IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received April 5, 2019, accepted April 16, 2019, date of publication April 25, 2019, date of current version May 6, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2913338

Group Decision Making Based on a Framework of Granular Computing for Multi-Criteria and Linguistic Contexts

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This work was supported in part by the Spanish Ministry of Economy and Competitiveness under Project DPI2016-77677-P, in part by the RoboCity2030-DIH-CM Madrid Robotics Digital Innovation Hub (''Robótica aplicada a la mejora de la calidad de vida de los ciudadanos. Fase IV''; S2018/NMT-4331), funded by the ''Programas de Actividades I+D de la Comunidad de Madrid,'' and co-funded by the Structural Funds of the EU, and in part by the research grant from the Asociación Universitaria Iberoamericana de Postgrado (AUIP) and Consejería de Economía y Conocimiento de la Junta de Andalucía.

ABSTRACT The usage of linguistic information involves computing with words, a methodology assuming linguistic values as computational elements, in group decision-making environments. In recent times, a new methodology founded on a framework of granular computing has been employed to manage linguistic information. An advantage of this methodology is that the distribution and the semantics of the linguistic values, in place of being initially established, are defined by the optimization of a certain criterion. In this paper, different from the existing approaches, we present a novel approach build on the basis of a granular computing framework that is able to cope with group decision-making problems defined in multi-criteria contexts, that is, those in which different criteria are considered to evaluate the possible alternatives for solving the problem. In particular, it models group decision-making problems in a more realistic way by taking into account that each criterion has an importance weight and by considering that each decision maker has a different importance weight for each criterion. This approach makes operational the linguistic values by associating them with intervals via the optimization of an optimization criterion composed of two important aspects that must be taken into account in this kind of decision problems, that is, the consensus at the level of group of decision makers and the consistency at the level of individual decision makers.

INDEX TERMS Consensus, consistency, granular computing, linguistic information, multi-criteria group decision making.

I. INTRODUCTION

In a group decision making (GDM) setting, a group of agents (usually called decision makers) must evaluate the suitability of different alternatives as a possible solution to a given problem [1], [2]. Considering the evaluations articulated by the decision makers, the purpose is to arrive at a ranking of the alternatives as possible solutions to the problem.

As decision making is a cognitive process carried out by humans that leads to the selection of a choice between some different ones, the computing with words (CW) methodology [3]–[5], which narrows the differences between computing and human reasoning, has been applied to enrich and create decision models manipulating information of qualitative nature [3]. In CW, linguistic values drawn from a natural language are the computation objects [5]. For instance, linguistic values like ''hot'', ''cool'', or ''nice'', could be used to evaluate the temperature of a room.

In the setting of GDM, each individual decision maker generally evaluates how good an alternative is regarding other one, that is, the decision makers perform pairwise comparisons [6]–[8]. When linguistic values are used to make comparisons, a key point is how making them operational. To deal with this issue, several computational models of linguistic information have been developed [3], [4], [9].

The associate editor coordinating the review of this manuscript and approving it for publication was Omar Khadeer Hussain.

Recently, information granulation [10], which is a key concept of granular computing [11], has been used in decision making to deal with linguistic information [12]–[14]. Information granulation is about the process of forming something into granules [11], [15], which are complex entities of information that must be handled in an efficient way in the computing setting that is relevant to a certain granulation framework. In particular, in the approaches presented in [12]–[14], the information granulation has been applied to associate the linguistic values with a family of intervals, which act as information granules, via an optimization process.

In the approaches proposed in [12]–[14], the decision makers evaluate the preference of an alternative over other one as a whole. However, this should be evaluated according to different criteria. For instance, to evaluate the best library among a group of them, several criteria as ‘‘library materials’’ or ‘‘community space for group study and learning’’ should be considered [16]. In addition, it must be considered that each decision maker participating in the decision problem could play a different role, that is, some decision makers should be more influential than others in some criteria as their knowledge degree, experience, and relevance, could be different among them. As a consequence, it cannot be supposed that each decision maker has the same importance concerning the decision being adopted. Continuing with the above example, the evaluations given by the professors on ‘‘community space for group study and learning’’ should be less relevant than the opinions given by the students because the students usually spend more time in the community space than the professors.

In this study, we aim to present a new approach modeling and supporting GDM processes in which linguistic information is employed. This new approach is able to deal with GDM scenarios in which different criteria are taken into account to evaluate the alternatives considered to solve the problem. In addition, to model GDM processes in a more realistic way, this approach is able to deal with heterogeneous contexts [17] from two points of view: (i) the criteria considered to assess the alternatives can have different importance weights, and (ii) each decision maker may have a different importance weight for each criterion. This new approach is structured into the following three stages:

- The first stage is devoted to gathering the linguistic pairwise comparisons expressed by the decision makers. In particular, the linguistic pairwise comparisons are modeled through linguistic preference relations [8].
- The second stage is vital to produce the ranking of the alternatives. This stage converts the linguistic values to formal constructs of information granules via an optimization process in which two optimization criteria are maximized. In particular, the linguistic values are converted to meaningful intervals so that the final solution is that of highest consensus [18]–[20] and consistency [21], two important aspects that must be considered in GDM scenarios. To address this optimization process, we make use of the particle swarm optimization (PSO) algorithm [22] because it has been proved as a viable technique to solve similar problems [12]–[14].
- The third, and final, stage consists in obtaining the final ranking of alternatives by considering the information contained in the linguistic preference relations (it is usually called selection process [23]). It takes into account that the criteria have different importance weights and that each decision maker has a different importance weight for each criterion.

This study is structured into five sections. In Section II, we recall in a concise way the granulation process of linguistic information, the PSO algorithm, and some aggregation operators. Section III focuses on the core part of this study, that is, the novel approach modeling and supporting GDM processes defined in multi-criteria and linguistic contexts. Section IV illustrates the proposed approach and analyzes its results. In Section V, we cover future research and main conclusions.

II. PRELIMINARIES

We start this section with a brief introduction to the granulation process of the linguistic information. Next, we recall some basic concepts of the PSO algorithm and, finally, we describe some aggregation operators.

A. GRANULATION OF LINGUISTIC INFORMATION

As aforementioned, linguistic values from a linguistic term set, $S = \{s_1, s_2, \ldots, s_g\}$ (being g its granularity [3]), are used for evaluating the degree of preference between alternatives if a domain of linguistic information is assumed. In this setting, a linear order $\prec$ between the linguistic values is generally supposed in which $\forall s_i, s_j \in S$, if $s_i \prec s_j$ $(j > i)$, then s_j indicates a higher preference degree than s_i. For instance, let us suppose a linguistic term set S formed by these linguistic values: s_1 = ‘‘Much Worse’’ (MW), s_2 = ‘‘Worse’’ (W), s_3 = ‘‘Equal’’ (E), s_4 = ‘‘Better’’ (B), and s_5 = ‘‘Much Better’’ (MB). The granularity g of this linguistic term set is five, and ‘‘Much Better’’ indicates a higher preference degree than ‘‘Equal’’.

The linguistic values themselves are not operational, which means that no further processing may be performed. Therefore, the linguistic values require a granulation [10], [24], that is, a process of forming them into information granules. For example, and just to refer to some options, shadowed sets, rough sets, intervals, or fuzzy sets, may be considered as formalisms of information granulation [25].

To arrive at the operational version of the linguistic values as information granules, an optimization process may be formulated to optimize an optimization criterion. For example, the consistency of individual decision makers was employed as an optimization criterion in [14] and [12], whereas both the consistency of individual decision makers and the consensus among the group were employed in [13].

B. PSO ALGORITHM

The PSO algorithm is a population based optimization method introduced by Kennedy and Eberhart in 1995 [22]. The social behavior of a flock of birds or a school of fish inspired them to develop this algorithm.

Let us suppose an only piece of food in a certain area and a flock of birds randomly looking for it. The birds are aware how far the piece of food is in each iteration, but they do not know its location. Here, to locate the piece of food, the best approach is to move behind the bird nearest to it.

PSO learns from the environment and employs that knowledge to solve optimization problems. Here, a candidate solution represents a bird, called particle, in the problem space. Each particle possesses a velocity directing its flying and a fitness value that is assessed by the fitness function to be optimized. Every particle flies through the search space by moving behind the current optimum particles.

The PSO algorithm starts initializing in a random manner a collection (swarm) of m particles, which symbolize solutions in the n-dimensional search space, and then explores for optima in every new iteration of the algorithm. Each particle i is composed of two n-dimensional vectors: (i) the velocity vector $\mathbf{v}_i = (v_{i,1}, v_{i,2}, \ldots, v_{i,n})$, and (ii) the position vector $\mathbf{x}_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,n})$. In each iteration of the algorithm, the particle's velocity and position are updated based on two best values: (i) the individual best fitness value the particle has obtained up to this point, called "pbest", and (ii) the global best fitness value achieved up to this point by any particle, called "gbest". To control the velocity, the inertia weight ω balancing the exploitation and exploration of the particles was introduced by Shi and Eberhart [26]. A lower value of ω speeds up the convergence to optima, and a higher value of ω encourages exploration of the total search space.

In iteration t, after finding the two best values, the update equations of the velocity and the position are [22], [27]:

$$v_{i,j}(t+1) = \omega(t) \cdot v_{i,j}(t) + c_1 \cdot r_{1,j} \cdot (x_{i,j}^{pbest}(t) - x_{i,j}(t)) + c_2 \cdot r_{2,j} \cdot (x_j^{gbest}(t) - x_{i,j}(t)) \tag{1}$$

$$x_{i,j}(t+1) = x_{i,j}(t) + v_{i,j}(t+1) \tag{2}$$

where $v_{i,j}$ represents the velocity of the particle i in the jth dimension, $x_{i,j}$ represents the position of the particle i in the jth dimension, and $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, being m the swarm size and n the dimension of the search space. The vector $\mathbf{x}^{gbest} = (x_1^{gbest}, x_2^{gbest}, \ldots, x_n^{gbest})$ corresponds to the global best position of the particle that has achieved the global best fitness value so far, and the vector $\mathbf{x}_i^{pbest} = (x_{i,1}^{pbest}, x_{i,2}^{pbest}, \ldots, x_{i,n}^{pbest})$ corresponds to the individual best position achieved so far by the particle i. The values $r_{1,j}$ and $r_{2,j}$ are two random values regenerated for each iteration from the uniform distribution on the unit interval. The value c_1 is the cognitive coefficient affecting the step size the particle takes in the direction of its individual best position, whereas the value c_2 is the social coefficient representing the step size the particle takes in the direction of the global best position the swarm has achieved up until now. Finally, the decrease of ω ensures the ability of a strong global exploration at the initial iterations of the search process and the ability of a strong local exploitation at the last iterations. Therefore, its value is usually decreased linearly according to:

$$\omega(t) = (\omega_{start} - \omega_{end}) \cdot \frac{t_{max} - t}{t_{max}} + \omega_{end} \tag{3}$$

where ω_{start} is the initial value of ω and ω_{end} is its final value, the current iteration number and the maximum iteration number are represented by t and t_{max}, respectively, and $\omega(t)$ is the value of ω in the current iteration.

Among the advantages of the PSO algorithm we can mention that there are few parameters to adjust and its ease of implementation [27].

C. AGGREGATION

For all types of knowledge-based systems, fusion and aggregation of information are fundamental matters of interest, in particular, for GDM. Aggregation has for purpose, from a general perspective, the concurrent usage of various pieces of information (obtained from different sources) so as to come to a decision or a conclusion. To perform an intelligent aggregation, numerical aggregation operators are employed, which are mathematical objects having the role of reducing a collection of numbers to an only significant one.

As GDM consists in expressing and fusing evaluations associated with some predetermined alternatives in order to rank them, aggregation operators are required for fusing the evaluations. Therefore, we recall two families of aggregation operators due to the fact that they are used by our approach.

1) ORDERED WEIGHTED AVERAGING OPERATORS

The family of the Ordered Weighted Averaging (OWA) operators was at first introduced by Yager to make available a means to aggregate assessments related to the satisfaction of multiple criteria. It unifies in one operator the disjunctive and conjunctive behavior.

Definition 1 [28]: A mapping ϕ from $\mathbb{R}^n \rightarrow \mathbb{R}$ is an OWA operator of dimension n if, related to ϕ, there exists a weighting vector $w = (w_1, w_2, \ldots, w_n)$ such that $w_i \in [0, 1]$, $\sum_{i=1}^{n} w_i = 1$, and where:

$$\phi(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j \cdot b_j \tag{4}$$

being b_j the jth largest component in the collection $a_1, a_2, \ldots, a_n$.

The OWA operators make available a parametric family of aggregation operators, including many of the well-known operators such as the median, the minimum, the arithmetic mean, and the maximum, which are obtained by choosing appropriate weights [29]. In fact, one issue of considerable interest related to the use of these operators is the development of an appropriate methodology for the derivation of the weights used in the OWA aggregation [28]–[31].

One approach to do so is by drawing upon the linguistic quantifiers proposed by Zadeh [32] and the application of this idea to multi-criteria decision making [28], [33]. OWA operators play a fundamental role in modeling linguistic quantifiers such as ''most'', ''almost all'', ''few'', or ''nearly half'', which are modeled by the weighting vector w. On the basis of this kind of quantifiers, Yager proposed to compute the weights using:

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), \quad i = 1, \ldots, n \tag{5}$$

where Q is the linguistic quantifier being modeled [28]–[31].

If the weights associated with the OWA operator are determined by using this approach, this is represented by ϕ_Q.

2) INDUCED WEIGHTED AVERAGING OPERATORS

A family of aggregation operators more general than the OWA operators is that of the Induced Ordered Weighted Averaging (IOWA) operators. The arguments taken by these operators are pairs, called OWA pairs, in which the first components bring about an arrangement of the second components that are then aggregated.

Definition 2 [34]: A mapping Φ from $\mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$ is an IOWA operator of dimension n if, related to Φ, there exists a weighting vector $w = (w_1, w_2, \ldots, w_n)$, such that $w_i \in [0, 1]$, $\sum_{i=1}^{n} w_i = 1$, and where:

$$\Phi(\langle u_1, a_1 \rangle, \langle u_1, a_2 \rangle, \ldots, \langle u_n, a_n \rangle) = \sum_{j=1}^{n} w_j \cdot b_j \tag{6}$$

being b_j the a_i component of the OWA pair $\langle u_i, a_i \rangle$ that has the jth largest u_i value. Due to the role of the components of the OWA pairs, a_i is known as the argument variable and u_i is known as the order inducing variable.

The same approach based on linguistic quantifiers can be also applied to generate the weights related to the IOWA operator, which is represented by Φ_Q.

III. A MULTI-CRITERIA GDM APPROACH IN LINGUISTIC CONTEXTS

In this section, we introduce a novel approach modeling and supporting GDM processes defined in multi-criteria and linguistic contexts.

A multi-criteria GDM problem can be formalized as one where a collection of n alternatives, $A = \{a_1, a_2, \ldots, a_n\}$, is evaluated by a group of m decision makers, $DM = \{dm^1, dm^2, \ldots, dm^m\}$, who are taking into account q criteria, $C = \{c^1, c^2, \ldots, c^q\}$, to arrive at a ranking of the alternatives as possible solutions to the problem under study [35]–[37]. In a linguistic context, the evaluations are modeled via linguistic values from a linguistic term set [3], [38].

To allow the modeling of real world problems in a more realistic way, we consider that each criterion, c^k $(k = 1, \ldots, q)$, has an importance weight, α^k, and each decision maker, dm^h $(h = 1, \ldots, m)$, has a different importance weight, β^{hk}, for each criterion, c^k $(k = 1, \ldots, q)$. This is done by representing the importance weights as linguistic values. In particular, we make use of three different linguistic term sets, one to represent the decision makers' evaluations, S^1, one to represent the importance of the criteria, S^2, and one to represent the decision maker's importance for each criterion, S^3. In addition, both the semantics and the number of the linguistic values can be different for each one of these three linguistic term sets.

In particular, the proposed approach carries out its activity in three stages that are elaborated on in the next subsections: (i) articulation of evaluations, (ii) granulation of the linguistic values, and (iii) selection process.

A. ARTICULATION OF EVALUATIONS

The first stage is devoted to gathering the evaluations articulated by the decision makers. Pairwise comparisons, utility values, or preference orderings, are usually employed to articulate evaluations in GDM [39]. In [7], the author analyzed different preference elicitation methods and came to the conclusion that a pairwise comparison is more accurate than a nonpairwise one (for example, a utility value or a preference ordering). Therefore, pairwise comparisons are assumed here.

In a pairwise comparison between alternatives: (i) the decision maker cannot compare the alternatives, (ii) the decision maker treats the alternatives as indifferent, or (iii) the decision maker selects one alternative to the other. Two mathematical approaches founded on the concept of a preference relation, a structure of preference on the collection of alternatives, have been proposed to model these situations. First, a preference relation can be determined for each one of the above three states. Second, an only one preference relation can integrate the above three states [40]. The last one has been widely used in GDM and, therefore, this study deals with this kind of preference relations.

Definition 3: A preference relation PR on a collection of alternatives $A = \{a_1, a_2, \ldots, a_n\}$ is characterized by a function $\mu_{PR} : A \times A \rightarrow D$, where D is the representation domain of preference degrees.

A preference relation PR is generally modeled by a $n \times n$ matrix $PR = (pr_{ij})$. In this representation, the degree in which a_i is preferred to a_j is represented by $pr_{ij} = \mu_{PR}(a_i, a_j)$, and the elements of the principal diagonal, that is, pr_{ii}, are commonly written as '-' because they are not important in this context [41]. According to the representation domain of preference degrees D, there exist several kinds of preference relations (refer to [8] for an exhaustive survey). Here, we make use of linguistic preference relations [42] due to the fact that we handle linguistic information.

In this stage, the linguistic term set S^1 that contains the linguistic values used to articulate the evaluations is provided to the group of decision makers prior to making any evaluation. Then, each decision maker dm^h $(h = 1, \ldots, m)$ offers a linguistic preference relation PR^{hk} for each criterion c^k $(k = 1, \ldots, q)$.

B. GRANULATION OF THE LINGUISTIC VALUES

Prior to carrying out the selection process, the linguistic values that come from the linguistic preference relations must be made operational via a granulation process. This second stage is devoted to do this.

The granular definition of the linguistic values is related to the realization of a family of information granules. In this study, similar to the approaches presented in [12]–[14], the granulation process is formulated in the language of intervals. It means that the granules of information come in the form of intervals over the unit interval. As a consequence, if we consider a linguistic term set consisting of g linguistic values, the vector of cut-off points, $\mathbf{p} = (p_1, p_2, \ldots, p_{g-1})$ forms and completely defines a family of intervals, $I_1, I_2, \ldots, I_g$, where $0 < p_1 < p_2 < \ldots < p_{g-1} < 1$ and $I_1 = [0, p_1)$, $I_2 = [p_1, p_2)$, …, $I_j = [p_{j-1}, p_j)$, …, $I_g = [p_{g-1}, 1]$.

This process presents the following three important characteristics:

- It retains the semantics of the linguistic values distributed in the granulation.
- The allocation of the corresponding intervals on the scale is not uniform, that is, the mapping is not linear.
- It arrives at the operational version of the linguistic values modeled as intervals by formulating an optimization task.

Next, we introduce the optimization criterion and the optimization process of this optimization criterion.

1) OPTIMIZATION CRITERION

In the setting of GDM, both the consensus and the consistency play an important role:

- To adopt a consensus decision by the group of decision makers is a major objective in GDM. If a consensual decision is not reached, some decision makers could think their evaluations have not been taken into account in a proper way and they might refuse the final decision. Therefore, the consensus has obtained a great attention in the development of GDM approaches (refer to [19], [43], and [44], for a better understanding of the meaning of consensus).
- Pairwise comparisons have as a principal advantage that of paying attention on merely two alternatives at once. This helps decision makers to verbalize evaluations in a better way [7]. However, pairwise comparisons generate more information than is actually needed and limit decision makers in the global understanding of the alternatives. Consequently, the evaluations could be inconsistent, leading to illogical decisions. Therefore, it is very important to analyze conditions satisfying consistency [21].

Given these two facts, the higher the consistency and the consensus achieved, the better the decision made. As a result, a weighted averaging of the consensus and the consistency is used as optimization criterion:

$$O = O_1 \cdot \gamma + (1 - \gamma) \cdot O_2 \tag{7}$$

where γ represents a parameter located in the unit interval [45], which establishes a tradeoff between the consensus, O_1, and the consistency, O_2.

2) OPTIMIZATION PROCESS

Considering the form of the optimization criterion, we may take into account several options to optimize it. However, as the PSO [22] has been proved as a good choice to solve this kind of optimization task [12]–[14], it is also used here. In this algorithm, as it is known, the two most essential parts are the definition of the particle and the fitness function employed to measure the quality of the particle.

Concerning the definition of the particle, we model it by means of a vector of cut-off points located in the unit interval, that is, the cut-off points specify the intervals into which the linguistic values are transformed. As an illustration, let us suppose the decision makers' evaluations are represented by the linguistic term set $S^1 = \{s_1^1 =$ "Much Worse" (MW^1), $s_2^1 =$ "Worse" (W^1), $s_3^1 =$ "Equal" (E^1), $s_4^1 =$ "Better" (B^1), $s_5^1 =$ "Much Better" (MB^1)$\}$, the importance of the criteria are represented by the linguistic term set $S^2 = \{s_1^2 =$ "Less Important" (LI^2), $s_2^2 =$ "Important" (I^2), $s_3^2 =$ "Very Important" (VI^2)$\}$, and the decision makers' importance for each criterion are represented by the linguistic term set $S^3 = \{s_1^3 =$ "Less Important" (LI^3), $s_2^3 =$ "Important" (I^3), $s_3^3 =$ "Very Important" (VI^3)$\}$. Then, the mapping formed is: MW^1: $[0, p_1^1)$, W^1: $[p_1^1, p_2^1)$, E^1: $[p_2^1, p_3^1)$, B^1: $[p_3^1, p_4^1)$, MB^1: $[p_4^1, 1]$, LI^2: $[0, p_1^2)$, I^2: $[p_1^2, p_2^2)$, VI^2: $[p_2^2, 1]$, LI^3: $[0, p_1^3)$, I^3: $[p_1^3, p_2^3)$, and VI^3: $[p_2^3, 1]$, being p_1^1, p_2^1, p_3^1, p_4^1, p_1^2, p_2^2, p_1^3, and p_2^3, the cut-off points that define the vector $\mathbf{p} = (p_1^1, p_2^1, p_3^1, p_4^1, p_1^2, p_2^2, p_1^3, p_2^3)$, which models each particle in this example. If the granularities of S^1, S^2, and S^3, are g^1, g^2, and g^3, respectively, each particle is composed of $(g^1 + g^2 + g^3 - 3)$ cut-off points.

Concerning the definition of the fitness function, we need to consider that intervals form the entries of the linguistic preference relations. However, the fitness function must return numeric values. That is, we have intervals as information granules that represent the linguistic values of the linguistic preference relations. Therefore, we form the entries by randomly producing numeric values coming from the intervals. We do this by sampling the linguistic preference relations PR^{hk} $(h = 1, \ldots, m;\ k = 1, \ldots, q)$ to generate the preference relations R^{hk} $(h = 1, \ldots, m;\ k = 1, \ldots, q)$, which are composed of entries whose values are drawn from the uniform distribution defined over the interval associated with the linguistic value that corresponds to each particular entry. As an illustration, let us suppose that, based on the preference degree expressed by the decision maker dm^1, the altenative a_4 is "Better" than the alternative a_2 over the criterion c^3. According to this evaluation, the linguistic value associated with the entry pr_{42}^{13} of PR^{13} is "Better".

Assuming that the interval related to "Better" is [0.71, 0.80), the entry of R^{13}, r^{13}_{42}, is calculated by the uniform distribution defined over [0.71, 0.80). Similarly, the importance weights, α^k $(k = 1, \ldots, q)$ and β^{hk} $(h = 1, \ldots, m;\ k = 1, \ldots, q)$, are also sampled to generate the weights u^k and v^{hk}, respectively, which are represented by numbers drawn from the uniform distribution defined over the corresponding interval related to the linguistic value representing the importance weight.

In summary, each linguistic preference relation and each importance weight are sampled N times and the average of the values assumed by the optimization criterion O over each collection of N samples determines the fitness function f:

$$f = \frac{1}{N} \sum_{i=1}^{N} O^i \tag{8}$$

In each sample i, the optimization criterion O^i is calculated using (7). Therefore, we must describe how to compute the consensus, O_1, and the consistency, O_2. Because the entries of the preference relations, PR^{hk} $(h = 1, \ldots, m;\ k = 1, \ldots, q)$, contain values belonging to the closed interval [0, 1], we can adapt the approaches existing in the literature to measure the consensus and the consistency when fuzzy preference relations are used.

On the one hand, we propose a new methodology, which is based on the coincidence concept [46] and the three levels of a preference relation [47], to compute O_1 in a multi-criteria GDM setting. It is as follows:

- First, we determine a similarity matrix $SM^{hlk} = (sm^{hlk}_{ij})$ for each critetion c^k and for each pair of decision makers dm^h and dm^l:

$$sm^{hlk}_{ij} = 1 - |r^{hk}_{ij} - r^{lk}_{ij}| \tag{9}$$

- Second, for each criterion c^k, we aggregate all the similatity matrices SM^{hlk} to determine a consensus matrix $CM^k = (cm^k_{ij})$:

$$cm^k_{ij} = \frac{1}{(m-1)\cdot(m-2)} \sum_{h=1}^{m-1} \sum_{l=h+1}^{m} sm^{hlk}_{ij} \tag{10}$$

- Third, for each criterion c^k, we compute three consensus measures related to the three levels of a preference relation:
  - Consensus degree associated with pairs of alternatives, cp^k_{ij}, evaluating the consensus achieved on a given pair of alternatives (a_i, a_j):

$$cp^k_{ij} = cm^k_{ij} \tag{11}$$

  - Consensus degree associated with alternatives, ca^k_i, evaluating the consensus achieved on a given alternative a_i:

$$ca^k_i = \frac{1}{2\cdot(n-1)} \sum_{j=1;\, j\neq i}^{n} (cp^k_{ij} + cp^k_{ji}) \tag{12}$$

  - Consensus degree associated with the relation, cr^k, evaluating the global consensus achieved on a given criterion c^k:

$$cr^k = \frac{1}{n} \sum_{i=1}^{n} ca^k_i \tag{13}$$

- Fourth, the consensus, O_1, is computed as the weighted average of the consensus degrees on the criteria:

$$O_1 = \frac{1}{\sum_{k=1}^{q} u^k} \sum_{k=1}^{q} u^k \cdot cr^k \tag{14}$$

On the other hand, O_2 is computed as the average of the consistency degrees related to each decision maker:

$$O_2 = \frac{1}{m} \sum_{h=1}^{m} cd^h \tag{15}$$

being cd^h the consistency degree related to the decision maker dm^h that is computed as the weighted average of the consistency degrees related to the decision maker for each criterion c^k:

$$cd^h = \frac{1}{\sum_{k=1}^{q} v^{hk}} \sum_{k=1}^{q} v^{hk} \cdot cd^{hk} \tag{16}$$

being cd^{hk} the consistency degree related to the decision maker dm^h for the criterion c^k, which is computed by using the methodology introduced by Herrera-Viedma et al. (refer to [23] for a detailed description of the procedure).

C. SELECTION PROCESS

The third stage is devoted to obtaining the ranking of alternatives via a selection process that is structured into two steps [23]: (i) aggregation, and (ii) exploitation. Next, we elaborate on both steps.

1) AGGREGATION

This step obtains a collective preference relation summarizing the evaluations verbalized by the decision makers. To do so, it must take into account that the criteria have different importance weights and that the decision makers also have different importance weights for each criterion, which can be modeled by an IOWA operator [34]. Again, as each linguistic value is formed as an interval, the entries of the linguistic preference relations, PR^{hk} $(h = 1, \ldots, m;\ k = 1, \ldots, q)$, the importance weights associated with the criteria, α^k $(k = 1, \ldots, q)$, and the importance weights associated with each decision maker for each criterion, β^{hk} $(h = 1, \ldots, m;\ k = 1, \ldots, q)$, are sampled N times. Then, the average is used as value for the associated entry in the preference relations, $\bar{R}^{hk} = (\bar{r}^{hk}_{ij})$ $(h = 1, \ldots, m;$ $k = 1, \ldots, q)$, for the importance weights associated with the criteria, $\bar{u}^k$ $(k = 1, \ldots, q)$, and for the importance weights associated with each decision maker for each criterion, $\bar{v}^{hk}$ $(h = 1, \ldots, m;\ k = 1, \ldots, q)$.

The process of obtaining the collective preference relation is as follows:

- For each criterion, c^k, a collective preference relation, $\bar{R}^{ck} = (\bar{r}_{ij}^{ck})$, is obtained by using an IOWA operator in which the importance weight associated with each decision maker for the criterion c^k is the order inducing variable:

$$\bar{r}_{ij}^{ck} = \Phi_Q(\langle \bar{v}^{1k}, \bar{r}_{ij}^{1k}\rangle, \langle \bar{v}^{2k}, \bar{r}_{ij}^{2k}\rangle, \ldots, \langle \bar{v}^{mk}, \bar{r}_{ij}^{mk}\rangle) \quad (17)$$

- The final collective preference relation, $\bar{R}^c = (\bar{r}_{ij}^c)$, is computed by using an IOWA operator in which the importance weights associated with the criteria are the order inducing variable:

$$\bar{r}_{ij}^{c} = \Phi_Q(\langle \bar{u}^{1}, \bar{r}_{ij}^{c1}\rangle, \langle \bar{u}^{2}, \bar{r}_{ij}^{c2}\rangle, \ldots, \langle \bar{u}^{k}, \bar{r}_{ij}^{ck}\rangle) \quad (18)$$

2) EXPLOITATION

This step, using the information contained in $\bar{R}^c$, ranks the alternatives in order to get the best one to solve the problem. To do so, we make use of two well-known choice degrees of alternatives [23], which are based on OWA operators and the concept of fuzzy majority:

- The quantifier-guided dominance degree, $QGDD_i$, measuring the dominance that the alternative a_i has over the reamining ones in a fuzzy majority sense:

$$QGDD_i = \phi_Q(\bar{r}_{i1}^c, \bar{r}_{i2}^c, \ldots, \bar{r}_{i(i-1)}^c, \bar{r}_{i(i+1)}^c, \ldots, \bar{r}_{in}^c) \quad (19)$$

- The quantifier-guided nondominance degree, $QGNDD_i$, measuring the degree in which the alternative a_i is not dominated by a fuzzy majority of the reamining ones:

$$QGNDD_i = \phi_Q(1-\bar{r}_{1i}^s, 1-\bar{r}_{2i}^s, \ldots, 1-\bar{r}_{(i-1)i}^s, 1-\bar{r}_{(i+1)i}^s, \ldots, 1-\bar{r}_{ni}^s) \quad (20)$$

  where the degree in which the alternative a_i is strictly dominanted by the alternative a_j is represented by $\bar{r}_{ji}^s = max\{\bar{r}_{ji}^c - \bar{r}_{ij}^c, 0\}$.

In particular, the selection process applies these two choice degrees of alternatives as follows:

- First, we obtain the following two collections of alternatives by applying each choice degree of alternatives to A:

$$A^{QGDD} = \left\{ a_i \in A \,|QGDD_i = \sup_{a_j \in A} QGDD_j \right\} \quad (21)$$

$$A^{QGNDD} = \left\{ a_i \in A \,|QGNDD_i = \sup_{a_j \in A} QGNDD_j \right\} \quad (22)$$

- Second, we obtain a new collection of alternatives as the intersection of these two collection of the alternatives:

$$A^{QG} = A^{QGDD} \bigcap A^{GQNDD} \quad (23)$$

  If $A^{QG} \neq \emptyset$, then this is the solution set of alternatives. Otherwise, continue.

- Third, if $\#(A^{QGDD}) = 1$, then this is the solution set of alternatives. Otherwise, we select the alternative of this set with the best quantified-guided nondominance degree.

IV. ILLUSTRATIVE EXAMPLE

This section aims to illustrate the proposal presented in this study and to discuss its results. Let us suppose that a group of four decision makers $DM = \{dm^1, dm^2, dm^3, dm^4\}$, in which the decision makers, dm^1 and dm^2, are students, the decision maker, dm^3, is a professor, and the decision maker, dm^4, is a staff member, evaluate five academic libraries at the University of Granada:

- Academic library at the School of Architecture (a_1).
- Academic library at the Faculty of Sciences (a_2).
- Academic library at the Faculty of Law (a_3).
- Academic library at the Faculty of Medicine (a_4).
- Academic library at the Faculty of Psychology (a_5).

The decision makers evaluate the five academic libraries according to three criteria:

- The library space inspires study and learning (c^1).
- Collections of electronic journals (c^2).
- Willingness to help users (c^3).

We also suppose that the decision makers' evaluations are represented by the linguistic term set $S^1 = \{s_1^1 =$ "Much Worse" (MW^1), $s_2^1 =$ "Worse" (W^1), $s_3^1 =$ "Equal" (E^1), $s_4^1 =$ "Better" (B^1), $s_5^1 =$ "Much Better" (MB^1)$\}$; the importance of the criteria is represented by the linguistic term set $S^2 = \{s_1^2 =$ "Less Important" (LI^2), $s_2^2 =$ "Important" (I^2), $s_3^2 =$ "Very Important" (VI^2)$\}$; and the decision makers' importance for each criterion is represented by the linguistic term set $S^3 = \{s_1^3 =$ "Less Important" (LI^3), $s_2^3 =$ "Important" (I^3), $s_3^3 =$ "Very Important" (VI^3)$\}$. In addition, the importance weights associated with the criteria are: $\alpha^1 = \text{I}^2$, $\alpha^2 = \text{LI}^2$, and $\alpha^3 = \text{VI}^2$; the importance weights associated with the decision maker dm^1 are: $\beta^{11} = \text{VI}^3$, $\beta^{12} = \text{LI}^3$, and $\beta^{13} = \text{VI}^3$; the importance weights associated with the decision maker dm^2 are: $\beta^{21} = \text{VI}^3$, $\beta^{22} = \text{LI}^3$, and $\beta^{23} = \text{VI}^3$; the importance weights associated with the decision maker dm^3 are: $\beta^{31} = \text{LI}^3$, $\beta^{32} = \text{VI}^3$, and $\beta^{33} = \text{I}^3$; and the importance weights associated with the decision maker dm^4 are: $\beta^{41} = \text{I}^3$, $\beta^{42} = \text{I}^3$, and $\beta^{43} = \text{LI}^3$.

In the following three subsections, we provide the parameter settings of the PSO algorithm, we show the results achieved by our proposal, and we analyze its performance.

A. PARAMETER SETTINGS

The parameters of the PSO algorithm, as a consequence of an intense experimentation, were set as follows:

- The swarm was composed of 200 particles. Similar results were reached in different runs of the PSO. Therefore, this size was found to produce steady results.
- The maximum iteration number was set to 400 because, after this number of iterations, the same values reported by the fitness function were observed.

- c_1 and c_2 were set to 2 as this value is usually used in the existing literature [48].
- ω was set to decrease linearly from 0.9 to 0.4 [49]:

$$\omega(t) = (0.9 - 0.4) \cdot \frac{t_{max} - t}{t_{max}} + 0.4 \qquad (24)$$

  where t and t_{max} represent the current iteration number and the maximum iteration number, respectively.
- Because we want to give more importance to the consensus, γ was set to 0.75 in (7).
- In (8), N was set to 500 as similar results were reported by higher values of N.

B. RESOLUTION PROCESS

The decision makers must compare the possible alternatives according to three criteria. Therefore, each decision maker must provide three linguistic preference relations:

$$PR^{11} = \begin{pmatrix} - & W & E & E & B \\ B & - & B & B & E \\ E & W & - & B & W \\ E & W & W & - & MW \\ MW & E & B & MB & - \end{pmatrix}$$

$$PR^{12} = \begin{pmatrix} - & B & B & B & W \\ MW & - & MB & W & B \\ E & W & - & MB & MW \\ E & E & W & - & E \\ MB & W & B & E & - \end{pmatrix}$$

$$PR^{13} = \begin{pmatrix} - & W & W & B & B \\ E & - & E & B & W \\ W & E & - & E & W \\ W & B & MB & - & B \\ MW & MB & W & MW & - \end{pmatrix}$$

$$PR^{21} = \begin{pmatrix} - & E & W & MB & E \\ E & - & MB & MB & MW \\ B & MW & - & E & MB \\ MW & MW & E & - & MW \\ E & B & MW & MB & - \end{pmatrix}$$

$$PR^{22} = \begin{pmatrix} - & MB & MB & B & W \\ W & - & B & E & B \\ W & MW & - & B & W \\ W & E & W & - & B \\ MB & MW & MB & MW & - \end{pmatrix}$$

$$PR^{23} = \begin{pmatrix} - & MW & MW & MB & B \\ E & - & B & E & E \\ B & W & - & MW & B \\ MW & E & B & - & E \\ MW & E & B & E & - \end{pmatrix}$$

$$PR^{31} = \begin{pmatrix} - & MW & B & W & MB \\ B & - & E & W & W \\ W & E & - & E & MW \\ B & B & E & - & B \\ MW & B & MB & W & - \end{pmatrix}$$

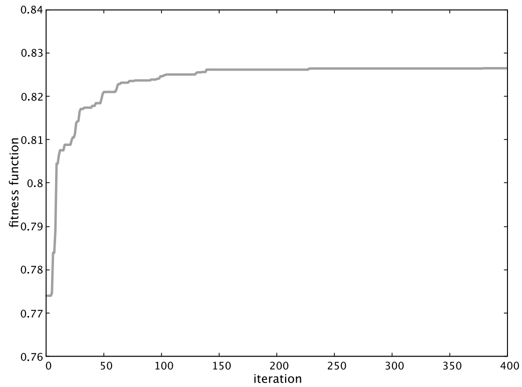

FIGURE 1. Plot of the values that the PSO algorithm returns.

$$PR^{32} = \begin{pmatrix} - & W & W & B & B \\ MB & - & W & W & E \\ MB & B & - & W & B \\ E & B & MB & - & B \\ W & E & MW & B & - \end{pmatrix}$$

$$PR^{33} = \begin{pmatrix} - & E & E & W & B \\ E & - & MW & MW & E \\ E & W & - & MB & MB \\ B & B & B & - & W \\ MW & E & W & B & - \end{pmatrix}$$

$$PR^{41} = \begin{pmatrix} - & W & MB & MB & B \\ B & - & MW & MW & E \\ MW & MB & - & B & MW \\ MW & MB & W & - & MB \\ W & E & B & MW & - \end{pmatrix}$$

$$PR^{42} = \begin{pmatrix} - & E & E & W & W \\ E & - & E & B & W \\ E & E & - & E & E \\ MB & MW & E & - & B \\ MB & B & E & W & - \end{pmatrix}$$

$$PR^{43} = \begin{pmatrix} - & MB & MB & W & MB \\ W & - & E & W & W \\ MW & E & - & B & E \\ E & MB & W & - & MB \\ W & B & E & W & - \end{pmatrix}$$

Once the four decision makers have communicated the linguistic preference relations, we run the approach introduced in Section III-B to make operational the linguistic values. Fig. 1 displays the performance of the PSO with regard to the values that the fitness function reports in consecutive iterations. The PSO returns 0.826 as the best value of the optimization criterion, being 0.005 its standard deviation.

The vector of cut-off points reported by the PSO is $\mathbf{p} = (0.56, 0.64, 0.72, 0.80, 0.06, 0.14, 0.62, 0.88)$. According to this vector of cut-off points, the intervals corresponding to the linguistic values of the sets S^1, S^2, and S^3, are:

- MW^1: $[0, 0.56)$, W^1: $[0.56, 0.64)$, E^1: $[0.64, 0.72)$, B^1: $[0.72, 0.80)$, and MB^1: $[0.80, 1]$.
- LI^2: $[0, 0.06)$, I^2: $[0.06, 0.14)$, and VI^2: $[0.14, 1]$.
- LI^3: $[0, 0.62)$, I^3: $[0.62, 0.88)$, and VI^3: $[0.88, 1]$.

Once we have obtained the intervals associated with the linguistic values, the last step consists in ranking the alternatives by means of the selection process.

First, the preference relations $\bar{R}^{hk} = (\bar{r}_{ij}^{hk})$ $(h = 1, \ldots, 4;\ k = 1, \ldots, 3)$, the importance weights associated with the criteria, $\bar{u}^k$ $(k = 1, \ldots, 3)$, and the importance weights associated with each decision maker for each criterion, $\bar{v}^{hk}$ $(h = 1, \ldots, 4;\ k = 1, \ldots, 3)$, are obtained:

$$\bar{R}^{11} = \begin{pmatrix} - & 0.60 & 0.68 & 0.68 & 0.76 \\ 0.76 & - & 0.76 & 0.76 & 0.68 \\ 0.68 & 0.60 & - & 0.76 & 0.60 \\ 0.68 & 0.60 & 0.60 & - & 0.28 \\ 0.28 & 0.68 & 0.76 & 0.90 & - \end{pmatrix}$$

$$\bar{R}^{12} = \begin{pmatrix} - & 0.76 & 0.76 & 0.76 & 0.60 \\ 0.28 & - & 0.90 & 0.60 & 0.76 \\ 0.68 & 0.60 & - & 0.90 & 0.28 \\ 0.68 & 0.68 & 0.60 & - & 0.68 \\ 0.90 & 0.60 & 0.76 & 0.68 & - \end{pmatrix}$$

$$\bar{R}^{13} = \begin{pmatrix} - & 0.28 & 0.28 & 0.76 & 0.76 \\ 0.68 & - & 0.68 & 0.76 & 0.60 \\ 0.60 & 0.68 & - & 0.68 & 0.60 \\ 0.60 & 0.76 & 0.90 & - & 0.76 \\ 0.28 & 0.90 & 0.60 & 0.28 & - \end{pmatrix}$$

$$\bar{R}^{21} = \begin{pmatrix} - & 0.68 & 0.60 & 0.90 & 0.68 \\ 0.68 & - & 0.90 & 0.90 & 0.28 \\ 0.68 & 0.28 & - & 0.68 & 0.90 \\ 0.28 & 0.28 & 0.68 & - & 0.28 \\ 0.68 & 0.76 & 0.28 & 0.90 & - \end{pmatrix}$$

$$\bar{R}^{22} = \begin{pmatrix} - & 0.90 & 0.90 & 0.76 & 0.60 \\ 0.60 & - & 0.76 & 0.68 & 0.76 \\ 0.60 & 0.28 & - & 0.76 & 0.60 \\ 0.60 & 0.68 & 0.60 & - & 0.76 \\ 0.90 & 0.28 & 0.90 & 0.28 & - \end{pmatrix}$$

$$\bar{R}^{23} = \begin{pmatrix} - & 0.28 & 0.28 & 0.90 & 0.76 \\ 0.68 & - & 0.76 & 0.68 & 0.68 \\ 0.76 & 0.60 & - & 0.28 & 0.76 \\ 0.28 & 0.68 & 0.76 & - & 0.68 \\ 0.28 & 0.68 & 0.76 & 0.68 & - \end{pmatrix}$$

$$\bar{R}^{31} = \begin{pmatrix} - & 0.28 & 0.76 & 0.60 & 0.90 \\ 0.76 & - & 0.68 & 0.60 & 0.60 \\ 0.60 & 0.68 & - & 0.68 & 0.28 \\ 0.76 & 0.76 & 0.68 & - & 0.76 \\ 0.28 & 0.76 & 0.90 & 0.60 & - \end{pmatrix}$$

$$\bar{R}^{32} = \begin{pmatrix} - & 0.60 & 0.60 & 0.76 & 0.76 \\ 0.90 & - & 0.60 & 0.60 & 0.68 \\ 0.90 & 0.76 & - & 0.60 & 0.76 \\ 0.60 & 0.76 & 0.90 & - & 0.76 \\ 0.60 & 0.68 & 0.28 & 0.76 & - \end{pmatrix}$$

$$\bar{R}^{33} = \begin{pmatrix} - & 0.68 & 0.68 & 0.60 & 0.76 \\ 0.68 & - & 0.28 & 0.28 & 0.68 \\ 0.68 & 0.60 & - & 0.90 & 0.90 \\ 0.76 & 0.76 & 0.76 & - & 0.60 \\ 0.28 & 0.68 & 0.60 & 0.76 & - \end{pmatrix}$$

$$\bar{R}^{41} = \begin{pmatrix} - & 0.60 & 0.90 & 0.90 & 0.76 \\ 0.76 & - & 0.28 & 0.28 & 0.68 \\ 0.28 & 0.90 & - & 0.76 & 0.28 \\ 0.28 & 0.90 & 0.60 & - & 0.90 \\ 0.60 & 0.68 & 0.76 & 0.28 & - \end{pmatrix}$$

$$\bar{R}^{42} = \begin{pmatrix} - & 0.68 & 0.68 & 0.60 & 0.60 \\ 0.68 & - & 0.68 & 0.76 & 0.60 \\ 0.68 & 0.68 & - & 0.68 & 0.68 \\ 0.90 & 0.28 & 0.68 & - & 0.76 \\ 0.90 & 0.76 & 0.68 & 0.60 & - \end{pmatrix}$$

$$\bar{R}^{43} = \begin{pmatrix} - & 0.90 & 0.90 & 0.60 & 0.90 \\ 0.60 & - & 0.68 & 0.60 & 0.60 \\ 0.28 & 0.60 & - & 0.76 & 0.68 \\ 0.68 & 0.90 & 0.68 & - & 0.90 \\ 0.60 & 0.76 & 0.68 & 0.60 & - \end{pmatrix}$$

$$\bar{u}^1 = 0.10 \quad \bar{u}^2 = 0.03 \quad \bar{u}^3 = 0.57$$
$$\bar{v}^{11} = 0.94 \quad \bar{v}^{12} = 0.31 \quad \bar{v}^{13} = 0.94$$
$$\bar{v}^{21} = 0.94 \quad \bar{v}^{22} = 0.31 \quad \bar{v}^{23} = 0.94$$
$$\bar{v}^{31} = 0.31 \quad \bar{v}^{32} = 0.94 \quad \bar{v}^{33} = 0.75$$
$$\bar{v}^{41} = 0.75 \quad \bar{v}^{42} = 0.75 \quad \bar{v}^{43} = 0.31$$

Second, the collective preference relation $\bar{R}^{c1}$ for the criterion c^1 is obtained by aggregating the preference relations $\bar{R}^{11}$, $\bar{R}^{21}$, $\bar{R}^{31}$, and $\bar{R}^{41}$. This aggregation is carried out by means of the IOWA operator in which the linguistic quantifier "most" defined as $Q(r) = r^{1/2}$ is used to generate the weights associated with the IOWA operator. The result obtained is:

$$\bar{R}^{c1} = \begin{pmatrix} - & 0.57 & 0.71 & 0.75 & 0.76 \\ 0.74 & - & 0.70 & 0.70 & 0.54 \\ 0.61 & 0.59 & - & 0.73 & 0.57 \\ 0.54 & 0.60 & 0.63 & - & 0.44 \\ 0.41 & 0.71 & 0.68 & 0.76 & - \end{pmatrix}$$

The same procedure is applied to obtain the collective preference relations $\bar{R}^{c2}$ and $\bar{R}^{c3}$ for the criteria c^2 and c^3, respectively:

$$\bar{R}^{c2} = \begin{pmatrix} - & 0.68 & 0.68 & 0.73 & 0.68 \\ 0.72 & - & 0.69 & 0.64 & 0.69 \\ 0.78 & 0.65 & - & 0.69 & 0.65 \\ 0.68 & 0.64 & 0.77 & - & 0.75 \\ 0.75 & 0.63 & 0.52 & 0.65 & - \end{pmatrix}$$

$$\bar{R}^{c3} = \begin{pmatrix} - & 0.42 & 0.42 & 0.74 & 0.78 \\ 0.67 & - & 0.63 & 0.65 & 0.63 \\ 0.60 & 0.64 & - & 0.64 & 0.69 \\ 0.57 & 0.76 & 0.82 & - & 0.74 \\ 0.32 & 0.80 & 0.64 & 0.48 & - \end{pmatrix}$$

Third, the final collective preference relation $\bar{R}^c$ is obtained by aggregating $\bar{R}^{c1}$, $\bar{R}^{c2}$, and $\bar{R}^{c3}$, using again the IOWA operator in which the same linguistic quantifier "most" is

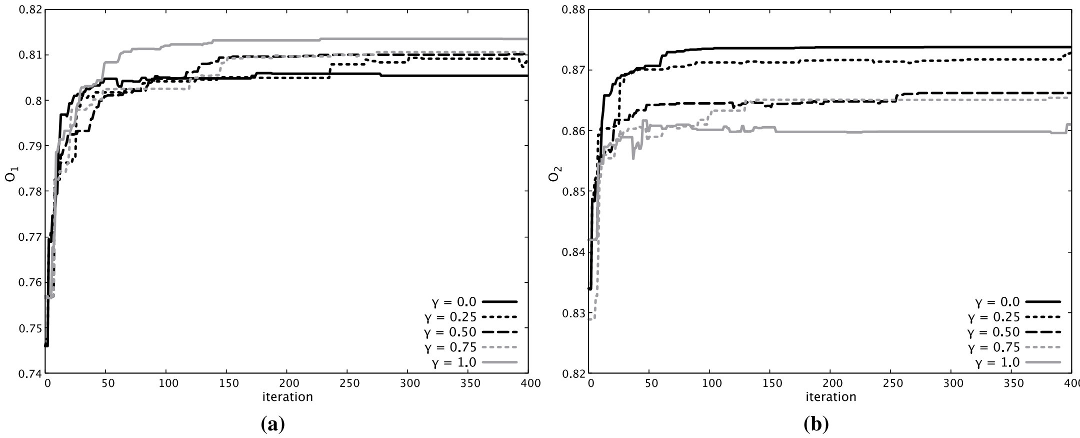

FIGURE 2. Plots of O_1 and O_2 for chosen values of γ. (a) O_1 for chosen values of γ. (b) O_2 for chosen values of γ.

employed to produce the weights:

$$\bar{R}^c = \begin{pmatrix} - & 0.50 & 0.54 & 0.74 & 0.76 \\ 0.70 & - & 0.66 & 0.66 & 0.62 \\ 0.64 & 0.63 & - & 0.67 & 0.65 \\ 0.58 & 0.70 & 0.77 & - & 0.67 \\ 0.42 & 0.75 & 0.62 & 0.58 & - \end{pmatrix}$$

Fourth, the quantifier-guided dominance degrees and the quantifier-guided nondominance degrees are calculated by means of the OWA operator that makes use of the same linguistic quantifier "most" to generate its weights:

$$\begin{aligned} QGDD_1 &= 0.69 \quad QGDD_2 = 0.67 \\ QGDD_3 &= 0.65 \quad QGDD_4 = 0.71 \\ QGDD_5 &= 0.65 \\ QGNDD_1 &= 0.96 \quad QGNDD_2 = 0.97 \\ QGNDD_3 &= 0.98 \quad QGNDD_4 = 0.98 \\ QGNDD_5 &= 0.93 \end{aligned}$$

Therefore, $A^{QGDD} = \{a_4\}$ and $A^{QGNDD} = \{a_3, a_4\}$, being $A^{QG} = \{a_4\}$. According to it, the academic library at the Faculty of Medicine (a_4) is the best one.

C. DISCUSSION

A comparison of the results generated by a GDM approach with others is not a simple task. Frequently, the linguistic values are represented in a different way or the context in which these approaches are defined is also different. Therefore, a quantitative comparison would not be meaningful. However, as mentioned previously, the proposed approach presents some important advantages concerning other approaches:

- In comparison with the traditional CWW approaches [3], [4], it makes operational the linguistic values via information granulation so that their distribution and semantics, in place of being defined a priori, are established by an optimization process. This allows the formation of linguistic values so that the solution to the problem is that of highest consensus and consistency.
- It increases the flexibility and richness of the GDM approaches based on granular computing [12]–[14] by allowing to cope with multi-criteria contexts in which each criterion has an importance weight and each decision maker has a different importance weight for each criterion. Therefore, it allows to model GDM problems in a more realistic way.

Anyway, with the intention of analyzing the performance of the proposed approach, we consider an approach in which the cut-off points are uniformly distributed over [0, 1], that is, $\mathbf{p} = (0.20, 0.40, 0.60, 0.80, 0.33, 0.66, 0.33, 0.66)$. Therefore, the intervals corresponding to the linguistic values of the sets S^1, S^2, and S^3, are:

- MW^1: [0, 0.20), W^1: [0.20, 0.40), E^1: [0.40, 0.60), B^1: [0.60, 0.80), and MB^1: [0.80, 1].
- LI^2: [0, 0.33), I^2: [0.33, 0.66), and VI^2: [0.66, 1].
- LI^3: [0, 0.33), I^3: [0.33, 0.66), and VI^3: [0.66, 1].

In this case, the optimization criterion takes a value of 0.722, being 0.010 its standard deviation. If we compare this value with the one reported by our proposal (0.826), we note that the optimization criterion achieves now a lower value.

We also analyze the impact of the values of γ standing in the optimization criterion O on the performance of the proposed approach (see Fig. 2). The optimization process focuses completely on the consistency when $\gamma = 0$, and, therefore, a higher value of O_2 is achieved, whereas it achieves lower values when γ assumes nonzero values (see Fig. 2b). This is not surprising as O is not O_2 itself but O also incorporates the effect of the consensus. In particular, when $\gamma = 1$, the optimization process focuses exclusively on the consensus and, therefore, higher values of O_1 are achieved. Different from O_2, the higher the value of γ, the higher the value of O_1 (see Fig. 2a). This is expected because more importance is given to the consensus.

V. CONCLUDING REMARKS AND FUTURE RESEARCH

We have presented an approach based on a framework of granular computing to cope with GDM scenarios in which linguistic values are used to evaluate the alternatives.

Different from the existing approaches founded on granular computing, it allows to deal with GDM scenarios in which different criteria are considered to evaluate the alternatives (multi-criteria contexts) and in which each criterion has an importance weight and each decision maker has a different importance weight for each criterion (heterogeneous contexts). We have shown that this proposal is able to formalize the linguistic values as intervals so that the final solution obtained is that of highest consistency and consensus.

We propose to continue this research in two directions. First, we have considered a joint treatment of the linguistic values, that is, the distribution and the semantics of the linguistic values are the same for all the decision makers. However, it should be considered that same words mean different things to different people [50], [51], especially in decision problems defined in scenarios like social networks, in which thousands of users could be involved in the decision problem and it is usual that the same word means a different thing to different users [52]. Second, we have considered all the decision makers employ the same linguistic term set. However, multi-granular linguistic contexts should be also considered [53], that is, scenarios in which the decision makers use different linguistic term sets.

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