

29 **1. Introduction**

30 Despite having many advantages, Reinforced Concrete (RC) members have one very
31 significant weakness: corrosion of steel reinforcement. More than a few RC structures
32 have been demolished before the end of their originally estimated useful life due to the
33 corrosion of reinforcement and numerous corroded RC structures are currently in
34 service, with or without being repaired. In this regard, the inspection, repair and
35 rehabilitation of existing RC structures affected by corrosion have become very
36 important in construction. Accordingly, the assessment of the residual capacity of a
37 corroded RC structure is a key task that reflects the current state of the structure.

38 Under normal circumstances, concrete provides protection to reinforcing steel through
39 both its high alkalinity (chemical protection) and its dense and relatively impermeable
40 structure (physical protection). The corrosion rate of rebars shows the efficiency of
41 protection against corrosion [1]. Corrosion is caused by the attack of chloride ions that
42 penetrate into the concrete matrix or by the carbonation of the concrete cover (or a
43 combination of both). The carbonation and/or chloride attacks cause the alkaline
44 concrete environment to deteriorate, disrupting the thin oxide layer that covers the steel
45 bars (known as the passive layer). This is the starting point of the corrosion process.
46 Among many other negative effects, corrosion causes a reduction of the cross-sectional
47 area of reinforcing bars [2], cracking of the concrete cover [3], reduction of the bond
48 strength between the reinforcing steel and the concrete [4–6] and softening of the
49 concrete [7]. All these effects acting together can weaken RC structures, thereby
50 reducing their load-carrying capacity and their service lives. Authors present a holistic
51 analysis of the deterioration due to the corrosion of D-Regions as a function of the crack
52 width in such a way that it provides the residual capacity of the D-Region. This has not
53 been performed before and is presented in this work.

54 The structural evaluation of an existing RC structure is a complex task. The use of
55 Destructive and Non-Destructive Tests (DT and NDT, respectively) in the
56 determination of the current conditions of RC members increases the accuracy of
57 structural evaluation. However, in most cases, the use of DT is not possible due to the
58 length of time it takes and the damage caused to a structure. To a lesser extent,
59 something similar occurs with NDT[8]. In this case it is because most of these tests
60 require special equipment and techniques that are often not readily available[9,10].
61 Numerical analyses have also been applied to study the crack propagation, particularly
62 in layered materials as in[11,12]. On the other hand, deterioration indicators such as
63 concrete cracks and the loss of concrete cover can easily be evaluated by visual
64 inspection or using simple NDT. This information provides the engineer with crucial
65 information about the real state of the structure.

66 An important aspect to be considered when applying an assessment method of the load
67 capacity of a potentially corroded structure is the type of region to be studied. It is
68 known that all RC structures consist of D (Disturbed or Discontinuity) and B (Bernoulli
69 or Beam) regions. In B-Regions it is assumed that plane sections remain plane after
70 deformation (linear strain distribution) whereas the strain distribution in D-Regions is
71 significantly nonlinear [13].

72 The design of B-Regions is well established in current structural practice codes, such as
73 the European [14] and North American ones [15], based on the classic beam theory. D-
74 Regions, which occur in zones close to corners, supports and concentrated loads such as
75 corbels and deep beams, can be designed with Finite Element Analysis (FEA) as in [16]
76 or alternatively with the Strut and Tie Method (STM)[13,17]. The STM is a design
77 method based on the lower-bound theorem of limit analysis for D-Regions in RC
78 structures. This method idealizes the structural behavior of a particular D-Region as a

79 system of struts (compression members) and ties (tension members) connected in nodes
80 (nodal zones). STM is included in Eurocode 2 (EC2) [14], AASTHO Bridge Design
81 Specifications [18] or ACI-318 [15] where some conditions on the definition of the truss
82 and on the calculation of the capacity and dimensions of its members are established.
83 Even though reinforcement corrosion has a considerable impact on the structural
84 behavior of both B- and D-Regions, the effect of corrosion on the residual capacity of
85 B-Regions has been covered much more extensively in relevant literature[7,19–21] than
86 that of D-Regions[22,23].

87 In Carbonell-Márquez et al. [20] authors proposed a procedure for the assessment of the
88 residual capacity of corroded B-Regions. In the present work, which corresponds to the
89 second part of the research, a procedure for the assessment of the residual capacity of
90 D-Regions in existing RC structures with corroded reinforcement is proposed. The
91 inputs of the method are the non-deteriorated or initial state of the structure (geometry
92 and reinforcement layout) and the actual geometry, properties of materials and
93 corrosion crack map (distribution and widths of the main concrete cracks due to
94 corrosion). The method is presented in detail together with an example. Finally, the
95 proposed procedure is verified by the comparison of its results with the corresponding
96 ones from experimental results existing in relevant literature. The present study, in
97 conjunction with the previous analysis of B-Regions, allows the effect of the corrosion
98 in the actual capacity of concrete structures to be evaluated.

99

100 **2. Structural modelling of corrosion effects in D-Regions**

101 All the corrosion-induced negative effects considered in the analysis of a corroded D-
102 Region are explained separately. All the aspects involved in the deterioration of D-
103 Regions can be expressed in terms of the geometry and the cracking pattern (i.e.

104 distribution and widths of the main corrosion cracks) of the RC member as described in
105 the following subsections.

106 **2.1.Loss of cross-sectional area of steel rebars**

107 A direct consequence of steel corrosion is the loss of the cross-sectional area of rebars.
108 The corrosion of the reinforcement can occur in an uniform form and/or in a localized
109 form [24]. Uniform corrosion (the most common type) leads to a homogeneous steel
110 cross-section reduction. On the other hand, localized corrosion or pitting consists of a
111 local iron dissolution that produces holes and cavities in the bar. This last type of
112 corrosion may cause a higher radial pressure on the surrounding concrete than with the
113 uniform one, accelerating the corrosion process [24]and making it more difficult to
114 detect and prevent.

115 The corrosion level, χ , is usually calculated by using the original or uncorroded mass m
116 of the rebar and the corresponding mass after the corrosion process m_{corr} obtained by
117 removing the corrosion products, see Eq. (1).

$$\chi = \frac{m - m_{corr}}{m} \quad (1)$$

118 Therefore, assuming as constant the steel density, the reduction of cross-sectional area,
119 ΔA_s , of the corroded steel bar is determined as a function of its corrosion level χ as:

$$\Delta A_s = A_s - A_{corr} = \chi A_s \Rightarrow A_{corr} = A_s(1 - \chi) \quad (2)$$

120 where A_s and A_{corr} are the uncorroded and corrosion-affected rebar areas, respectively.

121 According to this, the corrosion-affected rebar diameter ϕ_{corr} can be expressed as:

$$A_{corr} = A_s - \Delta A_s = A_s(1 - \chi) \Rightarrow \phi_{corr} = \phi\sqrt{1 - \chi} \quad (3)$$

122 with ϕ the diameter of the uncorroded steel bar.

123 In existing RC structures, the use of Eq. (3) to estimate the corrosion level would imply
124 the careful extraction of bars or coupons from the corroded member, which is risky,

125 costly and time consuming. Hence, an indirect computation of the corrosion level χ
 126 based on data obtained by a simple visual inspection of the corroded structure is very
 127 interesting from a practical point of view. Experimental results in the literature show
 128 that the surface crack width is closely related with the corrosion level of the steel
 129 bars[25,26]. Based on the results of two naturally corroded RC beams in a saline
 130 environment, subjected to wetting and drying cycles over periods of 14 and 17 years,
 131 Vidal et al. [26] proposed an empirical expression that relates the corrosion-induced
 132 rebar cross-section reduction ΔA_s (mm^2) to the width of corrosion-induced longitudinal
 133 cracks, w (mm):

$$w = 0.0575(\Delta A_s - \Delta A_{s0}) \quad (4)$$

134 where ΔA_{s0} is the reduction of the area of the cross section that initiates cracking
 135 expressed in mm^2 . As defined ΔA_s , in Eq. (2), ΔA_{s0} can be also expressed as a function
 136 of the corrosion level that initiates corrosion cracking χ_0 (i.e. $\Delta A_{s0} = \chi_0 A_s$).
 137 Finally, Eq. (4) can be combined with Eq. (2) in order to obtain a relationship between
 138 the corrosion level, χ , and the width of corrosion-induced longitudinal cracks, w (mm):

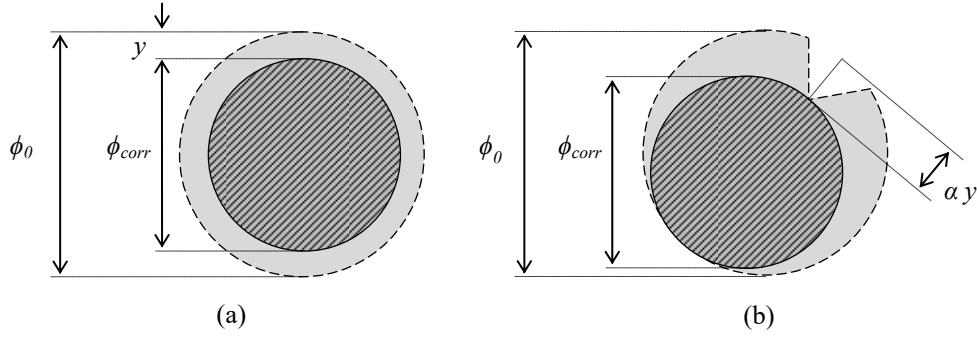
$$w = 0.0575 A_s (\chi - \chi_0) \leftrightarrow \chi = \frac{w}{0.0575 A_s} + \chi_0 \quad (5)$$

139 Rodríguez et al. [2] proposed a model that correlates the loss of cross-sectional area
 140 with the corrosion attack penetration or reduction of the radius of the cross-sectional
 141 rebar y so that the effective diameter of a rebar can be expressed as:

$$\phi_{corr} = \phi - \alpha y \quad (6)$$

142 where α is a corrosion parameter with value equal to 2 in case of uniform corrosion and
 143 with value between 4 and 8 in case of pitting corrosion [27] (see Figure 1). Naming y_0 as
 144 the value of the attack penetration that initiates cracking, combining Eqs. (3) and (6), a
 145 relationship between y_0 and the corresponding corrosion level χ_0 can be stated:

$$\chi_0 = 1 - \left(1 - \frac{\alpha y_0}{\phi}\right)^2 \quad (7)$$



146

147 **Figure 1. Reduced section of the steel bar produced by uniform (a) and localized (b) corrosion.**
 148 **Adapted from [2].**

149 Alonso et al. [28] proposed an empirical formulae to obtain the corrosion penetration
 150 that initiates cracking, y_0 (mm), as a function of the distance between the outer surface
 151 of the concrete and the corroded longitudinal steel bar c (mm) and the uncorroded
 152 diameter ϕ (mm):

$$y_0 = \left(7.53 + 9.32 \frac{c}{\phi}\right) 10^{-3} \quad (8)$$

153 Finally, combining now Eqs. (5), (7) and (8) the corrosion level can be estimated based
 154 on information about the original reinforcement configuration (c and ϕ) and visual
 155 inspection(w) as:

$$\chi = \frac{w}{0.0575 A_s} + 1 - \left[1 - \frac{\alpha}{\phi} \left(7.53 + 9.32 \frac{c}{\phi}\right) 10^{-3}\right]^2 \quad (9)$$

156 with w , c and ϕ expressed in mm and A_s in mm^2 . Once the corrosion level χ is
 157 estimated, the loss of cross-section of the corroded rebar A_{corr} can be calculated by
 158 means of Eq. (2).

159 **2.2. Bond strength model for corroded bars**

160 Eurocode 2 provides formulation to design or evaluate the anchorage in sound
 161 reinforced concrete members. According to this standard, the design anchorage length l_{bd}

162 (all the correction factors equal to 1) for a rebar with diameter ϕ is:

$$l_{bd} = \frac{\phi \sigma_{sd}}{4 f_{bd}} \quad (10)$$

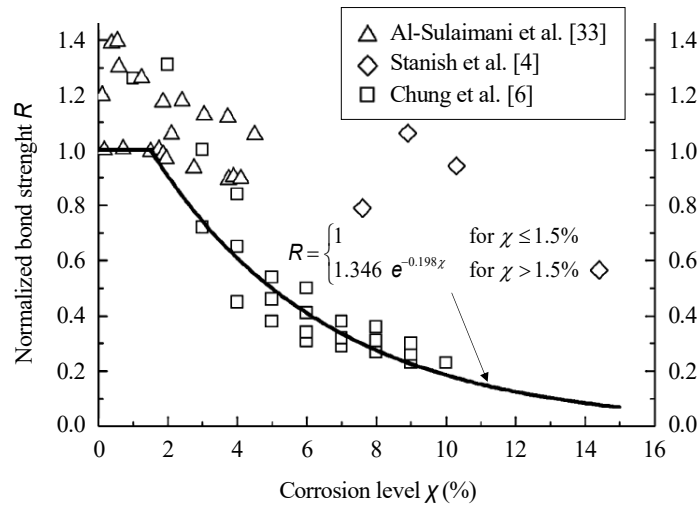
163 where σ_{sd} is the design stress of the reinforcing bar at the anchorage section and f_{bd} is
 164 the ultimate bond stress, defined in EC2 as:

$$f_{bd} = 2.25 \eta_1 \eta_2 f_{ctd} \quad (11)$$

165 In Eq. (11), f_{ctd} is the design value of the concrete tensile strength, η_1 is a coefficient
 166 related to the quality of the bond condition and the position of the bar during concreting
 167 and η_2 is a coefficient related to the diameter of the rebar.

168 A direct result of corrosion is the modification of the anchorage capacity of rebar given
 169 the reduction of bond strength. Many different models to relate bond strength with
 170 corrosion of reinforcement have been proposed by researchers[25,29,30]. Barghava et
 171 al. [31,32] proposed an empirical model based on data from flexural tests of specimens
 172 with stirrups (Al-Sulaimani et al.[33]) and without stirrups (Stanish et al. [6]and Chung
 173 et al. [4]), see Figure 2 and Eq.(12).

$$f_{bd,corr}(\chi) = R f_{bd} = \begin{cases} f_{bd} & \text{for } \chi \leq 1.5\% \\ 1.346 e^{-0.198\chi} f_{bd} & \text{for } \chi > 1.5\% \end{cases} \quad (12)$$



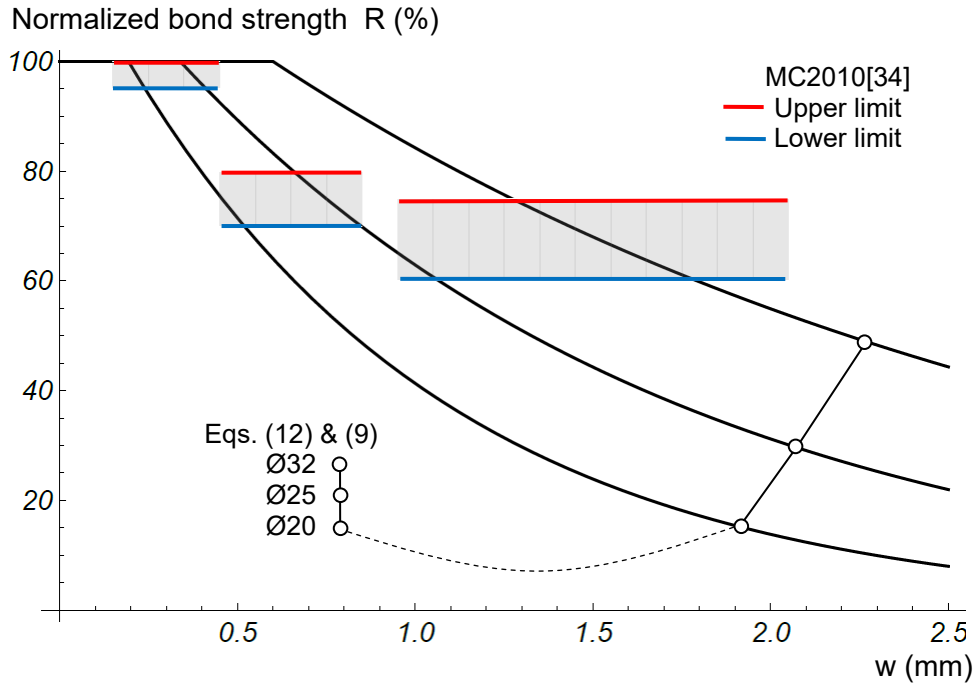
174

175 **Figure 2. Normalized bond strength R as a function of corrosion level χ for experimental data of**
 176 **flexural tests. Adapted from [31,32].**

177 The normalized bond strength R in Eq. (12) is defined as the ratio of the corrosion
 178 affected bond strength ($f_{bd,corr}$) to the original bond strength (f_{bd}), see Eq. (13).

$$f_{bd,corr} = R f_{bd} \quad (13)$$

179 Model Code 2010 (MC2010) [34] proposed a reduction of the bond strength of
 180 corroded reinforcement in function of the corrosion-induced crack width w . In Figure 3,
 181 the normalized bond strength R (in percentage) obtained from Eq.(12) and the upper and
 182 lower limits proposed by MC2010 have been plotted against the corrosion crack width
 183 for comparison. In Figure 3, Eq.(12) has been plotted in conjunction with Eq. (9) for
 184 $c=30\text{mm}$, $\alpha=2$ and $\phi=20, 25$ and 32 mm.



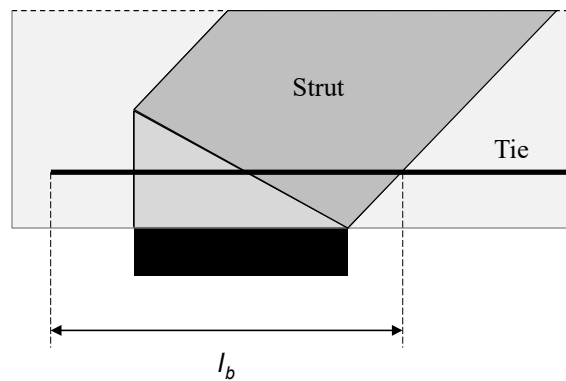
185
 186 **Figure 3. Comparison of the model proposed by Barghava et al. [31,32] and MC2010 [34].**
 187 It can be seen in Figure 3 that the normalized bond strength given by Eq.(12) is almost
 188 the same than the one proposed by MC2010 for $w \leq 0.4$ mm. For high degree of
 189 corrosion, the limits established by MC2010 are considerably greater than the value
 190 obtained from Eq.(12) whereas than for values of crack widths in the range 0.4 - 0.6 mm
 191 Eq.(12) overestimates the residual bond stress respect of the MC2010. Without loss of

192 generality, the expression for the normalized bond strength given by Eq.(12) has been
 193 adopted here.

194 In the expression of the anchorage length given by Eq.(10), two parameters are
 195 susceptible to be affected by corrosion: the diameter of the rebar and the bond strength,
 196 which are reduced according to Eq.(3) and Eq.(13), respectively. The corrosion-affected
 197 anchorage length, $l_{bd,corr}$, can be obtained from Eq. (10) as:

$$l_{bd,corr} = \frac{\phi_{corr}}{4} \frac{\sigma_{sd,corr}}{f_{bd,corr}} \quad (14)$$

198 where $\sigma_{sd,corr} \leq f_{yd}$ is the design stress of the reinforcing bar at the anchorage section but
 199 affected by corrosion. From a structural point of view, the effect of corrosion can be
 200 balanced by increasing the anchorage length [20] but this is not always possible. Such is
 201 the case of D-Regions, in which the anchorage length l_b is limited by the available
 202 distance as it is shown in Figure 4. For this reason, the way which has been proposed
 203 for estimating the loss of bond strength due to corrosion in D-Regions is by calculating
 204 the maximum stress that the reinforcing bar can withstand with the actual anchorage
 205 length, l_b (see Figure 4), but in corroded conditions, $\sigma_{sd,corr}$ (see Eq. (15)).



206
 207 **Figure 4. Example of available anchorage length limited by the geometry of STM.**

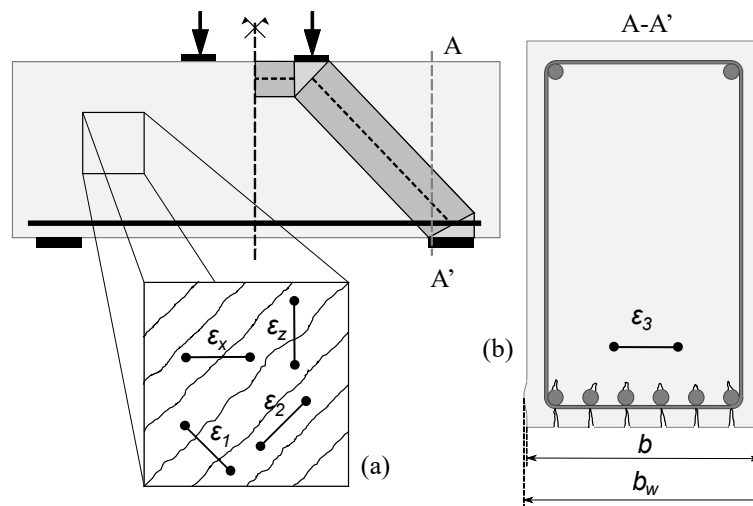
$$\sigma_{s,corr} = \text{Min} \left(\frac{4}{\phi_{corr}} l_b f_{bd,corr}; f_{yd} \right) \quad (15)$$

208 In Eq. (15) an upper limit has been imposed because the value of $\sigma_{sd,corr}$ cannot be
209 higher than the yield stress of the reinforcement bar, f_{yd} .

210 It should be considered that the bond strength loss considered in this section indirectly
211 supposed uniform corrosion. A safe assumption for the case of localized corrosion.

212 2.3. Softening effect in cracked concrete

213 Uncracked compressed concrete has exhibited higher compressive strength and stiffness
214 than cracked concrete in compression [35]. This effect, called compression softening, is
215 related to the principal tensile strain ε_1 and average tensile strain ε_3 (see Figure 5 (a) and
216 (b)). Both tensile strains form a right angle with the direction of the principal
217 compression strain ε_2 causing cracking. In the particular case of D-Regions, Tjhin and
218 Kuchma [36] stated that any disturbance in the struts significantly affects their capacity.
219 These disturbances include initial cracks, parallel or inclined, in the strut axis and
220 tensile transverse stress or strain (such as that produced by a crossing tie).



221

222 **Figure 5. Average strains in a cracked RC element. Example of D-Region (deep beam): (a) strain**
223 **perpendicular ε_1 and parallel ε_2 to the concrete strut axis; (b) average concrete strain at plane of**
224 **cross-section ε_3 .**

225 Corrosion-induced softening is mainly associated with the average tensile strain ε_3
226 (perpendicular to the principal compressive strain ε_2 , see Figure 5 (b)) produced by rust

227 accumulation which causes longitudinal micro-cracks [7]. According to Coronelli and
 228 Gambarova [7], the compressive strength of concrete affected by softening due to
 229 corrosion cracking f_c^* is given by the following expression:

$$f_c^* = \zeta_{corr} f_c \quad (16)$$

230 where ζ_{corr} is a factor called softening coefficient defined as:

$$\zeta_{corr} = \frac{1}{1 + K \frac{\varepsilon_3}{\varepsilon_{c0}}} \quad (17)$$

231 In Eq. (17), K is a coefficient related to the roughness of the bar and its diameter
 232 (according to Capè [37] $K = 0.1$ for medium-diameter ribbed bars) and ε_{c0} is the
 233 concrete strain corresponding to the peak compressive stress f_c .

234 According to Coronelli and Gambarova [7], the strain ε_3 can be calculated as:

$$\varepsilon_3 = \frac{b_w - b}{b} \approx \frac{\sum w_i}{b} \quad (18)$$

235 where b_w is the increased width by corrosion cracking (see example of a deep beam
 236 cross-section shown in Figure 5 (b)). The increase of the width of a D-Region ($b_w - b$)
 237 can be approximated by the sum of all the longitudinal crack widths w_i formed during
 238 the corrosion of the longitudinal reinforcing bars which intersect the strut.

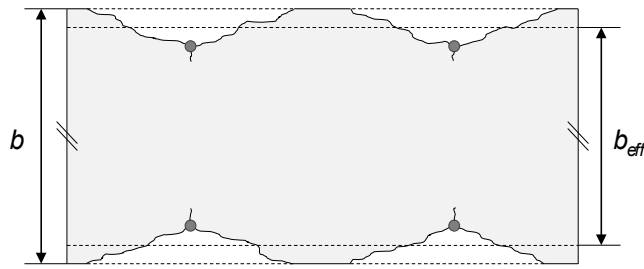
239 **2.4. Reduction of the concrete area**

240 Rust products formed during the corrosion of reinforcement can cause internal stresses
 241 on concrete due to its volume expansion. This internal pressure results in concrete cover
 242 cracking, delamination and spalling [38–40] that reduce the concrete area.
 243 Consequently, the effective dimensions of the RC element must be considered in order
 244 to account for these effects.

245 Empirical formulations can be used to calculate the non-damaged cross-sectional area of
 246 concrete. One of these empirical expressions is the one proposed by Higgins et al. [41]

247 from which the effective width, b_{eff} (see Figure 6), is obtained as function of the original
 248 undamaged beam width (b), the concrete cover (c), the stirrup diameter (ϕ_{st}) and the
 249 stirrup spacing (s_{st}):

$$\begin{aligned}
 b_{eff} &= b - 2(c + \phi_{st}) + \frac{s_{st}}{5.5} \quad \text{if } s_{st} \leq 5.5c \\
 b_{eff} &= b \frac{5.5}{s_{st}} (c + \phi_{st})^2 \quad \text{if } s_{st} > 5.5c
 \end{aligned}
 \tag{19}$$



250
 251 **Figure 6. Plan view of concrete cracking due to corrosion.**

252 In Eq. (19) all the dimensions are in inches (1 in= 25.4 mm). Certain precautions should
 253 be taken when using empirical formulations in order to guarantee their suitability for
 254 each particular case.

255
 256 **3. Critical review of the struts design in the STM methodology**

257 In the proposed method it is considered that the reinforcement layout is known when
 258 evaluating the capacity of a particular D-Region. Consequently, the location of the ties
 259 is initially defined. In order to determine the geometry of the struts and the nodes two
 260 different criteria can be followed. In the first criterion the width of the struts is defined
 261 supposing that they are working at the concrete strength capacity (§7.3.6.2 of the Model
 262 Code 2010[34]). On the other hand, the second criterion does not explicitly account for
 263 the struts dimensions based on that the struts are stronger than the nodes: "Since the
 264 compressive stress will be highest at a node there is no need to investigate compression
 265 elsewhere in the strut", from §C.5.8.2.2 AASHTO LRFD-8 [18].

266 Following AASHTO methodology, nodes are designed according to purely geometrical
267 conditions: bearing dimensions, reinforcement location and depth of the compression
268 zone calculated in the interface with the B-Region. The AASHTO philosophy is backed
269 by the classical Kani's campaign [42] and the experience of the authors [43].
270 Nevertheless, the latest version of the AASHTO LRFD-8 Bridge Design (2017), with
271 respect to the previous version of 2012, has modified some of the nodal geometries and
272 has removed the criteria for the effective breadth of the struts.

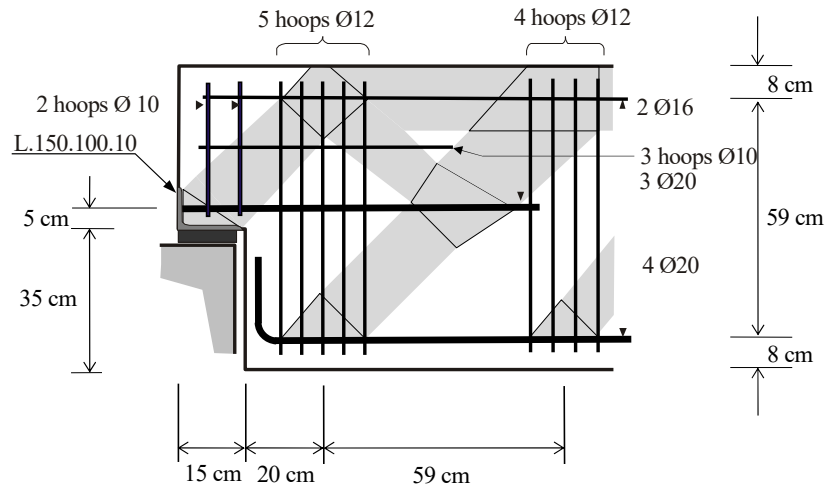
273 A close look to the formulation proposed by MC2010[34] and Eurocode 2[14] reveals
274 that, except for one case, the reduced concrete compressive strength in struts is always
275 smaller than in nodes. In this sense no formulation should be needed but just the
276 AASHTO LRFD-8 assertion quoted above. The said exception is for the case where the
277 struts is located in an undisturbed uniaxial compression stress state or in a region with
278 transverse compression.

279 In view of the aforementioned two options can be taken for the assessment and design
280 of the struts dimensions: 1) suppose the dimensions suggested by AASHTO and check
281 that demanded stresses are smaller than the capacity or 2) suppose the dimensions
282 deduced from the capacity. The former has been taken in this work.

283

284 **4. Capacity of the D-Region.**

285 An example from Hernández-Montes and Gil-Martín [44] has been taken to better
286 explain the procedure proposed for the assessment of the residual capacity of D-Regions
287 in RC structures. Figure 7 shows the reinforcement layout and the dimensions of the
288 nodes and the struts. These dimensions have been deduced according AASHTO LRFD-
289 8 [18]. A detailed explanation for a similar case can be seen in Martin and Sanders [45],
290 although they followed a former version of the same specifications.



291

292 **Figure 7. Reinforcement layout.**

293 Figure 8 shows the Strut and Tie (S&T) model of the D-Region. The thickness of strut

294 CD is designed following the AASHTO description of CTT nodes, applied to node C.

295 Strut BE has been supposed to be centered at 8 cm from the top, so that the thickness of

296 strut BE is 16 cm. For a fixed value of P (see Figure 8) the thickness of strut BE can be

297 calculated, being the depth of the compression zone. As P is considered to be a variable,

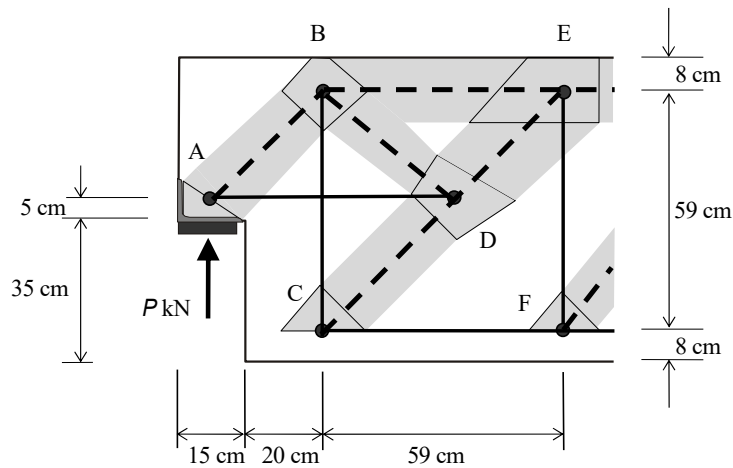
298 strut BE thickness could be also a variable, nevertheless in the present work and without

299 loss of generality it has been considered as a constant. Thickness of strut BE and

300 geometry of tie EF define the thickness of strut DE. Geometry of the tie AD and

301 thicknesses of struts CD, DE and BE define thickness of strut BD. Geometry of support

302 A and thicknesses of struts BD define strut AB (see Figure 8).



303

304 **Figure 8. S&T model of the example depicted in Figure 7.**

305 Solving the S&T model, the axial force N acting on each one of the strut or tie can be
 306 expressed as a function of P , i.e. $N_i = \delta_i P$, being i the number of the strut or tie, see Table
 307 1, where negative sign means tension and positive sign compression.

308 **Table 1. Forces in the truss structure**

	AB	AD	BC	BD	BE	CD	CF	DE	EF
Axial Force $N_i = \delta_i P$ (kN)	1.43 P	-1.02 P	-1.47 P	0.72P	0.47 P	2.07P	-1.47 P	1.41P	-1.00 P

309

310 **4.1. Capacity evolution of the ties**

311 The decay of the capacity of the ties is due to effects described in Subsections 2.1 (loss
 312 of area of the rebar as function of the crack width w) and 2.2 (reduction of the bond
 313 stress, function of w) of the present work.

314 Evolution of the capacity of the ties due to the loss of cross section as function of the
 315 crack width (w) is calculated following the expression:

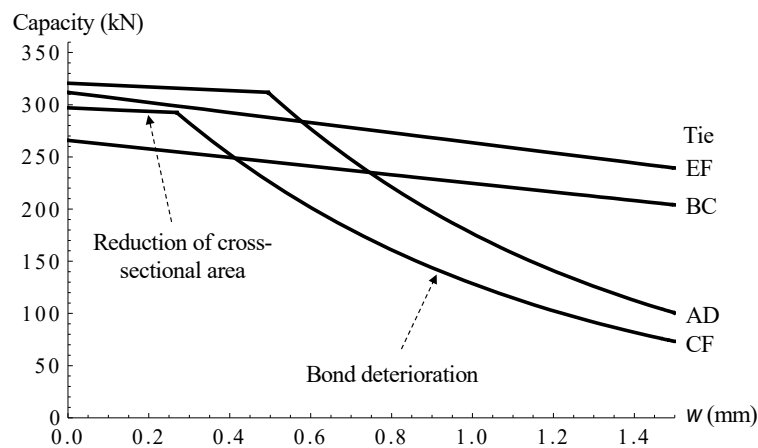
$$\left. \begin{aligned} P_i(w) &= N_i(w) / \delta_i \\ N_i(w) &= f_{yd} A_{i,corr}(w) \\ A_{i,corr}(w) &= (1 - \chi(w)) A_s \end{aligned} \right\} \rightarrow P_i(w) \quad (20)$$

316 The expression of the corrosion level $\chi(w, A_s, \phi)$ is given by Eq. (9). For each one of the
 317 ties the A_s and ϕ are the initial values of both area and diameter of the reinforcement,
 318 respectively. Because A_s and ϕ are constant χ is only function of crack width w , i.e.
 319 $\chi(w)$.

320 Evolution of the capacity of the ties due to decay of the bond because of corrosion is
 321 calculated supposing that anchorage length do not vary but the bond stress decreases
 322 taking the value of $f_{bd,corr}$. Anchorage of vertical ties (EF and BC) fulfils prescriptions
 323 for anchorage of links and shear reinforcement (e.g. EC2 §8.5) so that deterioration of
 324 bond is only considered for the AD and CF ties.

325 The evolution of the capacity due to both loss of cross-section and to bond deterioration

326 of ties can be observed in Figure 9. The flatter part of the curves corresponding to AD
 327 and CF ties shows the behavior due to the loss of cross-sectional area. The steepest part
 328 of these curves corresponds to bond deterioration. Ties EF and BC show steeper curves
 329 than the flatter parts of curves AD and CF because these reinforcing bars are of smaller
 330 diameter, and the loss of cross-sectional area is greater for the same crack width.
 331 It is assumed that concrete cover is 25mm and corrosion is uniform.



332
 333 **Figure 9. Loss of capacity of the ties as function of crack width.**

334 If the thickness of the crack w is supposed to be constant for the entire D-Region, Figure
 335 9 shows that the decay of the capacity is due to the reduction of the cross-sectional area
 336 of tie BC up to a $w \approx 0.4$ mm. Beyond this value, the decay of the capacity is due to bond
 337 deterioration of tie CF. Figure 9 can also be used if different values of w are considered
 338 for each tie of the D-Region, what supposes a greater precision in the application of the
 339 method.

340 **4.2. Capacity evolution of the nodes and struts**

341 Table 2 shows the classification of each node and thickness of its most stressed side.
 342 According to the AASHTO LRFD-8 recommendations, nodal check is enough to verify
 343 both nodes and struts. The thickness of the most stressed side times the breadth and the
 344 effective compression strength gives the capacity of both node and strut.

345 According to Eurocode 2 [14], the maximum stress applicable to the edge of the nodes

346 depends on the type of node:

$$\sigma_{Rd,max} = k\nu f_{cd} = \begin{cases} \nu f_{cd} & \text{for CCC node} \\ 0.85\nu f_{cd} & \text{for CCT node} \\ 0.75\nu f_{cd} & \text{for CTT node} \end{cases} \quad (21)$$

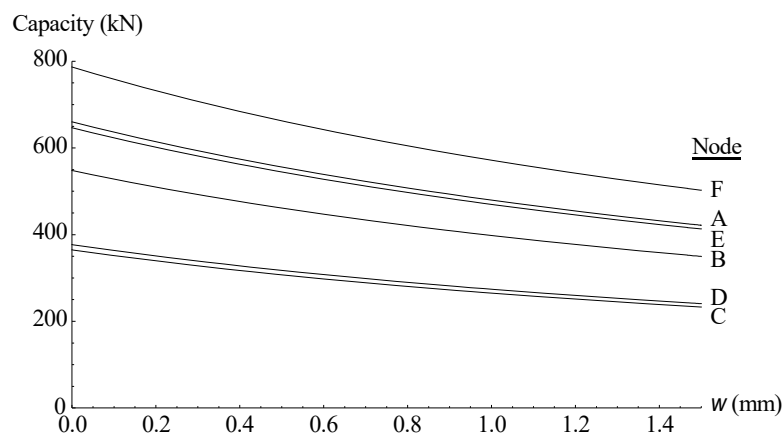
347 In Eq. (21) $\nu = 1 - f_{ck}/250$, being f_{ck} the characteristic compressive strength of concrete
 348 and $f_{cd} = f_{ck}/\gamma_c$, being γ_c the partial safety factor for concrete.

349 **Table2. Nodes characteristics**

	A	B	C	D	E	F
Thickness of the most stressed side (mm)	100	136.8	150	136.8	160	220.6
Force acting in the most stressed side	1.02 P	1.43 P	2.07P	2.07P	1.41P	1.41P
Type of node	CCC	CCT	CTT	CCT	CCT	CTT

350

351 The corrosion process produces a decay in the compressive strength of concrete due to
 352 softening and a reduction to the breadth of the struts because of the spalling of the
 353 cover. The first is expressed as function of the crack width while the second is
 354 considered to be a constant (Figure 6). Figure 10 shows the capacity of the nodes as
 355 function of the crack width.



356

357 **Figure 10. Loss of capacity of the nodes as function of crack width.**

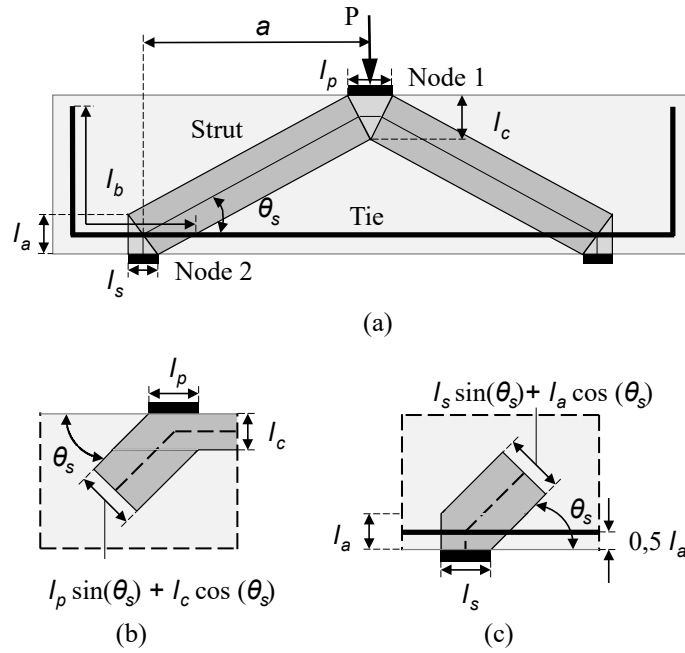
358 Comparison of Figure 9 and Figure 10 shows that in the studied case (D-Region in
 359 Figure 7) the capacity of the struts and nodes are always greater than those
 360 corresponding to the ties.

361

362 **5. Experimental example of the proposed approach**

363 In order to check the proposed model an experimental campaign from relevant literature
364 has been analyzed. In Azam and Soudki [22,46] four RC deep beams with corroded
365 longitudinal reinforcement were tested up to failure in three point bending. Deep beams
366 are RC beams with a shear span to depth ratio (a/d) lower than 2.5. Design codes such
367 as EC2 [14], AASHTO LRFD-8 [18] and ACI-318 [15] recommend S&T model for the
368 design of RC deep beams because the arch action has a considerable contribution to its
369 structural behavior. Two of the beams of the experimental campaign carried out in
370 [22,46] have not transverse reinforcement (L specimens). The other two beams (LS
371 specimens) have epoxy coated transverse reinforcement to prevent them from corrosion.
372 Relevant information such as corrosion crack widths, dimensions of the support and
373 loading plates and reinforcement detailing have been carefully examined.

374 The S&T model adopted is shown in Figure 11. In Figure 11, the dimension of the strut
375 at the nodal zones of both the support and the loading plate are computed based on
376 AASHTO LRFD-8 prescriptions. The capacity of each nodal zone is computed
377 according to Eurocode 2 (see Eq. (21)). Because the aim of the study is the assessment of
378 the residual capacity of the D-Region, a partial safety factor for concrete equal to one is
379 adopted (i.e. $\gamma_c=1$). The properties of the RC deep beams are the following [22,46]:
380 shear span $a = 500$ mm, effective depth $d = 307.5$ mm, concrete cover $c = 30$ mm, beam
381 depth $h = 350$ mm, beam breadth $b = 150$ mm, width of support bearing plates $l_s = 62.5$
382 mm, width of the loading plate $l_p = 100$ mm, available anchorage length $l_{b,av} = 584$ mm,
383 concrete strength $f_c = 47.3$ MPa (authors have considered it to be the expected, or mean,
384 value of the concrete compressive strength), longitudinal reinforcement 2-25M bars ($\phi =$
385 25.2 mm) and $f_y = 400$ MPa.



386

387 **Figure 11. Dimensions of S&T model employed to model the behavior of a RC deep beam. (a) S&T**
 388 **model, (b) strut anchored by bearing and reinforcement and (c) strut anchored by bearing and**
 389 **strut.**

390 The residual capacity of the corroded deep beams was estimated following the
 391 procedure shown in the previous section. The values of both experimental, P_{exp} , and
 392 predicted, P_{corr} , residual capacities of the four deep beams are in Table 3.

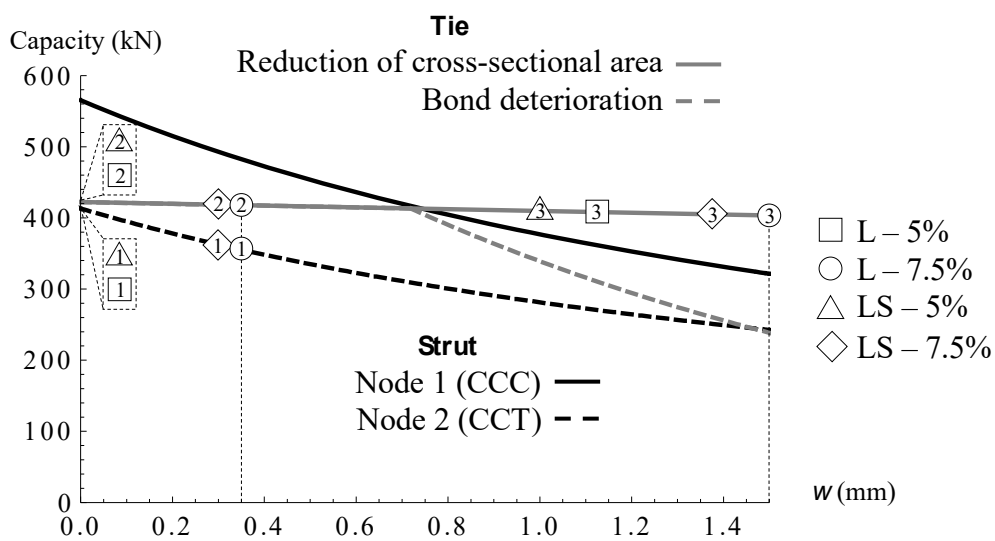
393 **Table 3. Comparison of predicted and experimental residual capacities of RC corroded deep beams**
 394 **presented in [22,46].**

Specimen	P_{corr} (kN)	P_{exp} (kN)	P_{exp}/P_{corr}
L-5%	413.24	476.40	1.15
L-7.5%	355.23	476.17	1.34
LS-5%	413.24	386.17	0.93
LS-7.5%	362.52	422.85	1.17

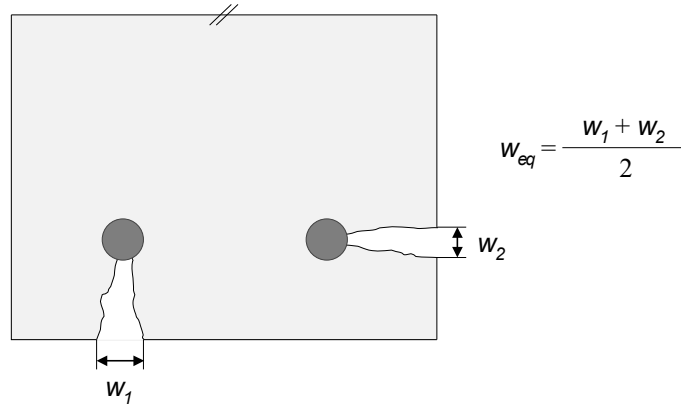
395

396 Figure 12 shows the loss of capacity of the tested specimens, notice that the only
 397 variation among the four beams is the level of corrosion, having the same geometry and
 398 reinforcement layout. The capacity of each component of the D-Region is shown as
 399 function of the crack width w . It can be seen that the decay of the capacity of the RC
 400 deep beam is due to the degradation of node 2 of the S&T model (see Figure 11 (a)).

401 The residual capacity of each corroded deep beam is computed following the proposed
 402 procedure. Let us consider for instance the specimen L-7.5%, shown by a circle in in
 403 Figure 12. From the corrosion cracking map [46] two crack widths have been taken to
 404 evaluate this specimen: crack width at midspan $w_m = 1.5$ mm and crack width at the
 405 supports nodal zone $w_s = 0.35$ mm. These values are the equivalent crack widths
 406 corresponding to both reinforcing bars obtained as it is indicated in Figure 13. The
 407 capacity of the beam as function of the reduction of cross-sectional area of the tie (see
 408 circle with 3 in it in Figure 12) is calculated considering the crack width at midspan w_m .
 409 On the other hand, the capacity of the beam depending on both, bond deterioration of
 410 the tie (see circle with number two in it in Figure 12) and capacity of node 2 (see circle
 411 with 1 in it, in Figure 12), have been estimated accounting for the crack width at the
 412 supports nodal zone w_s . Note that zone 1 is not affected by corrosion. Finally, the
 413 residual capacity of the corroded RC deep beam (specimen L-7.5%) corresponds to the
 414 minimum of the values defined by circles 1, 2 and 3 of Figure 12 (in this case
 415 $P_{corr}=355.23$ kN, corresponding to circle 1). The same procedure was applied for the rest
 416 of the specimens (see Figure 12).



417
 418 **Figure 12. Loss of capacity of the RC deep beams as function of crack width.**



419

420 **Figure 13. Computation of the equivalent crack width in the tie composed by two reinforcing bars.**

421 According to Azam and Soudki[22,46]the experimental failure mode of the four
 422 corroded deep beams considered in the study was splitting of the strut.Because in the
 423 observations of the mentioned campaign [22,46]the bearing face of node 2 was neither
 424 observed nor considered, in this work nodes are only checked at the interface with the
 425 struts. No tension failure (bond or area reduction) of the corroded tie was observed in
 426 any of the tests in [22,46]. This is due to the good initial anchoring conditions of the
 427 longitudinal reinforcement. Table 3 shows that the predicted failure modes and values
 428 correlated very well with the experimental ones.

429 Relationships between crack width and corrosion level are used only in two degradation
 430 phenomena: the reduction of the bond strength and the reduction of cross-section area of
 431 the reinforcing bar. Due to this fact, differences may appear when considering the crack
 432 width or the level of corrosion.

433

434 **6. Conclusions**

435 A general procedure for the assessment of the residual structural capacity of D-Regions
 436 in RC structures affected by corrosion is proposed. The residual capacity is evaluated
 437 for monotonic loading. This method takes into account the reduction of the bond
 438 strength, the reduction of area of the reinforcing bar cross-section, the cracking of the

439 concrete cover and the softening of concrete due to corrosion of the reinforcement. The
440 necessary inputs for the implementation of the proposed procedure are expressed in
441 terms of parameters that are obtained by Non-Destructive Tests and/or by visual
442 inspection. The most important inputs of the method are the concrete corrosion-induced
443 crack widths and their distribution, which can be easily obtained by a detailed visual
444 inspection of the RC structure.

445 The anchorage length is a constant value in D-Regions and it cannot be increased. For
446 this reason, the effect of the deterioration of bond strength between steel bars and the
447 surrounding concrete has been taken into account by reducing the maximum tensile
448 stress of the reinforcing bars. The worth of the method has been evaluated through its
449 application to a real experimental campaign obtaining good similarities between the
450 experimental and the predicted results. In addition, the computation of the residual
451 capacity of a corroded RC half-joint has been developed with high detail. The proposed
452 methodology can be considered as a good tool to assess the residual structural capacity
453 of D-Regions of RC structures affected by corrosion.

454

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459

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