD = \frac{1}{n}\sum_{a=1}^{n} PCD(e^a) \tag{14}$$

Obviously, the larger the value of $OPCD$ is, the higher the consensus degree among all experts will be. Specially, $OPCD = 1$ means that all experts have full and unanimous agreement. Suppose that ξ is the given threshold, if $OPCD \geq \xi$, then the consensus is achieved; Otherwise, if $OPCD < \xi$, then the consensus is not achieved and it is necessary to improve the consensus degree.

We develop a consensus improvement method, which consists of two steps: Firstly, we need to identify the experts and the pairs of alternatives that need to improve their consensus degrees, this step is called identification rules; Secondly, we also need to feedback the suggestions to the corresponding experts and tell them how to adjust their preferences, this step is called direction rules.

(I) Identification rules

Suppose that $OPCD < \xi$, then we can obtain a set of experts whose prospect consensus degrees are less than the given threshold ξ :

$$E^* = \left\{ e^a \middle| PCD(e^a) < \xi \right\} \tag{15}$$

Then, for any an expert $e^a \in E^*$, we need to identify the pairs of alternatives that should be adjusted and they are included in a set as follows:

$$PAL^a = \left\{ (i,j) \middle| e^a \in E^* \wedge \left(1 - \frac{\left| p_{ij}^a - p_{ij}^c \right| + \left| p_{ji}^a - p_{ji}^c \right|}{2} \right) < \xi \right\} \tag{16}$$

(II) Direction rules

Suppose that the adjusted preference relation of the expert $e^a \in E^*$ is $\mathbb{R}^* = (r_{ij}^{a*})_{m\times m}$. Based on Eqs. (15) and (16), the direction rules are designed as follows:

1. If $p_{ij}^a > p_{ij}^c$, then we should decrease the value of p_{ij}^a. Therefore, the expert should decrease his/her preference r_{ij}^a associated with the pair of alternatives (A_i, A_j), namely, $r_{ij}^{a*} < r_{ij}^a$;
2. If $p_{ij}^a = p_{ij}^c$, then $r_{ij}^{a*} = r_{ij}^a$;
3. If $p_{ij}^a < p_{ij}^c$, then we should increase the value of p_{ij}^a. Therefore, the expert should increase his/her preference r_{ij}^a associated with the pair of alternatives (A_i, A_j), namely, $r_{ij}^{a*} > r_{ij}^a$;

When the experts have got the direction rules discussed above, the next step is to determine the extent of the adjustments. Suppose that the $r_{ij}^{a*(\mathbb{Z})}$ and $r_{ij}^{a*(\mathbb{Z}+1)}$ are the preferences of the expert $e^a \in E^*$ in the $\mathbb{Z}-$th and the $\mathbb{Z}+1-$th iterations, respectively. Accordingly, we can obtain the prospect preference matrix $P^{a*(\mathbb{Z})} = (p_{ij}^{a*(\mathbb{Z})})_{m\times m}$ and the prospect preference matrix $P^{a*(\mathbb{Z}+1)} = (p_{ij}^{a*(\mathbb{Z}+1)})_{m\times m}$. Then the general range is

$$p_{ij}^{a*(\mathbb{Z}+1)} \in \left[\min\left\{p_{ij}^{a*(\mathbb{Z})}, p_{ij}^{c*(\mathbb{Z})}\right\}, \max\left\{p_{ij}^{a*(\mathbb{Z})}, p_{ij}^{c*(\mathbb{Z})}\right\}\right] \tag{17}$$

In fact, we can always find a parameter $\theta \in [0, 1]$, Eq. (17) can be equivalently transformed into

$$p_{ij}^{a*(\mathbb{Z}+1)} = \theta p_{ij}^{a*(\mathbb{Z})} + (1-\theta) p_{ij}^{c*(\mathbb{Z})} \tag{18}$$

3.3. The consensus process

Based on the selection process and consensus process discussed above, a MEDM consensus model with LPOs based on prospect theory is established as follows:

Algorithm 1. A MEDM consensus model with LPOs based on prospect theory

Input: The LPOs of all experts, the consensus threshold ξ, the parameters α, β and λ, and the maximum number of iterations $T_{\max}$.

Output: The number of iterations T, the final consensus degree, and the ranking order of all alternatives.

Step 1. Transform all the LPOs of experts into the corresponding DHLPRs $\mathbb{R}^a = (r_{ij}^a)_{m\times m} (a = 1, 2, \ldots, n)$.

Step 2. Based on Eqs. (10) and (11), we calculate and establish the prospect preference matrices P^a $(a = 1, 2, \ldots, n)$ of expert $e^a (a = 1, 2, \ldots, n)$.

Step 3. Based on Eqs. (13) and (14), we calculate the *PCD* of each expert and the *OPCD* among all experts, respectively. If $OPCD \geq \xi$, then go to Step 5; If $OPCD < \xi$, then go to Step 4.

Step 4. Identify the experts and the pairs of alternatives that need to improve their consensus degrees based on Eqs. (15) and (16). Then we feedback the adjustment suggestions to the corresponding experts according to the direction rules. The experts provide their novel LPOs or preferences on the basis of the adjustment suggestions. Go back to Step 1 or Step 2.

Step 5. Based on Model 1 and Eq. (12), calculate the priority vectors of all experts and the collective priority vector, respectively. Then the rank of all alternatives is obtained.

This MEDM consensus framework with LPOs based on prospect theory is described in Figure 2.

4. The application of the MEDM consensus model with LPOs based on prospect theory

In this section, firstly we apply the method proposed in this paper to deal with a practical MEDM problem involving the construction project investment, and then make some comparative analyses between the proposed method and the existing methods (Gou et al., 2018a, 2020c).

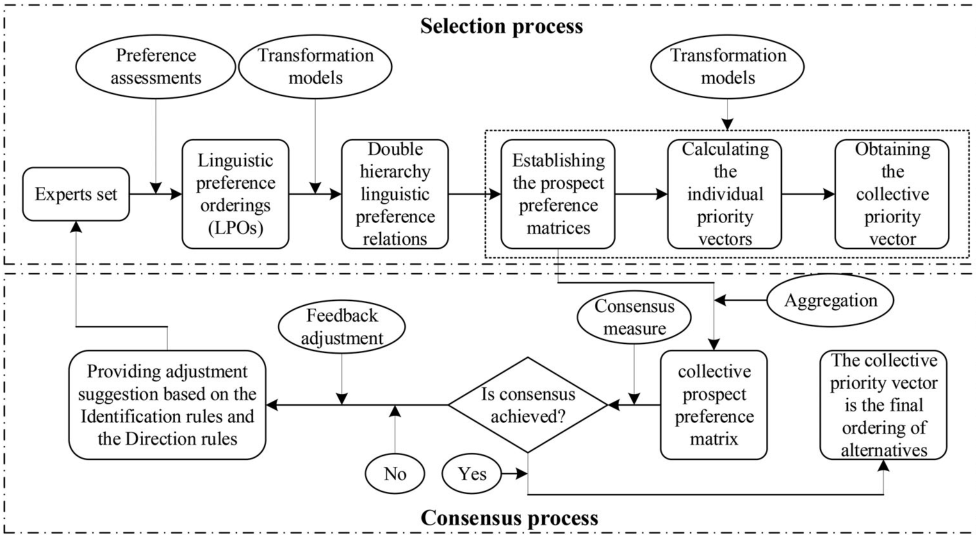

Figure 2. The MEDM consensus framework with LPOs based on prospect theory.
Source: The Authors.

4.1. The background of construction project investment with prospect theory

Construction project investment is a complex system engineering, the managers will face many risk factors and decision-making problems from project initiation to delivery. For a certain construction project, although its objective risk size and degree are certain, for different construction project investment, decision makers may choose different risk decision-making methods because of their different risk interest preferences. In the actual investment activities of construction projects, the attitudes of risk decision makers towards risk interests are not invariable. They may prefer risk interests in the face of some risks, but dislike risk interests in the face of other risks. Even in the same investment activity, no one will go to completely favor risk return, and do not consider risk loss; At the same time, no one is completely risk-averse without thinking about risk-return. Prospect theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992), as a major discovery in the field of decision-making, introduces psychological knowledge and integrates the value feeling factors of decision-makers into the analysis of decision-making behaviors, which hides the deficiency of expected utility theory and better explains decision makers' behaviors under uncertain conditions. Therefore, by introducing the risk attitudes and preferences of decision makers into the investment risk decision of construction projects through prospect theory, it can better reflect the decision-making behavior of the finite rational person under the condition of uncertain risk, thus improving the scientificity and objectivity of construction project investment.

Suppose that one real estate development company is facing five investment projects $A = \{A_1, A_2, \ldots, A_4\}$, and four experts $e^a(a = 1, 2, 3, 4)$ are invited to evaluate these investment projects. Let $S_O = \left\{s_{t<o_k>} | t = -4, \ldots, 4; \ k = -4, \ldots, 4\right\}$ be a DHLTS, where $S = \{s_{-4} = extremelybad, s_{-3} = very\ bad, s_{-2} = bad, s_{-1} = slightlybad, s_0 = medium, s_1 = slightlygood, s_2 = good, s_3 = verygood, s_4 = extremelygood\}$ and $O = \{o_{-4} = farfrom, \quad o_{-3} = scarcely, o_{-2} = onlyalittle, o_{-1} = alittle, o_0 = justright, o_1 = much, o_2 = verymuch, \ o_3 = extremelymuch, o_4 = entirely\}$. Then four invited experts provide their preferences by $LPO^a(a = 1, 2, 3, 4)$ shown as follows:

$$LPO^1 = \left\{ A_5 \overset{s_{0<o_2>}}{\succ} A_2 \overset{s_{1<o_1>}}{\succ} A_3 \overset{s_{1<o_{-1}>}}{\succ} A_1 \overset{s_{0<o_1>}}{\succ} A_4 \right\}$$

$$LPO^2 = \left\{ A_2 \overset{s_{1<o_1>}}{\succ} A_3, A_2 \overset{s_{2<o_1>}}{\succ} A_1, A_1 \overset{s_{1<o_{-1}>}}{\succ} A_4, A_5 \overset{s_{2<o_1>}}{\succ} A_4 \right\}$$

$$LPO^3 = \left\{ A_3 \overset{s_{0<o_1>}}{\succ} A_5 \overset{s_{2<o_0>}}{\succ} A_2 \overset{s_{1<o_{-2}>}}{\succ} A_4 \overset{s_{0<o_3>}}{\succ} A_1 \right\}$$

$$LPO^4 = \left\{ A_1 \overset{s_{2<o_1>}}{\succ} A_3, A_2 \overset{s_{1<o_1>}}{\succ} A_3, A_5 \overset{s_{1<o_{-1}>}}{\succ} A_1, A_2 \overset{s_{1<o_{-2}>}}{\succ} A_4 \right\}$$

4.2. Solving the MEDM problem with the proposed method

In this subsection, we can use the consensus framework with LPOs based on prospect theory proposed in this paper to deal with the above practical MEDM problem.

Step 1. Transform all LPOs of experts into the corresponding DHLPRs $\mathbb{R}^a = (r_{ij}^a)_{5\times 5}(a = 1, 2, 3, 4)$.

$$\mathbb{R}^1 = \begin{pmatrix} s_{0<o_0>} & s_{-2<o_0>} & s_{-1<o_1>} & s_{0<o_1>} & s_{-2<o_{-2}>} \\ s_{2<o_0>} & s_{0<o_0>} & s_{1<o_1>} & s_{2<o_1>} & s_{0<o_{-2}>} \\ s_{1<o_{-1}>} & s_{-1<o_{-1}>} & s_{0<o_0>} & s_{1<o_0>} & s_{-1<o_{-3}>} \\ s_{0<o_{-1}>} & s_{-2<o_{-1}>} & s_{-1<o_0>} & s_{0<o_0>} & s_{-2<o_{-3}>} \\ s_{2<o_2>} & s_{0<o_2>} & s_{1<o_3>} & s_{2<o_3>} & s_{0<o_0>} \end{pmatrix};$$

$$\mathbb{R}^2 = \begin{pmatrix} s_{0<o_0>} & s_{-2<o_{-1}>} & s_{-1<o_0>} & s_{1<o_{-1}>} & s_{-1<o_{-2}>} \\ s_{2<o_1>} & s_{0<o_0>} & s_{1<o_1>} & s_{3<o_0>} & s_{1<o_{-1}>} \\ s_{1<o_0>} & s_{-1<o_{-1}>} & s_{0<o_0>} & s_{2<o_{-1}>} & s_{0<o_{-2}>} \\ s_{-1<o_1>} & s_{-3<o_0>} & s_{-2<o_1>} & s_{0<o_0>} & s_{-2<o_{-1}>} \\ s_{1<o_2>} & s_{-1<o_1>} & s_{0<o_2>} & s_{2<o_1>} & s_{0<o_0>} \end{pmatrix}$$

$$\mathbb{R}^3 = \begin{pmatrix} s_{0<o_0>} & s_{-1<o_{-1}>} & s_{-3<o_{-2}>} & s_{0<o_{-3}>} & s_{-3<o_{-1}>} \\ s_{1<o_1>} & s_{0<o_0>} & s_{-2<o_{-1}>} & s_{1<o_{-2}>} & s_{-2<o_0>} \\ s_{3<o_2>} & s_{2<o_1>} & s_{0<o_0>} & s_{3<o_{-1}>} & s_{0<o_1>} \\ s_{0<o_3>} & s_{-1<o_2>} & s_{-3<o_1>} & s_{0<o_0>} & s_{-3<o_2>} \\ s_{3<o_1>} & s_{2<o_0>} & s_{0<o_{-1}>} & s_{3<o_{-2}>} & s_{0<o_0>} \end{pmatrix};$$

$$\mathbb{R}^4 = \begin{pmatrix} s_{0<o_0>} & s_{1<o_0>} & s_{2<o_1>} & s_{2<o_{-2}>} & s_{-1<o_1>} \\ s_{-1<o_0>} & s_{0<o_0>} & s_{1<o_1>} & s_{1<o_{-2}>} & s_{-2<o_1>} \\ s_{-2<o_{-1}>} & s_{-1<o_{-1}>} & s_{0<o_0>} & s_{0<o_{-3}>} & s_{-3<o_0>} \\ s_{-2<o_2>} & s_{-1<o_2>} & s_{0<o_3>} & s_{0<o_0>} & s_{-1<o_1>} \\ s_{1<o_{-1}>} & s_{2<o_{-1}>} & s_{3<o_0>} & s_{1<o_{-1}>} & s_{0<o_0>} \end{pmatrix}$$

Step 2. Based on Eqs. (10) and (11), calculate and establish the prospect preference matrices P^a $(a = 1, 2, 3, 4)$ of the experts $e^a(a = 1, 2, 3, 4)$:

$$P^1 = \begin{pmatrix} 0 & -0.6643 & -0.2802 & 0.0474 & -0.8084 \\ 0.2952 & 0 & 0.1952 & 0.3275 & -0.1961 \\ 0.1245 & -0.4393 & 0 & 0.1604 & -0.5907 \\ -0.1066 & -0.7369 & -0.3610 & 0 & -0.8792 \\ 0.3593 & 0.0872 & 0.2625 & 0.3907 & 0 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0 & -0.7369 & -0.3610 & 0.1245 & -0.5157 \\ 0.3275 & 0 & 0.1952 & 0.4218 & 0.1245 \\ 0.1604 & -0.4393 & 0 & 0.2625 & -0.1961 \\ -0.2802 & -0.9491 & -0.5907 & 0 & -0.7369 \\ 0.2292 & -0.2802 & 0.0872 & 0.3275 & 0 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0 & -0.4393 & -1.0870 & -0.2802 & -1.0184 \\ 0.1952 & 0 & -0.7369 & 0.0872 & -0.6643 \\ 0.4831 & 0.3275 & 0 & 0.3907 & 0.0474 \\ 0.1245 & -0.1961 & -0.8792 & 0 & -0.8084 \\ 0.4526 & 0.2952 & -0.1066 & 0.3593 & 0 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 0 & 0.1604 & 0.3275 & 0.2292 & -0.2802 \\ -0.3610 & 0 & 0.1952 & 0.0872 & -0.5907 \\ -0.7369 & -0.4393 & 0 & -0.2802 & -0.9491 \\ -0.5157 & -0.1961 & 0.1245 & 0 & -0.2802 \\ 0.1245 & 0.2625 & 0.4218 & 0.1245 & 0 \end{pmatrix}$$

Step 3. Based on Eqs. (13) and (14), calculate the *PCD* of each expert and the *OPCD*, respectively. The results are shown in Table 1.

Obviously, $OPCD = 0.6838 < 0.85$, the consensus is not achieved. Therefore, the experts should adjust their preferences and improve the group consensus.

Step 4. Based on Eqs. (15)–(17), the experts $e^a(a = 2, 3, 4)$ should improve their consensus degrees, and the adjusted preferences are shown as follows:

Table 1. The results of the prospect consensus degrees and the overall prospect consensus degree.

	e^1	e^2	e^3	e^4
$PCD(e^a)$	0.8577	0.7795	0.6248	0.5996
$OPCD$	0.6838			

Source: The Authors.

$$\mathbb{R}^{1(1)} = \begin{pmatrix} s_{0<o_0>} & s_{-2<o_0>} & s_{-1<o_1>} & s_{0<o_1>} & s_{-2<o_{-2}>} \\ s_{2<o_0>} & s_{0<o_0>} & s_{1<o_1>} & s_{2<o_1>} & s_{0<o_{-2}>} \\ s_{1<o_{-1}>} & s_{-1<o_{-1}>} & s_{0<o_0>} & s_{1<o_0>} & s_{-1<o_{-3}>} \\ s_{0<o_{-1}>} & s_{-2<o_{-1}>} & s_{-1<o_0>} & s_{0<o_0>} & s_{-2<o_{-3}>} \\ s_{2<o_2>} & s_{0<o_2>} & s_{1<o_3>} & s_{2<o_3>} & s_{0<o_0>} \end{pmatrix};$$

$$\mathbb{R}^{2(1)} = \begin{pmatrix} s_{0<o_0>} & s_{-1<o_{-2}>} & s_{-1<o_0>} & s_{1<o_{-1}>} & s_{-1<o_{-2}>} \\ s_{1<o_2>} & s_{0<o_0>} & s_{1<o_{-1}>} & s_{2<o_{-1}>} & s_{0<o_{-1}>} \\ s_{1<o_0>} & s_{-1<o_1>} & s_{0<o_0>} & s_{2<o_{-1}>} & s_{-1<o_0>} \\ s_{-1<o_1>} & s_{-2<o_1>} & s_{-2<o_1>} & s_{0<o_0>} & s_{-2<o_1>} \\ s_{1<o_2>} & s_{0<o_1>} & s_{1<o_0>} & s_{2<o_{-1}>} & s_{0<o_0>} \end{pmatrix}$$

$$\mathbb{R}^{3(1)} = \begin{pmatrix} s_{0<o_0>} & s_{-1<o_{-1}>} & s_{-1<o_0>} & s_{0<o_2>} & s_{-2<o_0>} \\ s_{1<o_1>} & s_{0<o_0>} & s_{0<o_{-3}>} & s_{1<o_{-2}>} & s_{-1<o_1>} \\ s_{1<o_0>} & s_{0<o_3>} & s_{0<o_0>} & s_{1<o_{-3}>} & s_{0<o_{-1}>} \\ s_{0<o_{-2}>} & s_{-1<o_2>} & s_{-1<o_3>} & s_{0<o_0>} & s_{-3<o_2>} \\ s_{2<o_0>} & s_{1<o_{-1}>} & s_{0<o_1>} & s_{3<o_{-2}>} & s_{0<o_0>} \end{pmatrix};$$

$$\mathbb{R}^{4(1)} = \begin{pmatrix} s_{0<o_0>} & s_{0<o_{-3}>} & s_{0<o_{-2}>} & s_{0<o_1>} & s_{-1<o_{-2}>} \\ s_{0<o_3>} & s_{0<o_0>} & s_{0<o_1>} & s_{1<o_{-2}>} & s_{-1<o_1>} \\ s_{0<o_2>} & s_{0<o_{-1}>} & s_{0<o_0>} & s_{0<o_{-1}>} & s_{-1<o_{-2}>} \\ s_{0<o_{-1}>} & s_{-1<o_2>} & s_{0<o_1>} & s_{0<o_0>} & s_{-1<o_{-2}>} \\ s_{1<o_2>} & s_{1<o_{-1}>} & s_{1<o_2>} & s_{1<o_2>} & s_{0<o_0>} \end{pmatrix}$$

Based on Eqs. (10) and (11), calculate and establish the new prospect preference matrices $P^{a(1)}$ $(a = 1, 2, 3, 4)$ of the experts $e^a (a = 1, 2, 3, 4)$. Then, based on Eqs. (13) and (14), calculate the new PCD ($PCD^{(1)}$) of each expert and the new OPCD ($OPCD^{(1)}$) among all experts. The results are shown in Table 2.

Obviously, $OPCD^{(1)} = 0.8681 > 0.85$, so the consensus is achieved.

Step 5. Based on Model 1 and Eq. (12), calculate the priority vectors of all experts and the collective priority vector, respectively. The results are shown in Table 3.

Therefore, the ranking order of all alternatives is $A_5 \succ A_2 \succ A_3 \succ A_1 \succ A_4$.

Table 2. The results of the prospect consensus degrees and the overall prospect consensus degree.

	e^1	e^2	e^3	e^4
$PCD^{(1)}(e^a)$	0.8579	0.8858	0.8592	0.8695
$OPCD^{(1)}$	0.8681			

Source: The Authors.

Table 3. The results of the prospect consensus degrees and the overall prospect consensus degree.

Experts	Priority vectors	Collective priority vector
e^1	$w^1 = (0.0147, 0.3181, 0.0385, 0.0107, 0.6180)^T$	$w = (0.0563, 0.251, 0.1383, 0.0544, 0.4999)^T$
e^2	$w^1 = (0.0484, 0.3353, 0.1504, 0.0196, 0.4463)^T$	
e^3	$w^1 = (0.0780, 0.1725, 0.2578, 0.0929, 0.3988)^T$	
e^4	$w^1 = (0.0842, 0.1782, 0.1065, 0.0945, 0.5366)^T$	

Source: The Authors.

4.3. Comparative analysis

In this subsection, we can make some comparative analyses between the proposed method and some existing methods.

(1) The comparative analysis about the CRP

Firstly, based the multi-stage interactive consensus reaching algorithm with LPOs developed by Gou et al. (2020c), the above MEDM problem can be solved after two times of iterations, and the ranking of alternatives is also $A_5 \succ A_2 \succ A_3 \succ A_1 \succ A_4$.

Secondly, we can use the consensus reaching method proposed by Gou et al. (2018a) to solve the above MEDM problem. Firstly, we obtain the overall consensus degree $ocd^{(0)} = 0.7854 < 0.85$. Therefore, we need to improve the preferences of the experts e^3 and e^4, and they provide their adjusted preferences:

$$\mathbb{R}'^{3(1)} = \begin{pmatrix} s_{0<o_0>} & s_{-1<o_{-1}>} & s_{-3<o_{-2}>} & s_{0<o_{-3}>} & s_{-3<o_{-1}>} \\ s_{1<o_1>} & s_{0<o_0>} & s_{0<o_1>} & s_{1<o_{-2}>} & s_{-1<o_0>} \\ s_{3<o_2>} & s_{0<o_{-1}>} & s_{0<o_0>} & s_{3<o_{-1}>} & s_{0<o_1>} \\ s_{0<o_3>} & s_{-1<o_2>} & s_{-3<o_1>} & s_{0<o_0>} & s_{-3<o_2>} \\ s_{3<o_1>} & s_{1<o_0>} & s_{0<o_{-1}>} & s_{3<o_{-2}>} & s_{0<o_0>} \end{pmatrix};$$

$$\mathbb{R}^4 = \begin{pmatrix} s_{0<o_0>} & s_{0<o_{-3}>} & s_{0<o_{-2}>} & s_{1<o_{-2}>} & s_{-2<o_0>} \\ s_{0<o_3>} & s_{0<o_0>} & s_{1<o_1>} & s_{1<o_{-2}>} & s_{-2<o_1>} \\ s_{0<o_2>} & s_{-1<o_{-1}>} & s_{0<o_0>} & s_{1<o_0>} & s_{-2<o_{-1}>} \\ s_{-1<o_2>} & s_{-1<o_2>} & s_{-1<o_0>} & s_{0<o_0>} & s_{-1<o_1>} \\ s_{2<o_0>} & s_{2<o_{-1}>} & s_{2<o_1>} & s_{1<o_{-1}>} & s_{0<o_0>} \end{pmatrix}$$

Then, we obtain the overall consensus degree $ocd^{(0)} = 0.8609 > 0.85$. Therefore, the consensus is achieved, and the synthetical value of each alternative is also $A_5 \succ A_2 \succ A_3 \succ A_1 \succ A_4$.

Based on the above three methods, some comparative analyses can be summarized as follows:

1. The fundamentals of these three methods are different. Firstly, the method proposed in this paper mainly utilizes prospect theory to calculate the prospect

Table 4. The priority vectors of all experts and the collective priority vector.

Methods	Iterations	The modification number of the DHLTs	The number of experts that have to change their preferences
The proposed method	Round 1	22	3
Gou et al. (2020c)'s methods	Round 1	20	4
	Round 2	11	4
Gou et al. (2018a)'s methods	Round 1	8	2

Source: The Authors.

preference matrices, the prospect consensus degree of each expert and the overall prospect consensus degree, and then checks whether the group consensus is achieved or not. Considering that the risk seeking for losses and risk aversion for gains are unsymmetrical in decision-making, so the proposed method is more in line with the bounded rational behavior of experts. Secondly, the method developed by Gou et al. (2020c) mainly consists of three stages consensus optimization processes. Therefore, this method can be used to achieve consensus using minimal changes in the size of the change, the number of modifications, and the number of individuals who need to revise their preferences. Finally, the method proposed by Gou et al. (2018a) makes the CRP only based on the DHLPRs instead of the LPOs. Therefore, it has limitation when dealing with group consensus reaching method with LPOs.

2. Even though the proposed method fully considers the risk seeking for losses and risk aversion for gains of experts, the extent of the changes is not effectively controlled. In contrast, the methods proposed by Gou et al. (2020c) develops optimization models to decrease the modification amplitudes of different aspects, and the method proposed by Gou et al. (2018a) also decreases the modification amplitudes of experts and preferences. However, both of them do not consider the bounded rational behavior of experts. The extent of the changes of these three methods are listed in Table 4. Clearly, the modification number of the DHLTs of the proposed method is larger than that of the other methods (Gou et al., 2018a, 2020c), and the number of experts that have to change their preferences of the method of this paper, Gou et al. (2020c), and Gou et al. (2018a) are 3, 4 and 2, respectively.

In specific decision-making processes, the determination of reference points is the main obstacle to be solved urgently in prospect theory, and it is influenced by the decision makers themselves, objective environment and other factors, so it is very difficult to accurately locate and evaluate them. In this paper, we suppose that $s_{0<o_0>}$ is the reference point. However, if decision makers do not want to use it, we can calculate the expected utility value of each alternative based on the expected utility theory and then use the mean expected utility value of alternatives to be the reference point. Therefore, using prospect theory and combining expected utility theory to make a risk decision can improve the scientificity of decision-making.

(2) The comparative analysis about decision-making methods without prospect theory

When the consensus is reached, this paper mainly obtains the ranking order of alternatives by calculating the priority vectors of all experts and the collective priority vector. In fact, without considering the prospect theory, there exist some other decision-making methods in other decision-making environment. However, there exist no decision-making method under double hierarchy linguistic preference environment. In order to show the advantages of the proposed method, we can make some comparative analyses between the proposed method and some methods provided in other decision-making environment such as the synthetical value-based method (Gou et al., 2018a, 2019) and the TOPSIS method (Gou et al., 2018b), etc.

Firstly, let $\{r^1, r^2, \ldots, r^n\}$ be a set of DHLTs, $(w_1, w_2, \ldots, w_n)^T$ be the weight vector of them. Based on (Gou et al., 2018a, 2019), the double hierarchy linguistic weighted average (DHLWA) operator can be shown as follows:

$$DHLWA\left(r^1, r^2, \ldots, r^n\right) = \overset{n}{\underset{i=1}{\oplus}} w_i r^i \tag{19}$$

Based on Eq. (9), for DHLPRs $\mathbb{R}^{a(1)} = (r_{ij}^{a(1)})_{5\times5} (a = 1, 2, 3, 4)$, the group DHLPR $\mathbb{R}^c = (r_{ij}^c)_{5\times5}$ can be calculated. Then we can aggregate the values of each row of $\mathbb{R}^c$ by $OV(A_i) = \sum_{j=1}^{5} f(r_{ij}^c)$ and obtain the overall values of all alternatives: $OV = \{2.0469, 2.8047, 2.5, 1.9375, 3.2109\}$. Therefore, the ranking order of alternatives is $A_5 \succ A_2 \succ A_3 \succ A_1 \succ A_4$.

Secondly, based on the traditional TOPSIS methods, the above MEDM problem can be solved. Firstly, the positive ideal solution $\mathbb{R}^+ = \{r_1^+, r_2^+, \ldots, r_5^+\}$ and the negative ideal solution $\mathbb{R}^- = \{r_1^-, r_2^-, \ldots, r_5^-\}$ of the group DHLPR $\mathbb{R}^c = (r_{ij}^c)_{5\times5}$ can be obtained, where $r_j^+ = \max\{r_{ij}^c\}$ and $r_j^- = \min\{r_{ij}^c\}$. Then, based on the following formula and let $\theta = 0.5$,

$$\begin{aligned}\phi(A_i) &= \frac{(1-\theta)\sum_{j=1}^{5} d\left(r_{ij}^c, r_j^-\right)}{\theta\sum_{j=1}^{5} d\left(r_{ij}^c, r_j^+\right) + (1-\theta)\sum_{j=1}^{5} d\left(r_{ij}^c, r_j^-\right)} \\ &= \frac{(1-\theta)\sum_{j=1}^{5}\left|r_{ij}^c, r_j^-\right|}{\theta\sum_{j=1}^{5}\left|r_{ij}^c, r_j^+\right| + (1-\theta)\sum_{j=1}^{5}\left|r_{ij}^c, r_j^-\right|}\end{aligned} \tag{20}$$

The satisfaction degree of each alternative can be got as $\{0.1078, 0.6886, 0.4551, 0.0240, 1\}$. Therefore, the ranking order of alternatives is $A_5 \succ A_2 \succ A_3 \succ A_1 \succ A_4$.

(3) The comparative analysis about decision-making methods with prospect theory

Different from the second part, we can make some comparations about decision-making methods with prospect theory.

Firstly, based on the prospect preference matrices $P^{a(1)}$ $(a = 1, 2, 3, 4)$ of the experts $e^a (a = 1, 2, 3, 4)$, we can also use the synthetical value-based method to solve this MEDM problem, and the overall values of all alternatives are $OV' =$

$\{-1.3239, 0.2081, -0.3448, -1.5535, 0.8595\}$. Therefore, the ranking order of alternatives is $A_5 \succ A_2 \succ A_3 \succ A_1 \succ A_4$.

Secondly, based on the traditional TOPSIS methods and prospect theory, we can obtain the satisfaction degree of each alternative as $\{0.1283, 0.7399, 0.5192, 0.0367, 1\}$. Therefore, the ranking order of alternatives is $A_5 \succ A_2 \succ A_3 \succ A_1 \succ A_4$.

Combining the second and the third parts, some discusses can be summarized as follows:

1. The ranking order of alternatives of all methods is $A_5 \succ A_2 \succ A_3 \succ A_1 \succ A_4$. However, the overall values, the satisfaction degrees of alternatives and the collective priority vector obtained by different methods are different. In fact, the synthetical value-based method and the TOPSIS method need some parameters which may change the decision-making result. In comparison, the proposed method is more simple and correct.
2. In second part, the synthetical value-based method and the TOPSIS method do not consider the prospect theory. Even though the decision-making result is same, it is more line with the bounded rational behavior of experts if we consider the risk seeking for losses and risk aversion for gains in decision-making. In addition, the methods in third part consider the prospect theory, but the models need more uncertain parameters and the decision-making processes are more complex.

5. Conclusions and future research directions

Considering that prospect theory has superiority in depicting risk attitudes during the uncertain decision-making process, and LPOs have the advantage of reflecting the ranking ordering of alternatives and the precise relationship between any two adjacent alternatives in the POs simultaneously. Therefore, this paper developed a consensus model based on prospect theory to deal with MEDM problems with LPOs. Firstly, to ensure the integrity of the original assessment information, the LPOs provided by experts was transformed into DHLPRs with complete consistency. In addition, the reference point of expert was determined and the prospect preference matrix was established by calculating the gains and losses with respect to alternatives for each expert. Moreover, the overall prospect consensus degree for a MEDM problem was developed based on the similarity degree between individual prospect preference matrix and the collective prospect preference matrix. Furthermore, a consensus improvement method was developed to complete the CRP. When all experts reach consensus, a model was set up to obtain the priority vector of each expert, and then the ranking of alternatives can be obtained. Finally, we applied the proposed method to deal with a practical MEDM problem involving the construction project investment and then made some comparative analyses with the existing methods.

As the future study, we will devote ourselves to the research of the prospect theory under different uncertain decision-making environment. Meanwhile, as some interesting topics, we will also study applications of LPOs in large-scale group decision-making or large-scale alternatives decision-making problems.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

This study was funded by the National Natural Science Foundation of China (No. 71771155), the China Postdoctoral Science Foundation (No. 2020M680151), the Sichuan Postdoctoral Science special Foundation, the Sichuan University Postdoctoral Interdisciplinary Innovation Startup Foundation, the Fundamental Research Funds for the Central Universities (Nos. YJ202015), the FEDER Financial Support (No. TIN2016-75850-R), and Sichuan Province System Science and Enterprise Development Research Center (Xq20B03).

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