Instrumented learning in a CODAP-enabled learning environment

Aprendizaje instrumentado en un entorno de aprendizaje habilitado para CODAP

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Abstract

In this study, we examined how seventh grade students used CODAP as a tool to make sense of roller coaster data while engaged in exploratory data analysis. Using instrumentation theory, we investigated their instrumentation approaches and examined the types of instrumental orchestration that emerged during whole class discussions.

Keywords: EDA, CODAP, instrumental genesis, instrumental orchestration

Resumen

En este estudio, examinamos cómo los estudiantes de séptimo grado utilizaron el CODAP como una herramienta para dar sentido a los datos de la montaña rusa mientras participan en el análisis exploratorio de datos. Usando la teoría de la instrumentación, investigamos sus enfoques de instrumentación y examinamos los tipos de orquestación instrumental que surgieron durante las discusiones de toda la clase.

Keywords: EDA, CODAP, génesis instrumental, orquestación instrumental

1. Introduction

Statistics has gained prominence in school curricula throughout the world (Franklin et al., 2007), which includes a focus on reasoning about data. Reforms in statistics education require that teachers carefully reconsider the design of learning environments to support students in developing productive statistical thinking in light of reform. Building on Cobb and McCain (2004), Ben-Zvi, Gravemeijer, and Ainley (2018) identify key characteristics of learning environments that support students in developing a deep, meaningful understanding of statistics: a) focusing on developing central statistical ideas rather than tools and procedures; b) using well-designed tasks to support statistical thinking; c) using real (or realistic) and motivating data sets; d) establishing norms that support statistical arguments; e) integrating technological tools that allow students to explore and analyze data; and, f) using assessment to monitor the development of students' statistical learning, as well as to evaluate instructional plans.

One way to encourage students to reason about data is providing opportunities to engage in Exploratory Data Analysis (EDA). EDA first developed by Tukey (1977), involves exploring data to summarize main characteristics. EDA is the "art of making sense of data by organizing, describing, representing, and analyzing data, with a heavy reliance on informal analysis methods, visual displays" (Ben-Zvi & Ben-Arush, 2014, p. 197). While approaches often use visual methods, such as graphs and other representations, statistical measures are sometimes calculated to make sense of data. Ben-Zvi (2004) points out that exploring data involves examining features such as shape, center, and spread; it involves considering graphs and looking for other characteristics of data like clusters, gaps, and outliers. Cobb and McClain (2004) recommend that EDA should be the focus of early experiences with instruction because of the emphasis on finding trends and patterns.

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EDA often involves the use of technology, in some cases dynamic statistical software. There is evidence that innovative technology tools aide students in developing statistical thinking (e.g., Biehler, Ben-Zvi, Bakker, & Makar, 2013). In this report, we are interested in understanding students' engagement with the Common Online Data Analysis Platform (CODAP), (https://codap.concord.org/), which has many powerful dynamic visualization and calculating capabilities that make it an ideal tool for engaging in EDA. Specifically, we investigated the following research questions: a) RQ1: How do students use CODAP as a tool to make sense of data while engaged in exploratory data analysis? a) RQ2: How does a teacher organize various aspects of a learning environment as students engage in exploratory data analysis using CODAP?

2. Theoretical perspectives

Our study draws on two theoretical perspectives, each focusing on fundamental aspects of instrumental theory: instrumental genesis and instrumental orchestration. In order to understand students' learning processes as they made sense of data during EDA using CODAP, we drew on Ben-Zvi and Ben-Arush's (2014) types of instrumentation. We used Drijvers et al.'s (2010) instrumental orchestration types to understand an instructor's interactions with students around CODAP.

2.1. Instrumental genesis

Instrumental genesis (IG) is comprised of five components (Ben-Zvi & Ben-Arush, 2014). The *subject* is a learner who accomplishes a task using an instrument. An *object* is a specific task. An *artifact* (a component of a tool) is a physical or virtual device that is used by the subject, which has no meaning for the learner in isolation. A *utilization scheme* is a cognitive scheme that the subject uses to accomplish a task using one or more artifacts. Once the subject has successfully used the utilization scheme to accomplish a task, the artifact becomes an *instrument* for the learner to use. Ben-Zvi and Ben-Arush (2014) indicate that IG occurs when a subject uses utilization schemes to transform an artifact into an instrument that can be used as a meaningful tool to achieve a particular goal.

Further, the researchers explicate two components of IG, *instrumentalization*, ways in which the subject's prior knowledge acts on the tool, and *instrumentation*, the way the instrument influences the subject's learning process. In our study, we are interested in instrumentation. Ben-Zvi and Ben-Arush (2014) identify three processes of instrumentation that learners use to investigate data: unsystematic, systematic, and expanding. An *unsystematic* approach involves actions that are not intentional or systematic, where learners make sense of a few basic artifacts and associated actions. *Systematic* instrumentation involves intentional and somewhat organized exploration, occurring after the learner has become familiar with artifacts, and may be more focused on the tool rather than the task. The third process involves *expanding* emerging instrumentalization (i.e., ways in which students' prior knowledge acts on the tool) of an artifact and associated actions that transform into a more usable and powerful instrument that can be used in a meaningful way in new contexts and situations.

2.2. Instrumental orchestration

Instrumental orchestration is the teacher's intentional and systematic organization and use of various artifacts in a learning environment to guide the learners' instrumental genesis in relation to a mathematical task (Drijvers et al., 2010; Trouche, 2004), or in our case a statistical task. The three elements within instrumental orchestration include the following: a) *didactical configuration*, referring to the design of the teaching setting and artifacts, b) *exploitation mode*, referring to the ways the teacher makes decisions to exploit the didactical configuration to achieve the learning goals, and c) *didactical performance,* referring to the in the moment decisions made by the teacher on how to act on the didactical configuration and enact the exploitation mode. After examining three lessons by 38 teachers, Drijvers et al. (2010) identified six orchestration types that we will use to investigate an instructor's purposeful use of artifacts to support learners' instrumental genesis as they use CODAP to engage in EDA with roller coaster data. Table 1 identifies the orchestration types and provides a brief definition for each.

Table 1. Six orchestration types identified by Drijvers et al. (2010)

Orchestration type	Definition
Technical-demo	The teacher demonstrates tool techniques. The classroom is arranged so students can follow the demonstration that is projected.
Explain-the-screen	The teacher, guided by what happens on the computer screen, provides an explanation to the whole-class. The explanation involves mathematical content, going beyond a focus on techniques. The classroom is configured so students can view what is projected.
Link-screen-board	The teacher emphasizes the relationship between what happened in the technological environment and how it is represented in conventional mathematics (e.g., paper and pencil, textbook, blackboard). A didactical configuration includes access to the technology and projecting facilities, and the blackboard/whiteboard and projecting screen are both visible.
Discuss-the-screen	In order to enhance collective instrumental genesis, a whole class discussion takes place focusing on what happens on the computer screen. There should be access to the technology and projecting facilities, as well as student work. The didactical configuration should also be conducive for promoting discussion.
Spot-and-show	Student's thinking is the central focus where interesting technology work is intentionally selected by the teacher during lesson planning and used in a classroom discussion. The classroom is arranged so students can view what is projected.
Sherpa-at-work	A student uses technology to present their own work, or to execute actions requested by the teacher. The classroom is arranged so that the student is in control of the technology and so that the class can follow both the Sherpa-student and teacher.

3. Methods

3.1. Context: Subject, object, instrument

The subjects in this study were 25 seventh grade students between the ages of 11 and 12-years old from a small urban school in the southeastern US. The school is racially diverse, and 48.6% of the students receive free/reduced lunch. Less than half of the students are proficient in mathematics (40.2%), as compared to 63.2% in their district. We report on one 72-minute mathematics lesson, where students engaged in EDA using CODAP to make sense of roller coaster data. This lesson took place during the second week of the school year, prior to any formal instruction on statistics and before students had exposure to CODAP. While the regular mathematics classroom teacher was present during the lesson, the class was taught by an experienced researcher, from a large research university in the southeastern US, with expertise in the teaching and learning of statistics, as well as using technology tools. We refer to the researcher as *instructor* for this paper. *During* the lesson, the regular classroom teacher observed students as they worked in pairs, and she interacted with some pairs.

The lesson consisted of four parts: 1) instructor launching the investigation (whole class); 2) instructor introducing CODAP as a tool using a small data set (whole class); 3) student pairs investigating larger data set using CODAP (small group work); and, 4) instructor facilitating discussion as student pairs present interesting findings (whole class). In Part 1, at the beginning of the lesson, the instructor asked students to consider aspects of roller coasters that might make the ride thrilling or scary. The instructor launched the lesson by showing a video of a wooden roller coaster from the data set, from the point of view (POV) of a rider, to introduce the context of the data. The video was projected onto a wall mounted television connected to her laptop, and the students were seated in pairs so that they could all view the screen. After leading a whole class discussion about different aspects that students thought might be thrilling or scary, the instructor introduced students to CODAP by facilitating the exploration of a small data set of 31 US roller coasters using a CODAP document (tinyurl.com/31UScoasters) in Part 2 of the lesson. This exploration was done as a class, where the instructor demonstrated features of CODAP using her laptop that was projected on the classroom television. Each student pair had access to and engaged with the data using a laptop. In Part 3, students opened the CODAP document of the larger data set of 157 US roller coasters (tinyurl.com/157UScoasters) and discussed attributes in the data set. The instructor then asked students to explore the data to find interesting things they could share about the coasters. They were encouraged to ask their own questions and use artifacts in CODAP, such as graphs, to investigate questions of interest to them. Finally, in Part 4, after student pairs completed their exploration, the instructor facilitated a whole class discussion, where students present their findings. Here, student pairs came to the front of the classroom and recreated their work on the instructor’s laptop that was presented on the classroom television and discussed their findings as the instructor stood to the side towards the front of the classroom.

The instrument used was CODAP, a free web-based data tool designed for students in grades 6-12+ that continues the tradition of TinkerPlots and Fathom by providing opportunities to engage in data exploration in an interactive environment. CODAP allows students to quickly manipulate data, augment graphs, visualize data, link multiple representations, and model probabilistic and sampling scenarios using a sampler. Students explored a data set of 157 US roller coasters with 15 numerical and categorical attributes (e.g., name, location, design, top speed, maximum height, etc.).

3.2. Data collection and analysis

While data collected for this study is part of a larger project, we focused on the class session as described in the context section of the paper above. The entire class session was video recorded using three cameras from multiple perspectives: one stationary camera focusing on the front of the class/television and a stationary side camera and floating camera focusing on the instructor and students as they spoke during whole class discussions. While student pairs used CODAP to investigate the roller coaster data, all three cameras recorded the instructor’s interactions with student pairs or focused on student pairs as they worked. Additionally, three student pairs’ laptop screens were recorded as screencasts throughout the entire class using Quicktime. The regular mathematics classroom teacher was asked to select the pairs to represent divergent student thinking. We used a deductive approach to selecting video for analysis (Derry et al., 2010). To examine how students use CODAP to make sense of data while engaged in EDA, we selected video recordings from the screencasts of students' laptops while they were engaged in EDA with the 157 roller coaster data set using CODAP. To investigate the types of instrumental orchestration that emerge as students engage in EDA using CODAP, we selected video of whole class discussions.

All selected video was initially viewed to identify episodes, our unit of analysis for both types of video data. For recordings of the whole class discussions, we define an episode as an action or set of actions that lead to the usage of a single feature of CODAP. Episodes of screencasts were defined as an action or group of actions that resulted in a process of instrumentation. Once episodes were identified, we created content logs to provide a time-indexed description of the events on the video (Derry et al., 2010). Each episode was coded by two of the three authors. Episodes of student pairs' screencasts were coded to identify the processes of instrumentation that learners used to investigate data (Ben-Zvi & Ben-Arush, 2014): unsystematic, systematic, and expanding. To identify the types of instrumental orchestration that emerged during whole class discussion, we coded for evidence of one of the six orchestration types (Drijvers et al., 2010). In addition to the evidence-based codes, we identified at least one orchestration type that emerged from the data. The three authors discussed any disagreements about codes until consensus was reached.

4. Results

4.1. Students' use of CODAP to make sense of data

To investigate how students used CODAP as a tool to make sense of data while engaged in EDA, we identified the instrumentation processes that three student pairs used to reason about 157 US roller coasters. We identified five episodes relating to Pair 1, four episodes to Pair 2 and three episodes to Pair 3. Table 2 shows the types of instrumentation process identified for each pair as they engaged in EDA using CODAP. All three pairs began using unsystematic instrumentation. Pair 1 moved from unsystematic to systematic instrumentation during the task. Pair 2 went from unsystematic to systematic for one episode, but the remainder of the time they used unsystematic instrumentation. Pair 3 was the only pair who engaged in all three types of instrumentation.

Table 2. Instrumentation Processes of Student Pairs

Pair	Episode 1	Episode 2	Episode 3	Episode 4	Episode 5
1	Unsystematic	Unsystematic	Systematic	Systematic	Systematic
2	Unsystematic	Systematic	Unsystematic	Unsystematic	
3	Unsystematic	Systematic	Expanding		

Pair 1 moved from unsystematic instrumentation to systematic instrumentation when they began creating graphs to answer questions such as "What's the highest height?". The context of the data motivated this student pair to investigate questions that they could explore and use the data to make inferences about their questions. They were able to create a graph with the purpose of finding the *maximum height* by creating a dot plot then using their mouse to click on the point to the furthest right to identify it in the case table (see Figure 1a).

Pair 2 spent most of their time in an unsystematic instrumentation process. Though they explored many different artifacts that are available in CODAP, such as creating a dot plot, moveable lines, finding measures of center (mean, median, mode), creating boxplots (see Figure 1b), they could not expand the use of the tools in an intentional way. Additionally, unlike the other two pairs, they never came up with a question to explore about the data. While they were very excited about discussing roller coasters, they spent most of their time discussing their favorite roller coasters that they had ridden or their favorite theme parks, regardless of whether the coasters or parks were in the data set. At one point the students questioned whether a specific coaster was in the data set, they clicked on the attribute name *coaster* in the case table. While sort ascending and sort descending were two options in the drop-down menu, the students chose not to sort and continued using an unsystematic approach.

Pair 3 were able to expand the use of the tools in an intentional way. They engaged in expanded instrumentation by creating a scatter plot comparing the *maximum drop* to the *top speed*, and then overlaying *type* onto the dots to investigate if the material a roller coaster is made of affects the *top speed* versus the *maximum drop* (see Figure 2). This made the use of the graph more powerful for them by allowing them to pose and answer a new question while using more features of the graph. Pair 3 was selected by the instructor to share their work with the class during the whole class discussion of students' interesting findings, and their contributions to the whole class discussion are shared in the Sherpa-at-work example in the next section of the results.

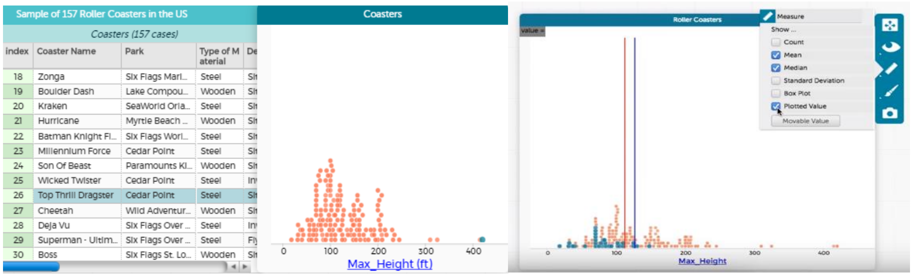

Figure 1a and 1b. Examples of systematic (1a) and unsystematic instrumentation (1b)

4.2. Types of instrumental orchestration that emerged during EDA using CODAP

To examine the types of instrumental orchestration that emerged as students engaged in EDA using CODAP, 37 episodes were coded (see Table 3 for frequencies).

Table 3. Frequencies of orchestration types

Orchestration type	Count	Percent
Technical-demo	13	35.14
Explain-the-screen	5	13.51
Link-screen-board	1	2.70
Discuss-the-screen	10	27.03
Spot-and-show	1	2.70
Sherpa-at-work	6	16.22
Link-tool	1	2.70
Total	37	100

About one-third (35.14%) of the types of instrumental orchestration that emerged as students engaged in EDA using CODAP were identified as a *Technical-demo*. An example of this type of instrumental orchestration occurred after the instructor asked students to make a conjecture about the typical *maximum height* of older coasters after displaying a dotplot showing *maximum heights*. After several students shared their conjectures and reasoning, the instructor demonstrated how to add measures of center (mean=80.9 and median=80) to the dotplot. Further, in another example, after adding these measures, the instructor showed students that they needed to hover over the lines representing the value of the mean and median to identify the numerical values.

We found that 13.51% of episodes were characterized as *Explain-the-screen* orchestration. For example, when the instructor clicked on a row in the case table to highlight a particular case, the Jack Rabbit, this went beyond a technical demonstration in CODAP because the instructor wanted to illustrate that highlighting the row (i.e., case) would provide information about different attributes related to the specific coaster, like the Jack Rabbit was made of wood and its maximum height is 40 feet. Understanding that data are not just numbers but numbers in context is a fundamental habit of mind that needs to be developed to support productive statistical thinking. We found only one instance (2.70%) of *Link-screen-board* orchestration throughout the lesson. Here, the instructor provided students with a handout listing all 15 attributes in the data set of 157 coasters, along with definitions of each attribute. The instructor showed students that if you hover over the name of the attribute in the case table, the definition appears. She was exemplifying how to link information that you would typically see in a conventional medium, like a handout, to its location in CODAP.

Almost one-third (27.03%) of the types of orchestration that emerged were identified as a *Discuss-the-screen*. An example of this type of instrumental orchestration occurred after the instructor asked students to make a conjecture about the typical *maximum height* of older roller coasters by examining a dotplot of *maximum heights*. A student conjectured that it was “around 80”. Most students provided a conjecture that involved an interval, or modal clump (Konold et al., 2002), such as 70 to 80 or 70 to 85. After sharing their conjectures and reasoning, the instructor followed up with a technical-demo by showing them how to add the mean and median to the graph of the dotplot.

There was only one instance (2.70%) of *Spot-and-show* orchestration identified during the class session. In this instance, the instructor had seen a student drag and drop the attribute *states* on the axis of a graph. Then the student drug and dropped the attribute *drop* in the center of the same graph. This action colored the dots representing cases a gradient of green, where the lightest shade of green represented the coasters with the shortest drops and darkest shade of green represented the coasters with the greatest drops. The student described his actions as the instructor recreated them and displayed them on the SMART Board. The student indicated he clicked on the darkest shades of green in the legend and concluded that Ohio is the best place for coasters because all of the coasters in Ohio were one of the two darkest shades of green. By spotting and showing this student’s work, the instructor was drawing the whole classes attention to the feature of using color to reason about the data.

Overall, 16.22% of the instrumental orchestration that emerged during the class session were identified as *Sherpa-at-work*. An example of this occurred when one student pair recreated one of their graphs during the whole class discussion. One of the students said, “We were just playing around, and we were comparing *drop* and *top speed*. And, then, I added in *type*, and this is how we came up with this graph [see Figure 2].” The instructor then asks, “Does that tell us any new information? What do you think that tells us?” A student reasons that, “wooden coasters seem to be in the same range of *drop*,” noticing that *maximum drop* of wooden coasters are clustered together. We argue that even though very few students considered the relationship between two numerical attributes and a categorical attribute simultaneously, many students benefited in a way that promoted collective instrumental genesis as evidenced by other students’ reasoning that was shared during the discussion.

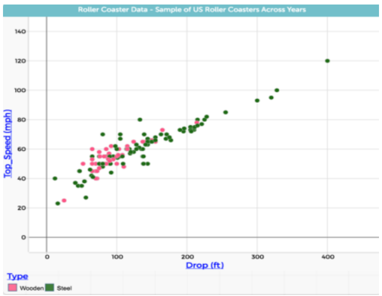

Figure 2. Student created scatterplot

While most episodes were identified as one of Drijvers et al.’s (2010) orchestration types, we found an episode (2.70%) that could not be classified. When introducing CODAP to the students for the first time, the instructor tried to draw on their prior experiences with spreadsheets. She specifically linked the structure in the case table in CODAP to the structure in a spreadsheet. Thus, we characterized this type of instrumental orchestration as *Link-tool*. As technology advances, especially dynamic data visualization tools, an important move by the teacher may be to connect students’ knowledge of less dynamic tools that they are more familiar with.

Table 4. Percentages of orchestration types during EDA lesson

Orchestration type	Part 2 (n=16)	Part 3 (n=25)	Part 4 (n=10)
Technical-demo	44	50	10
Explain-the-screen	20	0	0
Link-screen-board	0	50	0
Discuss-the-screen	32	0	20
Spot-and-show	0	0	10
Sherpa-at-work	0	0	60
Link-tool	4	0	0
Total	100	100	100

When we looked at orchestration types in different parts of the lesson, which were described in the context section, we noticed patterns within the parts. For example, in part two of the lesson when the instructor was introducing CODAP as a tool to make sense of data during EDA, only four orchestration types were identified (see Table 4): Technical-demo (44%), Explain-the-screen (20%), Discuss-the-screen (32%), and Link-tool (4%). While the focus of this part of the lesson was on showing the features of the tool, it is interesting that over half of the orchestration types involved explaining and discussing. A didactical decision was to show features of the tool while exploring a small data set of older roller coasters; thus, there were opportunities for students to reason about the data that went beyond a mere show and tell of the features of the technology tool. We also noticed that the majority of the episodes in the fourth part of the lesson, where student pairs shared their interesting findings were classified as Sherpa-at-work. In fact, of the 10 episodes in this part of the lesson, 60% where identified as Sherpa-at-work, while the rest of the episodes were coded in the following way: Technical-demo (10%), Discuss-the-screen (20%), and Spot-and-show (10%).

5. Discussion

The analysis of the three pairs of students conducting an EDA using CODAP has showed us how students make use of the artifacts in CODAP to create instruments that they use to answer meaningful questions of their own interest. Using three types of instrumentation to describe the processes that students used when engaged in EDA, we were able to determine that those who were able to transform the artifacts in CODAP to meaningful tools (i.e. going from unsystematic to systematic to expanding instrumentation) were able to pose and answer more robust questions that arose during EDA. While viewing the screencasts of the students' work, it was noted by the authors that often the switch between *different* types of instrumentation occurred when there was an intervention by an outsider, such as the instructor or classroom teacher, thus providing different learning opportunities for students to develop statistical thinking. Further research will be done to analyze the influence these interactions had on how students engage in EDA. While this was an exploratory study that focused on a small number of student pairs from one class, we believe it shows evidence that students' first experience with CODAP can support them as they make sense of data. We recognize that these findings cannot be generalized.

Like the prior work of Drijvers et al. (2010), we found that we were able to identify and classify orchestration types and found this framework to be a useful way to describe a teacher's intentional and systematic organization and use of various artifacts in a learning environment to guide students' instrumental genesis in relation to a statistical task. Drijvers et al. (2010) acknowledge that other types of orchestration might emerge in a different learning environment or while studying other teachers. We assert that we have identified another possible type, Link-tool. However, further research, involving more lessons and more students, needs to be conducted in order to confirm a systematic appearance of this additional construct.

While we were not surprised that a little over a third (35.14%) of all episodes involved a Technical-demo since this was students first introduction to CODAP, we think an interesting result is that 53% of orchestration types were characterized as Explain-the-screen and Discuss-the-screen, while 44% were Technical-demo. An important implication for designing learning environments is that given an appropriate tool and well-designed task that uses real data, students can learn to use a tool while engaging in EDA. While teachers often acknowledge the benefits of using technology to support student learning, they sometimes argue they have insufficient time to incorporate tools. These findings provide evidence that teachers do not need to teach students to use a tool first and then provide opportunities to engage in statistical thinking later.

In conclusion, in accordance with Cobb and McCain (2004) and Ben-Zvi et al. (2018), we believe that providing opportunities for students to engage with well-designed tasks that use real, motivating data are fundamental aspects of designing learning environments that support students' statistical thinking. We are also in agreement that providing opportunities for students to reason about data using dynamic statistical tools, like CODAP, is a fundamental component of learning environments that develop students statistical reasoning.

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