Doctoral Dissertation

SIMPLE ARITHMETIC: COACTIVATION AND INHIBITION

International Ph.D. Candidate

PATRICIA MEGÍAS GARCÍA

Advisor

PEDRO MACIZO SORIA

Ph.D. in Psychology

Department of Experimental Psychology

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UNIVERSIDAD DE GRANADA
DEPARTAMENTO DE PSICOLOGÍA EXPERIMENTAL
Campus Universitario de Cartuja, s/n
Teléf. +34 - 958 24 37 63 - Fax: +34 - 958 24 62 39
18071 - GRANADA - ESPAÑA

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Tesis Doctoral presentada por **Patricia Megías García**, en el Departamento de Psicología Experimental, para aspirar al grado de *Doctor Internacional*, en el programa de Doctorado RD99/2011 de Psicología de la Universidad de Granada.

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Fdo. Pedro Macizo Soria

Doctorando

Fdo. Patricia Megías García

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ÍNDICE

CAPÍTULO I

INTRODUCCIÓN Y OBJETIVOS DE LA TESIS

PREFACIO

Operaciones aritméticas simples, tales como sumas o multiplicaciones de un dígito (e.g., 2 + 4 o 2 x 4) están presentes con frecuencia en nuestra vida cotidiana. Hacemos uso de la aritmética en una gran variedad de tareas y contextos a los que nos enfrentamos diariamente (e.g., al recibir la vuelta tras comprar unos zapatos, cuando calculamos cuánto tiempo falta para que empiece una clase, o incluso cuando hay que compartir las porciones de una pizza entre varios amigos), aun sin percatarnos realmente de cómo esos hechos aritméticos están conformados en eso que llamamos "mente" y qué procesos mentales nos permiten llevar a cabo dichas operaciones.

La aritmética cognitiva es un campo de estudio que se ha ampliado y desarrollado de manera considerable desde los años 70 en adelante (Ashcraft, 1982; Groen y Parkman, 1972). En éste se asume que los hechos aritméticos simples se encuentran almacenados en la memoria a largo plazo formando redes asociativas con nodos interconectados, de modo que la fuerza asociativa entre los diferentes nodos que conforman la red se va configurando a través del aprendizaje y de la experiencia educativa (Ashcraft, 1992; 1987; Campbell y Graham, 1985; Siegler y Jenkins, 1989).

En base a esta arquitectura mental de los hechos aritméticos, a la hora de resolver un problema aritmético (e.g., una suma simple 2 + 4) los nodos de la red que representan el problema (2 y 4) y aquellos que representan la respuesta (6) se activarían permitiendo que la solución se recuperase desde la memoria (Campbell y Graham, 1985). Además, debido a la propagación de la activación a través de las diferentes conexiones que configuran la red, otros nodos que representan información aritmética relacionada podrían activarse de manera concurrente, como el resultado de multiplicar los operandos (8) o restarlos (2) (Ashcraft y Battaglia, 1978; Winkelman y Schmidt, 1974; Zbrodoff y Logan, 1986). Así pues, la representación de los hechos aritméticos en la memoria a largo plazo permite que diferente información aritmética relacionada se active conjuntamente y de manera automática. Además, esta activación concurrente plantea la cuestión de cómo los hechos aritméticos son finalmente seleccionados dentro de esa red para dar la respuesta correcta en cada caso (e.g., seleccionar el resultado 6 para resolver la suma 2 + 4).

En los siguientes apartados de la introducción, profundizaremos en los modelos teóricos que han sido formulados a lo largo del tiempo para explicar cómo los hechos aritméticos están representados en la memoria a largo plazo. Después, describiremos la amplia evidencia empírica que avala el fenómeno de coactivación de hechos aritméticos asociados a la suma y a la multiplicación, esto es, la activación automática y conjunta de hechos aritméticos que se encuentran relacionados en la red asociativa. Posteriormente, daremos paso a la ilustración de los mecanismos de selección de hechos aritméticos que han sido propuestos desde diferentes perspectivas. Continuaremos detallando una serie de factores que determinan el uso de los hechos aritméticos para, en último lugar, concretar los objetivos y la estructura de la serie experimental realizada en la presente tesis doctoral.

MODELOS DE REPRESENTACIÓN Y RECUPERACIÓN DE HECHOS ARITMÉTICOS

Desde los años 80 (Ashcraft, 1982) en adelante se ha propuesto una serie de modelos teóricos que intentan dar cuenta de cómo los hechos aritméticos se adquieren a través de su práctica y van configurando una red asociativa en la memoria a largo plazo. En este apartado, nos centramos en la revisión de los modelos asociativos clásicos más relevantes que han contribuido al entendimiento de tres cuestiones fundamentales de esta red aritmética: cómo se configura a través del aprendizaje, cómo se representan los hechos aritméticos en ella y cómo se recuperan y seleccionan finalmente desde la red para ofrecer la solución a un problema.

El modelo de la Red de Recuperación de Ashcraft

Uno de los primeros modelos que contribuyó tanto teórica como empíricamente al entendimiento de la arquitectura de los hechos aritméticos simples y cómo éstos se recuperan de la memoria a largo plazo fue el *modelo de la red de recuperación* (Ashcraft's network retrieval model; Ashcraft, 1982). Según este modelo, las personas con aprendizaje formal en aritmética tendrían una red de hechos aritméticos almacenada permanentemente en memoria. En esta red de hechos aritméticos coexistirían nodos que representan el primer operando de una operación (2 + N), aquellos que representan el segundo operando (N + 4) y los que representan la respuesta asociada a los operandos (6). Estos nodos se encontrarían relacionados en función de una fuerza asociativa que se iría estableciendo tras la práctica reiterada con cada operación. De este modo, los nodos que representan los operandos estarían relacionados con las diversas respuestas que los contienen (e.g., 2 + N, estaría asociado con las respuestas 3, 4, 5… de sumar 2 + 1, 2 + 2, 2 + 3, etc.) (ver Figura 1).

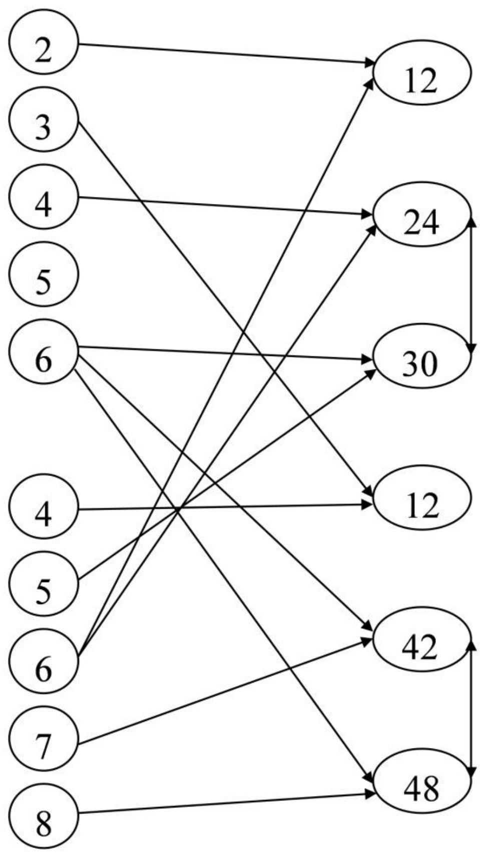

Figura 1. Ejemplificación del modelo de red de recuperación (Ashcraft, 1982). Los nodos que representan ambos operandos (1er operando arriba a la izquierda, 2º operando abajo a la izquierda) estarían asociados con sus respuestas correspondientes (a la derecha). A su vez, estas respuestas relacionadas estarían asociadas entre sí. Para ejemplificar, los nodos que representan los operandos 4 y 6 estarían asociados con la respuesta 24, la que a su vez establecería conexiones con respuestas relacionadas como 30.

En el modelo se proponía que el principio de propagación de la activación sería utilizado para acceder a la información aritmética en la red y recuperar el resultado correcto desde la memoria. De esta manera, la activación se propagaría en paralelo desde los diferentes conjuntos de nodos: los nodos que representan los operandos (2 + N y N + 4), aquellos que representan la respuesta (6) y otros nodos de respuesta que se encuentren relacionados con los anteriores (8); seleccionando la respuesta correcta del nodo que mayor activación recibiese finalmente.

Una de las contribuciones posteriores del modelo (Ashcraft, 1987) fue la inclusión de la perspectiva evolutiva. En concreto, en el modelo se asumió que la fuerza asociativa de las conexiones, entre los nodos que representan los operandos y aquellos que representan las respuestas, dependía de la frecuencia con la que cada problema era practicado, fundamentalmente, durante los primeros años de escolarización en la edad infantil. De este modo, los problemas de tamaño menor (e.g., 2 + 4) serían recuperados con mayor frecuencia que los problemas de mayor tamaño (e.g., 7 + 8), lo que a su vez produciría que se resolviesen con mayor precisión debido a la mayor fuerza asociativa de sus conexiones en la red de hechos aritméticos. Esta interpretación sentaba la base teórica del conocido efecto del tamaño, el cual consiste en menores tiempos de reacción y menor porcentaje de errores cuando se resuelven problemas de menor tamaño en comparación con problemas de mayor tamaño (Groen y Parkman, 1972).

Aunque el modelo de la red de recuperación (Aschcraft, 1982; 1987) fue relevante a la hora de entender cómo los hechos aritméticos se representan en la red asociativa y cómo ésta va tomando forma a través de la práctica educativa, este modelo no recogía al completo el conjunto de estrategias mediante las cuales los hechos aritméticos se van adquiriendo y conformando en la red. En concreto, el modelo de Aschcraft (1982, 1987) asumió la recuperación directa desde la memoria como la manera por defecto de solucionar una operación aritmética, sin contemplar otras estrategias de naturaleza procedimental que están presentes fundamentalmente en los primeros años de escolarización. El modelo que describiremos a continuación (el *modelo de distribución de asociaciones*; Siegler y Jenkins, 1989) ofreció una primera aproximación en la que se contemplaba el uso de diferentes estrategias a la hora de resolver un problema aritmético simple.

El modelo de Distribución de Asociaciones de Siegler

En el *modelo de distribución de asociaciones* (Siegler's distribution of associations model; Siegler y Jenkins, 1989) se reconocía el uso de estrategias procedimentales dentro del proceso de adquisición de la red aritmética. Para ejemplificar el proceso, cuando los niños comienzan su instrucción formal en la resolución de sumas simples (2 + 4 =), éstos pueden valerse del conteo para, partiendo del primer operando (2), contar según la magnitud del segundo (cuatro elementos) y llegar al resultado (6) (así, 2 + 4 = 2 y 3, 4, 5, 6). Igualmente, con la instrucción en multiplicaciones simples, a través del procedimiento de sumar repetidas veces el valor del primer operando se obtiene el resultado de la operación (2 x 4 = 2 + 2 + 2 + 2 = 8). En el modelo se propone que a través de la práctica repetida de estas estrategias (y de la recuperación directa) se irían estableciendo las conexiones entre los nodos que representan los operandos y aquellos que representan las respuestas (2 + 4 = 6, 2 x 4 = 8) en la memoria a largo plazo del niño.

Además, debido a que en la práctica de estrategias procedimentales podían cometerse errores en el proceso (e.g., añadir una suma más a la resolución de la multiplicación 2 + 4 = 2 + 2 + 2 + 2 + 2 = 10), Siegler propuso que en la red de hechos aritméticos también se establecían asociaciones entre los operandos y respuestas erróneas a los mismos (2 + 4 = 10). De este modo, la fuerza asociativa entre los nodos que representan los operandos y aquellos que representan las posibles respuestas variaba en función del aprendizaje y de la práctica con cada problema específico. Así, el efecto del tamaño podía concebirse igualmente desde este modelo: por ejemplo, al realizar repetidas sumas para ofrecer la respuesta de una multiplicación de tamaño pequeño, la probabilidad de tener asociaciones erróneas sería menor debido al menor número de sumas repetidas necesarias del multiplicando (2 x 2 = 2 + 2 = 4, dos sumas repetidas del número 2) en comparación con aquellas que se precisarían en operaciones de mayor tamaño (2 x 6 = 2 + 2 + 2 + 2 + 2 + 2= 12, seis sumas repetidas del número 2), o/y posibles errores de conteo uno a uno dentro de las sumas simples. Este proceso configuraba una relación asociativa de mayor fuerza entre las conexiones de los operandos y la respuesta correcta en operaciones de menor tamaño, mientras que las conexiones de los operandos de mayor tamaño con las posibles respuestas serían más distribuidas y, por ende, de menor fuerza asociativa con cada una de ellas, cometiendo un mayor número de errores y necesitando un mayor tiempo para ofrecer la solución (ver Figura 2).

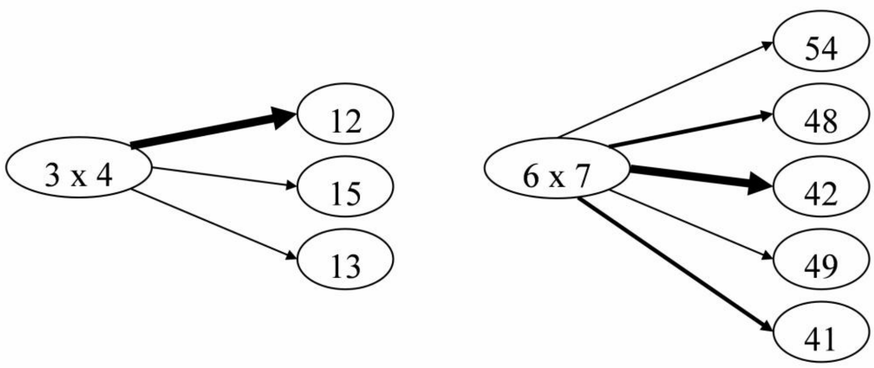

Figura 2. Ejemplificación del modelo de distribución de asociaciones (Siegler y Jenkins, 1989). Los nodos que representan los operandos (3 x 4) establecerían asociaciones con diversos nodos que representan no sólo la respuesta correcta (12) sino también otras posibles respuestas (15,13). Además, el número de conexiones entre los nodos que representan los operandos y aquellos que representan las posibles respuestas sería mayor a medida que aumentase el tamaño del problema (6 x 7, a la derecha).

Tras asumir la variedad de estrategias que los niños utilizan en el ámbito escolar, en el modelo de distribución de asociaciones (Siegler y Jenkins, 1989) se propuso un mecanismo de selección de estrategias (de recuperación o procedimentales) en función de dos criterios: la distribución de fuerzas asociativas entre operandos-respuestas y la longitud de la búsqueda (número de búsquedas necesarias para dar la respuesta correcta). En consecuencia, ante la presentación de un problema específico, el mecanismo seleccionaría con una mayor probabilidad la estrategia de recuperación directa desde la memoria conforme mayor fuese la fuerza asociativa entre operandos-respuesta correcta, y no se superase un número de búsquedas establecido. Cuando este número de búsquedas se sobrepasase sin dar la respuesta correcta o la distribución de asociaciones fuese más plana (esto es, las conexiones operandos-respuestas estuviesen más distribuidas), una estrategia procedimental sería la encargada de solucionar finalmente el problema aritmético. A medida que las asociaciones entre operandos-respuesta correcta cobraran mayor fuerza asociativa a consecuencia de la práctica, el mecanismo tendería a elegir con mayor probabilidad la estrategia de recuperación desde la memoria. De esta manera, este modelo no solo sirvió para aumentar nuestra comprensión sobre cómo los hechos aritméticos se representaban y se resolvían, sino para explicar cómo esta red aritmética se iba configurando a través de la experiencia educativa de los niños y cómo las estrategias procedimentales iban dando paso a estrategias de recuperación más rápidas y eficaces.

Sin embargo, son varias las críticas que el modelo de Siegler ha recibido a lo largo del tiempo: autores como Ashcraft (1992) han puntualizado que el modelo se ajustaba a un rango de edad escolar no generalizable a la edad adulta, donde las personas hacían un mayor uso de estrategias de recuperación frente a estrategias procedimentales. Además, entre las críticas más relevantes dentro de nuestro campo de estudio, estaba el hecho de que el modelo no podía acomodar una explicación plausible para los efectos de relación reportados en la literatura empírica, como es el caso del efecto de confusión asociativa (Stazyk, Ashcraft y Hamann, 1982; Winkelman y Schmidt, 1974; Zbrodoff y Logan, 1986). Como veremos más adelante, el efecto de confusión asociativa consiste en un peor rendimiento cuando se verifica una operación cuyo resultado es incorrecto pero está relacionado con el problema en cuestión: por ejemplo, se ha descrito un mayor tiempo en resolver una suma cuyo resultado es aquel de multiplicar sus operandos (2 + 4 = 8) en comparación con una suma cuyo resultado no está relacionado con la multiplicación de los operandos (2 + 4 = 10). Desde el modelo de Siegler, era posible explicar patrones de resultados que se debían a asociaciones erróneas consecuencia de fallos en las estrategias procedimentales, pero no era posible ofrecer una explicación sobre cómo la información procedente de diferentes operaciones aritméticas (sumas, multiplicaciones) podía estar interrelacionada en la memoria a largo plazo. En el siguiente apartado, describimos un nuevo modelo que da cuenta de efectos de confusión asociativa como los expuestos en ese párrafo.

El modelo de la Red de Interferencia de Campbell

En el *modelo de la red de interferencia* propuesto por Campbell (Campbell's nerwork interference model; Campbell, 1987; Campbell y Graham, 1985) se asumía, al igual que en el *modelo de la red de recuperación* de Ashcraft (1982), que la red de hechos aritméticos era accesible a través del principio de propagación de la activación y que los nodos se encontraban conectados con una fuerza asociativa que dependía de la práctica. En este modelo, se argumentaba que diferentes nodos en la red representarían no solo los operandos (2 x N, N x 4) o las respuestas asociadas a los mismos (8), sino también el problema como un todo (2 x 4). De este modo, ante la presentación de un problema, la propagación de la activación se daría en paralelo desde los nodos que representan los operandos y desde aquellos que representan el problema hacia un conjunto de nodos que representan diversas respuestas (la respuesta correcta y otras respuestas relacionadas tanto con los operandos como con el problema). De forma adicional, desde cada nodo de respuesta se establecerían conexiones con un conjunto de nodos que representan diversos operandos y problemas (ver Figura 3). Finalmente, como en el modelo de Ashcraft, la respuesta que mayor activación recibiera en la red era seleccionada y recuperada desde la memoria.

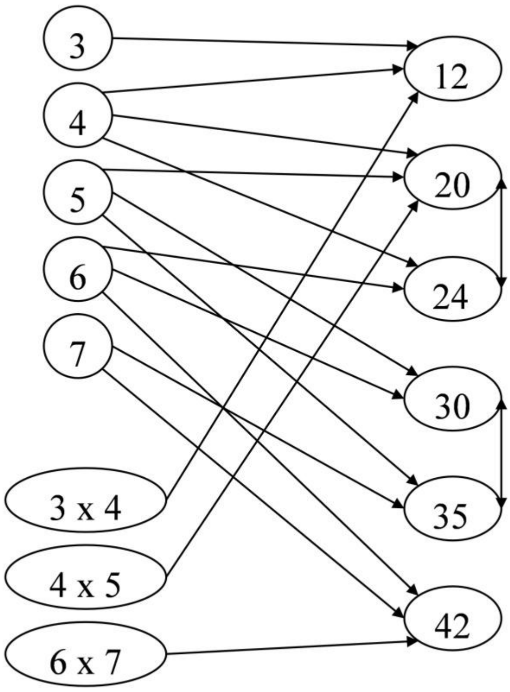

Figura 3. Ejemplificación del modelo de red de interferencia (Campbell, 1987; Campbell y Graham, 1985). Los nodos que representan los operandos (arriba a la izquierda), así como los nodos que representan los problemas (abajo a la izquierda) estarían asociados con sus respuestas correspondientes (a la derecha). A su vez, estas respuestas relacionadas estarían asociadas entre sí. Para ejemplificar, los nodos que representan los operandos 4 y 5, y aquel que representa el problema al completo (4 x 5) estarían asociados con el nodo que representa la respuesta 20. A su vez, el nodo que representa la respuesta 20 estaría asociado con respuestas relacionadas como 24.

Una de las aportaciones exclusivas del *modelo de la red de interferencia* fue que introdujo el concepto de interferencia en la recuperación de los hechos aritméticos. Cuando un problema aritmético era presentado (e.g., la multiplicación 2 x 4) no solo se activaría la respuesta correcta (8), sino que otras respuestas relacionadas con los operandos o con el problema (e.g., 12, asociado a los operandos 2 x 6 y 4 x 3) también podrían activarse, interfiriendo en el proceso de selección de la respuesta correcta. Este fenómeno de interferencia en la selección fue avalado empíricamente por el efecto conocido como facilitación del error (Campbell, 1987): cuando se presentaba una multiplicación simple, por ejemplo 6 x 4, se recuperaba la respuesta correcta, en este caso 24. Si inmediatamente después se presentaba otra multiplicación, por ejemplo 4 x 8, que tuviese conexiones en la red aritmética con el resultado dado previamente, en este caso los operandos 4 y 8 estaban asociados al resultado 24 (4 x 6 = 24, 3 x 8 = 24, Campbell y Graham, 1985), sería más probable que la respuesta dada fuese 24 en lugar de la respuesta correcta 32. Este efecto sugirió que la recuperación de una respuesta a una primera operación producía una activación en la red que se mantenía por un lapso de tiempo, de manera que si inmediatamente después se presentaba un segundo problema relacionado con la respuesta del primero, el nodo de la respuesta asociado a la primera operación recibiría aún mayor activación, interfiriendo con la selección de la respuesta al segundo problema.

Por otro lado, el efecto de confusión asociativa descrito previamente (Winkelman y Schmidt, 1974; Zbrodoff y Logan, 1986) encajaba dentro del modelo de Campbell. Ante la presentación de un problema aritmético (e.g., una suma 2 + 4) se activarían los nodos que representan los operandos, el problema, y aquellos que representan el resultado de la operación aritmética presentada (6). Además, debido al principio de propagación de la activación, otros nodos de la red también recibirían activación, como el resultado de la multiplicación de los dos operandos presentados (8). Este *fenómeno de coactivación* del resultado de la suma (6) y el de la multiplicación (8) podría dar lugar a interferencia a la hora de seleccionar la respuesta correcta (6, en caso de presentarse la suma 2 + 4 =).

El modelo de red de interferencia de Campbell ha sido bien aceptado en la aritmética simple. Es innovador al proponer la idea de interferencia en la resolución de problemas matemáticos y da cuenta de muchos efectos empíricos observados durante la resolución de operaciones simples (e.g., efecto de facilitación de error, efecto de confusión asociativa). Sin embargo, en el modelo no se abordó el posible mecanismo utilizado por las personas para resolver la interferencia generada por la coactivación de información en la red de hechos aritméticos. El siguiente modelo que describimos da cuenta explícitamente de dicho mecanismo de selección.

Modelo de la Recuperación de la Red Semántica de Whalen

En el modelo de *recuperación de la red semántica* (Whalen's semantic network retrieval model; Whalen, 2000) se incluía el componente inhibitorio como un elemento fundamental de la selección de hechos aritméticos dentro de la red asociativa. Según esta teoría, el sistema encargado de la recuperación representaría información procedente de tres fuentes: I) el problema aritmético que se pretende resolver, II) los hechos aritméticos asociados al problema en memoria, y III) la respuesta de salida. De este modo, al presentar un problema aritmético, se activarían en la red los nodos que lo representan y otros nodos relacionados con los operandos a través de conexiones excitatorias unidireccionales. Por ejemplo, al presentar la suma 2 + 4, se activarían los nodos que representan el problema (e.g., 2 + 4) y otros nodos relacionados con los operandos (e.g., 2 + 5, 2 x 4). De manera adicional, se establecerían conexiones inhibitorias con el resto de nodos no relacionados con el problema presentado (e.g., 3 + 6). A su vez, los nodos que representan los problemas en memoria establecerían conexiones excitatorias bidireccionales con aquellos nodos que representan las posibles respuestas de salida (e.g., 6, 7, 8) y conexiones inhibitorias con el resto de nodos de respuesta (e.g., 9). En el modelo se propuso un juego concurrente de procesos de activación e inhibición entre representaciones, de forma que la activación se iría focalizando al final del proceso de selección hacia los nodos correctos, permitiendo seleccionar una respuesta de salida: por ejemplo, si los nodos que representan los problemas activos (e.g., 2 + 4, 2 + 5, 2 x 4) comenzaran a inhibirse mutuamente, el problema que mayor activación acumulase en el tiempo (2 + 4) propagaría la activación al nodo que representa la respuesta asociada al mismo (6) y ésta sería finalmente seleccionada como la respuesta de salida. Existe evidencia empírica que apoya la idea de que procesos inhibitorios pueden estar al servicio del mecanismo de selección de hechos aritméticos en la red (Campbell, Chen y Maslany, 2013; Campbell y Dowd, 2012; Campbell y Thompson, 2012). Estos estudios se revisarán minuciosamente en el apartado "Selección de hechos aritméticos: El papel de la inhibición".

En resumen, los modelos que hemos presentado se encuadraron dentro de los modelos asociativos de aritmética simple. Todos ellos postularon que los hechos aritméticos se representan en la memoria a largo plazo formando una red cuyos nodos estarían conectados a través de diferentes fuerzas asociativas. En primer lugar, en el modelo de la red de recuperación (Ashcraft, 1982) se argumentó que la información que se encontraba relacionada en la red podía ser activada a través del principio de propagación de la activación. Además, se asumió que la estrategia utilizada por defecto a la hora de resolver operaciones aritméticas simples sería la recuperación de la respuesta directamente desde la red asociativa. Por otro lado, el modelo de distribución de asociaciones (Siegler y Jenkins, 1989) contempló un conjunto más amplio de estrategias. El modelo puso el énfasis en cómo se va configurando la red a medida que el niño usa estrategias de tipo procedimental que, a su vez, van dando paso a estrategias de recuperación, más rápidas y eficientes. Por su parte, el modelo de la red de interferencia (Campbell, 1987; Campbell y Graham, 1985) hizo hincapié en los procesos de interferencia que podían desencadenarse tras la activación de hechos aritméticos relacionados en la red, y cómo estos procesos afectarían a la selección de la respuesta correcta en cada caso. Además, se dio una mayor importancia a las conexiones entre representaciones de hechos aritméticos asociados a diferentes operaciones (sumas y multiplicaciones), abarcando fenómenos como el efecto de confusión asociativa (Winkelman y Schmidt, 1974; Zbrodoff y Logan, 1986). Por último, el modelo de recuperación de la red semántica (Whalen, 2000) se centró principalmente en determinar cómo se resolvían estas situaciones de interferencia en la red asociativa. Es decir, desde el modelo se enfatizó en los mecanismos de selección de hechos aritméticos, dando un papel crucial a procesos de naturaleza inhibitoria en la selección final de la respuesta correcta a cada problema específico.

Tras la revisión de los principales modelos asociativos que postulaban la existencia de una red aritmética, en el siguiente apartado nos detendremos en un aspecto fundamental del funcionamiento de dicha red, como es el fenómeno de coactivación. Este fenómeno viene a indicar que cuando resolvemos operaciones aritméticas simples, como por ejemplo la suma 2 + 4, en la red no solo se activaría la respuesta correcta (6) sino que otras respuestas asociadas a los operandos también recibirían activación, como el resultado asociado a la multiplicación (8).

COACTIVACIÓN DE HECHOS ARITMÉTICOS

En este apartado, nos centraremos en el fenómeno de coactivación de hechos aritméticos asociados a la suma y a la multiplicación. En primer lugar, examinaremos el efecto de confusión asociativa (Winkelman y Schmidt, 1974), el cual se ha interpretado como evidencia empírica del fenómeno de coactivación. Además, nos centraremos en aquellos estudios que han explorado el grado de automatización del fenómeno de coactivación (Galfano, Rusconi y Umiltà, 2003; LeFevre, Bisanz y Mrkonjic, 1988; LeFevre y Kulak, 1994; Lemaire, Fayol y Abdi, 1991; Rusconi, Galfano, Speriani y Umiltà, 2004; Zbrodoff y Logan, 1986). Posteriormente, nos detendremos en explorar si la coactivación de hechos aritméticos en memoria subyace realmente al efecto de confusión asociativa. En este sentido, pasaremos a explorar las evidencias aportadas desde estudios que utilizan técnicas de neuroimagen (De Visscher, Berens, Keidel, Noël y Bird, 2015; Grabner, Ansari, Koschutnig, Reishofer y Ebner, 2013) y electrofisiología cerebral (Domahs et al., 2007; Jost, Hennighausen y Rösler, 2004; Niedeggen y Rösler, 1996, 1999; Niedeggen, Rösler y Jost, 1999).

El efecto de confusión asociativa

El fenómeno de coactivación de hechos aritméticos se ha estudiado fundamentalmente mediante *la tarea de verificación* de operaciones (Winkelman y Schmidt, 1974; Zbrodoff y Logan, 1986; Lemaire et al., 1991). En esta tarea, operaciones simples (principalmente sumas o multiplicaciones de un dígito) son presentadas junto con un resultado que puede ser correcto o no. Los participantes han de verificar con la mayor precisión y velocidad posible si el resultado de cada operación es correcto o incorrecto. Es común encontrar que los problemas con respuestas correctas son verificados con una mayor rapidez que aquellos cuyas respuestas son incorrectas (Zbrodoff y Logan, 1986). Sin embargo, en lo que respecta al efecto de confusión asociativa, los estudios se han centrado en el análisis de los ensayos con respuestas incorrectas, comparando los problemas cuyas respuestas, aún siendo incorrectas, están relacionadas con otra operación aritmética (e.g., 2 + 4 = 8) frente a aquellos cuyas respuestas son incorrectas y no están relacionadas con otra operación aritmética (e.g., 2 + 4 = 10). Tomando la diferencia en precisión y tiempo de respuesta entre ambos conjuntos de problemas se obtiene la magnitud del efecto de confusión asociativa, índice a su vez de la coactivación de hechos aritméticos relacionados en la memoria a largo plazo.

El efecto de confusión asociativa fue reportado en la literatura empírica antes de que los modelos asociativos de la aritmética fueran descritos en detalle (Winkelman y Schmidt, 1974), sirviendo como soporte teórico a los mismos (Ashcraft, 1982). Este primer estudio de 1974 se realizó con la tarea de verificación en una pequeña muestra de estudiantes universitarios. La tarea de verificación estaba formada por un conjunto de problemas reducido que incluía cinco sumas y cinco multiplicaciones simples que compartían los operandos (3 + 3, 4 + 3, 3 + 5, 4 + 5 y 5 + 5; 3 x 3, 4 x 3, 3 x 5, 4 x 5 y 5 x 5). Para estudiar el efecto, se manipuló la respuesta que acompañaba a cada uno de los problemas. En este sentido, son de destacar las siguientes condiciones experimentales: I) resultados relacionados con la otra operación (e.g., 3 + 3 = 9, donde el resultado 9 es aquel resultante de multiplicar los operandos de la suma; 3 x 3 = 6, donde el resultado 6 es aquel resultante de sumar los operandos de la multiplicación), II) resultados no relacionados (e.g., 3 + 3 = 12, 3 x 3 = 7), y III) resultados correctos (e.g., 3 + 3 = 6, 3 x 3 = 9). Más allá de la operación (suma o multiplicación), los resultados mostraron que los sujetos tardaron de media unos 60 milisegundos más en verificar un problema con resultado relacionado en comparación con un problema con resultado no relacionado. Asimismo, el número de errores fue mayor en los problemas con resultados relacionados (221) en comparación con los de resultados no relacionados (46). Este peor rendimiento en problemas con resultados relacionados fue interpretado como la consecuencia de la coactivación de información aritmética relacionada y se denominó *efecto de confusión asociativa*. Además, los autores sugirieron que el efecto debía tener su origen en procesos asociativos involucrados en la resolución de operaciones simples, más allá del uso de estrategias procedimentales como el conteo uno a uno de los operandos.

La automaticidad de la coactivación

Una vez delimitado el efecto de confusión asociativa como índice de la coactivación de hechos aritméticos, investigaciones posteriores se centraron en replicar dicho efecto (Findlay, 1978; Zbrodoff, 1979) y determinar su automaticidad (Lemaire et al., 1991; Zbrodoff y Logan, 1986). Por ejemplo, Zbrodoff y Logan (1986) quisieron determinar en qué grado el efecto de confusión era un proceso automático que se desencadena sin la intención del sujeto y si era posible evitar este efecto de manera intencionada. A través de una serie experimental, los autores crearon condiciones donde la intención de los participantes para procesar la información aritmética relacionada pudiera verse comprometida. Utilizaron la tarea de verificación con sumas y multiplicaciones simples. Dentro del conjunto de resultados incorrectos se establecieron dos condiciones con igual número de ensayos: I) resultados relacionados (3 x 4 = 7) y II) no relacionados (3 x 4 = 11). Además, se utilizó todo el conjunto de operandos posibles de un solo dígito (desde el 1 hasta el 9, exceptuando el par 2 * 2). Para manipular la intención de los participantes de procesar la información aritmética relacionada, se manipuló el modo de presentación de los problemas entre-grupos (Experimentos 1 y 2): en un grupo, las operaciones fueron presentadas en bloques puros (sumas o multiplicaciones) por lo que los participantes no tendrían por qué tener la intención de procesar la información de la operación irrelevante para realizar correctamente la tarea. En el otro grupo, las operaciones fueron presentadas de manera combinada (sumas y multiplicaciones presentadas aleatoriamente) por lo que la intención de los participantes estaría centrada en resolver tanto una operación como la otra. Los resultados mostraron que el efecto de confusión asociativa fue mayor cuando las operaciones fueron presentadas de manera combinada (60 ms) que cuando eran presentadas en bloques puros (15 ms), lo que sugería que la coactivación de hechos aritméticos no era un proceso completamente automático, sino que se veía modulado por la intención del sujeto de procesar la información relacionada con cada operación. Continuando con la serie experimental (Experimentos 3 y 4), los autores manipularon la frecuencia de presentación de resultados relacionados entre-grupos: I) 20% de problemas relacionados, II) 80% de problemas relacionados. Se observó que el efecto de confusión asociativa fue modulado por la frecuencia de presentación de resultados relacionados, de modo que éste desaparecía en el grupo con un 80% de resultados relacionados (-3ms) en comparación con el grupo con un 20% de resultados relacionados (57 ms), mostrando que la intención de los participantes de procesar la información irrelevante para maximizar su rendimiento podía estar modulando el fenómeno.

En nuestra opinión, los resultados encontrados en esta serie experimental desarrollada por Zbrodoff y Logan (1986) podrían ser interpretados de una manera alternativa. Por ejemplo, la presentación combinada frente a la presentación en bloques puros podría favorecer la coactivación entre operaciones, de manera que tanto los hechos aritméticos asociados a la suma (e.g., 3 + 4 = 7) como aquellos asociados a la multiplicación (e.g., 3 x 4 = 12) recibirían una mayor activación en la red aritmética que podría permanecer durante un lapso temporal (Campbell, 1987). Esta mayor activación de hechos asociados a ambas operaciones se reflejaría en un mayor efecto de confusión asociativa en bloques combinados frente a la presentación de las operaciones en bloques puros, en los que no se está favoreciendo la activación de la otra operación asociada. Nótese que este patrón es explicable sin la necesidad de recurrir a la intencionalidad del sujeto, sino en términos de una activación residual en los diferentes nodos relacionados de la red aritmética. Por otro lado, los resultados obtenidos tras manipular la proporción de ensayos relacionados (20% vs. 80%) podrían ser explicados en términos de adaptación al conflicto (Botvinick, Braver, Barch, Carter y Cohen, 2001; Lindsay y Jacoby, 1994), de manera que cuando la proporción de ensayos relacionados es elevada, los participantes podrían aprender a resolver el conflicto de manera más eficaz, por ejemplo, desatendiendo o inhibiendo rápidamente la información conflictiva; de modo que el efecto de confusión asociativa desaparecería.

Por su parte, Lemaire et al. (1991) también aportaron evidencia empírica sobre el grado de automaticidad del efecto de confusión asociativa. Los autores evaluaron el rendimiento de estudiantes universitarios en la tarea de verificación de sumas y multiplicaciones (con los operandos desde 2 + 3 hasta 9 + 9), presentadas de manera aleatoria. Como en la investigación previa, los resultados que acompañaban a los problemas podían ser: I) resultados relacionados, II) resultados no relacionados y III) resultados correctos. Además, cuatro diferentes retrasos temporales fueron introducidos entre la presentación de los operandos del problema y la aparición de la respuesta, siendo manipulados entre-grupos: I) 0 ms, II) 100 ms, III) 300 ms o IV) 500 ms. Los autores replicaron el efecto de confusión asociativa en los grupos donde el retraso fue de 0 ms y 100 ms. Sin embargo, el efecto desapareció en los grupos donde el retraso introducido fue mayor (300 ms y 500 ms), apoyando la hipótesis de que el efecto de confusión asociativa era "parcialmente automático": "automático" porque ante la presentación del problema los hechos aritméticos relacionados fueron coactivados sin la intención de los participantes, "parcialmente" porque si los participantes disponían de suficiente tiempo para recuperar la respuesta correcta, las personas intencionalmente evitaban que la respuesta relacionada interfiriera en la resolución del problema.

Es nuestra opinión, el patrón de resultados reportado por Lemaire et al. (1991) puede ser interpretado de manera alternativa sin la necesidad de recurrir a la intencionalidad del sujeto. Si ante la presentación de un problema aritmético simple (e.g., 2 + 4), asumimos que se da una coactivación en la red del resultado correcto (e.g., 6) y del resultado asociado a la multiplicación (e.g., 8), éstos podrían competir en el proceso de selección de la respuesta correcta. Esta competición podría ser resuelta mediante la inhibición de la información irrelevante en cada caso (e.g., 8), un proceso que requiere tiempo. Al introducir un retraso temporal entre los operandos y el resultado, podría haber tiempo suficiente para que el proceso de inhibición se completase y se seleccionase la respuesta correcta. De esta manera, cuando apareciese el resultado correcto, el participante ya habría seleccionado la respuesta correcta y el proceso de coactivación e interferencia habría sido resuelto, no observándose el efecto de confusión asociativa. Esta hipótesis inhibitoria se planteará en detalle más adelante.

Los estudios revisados hasta ahora sobre el efecto de confusión asociativa fueron realizados en contextos aritméticos, donde los participantes necesitaban recuperar información aritmética para realizar correctamente la tarea. Estos estudios apoyaron la visión de que la coactivación de hechos aritméticos asociados a la suma y a la multiplicación tenía lugar, al menos parcialmente, de manera automática. Una nueva manera de evaluar si la coactivación de hechos aritmético se realizaba de manera automática era el estudio de dicha coactivación fuera de un contexto aritmético específico, es decir, incluso cuando ninguna operación aritmética era requerida (Galfano et al., 2003; García-Orza, Damas-López, Matas y Rodríguez, 2009; LeFevre et al., 1988; LeFevre y Kulak, 1994; Rusconi et al., 2004).

En este sentido, el estudio de LeFevre et al. (1988) se centró en evaluar si la activación de hechos aritméticos asociados a la suma se desencadenaba de manera automática fuera del contexto aritmético. Para ello, los autores se valieron de una tarea de comparación numérica simple, en la que se les pedía a los participantes que indicaran si un número de un dígito (e.g., 5) había sido presentado previamente como parte de una operación (e.g., 5 + 1, en este caso la respuesta era "si") y en ningún momento la tarea requería que los participantes realizaran la operación aritmética en cuestión. De manera similar a los estudios realizados dentro de un contexto aritmético, dos condiciones experimentales fueron de interés: I) estímulo de prueba relacionado con la suma, donde el número a comparar era el resultado de realizar la suma de los operandos (e.g., 6, precedido por 5 + 1); y II) estímulo de prueba neutral, en la que el número no estaba relacionado con la operación (e.g., 3, precedido por 5 + 1), en ambos casos los participantes tenían que contestar "no". Además, se introdujeron diferentes retrasos temporales entre el problema y la aparición del estímulo de prueba (60 ms, 120 ms, 180 ms, 240 ms, 480 ms). Los autores planteaban que tras la presentación de los operandos (e.g., 5 + 1) se produciría una activación automática del resultado de la suma en la red (e.g., 6), que podría interferir con los otros estímulos activados (e.g., 5 y 1) a la hora de decidir cuál había sido presentado previamente. Como se predecía, al comparar las condiciones relacionada y control se encontró un efecto de interferencia: los participantes tardaron un mayor tiempo en refutar un estímulo de prueba que era el resultado de sumar la operación que en refutar un estímulo neutral; sugiriendo que la respuesta asociada a la suma de la operación había sido activada de manera automática aunque no era necesaria para realizar la tarea e interfería a la hora de decidir si había sido presentada previamente o no. Este efecto de interferencia solo fue significativo cuando el retraso entre los estímulos fue corto (retrasos menores de 180 ms) pero no con retrasos de tiempo superiores, lo que fue interpretado como consecuencia de que dicha activación en memoria decaía con el tiempo o bien debido a que los participantes tuvieron un intervalo de tiempo suficiente para inhibir la información irrelevante de la suma. Es interesante señalar que el efecto se mantenía incluso si el problema era presentado sin el símbolo aritmético (e.g., 5 1), es decir, únicamente tras la presentación de pares de números. Lo anterior sugiere que, a pesar de no haber claves que favorecieran la coactivación de hechos aritméticos, la mera exposición a pares de números favoreció la activación del resultado de la sumatoria de ambos. En resumen, la activación de hechos aritméticos asociados a la suma en memoria parece ser un fenómeno robusto que se desencadena de manera obligatoria más allá del contexto aritmético, ¿ocurre lo mismo con hechos aritméticos asociados a la multiplicación?

Para contestar a esta pregunta y determinar si la activación de los hechos aritméticos asociados a la multiplicación también se desencadenaba de manera automática en contextos no aritméticos, Rusconi et al. (2004) usaron nuevamente la tarea de comparación numérica simple, en la que pares de números eran presentados sin el símbolo aritmético asociado a la multiplicación “x” (e.g., 3 8). Fue de interés la comparación de dos condiciones experimentales similares al estudio de LeFevre et al. (1988): I) estímulo de prueba relacionado con la multiplicación, donde el número a comparar era el resultado de multiplicar los operandos (e.g., 24, precedido por 3 8); y II) estímulo de prueba neutral, donde el número no estaba relacionado con la multiplicación de los operandos (e.g., 49, precedido por 3 8). Los autores encontraron un patrón de resultados análogo al caso de la activación del resultado de la suma (LeFevre et al., 1988) descrito anteriormente: los participantes tardaban más tiempo en refutar un estímulo de prueba relacionado con la multiplicación en comparación con un número no relacionado. De lo que se induce que tras la aparición del par de estímulos (e.g., 3 8) se produjo la activación automática del resultado asociado a la multiplicación de los mismos (e.g., 24), y a su vez la activación de éste interfería a la hora de decidir si había sido presentado previamente o no, al competir con la activación del par de estímulos realmente presentados (e.g., 3 y 8). Estos resultados apoyaban que, al igual que ocurre con los hechos aritméticos asociados a la suma, en la red aritmética de la memoria a largo plazo tiene lugar una activación obligatoria del conocimiento asociado a la multiplicación, incluso en aquellas condiciones en las que ninguna operación aritmética es necesaria para la ejecución de la tarea.

En conclusión, la coactivación de hechos aritméticos asociados a la suma y a la multiplicación parece ser un fenómeno robusto de naturaleza parcialmente automática, que se desencadena tras el procesamiento de estímulos relevantes. La mera presentación de pares de números, favorece la activación de diferentes nodos que se encuentran inter-conectados en la red aritmética de la memoria; con independencia de que los hechos aritméticos sean necesarios o no para la realización de una tarea. Para nuestro presente trabajo de investigación, es esencial destacar que este efecto ha sido interpretado como evidencia de la coactivación de hechos aritméticos asociados tanto a la suma como a la multiplicación en la memoria a largo plazo (Winkelman y Schmidt, 1974; Zbrodoff y Logan, 1986; Lemaire et al., 1991). Sin embargo, dicha premisa ha de ser corroborada empíricamente. En el siguiente apartado revisamos una serie de estudios que, directa o indirectamente, parece sugerir que la coactivación de hechos aritméticos subyace a los efectos de confusión asociativa reportados en la literatura.

¿La coactivación subyace a la confusión asociativa?

Aunque ha sido asumido que la coactivación de hechos aritméticos subyace al efecto de confusión asociativa, reconocemos que la investigación previa ha ofrecido apoyo indirecto a dicha interpretación (Galfano et al., 2003; LeFevre et al., 1994; Lemaire et al., 1991; Rusconi et al., 2004; Winkelman y Schmidt, 1974; Zbrodoff y Logan, 1986). Estudios actuales en que se han considerado técnicas de neuroimagen y electrofisiología cerebral parecen concretar más directamente la relación entre coactivación aritmética y confusión asociativa.

De forma específica, estudios recientes utilizando la técnica de neuroimagen por Resonancia Magnética Funcional (IRMf) se han centrado en explorar el correlato cerebral de los efectos de relación en la aritmética simple (De Visscher et al., 2015; Grabner et al., 2013). Concretamente, el estudio de Grabner et al. (2013) se centró en determinar los correlatos neuroanatómicos del efecto de confusión asociativa. Como en los estudios previos, a una muestra de adultos se les pidió que realizaran una tarea de verificación de sumas y multiplicaciones simples que se presentaban de manera combinada (pseudo-aleatoriamente). El conjunto de problemas presentados fue desde 2 + 3 hasta 9 + 9 y se establecieron las ya conocidas condiciones experimentales: I) resultados relacionados, II) resultados no relacionados y III) resultados correctos. A nivel comportamental, se replicó el efecto de confusión asociativa. Por su parte, el análisis de IRMf mostró que cuando los participantes resolvían problemas que eran acompañados de resultados relacionados (e.g., 3 + 4 = 12) se producía una mayor activación del giro angular izquierdo (anterior) que se extendía hacia el giro supramarginal y la corteza parietal superior y, a su vez, una mayor activación de la corteza prefrontal dorso-lateral del hemisferio izquierdo, en comparación con la condición en la que los participantes resolvían problemas con resultados no relacionados (e.g., 3 + 4 = 7).

Si consideramos que el giro angular izquierdo ha sido íntimamente ligado con la representación de hechos aritméticos (Delazer, Domahs, Bartha, Brenneis, Lochy, Trieb, & Benke, 2003; Grabner, Ansari, Koschutnig, Reishofer, Ebner, & Neuper, 2009), este estudio parece sugerir que, en efecto, la activación de la red aritmética subyace al efecto de confusión asociativa. Además, los autores (Grabner et al., 2013) disociaron el efecto de confusión asociativa de los efectos asociados a la dificultad del problema en el giro angular izquierdo (el cual presentaba una menor activación durante la resolución de problemas difíciles en comparación con la resolución de problemas fáciles), apoyando que esta área cerebral estaría comprometida con los procesos de mapeo entre el problema presentado y las posibles respuestas asociadas al mismo. De este modo, cuando un problema con resultado relacionado es presentado, se produciría de manera automática una mayor activación de dichos resultados en la memoria a largo plazo del sujeto.

Por otro lado, en estudios de registro de la actividad electrofisiológica cerebral (Domahs et al., 2007; Guthormsen, Fisher, Bassok, Osterhout, DeWolf y Holyoak, 2015; Jost et al., 2004; Niedeggen y Rösler, 1996, 1999; Niedeggen et al., 1999) también se ha evaluado el fenómeno de coactivación ligado al procesamiento de multiplicaciones simples. Por ejemplo, en el estudio de Niedeggen y Rösler (1999) se utilizó una tarea de verificación de multiplicaciones simples en la que los participantes tenían que indicar si cada multiplicación presentada era correcta o no. En lo que respecta a las multiplicaciones incorrectas, se establecieron dos condiciones experimentales: I) condición relacionada, en la que el resultado presentado era múltiplo del primer o del segundo operando (e.g., 5 x <u>8</u> = <u>32</u>, dado que 4 x <u>8</u> = <u>32</u>), y II) condición control, en la que el resultado presentado no estaba relacionado con la operación (e.g., 5 x 8 = 34). Los autores consideraron tanto medidas comportamentales (e.g., tiempo de reacción), como de promediado de la actividad eléctrica cerebral (ERPs, por sus siglas en inglés Event-Related Potentials). Por una parte, se encontró un efecto de interferencia similar al efecto de confusión asociativa, pero entre resultados asociados únicamente a multiplicaciones simples. Así, los participantes tardaban más tiempo en descartar una multiplicación incorrecta relacionada (e.g., 5 x <u>8</u> = <u>32)</u> frente a una multiplicación no relacionada (e.g., 5 x 8 = 34). Este resultado fue interpretado como consecuencia de la coactivación de varias respuestas relacionadas con la multiplicación en la red aritmética; coactivación que interfería en la selección de la respuesta correcta al problema (e.g., 40). Por otra parte, se encontraron modulaciones del componente N400, una onda negativa cuyo pico se encuentra aproximadamente entre los 350-450 ms tras la presentación del estímulo, y que ha sido considerado como un índice ligado a la dificultad/facilidad en el acceso a la información semántica (Kutas y Hillyard, 1980, 1984). En concreto, se observó una atenuación del componente N400 en la condición relacionada frente a la condición control. Los autores interpretaron que la atenuación en el componente N400 en la condición relacionada se debía a la propagación de la activación en la red aritmética en la condición relacionada, de manera que la activación de la respuesta relacionada facilitaba la activación de la respuesta correcta. A su vez, este proceso de coactivación entre respuestas relacionadas interferiría posteriormente en la selección de la respuesta correcta. Esta interferencia fue observada en los resultados comportamentales, no pudiendo ser capturada mediante medidas electrofisiológicas. Por lo tanto, parece existir evidencia empírica que vincula el fenómeno de coactivación con el posterior efecto de interferencia entre hechos aritméticos relacionados y asociados a la multiplicación.

Por otro lado, en un estudio reciente de Avancini et al. (Avancini, Soltész y Szűcs, 2015) se ha mostrado que las modulaciones del N400 asociadas al procesamiento aritmético se desencadenaban incluso cuando no es necesaria la recuperación del hecho aritmético desde la memoria. Estos autores crearon un nuevo paradigma experimental en el que, inicialmente, se presentaba una suma simple (e.g., 3 + 4) seguida de un tercer número (e.g., 9). Los participantes debían indicar la paridad del tercer número (e.g., 9, impar). Además, los autores manipularon la relación entre el tercer número y los dos operandos presentados previamente de forma que dicho número podía ser el resultado de la suma (7) o no (9). A pesar de que los sujetos no tenían que realizar operaciones aritméticas sino juicios de paridad, se encontró una modulación del componente N400, de manera que su amplitud fue menos negativa cuando el tercer número era el resultado de la suma frente a cuando no lo era. Esta atenuación del componente N400 fue interpretada como evidencia de la recuperación del hecho aritmético asociado a la suma aunque no fuese necesario para la resolución del problema.

Hemos visto, por lo tanto, que la atenuación del componente N400 se ha tomado como un indicador del acceso tanto a hechos aritméticos asociados a sumas (Avancini et al., 2015) como a multiplicaciones (Niedeggen y Rösler, 1996, 1999; Niedeggen et al., 1999). Sin embargo, es importante destacar que en estas investigaciones previas de corte electrofisiológico se ha evaluado la coactivación de hechos aritméticos dentro de una misma categoría de problemas (entre resultados de multiplicaciones, o entre resultados de sumas). Sería de especial interés explorar si las mismas modulaciones del componente N400 se dan como consecuencia de la coactivación entre hechos aritméticos asociados a diferentes operaciones aritméticas (sumas y multiplicaciones) y si esta coactivación subyace al efecto de confusión asociativa descrito hace cuarenta años (Winkelman y Schmidt, 1974). Estas cuestiones se abordarán en el Capítulo IV de la serie experimental.

SELECCIÓN DE HECHOS ARITMÉTICOS: EL PAPEL DE LA INHIBICIÓN

En el apartado anterior, hemos explorado un aspecto fundamental de la representación de hechos aritméticos en la memoria a largo plazo: el fenómeno de coactivación, es decir, la activación conjunta de diversos hechos aritméticos que se encuentran relacionados en la red asociativa. Sin embargo, resulta importante destacar que, a pesar de la interferencia que el fenómeno de coactivación puede generar a la hora de seleccionar la respuesta correcta (e.g., es más difícil contestar a una suma cuyo resultado es aquel de multiplicar sus operandos, 2 + 4 = 8; Zbrodoff y Logan, 1986), la mayor parte de las personas realizamos correctamente este tipo de operaciones aritméticas simples. Así pues, cabe preguntarse de qué modo se selecciona finalmente la respuesta correcta y se descartan otras posibles respuestas competidoras. En este apartado, centraremos nuestra atención justamente en los procesos de selección que permiten finalmente recuperar la respuesta correcta desde la memoria, dándole un papel fundamental a procesos de naturaleza inhibitoria.

A lo largo de los años, resultados de diferentes estudios han sugerido que procesos de naturaleza inhibitoria podrían estar implicados en la selección de hechos aritméticos (Lemaire et al., 1991; LeFevre et al., 1988; LeFevre y Kulak, 1994). Además, modelos asociativos centrados en la recuperación, como el modelo de *recuperación de la red semántica* (Whalen, 2000) descrito en páginas precedentes, incluían procesos inhibitorios como elementos fundamentales de la selección de hechos aritmético. Recordemos que en el modelo se proponía que la selección de una respuesta concreta se debía a la sucesión de conexiones tanto excitatorias como inhibitorias entre diversos nodos de la red, de manera que la activación se iría focalizando en los nodos relevantes permitiendo seleccionar la respuesta correcta.

Una primera línea de evidencia empírica que parece demostrar la importancia de los procesos inhibitorios en la aritmética simple es de corte evolutivo. De hecho, en una gran variedad de estudios se ha mostrado la relación entre diferentes medidas de control inhibitorio y el rendimiento en matemáticas de niños en edad escolar (Adams y Hitch, 1997; Bull, Johnston y Roy, 1999; Bull y Scerif, 2001; Fürst y Hitch, 2000; Geary, Hamson y Hoard, 2000; Gilmore et al., 2013; Lubin, Vidal, Lanoë, Houdé y Borst, 2013; McLean y Hitch, 1999; Van der Sluis, De Jong y Van der Leij, 2004). Dicha evidencia empírica será revisada en el apartado “El desarrollo de la red asociativa”.

Por otro lado, se ha encontrado evidencia empírica a favor de la existencia de procesos de auto-inhibición encargados de regular el rendimiento en tareas secuenciales utilizando operaciones aritméticas simples (Arbuthnott y Campbell, 2000; 2003; Campbell y Arbuthnott, 1996). Los procesos de auto-inhibición son los encargados de suprimir el estímulo que deja de ser relevante para procesar el siguiente estímulo y conseguir así que una tarea secuencial se ejecute de manera fluida. En el estudio de Arbuthnott y Campbell (2003) el objetivo era evaluar si estos procesos de auto-inhibición ocurrían tras la mera activación de la representación del estímulo en memoria o si era necesaria una producción motora asociada al estímulo para su posterior inhibición. Para ello, los autores usaron una tarea de producción de sumas en la que se manipularon dos condiciones: I) condición relacionada, en la que dos sumas consecutivas estaban relacionadas, de manera que algún operando de la primera (e.g., 3 + <u>7</u>) coincidía con el resultado de la segunda (e.g., 2 + 5 = <u>7</u>), y II) condición control, en la que la primera suma (e.g., 3 + 9) no estaba relacionada con la segunda (e.g., 2 + 5 = 7). Téngase en cuenta que en la condición relacionada, si tras la activación de la representación de los operandos de la primera operación se producía la inhibición de los mismos como posibles respuestas (e.g., 3 + <u>7</u>), se tardaría un mayor tiempo en responder a una segunda operación cuyo resultado coincidiese con alguno de los operandos (e.g., 2 + 5 = <u>7</u>) puesto que se requeriría más tiempo para recuperar este estímulo nuevamente desde la memoria. Los resultados mostraron este efecto de interferencia en la condición relacionada en comparación con la condición control, sugiriendo que la activación de la representación del operando en la primera suma era suficiente para que se produjese la inhibición del mismo como respuesta, interfiriendo en la ejecución de una suma consecutiva cuya respuesta coincidía con el número inhibido. Este estudio resultó de gran relevancia a la hora de mostrar que los procesos de inhibición no necesitaban de una respuesta motora, sino que podían desencadenarse tras la activación de la representación del número en memoria, lo que era acorde con el modelo de recuperación de la red semántica (Whalen, 2000). Sin embargo, una vez más, estos estudios no especificaban el posible papel del control inhibitorio en la selección de los hechos aritméticos cuando hay coactivación y competición de resultados en la red asociativa.

Recientemente, en una serie de estudios conducidos por Campbell (Campbell, Chen y Maslany, 2013; Campbell y Dowd, 2012; Campbell y Thompson, 2012) se ha examinado directamente el papel de los procesos inhibitorios en la selección de hechos aritméticos. Para ello, en estos estudios se ha utilizado una adaptación del paradigma de práctica en la recuperación, el cual se utiliza normalmente para mostrar cómo la interferencia producida por diferentes representaciones en la memoria es resuelta mediante un mecanismo de inhibición de las representaciones competidoras (Anderson, Bjork y Bjork, 1994).

En los estudios realizados por Campbell et al., los participantes pasaban previamente por una fase de entrenamiento, en la cual se presentaba una serie de multiplicaciones simples y había que dar el resultado correcto en voz alta a cada una de ellas (e.g., 2 x 3 = ?). Tras esta fase de entrenamiento, los participantes pasaban a la fase de prueba, en la que se presentaban sumas simples y los participantes igualmente tenían que resolverlas en voz alta. Lo interesante es que entre las sumas, se incluían problemas cuyos operandos habían sido practicados previamente en la fase de entrenamiento en multiplicaciones (e.g., 2 + 3 = ?) y problemas cuyos operandos no habían sido practicados en la fase de entrenamiento (e.g., 2 + 5 = ?). El principal resultado encontrado fue un efecto de interferencia en la segunda fase, de manera que los participantes contestaron más lentamente a las sumas practicadas frente a las sumas cuyos operandos no habían sido practicados previamente en la multiplicación. Este efecto, denominado como Olvido Inducido por la Recuperación (OIR o efecto RIF, en inglés: Retrieval-Induced Forgetting) fue interpretado en términos inhibitorios, de modo que la recuperación de los hechos asociados a la multiplicación en la fase de entrenamiento produjo una inhibición de los hechos competidores asociados a la suma en la red aritmética. Como consecuencia, en la fase de prueba, los participantes tardaron un mayor tiempo en recuperar aquellos hechos asociados a la suma que habían sido previamente inhibidos en comparación con los aquellos no inhibidos.

Por otro lado, Campbell et al. (Campbell et al., 2013; Campbell y Dowd, 2012; Campbell, Dufour y Chen, 2015; Campbell y Thompson, 2012) han especificado diversas características del efecto OIR en la aritmética simple, entre ellas: el fenómeno OIR se hace más evidente en los problemas de tamaño pequeño (la suma de sus operandos es ≤ 10), es decir, en aquellos cuya fuerza asociativa es mayor en la red aritmética y, por lo tanto, producen una gran competición. Además, el fenómeno ocurre con independencia del formato en el que los hechos aritméticos se presentan, se ha encontrado el mismo patrón tanto si los problemas eran presentados en un formato numérico familiar (e.g., 2 x 3 =) o escritos en palabras (e.g., dos más tres igual).

Llegados a este punto, podríamos preguntarnos sobre el locus del proceso inhibitorio. Campbell et al. (Campbell et al., 2013; Campbell y Dowd, 2012; Campbell y Thompson, 2012) no especifican qué es exactamente lo que se está inhibiendo al seleccionar la respuesta correcta. En nuestra opinión, la inhibición podría producirse en las respuestas competidoras relacionadas con el problema en cuestión, de manera que; tras la presentación de un hecho aritmético asociado a la multiplicación (e.g., 2 x 3 =) se activarían la respuesta correcta (6) y otras respuestas asociadas, entre ellas, el resultado de la suma de los operandos (5). Esta competición podría resolverse inhibiendo la respuesta incorrecta asociadas a la suma (5). De este modo, cuando la respuesta inhibida es relevante en la fase de prueba, se precisaría un tiempo adicional para recuperarla nuevamente desde la memoria.

En resumen, los estudios revisados en esta sección sugieren la existencia de un mecanismo de carácter inhibitorio durante la resolución de operaciones aritméticas simples. Es importante destacar que este mecanismo inhibitorio tiene consecuencias a largo plazo puesto que se asume que la inhibición descrita por Campbell et al. se produce en una fase de entrenamiento con multiplicaciones y, pasada esta tarea, se evalúan las consecuencias del proceso inhibitorio durante la tarea de sumas. A lo largo de nuestra serie experimental, nosotros evaluaremos si este mecanismo inhibitorio actúa de manera continua, es decir, ensayo a ensayo, cuando las personas resuelven operaciones aritméticas simples.

EL USO DE HECHOS ARITMÉTICOS: FACTORES MODULADORES

En los apartados precedentes nos hemos centrado en dos aspectos fundamentales de la recuperación de hechos aritméticos desde la memoria: la coactivación de información aritmética relacionada en la red, y el posterior proceso de selección de la respuesta correcta en cada caso. Sin embargo, tal y como señalaba Siegler (Siegler y Jenkins, 1989), es importante tener en cuenta que en la resolución de operaciones aritméticas simples intervienen tanto estrategias de recuperación como estrategias de tipo procedimental. En este apartado, pasaremos a preguntarnos qué factores influyen en el uso de hechos aritméticos desde la memoria. En un primer momento, nos centraremos en aquellos modelos que han defendido el uso de mecanismos procedimentales en la aritmética simple (Baroody, 1983) y la evidencia empírica que han recibido (Barrouillet y Thevenot, 2013; Della Puppa et al., 2015; LeFevre, Sadesky y Bisanz, 1996; Roussel, Fayol y Barrouillet, 2002; Thevenot, Barrouillet, Castel y Uittenhove, 2016; Thevenot, Castel, Danjon y Fayol, 2015; Thevenot, Fanget y Fayol, 2007). Posteriormente, exploraremos cómo el formato en que las operaciones aritméticas son presentadas puede modular el uso de estrategias en la resolución de operaciones aritméticas simples (Campbell y Epp, 2004). En último lugar, contemplaremos el desarrollo evolutivo de los niños como otro de los factores que influye en el uso de hechos aritméticos al tiempo que se va conformando la red aritmética en la memoria (Siegler y Jenkins, 1989).

Mecanismos procedimentales

En el modelo de distribución de asociaciones de Siegler (Siegler y Jenkins, 1989), descrito más arriba, se contempló el uso de estrategias procedimentales para la resolución de problemas aritméticos simples incluso en la población adulta. A través del uso de auto-informes, en los que se les pedía a adultos de diferentes rangos de edad que indicaran cómo habían resuelto operaciones aritméticas simples, se mostró que todos reportaban haber utilizado estrategias procedimentales en mayor o menor grado (Geary y Wiley, 1991; Healy, Rickard y Bourne, 1993; Núñez-Peña, Colomé y Tubau, 2015). Por ejemplo, el estudio de Geary y Wiley (1991) mostró que adultos jóvenes resolvían un 10% de las operaciones a través de estrategias procedimentales, frente al 90% del uso de recuerdo directo de la respuesta desde la memoria. Sin embargo, los modelos asociativos asumieron que el uso de estrategias procedimentales era más lento en comparación con la rápida recuperación desde la memoria (Ashcraft, 1992), apostando por la eficacia de esta última estrategia.

Un planteamiento alternativo a los modelos asociativos (Ashcraft, 1982; 1987; Campbell y Graham, 1985; Siegler y Jenkins, 1989) fue el *modelo basado en esquemas* de Baroody (Baroody's schema-based model; Baroody, 1983; 1994) que defendía el uso de estrategias procedimentales como la estrategia por defecto en la resolución de las operaciones aritméticas simples. En un primer momento, Baroody (1983) criticó la asunción de los modelos asociativos sobre la rapidez del uso del recuerdo frente a estrategias de carácter procedimental, argumentando que la práctica continuada de estrategias procedimentales en edad escolar podría hacer que éstas se ejecutaran de una manera automática, y por ende, con gran rapidez. Además, el autor argumentó que un modelo basado en estrategias procedimentales era más económico en términos cognitivos: por un lado, bajo su planteamiento, no era necesaria la representación de un amplio rango de hechos aritméticos en la memoria; sino que un rango más concreto de reglas, heurísticos y principios generales podría ser almacenado y aplicado a una gran variedad de operaciones aritmética simples. Por ejemplo, el uso de la regla N + 1 (*el resultado es el siguiente número a N*) sería más fácil de aplicar que la búsqueda en memoria del resultado asociado a ambos operandos.

Por otro lado, resulta relevante indicar que Baroody (1983) puso en duda que el efecto de confusión asociativa (Winkelman y Schmidt, 1974) fuese evidencia de que los hechos aritméticos están almacenados en la memoria. El fenómeno, según el autor, podría entenderse como consecuencia de la utilización de reglas inapropiadas para la operación en cuestión. Por ejemplo, ante la resolución de la multiplicación 7 x 0, el uso de una regla correspondiente a la suma como N + 0 = N (en lugar de la regla relevante para la multiplicación N x 0 = 0) podría derivar en el fenómeno de confusión. Sin embargo, tal y como objeta Ashcraft (1983), Baroody no plantea explícitamente qué reglas inapropiadas podrían estar a la base del fenómeno en operaciones aritméticas de mayor tamaño, que no se siguen de una regla aparente (3 + 5 = 15) y que son distintas de aquellas reglas aplicadas cuando los operandos son 0 ó 1 (*N + 0, N x 0, N + 1 o N x 1).* Además, el autor no justifica el porcentaje tan elevado de errores (implementación de reglas inapropiadas) que deberían usar los participantes para evidenciar un fenómeno tan robusto en la literatura empírica como el efecto de confusión asociativa (Findlay, 1978; Lemaire et al., 1991; Winkelman y Schmidt, 1974; Zbrodoff y Logan, 1986; Zbrodoff, 1979).

En una formulación posterior del modelo basado en esquemas de Baroody (1994), el autor especificó que además de la representación de algunos hechos aritméticos en la memoria a largo plazo, las diferentes estrategias procedimentales a las que es posible acceder para resolver un problema aritmético también se representarían en memoria, conformando la estructura de esquemas. Por ejemplo, ante la resolución de una multiplicación, un primer esquema podría activarse para identificar la operación concreta del problema (en este caso, una multiplicación). A partir de este paso inicial, diferentes esquemas se activarían dependiendo del problema: I) Si el problema fuese del tipo N x 0, se ejecutaría el esquema de la regla N x 0 = 0 (*Si* 0 es multiplicado a otro número, *entonces* el resultado es 0); II) Si el problema fuese del tipo N x 1, se ejecutaría el esquema de la regla N x 1 = N (*Si* 1 es multiplicado a otro número, *entonces* el resultado es igual al número). Además, en el modelo se incluían esquemas cuya premisa era acceder a la red aritmética: III) Si el problema fuese del tipo N_1 x N_1 (Si un número es multiplicado por él mismo, entonces accede a la red asociativa). En otras palabras, esta formulación del modelo estaba a medio caballo entre el uso de reglas procedimentales y la recuperación en memoria como maneras de resolver operaciones aritméticas simples.

Diferentes investigaciones empíricas (Barrouillet y Thevenot, 2013; LeFevre et al., 1996; Roussel et al., 2002; Thevenot et al., 2007, 2015, 2016) parecen apoyar el modelo basado en esquemas (Baroody, 1983; 1994) en población adulta. En términos metodológicos, estas investigaciones comparten la idea de que las medidas de auto-informe en que se pregunta a los sujetos sobre la manera en que han resuelto un problema, podrían ser medidas cuestionables para determinar la recuperación en memoria frente al uso de procedimientos. En concreto, en estos estudios se plantea que si las estrategias procedimentales son automáticas, el sujeto puede resolver el problema rápidamente sin tener consciencia de haberlas usado. En consecuencia, una medida de auto-informe no evidenciaría el uso de procedimientos al ser ejecutados de manera no consciente. Por estas razones, Thevenot et al. (2007) utilizaron un paradigma de reconocimiento de operandos para evaluar el uso de estrategias procedimentales en la resolución de sumas de tamaño pequeño (la suma es < 10), mediano (la suma es > 10) y grande (los operandos de la suma son de dos dígitos). Los autores querían corroborar si las sumas eran resueltas mediante estrategias procedimentales como la transformación de los operandos (e.g., 5 + 7 = 5 + 5 + 2 = 12). Se argumentó que, de ser así, si tras la resolución del problema los operandos originales volviesen a presentarse (5 y 7) y el participante tuviese que indicar si éstos aparecieron previamente o no, el reconocimiento de los mismos se vería afectado, comparándolo con una tarea de comparación numérica (e.g., *¿Está el número 6 entre 5 y 7?*) en la que no se precisaba la transformación de los números. Los autores encontraron un peor desempeño en el reconocimiento de los operandos cuando las sumas eran de tamaño mediano y grande, lo que fue interpretado como consecuencia del uso de estrategias procedimentales en algunas de las operaciones. Sin embargo, el reconocimiento no se vio afectado con sumas pequeñas (e.g., 3 + 5 = 8), sugiriendo que éstas se realizaban a través de la recuperación de la respuesta desde la memoria, sin necesidad de transformar los operandos. En un segundo experimento, los autores mostraron que los participantes con una alta habilidad aritmética únicamente empeoraron su rendimiento en la tarea de reconocimiento con los problemas grandes, sugiriendo que el resto de sumas simples eran resueltas principalmente a través del recuerdo desde la red aritmética; y en nuestra opinión, mostrando que el recuerdo reemplaza a las estrategias procedimentales a medida que la práctica incrementa.

Por otro lado, Fayol y Thevenot (2012) pidieron a participantes adultos que resolviesen sumas, restas o multiplicaciones simples cuyos operandos iban del 1 al 9 y que eran precedidos por el signo de la operación 150 ms antes (e.g., +, 9 + 6 = ?) o aparecían directamente junto con los operandos. Los autores partían de la idea de que si las operaciones se resolvían a través de estrategias procedimentales, éstas debían activarse tan rápido como fuese posible, independientemente del problema concreto a resolver. Los resultados mostraron que tanto las sumas como las restas se contestaron más rápidamente cuando fueron precedidas por el signo aritmético (+ y - respectivamente), mientras que las multiplicaciones fueron contestadas con la misma rapidez independientemente si eran precedidas del signo aritmético (x) o no. Además, el patrón de resultados fue similar para todas las sumas, exceptuando los problemas "tie" con operandos iguales (e.g., 3 + 3), sobre los que se asume una especial representación en la memoria a largo plazo. Este patrón fue interpretado como consecuencia de una pre-activación de estrategias procedimentales que facilitaron la resolución de operaciones aritméticas simples como la resta o la suma. Por ejemplo, en el caso de la suma, tras la presentación del signo + la "min estrategia" (*contar desde el operando de mayor tamaño*) podría activarse en la memoria, facilitando la posterior aplicación de la misma.

Basándose en los resultados expuestos arriba, Fayol y Thevenot (2012) defendieron que las estrategias procedimentales eran el método por defecto utilizado a la hora de resolver sumas simples; pero no en el caso de las multiplicaciones simples, siendo éstas resueltas a través de la recuperación. Sin embargo, en nuestra opinión, es posible que el paradigma empleado por los autores favoreciese artificialmente el uso de estrategias procedimentales. Es decir, al presentarse previamente el signo aritmético, los participantes podrían decantarse por el uso de estrategias en vez de por la recuperación, puesto que la preactivación de estrategias favorecería la rápida resolución del problema una vez que los operandos fuesen presentados. Es interesante señalar que los autores también encuentran un menor tiempo en responder a sumas, restas y multiplicaciones cuando unos 150 ms antes del problema aparecen los operandos sin el signo aritmético (e.g., 9 6, 9 + 6 = ?). En este caso, podría suceder que estos operandos preactivasen hechos aritméticos en memoria, facilitando el recuerdo directo como manera de resolver el problema. Es decir, en nuestra opinión, la investigación realizada por Fayol y Thevenot (2012), presenta posibles explicaciones alternativas a la idea del uso de estrategias procedimentales usadas por defecto en la resolución de problemas aritméticos simples.

En resumen, existe evidencia empírica que apoya tanto el uso de estrategias de recuerdo (Geary y Wiley, 1991) como el uso de estrategias procedimentales (Barrouillet y Thevenot, 2013; LeFevre et al., 1996; Núñez-Peña et al., 2015; Roussel et al., 2002) en la resolución de operaciones aritméticas simples. En un capítulo de nuestra sección experimental, directamente evaluaremos el uso de dichas estrategias frente a la recuperación de hechos aritméticos (ver Capítulo V). Por otro lado, sería interesante preguntarse por factores que indujesen a la elección de una manera de resolver las operaciones aritméticas (recuerdo frente a procedimientos). En este sentido, cambios en la familiaridad del formato en el que las operaciones son presentadas, por ejemplo, escritas en palabras (e.g., dos + cuatro) en lugar de en dígitos (e.g., 2 + 4), puede determinar el mayor uso de estrategias procedimentales frente a estrategias de recuerdo (Schunn, Reder, Nhouyvanisvong, Richards y Stroffolino, 1997). En el siguiente apartado, describiremos brevemente tanto los estudios como los principales modelos teóricos que han sido propuestos para explicar las diferencias encontradas en aritmética simple dependiendo del formato numérico.

El papel del formato de presentación de las operaciones aritméticas

Como acabamos de indicar en el apartado previo, en varias investigaciones se ha demostrado que el uso de la red de hechos aritméticos (recuerdo desde la memoria) y el uso de reglas procedimentales parecen coexistir al resolver operaciones aritméticas simples. En este apartado, nos centraremos en cómo el formato en el que se presentan las operaciones aritméticas simples puede influir en la manera en la que éstas son resueltas.

Investigaciones previas han mostrado que el uso de estrategias procedimentales era mayor cuando las operaciones eran presentadas en un formato poco familiar (e.g., escritas, dos + cuatro) frente a un formato familiar (e.g., en dígitos, 2 + 4) (Campbell y

Alberts, 2009; Campbell y Epp, 2004; Campbell y Fugelsang, 2001; Schunn et al., 1997). Para ejemplificar, en el estudio de Campbell y Fugelsang (2001) se presentaban sumas simples junto con un resultado y los participantes tenían que decidir si éste era correcto o incorrecto. Tras dar una respuesta, se les pedía que indicaran si habían resuelto la operación por medio de estrategias de recuerdo o por estrategias de tipo procedimental. Se manipularon dos formatos, de manera que las operaciones podían ser presentadas en dígitos (e.g., 2 + 4 = 6) o escritas en palabras (e.g., dos + cuatro = seis). Los participantes tardaron un mayor tiempo en responder a las operaciones escritas en palabras frente al formato en dígitos. Además, mientras que los participantes únicamente hicieron uso de estrategias procedimentales en un 25% de las operaciones presentadas en dígitos, el uso de estas estrategias se incrementó de manera significativa en un 41% de los casos con operaciones escritas en palabras. Estos resultados sugirieron que el formato tenía un papel crucial en el uso de estrategias procedimentales frente al recuerdo desde la memoria a la hora de resolver un problema aritmético simple.

Ahora bien, a pesar de que las estrategias procedimentales parecían ser usadas en mayor medida cuando los problemas eran presentados en formato escrito frente a números arábigos, lo cierto es que un porcentaje de estos problemas también era resuelto mediante la recuperación en memoria (un 59% de los problemas en formato verbal, Campbell y Fugelsang, 2001). Así pues, cabe preguntarse si, cuando se utiliza la recuperación en memoria, el formato de presentación del problema tiene un impacto directo en la recuperación de hechos aritméticos o no. Desde varios modelos teóricos se han dado respuestas diferentes a esta pregunta (Campbell y Clark, 1992; Campbell y Epp, 2004; Dehaene, 1992; McCloskey, 1992; McCloskey, Sokol y Goodman, 1986; Noël y Seron, 1992). En el *modelo modular abstracto* de McCloskey (McCloskey's abstract-modular model, McCloskey, 1992) se defendía que los hechos aritméticos están representados de manera abstracta en la memoria. De este modo, cuando un problema aritmético es presentado en un formato determinado (e.g., en dígitos, 2 + 4), en el proceso de codificación éste sería transformado a una forma abstracta, y su resolución se llevaría a cabo de manera independiente a las características periféricas del formato en que se presentó. Este modelo ofrecía explicaciones plausibles al mayor tiempo necesario para resolver un problema en un formato poco familiar (e.g., dos + cuatro) frente a un problema en formato familiar (e.g., 2 + 4) (Blankenberger y Vorberg, 1997): La familiaridad del formato facilitaría el procesamiento del problema en una etapa temprana de análisis, cuando el problema está siendo codificado, siendo más rápida la transformación de éste a su representación abstracta en caso de presentarse en un formato practicado (e.g., números arábigos). Así pues, desde esta perspectiva, el acceso y representación de hechos aritméticos sería independiente del formato del problema. El formato solamente afectaría a los procesos iniciales de codificación.

En resumen, en el modelo modular abstracto (McCloskey, 1992) se asumía que la resolución de las operaciones se efectuaría en una misma forma abstracta, independiente del formato, por lo que no se esperaría encontrar ninguna modulación del formato sobre fenómenos ligados a la representación de los hechos aritméticos en memoria. Sin embargo, diferentes estudios demostraron que el formato sí influía en el procesamiento central de la aritmética (Campbell y Alberts, 2009; Campbell y Clark, 1992; Campbell y Fugelsang, 2001; Jackson y Coney, 2007; McNeil y Warrington, 1994). Por ejemplo, el efecto del tamaño (recordemos que consiste en una peor ejecución en la resolución de problemas de tamaño mayor frente a problemas de menor tamaño ligado a la mejor representación de los últimos en la red) era mayor cuando las operaciones eran presentadas escritas en palabras frente al formato en dígitos (Campbell y Clark, 1988). Estos datos parecían indicar que la resolución de operaciones aritméticas simples sí era dependiente del formato en el que se presentaban, puesto que el efecto del tamaño del problema es índice de la representación de hechos aritméticos y no de etapas de codificación tempranas. En este sentido, los datos empíricos parecían ir a favor del *modelo de codificación compleja* de Campbell (Campbell's encoding-complex model, Campbell y Clark, 1988; Campbell y Clark, 1992). En este modelo, se postuló que tanto la representación de los hechos aritméticos en memoria como la resolución de los mismos eran dependientes del formato. De esta manera, las características periféricas del estímulo, en este caso el formato, influirían en el procesamiento central del mismo. Desde el modelo, se asumió que la resolución de operaciones en dígitos (e.g., 2 + 4) era más automática por la práctica reiterada de operaciones en este formato a lo largo de la vida, lo que facilitaba la recuperación de la respuesta correcta desde la red asociativa (e.g., 6).

En resumen, mientras que los primeros modelos teóricos y la evidencia empírica favorecieron la idea de que el formato no afectaba a la representación de hechos aritméticos, la investigación más reciente sugiere que el formato del problema determina la manera en que se recuperan los hechos aritméticos asociados. En nuestra serie experimental directamente evaluamos el papel del formato de presentación de operaciones aritméticas simples tanto en la coactivación como en la selección de hechos aritméticos (ver Capítulos V y VI).

El desarrollo de la red asociativa

Volviendo al modelo de distribución de asociaciones de Siegler (Siegler y Jenkins, 1989), en éste se postulaba la importancia del uso de estrategias procedimentales en la adquisición de la red asociativa durante los primeros años de educación formal. Así pues, en un primer momento, los niños se valdrían de estrategias procedimentales para resolver operaciones aritméticas simples; por ejemplo, contando desde el primer operando de una suma para dar la respuesta correcta (e.g., 2 + 4 = 2 y 3, 4, 5, 6). Tras la práctica reiterada de este tipo de estrategias, la respuesta a cada problema específico (e.g., 6) se almacenaría en la memoria. A medida que la red aritmética va configurándose, los niños pasarían de un mayor uso de estrategias procedimentales al uso prioritario del recuerdo de la respuesta directamente desde la memoria.

Esta transición desde estrategias de tipo procedimental hasta la recuperación de hechos aritméticos desde la memoria a medida que el niño configura el conocimiento aritmético en la red ha sido avalada empíricamente (Cooney, Swanson y Ladd, 1988; Imbo y Vandierendock, 2007, 2008; Lemaire y Siegler, 1995). Por ejemplo, Imbo y Vandierendock (2008) comprobaron que el uso de la recuperación desde la memoria para resolver operaciones aritméticas simples aumentaba entre niños de diferentes cursos educativos. Mientras que niños en el 2º curso de Educación Primaria usaban la recuperación únicamente en un 60% de las ocasiones; el porcentaje aumentaba significativamente en niños de 4º o 6º curso (80,5% y 79,5% respectivamente). Así pues, la probabilidad de elegir una estrategia de recuperación directa frente a estrategias de carácter procedimental se incrementaría a través de la experiencia educativa.

Por otro lado, en apartados anteriores nos centramos en la evidencia empírica sobre el fenómeno de coactivación de hechos aritméticos en la población adulta (Ashcraft y Battaglia, 1978; Winkelman y Schmidt, 1974). En los estudios revisados previamente se llegaba a la conclusión de que la coactivación de hechos aritméticos relacionados en la red ocurría de manera, al menos parcialmente, automática; sirviendo de apoyo a los modelos que planteaban la recuperación directa como estrategia prioritaria en la edad adulta (Zbrodoff y Logan, 1986). De manera adicional a estos estudios en población adulta, para determinar si el grado de automaticidad variaba dependiendo del conocimiento en aritmética simple, Lemaire et al. (1991) llevaron a cabo un estudio con niños de 9 y 10 años de edad (4º y 5º de Educación Primaria, respectivamente). Los autores encontraron que mientras los niños con 9 años de edad fueron capaces de suprimir el efecto de confusión asociativa cuando el retraso entre el problema y la respuesta fue de 500 ms, en los niños con 10 años de edad el efecto desaparecía con retrasos de 300 ms y 500 ms, comportándose igual que los adultos. Este patrón sugirió que el efecto de confusión asociativa y, por ende, el fenómeno de coactivación de hechos aritméticos, se va desarrollando con la edad a la par de aquellos mecanismos que permiten controlarlo y seleccionar la respuesta correcta en cada caso. Además, en esta investigación se ofrece una visión de la coactivación como un fenómeno que no es completamente automático, sino sobre el cual podemos ejercer un control aun de manera parcial, con el fin de maximizar nuestro rendimiento en la resolución de operaciones aritméticas simples. Como venimos diciendo y en nuestra opinión, este patrón de resultados podría ser explicado por la implementación de un posible mecanismo inhibitorio que se desarrollase durante el curso evolutivo del niño; de manera que a mayor edad, mayor eficacia del mecanismo, permitiendo seleccionar rápidamente la respuesta correcta. Esta hipótesis será evaluada empíricamente en el Capítulo VII del presente trabajo.

En este sentido, existe evidencia empírica que sugiere la implicación de procesos inhibitorios en el rendimiento aritmético de los niños en edad escolar (Adams y Hitch, 1997; Bull et al., 1999; Bull y Scerif, 2001; Dooren y Inglis, 2015; Fürst y Hitch, 2000; Geary et al., 2000; Gilmore et al., 2013; Lubin et al., 2013; McLean y Hitch, 1999; Van der Sluis et al., 2004). Para ejemplificar, en el estudio de Bull et al. (1999) los autores clasificaron a una muestra de niños en dos grupos de alta y baja habilidad matemática según su rendimiento en una prueba aritmética (la cual contenía sumas y restas con operandos de uno o varios dígitos) y les administraron el Wisconsin Card Sorting Test (WCST) como medida de control inhibitorio. En esta tarea, los participantes van recibiendo tarjetas una a una que pueden variar en tres criterios (color, forma o número) y tienen que ir clasificándolas según uno de los criterios. El experimentador no informa del criterio de clasificación, sino que únicamente les indica si han clasificado correctamente cada tarjeta después de cada ensayo (correcto o incorrecto). Una vez que el participante ha clasificado correctamente diez tarjetas dentro de una categoría (e.g., color), se cambia sin previo aviso a otra categoría (e.g., forma). Al cambiar el criterio pueden aparecer errores perseverativos, cuando los participantes siguen intentando clasificar las respuestas según el antiguo criterio, aunque el experimentador indique que es incorrecto. Los errores perseverativos son tomados como evidencia de un fallo en la inhibición de la categoría anterior. Los autores encontraron que los niños con baja habilidad matemática presentaban un mayor número de errores tanto perseverativos como no perseverativos en el WCST en comparación con el grupo de alta habilidad matemática, lo que sugería que estos niños tenían una mayor dificultad para inhibir estrategias aprendidas que interferían con la ejecución de la prueba. Además, las puntuaciones de las medidas perseverativas del WCST correlacionaron con el rendimiento en aritmética, sugiriendo una relación entre los procesos de control inhibitorio y el desempeño en tareas aritméticas.

En otro estudio, siguiendo un procedimiento similar, Bull y Scerif (2001) encontraron resultados análogos en niños de 7 años al introducir una tarea Stroop como medida de capacidad inhibitoria. De este modo, aquellos niños clasificados dentro del grupo de baja habilidad matemática tenían una mayor dificultad para inhibir la dimensión irrelevante en la tarea Stroop. Sin embargo, a pesar de que estos estudios sugieren una relación funcional entre los procesos de control inhibitorio y el desempeño de la aritmética, no permiten determinar el papel concreto de la inhibición en la resolución de operaciones aritméticas simples y se hace necesario investigar esta cuestión de manera más directa. En el Capítulo VII de nuestra serie experimental nos dedicaremos al estudio del desarrollo evolutivo de este posible mecanismo inhibitorio encargado de la selección de hechos aritméticos.

Tras revisar exhaustivamente tanto los modelos teóricos como la evidencia experimental que avala la existencia de una red asociativa de hechos aritméticos, en el último apartado pasamos a describir los objetivos y la estructura de nuestra serie experimental.

OBJETIVOS Y ESTRUCTURA DE LA SERIE EXPERIMENTAL

Nuestra serie experimental se engloba en la aritmética cognitiva simple. Más en concreto, en la representación y recuperación de hechos aritméticos durante la resolución de operaciones aritméticas tan simples como las sumas. Dos fueron los objetivos generales de toda nuestra serie experimental. En primer lugar, queríamos caracterizar el fenómeno de coactivación en la red asociativa (hechos aritméticos asociados a sumas y multiplicaciones) como fenómeno subyacente al efecto de confusión asociativa (Winkelman y Schmidt, 1974; Zbrodoff y Logan, 1986). En segundo lugar, estábamos interesados en determinar el mecanismo de selección de hechos aritméticos utilizado a la hora de resolver sumas simples. Hasta el momento, conocíamos que la presentación de una operación simple (e.g., 2 + 4 =) podía desencadenar la activación de diversas respuestas relacionadas (e.g., 6, 8) en la memoria (Ashcraft y Battaglia, 1978; Winkelman y Schmidt, 1974; Zbrodoff y Logan, 1986); y que a su vez, esta coactivación podría interferir a la hora de seleccionar la respuesta correcta (e.g., 6) (Campbell y Graham, 1985). Sin embargo, la investigación no se había encargado de dar la respuesta a cómo finalmente se resuelve la coactivación y se selecciona la respuesta correcta.

Para abordar nuestros dos objetivos, en toda nuestra serie experimental hicimos uso de un nuevo paradigma, diseñado por nosotros, para evaluar la coactivación y la selección de hechos aritméticos. En nuestro paradigma, los participantes realizaban una tarea de verificación, en la se presentaban sumas simples junto con un resultado y había que indicar la veracidad del mismo. La tarea de verificación estaba compuesta por bloques de dos ensayos consecutivos (ver Figura 4). El primer ensayo nos sirvió para indexar el efecto de confusión asociativa. En este ensayo, los resultados eran incorrectos y podían estar relacionados con la multiplicación de los operandos (e.g., 2 + 4 = 8) o no estar relacionados (e.g., 2 + 4 = 10). Esperábamos, por lo tanto, encontrar un mayor tiempo de reacción en la condición relacionada frente a la condición control. La comparación de estas dos condiciones nos ofreció un índice de la coactivación del resultado de la multiplicación durante la suma. Por otro lado, el segundo ensayo estaba destinado a evaluar el posible mecanismo inhibitorio utilizado por los participantes para seleccionar la respuesta correcta en el ensayo previo. En este ensayo, se presentaron sumas cuya respuesta era correcta y podía coincidir con el resultado de multiplicar los operandos del ensayo previo (e.g., 2 + 6 = 8; precedido de 2 + 4) o no estar relacionado con el ensayo previo (e.g., 4 + 6 = 10, precedido de 2 + 4). Esperábamos un mayor tiempo de reacción en la condición relacionada frente a la condición control. En caso de que los participantes inhibiesen el resultado de la multiplicación en el primer ensayo (8) para seleccionar correctamente el resultado correcto asociado a la suma (6), éstos tardarían un mayor tiempo en recuperar nuevamente el resultado inhibido (8) para contestar correctamente al segundo ensayo (e.g., 2 + 6 = 8) frente a una condición no relacionada.

La serie experimental al completo se estructura en cinco capítulos (Capítulos III, IV, V, VI y VII) que se describen por objetivos concretos de investigación.

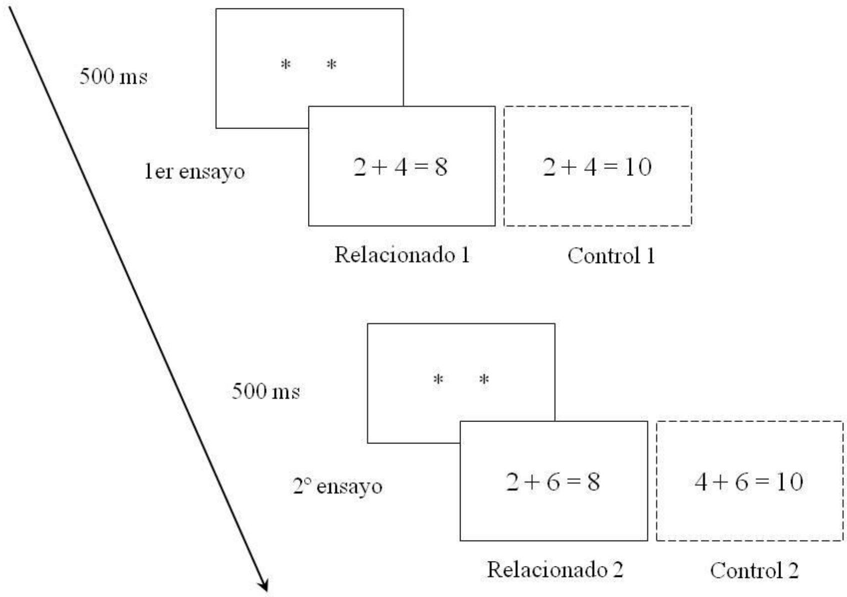

Figura 4. Adaptación del paradigma del priming negativo (Tipper y Driver, 1998) a nuestro estudio. Se presentaron bloques de sumas con dos ensayos consecutivos. En el primer ensayo, el resultado era incorrecto y podía coincidir con el resultado de multiplicar los operandos (relacionado 1) o no (control 1). En el segundo ensayo, el resultado era correcto y podía coincidir con el resultado de multiplicar los operandos del ensayo previo (relacionado 2) o no (control 2).

En el Capítulo III desarrollamos dos experimentos guiados por el objetivo principal de la presente serie experimental. En el Experimento 1, evaluamos la coactivación de hechos aritméticos y el posible carácter inhibitorio del mecanismo de selección, poniendo a prueba para ello, el paradigma experimental diseñado por nosotros. En el Experimento 2, evaluamos si este mecanismo inhibitorio encargado de la selección de hechos aritméticos depende de características contextuales, como la presentación de multiplicaciones simples durante la verificación de sumas.

En el Capítulo IV, a través del registro del electroencefalograma (ERPs), nos centramos en demostrar si el efecto de confusión asociativa se encuentra realmente ligado a la coactivación de hechos aritméticos en memoria. Además, queríamos explorar las consecuencias de la inhibición en la red asociativa para conocer qué ocurre a la hora de recuperar desde la memoria información que ha sido inhibida previamente. Con este objetivo en mente, adaptamos la tarea experimental para el registro de medidas electrofisiológicas y, finalmente, indexar el efecto de coactivación mediante marcadores electrofisiológicos (potenciales evocados).

Por su parte, en el Capítulo V, se desarrollaron dos experimentos encargados de examinar el papel del formato tanto en la coactivación como en el mecanismo inhibitorio implicado en la selección de la respuesta correcta. En ambos experimentos, las operaciones se presentaron en formato numérico (e.g., 2 + 4 = 8) o en formato escrito (e.g., dos + cuatro = ocho). Además, en el Experimento 2, se analizó el papel modulador de las estrategias que utilizaban los participantes (procedimentales vs. recuerdo directo) en la coactivación y selección de hechos aritméticos en los formatos investigados (números arábigos y formato verbal escrito).

El Capítulo VI está dedicado a la evaluación del fenómeno de coactivación y del mecanismo inhibitorio en el formato auditivo. Este capítulo consta de dos experimentos. En el Experimento 1, el objetivo fue determinar si ambos procesos se dan en el formato auditivo, el cual es el formato elegido por defecto para aprender las tablas de multiplicar en la educación formal. En el Experimento 2, determinamos si el patrón de resultados encontrados con el formato auditivo es consecuencia de la secuencia temporal en que aparecen los operandos y el resultado de un problema en esta modalidad de presentación.

Por último, en el Capítulo VII nos centramos en determinar el desarrollo evolutivo de los dos procesos de interés en nuestra serie experimental a través de la educación formal. Para tal fin, evaluamos a alumnos/as desde los 8 hasta los 13 años de edad, circunscritos en tres ciclos educativos bien diferenciados (2º ciclo de Educación Primaria, 3er ciclo de Educación Primaria y 1er ciclo de Educación Secundaria Obligatoria).

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CHAPTER II

INTRODUCTION AND AIMS OF THE THESIS

PREFACE

In the field of arithmetic cognition, it is assumed that arithmetic facts are stored in long-term memory, in an associative network whose nodes are interconnected (Ashcraft, 1982). When a problem is presented (e.g., a simple addition 2 + 4), the nodes that represent the problem (2 and 4) and the answer (6) are activated and the correct result is retrieved from memory directly (Campbell & Graham, 1985). Furthermore, due to the spreading of activation, other related nodes are activated too such as the result of multiplying (8) or subtracting (2) the operands (Ashcraft & Battaglia, 1978; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). This concurrent coactivation of related arithmetic facts might produce interference when individuals solve the problem (Campbell & Graham, 1985), opening the question about the mechanism responsible to finally select the correct answer.

In the present chapter, we review theoretical models proposed to explain the arithmetic network. We report empirical evidence of coactivation associated to additions and multiplications. Afterwards, we talk about the selection mechanisms of arithmetic facts. Then, we discuss factors that determine the use of arithmetic facts. Finally, we describe the aims and the structure of the empirical research carried out in the current doctoral dissertation.

REPRESENTATION AND RETRIEVAL MODELS OF ARITHMETIC FACTS

From the 80s (Ashcraft, 1982) onwards, it has been proposed several theoretical models to explain how arithmetic facts are acquired through practice and how they configure an arithmetic network in long-term memory. In this section, we focus on most relevant theoretical models.

Ashcraft’s network retrieval model

In *Ashcraft’s network retrieval model* (Ashcraft, 1982), it was proposed an arithmetic network composed by nodes that represent the first operand of an operation (2 + N), nodes that represent the second operand (N + 4) and the node that represents the answer associated to the operands (6). Moreover, the associative strength between these nodes would depend on the repeated practice with each problem in formal learning (Ashcraft, 1897). Furthermore, the principle of spreading activation would enable the access to the arithmetic network, activating the node that represents the correct answer (6) and other nodes that represent related results (for example, the one of multiplying the operands, 8). Finally, the node with a higher activation would be selected as the correct result of the problem.

Although this model was relevant to understand how arithmetic facts are represented in the network, it did not contemplate all possible strategies used to shape the arithmetic network through educational experience (e.g., counting one-by-one the operands 2 + 4 = 2, 3, 4, 5, 6). In this model, it was assumed that the retrieval of the correct result from memory was the only way to resolve simple arithmetic problems. However, it has been observed that other procedural strategies have an important role in the acquisition of the network in first years of elementary school. In the next model (Siegler’s distribution of associations model; Siegler & Jenkins, 1989), it was considered the use of diverse strategies along with retrieval from memory.

Siegler’s distribution of associations model

In *distribution of associations model* proposed by Siegler and Jenkins (1989), it was postulated the use of both retrieval and procedural strategies in the acquisition of the arithmetic network. For example, when children begin formal instruction in the resolution of simple additions (2 + 4 =), they use procedural strategies as the counting one-by-one the magnitude of the second operand (4) from the first operand (2) to obtain the result (6) (e.g., 2 + 4 = 2 and 3, 4, 5, 6). According to this model, the connections between the nodes that represent operands and answers would be established by the repeated practice of these procedures (and the retrieval from memory). Furthermore, since the use of procedures is subject to errors in some occasions (e.g., 2 + 4 = 2 and 3, 4, 5, 6, 7); Siegler proposed that these incorrect results would be stored in the network associated to the operands of the problem (e.g., 2 + 4 and the answer 7).

In this model, a mechanism to select the way of solving the problem (retrieval or procedures) is proposed. This mechanism works with two criterions: the distribution of associative strengths operands-answer and the time consumed in memory search (number of searches needed to get the correct answer). When an arithmetic problem is presented, the mechanism would select retrieval from memory when the associative strength between operands-answer is higher and the number of searchers is not too high. When individuals take a long time in the memory search process, the selection mechanism would opt for a procedural strategy. Moreover, it is important to note that practice with arithmetic problems would increase associative strength between operands-answer and this could facilitate the selection of retrieval from memory over procedures. Therefore, individuals with a good knowledge of arithmetic would use retrieval from memory since it would be faster and more efficient than procedural strategies (Ashcraft, 1992).

Siegler's distribution of associations model is a nice approach to explain how individuals establish the network of arithmetic facts. However, within this account it is difficult to explain some effects observed in cognitive arithmetic such as the associative confusion effect (Stazyk, Ashcraft, & Hamann, 1982; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). This effect consists in worse performance when people have to verify an incorrect operation whose result is related with the problem: for example, it has been described longer time to respond to an addition problem presented with a proposed result that is incorrect but is the result of multiplying the operands (e.g., 2 + 4 = 8) compared to an addition problem presented with a proposed result that is incorrect and unrelated (e.g., 2 + 4 = 10). Siegler did not describe how information of different arithmetic operations (additions and multiplications) is interconnected in long-term memory. Therefore, the associative confusion effect could not be explained. In the next section, we describe another model of cognitive arithmetic which accommodates relation effects between arithmetic operations.

Campbell's network interference model

In *Campbell's network interference model* (Campbell, 1987; Campbell & Graham, 1985), it was assumed that the arithmetic network involved spreading activation. Similarly to Ashcraft's network retrieval model (Ashcraft, 1982), nodes were described to represent operands (2 x N, N x 4) and answers (8). Furthermore, in this model, it was included an additional set of nodes to represent the problem as a whole (2 x 4).

One of the unique contributions of Campbell's network interference model was the concept of "interference" in the retrieval process of arithmetic facts. When one arithmetic problem is presented (e.g., the multiplication 2 x 4), both the correct answer (8) and other related answers (e.g., 12, because it is associated with operands, 2 x 6 and 4 x 3) could be activated, which would produce interference in the selection of the correct response. This interference phenomenon in the arithmetic network was supported by several relation effects, as the error priming effect (Campbell, 1987) or the associative confusion effect (Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). For example, in the associative confusion effect, after the presentation of an arithmetic problem (e.g., the addition 2 + 4), the nodes that represent the operands, the problem as a whole and the correct answer (6) would be activated. Moreover, other related nodes, as the result of multiplying the operands (8) would be activated by the principle of spreading activation. This coactivation of facts associated to additions (6) and multiplications (8) would produce interference to select the needed to perform a task. However, in this model, the mechanism used to solve this interference between coactivated arithmetic facts was not discussed. The next model focused on this selection mechanisms.

Whalen´s semantic network retrieval model

In *Whalen's semantic network retrieval model* (Whalen, 2000), the author argued that inhibitory processes have a crucial role in the selection of arithmetic facts. In this model, the retrieval system would represent three type of information: I) the problem to solve, II) the arithmetic fact associated to the problem and, III) the output answers. When an arithmetic problem is presented, in the network there are excitatory connections between the nodes that represent the problem (e.g., 2 + 4), other related nodes (e.g., 2 + 5, 2 x 4) and nodes representing the answer (e.g., 6, 7, 8). Additionally, there are inhibitory connections with unrelated nodes (e.g., 3 + 6). Through these excitatory and inhibitory connections, activation would focus on correct representations at the end of the selection process, so participants would be able to perform the task. There is empirical evidence of an inhibitory mechanism responsible to the selection of arithmetic facts in simple arithmetic (Campbell, Chen & Maslany, 2013; Campbell & Dowd, 2012; Campbell & Thompson, 2012). These studies will be reviewed in the section "Selection of arithmetic facts: The role of inhibition".

To conclude, the models discussed in this section propose that arithmetic facts are represented in an associative network in long-term memory that takes shape through educational experience (Ashcraft, 1982; Siegler & Jenkins, 1989). Moreover, in Campbell's network interference model, it was introduced the concept of interference in the selection of arithmetic facts, and in Whalen's semantic network retrieval model, it was proposed an inhibitory mechanism responsible for resolving interference to select the correct answer to arithmetic problems.

After the review of the main associative models, we will focus on one characteristic of the network of arithmetic facts: the concurrent coactivation of related arithmetic facts.

COACTIVATION OF ARITHMETIC FACTS

In this section, we describe the coactivation phenomenon associated to additions and multiplications. First, we examine the associative confusion effect (Winkelman & Schmidt, 1974) and the automaticity of this effect (Galfano, Rusconi, & Umiltà, 2003; LeFevre, Bisanz, & Mrkonjic, 1988; LeFevre & Kulak, 1994; Lemaire, Fayol, & Abdi, 1991; Rusconi, Galfano, Speriani, & Umiltà, 2004; Zbrodoff & Logan, 1986). Afterwards, we evaluate whether this coactivation underlies the associative confusion effect. To this end, we discuss studies in which neuroimage techniques are used (De Visscher, Berens, Keidel, Noël, & Bird, 2015; Grabner, Ansari, Koschutnig, Reishofer, & Ebner, 2013) and cerebral electrophysiology registered (Domahs et al., 2007; Jost, Hennighausen, & Rösler, 2004; Niedeggen, & Rösler, 1996, 1999; Niedeggen, Rösler, & Jost, 1999).

The associative confusion effect

The coactivation of arithmetic facts has been studied mainly with the *arithmetic verification task* (Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986; Lemaire et al., 1991). In this task, simple problems are presented with a proposed result and participants have to decide if the result is correct o no. This task was used in the study of Winkelman and Schmidt (1974), the first time the associative confusion effect was reported. In this study, the verification task included a small set of simple addition and multiplication problems (3 + 3, 4 + 3, 3 + 5, 4 + 5 y 5 + 5; 3 x 3, 4 x 3, 3 x 5, 4 x 5 y 5 x 5). In order to determine the associative confusion effect, two critical experimental conditions were used in which the proposed result was manipulated, so that it could be: I) a related result (e.g., 3 + 3 = 9, where 9 was the result of multiplying the operands of the addition problem; 3 x 3 = 6, where 6 was the result of adding the operands of the multiplication problem), or II) an unrelated result (e.g., 3 + 3 = 12; 3 x 3 = 7). Regardless of the operation (addition or multiplication problems), participants took more time to respond to related results compared to unrelated results (60 ms). This interference effect was interpreted as a consequence of the coactivation of related information in the network of arithmetic facts and it was named the *associative confusion effect*.

Automaticity in the coactivation

In further research the associative confusion effect was replicated (Findlay, 1978; Zbrodoff, 1979) and its automaticity was determined (Lemaire et al., 1991; Zbrodoff & Logan, 1986). For example, Zbrodoff and Logan (1986) explored in a series of experiments if the associative confusion effect would depend on the intentionality of participants. To illustrate, in Experiments 1 and 2, the authors manipulated the presentation of problems: problems could be presented in a blocked condition (only additions or only multiplications) or in a mixed condition (additions and multiplications randomly presented). The prediction was that in the blocked condition, for example with additions only, the participants would not have the intention of performing multiplications. Thus, no coactivation would be produced and no associative confusion effect observed. In agreement with this hypothesis, the results showed that the associative confusion effect was smaller when problems were presented in the blocked condition (15 ms) compared to the presentation of additions and multiplications in the mixed condition (60 ms). These results supported the partial automaticity of the associative confusion effect. It was only partially automatic because the effect was found in all cases (the blocked and the mixed condition). However, it was subject to some intentional control since the associative confusion effect was modulated by the context in which operations were presented. However, in our opinion, it is possible that this pattern of results had an alternative explanation. For example, it is possible that the presentation of problems in a mixed condition (compared to a blocked condition) facilitated the coactivation between operations, so that both arithmetic facts associated to additions (e.g., 3 + 4 = 7) and multiplications (e.g., 3 x 4 = 12) would receive a higher activation. Therefore, the mixed condition fostered the associative confusion effect compared with the blocked condition without taking into account the intentionally of participants in performing the task.

In other study, Lemaire et al. (1991) found that the associative confusion effect disappeared when a delay (higher than 300 ms) was introduced between presenting the operands and the result. The authors suggested that the associative confusion effect was partially automatic: "automatic" because after presenting the problem, related arithmetic facts were activated without the participant's intention, "partially" because if participants had enough time to retrieve the correct answer, the related result did not interfere with the resolution of the problem. However, it is possible that this pattern of results could be interpreted in other terms: when the operands were presented (e.g., 2 + 4), they produced the activation of both the correct result (e.g., 6) and the result associated to the multiplication (e.g., 8) and these results could compete in the selection processes. This competition could be resolved by the inhibition of the irrelevant information (e.g., 8), which is a time consuming process. When a temporal delay was introduced between the operands and the result, participants would have enough time to perform the selection-by-inhibition process so no interference effect would be observe. This inhibitory hypothesis will be described later.

The studies reviewed so far showed that the coactivation of arithmetic facts was, at least, partially automatic. These studies were performed inside an arithmetic context, where participants needed to retrieve arithmetic information in order to perform the task successfully. However, another way of evaluating the automaticity of the coactivation phenomenon is outside arithmetic contexts, where no arithmetic facts are needed to perform the task (Galfano et al., 2003; García-Orza, Damas-López, Matas, & Rodríguez, 2009; LeFevre et al., 1988; LeFevre & Kulak, 1994; Rusconi et al., 2004). In this regard, there is empirical evidence of the automatic coactivation of arithmetic facts associated to additions and multiplications even when the task does not require arithmetic processing. To illustrate, Rusconi et al. (2004) used a simple numerical comparison task in which participants had to indicate if a single number (e.g., 8) was presented previously in a set of two numbers (e.g., 3 8). These pairs of numbers were presented without arithmetic signs (e.g., 3 8) and participants did not perform any operation with them. There were two experimental conditions of interest: I) the single number coincided with the result of multiplying the pair of numbers (e.g., 24, preceded by 3 8), and II) the single number was unrelated with the previous pair (e.g., 49, preceded by 3 8). The results showed that participants took more time to reject the single number as one of the numbers presented previously when it was the result of multiplying the two numbers of the pair. The authors interpreted this pattern as due to the automatic activation of multiplication facts (e.g., 24), so that it interfered in the decision processes, competing with the activation of the other numbers (e.g., 3 and 8). Furthermore, similar results have been found with arithmetic facts associated to additions (LeFevre et al., 1988). These results supported that, in the arithmetic network, the activation of arithmetic knowledge associated to additions and multiplications is automatic and mandatory, even when no arithmetic operations are needed to perform the task.

It is important to note for our research work that the associative confusion effect (Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986; Lemaire et al., 1991) was considered an index of coactivation in all studied reviewed above. However, this assumption needs to be corroborated empirically. In the following section, we review several studies which support that coactivation of arithmetic facts underlies the associative confusion effect.

Does coactivation underlie the associative confusion?

Recent studies which use functional Magnetic Resonance Imaging (fMRI), have explored cerebral correlates of relation effects in simple arithmetic (De Visscher et al., 2015; Grabner et al., 2013). For example, Grabner et al. (2013) described brain areas involved in the associative confusion effect. The authors found that when participants had to reject related problems (e.g., 3 + 4 = 12) compared to unrelated problems (e.g., 3 + 4 = 7), there was higher activation from the left angular gyrus to the supramarginal gyrus, the superior parietal cortex and the left dorsolateral prefrontal cortex. Importantly, the left angular gyrus has been related with the retrieval of arithmetic facts

(Delazer, Domahs, Bartha, Brenneis, Lochy, Trieb, & Benke, 2003; Grabner, Ansari, Koschutnig, Reishofer, Ebner, & Neuper, 2009). Thus, the coactivation of arithmetic facts seems to underlie the associative confusion effect.

Moreover, in electrophysiological studies (Domahs et al., 2007; Guthormsen, Fisher, Bassok, Osterhout, DeWolf & Holyoak, 2015; Jost et al., 2004; Niedeggen & Rösler, 1996, 1999; Niedeggen et al., 1999), it has been evaluated the coactivation of arithmetic facts associated to multiplications. To illustrate, in the study of Niedeggen and Rösler (1999), participants had to verify simple multiplication problems that could be presented with a correct result (e.g., 5 x 8 = 40), an incorrect related result (e.g., 5 x $\underline{8}$ = $\underline{32}$, so that 4 x $\underline{8}$ = $\underline{32}$) or an incorrect unrelated result (e.g., 5 x 8 = 34). Similar to what is found with the associative confusion effect, participants took more time to reject incorrect multiplication problems whose results were related compared to unrelated multiplication problems. This result was interpreted as due to the coactivation of several related results associated to multiplication in the arithmetic network. Moreover, there was a modulation of the N400 component, a negative wave peaking at 350-450 ms after stimulus presentation, which has been considered an index of semantic processing (Kutas & Hillyard, 1980, 1984). Specifically, there was an attenuation of the N400 component in the related condition, suggesting that the activation of the related result could facilitate the activation of the correct result due to the spreading of activation in the network of multiplication facts. This coactivation would produce a subsequent interference between related answers in a late selection process which was captured in the latency analysis. Furthermore, it has been also found N400 modulations during the retrieval of additions (Avancini, Soltész y Szűcs, 2015). Therefore, these studies suggest that the attenuation of the N400 component is an index of the accessing to interrelated arithmetic facts. However, these studies evaluated the coactivation of arithmetic facts within a category of problems (e.g., only multiplications). Therefore, it needs to be explored if N400 component is also sensitive to the coactivation of arithmetic facts between operations (e.g., additions and multiplications). This question will be explored in Chapter IV of the experimental series.

SELECTION OF ARITHMETIC FACTS: THE ROLE OF INHIBITION

As we have shown in previous sections, coactivation of several arithmetic facts can produce interference when individuals solve simple arithmetic problems. However, most of the time people resolve them successfully. Therefore, it is important to ask about the mechanism responsible for the selection of correct answers when people have to resolve simple arithmetic problems.

In Whalen's semantic network retrieval model (Whalen, 2000), it was proposed that the selection of simple arithmetic problems was carried out by a game of excitatory and inhibitory connections between several nodes of the network of arithmetic facts. Furthermore, over the years, results from different studies have suggested that inhibitory process could be involved in the selection of arithmetic facts (Lemaire et al., 1991; LeFevre et al., 1988; LeFevre & Kulak, 1994). Firstly, children show a relationship between inhibitory control and performance in mathematics (Adams & Hitch, 1997; Bull, Johnston, & Roy, 1999; Bull & Scerif, 2001; Dooren & Inglis, 2015; Fürst & Hitch, 2000; Geary, Hamson, & Hoard, 2000; Gilmore et al., 2013; Lubin, Vidal, Lanoë, Houdé, & Borst, 2013; McLean & Hitch, 1999; Van der Sluis, De Jong, & Van der Leij, 2004). This empirical evidence will review in the section "The development of the arithmetic network". Moreover, empirical evidence suggests the involvement of self-inhibition processes in the performance of arithmetic sequential tasks (Arbuthnott & Campbell, 2000; 2003; Campbell & Arbuthnott, 1996). In this regard, self-inhibition processes are responsible for suppressing one stimulus in order to process the next one and finally perform a numerical task fluently. However, these studies do not specify the role of inhibitory control in the selection of arithmetic facts from the associative network.

Recently, one series of studies performed by Campbell et al. (Campbell, Chen, & Maslany, 2013; Campbell & Dowd, 2012; Campbell & Thompson, 2012) examined directly the role of inhibition in the selection of arithmetic facts. To this end, the authors used an adapted version of the retrieval-practice paradigm (a paradigm typically used to demonstrate inhibition of irrelevant information in memory, Anderson, Bjork, & Bjork, 1994). In these studies, participants performed a training phase in which simple multiplication problems were presented (e.g., 2 x 3 = ?) and they had to say aloud the correct answer to each one. Afterwards, the same operands were used in a second test phase with simple addition problems (e.g., 2 + 3 = ?). The main result was that training the multiplication problems slowed the response times to the addition counterpart compared to additions whose operands were not presented in the training phase. This interference effect, named Retrieval-Induced Forgetting (RIF), was interpreted in terms of inhibition: The retrieval of arithmetic facts associated to multiplications in the training phase would produce the inhibition of related additions that competed for selection in the network.

Campbell et al. (Campbell et al., 2013; Campbell & Dowd, 2012; Campbell & Thompson, 2012) did not specify what was inhibited exactly in the selection process. In our opinion, inhibition could take place on answers, so that; after the presentation of the problem (e.g., 2 x 3 =), both the correct answer (6) and other related answers (e.g., the one of adding the operands, 5) would be activated. This competition between answers could be resolved by inhibiting the incorrect answer associated to the addition (5). Thus, when the inhibited answer was relevant in the test phase, an additional time was needed to retrieve it from memory again.

In all, the studies reviewed in this section suggested the involvement of an inhibitory mechanism in the resolution of simple arithmetic facts. Furthermore, it seems that this inhibitory mechanism has long-term consequences. Concretely, in the retrieval-practice paradigm, it is assumed that inhibition occurs in the training phase and it influences a posterior test phase in which the consequences of applying inhibition are evaluated. In our experimental series, we evaluate if this inhibitory mechanism acts in a continuous manner when people solve simple arithmetic facts.

USE OF ARITHMETIC FACTS: MODULATING FACTORS

In previous sections, we have talk about two crucial aspects of the arithmetic network: the coactivation of related arithmetic information and the selection of the correct answer needed to resolve a problem. However, it is important to note that during the resolution of simple arithmetic problems, people can use both retrieval from memory and procedural strategies (Siegler & Jenkins, 1989). In this section, we focus on one model of procedural mechanisms (Baroody, 1983) and the empirical evidence that supports it (Barrouillet & Thevenot, 2013; Della Puppa et al., 2015; LeFevre, Sadesky, & Bisanz, 1996; Roussel, Fayol, & Barrouillet, 2002; Thevenot, Barrouillet, Castel, & Uittenhove, 2016; Thevenot, Castel, Danjon, & Fayol, 2015; Thevenot, Fanget, & Fayol, 2007). Afterwards, we explore several variables that can determine the use of simple arithmetic facts, such as the numerical format in which arithmetic problems are presented (Campbell & Epp, 2004) and the development of the network in childhood (Siegler & Jenkins, 1989).

Procedural mechanisms

In Siegler's distribution of associations model (Siegler & Jenkins, 1989), it was considered the use of procedural strategies to resolve arithmetic problems. Furthermore, empirical research showed the use of these strategies in adult population (Geary & Wiley, 1991; Healy, Rickard, & Bourne, 1993; Núñez-Peña, Colomé & Tubau, 2015). For example, in the study of Geary and Wiley, young adults reported that they resolved 10% of simple problems by procedural strategies.

In associative models, it was assumed that adults do not use procedural strategies to resolve arithmetic problems since they are less efficient than retrieval from memory processes (Ashcraft, 1992). However, other authors have highlighted the relevance of procedural strategies in mental arithmetic. Baroody (1983) defended that practice with procedural mechanisms through educational experience would let procedures to be used automatically, quickly and without awareness. Also, Baroody argued that the use of procedures would consume less cognitive resources since they could be applied to a large set of arithmetic problems without representing all possible problems in memory as it was proposed in associative models of mental arithmetic.

In *Baroody's schema-based model* (Baroody, 1983; 1994), it was assumed the representation of procedural strategies stored as schemas in long-term memory. When a simple problem is encountered, a first scheme would be activated to identify the arithmetic operation (e.g., multiplication). Afterwards, different schemas could be activated depending on the problem. For example, if the problem follows the structure *N x 0*, the rule *N x 0* would be executed ("*If* 0 and N are multiplied, *then* the product is equal to N"). Furthermore, this model also included the use of arithmetic facts; for example, in the case of tie problems the N_1 x N_1 rule would be used ("*If* a number is multiplied by itself, *then* access to the network of arithmetic facts").

Empirical research (Barrouillet & Thevenot, 2013; LeFevre et al., 1996; Roussel et al., 2002; Thevenot et al., 2007, 2015, 2016) supports Baroody's schema-based model (Baroody, 1983; 1994). To illustrate, Thevenot et al. (2007) used an operand recognition paradigm to evaluate the use of procedural strategies in the resolution of addition problems with different sizes: small (the result was < 10), medium (the result was > 10) or large (two-digit operands). The authors wanted to demonstrate that addition problems were resolved by procedural mechanisms, as the transformation of their operands (e.g., 5 + 7 = $\underline{5 + 5 + 2}$ = 12). To this end, after the resolution of each problem, the operands were presented again (5 and 7) and participants had to indicate if these numbers appeared previously. This task was compared to a number comparison task in which participants decided if a number was between the two operands (e.g., *is the number 6 between 5 and 7?*). If participants used procedures to perform the arithmetic facts, recognition would be impaired compared to the comparison task due to the additional computation done with these numbers in the arithmetic task. The results showed worse recognition of operands after the resolution of medium and large addition problems; however, the recognition was not disrupted in the case of small addition problems (e.g., 3 + 5 = 8), suggesting that they were performed by retrieval from memory. Therefore, this study leaves open the possibility of using procedural strategies to resolve simple arithmetic problems, at least those with medium and large size.

It is important to note that empirical evidence about the resolution of arithmetic operations supports the use of both retrieval from memory (Geary and Wiley, 1991) and procedural strategies (Barrouillet & Thevenot, 2013; LeFevre et al., 1996; Núñez-Peña et al., 2015; Roussel et al., 2002). Therefore, it would be interesting to examine factors that might determine the use of retrieval over procedures in mental arithmetic. Familiarity of numerical format in which operations are presented (e.g., 2 + 4 vs. two + four) might be one of these factors (Schunn, Reder, Nhouyvanisvong, Richards, & Stroffolino, 1997). In the next section, we describe studies and theoretical models proposed to explain the role of numerical format in cognitive arithmetic.

The role of numerical format

Previous research have been shown that the use of procedural strategies is higher when operations are presented in an unfamiliar format (e.g., written number words, two + four) compared to a familiar format (e.g., Arabic digits, 2 + 4) (Campbell & Alberts, 2009; Campbell & Epp, 2004; Campbell & Fugelsang, 2001; Schunn et al., 1997). To illustrate, in the study of Campbell and Fugelsang (2001), simple addition problems were presented and participants decided whether the proposed result was correct or not. Afterwards, participants reported the strategy used to solve the problem. The format of addition problems was manipulated, so that operations were presented with Arabic digits (e.g., 2 + 4 = 6) or written number words (e.g., two + four = six). Participants took more time to respond to verbal operations compared to operations with Arabic digits. Furthermore, procedural strategies were used to a less extend (25% of total) when operations were presented in the digit format relative to verbal arithmetic problems (41% of total). This result suggested that numerical format determined the strategy used to solve simple arithmetic problems.

Even when procedural strategies are used with verbal problems, it is true that retrieval from memory is the preferred way of resolving these operations (e.g., 59% of total with verbal problems in Campbell and Fugelsang, 2001). Therefore, it is important to evaluate the role of numerical format in occasions where retrieval from memory is used to resolve arithmetic problems. Different theoretical models have proposed alternative answers to this question (Campbell & Clark, 1992; Campbell & Epp, 2004; Dehaene, 1992; McCloskey, 1992; McCloskey, Sokol & Goodman, 1986; Noël & Seron, 1992). In *McCloskey's abstract-modular model* (McCloskey, 1992), it was argued that arithmetic facts are abstract entities whose representation do not depend on the format in which the arithmetic problem is presented. Therefore, numerical format effects would be located at the encoding stage of processing.

However, different studies show that numerical format influences central processing of arithmetic problems (Campbell & Alberts, 2009; Campbell & Clark, 1992; Campbell & Fugelsang, 2001; Jackson & Coney, 2007; McNeil & Warrington, 1994). For example, the size problem effect (worse performance with large problems compared to small problems) is larger when operations are presented with words than with Arabic digits (Campbell & Clark, 1988). This pattern suggests that the resolution of simple arithmetic problems depends on numerical format. Moreover, this sort of evidence supports *Campbell's encoding-complex model* (Campbell & Clark, 1988; Campbell & Clark, 1992). In this model, it is proposed that representation and resolution of arithmetic problems depend on numerical format. Concretely, it is assumed that resolution of problems presented with Arabic digits is more automatic, which facilitates retrieval of correct answers in an efficient way. In our experimental series, we evaluate the role of numerical format in the coactivation and selection of simple arithmetic facts (see Chapters V and VI).

The development of the arithmetic network

In childhood there is a progressive change from procedural strategies to retrieval from memory as the way of resolving simple arithmetic problems (Cooney, Swanson, & Ladd, 1988; Imbo & Vandierendock, 2007, 2008; Lemaire & Siegler, 1995). For example, Imbo and Vandierendock (2008) demonstrated that the use of retrieval from memory increased with educational instruction: children in 2nd grade of elementary school used retrieval in 60% of problems, whereas those children in 4th and 6th grade used retrieval to resolve most of the problems (80.5% and 79.5% respectively).

We have reviewed in previous section the coactivation of arithmetic facts in adults (Ashcraft & Battaglia, 1978; Winkelman & Schmidt, 1974), and it has been demonstrated that the coactivation is partially automatic (Zbrodoff & Logan, 1986). To determine if automaticity depends on knowledge about simple arithmetic, Lemaire et al. (1991) carried out a study with 9-10 years-old children. The authors found that 9 years-old children did not show the associative confusion effect when a delay of 500 ms was introduced between the operands and the answer; however, in 10 years-old children, the associative confusion effect disappeared with delays of 300 ms and 500 ms, showing the pattern found in adults. This observation suggests that the associative confusion effect and, therefore, the coactivation of several arithmetic facts, develop with age. Furthermore, these results can be explained with the inclusion of inhibition as the mechanism responsible to select arithmetic facts. It is possible that this mechanism develops with education in mathematics. This hypothesis will be evaluated in Chapter VII.

Furthermore, empirical evidence supports the relationship between inhibitory control and arithmetical performance in children (Adams & Hitch, 1997; Bull et al., 1999; Bull & Scerif, 2001; Dooren & Inglis, 2015; Fürst & Hitch, 2000; Geary et al., 2000; Gilmore et al., 2013; Lubin et al., 2013; McLean & Hitch, 1999; Van der Sluis et al., 2004). For example, in the study of Bull et al. (1999), the authors classified children in two groups (low and high arithmetic ability) and the Wisconsin Card Sorting Test (WCST) was administrated to index inhibitory control. Children with low arithmetic ability presented higher number of non-perseverative and perseverative errors compared to children with high arithmetic ability. Furthermore, perseverative measures correlated with performance in arithmetic, suggesting the relationship between inhibitory control and arithmetical performance. In Chapter VII of our experimental series, we evaluate the development of a possible inhibitory mechanism responsible to select simple arithmetic facts.

After the review of both theoretical models and experimental evidence about the network of arithmetic facts we continue with the description of the aims and structure of our research work.

AIMS AND STRUCTURE OF EXPERIMENTAL SERIES

Our experimental series is framed within arithmetic cognition. Specifically, this dissertation focused on coactivation and selection of simple arithmetic facts. The main goal of our research work was two-fold. Firstly, we aimed at determining coactivation across arithmetic facts (additions and multiplications) as the phenomenon underlying the associative confusion effect (Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). Secondly, we explored the mechanism used to select arithmetic facts during the resolution of simple addition problems.

In order to address our goal, we designed a new experimental paradigm to evaluate the coactivation and selection of arithmetic facts. In this paradigm, participants performed a verification task in which simple addition problems were presented and they decided whether the proposed result was correct or not. The task comprised two consecutive trials. In the first trial, we evaluated the associative confusion effect. In this trial, the proposed result of an addition problem was incorrect and it could be related with the multiplication (e.g., 2 + 4 = 8) or not (e.g., 2 + 4 = 10). We expected slower reaction time in the related condition compared to the control condition. Therefore, the comparison between these two conditions was an index of the coactivation of arithmetic facts associated to multiplications and additions. Furthermore, in the second trial, we evaluated the possible inhibitory mechanism used by participants to select the correct answer in the previous trial. Here, addition problems were presented with a correct result, and this result coincided with that of multiplying the operands of the previous trial (e.g., 2 + 6 = <u>8</u>; preceded by 2 + 4) or it was unrelated (e.g., 4 + 6 = 10, preceded by 2 + 4). We expected slower reaction time in the related condition compared to the control condition. If participants inhibited in the first trial the result of multiplying the operands of the problem (8) to select the correct answer (6); they would take more time to retrieve the inhibited result (8) to respond to the second trial (e.g., 2 + 6 = 8).

Our experimental series was structured in five chapters (Chapters III, IV, V, VI and VII). In all chapters we embraced the paradigm described above to evaluate several research questions about the coactivation and selection of arithmetic facts.

In Chapter III, we developed two experiments to evaluate our main goal. In Experiment 1, we examined the coactivation and the possible inhibitory mechanism responsible to select arithmetic facts. In Experiment 2, we evaluated if this inhibitory mechanism depended on contextual factors, as the presentation of multiplication problems in the task.

In Chapter IV, we gathered electrophysiological indexes to evaluate whether the associative confusion effect really involved coactivation of arithmetic facts. Furthermore, we wanted to explore the consequence of inhibiting arithmetic knowledge in the network of arithmetic facts.

In Chapter V, we performed two experiments in order to determine the role of numerical format in the coactivation of arithmetic facts and selection-by-inhibition. In both experiments, problems were presented with Arabic digits (e.g., 2 + 4 = 8) or written number words (e.g., two + four = eight). Furthermore, in Experiment 2, we evaluated the strategies used by participants (retrieval or procedural strategies) to resolve the problem as a modulating factor.

In Chapter VI, we evaluated coactivation and inhibition when problems were presented in auditory format. Here, we performed two experiments too. In Experiment 1, the goal was to determine if both processes (coactivation and selection) would occur in oral calculation. In Experiment 2, we examined if the pattern of results found with this format was due to the way in which individuals receive oral problems.

Finally, in Chapter VII, we explored the development of coactivation and selection-by-inhibition through formal instruction in arithmetic knowledge. To this end, we evaluated 8 to 13 years-old children (from 2^{nd} cycle of elementary school to 1^{er} cycle of high school).

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CHAPTER III

SIMPLE ARITHMETIC: EVIDENCE OF AN INHIBITORY MECHANISM TO SELECT ARITHEMTIC FACTS[1]

In two experiments we evaluated the coactivation of arithmetic facts and the possible inhibitory mechanism used to select the correct one. To this end, we introduced an adapted version of the negative priming paradigm in which participants received additions and they decided whether they were correct or not. When the addition was incorrect but the result was that of multiplying the operands (e.g., 2 + 4 = 8) participants took more time to respond relative to control additions with unrelated results. This finding corroborated that participants coactivated arithmetic facts of multiplications even when they were irrelevant to perform the task. Moreover, the participants were slower to respond to an addition whose result was that of multiplying the operands of the previous trial (e.g., 2 + 6 = 8). These results support the existence of an inhibitory mechanism involved in the selection of arithmetic facts.

1 This paper was published in *Psychological Research* and it was co-authored by Pedro Macizo and Amparo Herrera.

SIMPLE ARITHMETIC: EVIDENCE OF AN INHIBITORY MECHANISM TO SELECT ARITHEMTIC FACTS

There is a wide consensus about how we represent and solve arithmetic problems. Various theoretical models state that arithmetic facts are stored in long-term memory in an associative network with interrelated nodes, whose associative strength changes depending on learning and educational experience (Ashcraft, 1992). In addition, it is assumed that the solutions of simple arithmetic facts are automatically retrieved when an arithmetic problem is presented (although see Barrouillet &Thevenot, 2013; Fayol & Thevenot, 2012, for a suggestion that simple additions are solved through procedures). For example, the Network-Interference Model proposed by Campbell (Campbell & Graham, 1985) assumes that when an arithmetic fact is presented (i.e., an addition, 2 + 4), the nodes that represent the operands of the problem are activated (2 and 4), along with that representing the solution (6). Furthermore, according to the principle of spreading activation, other nodes of the network would be activated, such as the result of multiplying the two operands (i.e., 8), even when the arithmetic problem was an addition. This automatic activation of several arithmetic facts would produce interference, since only one is required to solve the problem. Thus, when individuals are solving additions, the coactivation of the arithmetic facts associated to the multiplication would produce interference during the selection of the correct addition result.

There is empirical evidence to support the concurrent coactivation of arithmetic facts associated to additions and multiplications (Winkelman & Schmidt, 1974). One procedure frequently used to corroborate this coactivation is the verification of additions (Zbrodoff & Logan, 1986). In this task, a simple addition is presented (i.e., a pair of one-digit operands and a result) and participants have to decide whether the result is the correct solution of the addition problem. The most interesting trials are those associated to negative responses (incorrect addition problems). In these occasions, participants take more time to respond when the result presented with the addition is incorrect but is the one of multiplying the operands (2 + 4 = 8) relative to a condition in which the result is unrelated (2 + 4 = 10). This longer reaction time when the incorrect addition result is the one of multiplying the operands has been taken as an index of the simultaneous activation of addition and multiplication arithmetic facts. Furthermore, it is worth mentioning that it is assumed that coactivation is automatically produced after presenting the operands in any case. For example, when the problems 2 + 4 = 8 or 2 + 4 = 10 are presented (incorrect additions), the result associated with multiplying the operands (8) would be activated. However, since the incorrect result (8) is visually presented in the first case (2 + 4 = 8), it would receive more activation and thus, it would compete more strongly with the correct solution of the addition problem (6), relative to the addition in which the incorrect result associated to the multiplication of the operands (8) is not visually presented (2 + 4 = 10).

Despite the fact that participants perform the verification of additions more poorly when the result is the one of multiplying the operands (2 + 4 = 8), (Zbrodoff & Logan, 1986); most of the time they are able to perform the task successfully. Therefore, the question to answer is about the mechanism used to select the correct arithmetic fact to solve the problem efficiently. We suggest that this mechanism might be similar to that proposed in other cognitive fields to select correct representations among several competing alternatives. For instance, in the field of bilingualism, it has been observed that bilinguals coactivate words in their two languages even when they need only one language to communicate (Macizo, Bajo, & Martín, 2010). The bilinguals seem to select representations in the correct language by inhibiting those of the irrelevant language (e.g., Green, 1998; Macizo et al., 2010; see Kroll, Bobb, & Wodniecka, 2006, for a review). For example, Macizo et al. observed that Spanish-English bilinguals were slower to process the word *foot* in English when they received previously the interlexical homographs *pie* (meaning foot in Spanish). The authors interpreted that the bilinguals activated the irrelevant Spanish meaning of *pie* (foot in Spanish) and later it was inhibited. The bilinguals took additional time to reactivate the inhibited word when it was presented later. Therefore, the authors proposed an inhibitory mechanism responsible to select correct representations by suppressing the activation of competing candidates.

In the field of mathematical cognition there are some theoretical models that describe the way in which arithmetic facts are selected with the involvement of inhibitory processes such as the Network Retrieval Theory (Whalen, 2000). This model identifies three types of nodes: (a) nodes to represent the operands of an arithmetic problem, (b) nodes to specify the type of problem (addition, multiplication), and (c) nodes associated to the results of the problems. When an arithmetic problem is presented (2 + 4), the network would spread activation to the operands, the possible arithmetic problems associated to the operands (i.e., addition 2 + 4, multiplication 2 x 4), and the solutions (6 and 8 for the addition and multiplication problems, respectively). Moreover, the network also includes inhibitory connections to reduce the activation of unrelated nodes so finally, the operands, the type of arithmetic problem and the correct solution are selected (2 + 4 = 6). Hence, Whalen (2000) describes an architecture in which several arithmetic facts are automatically activated and inhibitory processes regulate the activation of the network to reach the correct solution.

The available empirical evidence about the existence of an inhibitory mechanism underlying the selection of correct arithmetic facts is contradictory. Censabella and Noël (2004) asked whether simple mental arithmetic involved the suppression of irrelevant arithmetic facts. To this end, the authors evaluated the relationship between the production of additions and multiplications and three tasks aimed to index exogenous suppression, endogenous suppression and activation-based interference. These tasks were, respectively: (a) a Stroop tasks in which participants named the ink color of words that might denote a different color (i.e., incompatible trials such as the word *green* colored in red). The authors assumed that participants solved the interference in incompatible Stroop trials by the inhibition of the irrelevant dimension (i.e., reading of the word), after perceiving the exogenous stimulus; (b) a task in which participants first read sentences with high/low predictable final words and later they recognized whether a target word was presented in the sentence or not. The authors predicted that high predictable final words that were not presented would be more difficult to reject in the recognition test because they would be activated and subsequently they had to be suppressed; (c) finally the authors used a task based on the *fan effect* (Anderson, 1974) to evaluate the activation-based interference. The participants learned sentences containing a person (i.e., 'the guard') and a location (i.e., 'is in the inn') while the number of locations was manipulated to create different fan sizes. The authors assumed that the recall of sentences would be higher with small-fan contexts relative to large-fan contexts due to the reduction of the target context activation with the increase of the number of competing contexts. Censabella and Noël observed that performance on the simple arithmetic tasks was related with the task based on the fan effect so participants with more errors when solving arithmetic problems showed a large fan effect. However, no associations were obtained between the arithmetic tasks and (a) the Stroop task or (b) the sentence reading and word recognition task. The authors interpreted these results as evidence that the solving of arithmetic problems did not involve inhibitory processes. Instead, they proposed that the interference associated to the coactivation of arithmetic facts was due to the reduced level of activation of the correct result because of the increased activation of associated responses.

However, in our opinion, the study reported by Censabella and Noël (2004) is open to other interpretations. For example, some authors have suggested that the fan effect involves the inhibition of irrelevant contexts rather than the decrease of activation of the target context (i.e., Radvansky, Zacks, & Hasher, 1996). Thus, the relationship between the arithmetic task and the fan effect obtained in the Censabella and Noel's study might be reflecting common inhibitory processes underlying both tasks.

Recently, other empirical studies have given support to the existence of inhibitory processes in the selection of arithmetic facts. In several studies, Campbell and colleagues (Campbell & Dowd, 2012; Campbell & Thompson, 2012) have used an adaptation of the retrieval practice (RP) paradigm typically employed to demonstrate the active inhibition of irrelevant information (Anderson, 2003; Anderson, Bjork & Bjork, 1994). In these studies, participants perform a practice phase of simple multiplication problems (i.e., 2 x 3 = ?; 4 x 6 = ?) and, afterward, the same operands are used in a second test phase with simple addition problems (i.e., 2 + 3 = ?; 4 + 6 = ?). The overall finding is that practicing the multiplication problems slows the response times to the corresponding addition problems relative to additions problems whose operands were not presented in the practice phase with multiplication problems. This retrieval induced forgetting (RIF) effect is interpreted in terms of inhibitory processes. When participants solved the multiplication problems in the practice phase, the addition competitors need to be inhibited. Therefore, participants would take more time to solve addition problems in the test phase because of the previous inhibition of these problems. Several studies with the RP paradigm have corroborated and delimited some characteristics of RIF effects in the resolution of simple arithmetic (i.e., stronger for small size problems, etc.); thus corroborating that an inhibitory mechanism is involved in the resolution of some arithmetic problems (Campbell & Dowd, 2012; Campbell & Thompson, 2012).

The studies showing RIF effects in simple arithmetic with the RP paradigm nicely demonstrate inhibitory effects in mental arithmetic. More specifically, these studies show long-term inhibition since the result of inhibiting competing arithmetic problems in the practice phase is evaluated in a test phase which is performed several minutes later. The goal of the current study is to explore the existence of a continuous inhibitory mechanism which modulates the selection of arithmetic facts trial-by-trial. To this end, we made use of a paradigm employed in the field of bilingualism to explore inhibitory processes associated to the selection of a language, a version of the negative priming paradigm (Macizo et al., 2010; Tipper & Driver, 1988).

Previous studies on arithmetical cognition have used also the negative priming paradigm to nicely demonstrate the involvement of inhibitory processes when participants solve additions (Arbuthnott & Campbell, 2000). The authors asked participants to solve probe additions while ignoring distractor additions. The relevant result for the present study was the observation of negative priming so the responses were slower to a probe addition when it was the distractor addition in the preceding trial. Hence, negative priming paradigm seems to be sensitive to index inhibition during arithmetic resolution.

It is important to note that in the Arbuthnott and Campbell's (2000) study and other relevant studies on cognitive arithmetic, the existence of inhibitory processes in simple arithmetic has been evaluated within categories of arithmetic facts such as sets of additions (Arbuthnott & Campbell, 2000, 2003) and sets of multiplications (Campbell & Arbuthnott, 1996) . For instance, Arbuthnott and Campbell showed that participants took longer to respond to an addition (e.g., 3 + 5) whose result was one of the operands contained in a prime addition (4 + 8), suggesting that participants inhibited the previously encoded unit.

The current study was aimed to evaluate the existence of a continuous inhibitory mechanism acting trial-by-trial to select addition facts among coactivated multiplication facts. To this end, a verification task was used in which arithmetic problems were presented and participants decided whether the result was correct or incorrect (see Table 1). The task structure comprised blocks of two trials. In the first trial, the result of the addition problem could be the one of multiplying the operands (i.e., 2 + 4 = 8) or not (i.e., 2 + 4 = 10). If participants coactivate multiplication facts, they would take longer to respond to 2 + 4 = 8 relative to 2 + 4 = 10. In the second trial, the result of multiplying the operands in Trial 1 was presented again (2 + 6 = 8) or not (4 + 6 = 10). If participants inhibited the result of multiplying the operands of the first trial, they would take longer to respond to 2 + 6 = 8 relative to 4 + 6 = 10, since they would needed additional time to reactivate this solution when it became relevant in Trial 2. This pattern of results would indicate that when participants verify addition problems, they coactivate multiplication facts and they use an inhibitory mechanism to select the correct addition solution.

Table 1. Examples of blocks of two trials used in the study.

	Example 1
First trial	2 + 4 = 8 (Related 1 condition)
Second trial	2 + 6 = 8 (Related 2 condition)
	Example2
First trial	2 + 4 = 10 (Unrelated 1 condition)
Second trial	4 + 6 = 10 (Unrelated 2 condition)

Note. Problems in the first trial were always incorrect additions. Problems in the second trial were always correct additions.

EXPERIMENT 1

Method

Participants. Forty-eight students from the University of Granada (36 women and 12 men) took part in the study. Their mean age was 23 years (*SD* = 4.23). Five were left-handed and 43 were right-handed. The participants gave informed consent and they were remunerated by academic credits. Before the experimental task, the participants completed a questionnaire to determine their use of simple arithmetic (Colomé, Bafalluy, & Noël, 2011). Most participants learned the multiplication tables orally (77.81%). Furthermore, 64.58% of participants made simple calculations on a daily basis (see Table 2).

Table 2. Use of simple arithmetic of participants.

	Experiment 1	Experiment 2
Calculation frequency		
Daily	64.58%	48.48%
Weekly	31.25%	39.39%
Monthly	4.17%	12.12%
Type of calculation		
Divisions	12.77%	19.73%
Multiplications	20.10%	20.18%
Additions	44.27%	38.00%
Subtractions	22.85%	22.09%
Calculation strategies		
Saying numbers mentally or aloud	35.67%	34.47%
Visualising Arabic numbers mentally	30.28%	36.33%
Writing numbers with pencil and paper	12.38%	11.44%
With a calculator	19.93%	17.35%
Other strategies	1.60%	0.38%
Learning method (multiplication tables)		
Repeating it orally	77.81%	77.27%

Exercises with Arabic numbers	18.65%	19.39%
Others methods	3.54%	3.33%

In addition, to evaluate the participants' knowledge about multiplications tables, after the main experiment they performed a multiplication task in which the operands used in the main experiment were presented (i.e., 2 x 4 = ?) and participants had to say aloud the correct result (i.e., 8). The mean correct responses in the multiplication task was 88.04%, and there were no significant differences between males and females, $t(46) = 1.55$, $p = .13$.

Design and Materials. We used a verification task in which participants received one-digit additions and they decided whether they were correct or incorrect. The additions were presented in blocks of two trials. In the critical blocks, the first trial was composed of incorrect additions whereas in the second trial the additions were correct.

Two conditions were used in the first trial. In the related 1 condition, the result of the addition problem was not the result of adding the operands but the one of multiplying them. This condition was compared with an unrelated 1 condition in which the result of the addition was incorrect and it was not the result of multiplying the operands. In the second trial all additions were correct and the result of each addition could be the result of multiplying the operands of the previous trial (related 2 condition) or not (unrelated 2 condition).

Therefore, in the first trial the Type of addition result was manipulated: (a) Related 1: An addition problem of one-digit is presented with an incorrect result which coincides with the result of multiplying the operands (i.e., 2 + 4 = 8). (b) Unrelated 1: An addition problem of one-digit is presented with an incorrect result which is not related with the result of multiplying the operands (i.e., 2 + 4 = 10). In the second trial, other two conditions were established: (c) Related 2: An addition problem of one-digit is presented (i.e., 2 + 6 = 8) with a correct result (8) which coincides with the result of multiplying the operands of the previous trial (2 + 4), regardless of whether this result was explicitly presented in the previous trial (the related 1 condition: 2 + 4 = 8) or not (2 + 4 = 10). (d) Unrelated 2: An addition of one-digit is presented (i.e., 4 + 6 = 10) with a correct result (i.e., 10) which is not the one of multiplying the operands of the previous trial (i.e., 2 + 4), regardless of whether this result was explicitly present in the previous trial (i.e., 2 + 4 = 10) or not (2 + 4 = 8). An example of trials in each experimental condition is reported in Table 1.

The stimulus material in the first trial was composed of 10 incorrect additions in the related 1 condition and other 10 incorrect additions in the unrelated 1 condition. In the second trial, 10 correct additions were selected for the related 2 condition and other 10 correct additions for the unrelated 2 condition. The complete set of experimental trials used in the experiment is reported in Appendix 1.

The verification task was composed of blocks of two trials. Forty experimental blocks were presented twice to each participant. The structure of the first trial and second trials of these blocks was as follows: The 10 incorrect additions in the related 1 condition were followed by the 10 additions in the related 2 conditions. The 10 incorrect additions in the related 1 condition were also followed by the 10 additions in the unrelated 2 condition. Similarly, the 10 incorrect additions in the unrelated 1 condition were followed by the 10 additions in the related 2 condition and the same 10 incorrect additions from the unrelated 1 condition were also followed by 10 additions in the unrelated 2 conditions. Hence, across the verification task, the related 2 condition and the unrelated 2 condition were presented an equal number of times preceded by the related 1 condition and the unrelated 1 condition.

The additions used in the experimental task were carefully selected to equate them in several factors that might determine possible differences between the conditions in the first and second trial of the experiment. All additions were composed of one-digit operands and the two operands of each problem were presented in ascending order (i.e., 2 + 6) and never in descending order (i.e., 6 + 2 was not used). The parity (even and odd digits) of operands and results was equally distributed across the conditions of the first trial and second trial of the experimental blocks. In each trial, the solution corresponded to multiplication tables from 1 to 4 and it was never one of the two operands presented in the addition (i.e., 2 + 1 = 2 was not presented).

In the first trial, the related 1 condition and the unrelated 1 condition were equated in the problem size (the sum of the two operands in both conditions was 7.40). The size of the incorrect results presented in the related 1 condition and the unrelated 1 condition was also similar (11.80 and 11.60, respectively). Also, the distance between the incorrect result presented to the participants and the correct result of the addition in the two conditions of the first trial was the same (4.40). In the second trial, the problem size was equated in the related 2 condition (11.80) and the unrelated 2 condition (11.60). In order to maintain the same problem size in the two conditions of trial 2, one addition problem in the related 2 condition (7 + 9 = 16) and one problem in the unrelated 2 condition (4 + 6 = 10) were repeated. Other problems could be repeated to maintain this criterion. Thus, the repeated problems were randomly selected.

In addition, in order to check that there were no differences in response latency and accuracy when individuals answer to the additions problems used in the related 2 and unrelated 2 condition without any manipulation, we made use of the database developed by Campbell and Xue (2001). From this database; we selected the mean reaction times based on correct responses, median response time including outliers and error percentages when non-Asian Canadian individuals solved additions regardless of the strategy used to solve them. When mean RT was considered, there were no differences between the related 2 additions (928 ms) and the unrelated 2 additions (860 ms), $t(18) = 0.95$, $p = .36$. Similarly, the median RT was equated in the related 2 condition (958 ms) and the unrelated 2 condition (860 ms), $t(18) = 1.02$, $p = .32$. Finally, there were no differences in the percentage of errors associated to additions in the related 2 condition (6.26%) and the unrelated 2 condition (4.59%), $t(18) = 0.62$, $p = .54$.

Moreover, we controlled for the amount of similarities between the additions presented in the first trial and those corresponding to the related 2 condition and the unrelated 2 condition of the second trial. The numerical distance between the incorrect result presented in the first trial and the second trial was the same in the related 2 condition and the unrelated 2 condition (1.40). The difference between the problem size in the first trial and the second trial was the same in the related 2 condition and the unrelated 2 condition (4.40). In addition, the number of repetitions between the digits presented in the first trial and the second trial (i.e., 2 was repeated in the block composed of the first trial 2 + 3 = 6 followed by 2 + 4 = 6), was the same in the related 2 condition and the unrelated 2 condition (8 repetitions).

To avoid the participant noticed the structure of the experimental blocks (a sequence of an incorrect operation in the first trial and a correct operation in the second trial), these blocks were randomly intermixed with 10 filler blocks of trials. The correct responses in the first and second trial of these blocks were ‘yes’-’yes’, ‘no’-’no’, and ‘yes’-’no’, respectively. Therefore, the sequence of responses within each block of two trials was unpredictable through the experiment. The order in which the experimental blocks and filler blocks were presented was randomized for each participant.

Previous studies have shown that the practice with multiplication problems interferes with the resolution of addition problems (i.e., Campbell & Arbuthnott, 2010). Since we were interested in the mechanism associated to the selection of arithmetic facts after competition, four out of the ten filler trials were multiplication problems which were introduced to foster competition among arithmetic facts. These multiplication trials were composed of two one-digit operands that were not used together as operands in the experimental trials. The complete set of filler trials used in the experiment is reported in Appendix 2.

Before starting the verification task, the participants performed four blocks of practice trials (2 pairs of additions and 2 pairs of multiplications) with problems that were not used in the main experiment.

Procedure. The experiment was designed and controlled by E-prime experimental software, 1.1 version (Schneider, Eschman, & Zuccolotto, 2002). The stimuli were always presented in the middle of the screen in black color (Arial font, 30 point size) on a white background. Participants were tested individually and they were seated at approximately 60 cm from the computer screen.

The experimental task was a verification of arithmetic problems presented in blocks of two trials. Participants had to decide if the result of each problem was correct or incorrect. The first trial began with a fixation point in the middle of screen for 500 ms; followed by the arithmetic problem until the participant's response. After giving the answer, the second trial appeared with the same sequence of events as that of the first trial: a fixation point for 500 ms and the arithmetic problem until the participant responded. After each block of two trials, the participants were instructed to press the space bar to continue with the following block. Participants were instructed to respond by pressing the Z and M keys of the keyboard. The Z and M keys to 'correct' and 'incorrect' assignment were counterbalanced across participants. The duration of the experiment was approximately 25 minutes.

Results

Trials in which participants committed an error were eliminated from the latency analysis and submitted to the accuracy analyses (4.37% of the data in the first trial and 4.19% of the data in the second trial). Furthermore, the trimming procedure excluded reaction times (RTs) that were 2 *SD* above and below the mean, separately for each participant and for each experimental condition. The percentage of outliers was similar in the unrelated 1 condition (4.97%) and the related 1 condition (4.22%), $F(1, 47) = 3.35$, $p = .07$. Similarly, the percentage of outliers did not differ in the unrelated 2 condition (5.22%) and the related 2 condition (5.16%), $F < 1$.

Since we were interested in possible differences between conditions within each trial, the two conditions of the first trial and the second trial were separately analyzed. A factorial design including the condition (related vs. unrelated) and the trial (first and second) could not be considered because the problem size of additions in the second trial was significantly larger (11.70) than that of the first trial (7.40), $t(38) = 5.09$, $p < .001$. This difference might produce a problem size effect (Ashcraft, 1992; Groen &

Parkman, 1972) which consists in longer reaction times and more errors when solving additions with large problem size relative to problems with small problem size. Therefore, we report first the results obtained in the first trial (related 1 condition vs. unrelated 1 condition) and then the results found in the second trial (related 2 condition vs. unrelated 2 condition)[1].

First Trial.[2] We performed analyses of variance (ANOVAs) on the RTs and percentage of errors with the variable Type of addition result: related 1 (i.e., 2 + 4 = 8) and unrelated 1 (i.e., 2 + 4 = 10) as a within-subject factor. The difference between these conditions on the RT analysis was significant, $F(1, 47) = 6.99$, $p = .01$, $\eta^2 = .13$,

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[1]The analysis of second trial depending on the type of first trial. It is important to note that the type of second trial (related 2 vs. unrelated 2) could not be analyzed depending on the type of first trial (related 1 vs. unrelated 1) due to a repetition effect that might have a different impact on the two conditions of trial 2. For example, while the solution 8 is repeated in the related 2 condition: 2 + 6 = 8, after the related 1 condition 2 + 4 = 8; the solution 10 is repeated in the unrelated 2 condition 4 + 6 = 10 after the unrelated 1 condition 2 + 4 =10. Note, however, that this unbalanced repetition effect is avoided when related 2 and unrelated 2 conditions are directly compared since in both conditions, half of the solutions were explicitly presented in the previous trial. Nevertheless, we performed additional analyses to evaluate the influence of Trial 1 on Trial 2 by avoiding the unbalanced repetition effect. Firstly, the Trial 1 (related 1, unrelated 1 condition) x Trial 2 (related 2, unrelated 2) interaction was significant, $F(1, 47) = 17.55$, $p < .001$, $\eta^2 = .27$. Afterward, we compared the two conditions that involved repetition: (a) related 1 - related 2 condition vs. (b) unrelated 1 - unrelated 2 condition. The RTs in Trial 2 were 100 ms slower in (a) vs. (b), $F(1, 47) = 6.18$, $p = .02$, $\eta^2 = .12$, suggesting that, in spite of the repetition effect, Trial 2 was difficult to perform when it included the result of multiplying the operands of Trial 1. However, the comparison between (c) related 1 - unrelated 2 vs. (d) unrelated 1 vs. related 2 was not significant, $F(1, 47) = 2.71$, $p = .11$, $\eta^2 = .05$ (57 ms difference). These two conditions did not involve repeating the result. Hence, the consequences of inhibiting the result of multiplying the operands of Trial 1 were only evident in Trial 2 when the result was visually present in both trials, the (a) condition.

such that responses to related 1 trials were slower than responses to unrelated 1 trials (see Table 3).

Furthermore, the ANOVA performed on the percentage of errors showed significant the difference between the related 1 trials (6.30%, *SE* = 1.1) and the unrelated 1 trials (2.45%, *SE* = 0.52), $F(1, 47) = 14.82$, $p < .001$, $\eta^2 = .24$.

Second trial. We performed an ANOVA on the RTs and percentage of errors with the variable Type of result of previous trial (related 2 trials and unrelated 2 trials). There were significant differences between these two conditions, $F(1, 47) = 8.17$, $p = .006$, $\eta^2 = .15$. As showed in Table 3, responses to related 2 trials were slower than responses to unrelated 2 trials. However, the accuracy analyses did not show significant differences between the related 2 condition (4.17%, *SE* = 0.8) and the unrelated 2 condition (4.22%, *SE* = 0.86), $F < 1$.

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[2]Possible differences due to the gender of participants were examined. In Experiment 1, the results obtained in the first trial did not show differences between females and males, $F(1, 46) = 1.61$, $p = .21$, $\eta^2 = .03$, and the Gender x Type of addition result was not significant $F < 1$. In the second trial, Gender was not a significant variable, $F(1, 46) = 2.35$, $p = .13$, $\eta^2 = .05$, nor this variable interacted with type of results of previous trial, $F < 1$. In Experiment 2, the results of trial 1 did not show a significant effect of Gender, $F < 1$, and this variable did not interact with type of addition results, $F < 1$. In the second trial, Gender was not significant, $F(1, 31) = 1.12$, $p = .30$, $\eta^2 = .03$, and this variable did not interact with type of result of previous trial, $F(1, 31) = 1.20$, $p = .28$, $\eta^2 = .04$. Hence, there were no differences due to the gender of participants in this study.

Table 3. Results (RT) in Experiment 1.

	First Trial		
	Related 1 condition	Unrelated 1 condition	RT Diff.
	(2 + 4 = 8)	*(2 + 4 = 10)*	
	1335 (48.00)	1301 (48.18)	34
	Second Trial		
	Related 2 condition	Unrelated 2 condition	
	(2 + 6 = 8)	*(4 + 6 = 10)*	
	1467 (63.28)	1384 (64.3)	83

Note. Mean reaction times in milliseconds (ms) for each condition in first and second trial of Experiment 1. Standard errors are reported into bracket.

To evaluate whether the magnitude of the interference effect varies between the first trial and the second trial, we performed additional analyses computing the interference effect (unrelated condition vs. related condition) for each participants in the two trials of the experiment. The results showed that the magnitude of the interference effect was similar in the first trial (34 ms) and the second trial (83 ms) of the experiment, $F(1, 47) = 2.33$, $p = .13$.

Discussion

The aim of Experiment 1 was to evaluate the possible inhibitory mechanism involved in the selection of arithmetic facts. However, in order to be able to address this goal, a previous step was to corroborate that participants activated arithmetic facts that were irrelevant to solve the addition problems. The results obtained in the first trial of the experimental blocks confirmed that participants coactivated the result of multiplying two operands when they decided on the correctness of adding these two operands. The participants were slower to answer when the result of the incorrect addition problems (2 + 4 = 8) was that of multiplying the operands relative to a control condition with an unrelated result (2 + 4 = 10).

The coactivation of arithmetic facts found in Experiment 1 is not new and it has been observed in previous studies (Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). Zbrodoff and Logan suggested that the coactivation of arithmetic facts is an automatic process which would be obtained any time participants are presented with an arithmetic problem. This statement would imply that, in the current study, the participants activated the result of multiplying the two operands (the result 8 when 2 + 4 was presented) in the two conditions of the first trial. However, the competition would be greater in the related 1 condition relative to the unrelated 1 condition since in the first case the result of multiplying the operands (8) was visually presented (2 + 4 = 8), enhancing thus its activation relative to the second case in which the result of multiplying the operands was not visually presented (2 + 4 = 10).

More important for the purpose of the current study was the observation that participants took more time to respond when the result of multiplying the operands of the first trial (2 and 4) was the correct result of the problem presented in the second trial (2 + 6 = 8), relative to the unrelated 2 condition. This pattern of results suggests that participants inhibited the result that was irrelevant in the first trial so they needed additional time to retrieve it when it was presented again in the related 2 condition of the second trial. Hence, the results of Experiment 1 suggest that participants used an inhibitory mechanism to select the correct arithmetic facts.

The paradigm introduced in Experiment 1 to study the possible inhibitory mechanism underlying the selection of arithmetic facts has not been used before in the field of mathematical cognition. Therefore, we decided to gather additional evidence with the same paradigm under circumstances that did not favor the activation of irrelevant arithmetic facts. Previous research has shown that participants are sensitive to contextual information presented in the course of an experiment. For example, in a comparison task in which participants decide the larger of two-digit number pairs (i.e., 42-58), although the decade digit suffices to answer (58 is larger than 42 because 5 > 4), the participants activate the irrelevant unit digits and this activation depends on the amount of filler trials presented in the experiment (Macizo & Herrera, 2011, 2013): When there is a large number of filler trials in which the unit digit needs to be necessarily processed (i.e., within-decade number pairs with the same decade digit; 42-48), participants strongly activate the unit digit even in trials in which the units are irrelevant and might produce competition (the unit-decade compatibility effect, Macizo & Herrera, 2011, 2013). Therefore, these studies indicate that the activation of irrelevant numerical representations depends on the nature of filler trials introduced in the study. In Experiment 1, we included filler trials with multiplication problems in order to favor the coactivation of irrelevant arithmetic facts (multiplications) when they solved addition problems. In Experiment 2, we continued evaluating the selection of arithmetic facts under circumstances that did not promote the retrieval of irrelevant arithmetic facts.

EXPERIMENT 2

This experiment was aimed to gather additional evidence of the way individuals coactivate irrelevant multiplication facts when they verify the correctness of addition problems. Previous studies showing the coactivation of several arithmetic facts have used a task in which multiplications and addition are presented intermixed so both are relevant to perform the task (Grabner, Ansari, Koschutnig, Reishofer, & Ebner, 2013; Lemaire, Fayol, & Abdi, 1991, Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). Similarity, in Experiment 1 the verification task included filler trials with multiplication problems to foster the coactivation of irrelevant arithmetic facts associated to multiplication problems. In Experiment 2 only addition problems were presented so multiplication facts were completely irrelevant to perform the task. Hence, we had the opportunity to evaluate the coactivation of multiplication facts when they were completely irrelevant to verify the correctness of addition problems. In addition, Experiment 2 was a replication of Experiment 1 with the exception that multiplication filler trials were excluded. Thus, we were able to examine again the involvement of inhibitory processes during the verification of simple additions.

Method

Participants. A new set of thirty-three students from the University of Granada (26 women and 7 men) took part in Experiment 2. None of them participated in Experiment 1. Their mean age was 21 years (*SD* = 2.71). Three were left-handed and 30 were right-handed. The participants gave informed consent and their participation was remunerated by academic credits. After the experimental task, the participants completed the same questionnaire used in Experiment 1 to determine their knowledge of simple arithmetic (Colomé et al., 2011). *T*-tests analyses did not show differences between participants of Experiments 1 and 2 in the scores of this questionnaire (all p values > .15), except an increased use of division in daily life in participants of Experiment 1 (M = 19.73%) relative to participants of Experiment 2 (M = 12.77%), $t(79) = 2.49$, $p = .01$. Most of the participants of Experiment 2 learned the multiplication tables orally (77.27%). In addition, 48.48% of participants made simple calculations on a daily basis (see Table 2).

As in Experiment 1, the participants performed a multiplication task with multiplication tables 1-4 which was performed at the end of the experimental session to avoid possible influences of the multiplication problems on the main tasks of verification of additions. The mean correct responses in the multiplication task was 89.2%, and there were no significant differences between males and females, $t(31) = 1.34$, $p = .19$. The correct responses in the multiplication task was similar in Experiment 1 (88.04%) and Experiment 2 (89.2%), $t(79) = 0.67$, $p = .50$.

Design and Materials. The experimental task and the experimental conditions in the first trial and second trial of this experiment were the same as those of Experiment

1. The only difference between experiments concerned the blocks of filler trials. While in Experiment 1 the 10 filler blocks included 4 blocks of multiplications, in Experiment 2, the 10 filler blocks were composed of additions only. The filler additions were never used as experimental trials. As in Experiment 1, the two trials of these filler blocks followed the sequence of 'yes'-'yes', 'no'-'no', and 'yes'-'no' responses; so the sequence of responses within each block of two trials was unpredictable across the experiment.

Procedure. The same procedure employed in Experiment 1 was used in Experiment 2.

Results

Trials in which participants committed an error were excluded from the latency analyses and submitted to the accuracy analyses (1.14% of the data in the first trial and 2.12% of the data in the second trial). As in Experiment 1, the trimming procedure excluded RT data that were 2 *SD* above and below the mean, separately for each participant and for each experimental condition. The percentage of outliers was similar in the unrelated 1 condition (4.98%) and the related 1 condition (4.75%), $F < 1$. Similarly, the percentage of outliers did not differ in the unrelated 2 condition (4.78%) and the related 2 condition (4.53%), $F < 1$. As in Experiment 1, we performed an ANOVA on the RTs and percentage of errors of the first trial and then the same was analyzed in the second trial.

First Trial. The ANOVA on the RTs with the variable Type of addition result showed significant differences between the related 1 condition and the unrelated 1 condition, $F(1, 32) = 3.86$, $p = .05$, $\eta^2 = .11$, so related 1 trials were answered to more slowly than unrelated 1 trials (see Table 4).

However, the accuracy analyses showed similar error rates in the related 1 condition (1.06%, $SE = 0.34$) and the unrelated 1 condition (1.21%, $SE = 0.33$), $F < 1$.

***Second Trial*.** The ANOVA on the RTs with the variable Type of result of previous trial showed significant differences between the related 2 condition and the unrelated 2 condition, $F(1, 32) = 35.44$, $p < .001$, $\eta^2 = .53$, such that related 2 trials were answered to more slowly than unrelated 2 trials (see Table 4).

Table 4. Results (RT) in Experiment 2.

	First Trial		
	Related 1 condition *(2 + 4 = 8)*	Unrelated 1 condition *(2 + 4 = 10)*	RT Diff.
	1183 (50.66)	1151 (41.83)	32
	Second Trial		
	Related 2 condition *(2 + 6 = 8)*	Unrelated 2 condition *(4 + 6 = 10)*	
	1563 (90.64)	1433 (79.75)	130

Note. Mean reaction times in milliseconds (ms) for each condition in first and second trial of Experiment 2. Standard errors are reported into brackets.

Nevertheless, the accuracy analyses did not show significant differences between these two conditions: related 2 condition (2.5%, $SE = 0.36$) and the unrelated 2 condition (1.74%, $SE = 0.4$), $F(1, 32) = 3.13$, $p = .09$, $\eta^2 = .09$.

Additional analyses showed that the magnitude of the interference effect in the second trial was larger (130 ms) than that observed in the first trial (32 ms), $F(1, 32) = 15.38$, $p < .001$ $\eta^2 = 0.32$.

Joint analyses of Experiment 1 and 2. Cross-experiment analyses were performed to compare the effects observed in the first and second trial. In these analyses, the experiment was considered a between-participants variable. In the first trial, the main effect of the Type of addition result was significant, $F(1, 79) = 10.31$, $p =$

.002, η^2 = 0.12. The related 1 condition was answered to more slowly (1259 ms, *SE* = 35.77) than the unrelated 1 condition (1225 ms, *SE* = 33.86), (33 ms difference). However, although the main effect of the Experiment was significant, $F(1, 79) = 4.79$, $p = .03$, η^2 = 0.06 (151 ms difference between Experiment 1 and 2), this variable did not interact with the Type of addition result, $F < 1$.

Moreover, the ANOVA on the second trial showed significant differences between the related 2 condition and the unrelated 2 condition, $F(1,79) = 29.19$, $p < .001$, η^2 = 0.27, so related 2 trials were answered to more slowly (1515 ms, *SE* = 53.53) that unrelated 2 trials (1409 ms, *SE* = 50.95), (106 ms difference). Nevertheless, the main effect of the Experiment was not significant and this variable did not interact with the Type of result of previous trial (*p*s > .23).

Discussion

The results obtained in this experiment were exactly the same as those found in Experiment 1. In the first trial, the participants took more time to respond to the related 1 condition relative to the unrelated 1 condition, which suggests that they coactivated the arithmetic fact associated to the multiplication when they performed the addition verification task. In the second trial, the participants responded to more slowly in the related 2 condition relative to the unrelated 2 condition. This result suggests that the irrelevant multiplication facts coactivated in the first trial were inhibited so participants required additional time to reactivate them when they became relevant in the second trial.

The similar pattern of results obtained in Experiment 1 and 2 strongly indicates that there is a coactivation of different arithmetic facts (additions, multiplications) even when some of them are irrelevant to perform the experimental task (i.e., the multiplication facts). Moreover, the data obtained in the second trial of Experiment 1 and 2 suggests the existence of an inhibitory mechanism responsible for the selection of the correct result of simple arithmetic problems.

Although the results of Experiment 2 replicate those of Experiment 1, we expected reduced interference effects in the first and second trial of Experiment 2 relative to those found in Experiment 1. In Experiment 2 there were no contextual factors to promote the activation of irrelevant multiplication facts (all trials were simple addition problems). In this scenario, we expected a reduced coactivation of multiplication facts in the first trial of Experiment 2 which would be reflected in a small difference between the related 1 and the unrelated 1 condition. However, these between-experiments differences were not found. We will elaborate further on this point in the next section.

GENERAL DISCUSSION

It is widely agreed that individuals have represented in long-term memory arithmetic facts associated to simple mental arithmetic such as addition and multiplication problems (e.g., Ashcraft, 1987; Campbell & Graham, 1985; Siegler & Shrager, 1984). It is also assumed that these arithmetic facts are represented in an associative network with interconnections among different arithmetic problems, their operands and results. When an individual is performing an addition problem (i.e., 2 + 4 = ?) or evaluating whether it is correct or not (i.e., 2 + 4 = 8, false solution), the activation flows through the arithmetic network so the nodes associated to the correct addition solution become active. Moreover, due to this automatic spreading of activation, other related nodes (i.e., multiplication facts) receive activation too (i.e., 2 x 4 = 8). The concurrent activation of the correct addition and the incorrect but related multiplication produces competition among representations. This competition causes interference which is indexed in experimental studies as longer response latencies to decide the correctness of an addition problem (2 + 4 = 8) when the solution presented to the participant is false but associated to other related arithmetic problem (8 which is the result of multiplying 2 x 4) (Zbrodoff & Logan, 1986). The question addressed in the current study was about the cognitive mechanism involved in the resolution of this conflict among coactivated arithmetic facts. The answer that can be drawn from previous studies is not unequivocal. Within the activation-based account, Censabella and Nöel (2004) argue that the conflict among coactivated arithmetic facts derivates from the limited amount of activation that might spread through the arithmetic network when participants perform simple arithmetic. In a given trial of a verification task (i.e., 2 + 4 = 10), the activation spreads to other related nodes (also the one associated to multiplying the operands 2 x 4 = 8); however, competition is not too high since the coactivated irrelevant result (8) is not presented to the participant. On the contrary, when participants receive a false addition with a related solution (2 + 4 = 8), the related node (2 x 4 = 8) receives a large amount of activation while the correct one would be weakly activated producing, thus, the competition and interference effect. In other words, the passive activation of false but related nodes produces an overfacilitation of the competing nodes.

There is empirical evidence supporting the passive activation account such as the relationship found between the accuracy in solving arithmetic facts and the fan effect (Censabella & Noël, 2004), a phenomenon which is usually explained by the principle of passive activation. However, as we commented on earlier sections of this paper, the fan effect is open to several interpretations. Hence, the explanation of conflict and selection of arithmetic facts based on the passive activation is not conclusive. An alternative view is offered by Campbell and colleagues (Campbell & Dowd, 2012; Campbell & Thompson, 2012). The authors propose that the selection of arithmetic facts is mediated by inhibitory processes. After the coactivation of irrelevant nodes in the semantic network, the individuals are able to correctly perform the arithmetic problem by inhibiting the irrelevant competing alternatives. In several studies the authors have corroborated this inhibitory mechanism with the RP paradigm (Anderson, 2003; Anderson, Bjork & Bjork, 1994). The retrieval of multiplication facts in a practice phase (2 x 4 = ?), slows down the retrieval of their addition counterparts on a posterior test phase (2 + 4 = ?). This RIF effect shows the long-term consequences of inhibiting the competing arithmetic facts since their accessibility is evaluated after minutes of delay from the competing situation (the practice phase in which it is supposed that the irrelevant arithmetic facts were inhibited).

The main goal of this study was to gather direct evidence of this inhibitory mechanism after the coactivation of arithmetic facts and the selection of the correct one. To this end, the first step was to design a paradigm with which we could capture the continuous retrieval and coactivation of arithmetic facts when participants performed a verification task of addition problems. In two experiments we observed interference effects that seem to be due to the retrieval of irrelevant multiplication counterparts. Firstly, participants took longer to decide that the problem 2 + 4 = 8 was incorrect relative to the problem 2 + 4 = 10, which might be interpreted as due to the competition between the correct result 6 and the incorrect result 8, which is the one of multiplying the operands (2 x 4 = 8). Importantly, an interference effect was also found when the irrelevant result (8) was presented again and it became relevant to decide that 2 + 6 = 8 was a correct addition, relative to the control problem 4 + 6 = 10. This interference effect was interpreted as reflecting inhibitory processes during the selection of addition facts. Therefore, we suggest that in the first moment, participants correctly responded 'no' to 2 + 4 = 8 by inhibiting the competing result (8). Afterward, participants needed to overcome inhibition as reflected by the additional time required to respond to the previous conflicting result when it became relevant later (2 + 6 = 8).

In this study, evidence for the coactivation and the posterior inhibition of irrelevant arithmetic facts was obtained in two experiments. Nevertheless, we expected some differences between experiments. In Experiment 1, we fostered the coactivation of irrelevant multiplication facts because our main interest was to corroborate the involvement of inhibitory processes to resolve the competition. To this end, multiplication problems were intermixed with additions. In Experiment 2, only additions problems were used to evaluate the consequences of presenting multiplication filler problems in our first experiment. In Experiment 2, we expected reduced coactivation of irrelevant multiplication facts in a task that could be resolved with addition facts solely, and thus, we predicted less interference. However, the similar pattern of results obtained in Experiment 1 and 2, indicates that contextual multiplication problems did not influence the retrieval of multiplication facts when participants resolved addition problems. This finding is particularly odd because it is usually found that when participants are engaged in a number cognition tasks (i.e., number comparison) their performance is modulated by the type and number of filler trials used in the experiment (Huber, Mann, Nuerk, & Moeller, 2014; Macizo & Herrera, 2011, 2013). Several explanations might be offered for the lack of contextual effects in our study. These explanations are based on the degree of activation and competition in the current study between the target arithmetic facts (addition facts) and the competitors (multiplication facts). The addition problems used in our study were very simple; they were composed of one-digit operands. Thus, it might be argued that the retrieval of the irrelevant multiplication facts in this situation would be largely automatic, so they would become easily activated regardless of the experimental context in which they were immersed. As a consequence, they would strongly compete with the addition facts producing the interference effect in Experiments 1 and 2. In other words, competition between addition and multiplication facts might be promoted by the type of addition problems used in the study (small problems with sums < 18). It has been observed that small problems have greater memory strength than large problems (i.e., Campbell & Xue, 2001; LeFevre, Sadesky & Bisanz, 1996; Zbrodoff & Logan, 2005). Hence, the competition of arithmetic facts would be observed more easily with small addition problems than with large problems, because the coactivation would be stronger in the first case irrespective of the experimental context used in Experiments 1 and 2. Future research will shed light on these explanations. However, the main point to draw up is that the current study supports the existence of an inhibitory mechanism that is involved in the continuous selection of simple arithmetic facts.

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CHAPTER IV

SIMPLE ARITHMETIC: ELECTROPHYSIOLOGICAL EVIDENCE OF COACTIVATIONA AND SELECTION OF ARITHMETIC FACTS[1]

This study aimed at demonstrating that the associative confusion effect found in simple arithmetic involves the coactivation of arithmetic facts in semantic memory. We also evaluated the consequences of selecting arithmetic facts to resolve addition problems. We gathered electrophysiological evidence when participants performed a verification task. Simple addition problems were presented in blocks of two trials and participants decided whether they were correct or not. The N400-like component was considered an index of semantic access (i.e, the retrieval of arithmetic facts) and the P200 component was used to determine the difficulty of retrieving arithmetic facts after the answer to an addition problem. When an addition problem was incorrect but the result presented to the participant was that of multiplying the operands (e.g., 2 + 4 = 8), N400-like amplitude was reduced relative to an unrelated condition (e.g., 2 + 4 = 10). This finding suggested that the coactivation of addition and multiplication facts took place. Furthermore, the P200 amplitude was more positive when participants answered to addition problems whose result was that of multiplying the operands of the previous trial (e.g., 2 + 6 = 8). This suggests that irrelevant results were inhibited and it was difficult to retrieve them later.

1 This paper was submitted to *Experimental Brain Research* and it was co-authored by Pedro Macizo.

SIMPLE ARITHMETIC: ELECTROPHYSIOLOGICAL EVIDENCE OF COACTIVATIONA AND SELECTION OF ARITHMETIC FACTS

Simple arithmetic facts are stored in semantic memory as an associative network whose nodes are interrelated (Campbell & Graham, 1985). When a simple problem is presented (i.e., the addition problem 2 + 4), the nodes that represent the operands (i.e., 2 and 4) and the solution (i.e., 6) of the problem are activated automatically. Furthermore, due to the principle of spreading activation, other related nodes become activated too (i.e., 8, the result of multiplying the operands 2 and 4) (Ashcraft, 1992). This concurrent activation produces competition between arithmetic facts (Winkelman & Schmidt, 1974). For instance, when individuals resolve an addition problem (i.e., 2 + 4), the arithmetic fact associated with the multiplication (i.e., 8) produces interference and slows down the time needed to select the correct answer (i.e., 6).

There is empirical evidence of this interference effect during the verification of addition problems (Grabner, Ansari, Koschutnig, Reishofer, & Ebner, 2013; Lemaire, Fayol, & Adbi, 1991; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). In this task, a simple addition problem is presented (i.e., a pair of one–digit operands and a result) and participants have to decide whether the proposed result is correct or not. The critical trials are those associated with negative responses (incorrect addition problems). In these trials, participants take more time to respond when the proposed result is incorrect but it is the one of multiplying the operands (2 + 4 = 8) relative to an unrelated condition (2 + 4 = 10). This so-called associative confusion effect (Winkelman & Schmidt, 1974) has been taken as an index of the simultaneous activation of addition and multiplication facts in semantic memory (Grabner et al., 2013; de Visscher, Berens, Keidel, Noël, & Brid, 2015). The current study was aimed at evaluating this assumption.

In our study, we recorded electrophysiological activity when participants resolved addition problems in order to demonstrate that the coactivation of semantic information in simple arithmetic (i.e., the retrieval of arithmetic facts) underlies the associative confusion effect. Specifically, we focused on the N400, a negative-going waveform peaking at approximately 350-450 ms after stimulus onset. Importantly for the current study, the amplitude of this component is sensitive to the processing of semantic information (Domahs et al., 2007; Jost, Henninghausen, & Rösler, 2004; Macizo, Van Petten, & O'Rourke, 2012; Niedeggen & Rösler, 1996, 1999; Niedeggen, Rösler, & Jost, 1999). For instance, in psycholinguistic studies, it has been corroborated that N400 amplitude is attenuated (less negative) when a target stimulus is preceded by a semantically related context relative to an unrelated context (Kutas & Hillyard, 1980, 1984). This N400 attenuation has been interpreted as due to the spreading of activation in semantic memory which facilitates the processing of target stimuli preceded by related primes (Kutas & Federmeier, 2011). The N400 is not specific to the processing of linguistic stimuli. In fact, N400-like potentials have been found when individuals process meaningful stimuli in the non-verbal domain (pictures, faces, etc.), suggesting that members of this family of N400-like potentials are found whenever stimuli tap into semantic memory (Kutas & Federmeier, 2011).

To our knowledge, ERPs have not been used to study the associative confusion effect in simple arithmetic. However, amplitude modulations of N400-like components have been reported in studies about the processing of multiplication problems (Domahs et al., 2007; Jost et al., 2004; Niedeggen & Rösler, 1996, 1999; Niedeggen et al., 1999)[1]. To illustrate, Niedeggen and Rösler (1999) asked participants to decide whether simple multiplication problems were correct or not. The result of incorrect multiplication problems could be related (i.e., the results were multiples of either the first or the second operand, 5 x 8 = 32) or unrelated (i.e., 5 x 8 = 34). The authors found behavioral interference so related problems were responded slower than unrelated problems. In contrast, when the ERP pattern was considered, an attenuation of the N400-like component was obtained for related results relative to unrelated results. Hence, the authors observed dissociation between decision times and ERPs measures where behavioral interference was accompanied by an attenuation of the N400-like amplitude. The authors concluded that N400-like effects indexed the spreading of activation in the network of arithmetic facts so related results facilitated the retrieval of the correct multiplication results. In contrast, behavioral interference was interpreted as the consequence of a late competition process which was not captured in ERP measures but it was observed in response times.

When we revisit the associative confusion effect, we observe that even when people take more time to verify a problem whose result is the one of multiplying its operands (i.e., 2 + 4 = 8), they are able to resolve it correctly most of the time (i.e., to say that 2 + 4 = 8 is incorrect). It has been proposed that the conflict produced by the co-activation of arithmetic facts is solved by an inhibitory mechanism (Campbell & Dowd, 2012; Campbell & Thompson, 2012; Megías, Macizo, & Herrera, 2014; Megías & Macizo, 2015a, 2015b). In a recent study, Megías et al. demonstrated that this inhibitory mechanism acts in a continuous manner in order to reduce interference when competition between arithmetic facts takes place. To address this issue, Megías et al. (2015a) designed a new paradigm in which additions were presented in blocks of two trials and participants had to decide whether the proposed result of an addition problem was correct or not. In the first trial, participants took more time to respond to an incorrect addition problem whose result was the one of multiplying the operands (i.e., 2 + 4 = 8) relative to an unrelated condition (i.e., 2 + 4 = 10). This interference effect suggested that participants activated multiplication facts when they verified addition problems. In the second trial, the participants took more time to respond to another addition problem whose result was the one of multiplying the operands of the previous trial (i.e., 2 + 6 = 8 preceded by 2 + 4) relative to an unrelated condition (i.e., 4 + 6 = 10

[1]Apart from the ERP components considered here, there are other potentials associated to the processing of numerical information. N100 is sensitive to variations in non-symbolic magnitudes (Hyde & Spelke, 2009), P100 modulations seem to be related to the implicit estimation of ordinal information (Rubinsten, Dana, Lavro, & Berger, 2013). In verification tasks, large vs. small distance between the proposed result and the correct result is related to more negative N2b amplitude and more positive P3b amplitude (Avancini, Soltész, & Szűcs, 2015, for a review). In the current study, we did not consider these indexes since the underlying cognitive processes did not apply (e.g., we did not use non-symbolic quantities nor magnitude estimation), or they were equated across conditions of our experiment (e.g., the distance between the incorrect and correct results was equated in the conditions of trial 1, and the distance between the results in trial 1 and 2 was also equated in the two conditions of trial 2).

preceded by 2 + 4). This interference effect obtained in the second trial was interpreted as the consequence of inhibiting the irrelevant multiplication result when participants responded to the first trial. Hence, they needed additional time to reactivate the inhibited result (i.e., 8) when it was presented again in the second trial and it was the one needed to perform the task (i.e., 2 + 6 = 8).

The second goal of the current study was to gather electrophysiological evidence of the consequences associated to the selection of arithmetic facts. To this end, we focused on the P200 potential, a complex component peaking at about 200 ms after stimulus onset. This component is sensitive to several cognitive processes such as the analysis of facial expressions (Paulmann & Pell, 2009), the early processing of lexical stimuli (Dehaene, 1995; McCandliss, Posner, & Givon, 1997), and the encoding and retrieval of the meaning of stimuli in semantic memory (Chapman, McCrary, & Chapman, 1978; Dunn, Dunn, Languis, & Andrews, 1998; Friedman, Vaughn, & Erlenmeyer-Kimling, 1981). Therefore, the cognitive interpretation of the P200 is not straightforward and it depends on what is being studied. In the current research we considered the sensitivity of the P200 potential to index the difficulty to retrieve information from semantic memory (Raney, 1993; Smith, 1993). For instance, when participants with high or low recall of a list of words are compared (Dunn et al., 1998), low recall participants show larger P200 amplitude in anterior regions and smaller posterior amplitudes than high recall participants. The authors suggest that frontal P200 would be associated to the ease of encoding a stimulus whose meaning has to be retrieved while the posterior P200 would be linked to the complete access to long-term memory.

In the field of arithmetic cognition, the P200 component has been related also to the difficulty of retrieving semantic information with numerical stimuli (Kong, Wang, Shang, Yang, & Zhuang, 1999; Muluh, Vaughan, & John, 2011; Szűcs & Csépe, 2004). For example, when participants have to verify the correctness of addition problems, the P200 amplitude is larger in frontal regions when the addition problem is difficult (i.e., large addition problems with carrying in solution; e.g., 7 + 8 =) relative to easy addition problems (small addition problems without carrying in solution; e.g., 2 + 4 =) (Kong et al., 1999). This problem size effect seems to indicate that arithmetic facts associated with large problems are less accessible than those associated with small problems (Ashcraft, 1992). Therefore, the results of these studies suggest that P200 component can be considered an index of the difficulty in resolving simple arithmetic problems.

The current study. The goal of the current study was two-fold. Firstly, we aimed at demonstrating that the associative confusion effect reported in the past when participants performed arithmetic tasks involves semantic activation (arithmetic facts stored in semantic memory; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). To this end, participants verified the correctness of addition problems. In a first trial, we expected to corroborate the behavioral interference effect reported in previous research (Megías et al., 2014; Megías & Macizo, 2015a, 2015b), so participants would take more time to verify an incorrect related addition problem presented with a proposed result that was the one of multiplying the operands (i.e., related 1 condition: 2 + 4 = 8) relative to an unrelated condition. This effect would capture the automatic co-activation of addition and multiplication facts in long-term memory. Moreover, if participants coactivate arithmetic facts, a N400-like attenuation would be observed in the related 1 condition due to the spreading of activation in the associative network of arithmetic facts which would facilitate the activation of related nodes.

Our second goal was to examine the consequences associated to the selection of arithmetic facts in order to resolve the addition problems. We expected to observe longer reaction times in a second trial when participants verify a correct addition problem whose result is the one of multiplying the operands of the previous trial (related 2 condition, i.e., 2 + 6 = 8, preceded by 2 + 4 =) compared to an unrelated condition. This interference effect has been interpreted as due to the inhibition of the irrelevant result in the previous trial, so the difficulty to retrieve the result increases when it is presented afterwards (Megías et al., 2014; Megías & Macizo, 2015a, 2015b). If this argument is correct, the behavioral interference effect in the second trial would be accompanied by a modulation of the P200 component which is associated with the difficulty of retrieving information in long-term memory.

METHOD

Participants. Seventeen students from the University of Granada (10 women and 7 men) took part in the study. Their mean age was 22 years (*SD* = 4.12). Sixteen participants were right-handed and 1 participant was left-handed. All participants had normal or corrected-to-normal visual acuity. None had any reported history of neurological or psychiatric disorders. The experiment was undertaken in accordance with the Declaration of Helsinki. The Ethics Committees of the University of Granada approved the experimental procedures and each subject provided written informed consent before performing the experiment. Their participation was remunerated with academic credits. Before the experimental task, they completed a questionnaire to determine their use of simple arithmetic (Colomé, Bafalluy, & Noël, 2011) (see Table 1). The percentage of calculation of addition problems on a daily basis was 43.8% (*SD* = 11.80). Moreover, 81.18% (*SD* = 27.36) of the participants learned the multiplication tables orally.

Table 1. Use of Simple Arithmetic

Calculation frequency	
Daily	58.82%
Weekly	41.18%
Monthly	0%
Type of calculation	
Multiplications	19.71%
Divisions	12.76%
Additions	43.82%
Subtractions	23.12%
Calculation strategies	
Saying numbers mentally or aloud	48.01%

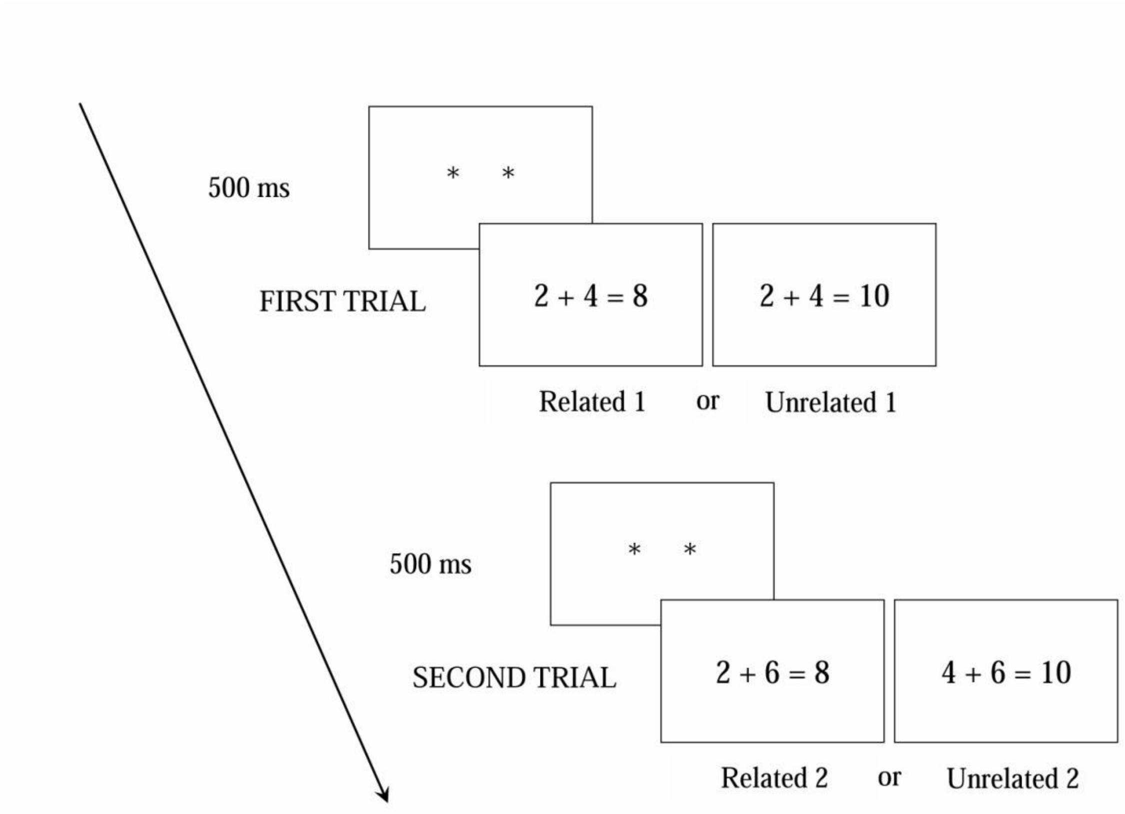
Visualizing Arabic numbers mentally	21.29%
Writing numbers with pencil and paper	16.95%
With a calculator	13.75%
Learning method (multiplication tables)	
Repeating orally	81.18%
Exercises with Arabic numbers	17.65%
Others methods	1.18%

In order to control that participants had a good knowledge about multiplication tables, they performed a production multiplication task. In this task, tables from 1 to 4 were presented (i.e., 2 x 4 = ?) and participants had to say aloud the correct result (i.e., 8). This task was performed after the experiment and electrophysiological data were not recorded. Participants showed a good knowledge of simple multiplication problems, with 92.84% of correct responses (*SD* = 5.74).

Design and Materials. We used a verification of arithmetic problems task (see Figure 1) in which participants received addition problems and they decided whether they were correct or incorrect. The addition problems were presented in blocks of two trials. In the first trial, the variable Relation 1 was manipulated as a within-subject factor with two conditions: the related 1 condition included an incorrect addition problem whose result was that of multiplying the operands (i.e., 2 + 4 = 8), and the unrelated 1 condition contained an incorrect addition problem whose result was not the one of multiplying the operands (i.e., 2 + 4 = 10). In the second trial, the variable Relation 2 was manipulated as a within-subject factor with two conditions: the related 2 condition contained a correct addition problem whose result was the one of multiplying the operands of the previous trial (i.e., 2 + 6 = 8), and the unrelated 2 condition included a correct addition problem with a result which was not the one of multiplying the operands of the previous trial (4 + 6 = 10).

To make the experimental blocks of trials, 20 false addition problems were selected in the first trial (10 related 1 addition problems and 10 unrelated 1 addition problems), and 20 correct addition problems were used in the second trial (10 related 2 addition problems and 10 unrelated 2 addition problems) (see Appendix 1). Across participants, each problem in each condition of the first trial (related 1 and unrelated 1 addition problems) was presented half of the times followed by related 2 addition problems and the other half they were followed by unrelated 2 addition problems. Therefore, the related 2 and unrelated 2 addition problems were preceded an equal number of times by related 1 and unrelated 1 trials. Each participant received the experimental block of trials three times in order to have more trials per condition. Therefore, the total number of observations was 60 in each condition of the first trial (related 1 and unrelated 1) and in each condition of the second trial (related 2 and unrelated 2). The complete set of experimental trials used in the experiment is reported in Appendix 1.

Figure 1. Adaptation of the negative priming parading.



Note. The arithmetic verification task was presented in blocks of two trials. The first trial started with a fixation point of 500 ms followed by an addition problem. Two addition problems could be presented: Related 1 addition problems (i.e., 2 + 4 = 8) or Unrelated 1 addition problems (i.e., 2 + 4 = 10). After the participant's response, the second trial started with a fixation point of 500 ms followed by the second addition problem which could belonged to the Related 2 condition (i.e., 2 + 6 = 8) or the Unrelated 2 condition (4 + 6 = 10).

The addition problems used in the experimental task were carefully selected to equate them in several factors that might determine possible differences between conditions in the first and second trial of the experiment. All addition problems were composed of single-digit operands and the two operands of each problem were presented in ascending order (i.e., 2 + 6). The parity (even and odd digits) of operands and results was equally distributed across the conditions of the first and second trials of the experimental blocks. In each trial, the solution corresponded to multiplication tables from 1 to 4 and it was never one of the two operands presented in the problem (i.e., 2 + 1 = 2 was not presented).

In the first trial, the related 1 condition and the unrelated 1 condition were equated in problem size (the sum of the two operands in both conditions was exactly the same: $M = 7.40$). The size of the incorrect results presented in the related 1 condition and the unrelated 1 condition was also similar ($M = 11.80$ and $M = 11.60$, respectively), $t(18) = 0.12$, $p = .90$. Furthermore, the distance between the incorrect result presented to the participants and the correct result of the addition problem in the two conditions of the first trial was exactly the same ($M = 4.40$). In the second trial, the problem size was equated in the related 2 condition ($M = 11.80$) and the unrelated 2 condition ($M = 11.60$), $t(18) = 0.12$, $p = .90$. In order to maintain the same problem size in the two conditions of the second trial, one addition problem in the related 2 condition (7 + 9 = 16) and one addition in the unrelated 2 condition (4 + 6 = 10) were repeated. The selection of these problems was random.

Moreover, we controlled for the degree of similarities between the addition problems presented in the first trial and those corresponding to the related 2 and the unrelated 2 condition of the second trial. The numerical distance between the incorrect result presented in the first trial and the second trial was exactly the same in the related

2 condition and the unrelated 2 condition (*M* = 1.40). The difference between the problem size in the first trial and the second trial was exactly the same in the related 2 condition and the unrelated 2 condition (*M* = 4.40). The number of repetitions between the digits presented in the first trial and the second trial (i.e., 2 was repeated in the block composed of the first trial 2 + 3 = 6 followed by 2 + 4 = 6), was exactly the same in the related 2 condition and the unrelated 2 condition (8 repetitions).

In order to check that there were no differences in response latency and accuracy when individuals answered to the addition problems used in the related 2 and unrelated 2 condition without any manipulation, we performed a pilot study (Megías & Macizo, 2015b). Participants performed a production task that contained the addition problems presented in the related 2 and unrelated 2 conditions. There were no differences in the percentage of errors associated with related 2 addition problems (13.53%) and unrelated 2 addition problems (11.59%), $F < 1$. Furthermore, there were no differences in reaction times associated to the related 2 (990 ms) and the unrelated 2 conditions (984 ms). Therefore, the two conditions of the second trial were equated.

To prevent the participants from noticing the structure of the experimental blocks (a sequence of an incorrect operation in the first trial and a correct operation in the second trial), each list of experimental blocks was randomly intermixed with 10 filler blocks of trials which were repeated four times. The correct responses in the first and second trial of these blocks were ‘yes’-’yes’, ‘no’-’no’, and ‘yes’-’no’, respectively. Therefore, the sequence of responses within each block of two trials was unpredictable through the experiment. The filler blocks included 6 addition problems and 4 multiplication problems. Before starting the arithmetic verification task, the participants performed four blocks of practice trials (2 pairs of addition problems and 2 pairs of multiplication problems) with problems that were not used in the main experiment.

Procedure. The experiment was designed and controlled by E-prime experimental software (Schneider, Eschman, & Zuccolotto, 2002). The stimuli were always presented in the middle of the screen in black color (Arial font, 40 point size) on a white background. Participants were tested individually and they were seated at approximately 60 cm from the computer screen. At this viewing distance, one character subtended a maximum vertical visual angle of 0.86 degrees and a maximum horizontal visual angle of 0.76 degrees.

The experimental task was a verification of arithmetic problems presented in blocks of two trials. Participants had to decide if the result of each problem was correct or incorrect. We used the same procedure described by Megías et al. (2014; Megías & Macizo, 2015a, 2015b) in order to make comparable the current electrophysiological experiment with behavioral studies previously done with the same paradigm: The first trial began with a fixation point in the middle of screen for 500 ms; followed by the arithmetic problem until the participant's response. After giving the answer, the second trial appeared with the same sequence of events as that of the first trial: a fixation point for 500 ms and the arithmetic problem until the participant's response. After each block of two trials, the participants were instructed to press the space bar to continue with the following block. Participants were instructed to respond by pressing the keys 'M' and 'Z', which were labeled as 'correct' and 'incorrect'. The 'correct' and 'incorrect' position assignment was counterbalanced across participants. The duration of the complete experimental session was approximately 90 minutes.

Electrophysiological recording and analysis. The EEG was recorded from 15 scalp electrodes (left frontal, F3, F1; medial frontal, FZ; right frontal, F2, F4; left central, C3, C1; medial central, CZ; right central, C2, C4; left parietal, P3, P1; medial parietal, PZ; and right parietal, P4, P2) mounted on an elastic cap according to the international 10-20 system (Jasper, 1958). The continuous electrical activity was recorded with Neuroscan Synamps2 amplifiers (El Paso, TX). The EEG was initially recorded against an electrode placed in the midline of the cap (between Cz and CPz) and re-referenced off-line against a common average reference. To control for vertical and horizontal eye movements two additional pairs of electrodes were used: a) Bipolar pairs of electrodes placed above and below the left eye and on the outer canthi, allowed blink artefact to be corrected; b) two electrodes placed in the external canthi, with one electrode on the left and another on the right side, allowed eye movements to be rejected. Each EEG channel was amplified with a band pass of 0.01–100 Hz and digitized at a sampling rate of 500 Hz. Impedances were kept below 5 kΩ.

Trials contaminated by eye movements, or amplifier saturation artefacts were rejected. Eye blinks were corrected from EEG using a voltage threshold method in which a voltage threshold was computed for each participant after a careful visual inspection between 100μV and 300μV. Afterwards, blinks were averaged using a minimum of 73 blinks for each participant and later corrected with Neuroscan Scan 4.5 software (El Paso, TX). Individual epochs were performed for each experimental condition beginning with a 100 ms pre-stimulus baseline. Average ERP waveforms were time-locked to the presentation of the arithmetic problem. Trials with incorrect responses in the arithmetic verification task were excluded from average ERP and they were submitted to the behavioral analysis of accuracy (2.01% of the data in the first trial and 3.14% of the data in the second trial). Afterward, averages in each condition of the study were comprised of a mean of 58.46 trials out of 60 trials (with a minimum of 58 trials per condition).

Statistical analyses were performed on the mean amplitude in two time windows. These time windows were established after visual inspection and were intended to evaluate two ERP components: The 170-230 ms time window was used to assess the P200 component (Jiang & Zhou, 2009; Paulmann, Bleichner, & Kotz, 2013) and the 350-450 ms time window was used to evaluate the N400 component (Carreiras, Duñabeitia, & Molinaro, 2009; Galfano, Penolazzi, Vervaeck, Angrilli, & Umiltà, 2009). For the repeated-measure analyses of variance (ANOVAs), the Greenhouse-Geisser correction (Greenhouse & Geisser, 1959) for nonsphericity of variance was used for all *F*-ratios with more than one degree of freedom in the denominator; reported are the original df, the corrected probability level, and the ε correction factor.

RESULTS

Behavioural. The reaction times (RTs) associated with correct responses were trimmed following the procedure described by Tabachnick and Fidell (2001) to eliminate univariate outliers (data points that after standardization were 3 *SD* outside the normal distribution of the data in each trial): 5.45% and 6.28% of the data were excluded in the first and second trials respectively. Since we were interested in possible differences between conditions within each trial, the two conditions of the first trial and the second trial were analyzed separately. Therefore, we report firstly the results obtained in the first trial (related 1 condition vs. unrelated 1 condition) and then the results found in the second trial (related 2 condition vs. unrelated 2 condition).

***First Trial*.** We performed ANOVAs on the RTs and percentage of errors with the variable Relation 1 (related 1 and unrelated 1) as a within-subject factor. The RT analysis showed a main effect of Relation 1, $F(1, 16) = 4.29$, $p = .05$, $\eta^2 = .21$, so that responses to related 1 trials (1074 ms, $SE = .46$) were slower than responses to unrelated 1 trials (1051 ms, $SE = .45$) (see Table 2). Moreover, the ANOVA on the percentage of errors showed a significant difference between the related 1 trials (3.14%, $SE = 1.05$) and the unrelated 1 trials (0.88%, $SE = .48$), $F(1, 16) = 6.08$, $p = .03$, $\eta^2 = .28$.

***Second Trial*.** We performed ANOVAs on the RTs and percentage of errors with the variable Relation 2 (related 2 and unrelated 2) as a within-subject factor. In the RT analysis, we found significant differences between these two conditions, $F(1, 16) = 23.73$, $p < .001$, $\eta^2 = .60$, such that responses to related 2 trials (1239 ms, $SE = .67$) were slower than responses to unrelated 2 trials (1140 ms, $SE = .62$) (see Table 2). However, the ANOVA on the percentage of errors did not show significant differences between the related 2 (3.33%, $SE = .77$) and unrelated 2 conditions (2.94%, $SE = .72$), $F < 1$.

Table 2. *Behavioral Results*

Condition		RT Diff.
	First trial	
Unrelated 1	1051 (45)	

Related 1	1074 (46)	22*
	Second trial	
Unrelated 2	1140 (62)	
Related 2	1239 (67)	99***

Note. Mean reaction times in milliseconds for each condition in first and second trial. Standard errors are reported into brackets. RT Diff.: Reaction time difference between the two conditions in milliseconds. $^{*}p < .05$, $^{***}p < .001$.

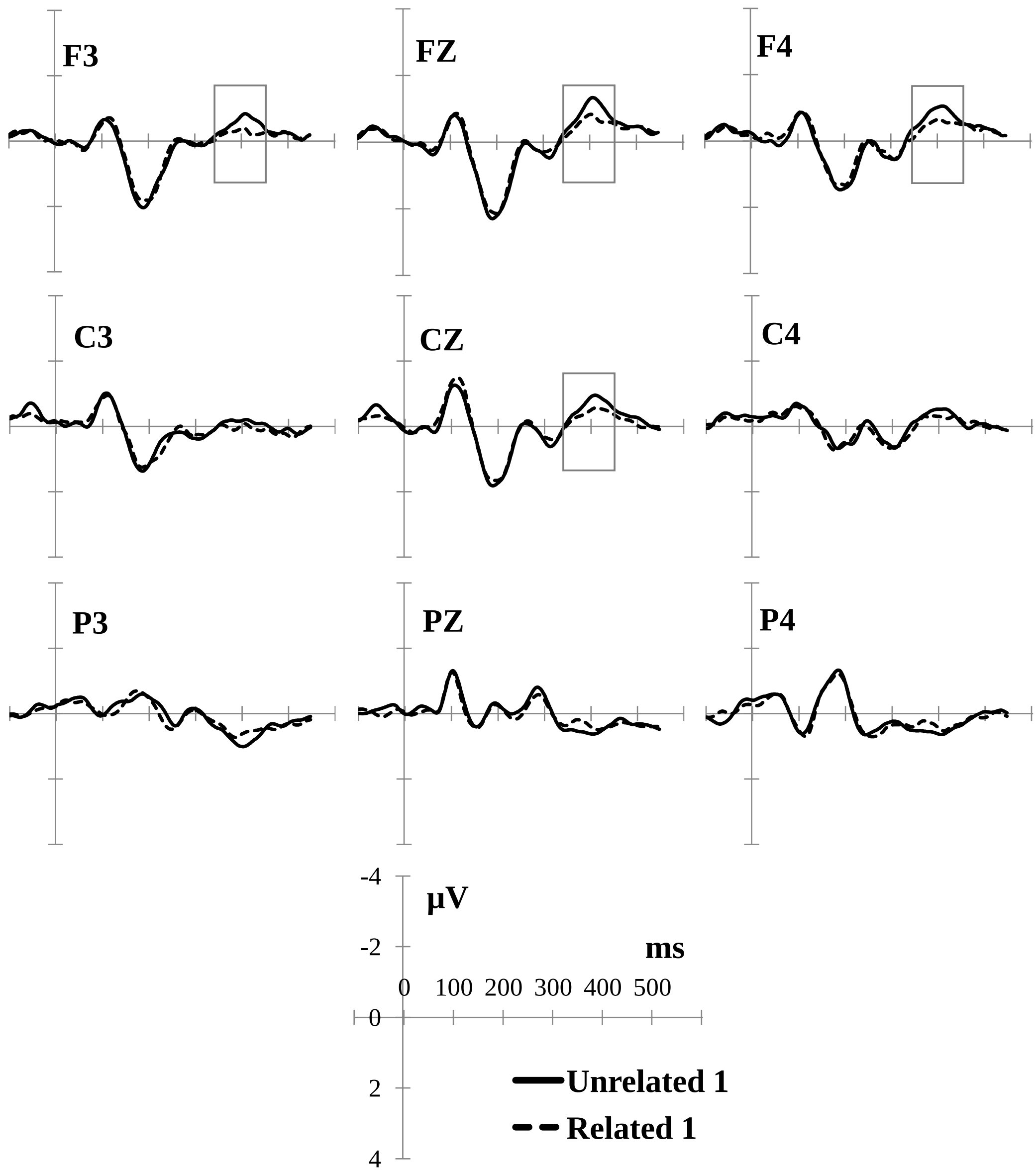
Event-Related Potentials. Analyses are reported in the same order in which each component is discussed in the introduction section, N400-like and P200. As with the behavioural data, for each component we report analysis of the first trial and then, analysis of the second trial.

N400-like component

First Trial. We performed ANOVAs on the mean amplitude in the 350-450 ms time window, with Relation 1 (related vs. unrelated conditions) and ROIs (left frontal, medial frontal, right frontal, left central, medial central, right central, left parietal, medial parietal and right parietal) as within-subject factors. The analysis showed a main effect of Relation 1, $F(1, 16) = 4.31$, $p = .05$, $\eta_p^2 = .21$. Furthermore, there was a main effect of ROIs, $F(8, 128) = 14.43$, $p < .001$, $\varepsilon = .20$, $\eta_p^2 = .47$. Importantly, the Relation 1 x ROIs interaction effect was significant, $F(8, 128) = 10.82$, $p < .001$, $\varepsilon = .38$, $\eta_p^2 = .40$. A posteriori analysis with Bonferroni correction for multiple comparisons was performed to evaluate the Relation 1 effect in all ROIs. The N400-like amplitude was less negative when participants responded to related 1 trials relative to unrelated 1 trials in the left frontal region ($p = .004$), the medial frontal region ($p = .001$), the right frontal region ($p = .008$) and the medial central region ($p = .05$). The Relation 1 effect was not significant in other regions (all ps > .53) (Figure 2).

Second Trial. The ANOVA in the second trial with Relation 2 and ROIs as within-subject factors, did not show a main effect of relation 2, $F < 1$. There was a main effect of ROIs, $F(8, 128) = 8.63$, $p = .002$, $\varepsilon = .20$, $\eta_p^2 = .35$. The Relation 2 x ROIs interaction effect was not significant, $F(8, 128) = 1.43$, $p = .24$, $\varepsilon = .44$, $\eta_p^2 = .08$.

Figure 2. Grand average ERPs for Related 1 condition (i.e., 2 + 4 = 8) and Unrelated 1 condition (i.e., 2 + 4 = 10) of the first trial.



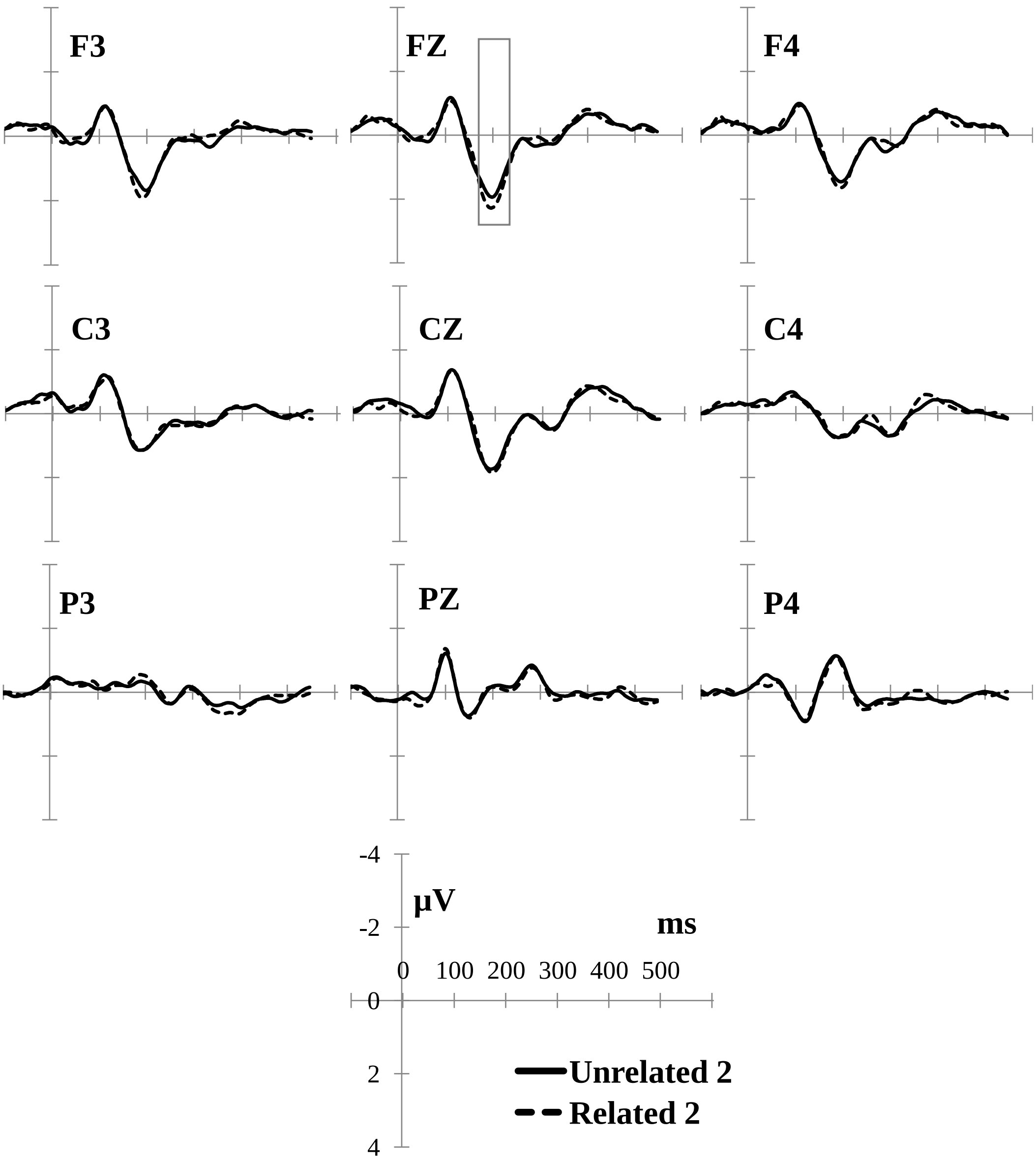
P-200 component

First Trial. The ANOVA on the mean amplitude in the 170-230 ms time window with the Relation 1 and ROIs as within-subject factors did not show a main effect of Relation 1, $F < 1$. There was a main effect of ROIs, $F(8, 128) = 16.46$, $p < .001$, $\varepsilon = .22$, $\eta_p^2 = .51$. The Relation 1 x ROIs interaction was not significant, $F < 1$.

Second Trial. We performed ANOVAs on the mean amplitude with the Relation 2 and the ROIs as within-subject factors. The analysis did not show a main effect of Relation 2, $F(1, 16) = 2.39$, $p = .14$. There was a main effect of ROIs, $F(8, 128) = 13.57$, $p < .001$, $\varepsilon = .19$, $\eta_p^2 = .46$. Moreover, the Relation 2 x ROIs interaction showed a trend toward significance, $F(8, 128) = 2.57$, $p = .07$, $\varepsilon = .34$, $\eta_p^2 = .14$. A posteriori analysis with Bonferroni corrected probabilities showed a marginal Relation 2 effect in the medial frontal region ($p = .07$). The amplitude of the P200 component seemed to be more positive in the related 2 condition compared to the unrelated 2 condition. The Relation 2 effect was not significant in any other region (all *p*s > .90) (see Figure 3).

Finally, we explored the possible relationship between the N400-like attenuation associated to the Relation 1 effect and the increased P200 positivity associated to the Relation 2 effect. To this end, we computed the N400-like effect in the first trial (related 1 vs. unrelated 1) and the P200 effect found in the second trial (related 2 vs. unrelated 2). There was a positive correlation between these two electrophysiological indexes ($r = .75$, $p = .02$). Thus, when the N400-like component increased its attenuation in the first trial, the P200 potential increased its positivity in the second trial.

Figure 3. Grand average ERPs for Related 2 condition (i.e., 2 + 6 = 8) and Unrelated 2 condition (4 + 6 = 10) of the second trial.



DISCUSSION

During the 1970s, it was observed an associative confusion effect in mental arithmetic: The verification of an addition problem presented with an incorrect result which was the result of multiplying the operands (2 + 4 = 8) was difficult to be performed (Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). It was assumed that this effect reflected the existence of an interrelated network of arithmetic facts in semantic memory: multiplication facts are activated even when individuals resolve addition problems (Ashcraft, 1992). Although this axiom has been largely assumed in cognitive arithmetic (Grabner et al., 2013; Lemaire et al., 1991; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986), direct empirical evidence needed to be offered.

In the first trial of our study, we replicated the associative confusion effect at the behavioral level. The participants took more time to verify an incorrect addition problem whose result was the one of multiplying the operands (2 + 4 = 8, the related 1 condition) compared to an incorrect addition problem whose result was unrelated (2 + 4 = 10, the unrelated 1 condition). Importantly, electrophysiological analyses helped us to determine whether this effect was associated to the spreading of activation in the network of arithmetic facts. In the 350-450 time window, N400-like amplitude was less negative in frontal-central regions in the related 1 condition relative to the unrelated 1 condition. This pattern of results corroborates that the associative confusion effect involves the co-activation of related addition and multiplication facts in semantic memory.

To our knowledge, this is the first study in which the associative confusion effect has been indexed with electrophysiological markers. However, other studies have reported N400-like modulations as evidence of coactivation of arithmetic facts (Domahs et al., 2007; Jost et al., 2004; Niedeggen & Rösler, 1996, 1999; Niedeggen et al., 1999). In these studies, an attenuation of the N400 amplitude was found along with a behavioral interference when individuals resolved a multiplication whose result was incorrect but related (it was a multiple of one operand; 5 x 8 = 32) compared to an unrelated condition (5 x 8 = 34). The critical difference between this previous evidence and the research presented here is that the former offered evidence of coactivation within-operations (several related multiplication facts are activated together) while our study demonstrates coactivation of related arithmetic facts across operations (additions and multiplications). It is important to note that the N400-like attenuation found in our study and those exploring coactivation effects in mental arithmetic are accompanied by a behavioral interference (slower responses in related problems relative to unrelated problems). The behavioral interference is interpreted as the consequence of a late competition process after the co-activation of irrelevant multiplication facts, a process that was not captured with EEG measures. The same dissociation between N400 amplitudes and response times, and a similar interpretation of this dissociation has been offered in other fields (language production, Blackford, Holcomb, Grainger, & Kuperberg, 2012). The main point to highlight from the first trial of our study is that coactivation of related arithmetic facts across operations (additions and multiplications) underlies the associative confusion effect in simple arithmetic.

In our study, we also wanted to gather electrophysiological evidence of the consequences of selecting arithmetic facts. The results found in the second trial showed that participants were slower to verify an addition problem whose result was that of multiplying the operands of the first trial (the related 2 condition: 2 + 6 = 8, preceded by 2 + 4) compared to an unrelated condition (the unrelated 2 condition: 4 + 6 = 10, preceded by 2 + 4). This interference effect has been found in previous research (Megías et al., 2014; Megías & Macizo, 2015a, 2015b) and it has been interpreted as the result of inhibiting irrelevant arithmetic facts: To resolve the competition between addition and multiplication facts in the first trial, the incorrect multiplication result (8) was inhibited in order to select the correct addition result (6). Hence, when the inhibited result was presented again and it was relevant to perform the second trial (2 + 4 = 8) an additional time was required to retrieve it from semantic memory.

When the electrophysiological pattern was considered in the second trial, we observed that the P200 amplitude was larger in the middle frontal region in the related 2 condition relative to the unrelated 2 condition. As stated in the introduction section, it is difficult to offer a unique interpretation of P200 modulations since this component is related to several cognitive processes. To illustrate, the P200 amplitude varies as a function of visual complexity of stimulus in language processing (Dehaene, 1995; McCandliss et al., 1997). However, this factor cannot account for the P200 pattern found here since the addition problems were presented in the same visual format (Arabic digits) in all conditions. Moreover, it could be argued the differences we found in the P200 amplitude were related to magnitude processing. For example, P200 amplitudes are sensitive to distance effect in comparison tasks with numbers close to the numerical standard eliciting a larger P200 amplitude than numbers far from the standard (Turconi, Jemel, Rossion, & Seron, 2004; see also Hyde & Spelke, 2009; Hyde & Wood, 2011; for P200 modulations in non-symbolic comparison tasks). Nevertheless, this explanation would not account for the results found in our study since the magnitude of the addition results presented in the second trial were equated in the related 2 and unrelated 2 conditions (problem size) as well as the distance between these results and those presented in the previous trial.

Although tentative, we suggest that P200 modulations found in our study were associated to the difficulty in the retrieval of arithmetic facts when they were irrelevant in the previous trial. As we explained in the introduction section, P200 modulations have been related to the ease to which semantic information is retrieved form semantic memory (Dunn et al., 1998; Raney, 1993; Smith, 1993). Large P200 amplitude in anterior regions is associated to the difficulty in the encoding of stimuli to access semantic memory while a posterior P200 seems to reflect the complete retrieval process in long-term memory. The medial frontal P200 effect found in the second trial of our study suggests hence, that it is difficult to encode an addition problem whose result was irrelevant in the preceding trial. Support for this interpretation comes from the correlation between the N400 modulations found in the first trial and the P200 modulations observed in the second trial. A greater N400 modulation was connected to a greater P200 effect suggesting that, the difficulty in the encoding of arithmetic problems depends on the degree to which they were activated when they were irrelevant in the preceding trial.

To conclude, this study shows that the presence of an associative confusion effect in decision times is related to N400-like modulations which support the underlying coactivation of arithmetic facts in semantic memory. Moreover, once the addition problem is resolved, P200 modulations suggest that it is difficult to encode a posterior addition problem with a result which was previously irrelevant.

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CHAPTER V

ACTIVATION AND SELECTION OF ARITHMETIC FACTS: THE ROLE OF NUMERICAL FORMAT[1]

We examined the role of numerical format in the activation and selection of arithmetic facts. We also explored the inhibitory nature of this mechanism. To this end, in two experiments we manipulated the format of the operations (digit format and word format) while participants decided whether simple additions were correct or not. In Experiment 1, when an addition was incorrect but the result was that of multiplying the operands (e.g., 2 + 4 = 8) participants took more time to respond relative to a control condition where the addition's result was incorrect but unrelated. Afterward, participants took more time to respond when the result of multiplying the operands was presented again in a correct addition problem (e.g., 2 + 6 = 8); suggesting that the related multiplication result in the previous trial (e.g., 8) was inhibited to select the correct response (e.g., 6); thus, when it was presented again in the next problem, additional time was necessary to reactivate it. These effects were found in the digit format but not in the word format. In Experiment 2, we considered the degree to which participants used memory retrieval to perform the task. In participants with high retrieval usage the interference effects in the first and second trials were larger for the digit format than for the word format. However, the participants with low retrieval usage showed interference effects only for problems with digits. These findings are discussed in terms of automaticity in retrieving arithmetic facts to perform simple arithmetic.

[1] This paper was published in *Memory and Cognition* and it was co-authored by Pedro Macizo.

ACTIVATION AND SELECTION OF ARITHMETIC FACTS: THE ROLE OF NUMERICAL FORMAT

It is widely agreed that individuals have arithmetic facts represented in long-term memory (e.g., Ashcraft, 1992; Campbell & Graham, 1985; Siegler & Shrager, 1984), which are automatically retrieved when an arithmetic problem is presented (although see Barrouillet & Thevenot, 2013; Fayol & Thevenot, 2012, for a suggestion that simple additions are resolved through procedures). To illustrate, when a simple addition problem appears (i.e., 2 + 4), due to the principle of spreading activation, there is activation of the correct answer (i.e., 6) and other results related to the operands such as the result of multiplying them (i.e., 8, Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986) or subtracting them (i.e., 2; Ashcraft & Battaglia, 1978). There is empirical evidence supporting the coactivation of several arithmetic facts when people resolve arithmetic problems (Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986): When participants perform an arithmetic verification task where they have to decided whether the result of an addition is correct or not, they show higher response latencies for false problems when the stated result is correct for the multiplication operation (i.e., 2 + 4 = 8) compared to when it is not (2 + 4 = 10). This effect was named confusion-product effect and it seems to indicate that participants coactivate the results associated to the addition and the multiplication problem. Furthermore, Lemaire, Fayol, and Adbi (1991) showed that this confusion-product effect was automatic because the multiplication answer was activated unintentionally after presenting the operands of the addition problem. Importantly, this effect disappeared when there was a 300 ms delay between the operands and the result, suggesting that people had time to resolve the competition among the correct addition result and the irrelevant multiplication result.

It has been proposed that the resolution of conflict after the coactivation of several arithmetic responses is resolved by an inhibitory mechanism (Campbell & Dowd, 2012; Campbell & Thompson, 2012; although see Censabella & Noël, 2004, for an alternative explanation). Campbell et al. (Campbell & Dowd, 2012; Campbell & Thompson, 2012) used an adaptation of the retrieval practice paradigm. This paradigm is typically employed to demonstrate the inhibition of irrelevant information that competes for selection (Anderson, 2003; Anderson, Bjork & Bjork, 1994). Participants performed a practice phase with simple multiplication problems (i.e., 2 x 3 = ?; 4 x 6 = ?) and, afterward, the same operands were used in a second test phase with simple addition problems (i.e., 2 + 3 = ?; 4 + 6 = ?). The overall finding was that practicing the multiplication problems slowed down the response times to resolve additions whose operands were presented in the practice phase relative to additions problems whose operands did not appear before. This retrieval induced forgetting effect was interpreted in terms of inhibitory processes. When participants resolved the multiplication problems in the practice phase, the addition problems which competed with the multiplications needed to be inhibited. Therefore, participants took more time to reactivate the additions when they were presented in the test phase.

In addition, recent evidence suggests that individuals apply this inhibitory mechanism in a continuous manner when competition between arithmetic facts takes place during the course of an arithmetic task (Megías, Macizo, & Herrera, 2014). Megías et al. designed an adaptation of the negative priming paradigm (Macizo, Bajo, & Matín, 2010; Tipper & Driver, 1988) in which additions were presented and participants decided whether they were correct or incorrect. The task structure comprised blocks of two trials. In the first trial, participants took more time to respond to an incorrect addition whose result was that of multiplying the operands (i.e., 2 + 4 = 8) relative to a control condition with an unrelated result (i.e., 2 + 4 = 10). This interference effect corroborated that participants activated multiplication facts when they verified addition problems. Moreover, participants took more time to respond in a subsequent trial when a correct addition was presented and its result was that of multiplying the operands of the previous trial (i.e., 2 + 6 = 8) relative to a control condition with an unrelated result (i.e., 4 + 6 = 10). This interference effect obtained in the second trial was the consequence of inhibiting the irrelevant multiplication result in the first trial. Hence, participants needed additional time to reactivate it when it was presented again and became relevant to perform the task.

The main goal of the current study was to evaluate if the coactivation of arithmetic facts and this inhibitory mechanism depended on the numerical surface format with which arithmetic problems were presented. To address this point we compared the processing of arithmetic problems presented in different numerical formats.

The role of the numerical format in mathematical cognition. There is no consensus on whether the representation of number magnitude which is needed to decide, for instance, the larger of two numbers, is format dependent. Similarly, there is no agreement on whether the representation of arithmetic facts used to resolve simple mathematical operations depends on the format in which they are presented (Cohen Kadosh, Henik, & Rubinsten, 2008).

Some models of mathematical cognition have proposed that magnitude information and arithmetic facts are *abstract representations* that do not depend on the format of the problem (i.e., the abstract-modular model, McCloskey, 1992; see also, Blankenberger & Vorberg, 1997). These models assume that regardless of the format, the processing of a numerical input involves the transcoding to an amodal representation. Hence, any difference observed between numerical formats would be located at the encoding stage of processing. For example, individuals take more time to resolve problems in the word format relative to problems in the digit format (Blankenberger & Vorberg, 1997; Campbell & Fugelsang, 2001). The amodal perspective would explain this difference as due to the additional time needed to encode the operands presented in the word format relative to the digit format.

Moreover, this amodal perspective would predict that an effect directly related to the representation of magnitude information or arithmetic facts in long-term memory would not depend on the format in which the problems are presented (i.e., McCloskey, Macaruso, & Whetstone, 1992). However, empirical data do not support this claim. To illustrate, the problem-size effect, which consists in longer reaction times and more errors when individuals resolve operations with large problem size relative to problems with small problem size (Ashcraft, 1992; Groen & Parkman, 1972), seems to depend on the numerical format of the problem (Campbell & Clark, 1988). In fact, the problem size effect is larger with operations presented in the word format relative to the same operations presented with Arabic digits. This pattern of evidence is easy to accommodate within models suggesting that the representation of arithmetic facts indeed depends on the format of the problem, for instance, the encoding-complex model (Campbell, 1992; Campbell & Clark, 1988) in which it is assumed that the processing and representation of number magnitude and arithmetic facts is format-dependent.

Therefore, although there are studies suggesting the existence of an abstract representation of numbers at the behavioral level (e.g., Dehaene & Akhavein 1995; Naccache & Dehaene 2001a; Schwarz & Ischebeck, 2000) and at the neuronal level (Dehaene 1996; Libertus, Woldrorff, & Brannon, 2007; Naccache & Dehaene, 2001b); many recent studies support the format-dependent representation of number magnitude and arithmetic facts (Bernardo, 2001; Blankenberger & Vorberg, 1997; Campbell & Alberts, 2009; Campbell & Clark, 1992; Jackson & Coney, 2007; McNeil & Warrington, 1994; although see Cohen & Dehaene, 1994; Noël & Seron, 1992; for evidence of format-independent arithmetic processing). For instance, Jackson and Coney used a priming procedure to evaluate format dependent differences in the resolution of simple arithmetic problems. Participants had to name numbers (e.g., 5) that were preceded by congruent operations (e.g., 2 + 3 = 5), incongruent operations (9 + 7 = 5) or neutral operations (X + Y = 5). The overall priming effect (congruent vs. incongruent condition) was greater when the primes were presented with digit operands than with word operands.

In short, there is previous research demonstrating reliable format dependent effects in mathematical cognition (e.g., number magnitude and arithmetic facts). Therefore, the question to be addressed is the reason for the differences observed in the resolution of mathematical operations depending on the numerical surface format of the problems. We address this point in the following section.

Format effects and automaticity. The constant and continued practice along time on a specific task allows cognitive operations to be automatic (Posner & Snyder, 1975). Automaticity can be considered a relatively low-effort cognitive process that leads to faster and more stable responding (Shiffrin & Schneider, 1977). In the field of mathematical cognition, it has been proposed that the resolution of problems presented in the digit format might be more automatic than problems presented in the word format due to the continuous practice of individuals with Arabic digits (Campbell & Epp, 2004; 2005). For example, as we commented in the previous section, the problem-size effect is larger for problems written in word format relative to problems with Arabic digits (Campbell, 1994; Campbell & Alberts, 2009; Noël, Fias & Brysbaert, 1997). The usual interpretation of this format dependent effect is that small problems are encountered more frequently in the digit format than in the word format so they are more likely to have high memory strength and to be retrieved more automatically relative to large problems.

The current study. Given that previous research has provided evidence of the role of surface format on the processing of numerical information and the resolution of arithmetic facts (Campbell, 1994; Campbell & Alberts, 2009; Lemaire & Reder, 1999; Noël, Fias & Brysbaert, 1997; Schunn & Reder, 2001; Siegler & Shipley, 1995), the main goal of the current study was to evaluate whether the coactivation of arithmetic facts and the inhibitory mechanism which seems to be responsible to select the correct answer (Campbell & Dowd, 2012; Campbell & Thompson, 2012; Megías, Macizo, & Herrera, 2014) depend on the format of the problem.

In the current study, we manipulated variables to index two processing stages during the resolution of arithmetic problems. The numerical format in which equations were presented directly tapped an encoding stage of processing, while the relationship between additions and multiplications was intended to evaluate two processes at a central level (the coactivation and the inhibition of nodes in the associative network of arithmetic facts). We expected that the activation and the selection of arithmetic facts through an inhibitory mechanism would depend on the format in which simple arithmetic problems were presented. This interaction would support the idea that encoding processes (the type of format in which the problems are presented) determines a central stage of processing (the spread of activation in the associative network represented in long term memory). To evaluate whether the format of problems determined the coactivation and selection of arithmetic facts, we used the paradigm developed by Megías et al. (2014) and we extended it to the case of arithmetic equations presented with words. The participants verified the correctness of additions presented in the digit format (i.e., 2 + 6 = 8) and the word format (i.e., two + six = eight). The first trial was intended to evaluate the automatic coactivation of multiplication facts. We expected to find longer response latency to verify incorrect additions whose result was that of multiplying the operands (2 + 4 = 8) relative to a control condition with unrelated results (2 + 4 = 10). More importantly, if we assume that the automaticity in the resolution of simple arithmetic depends on the format of the problem, this effect would be observed with problems presented in the digit format but not with problems in the word format. These results would support the idea that the coactivation of arithmetic facts depends on the format in which the operations are presented. In addition, participants would inhibit the coactivated result in the digit format so they would take additional time to verify a correct addition whose result was that of multiplying the operands of the previous trials. Again, this effect would interact with the format of the problem so it might be only observed in the digit format condition but not in the word format condition, indicating that inhibition is applied when competition between coactivated arithmetic facts takes place.

EXPERIMENT 1

In Experiment 1 we evaluated the coactivation and selection of arithmetic facts with problems presented in digit format and word format. Evidence for the coactivation of arithmetic facts with the adaptation of the negative priming paradigm was reported in Megías et al. (2014). Since the results reported by the authors with problems in the digit format were innovative, we wanted to replicate them here. Importantly, this condition was directly compared to a new condition where problems were presented using word format for numbers. This was done in order to explore the role played by presentation format in the retrieval of arithmetic facts. If the retrieval of arithmetic facts is less automatic in the case of operations presented with words relative to problems with digit operands, the coactivation of arithmetic facts would be weaker in the written word format relative to the Arabic digit format.

Method

Participants. Twenty six students from the University of Granada took part in this study. Thirteen participants (12 women and 1 man) were assigned randomly to the group of digit format. Their mean age was 20 years (*SD* = 2.22). All participants of the digit format condition were right-handed. Similarly, thirteen participants (12 women and 1 man) were assigned randomly to the group of word format. Their mean age was 21 years (*SD* = 1.99). As in the digit format group, all participants were right-handed. All the participants gave informed consent to participate in the study at the beginning of the experimental session and their participation was remunerated with academic credits. The participants completed a questionnaire to determine their use of simple arithmetic (Colomé, Bafalluy, & Noël, 2011) before performing the experimental task (see Table 1). All the participants made simple calculations on a daily basis, at least, once daily. Furthermore, *t*-tests analyses did not show differences between the participants of the digit format group and the participants of the word format group in the scores of this questionnaire, so they had the same knowledge of simple arithmetic (see Table 1). The percentage of calculation of additions on a daily basis was similar in both groups, $t(24) = 0.07$, $p = .94$ (43.46% for the digit format group and 43.08% for the word format group). Similarly, most of the participants in both groups learned the multiplication tables orally (75.38% for the digit format group and 83.08% for the word format group, $t(24) = 0.77$, $p = .45$).

Table 1. Use of simple arithmetic of participants in Experiment 1

	Digit format	Word format
Calculation frequency		
Daily	100%	100%
Once daily	15.38%	38.46%

Twice daily	30.77%	23.08%
Three o more times a day	53.85%	38.47%
Type of calculation		
Divisions	16.54%	13.08%
Multiplications	18.08%	20.77%
Additions	43.46%	43.08%
Subtractions	21.92%	23.08%
Calculation strategies		
Saying numbers mentally or aloud	26.97%	42.36%
Visualizing Arabic numbers mentally	34.04%	21.06%
Writing numbers with pencil and paper	16.59%	12.07%
With a calculator	18.41%	23.56%
Learning method (multiplication tables)		
Repeating orally	75.38%	83.08%
Exercises with Arabic numbers	13.85%	16.92%
Other methods	10.77%	0%

Note. The numerical format in which operations were presented (Digit format or Word format conditions) was manipulated as a between-participants variable.

To evaluate the participants' knowledge about multiplication tables, they performed a multiplication task in which the operands used in the main experiment were presented (i.e., 2 x 4 = ?) and they had to say aloud the correct result (i.e., 8). The mean correct responses in the multiplication task was similar in both groups (94.65% for the digit format group and 92.31% for the word format group), $t(24) = 1.08$, $p = .29$. Response times in the multiplication task were also similar in the digit and word format groups (1062 ms and 1306 ms, respectively), $t(24) = 2.02$, $p > .05$.

Design and Materials. We used a verification task in which participants received additions and they decided whether they were correct or incorrect. The format of problems, digit format (2 + 4 = 8) and word format (two + four = eight), was manipulated between-participants. The additions were presented in blocks of two trials.

In the first trial, two conditions were manipulated within-participant. The related 1 condition included incorrect additions whose result was that of multiplying the operands (i.e., 2 + 4 = 8). The unrelated 1 condition contained incorrect additions whose result was not that of multiplying the operands (i.e., 2 + 4 = 10). In the second trial, two conditions were also manipulated within-participant. The related 2 contained correct additions whose result was that of multiplying the previous trial (i.e., 2 + 6 = 8). The unrelated 2 condition included correct additions with a result which was not that of multiplying the previous trial (4 + 6 = 10). An example of trials in each experimental condition is reported in Table 2.

Table 2. Example of trials used in the experiments

Experimental condition	Digit format	Word format
	First Trial	
Related 1	2 + 4 = 8	two + four = eight
Unrelated 1	2 + 4 = 10	two + four = ten
	Second Trial	
Related 2	2 + 6 = 8	two + six = eight
Unrelated 2	4 + 6 = 10	four + six = ten

To create the experimental blocks of trials, 20 false additions were selected in the first trial (10 related 1 additions and 10 unrelated 1 additions), and 20 correct additions were selected in the second trial (10 related 2 additions and 10 unrelated 2 additions).Across participants, each addition in each condition of trial 1 (related 1 and unrelated 1) was presented half of the times followed by a related 2 addition and the other half it was followed by an unrelated 2 addition. Therefore, the related 2 and unrelated 2 additions were preceded an equal number of times by related 1 trials and unrelated 1 trials. Each participant received the experimental block of trials twice. Hence, for each participant there was a total number of 40 observations in each condition of trial 1 (related 1 and unrelated 1) and each condition of trial 2 (related 2 and unrelated 2 condition). The complete set of stimuli used in the experiment is reported in Appendix 1.

The additions used in the experimental task were carefully selected to equate them in several factors that might determine possible differences between the conditions in the first and second trial of the experiment. All additions were composed of single-digit operands. The two operands of each problem were presented in ascending order (i.e., 2 + 6) and never in descending order (i.e., 6 + 2 was not used). The parity (even and odd digits) of operands and results was equally distributed across the conditions of the first and second trial of the experimental blocks. In each trial, the solution corresponded to multiplication tables from 1 to 4 and it was never one of the two operands presented in the addition (i.e., 2 + 1 = 2 was not presented).

In the first trial, the related 1 condition and the unrelated 1 condition were equated in problem size (the sum of the two operands in both conditions was exactly the same: $M = 7.40$). The size of the incorrect results presented in the related 1 condition and the unrelated 1 condition was also similar ($M = 11.80$ and $M = 11.60$, respectively), $t(18) = 0.12$, $p = .90$. Also, the distance between the incorrect and the correct result in trial 1 was exactly the same ($M = 4.40$). In the second trial, the problem size was equated in the related 2 condition ($M = 11.80$) and the unrelated 2 condition ($M = 11.60$), $t(18) = 0.12$, $p = .90$. In order to maintain the same problem size in the two conditions of trial 2, one addition problem in the related 2 condition (7 + 9 = 16) and one problem in the unrelated 2 condition (4 + 6 = 10) were repeated. The selection of these problems to maintain this criterion was random.

Moreover, we controlled for the degree of similarity between the additions presented in the first trial and those corresponding to the related 2 condition and the unrelated 2 condition of the second trial[1]. The numerical distance between the incorrect result presented in the first and second trial was the same in the related 2 condition and the unrelated 2 condition ($M = 1.40$). The difference between the problem size in the first and second trial was the same in the related 2 condition and the unrelated 2 condition ($M = 4.40$). The number of repetitions between the digits presented in the first and second trial (i.e., 2 was repeated in the block composed of the first trial 2 + 3 = 6 followed by 2 + 4 = 6), was the same in the related 2 condition and the unrelated 2 condition (8 repetitions).

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[1]Tie problems (e.g., 4 + 4 =) are solved faster than non-tie problems (Campbell & Xue, 2001). The stimulus set in trial 1 included two tie problems that were presented in the related 1 and unrelated 1 condition so this variable was controlled for. However, the stimulus set in trial 2 was different in the related 2 and unrelated 2 condition and there was only a tie problem (i.e., 9 + 9 = 18) in the unrelated 2 condition. Thus, it could be argued that longer reaction times in the related 2 condition relative to the unrelated 2 condition might be modulated by the inclusion of this tie problem in the unrelated 2 condition only which would decrease the response time in this condition. However, analyses performed after eliminating this stimulus produced the same pattern of results as that reported in text. In Experiment 1, there was a main effect of relation, $F(1, 24) = 20.11$, $p < .001$, $\eta_p^2 = .46$, such that participants took more time to respond to related trials ($M = 1334$ ms, $SE = 27$) compared to unrelated trials ($M = 1298$ ms, $SE = 25$). Moreover, the effect of numerical format was significant, $F(1, 24) = 29.43$, $p < .001$, $\eta_p^2 = .55$, so that the word format group was slower ($M = 1453$ ms, $SE = 36$) than the digit format group ($M = 1178$ ms, $SE = 36$). Furthermore, the Relation x Format interaction was significant, $F(1, 24) = 16.07$, $p < .001$, $\eta_p^2 = .40$. Planned comparisons showed significant differences between the related ($M = 1213$ ms, $SE = 38$) and the unrelated conditions ($M = 1144$ ms, $SE = 35$) in the digit format group, $F(1, 24) = 36.07$, $p < .001$, $\eta_p^2 = .60$; but not in the word format group, $F < 1$. In Experiment 2, there was a main effect of relation, $F(1, 56) = 38.17$, $p < .001$, $\eta_p^2 = .41$, so that responses to related trials were slower ($M = 1366$ ms, $SE = 21$) compared to unrelated trials ($M = 1326$ ms, $SE = 19$). Furthermore, there was a main effect of the numerical format, $F(1, 56) = 45.37$, $p < .001$, $\eta_p^2 = .45$, so that the word format group was slower ($M = 1477$ ms, $SE = 27$) than the digit format group ($M = 1215$ ms, $SE = 27$). In the same way, there was a main effect of direct memory retrieval usage, $F(1, 56) = 12.32$, $p < .001$, $\eta_p^2 = .18$, so that the low retrieval usage group was slower to respond ($M = 1414$ ms, $SE = 27$) compared to the high retrieval usage group ($M = 1278$ ms, $SE = 27$). More important, the Format x Direct memory retrieval usage interaction effect was significant, $F(1, 56) = 5.52$, $p = .02$, $\eta_p^2 = .09$, and the Relation x Direct memory retrieval interaction was significant too, $F(1, 56) = 4.38$, $p = .04$, $\eta_p^2 = .07$.

Furthermore, we controlled for the length of the written number words when the additions were presented in word format. In the first trial, the related 1 condition and the unrelated 1 condition were equated in the number of letters of the first operand (M = 3.6 in both conditions), the number of letters of the second operand (M = 4.9 in both conditions) and the number of letters of the result presented to the participant (M = 5.6 and M = 5.4, respectively), $t(18) = .23$, $p = .82$. In the second trial, the length of the written number words was similar in the related 2 condition and the unrelated 2 condition for the first operand (M = 4.4 and M = 4.7, respectively), $t(18) = .25$, $p = .49$, the second operand (M = 4.6 and M = 4.7, respectively), $t(18) = .32$, $p = .75$, and the result of the addition problem (M = 5.6 and M = 5.4, respectively), $t(18) = .23$, $p = .82$.

In order to check that there were no differences in response latency and accuracy when individuals answered to the additions problems used in the related 2 and unrelated 2 condition without any manipulation, we performed a pilot study. We evaluated 35 students from the same population than those participating in the main experiment. The participants performed a production task that contained the addition problems presented in the related 2 and unrelated 2 conditions. In this task, the order of presentation of additions was pseudorandom, so we controlled that the result of one addition was different from the operands and the result of the preceding addition. We analyzed error percentages, mean RT, and median RT on correct responses with Relation 2 (Related 2 or Unrelated 2) as a within-participant factor. There were no differences in the percentage of errors associated to related 2 additions (13.53%) and unrelated 2 additions (11.59%), $F < 1$. Furthermore, the results on the mean RT did not show significant differences between the related 2 (990 ms) and the unrelated 2 conditions (984 ms), $F < 1$. Similarly, the median RT was equated in the related 2 condition (970 ms) and the unrelated 2 condition (946 ms), $F < 1$.

To prevent the participants from noticing the structure of the experimental blocks (a sequence of an incorrect operation in the first trial and a correct operation in the second trial), each list of experimental blocks was randomly intermixed with 10 filler blocks of trials which were repeated four times. The correct responses in the first and second trial of these blocks were ‘yes’-’yes’, ‘no’-’no’, and ‘yes’-’no’, respectively. Therefore, the sequence of responses within each block of two trials was unpredictable through the experiment. The filler blocks included 6 addition problems and 4 multiplication problems (see Appendix 2). These filler blocks were presented as Arabic digits or as written number words in the digit condition and the word condition, respectively.

Before starting the verification task, the participants performed four blocks of practice trials (2 pairs of additions and 2 pairs of multiplications) with problems that were not used in the main experiment.

***Procedure*.** The experiment was designed and controlled by E-prime experimental software, 1.1 version (Schneider, Eschman, & Zuccolotto, 2002). The stimuli were always presented in the middle of the screen in black color (Arial font, 30 point size) on a white background. Participants were tested individually and they were seated at approximately 60 cm from the computer screen.

The experimental task was a verification of arithmetic problems presented in blocks of two trials. All the problems were presented with Arabic digits (digit condition) or written number words in Spanish (word condition). Participants had to decide if the result of each problem was correct or incorrect. The first trial began with a fixation point in the middle of screen for 500 ms; followed by the arithmetic problem until the participant's response. After giving the answer, the second trial appeared with the same sequence of events as that of the first trial: a fixation point for 500 ms and the arithmetic problem until the participant's response. After each block of two trials, the participants were instructed to press the space bar to continue with the following block. Participants were instructed to respond by pressing the keys labeled as 'correct' and 'incorrect'. The duration of the experiment was approximately 25 minutes.

Results

The percentage of errors was 2.67%. Accuracy analyses were not performed due to the reduced variability of errors in two conditions of the study (only 3 out of 13 participants committed errors in the unrelated 1 condition with digit numbers and only 5 out of 13 participants committed errors in the unrelated 1 condition with verbal numbers). Data points below 200 ms and above 2000 ms were considered outliers and analyses of variance (ANOVAs) were performed on mean reaction times with trial (first and second trial) and relation (related and unrelated) as within-participant variables and the numerical format (digit vs. numerical words) as a between-participants variable. These analyses showed a main effect of relation, $F(1, 24) = 25.22$, $p < .001$, $\eta_p^2 = .51$, so that participants took more time to respond to related trials ($M = 1334$ ms, $SE = 27$) relative to unrelated trials ($M = 1294$ ms, $SE = 25$). Furthermore, there was a main effect of numerical format, $F(1, 24) = 29.29$, $p < .001$, $\eta_p^2 = .55$, such that responses in the word format group were slower ($M = 1452$ ms, $SE = 40$) in comparison to responses in the digit format group ($M = 1176$ ms, $SE = 36$). However, there was not a main effect of trial, $F < 1$. Importantly, the Relation x Format interaction was significant, $F(1, 24) = 17.61$, $p < .001$, $\eta_p^2 = .42$. Planned comparisons showed significant differences between the related condition ($M = 1213$ ms, $SE = 38$) and the unrelated condition ($M = 1139$ ms, $SE = 35$) in the digit format group, $F(1, 24) = 23.52$, $p < .001$, $\eta_p^2 = .49$. However, in the word format group, there were no differences between the related condition ($M = 1455$ ms, $SE = 38$) and the unrelated condition ($M = 1448$ ms, $SE = 35$), $F < 1$, $\eta_p^2 = .03$. Other effects were not significant (all *p*s > .27) (see Table 3).

Table 3. Results obtained in Experiment 1.

	Digit format	Word format
	First trial	
Unrelated 1	1134 (38)	1445 (38)
Related 1	1217 (39)	1442 (39)
Int. Effect	83^{***}	-3^{ns}
	Second trial	
Unrelated 2	1145 (35)	1451 (35)
Related 2	1209 (40)	1468 (40)
Int. Effect	64^{***}	17^{ns}

Note. Mean reaction times in milliseconds for each condition in the first and second trial depending on the digit and word format in which operations were presented. Standard errors pooled across the digit and word format are reported into brackets. Int. Effect: Interference effect (related condition minus unrelated condition). $^{***}p < .001$, $^{ns}p > .30$

Further analyses were performed. Firstly, we evaluated whether the interference effect depended on the problem size. To this end, the additions were categorized within each trial (trial 1 and trial 2) into small and large problems based on the size of the correct addition result (below and above the median problem size of the stimulus set). Afterwards, this problem size (large and small) was introduced in the analyses along with trial (first, second), format (digit, words) and relation (related, unrelated). The outcome of this analysis showed a significant problem size effect, $F(1, 24) = 88.44$, $p < .001$, $\eta_p^2 = .78$. Small problems were resolved faster (1236 ms) than large problems (1391 ms). However, problem size did not interact with any other variable (all *p*s > .12). Importantly, the Relation x Format interaction was significant again, $F(1, 24) = 7.76$, $p = .01$, $\eta_p^2 = .24$, indicating that after controlling for the problem size, the interference effect depended on the format of the addition problems. Secondly, we evaluated the possible relationship between the interference effect found in the first and second trial of the study. In the digit format group, the correlation was not significant, $r = -.06$, $p = .83$. In the word format condition, the correlation was not significant either, $r = -.07$, $p = .83$. Finally, the interference effect depended on the performance of participants in the experimental task. Thus, there was a negative correlation between the response time of participants to true addition problems in trial 2 and the interference effect (difference between related minus unrelated trials), $r = -.39$, $p = .05$.

Discussion

In Experiment 1 we observed interference effects that were modulated by the format in which the addition problems were presented. In the first trial, the participants in the digit group were slower in the related 1 condition relative to the unrelated 1 condition which seems to indicate that they coactivated the result of multiplying the operands. However, this effect was not observed with additions presented in the word format. Similarly, with Arabic digits, participants were slower in the related 2 condition relative to the unrelated 2 condition suggesting that they inhibited the irrelevant result of multiplying the operands of the first trial so they needed additional time to reactivate it in the second trial. Again, this effect was not found with problems presented in the word format. The results obtained with problems presented in the digit format replicate the data reported by Megías et al. (2014) with the same paradigm and surface form of the problems. Hence, the effects found with the adaptation of the negative priming paradigm seem to be a reliable phenomenon. Additionally, the interference effect did not depend on the size of the addition problems presented in the experiment which seems to indicate that overall, the problem size of addition problems we used was small so the automatic processing of additions fostered the coactivation of related nodes in the network of arithmetic facts. Moreover, in the digit condition of this experiment, the interference effects in trial 1 and trial 2 were unrelated, which seems to indicate that, in the context of the current study, the inhibition applied to select the correct solution was not proportional to the amount of conflict among coactivated arithmetic facts. Furthermore, if we consider the interference effect found in this experiment as an index of the degree of activation spreading through the network of arithmetic facts, it was related to the proficiency of participants in resolving addition problems. A stronger interference effect was associated to faster responses given to correctly resolved additions.

Importantly, the current experiment suggested that coactivation and inhibition of irrelevant arithmetic facts depend on the degree of automaticity with which problems are retrieved from memory. Problems with Arabic digits would be more automatically recovered relative to problems presented in the word format. This automatic retrieval from memory would be accompanied by the spread of activation though the network of arithmetic facts which would produce the coactivation of related nodes (i.e., multiplications) when participants verified addition problems. The results obtained in this experiment showed indeed faster response times in the digit format condition relative to the word format condition. However, the format dependent effect found here (interference effect in the digit format group only) might be due to differences in response speed between the two format groups. In fact, it has been documented that negative priming effects are modulated by the time needed to perform the task (fast vs. slow responses, e.g., Neill & Westberry, 1987). Hence, in order to evaluate whether the presence or absence of interference was due to the format of the problem and not to differences in the speed of response, further analyses were performed. For each participant, the median RT was computed and RT data above and below the median were assigned to a fast speed condition and a slow speed condition. This factor was introduced in the analyses as a within-participant variable (fast responses, slow responses) along with trial (first, second), relation (related, unrelated) and format (digit, words). None of the two-way interactions including processing speed were significant (all *p*s > .05) and the interaction among all factors was not significant either, $F < 1$. Importantly, the critical Relation x Format interaction was marginal, $F(1, 48) = 3.66$, $p = .06$, $\eta_p^2 = .07$. The interference effect was significant in the digit format group, $F(1, 48) = 7.65$, $p = .008$, $\eta_p^2 = .14$, while it was not in the word format group $F < 1$. Therefore, the absence of interference effects obtained with problems presented in the word format seems to be not explained by the slow RT of participants in this condition.

The pattern of results obtained in this experiment fits well with the idea that the automaticity in the retrieval of arithmetic facts underlies the presence of interference obtained with problems in the digit format and its absence with problems in the word format.

Nevertheless, it could be argued that the absence of interference effects obtained with word format operations were not due to a reduced automaticity in the retrieval of arithmetic facts but to the use of other ways to resolve these problems. The participants in the verbal format might be using non-retrieval strategies to verify the additions and thus, no evidence of coactivation and selection during the retrieval of arithmetic facts was observed. Previous proposals supposed that adult individuals always used direct retrieval from memory to resolve arithmetic problems such as additions and multiplications regardless of the numerical format (e.g., Ashcraft, 1992; Ashcraft & Christy, 1995; McCloskey, 1992). However, many recent studies have shown that even simple arithmetic problems might be solved with non-retrieval or procedural strategies such as counting (e.g., 4 + 3 = 4 + 1 + 1 + 1) and transformation (e.g., 4 + 7 = 4 + 4 + 3) (Campbell & Fugelsang, 2001; Fayol & Thevenot, 2012; Imbo & Vandierendonck, 2008; Metcalfe & Campbell, 2007; Thevenot, Fanget, & Fayol, 2007; see Ashcraft & Guillaume, 2009, for a review of strategies in mental arithmetic).

Campbell and Alberts (2009) evaluated whether the format of arithmetic problems (digit format and word format) influenced the degree to which participants used direct memory retrieval to resolve additions and subtractions. After performing these arithmetic problems, the participants indicated the way they resolved them. Overall, the participants used retrieval from memory to resolve problems presented in the digit format in 67% of cases while this percentage was reduced to 57% with problems presented in the word format.

Therefore, the absence of interference effects obtained in the word format group of Experiment 1 would be explained because participants were using non-retrieval (procedural) strategies to resolve the task so potential effects associated to direct memory retrieval were not observed. Thus, in order to conclude that the modulation of the interference effect by the surface format was due to automaticity (less automatic retrieval in the word format group), this modulation should be found in participants from the digit and word format groups that used retrieval from memory to perform the task. In the next experiment we addressed directly this issue.

EXPERIMENT 2

The goal of Experiment 2 was to evaluate whether differences due to the format of the problem modulated the activation and selection of arithmetic facts in participants that mainly used direct memory retrieval to resolve the task. A modulation of the interference effects due to the numerical format in these participants would indicate that differences in automaticity would be the underlying factor explaining the effect of the surface form of the problems found in Experiment 1.

Method

Participants. A new set of sixty students from the University of Granada (33 women and 27 men) took part in Experiment 2. None of them participated in Experiment 1. Thirty participants (18 women and 12 men) were assigned randomly to the group of digit format. Their mean age was 23 years (*SD* = 4.40). Twenty-eight participants of this condition were right-handed and 2 were left-handed. Thirty participants (15 women and 15 men) were assigned randomly to the group of word format. Their mean age was 23 years (*SD* = 4.45). In this group, twenty-seven participants were right-handed and 3 were left-handed. All the participants gave informed consent to participate in the study at the beginning of the experimental session and their participation was remunerated with academic credits.

In this experiment, we were interested in evaluating possible differences depending on the degree to which participants used direct memory retrieval to perform the task with problems presented in the digit format and word format. To this end, we formed a high retrieval usage group and a low retrieval usage group in each numerical format (digit and word) with the same sample size (15 participants of high retrieval usage and 15 of low retrieval usage in the digit and word format groups) by sorting the participants depending on the percentage of direct memory retrieval strategy reported after the finishing the experimental task. The criterion of selection was established according to the median value of direct memory retrieval reported by the participants in each format group. In the digit format condition, there were differences in the use of direct memory retrieval between the high retrieval usage group (90%) and the low retrieval usage group (44%), $F(1, 28) = 79.28$, $p < .001$, $\eta^2 = .74$. The same difference was found in the word format condition between the high retrieval usage group (95%) and the low retrieval usage group (50%), $F(1, 28) = 55.62$, $p < .001$, $\eta^2 = .66$. The interaction between format and strategy was not significant, $F < 1$, so the difference between the high retrieval usage group and the low retrieval usage group was similar in the two format groups.

Similarly to Experiment 1, the participants completed a questionnaire to determine their use of simple arithmetic (Colomé et al., 2011) before performing the experimental task (see Table 4). All the participants made simple calculations on a daily basis, most of the participants in both groups learned the multiplication tables orally and no differences were found in other questions regarding additions and multiplications between the two format groups (all *p* values > .12). In addition, the participants of Experiments 1 and 2 were equated in the use of simple arithmetic (all *p*s > .12).

Table 4. Use of simple arithmetic of participants in Experiment 2

	Digit format	Word format
Calculation frequency		
Daily	100%	100%
Once daily	20.00%	23.33%
Twice daily	40.00%	20.00%
Times a day	40.00%	56.67%
Type of calculation		
Divisions	15.15%	17.00%
Multiplications	19.02%	24.33%
Additions	38.75%	37.67%
Subtractions	27.08%	20.67%
Calculation strategies		
Saying numbers mentally or aloud	34.46%	41.14%
Visualizing Arabic numbers mentally	32.99%	36.31%
Writing numbers with pencil and paper	14.19%	7.35%
With a calculator	18.36%	15.20%
Learning method (multiplication tables)		
Repeating orally	80.67%	86.17%
Exercises with Arabic numbers	18.00%	13.17%
Other methods	1.33%	0.67%

Note. The numerical format in which operations were presented (Digit format or Word format conditions) was manipulated as a between-participants variable.

As in Experiment 1, we evaluated the participants' knowledge about multiplication tables with the multiplication production task. The mean correct responses in this task was similar in both format groups (92.46% for the digit format group and 91.30% for the word format group), $t(58) = 0.67$, $p = .50$. Response times were also similar in the digit and word format groups (1086 ms and 1114 ms, respectively), $t(58) = 0.37$, $p = .70$. Moreover, the percentage of correct responses in the multiplication product task was similar in participants of Experiment 1 (93.48%) and participants of Experiment 2 (91.88%), $t(84) = -1.07$, $p = .29$. Response times did not differ in participants of Experiment 1 and 2 (1184 ms and 1100 ms, respectively), $t(84) = 1.19$, $p = .24$.

Design and Materials. The experimental task and the experimental conditions in the first and second trial of this experiment were the same as those of Experiment 1. Additionally, in this experiment we gathered information about the way participants performed the task at the end of the experiment through self-reports of strategies used to resolve arithmetic problems. The participants had to indicate the degree to which they used direct memory retrieval vs. non-retrieval strategies in a seven point Likert scale from 0 (never used) to 7 (always used) to perform the experimental task.

Procedure. The same procedure employed in Experiment 1 was used here; except that in Experiment 2 participants had to indicate the degree to which they used retrieval from memory vs. nonretrieval strategies (transformation and counting) to resolve the task. The use of retrieval from memory included this explanation: when a problem such as 2 + 3 = is presented, you know from memory that 5 is the correct answer. Non-retrieval strategies included the explanation for counting (when a problem such as 2 + 3 = is presented, you count mentally from 2… 3, 4 and 5 to get the answer), transformation (when a problem such as 2 + 3 = is presented, you decompose it in other easy problems, e.g., 2 + 2 + 1) and other strategies different from those explained before.

Results

The mean percentage of incorrect responses was 3.18%. The mean percentage with which participants used direct memory retrieval over procedural strategies was 70%, and there were no differences between the digit format group (67%) and the word format group (73%), $F < 1$. Data points below 200 ms and above 2000 ms were considered outliers and analyses of variance (ANOVAs) were performed on means of the reaction times with trial (first and second trial) and relation (related and unrelated) as within-participant variables (related and unrelated conditions), numerical format as a between-participants variable (digit format vs. word format) and direct memory retrieval usage as a between-participants variable (high retrieval usage vs. low retrieval usage). There was a main effect of relation, $F(1, 56) = 53.00$, $p < .001$, $\eta_p^2 = .49$. As in Experiment 1, responses to related trials were slower ($M = 1366$ ms, $SE = 21$) than responses to unrelated trials ($M = 1321$ ms, $SE = 19$). Similarly, differences between the two numerical formats were significant, $F(1, 56) = 44.25$, $p < .001$, $\eta_p^2 = .44$, so that participants in the word format group were slower to give the response ($M = 1473$ ms, $SE = 28$) relative to participants in the digit format group ($M = 1213$ ms, $SE = 28$). Furthermore, there was a main effect of direct memory retrieval usage, $F(1, 56) = 12.31$, $p < .001$, $\eta_p^2 = .18$, such that participants in the low retrieval usage group were slower to respond ($M = 1412$ ms, $SE = 28$) in comparison to participants in the high retrieval usage group ($M = 1275$ ms, $SE = 28$). On the other hand, the Format x Direct memory retrieval usage interaction effect was significant, $F(1, 56) = 5.26$, $p = .02$, $\eta_p^2 = .09$ as well as the Relation x Direct memory retrieval interaction, $F(1, 56) = 5.59$, $p = .02$, $\eta_p^2 = .09$.

In order to further investigate these interactions including ‘direct memory retrieval’ as a variable, we conducted the same analyses performed in Experiment 1 for the two retrieval groups separately. The detailed results found in each cell of the current experiment are reported in Table 5.

Table 5. *Results obtained in Experiment 2*

	High Retrieval usage		Low Retrieval usage	
	Digit format	Word format	Digit format	Word format
		First trial		
Unrelated 1	1079 (44)	1425 (44)	1330 (41)	1489 (41)
Related 1	1128 (42)	1459 (42)	1365 (39)	1497 (39)
Int. Effect	49^{**}	34^{*}	$36^{\sim}$	8^{ns}
		Second trial		
Unrelated 2	1044 (40)	1432 (40)	1276 (35)	1493 (35)
Related 2	1148 (49)	1483 (49)	1336 (42)	1510 (42)
Int. Effect	105^{***}	51^{**}	60^{**}	17^{ns}

Note. Mean reaction times in milliseconds for each condition in the first and second trial depending on the high and low use of direct memory retrieval to resolve the task. Standard errors pooled across the digit and word format are reported into brackets. Int. Effect: Interference effect (related condition minus unrelated condition). $^{***}p < .001$, $^{**}p < .01$, $^{*}p < .05$, $^{\sim}p = .07$, $^{ns}p > .40$

High retrieval usage group. As done in Experiment 1, we performed an ANOVA with trial (first trial and second trial), relation (related condition, unrelated condition) and format (digit format vs. word format) (see Table 5). The analyses showed a main effect of relation, $F(1, 28) = 42.78$, $p < .001$, $\eta_p^2 = .60$, such that related trials were answered to more slowly (M = 1305 ms, SE = 31) than unrelated trials (M = 1245 ms, SE = 28). On the other hand, there was a significant difference between the two numerical formats, $F(1, 28) = 35.57$, $p < .001$, $\eta_p^2 = .56$, so that participants in the word format group took more time to respond (M = 1450 ms, SE = 41) than participants in the digit format group (M = 1100 ms, SE = 41). Furthermore, the Relation x Format interaction effect was marginal, $F(1, 28) = 3.58$, $p = .07$, and it was associated to a medium effect size, $\eta_p^2 = .11$. Planned comparison showed a large relation effect in the digit format group, $F(1, 28) = 35.56$, $p < .001$, $\eta_p^2 = .56$ (77 ms difference); compared to the relation effect in the word format group, $F(1, 28) = 10.81$, $p = .003$, $\eta_p^2 = .28$ (42 ms difference). The Trial x Relation x Format interaction effect was not significant, $F(1, 28) = 1.65$, $p = .21$. No other effects were significant either (all ps > .41).

Low retrieval usage group. Regarding the analysis in the low retrieval usage group, there was a main effect of relation, $F(1, 28) = 13.23$, $p = .001$, $\eta_p^2 = .32$, so that participants took more time to respond to related trials ($M = 1427$ ms, $SE = 27$) compared to unrelated trials ($M = 1397$ ms, $SE = 25$). Furthermore, there was a main effect of numerical format, $F(1, 28) = 10.85$, $p = .003$, $\eta_p^2 = .28$, so that problems in the word format were answered to more slowly ($M = 1497$ ms, $SE = 37$) than problems in the digit format ($M = 1327$ ms, $SE = 37$). In this case, the Relation x Format interaction effect was significant, $F(1, 28) = 4.46$, $p = .04$, $\eta_p^2 = .14$. Planned comparison showed a relation effect in the digit format group, $F(1, 28) = 16.53$, $p < .001$, $\eta_p^2 = .37$ (48 ms difference); whereas the relation effect in the word format group was not significant, $F(1, 28) = 1.16$, $p = .29$. The Trial x Relation x Format interaction effect was not significant, $F < 1$. No other effects were significant either (all ps > .11).

Discussion

In Experiment 2, two interference effects were obtained when participants performed the verification of addition problems. These two effects seem to indicate, firstly, that participants coactivated multiplication facts when they checked the additions and secondly, that they used an inhibitory mechanism to suppress the activation of irrelevant arithmetic facts to correctly perform the task. However, these effects depended on the numerical format of the operations even in participants that mainly used direct memory retrieval to resolve the problem. In the next section we discuss this pattern of results in terms of the automaticity in the retrieval of arithmetic facts.

In the current experiment, we evaluated the way in which participants resolved the addition problems (memory retrieval, procedural strategies), by asking them to indicate how they accomplished the task at the end of the experiment. It might be argued that a final report is not valid to determine strategy use since traces in working memory of what participants did on each trial would not be available at the end of the task. Hence, a procedure in which participants reported the strategy used to resolve each trial would be preferred. However, negative priming effects strongly depend on the inter-stimulus interval (i.e., Martín, Macizo, & Bajo, 2010). Therefore, sequential negative priming effect required that problems were solved in succession without an interfering task in-between. Moreover, in previous studies, the usual way of evaluating strategy selection is a choice procedure (e.g., Lemaire, Arnaud, & Lecacheur, 2004; Lemaire & Lecacheur, 2001, 2002; see Thevenot et al., 2007, for a critical discussion about the use verbal reports), where participants have to indicate whether they solve a problem by retrieving the result from memory or by using procedures. However, in the current study participants were asked to report the degree to which they used retrieval and procedures on a Likert scale. Hence, it could be argued also that the way in which we measured strategy selection with the use of a Likert scale was not appropriated.

We performed an additional control experiment to determine the validity of the self-report measure of strategy selection used in Experiment 2. We evaluated a new set of sixty students from the same pool that participated in Experiment 2. Individuals had to verify the correctness of all addition problems used in Experiment 1 and 2. Thirty participants performed the task with digit numbers, and another thirty with number words. Since we were not interested in the responses to the addition problems but in the strategy used to resolve them, the addition problems were randomly presented. Participants reported the strategy used to resolve each problem on a trial-by-trial basis with a two-choice procedure: After the answer to each addition, individuals decided whether they resolved it by direct memory retrieval or by a non-retrieval strategy. Furthermore, in order to compare the two-choice report measure to that used in Experiment 2, participants had to indicate at the end of the arithmetic task the degree to which they used direct memory retrieval vs. non-retrieval strategies in a seven point Likert scale from 0 (never used) to 7 (always used); a measure which was exactly the same as that used Experiment 2. We examined possible differences in the percentage of retrieval from memory usage due to the measure of strategy selection (trial-by-trial vs. final report) and the possible interaction with format (digit format vs. word format). The participants reported the use of retrieval from memory to a greater extent in the final report test (72%) than in the trial-by-trial test (65%), $F(1, 58) = 4.38$, $p = .04$, $\eta_p^2 = .07$. The retrieval from memory percentage was similar in the digit format group (69%) and the word format group (68%), $F < 1$. Importantly, the Strategy test x Format interaction was not significant, $F < 1$. In the trial-by-trial test, the percentage of retrieval from memory was 66% in the digit group and 65% in the word group. In the final report test, these percentages were 71% and 72% for the digit and word groups, respectively. Moreover, the retrieval from memory percentage obtained in the trial-by-trial test (65%) correlated with that obtained in the final report test (72%), $r = .44$, $p < .001$; and this correlation was significant in the digit format group, $r = .38$, $p = .04$; and in the word format group, $r = .49$, $p = .006$. Furthermore, when we compared the final report of this control experiment with that of Experiment 2, there were no differences in the percentage of retrieval from memory (70% and 72% respectively), nor the experiment interacted with format, *F*s < 1. Thus, the measure used in Experiment 2 to evaluate strategy selection seems to be valid for set of addition problems used in the study.

A fine-grained examination of the results obtained in Experiment 2 leaves open another question that needs to be attended: There were no differences in the usage of direct retrieval between numerical formats. In fact, the use of direct memory retrieval was similar in the digit format (67%) and the word format (73%). This finding differs from previous studies showing that direct memory retrieval is more frequently used with word operands than with digit operands (Campbell, 1994; Campbell & Alberts, 2009; Campbell, Kanz, & Xue, 1999; McNeil & Warrington, 1994). For example, Campbell and Alberts showed that when addition problems were presented in digit format, participants reported the use of direct memory retrieval more often than the use of procedural strategies (i.e., counting); while the opposite was found when participants resolved operations in the word format. The lack of differences in the use of the direct memory retrieval strategy might be due to the small size of the problems we selected (the addends produced a result equal or less than 18). In fact, there is evidence of reduced format effects on direct retrieval usage for equations with small problem size. Additionally, differences between the current study and previous research might be due to the way of manipulating the format of the problem. Thus, numerical format is usually considered as a within-participants variable (Campbell & Alberts, 2009; Campbell & Fugelsang, 2001; Campbell, Kanz, & Xue, 1999; McNeil & Warrington, 1994); while it was a between-participants factor in our study.

GENERAL DISCUSSION

The purpose of the current study was to evaluate if the retrieval and selection of arithmetic facts depended on the numerical surface format in which operations were presented. To this end, participants verified the correctness of simple additions presented either in a digit format or in a word format. The results obtained in Experiment 1 with digits showed that participants were slower to verify additions when the result was incorrect but it was that of multiplying the operands (2 + 4 = 8) relative to a control condition with an unrelated result (2 + 4 = 10). This interference effect is usually interpreted as due to the coactivation and competition of related multiplication answers when participants retrieve the addition facts needed to perform the task (Zbrodoff & Logan, 1986). Previous research has suggested that this competition is solved with the involvement of an inhibitory mechanism responsible to suppress the irrelevant arithmetic response (Campbell & Dowd, 2012; Campbell & Thompson, 2012; Megías et al., 2014). In the current study, this view would imply that participants inhibited the multiplication answer when they verified the correctness of addition problems in the first trial. As a consequence, participants would take additional time to resolve a subsequent addition when the result was that of multiplying the operands of the previous trial. In agreement with this hypothesis, participants responded more slowly to additions presented in the digit format when the result of multiplying the operands of the first trial (2 and 4) was the correct result of the problem presented in the second trial (2 + 6 = 8) relative to an unrelated condition.

Nevertheless, other explanations might account for the interference effects found in the current study when the additions were presented in the digit format. To illustrate, when an incorrect addition was presented in the first trial (e.g., 2 + 5 = 10), participants might coactivate and inhibit the multiplication fact (2 x 5 = 10) along with other addition facts to which the presented result was also true (e.g., 7 + 3 = 10). The current study cannot determine whether other addition facts were coactivated when participants performed the task. However, in our opinion, the interference effects obtained here mainly came from the competition associated to coactivated multiplication facts: Firstly, the critical difference between the related 1 trials (2 + 5 = 10) and unrelated 1 trials (2 + 5 = 14) was that the result of a related addition was exactly that of multiplying its operands while it was not the case in unrelated 1 additions. On the contrary, the possible coactivation of related additions with the same result (e.g., 7 + 3 = 10) would occur in both a related condition (2 + 5 = 10) and an unrelated condition (e.g., 3 + 4 = 10). Secondly, coactivation of arithmetic facts directly depends on the strength of connections among problems; and it is assumed that multiplications, which are learnt by rote in school, have a higher associative strength than additions (e.g., Ashcraft, 1992; Campbell & Xue, 2001). Therefore, it is reasonable to assume that coactivated multiplications might compete strongly relative to other additions potentially activated.

Another issue to be considered is that interference might not be located at the network of arithmetic facts but at the response level. Thus, in related 1 trial (2 + 4 = 8), participants might learn the association 8-false, so when a related 2 trial was presented afterwards (2 + 6 = 8), the result was associated to a true response (8-true) and thus, it was hard to overcome the previous incongruous association. Nevertheless, this explanation is difficult to reconcile with previous research showing that interference effects do not depend on the congruency of responses (same/different) in the first and second trial of a negative priming paradigm (Macizo et al., 2010, Experiment 2).

The interference effects obtained in Experiment 1 were only observed when the additions appeared in the digit format but not when they were presented with words. Hence, these results seemed to indicate that coactivation and selection of arithmetic facts was determined by the numerical surface format of problems. We argued that the format effect in the coactivation and selection of arithmetic facts was related to the degree of automaticity with which arithmetic facts are retrieved from memory. It has been proposed that the retrieval and selection of arithmetic facts is associated to practice in the solution of everyday mathematical problems (Besner & Coltheart, 1979). Individual are encountered with operations in the digit format more often than with operations in the word format. Therefore, the resolution of arithmetic problems in the digit format would be associated to an effortless processing of the task and a ballistic retrieval of arithmetic facts from memory. This explanation would imply that when participants were presented with operations in the word format the processing was less automatic so the spreading of activation in the network of arithmetic facts was reduced and thus, participants did not coactivate arithmetic facts and no inhibitory processes were needed to resolve competition. As a consequence, no interference effects were found in the first trial and second trial of Experiment 1 with equations presented with words.

Nevertheless, the absence of interference effects in the word format group might be accounted simply by the fact that participants in this group did not use retrieval from memory as the way to resolve the problems. As a consequence, no interference effects would be expected. However, when we controlled for the way in which participants performed the arithmetic task in Experiment 2, the interference effects were still modulated by the numerical format. Specifically, the participants with a high use of retrieval from memory showed the interference effect in the first and second trial of the study. However, the magnitude of these effects was smaller in the word format group (42 ms) relative to the digit format group (77 ms). Since these participants were equated in their high use of direct retrieval to resolve the problems, the differences due to the numerical format seem to be related to the automaticity in the activation of arithmetic facts.

Moreover, we considered also participants with reduced use of direct memory retrieval (less than half of cases) and the interference effects were observed again in the digit format but it was not present in the word format. This last result suggests that even when individuals used retrieval from memory to a lesser extent, the automatic access to the calculation network with digit problems sufficed to observe the interference effect due to the coactivation of arithmetic facts. In contrast, in the word format, the less automatic spreading of activation in the calculation network reduced the probability of finding this interference effect.

Together, the results found in the current study suggest that the format of arithmetic problems and the degree to which participants use retrieval from memory, determine the resolution of simple additions. Both variables work together to foster the spread of activation in the network of arithmetic facts (coactivation effects) and the subsequent selection of what is needed to resolve the problem. The highest coactivation in the network of arithmetic facts is produced when the use of retrieval predominates and the problem is presented in the digit format. On the contrary, reduced or no coactivation of arithmetic facts is observed when the use of retrieval is low and additions are presented in the word format. From this view, we can explain the interference effect associated to problems presented in the word format (Experiment 2). Even when the automaticity with which these problems are solved is low relative to problems in the digit format, interference arises when participants prefer the use of retrieval from memory to perform the task.

Implications for perspectives of arithmetic processing. The results obtained in the current study have relevant implications for current models of arithmetic processing discussed in the introduction section. Overall, the amodal view of arithmetic processing (the abstract-modular model, McCloskey, 1992; see also, Blankenberger & Vorberg, 1997) would assume that the resolution of simple arithmetic would not depend on the surface form of the problem. In contrast, from a format-dependent perspective (i.e., the encoding complex view, Campbell & Clark, 1992; Campbell & Epp, 2004) arithmetic processing and representation would vary with the surface form.

The main effect of surface format observed in this study could be accommodated within the abstract perspective (Blankenberger & Vorberg, 1997; McCloskey, 1992). The participants were faster in verifying the correctness of additions when they were presented in the digit format relative to the word format. This faster response time associated to problems presented with digits might be due to the familiarity of this format which would make easier to process the problem at the initial encoding stage of processing. Nevertheless, the results found in this study suggesting that the coactivation of arithmetic facts depended on the numerical format, is difficult to reconcile with the abstract perspective since these effects go beyond the encoding stage by impacting the retrieval of arithmetic facts. The abstract view assumes the existence of a problem-encoding mechanism to convert several numeral surface forms into a common internal code for calculation and thus, the retrieval of arithmetic facts is not expected to differ with surface form. Accordingly, regardless of the main effect of format, which could be explained by differences at the encoding stage of processing, no other differences would be observed in the coactivation of arithmetic facts across formats since coactivation is a direct image of how calculation knowledge is accessed within the network of arithmetic facts. In contrast, the surface format x relation interaction effects can be accommodated within a format-dependent perspective in arithmetic cognition (Campbell & Clark, 1992; Campbell & Epp, 2004). When individuals mainly use retrieval from memory to resolve simple addition problems, calculation is less automatic with written number words relative to problems presented with Arabic digits.

Overall, we can consider two stages of processing involved in the resolution of arithmetic problems: The encoding level where the operands and the results are processed and a central level where activation spreads in the associative network of arithmetic facts. A main contribution of the current study is the demonstration of an interactive process by which encoding and central stages do not work in an independent manner. On the contrary, we observed an interaction between numerical formats, which tapped the encoding level, and coactivation and inhibition effects which were located at the central level. This pattern of results suggests that the resolution of simple arithmetic do not involve strictly serial processes performed in isolation but it supports a dynamic view of simple arithmetic in which interactions between peripheral and central processes take place.

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CHAPTER VI

THE RETRIEVAL AND SELECTION OF ARITHMETIC FACTS IN ORAL ARITHMETIC

We examined the co-activation and the selection of arithmetic facts in oral arithmetic. In two experiments, participants had to verify whether simple additions were correct or not. In Experiment 1, additions were presented in the auditory-verbal format; in Experiment 2, additions were presented in the digit format but simulating the temporal sequence of auditory problems of Experiment 1. Results were similar in both experiments. Firstly, participants took the same time to respond when an addition was incorrect but the result was that of multiplying the operands (e.g., 2 + 4 = 8) relative to a control addition with unrelated results. Secondly, participants took more time to respond when the result of multiplying the operands of the first trial was presented again in a correct addition problem (e.g., 2 + 6 = 8) relative to a control addition. This pattern of results is discussed in terms of the temporal resolution to which auditory problems are resolved and the role of an inhibitory mechanism involved in the selection of arithmetic facts.

[1] This paper was submitted to *Acta Psychologica* and it was co-authored by Pedro Macizo.

THE RETRIEVAL AND SELECTION OF ARITMETIC FACTS IN ORAL ARITHMETIC

Simple arithmetic facts, such as one-digit additions and multiplications, are represented within an associative network in long-term memory (e.g., Ashcraft, 1992; Campbell & Graham, 1985; Siegler & Shrager, 1984). Arithmetic facts are automatically retrieved when individuals perform simple math problems (although see Barrouillet & Thevenot, 2013; Fayol & Thevenot, 2012, for a suggestion that simple additions are resolved through procedures). Moreover, individuals co-activate several arithmetic facts when they resolve a problem. The associative confusion effect supports this coactivation process (Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986): When people perform an arithmetic verification task in which they have to decide whether an addition is correct or incorrect, they show slower response latencies for false problems when the stated result is correct for the multiplication operation (i.e., 2 + 4 = 8) compared to when it is not (i.e., 2 + 4 = 10). This associative confusion effect seems to indicate that participants co-activate the facts associated to additions and multiplications automatically, even when the problem to be resolved is an addition. This co-activation would produce competition during the selection of the correct answer, since only one solution is needed to answer the problem successfully.

There is empirical evidence to support that competition produced by the co-activation of several arithmetic facts is resolved by inhibition (Campbell & Dowd, 2012; Campbell & Thompson, 2012; Megías, Macizo, & Herrera, 2014; Megías & Macizo, 2015a; 2015b; although see Censabella & Noël, 2004, for an alternative explanation). Campbell et al. used an adaptation of the retrieval practice paradigm. This paradigm is frequently employed to demonstrate the inhibition of irrelevant information that competes for selection (Anderson, 2003; Anderson, Bjork, & Bjork, 1994). Participants performed a practice phase with simple multiplication problems (i.e., 2 x 3 = ?; 4 x 6 = ?) and, afterward, the same operands were used in a second test phase with simple addition problems (i.e., 2 + 3 = ?; 4 + 6 = ?). The overall finding was that practicing the multiplication problems slowed the response times to resolve additions whose operands were presented in the practice phase relative to additions problems whose operands did not appear previously. This retrieval induced forgetting effect was interpreted in terms of inhibitory processes: When participants resolved the multiplication problems in the practice phase, the addition counterparts which competed with the multiplications needed to be inhibited. Therefore, participants took more time to reactivate these additions when they were presented in the test phase.

In addition, recent evidence suggests that individuals apply this inhibitory mechanism in a continuous manner when competition between arithmetic facts takes place during the course of an arithmetic task (Megías et al., 2014; Megías & Macizo, 2015a, 2015b). Megías et al. designed an adaptation of the negative priming paradigm (Macizo, Bajo, & Martín, 2010; Tipper & Driver, 1988) in which additions in digit format were presented and participants decided whether they were correct or incorrect. The task structure comprised blocks of two trials. In the first trial, participants took more time to respond to incorrect additions whose result was the one of multiplying the operands (i.e., 2 + 4 = 8) relative to a control condition with an unrelated result (i.e., 2 + 4 = 10). This interference effect captured the coactivation and competition of arithmetic facts (multiplication and addition problems). Moreover, participants took more time to respond in a subsequent trial when a correct addition was presented and its result was the one of multiplying the operands of the previous trial (i.e., 2 + 6 = 8) relative to a control condition with an unrelated result (i.e., 4 + 6 = 10). This interference effect obtained in the second trial was interpreted as the result of inhibiting the irrelevant multiplication result in the first trial. Hence, participants needed additional time to reactivate it when it was presented again and was relevant to perform the task.

The goal of the current study was to examine how oral arithmetic is carried out. Oral arithmetic is the preferred way of solving math problems when children learn multiplication tables (Colomé, Bafalluy, & Noël, 2011). To illustrate, Colomé et al. asked Spanish and Belgian people to report the format in which they learned the multiplication tables at school. The 63% and 66% of Spanish and Belgian participants, respectively, reported the learning of multiplication tables by rote. Similarly, most of the participants in the study by Megías et al. (2014) reported the memorization of multiplication tables orally in elementary school (77%). Hence, oral arithmetic plays an important role when individuals acquire arithmetic facts used to resolve simple addition and multiplication problems. Therefore, the underlying mechanism involved in oral arithmetic deserves to be examined in deep.

Oral arithmetic. Oral arithmetic has been considered in several models of arithmetic processing (Campbell & Clark, 1988, 1992; Dehaene, 1992; McCloskey, Sokol, & Goodman, 1986; Noël & Seron, 1992). For example, the Triple code model (Dehaene, 1992) defends that mental operations depend on the numerical format in which they are presented. Thus, oral arithmetic would be associated to specific representations in long-term memory and to specific mechanisms involved in the resolution of simple additions and multiplications. In the Abstract-modular model (McCloskey et al., 1986; McCloskey, Macaruso, & Whetstone, 1992) oral arithmetic would be similar to other ways of solving arithmetic problems (e.g., those visually presented with Arabic digits). The differences between numerical formats would be located at the encoding stage of processing. In other models, such as the Encoding-complex model (Campbell & Clark, 1988, 1992) or the Preferred entry code model (Noël & Seron, 1992), it is argued that numbers are represented in different formats in long-term memory and the use of the oral format in the resolution of arithmetic problems depends on idiosyncratic factors, as the previous arithmetic learning.

Importantly, from a neuropsychological approach, it has been found dissociation between oral arithmetic and the resolution of problems presented in other formats (Cohen & Dehaene, 1995; Martin et al., 2003; McNeil & Warrington, 1994; Salguero-Alcañiz & Alameda-Bailén, 2014). For example, in the study of Salguero-Alcañiz and Alameda-Bailén, six patients with acquired brain injury performed a written arithmetic test (e.g., two + four =) and an oral arithmetic test. Results showed a double dissociation between oral arithmetic and written arithmetic, suggesting the functional independence of the processes involved in these two types of problems. To illustrate, two patients, BET and MC, presented a lower percentage of correct responses when they had to resolve additions in the written format (66.5% and 69.5%, respectively) compared to the percentage of accuracy with oral addition problems (80% and 100%, respectively); whereas the patient MNL showed the inverse pattern: a good performance with written additions (94.5%) compared to oral additions (60%). In another study, Martin et al. (2003) investigated possible oral and written arithmetic deficits in Alzheimer disease, comparing the performance of patients at different stages of the disease (mild and moderate) and healthy elderly individuals. In general, patients had a worse performance with both oral and written problems compared to healthy participants. More interesting was the results observed in patients with moderate Alzheimer disease; they had worse performance in written arithmetic compared to patients in a mild stage of the disease; however the performance in oral arithmetic was similar in both stages of the Alzheimer disease. In line with the previous study, these results support a functional independence of written and oral arithmetic processes.

Despite the relevance of oral arithmetic, there is little research about the processes underlying the resolution of oral problems in healthy adults (LeFevre, Lei, Smith-Chant, & Mullins, 2001), relative to the large number of studies in which other verbal formats have been considered (e.g., written numbers, i.e., two + four = six; Blankenberger & Vorberg, 1997; Bernardo, 2001; Campbell, 1994; 1997; 1999; Campbell & Alberts, 2009; Campbell & Fugelsang, 2001; Megías & Macizo, 2015b; Noël, Fias, & Brysbaert, 1997; Noël, Robert, & Brysbaert, 1998). LeFevre et al. defend the necessity of exploring the resolution of problems in the auditory format because they are more ecological than problems written with number words. The authors examined if the oral presentation of operations determined the resolution processes. The authors used a multiplication production task where problems were presented in an auditory format or in Arabic digit format (e.g., 2 x 4 =) while healthy participants had to give the solution to each one. Results showed that participants committed more errors in oral problems compared to problems with Arabic digits. More important, the percentage of phonological errors (e.g., naming errors, 2 x 4 = 4; operand intrusion errors, 2 x 4 = 21) was higher in the auditory format too. The authors interpreted these results as evidence of an additional activation of phonological codes when multiplication problems were presented in the auditory format.

The current study. The current study aimed at investigating the way in which oral arithmetic is performed. Specifically, we evaluated whether the co-activation and selection of arithmetic facts as indexed by the associative confusion effect (Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986) and widely corroborated with visual arithmetic problems, also applies when additions are presented in the auditory-verbal format. To our knowledge, there is no previous research in which this phenomenon has been evaluated in oral arithmetic. We addressed this issue by using the paradigm developed by Megías et al. (2014) and we extended it to the case of problems presented orally. The first trial was intended to evaluate the automatic co-activation of multiplication facts. If oral arithmetic is done as problems with Arabic digits, we expected to find longer response latency to verify an incorrect addition whose result was that of multiplying the operands (2 + 4 = 8) relative to an unrelated condition (2 + 4 = 10). This associative confusion effect would indicate that the way in which addition facts are retrieved and related multiplication facts coactivated would be similar in digit and oral arithmetic. However, it might be possible that the coactivation and subsequent selection of arithmetic facts in oral arithmetic might differ from Arabic digit problems due to the inherent temporal sequence in which problems are processed in the auditory-verbal format. Concretely, when an individual perceives a simple addition problem orally presented, the first operand, the second operand and the result are received in a temporal ordered sequence. Hence, the participant might have time to coactivate the multiplication result before the addition result was coded. For example, after presenting the two operands (2 + 4) participants would have time to retrieve the correct addition result (6) and the competing multiplication result (8). Under this situation, the individual might have time to resolve the competition among coactivated facts before finishing the stimulus presentation (the result presented in the auditory format). Thus, no differences between the related and unrelated conditions of the first trial would be observed since coactivation and competition might be resolved already.

There is previous evidence indicating that the interference due to the coactivation of arithmetic facts disappears when participants have enough time between the presentation of the operands and the result to resolve competition. Lemaire, Fayol, and Adbi (1991) explored whether the associative confusion effect with addition problems presented with Arabic digits depended on the delay between the presentation of the operands and the result. The authors observed the interference effect when the stimulus were presented simultaneously or when a delay of 100 ms was introduced between the operands and the result. However, the interference effect disappeared when the delay increased to 300 ms or higher (500 ms).

Furthermore, the second trial of Experiment 1 was intended to evaluate the consequences of applying inhibition to resolve the competition between arithmetic facts in the first trial. We expected to find longer response latency to verify a correct addition whose result was the same of multiplying the operands of the previous trial (2 + 6 = 8, preceded by 2 + 4) relative to an unrelated condition (4 + 6 = 10, preceded by 2 + 4). This effect would indicate that participants inhibited the incorrect result in the first trial so they needed time to reactivate the inhibited answer when it was needed to resolve the second trial.

EXPERIMENT 1

Method

Participants. Thirty students from the University of Granada (26 women and 4 men) took part in this study. The mean age of participants was 20 years (*SD* = 1.79). All participants were right-handed. They did not report history of numerical or auditory problems. They gave informed consent to participate in the study and their participation was remunerated with academic credits. The participants completed a questionnaire to determine their use of simple arithmetic (Colomé et al., 2011) before performing the experimental task (see Table 1). The percentage of calculation of addition problems on a daily basis was 42.5% (*SD* = 16.18). Moreover, 74.67% (*SD* = 36.83) of participants learned multiplication tables in the auditory-verbal format by rote.

Table 1. Use of simple arithmetic of participants in both experiments

	Experiment 1	Experiment 2
Calculation frequency		
Daily	50.00%	52.27%
Weekly	43.33%	45.45%
Monthly	3.33%	2.73%
Less than once per month	3.33%	0%
Type of calculation		
Multiplications	21.50%	22.00%
Divisions	14.50%	14.02%
Additions	42.50%	40.34%
Subtractions	21.50%	23.64%
Calculation strategies		
Saying numbers mentally or aloud	37.08%	31.63%
Visualizing Arabic numbers mentally	26.10%	29.23%
Writing numbers with pencil and paper	11.95%	15.05%
With a calculator	23.58%	22.65%
Others strategies	1.30%	1.45%
Learning method (multiplication tables)		
Repeating orally	74.67%	78.14%
Exercises with Arabic numbers	24.67%	20.70%
Others methods	0.67%	1.16%

Furthermore, participants performed a multiplication task to evaluate their knowledge about multiplication tables. In this task, tables from 1 to 4 were presented in ascending order (i.e., 2 x 4 = ?) and participants had to say aloud the correct result (i.e., 8). Participants showed a good knowledge of simple multiplications, with 88.12% of correct responses (*SD* = 7.48).

Design and Materials. We used a verification task in which participants received additions and they decided whether they were correct or not. The problems were presented in auditory-verbal format, so that participants were listening simple additions in Spanish language. The additions were presented in blocks of two trials. In the first trial, two conditions were manipulated within-participant. The related 1 condition included incorrect additions whose result was that of multiplying the operands (i.e., 2 + 4 = 8). The unrelated 1 condition contained incorrect additions whose result was not the one of multiplying the operands (i.e., 2 + 4 = 10). In the second trial, two conditions were also manipulated within-participant. The related 2 condition contained correct additions whose result was the one of multiplying the operands of the previous trial (i.e., 2 + 6 = 8). The unrelated 2 condition included correct additions with a result which was not the one of multiplying the operands of the previous trial (4 + 6 = 10). An example of trials in each experimental condition is reported in Table 2.

Table 2. Example of trials used in the study

Experimental condition	Experimental trial
	First Trial
Related 1	2 + 4 = 8
Unrelated 1	2 + 4 = 10
	Second Trial
Related 2	2 + 6 = 8
Unrelated 2	4 + 6 = 10

To create the experimental blocks of trials, 20 false additions were selected in the first trial (10 related 1 additions and 10 unrelated 1 additions), and 20 correct additions were selected in the second trial (10 related 2 additions and 10 unrelated 2 additions). Across participants, each addition in each condition of trial 1 (related 1 and unrelated 1) was presented half of the times followed by a related 2 addition and the other half it was followed by an unrelated 2 addition. Therefore, the related 2 and unrelated 2 additions were preceded an equal number of times by related 1 trials and unrelated 1 trials. Each participant received the experimental block of trials twice. Hence, for each participant there were 40 observations in each condition of trial 1 (related 1 and unrelated 1) and in each condition of trial 2 (related 2 and unrelated 2). The complete set of experimental trials used in the experiment is reported in Appendix 1.

Each simple problem used in the study was recorded by a female speaker in a quiet environment and digitized at 44 kHz. Audio files for each auditory problem were edited to align the acoustic onset of the word denoting the first operand with the onset of the audio file. Furthermore, the addition problems were carefully selected to equate them in several factors that might determine possible differences between the conditions in the first and second trials of the experiment. All additions were composed of one-digit operands and the two operands of each problem were presented in ascending order (i.e., 2 + 6). The parity (even and odd digits) of operands and results was equally distributed across the conditions of the first and second trials of the experimental blocks. In each trial, the solution corresponded to multiplication tables from 1 to 4 and it was never one of the two operands presented in the addition (i.e., 2 + 1 = 2 was not presented).

In the first trial, the related 1 condition and the unrelated 1 condition were equated in problem size (the sum of the two operands in both conditions was exactly the same: $M = 7.40$). The size of the incorrect results presented in the related 1 condition and the unrelated 1 condition was also similar ($M = 11.80$ and $M = 11.60$, respectively), $t(18) = 0.12$, $p = .90$. Also, the distance between the incorrect result presented to the participants and the correct result of the addition in the two conditions of the first trial was exactly the same ($M = 4.40$). In the second trial, the problem size was equated in the related 2 condition ($M = 11.80$) and the unrelated 2 condition ($M = 11.60$), $t(18) = 0.12$, $p = .90$. In order to maintain the same problem size in the two conditions of trial 2, one addition problem in the related 2 condition (7 + 9 = 16) and one problem in the unrelated 2 condition (4 + 6 = 10) were repeated. The selection of these problems was random.

Moreover, we controlled for the amount of similarities between the additions presented in the first trial and those corresponding to the related 2 condition and the unrelated 2 condition of the second trial. The numerical distance between the incorrect result presented in the first trial and the second trial was exactly the same in the related 2 condition and the unrelated 2 condition (M = 1.40). The difference between the problem size in the first trial and the second trial was exactly the same in the related 2 condition and the unrelated 2 condition (M = 4.40). The number of repetitions between the digits presented in the first trial and the second trial (i.e., 2 was repeated in the block composed of the first trial 2 + 3 = 6 followed by 2 + 4 = 6), was exactly the same in the related 2 condition and the unrelated 2 condition (8 repetitions).

Furthermore, we controlled for the stimulus duration in each condition of the study. In the first trial, the duration of oral problems was similar in the related 1 condition and the unrelated 1 condition (M = 2217 ms and M = 2212 ms, respectively), $t(18) = 0.07$, $p = .95$. In the second trial, the duration of the problem was similar in the related 2 condition and the unrelated 2 condition (M = 2235 ms and M = 2198 ms, respectively), $t(18) = 0.43$, $p = .67$.

To avoid the participants noticed the structure of the experimental blocks (a sequence of an incorrect operation in the first trial and a correct operation in the second trial), each list of experimental blocks was randomly intermixed with 10 filler blocks of trials which were repeated four times. The correct responses in the first and second trials of these blocks were 'yes'-'yes', 'no'-'no', and 'yes'-'no', respectively. Therefore, the sequence of responses within each block of two trials was unpredictable through the experiment. The filler blocks included 6 addition problems and 4 multiplication problems which were presented orally to the participants (see Appendix 2).

Before starting the verification task, the participants performed four blocks of practice trials (2 pairs of additions and 2 pairs of multiplications) with problems that were not used in the main experiment.

Procedure. The experiment was designed and controlled by E-prime experimental software, 1.1 version (Schneider, Eschman, & Zuccolotto, 2002). The stimuli were presented through headphones. Participants were tested individually.

The experimental task was a verification of arithmetic problems arranged in blocks of two trials. All problems were presented orally to the participants in Spanish language. Participants had to decide if the result of each problem was correct or incorrect. The first trial began with a visual fixation point in the middle of screen for 500 ms; followed by the auditory arithmetic problem (e.g., 2 + 4). Afterward, the result was presented until the participant's response. After giving the answer, the second trial appeared with the same sequence of events that the first trial: a visual fixation point for 500 ms and the auditory arithmetic problem followed by the result until the participant's response. After each block of two trials, the participants were instructed to press the space bar to continue with the following block. Participants were instructed to respond by pressing the Z and M keys of the keyboard. The Z and M keys to 'correct' and 'incorrect' assignment was counterbalanced across participants. The duration of the experiment was approximately 25 minutes.

Results

Trials answered incorrectly were eliminated from the latency analysis and submitted to the accuracy analysis (1.75% of the data in the first trial, and 3.67% of the data in the second trial). Moreover, the RTs of correct responses were trimmed following the procedure described by Tabachnick and Fidell (2001) to eliminate univariate outliers (data points that after standardization were 3 *SD* outside of the normal distribution of the data). The percentage of outliers was 9.20% of the data in the first trial, and 12.50% of the data in the second trial. Firstly, we report the results obtained in the first trial and then the results found in the second trial (see Table 3).

Table 3. Results obtained in Experiment 1

		RT Diff.
	First Trial	
Unrelated 1	909 (33)	
Related 1	898 (30)	-11^{ns}
	Second Trial	
Unrelated 2	981 (46)	
Related 2	1065 (48)	84*

Note. Mean reaction times in milliseconds for each condition in the first and second trials. Standard errors are reported into brackets. RT Diff. indicates the difference in milliseconds between related minus unrelated conditions.*$p < .05$, $^{ns}p > .05$

First Trial. We performed analysis of variance (ANOVA) on reaction times and percentage of errors with relation 1 as the within-participant variable: related 1 condition (i.e., 2 + 4 = 8) and unrelated 1 condition (i.e., 2 + 4 = 10). We did not find an effect of relation 1, $F(1,29) = 1.57$, $p = .22$. So that, participants took similar time to verify related 1 trials (M = 898 ms, SE = 29.83) and unrelated 1 trials (M = 909 ms, SE = 32.53).

On the other hand, the accuracy analysis showed a main effect of relation 1, $F(1, 29) = 11.15$, $p = .002$, $\eta^2 = .28$. Participants committed significantly more errors when they had to verify related trials (M = 3%, SE = 0.72) compared to unrelated trials (M = 0.5%, SE = 0.28).

Second Trial. We performed ANOVAs on reaction times and percentage of errors with relation 2 as a within-participant variable: related 2 condition (i.e., 2 + 6 = 8) and unrelated 2 condition (i.e., 4 + 6 = 10). The results showed significant differences between the two conditions, $F(1, 29) = 15.13$, $p < .001$, $\eta_p^2 = .34$. Participants took more time to respond to related 2 trials (M = 1065 ms, SE = 48.37) relative to unrelated 2 trials (M = 981 ms, SE = 45.67).

However, the accuracy analysis did not show significant differences between related 2 (*M* = 4.17%, *SE* = 0.78) and unrelated 2 conditions (*M* = 3.17%, *SE* = 0.63), $F(1, 29) = 2.07$, $p = .16$.

Discussion

The associative confusion effect is a robust phenomenon previously described in many studies where simple problems are visually presented with Arabic digits (Grabner, Ansari, Koschutnig, Reishofer, & Ebner, 2013; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986): Participants take more time to verify incorrect additions whose result is that of multiplying the operands (2 + 4 = 8) relative to an unrelated condition (2 + 4 = 10); an effect which supports the co-activation of related arithmetic facts in long-term memory. This effect has been observed with the paradigm used in the current study when additions were presented with Arabic digits (Megías et al., 2014; Megías & Macizo, 2015a, 2015b). On the contrary, in the latency analyses, we did not find differences between the related 1 and unrelated 1 conditions of this experiment with oral presentation of additions. As we have indicated, the paradigm and the stimulus set used by Megías et al. (2014, Megías & Macizo, 2015a, 2015b) were exactly the same as those employed here. The only difference between studies was the format in which participants received the arithmetic problems. Therefore, it is reasonable to assume that the lack of effect in the first trial of the current study was due to the oral presentation of problems. We take up this argument in the next paragraph. When we examined the second trial, we observed that participants took more time to respond to additions whose results were those of multiplying the operands of the first trial (2 + 6 = 8, preceded by 2 + 4) compared to unrelated additions (4 + 6 = 10, preceded by 2 + 4). This pattern suggests that the irrelevant multiplication result was inhibited in the first trial. Thus, when it was presented again in the second trial, participants needed additional time to retrieve it from long-term memory.

One key difference between the processing of a visual and an oral problem is that the last is distributed in a temporal ordered sequence. This temporal constraint determines the processing of problems since participants would have time to activate the addition result and the related multiplication result before the problem was completely listened. This explanation might account for the presence of associative confusion effect in previous studies (Megías et al., 2014, Megías & Macizo, 2015a; 2015b) and its absence in the current experiment. In other words, participants might resolve the coactivation of arithmetic facts after listening the two operands of an oral addition so the competition would be resolved before the oral result was presented and no interference effect would be found in the reaction time analyses. This explanation agrees with studies in which the associative confusion effect is not found with visual arithmetic problems when there is a delay between the presentation of the Arabic digit operands and the addition result (Lemaire et al., 1991). In Experiment 2, we directly addressed whether temporal constraints of auditory presentation determined the differences found between oral arithmetic (Experiment 1) and visual arithmetic (Megías et al., 2014; Megías & Macizo, 2015a, 2015b) in the coactivation of arithmetic facts.

EXPERIMENT 2

This experiment aimed at determining if the lack of the associative confusion effect in the first trial of Experiment 1 was due to the temporal sequence of arithmetic problems when they were presented orally. Previous studies have corroborated the associative confusion effect when additions are visually presented with Arabic digits (Megías et al., 2014; Megías & Macizo, 2015a, 2015b; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 19860). In these previous studies the operands and the result of the problem are always presented simultaneously. On the contrary, in Experiment 2, we used visual problems with Arabic digits and we adapted the timing in which the operands and the result appeared. Thus, the operands and the result of each addition were shown for the same specific duration of each auditory problem used in Experiment 1. If the sequential presentation determined the pattern of data found in Experiment 1 with oral additions, the same results would be found here with problems presented visually. Furthermore, in the second trial of the study, which was intended to evaluate the consequences of selection-by-inhibition, we also expected to find the same results as those reported in Experiment 1: Participants would take longer time to respond to related 2 trials relative to unrelated 2 trials as sing of the time needed to overcome inhibition.

Method

Participants. A new set of forty-four students from the University of Granada (35 women and 9 men) took part in Experiment 2. None of them participated in Experiment 1. The mean age of participants was 21 years (*SD* = 2.05). Forty-three participants were right-handed and only 1 was left-handed. They did not report history of numerical or auditory problems. All the participants gave informed consent to participate in the study and their participation was remunerated with academic credits. Similarly to Experiment 1, the participants completed a questionnaire to determine their use of simple arithmetic (Colomé et al., 2011) before performing the experimental task (see Table 1). Their percentage of calculation of addition problems on daily basis was 40.34% (*SD* = 15.79), and most of participant learned multiplication tables orally (78.14%, *SD* = 29.78). More important, the participants of Experiments 1 and 2 were equated in the use of simple arithmetic (all *p*s > .05). As in Experiment 1, we evaluated the participants' knowledge about multiplication tables. The percentage of correct responses in the multiplication production task was 87.75% (*SD* = 6.78), similar to the percentage obtained by participants of Experiment 1 (88.12%), $t(72) = -.22$, $p = .83$.

Design and Materials. The task and the experimental conditions in the first and second trials of this experiment were the same that those of Experiment 1. The only difference was that the arithmetic problems were presented with Arabic digits in this experiment.

Procedure. In order to simulate the temporal sequence in which oral problems were presented in Experiment 1, the audio file associated to each problem was carefully examined and then, it was divided in the constituents of the problem. We computed the duration of the first operand, the symbol (+) and the second operand. Furthermore, we took the duration of the oral results of problems in Experiment 1.

The duration of the additions (first operand, symbol and second operand) was equated in the two conditions of the first and second trials. In the first trial, there were not differences between related 1 and unrelated 1 conditions (M = 1663 ms and M = 1659 ms, respectively), $t(18) = 0.08$, $p = .94$. Similarly, in the second trial, there were not differences between related 2 and unrelated 2 conditions (M = 1638 ms and M = 1641 ms, respectively), $t(18) = -0.04$, $p = .97$. We also controlled for the duration of the results in the two conditions of trial 1 and 2. In the first trial, there were no difference between the related 1 condition and the unrelated 1 condition, $t(18) = .14$, $p = .89$. In the second trial, the duration of the result problem was similar in related 2 condition and the unrelated 2 condition (M = 551 ms and M = 543 ms, respectively), $t(18) = .13$, $p = .90$.

In Experiment 2, the stimuli (problems presented with Arabic digits) were always presented in the middle of the screen in black color (Arial font, 30 point size) on a white background. As in Experiment 1, the first trial began with a fixation point in the middle of screen for 500 ms. Afterwards, the first operand, the symbol and the second operand were presented in succession with the same durations as those of the same elements (in the same problem) used in Experiment 1. The mean duration of these elements were M = 370 ms (SD = 59.52) for the first operand, M = 441 ms (SD = 75.25) for the symbol (+), and M = 850 ms (SD = 102.17) for the second operand. After the end of the second operand, the result of the problem was presented until the participant's response. Once the participant responded to the first trial, the second trial appeared with the same sequence of events as that of the first trial: a fixation point for 500 ms, the operand 1 (M = 526 ms, SD = 124.42), the symbol (M = 333 ms, SD = 84.47), the operand 2 (M = 781 ms, SD = 168.39), and the result until the participant's response. Other details were similar to those of Experiment 1.

Results

The percentage of data eliminated from the latency analysis and submitted to the accuracy analysis was 6.19% in the first trial and 5.11% in the second trial. Furthermore, the trimming procedure was the same that in Experiment 1 (data points that after standardization were 3 *SD* outside of the normal distribution of the data, Tabachnick & Fidell, 2001). The percentage of outliers was 8.36% in the first trial, and 11.53% in the second trial. As in Experiment 1, we performed ANOVAs on reaction times and percentage of errors separately for the first and second trials (see Table 4).

Table 4. Results obtained in Experiment 2

		RT Diff.
	First Trial	
Unrelated 1	764 (23)	
Related 1	756 (23)	-8^{ns}
	Second Trial	
Unrelated 2	786 (26)	
Related 2	729 (22)	58*

Note. Mean reaction times in milliseconds for each condition in the first and second trials. Standard errors are reported into brackets. RT Diff. indicates the difference in milliseconds between related minus unrelated conditions. *$p < .05$, $^{ns}p > .05$

First Trial. ANOVA on reaction times with relation 1 as a within-participant variable did not show significant differences between related 1 condition (M = 756 ms, SE = 23.21) and unrelated 1 condition (M = 764 ms, SE = 23.45), $F < 1$. As in Experiment 1, participants took the same time in verifying related 1 trials (i.e., 2 + 4 = 8) and unrelated trials (i.e., 2 + 4 = 10).

On the other hand, the ANOVA on percentage of errors showed differences between related 1 condition (M = 9.26%, SE = 1.43%) and unrelated 1 condition (M = 3.13%, SE = 0.77), $F(1, 43) = 25.50$, $p < .001$, $\eta_p^2 = .37$, so that participants committed a higher percentage of errors when the result presented with the addition problem was the same of multiplying their operands (i.e., 2 + 4 = 8) in comparison to unrelated trials (i.e., 2 + 4 = 10).

Second Trial. ANOVA on reaction times with relation 2 as a within-participant variable showed significant the difference between related 2 trials (*M* = 786 ms, *SE* = 25.97) and unrelated 2 trials (*M* = 729 ms, *SE* = 22.02), $F(1, 43) = 16.77$, $p < .001$, $\eta_p^2 = .28$. Participants took more time to respond when the result coincided with the one of multiplying the operands of the previous trial (i.e., 2 + 6 = 8) compared to trials in which the result was not the multiplication of the previous trial (i.e., 4 + 6 = 10).

However, accuracy analysis did not show difference between the related 2 condition (*M* = 5.34%, *SE* = 0.79) and the unrelated 2 condition (*M* = 4.89%, *SE* = 0.86), $F < 1$.

Discussion

The pattern of results found in the current study paralleled that of Experiment 1. In the first trial, participants took the same time to respond to additions whose result was that of multiplying the operands (2 + 4 = 8) compared to unrelated additions (2 + 4 = 10). In the second trial, additions whose results were those of multiplying the operands of the first trial (2 + 6 = 8, preceded by 2 + 4) were answered more slowly than unrelated additions (4 + 6 = 10, preceded by 2 + 4).

Previous studies on mental calculation with Arabic digits in which the operands and the result of the problem are presented simultaneously show the associative confusion effect (Megías et al., 2014, Megías & Macizo, 2015a; 2015b; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). In contrast, when we presented the Arabic digit operands and the result in a sequential manner no interference effect was found. The temporal order in which the Arabic digit problems were presented in Experiment 2 paralleled that used in Experiment 1 with oral problems. The similar pattern of results found in both experiments suggests that participants had time to retrieve addition and multiplication facts and they resolved competition before finishing the presentation of the result of the problem. We discuss this point further in the next section.

GENERAL DISCUSSION

When individuals resolve an addition problem, they coactivate related multiplication facts in memory which increases the time needed to resolve the operation. For example, when individuals receive an incorrect addition (e.g., 2 + 4 = 8), they take longer to decide that it is wrong since the result (8) is that of multiplying the operands (2 and 4). This associative confusion effect has been observed in many studies when the problems contain Arabic digits (Grabner et al., 2013; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). Furthermore, individuals seem to inhibit irrelevant arithmetic facts in order to select the one needed to resolve the problem. Thus, participants respond more slowly when another addition with the inhibited result is presented afterwards (2 + 6 = 8, preceded by 2 + 4) (Megías et al., 2014; Megías & Macizo, 2015a, 2015b).

The coactivation and selection of arithmetic facts in simple arithmetic has been explored with problems whose operands and results are presented in Arabic digit format (Megías et al., 2014; Megías & Macizo, 2015a) and written number words (Megías & Macizo, 2015b). The study with Arabic digits is justified since it is the most conventional format in simple arithmetic. However, the study of the auditory format is of interest since children typically learn to do simple arithmetic orally in elementary school (Colomé et al., 2011). Moreover, different theories about arithmetic representation highlight the relevance of the format in which problems are coded (Campbell & Clark, 1988, 1992; Dehaene, 1992; McCloskey, Sokol, & Goodman, 1986; Noël & Seron, 1992). Thus, the current study was intended to fill the gap in the study of the processes underlying oral arithmetic.

The results of Experiment 1 with problems presented in auditory format showed similar response times in problems where the result was the one of multiplying the operands (2 + 4 = 8) compared to an addition problem whose result was unrelated (2 + 4

= 10). The absence of associative confusion effect in Experiment 1 contrasts with many studies in which the effect is found with Arabic digits. Since this effect indexes processes associated to representation of arithmetic facts, the different pattern of results found with oral problems in Experiment 1 seems to support models in which format effects influence the processing beyond encoding stages of analysis (e.g., the encoding complex model, Campbell & Clark, 1988, 1992; Dehaene & Cohen, 1995). Other studies have supported format dependent processing of arithmetic facts too (e.g., Campbell, 1994; 1999). To illustrate, the problem size effect (responses are faster for problems with small vs. large operands) is larger for verbal versus digit format operations. Since the problem size effect is associated to the retrieval of arithmetic facts, the format x size interaction would indicate that the processing of mental arithmetic is format dependent.

The results found in Experiment 1 with additions in the auditory format are difficult to reconcile with encoding accounts of simple arithmetic (McCloskey, 1992). According to this view, format effects are attributed entirely to differences at the encoding level where the input is translated to an abstract representation to access the answer of the problem. Thus, under this perspective, the same associative confusion effect found with Arabic digits (Megías et al., 2014; Megías & Macizo, 2015a, 2015b) should be observed with problems in the auditory format since format dependent effects would be restricted to the analyses of the auditory input only. In other words, the encoding view assumes an additive processing in which the encoding stage is functionally independent of the retrieval stage. On the contrary, the encoding complex model (Campbell, 1994; 1999) assumes an interactive viewpoint, so encoding conditions might affect the subsequent retrieval of arithmetic facts. The results found in Experiment 1 seem to support this interactive perspective. Arithmetic problems in the auditory format are received in a temporal sequence and this encoding condition constrained the way to which arithmetic facts were retrieved. In fact, we considered this characteristic of the auditory input as the critical factor that determined the results found in the first trial of Experiment 1. When participants were listening the two operands of the addition problem, they were able to retrieve the solution of the problem as well as the multiplication counterpart. Thus, before finishing the listening of the auditory problem, the participants already solved the competition between coactivated arithmetic facts and no differences were found between the related and unrelated condition of the first trial. The results found in Experiment 2 support this argumentation. The associative confusion effect has been replicated in several studies with Arabic digits presented with the same paradigm as that used here (Megías et al., 2014; Megías & Macizo, 2015a, 2015b). However, the effect was not found in Experiment 2 when the problems were presented in the Arabic digit format simulating the timing in which auditory problems were listened in Experiment 1 (temporal sequence of operands and solution). The absence of associative confusion effect with Arabic digits presented in a sequence of operands and results agrees with previous observations (Lemaire et al., 1991): When there is a sufficient delay between the presentation of operands and result (300 ms and longer time intervals) participants do not show interference due to the concurrent coactivation of arithmetic facts.

The assumption that participants retrieved arithmetic facts before the addition solution was presented in Experiment 1 and 2 requires further elaboration. Since the two addition operands were presented in a sequential manner, the access to arithmetic facts might took place during the processing of the first operand or the second operand. Previous research supports the second alternative. Zhou et al. (2007) examined event-related potentials elicited by single-digit problems to evaluate the access to arithmetic facts. To this end, they considered the operand-order effect as evidence of arithmetic fact retrieval: Participants take shorter times to respond to smaller-operand-first problems (e.g., 2 x 8) than to larger-operand-first problems (e.g., 8 x 4). In this study, multiplication problems were presented in auditory format and participants were asked to decide whether the proposed answer was correct or not. The results did not show operand-order effect during the presentation of the first operand. On the contrary, this effect appeared as early as 120 ms after the onset of the second operand (a greater negativity for large-operand-first problems relative to smaller-operand-first problems). Hence, these results suggest that the processing of the first operand did not lead to automatic activation of arithmetic facts. However, the retrieval of arithmetic facts might begin before the auditory second operand was finished.

Future research will shed light on whether the coactivation of arithmetic facts with problems in the auditory format takes place during the processing of the second operand. Nevertheless, the studies commented above seems to suggest that when participants resolve problems whose operands and result appears one at a time (auditory format and Arabic digits presented in a sequential manner) arithmetic facts are quickly accessed. Moreover, the results obtained in the second trial of Experiment 1 and 2 suggest that participants coactivated addition and multiplication arithmetic facts in the first trial so they used an inhibitory process to select the correct addition result. Participants took more time to respond when the result of multiplying the operands of the first trial was the correct result of the second trial (2 + 6 = 8, preceded by 2 + 4) relative to an unrelated condition (4 + 6 = 10, preceded by 2 + 4). Under the inhibitory account, participants suppressed the irrelevant result in the first trial, so an additional time was needed in the second trial to reactivate it and to answer the arithmetic problem correctly. This interference effect corroborates that found in many other studies with the same paradigm and suggests that inhibition is the underlying mechanism responsible to suppress irrelevant solutions in simple arithmetic.

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CHAPTER VII

SIMPLE ARITHMETIC DEVELOPMENT IN SCHOOL AGE: THE COACTIVAION AND SELECTION OF ARITHMETIC FACTS[1]

We evaluated the possible inhibitory mechanism responsible to select arithmetic facts in children from 8-9 to 12-13 years of age. To this end, we used an adapted version of the negative priming paradigm (NP paradigm) in which children received additions and they decided whether they were correct or not. When an addition was incorrect but the result was that of multiplying the operands (e.g., 2 + 4 = 8) only children from 10-11 years of age onwards took more time to respond compared to control additions with unrelated results, suggesting that they coactivated arithmetic knowledge of multiplications even when it was irrelevant to perform the task. Furthermore, children from 10-11 years of age onwards were slower to respond when the result of multiplying the operands was presented again in a correct addition problem (e.g., 2 + 6 = 8). This result showed the development of an inhibitory mechanism involved in the selection of arithmetic facts through formal education.

[1] This paper was published in *Journal of Experimental Child Psychology* and it was co-authored by Pedro Macizo.

SIMPLE ARITHMETIC DEVELOPMENT IN SCHOOL AGE: THE COACTIVATION AND SELECTION OF ARITHMETIC FACTS

In the field of numerical cognition, it is assumed that arithmetic facts are stored in long-term memory within an associative network whose nodes are interrelated. When a simple problem is presented (i.e., an addition, 2 + 4), nodes that represent the operands of the problem (2 and 4) and that representing the solution (6) are activated, so people can provide the correct solution directly by retrieving the problem from memory (Campbell & Graham, 1985).

In early years of schooling, children expend a lot of time memorizing multiplication facts to solve the problems by the retrieval of the stored answers in memory (Siegler, 1986). Several studies have evaluated the changes in the use of retrieval of arithmetic facts with age (Cooney, Swanson & Ladd, 1988; Imbo & Vandierendock, 2007, 2008; Lemaire & Siegler, 1995). Imbo and Vandierendock (2008) examined this question in children from 2^{nd}, 4^{th} and 6^{th} grade of elementary school; and showed that the use of retrieval from memory to resolve multiplication and addition problems increased with the educational level in a progressive manner (60%, 80.5% and 79.5% in 2^{nd}, 4^{th} and 6^{th} grade, respectively). Hence, the probability of using the retrieval from memory seems to increase when children advance in educational cycles.

When the associative network of arithmetic facts is established, adult individuals coactivate several related nodes during the resolution of mathematical problems (Ashcraft, 1992). Hence, when two operands of an addition are presented (i.e., 2 + 4), the result of multiplying the two operands (i.e., 8) is activated even when the multiplication result is not needed to resolve the addition problem. Thus, when individuals are solving additions, the coactivation of the arithmetic fact associated to the multiplication would compete and an interference effect would be observed so the participants' performance in the addition task would be impaired.

There is empirical evidence to support the concurrent coactivation of arithmetic facts associated to additions and multiplications in adults (Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986; see Grabner, Ansari, Koschutnig, Reishofer & Ebner, 2013, for the neural correlate of this coactivation effect). One procedure frequently used to corroborate this coactivation is the verification of additions (Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). In this task, a simple addition is presented (i.e., a pair of one-digit operands and a result) and participants have to decide whether the result is the correct solution of the addition problem. The critical trials are those associated to negative responses (incorrect addition problems). In these trials, participants show an interference effect, so they take more time to respond when the result presented with the addition is incorrect but it is the one of multiplying the operands (2 + 4 = 8) relative to a control condition in which the result is unrelated (2 + 4 = 10). This longer reaction time when the incorrect addition result is the one of multiplying the operands has been taken as an index of the simultaneous activation of addition and multiplication arithmetic facts (Grabner et al., 2013; Lemaire, Fayol, & Adbi, 1991; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986).

With regard to child population, Lemaire et al. (1991) found that children from 4^{th} to 5^{th} grade (9-10 year-old children) already presented interference effects due to the coactivation of arithmetic facts when they had to verify simple addition and multiplication problems. Moreover, these authors showed that this phenomenon was partially automatic because the activation of related answers was produced unintentionally in all cases after the presentation of the operands; and only *partially* because the coactivation effect disappeared when the experimental procedure included a delay between the operands and the result of the problem above 300 ms or 500 ms. This abolishment of the coactivation effect depended on grade: children of 4^{th} grade showed the interference effect when the delay between the operands and the result was 500 ms, but not with a small delay (300 ms); whereas the interference effect was eliminated with both delays in 5^{th} grade children. These results suggest that when educational grade increases, the strength of associations between operands and answers becomes stronger. Thus, children at a higher grade can select the correct answer quickly. Furthermore, the authors proposed that suppression of irrelevant answers underlie the selection of correct arithmetic facts, however, they did not explore this conclusion. The current study aimed to address this mechanism and its development with age.

It is difficult to answer to a problem whose result is the one of multiplying their operands (i.e., 2 + 4 = 8). Nevertheless, people are able to resolve it correctly most of the time (i.e., to say that 2 + 4 = 8 is incorrect). It has been proposed that the conflict produced by the coactivation of several arithmetic facts is solved by an inhibitory mechanism (Campbell & Dowd, 2012; Campbell & Thompson, 2012; although see Censabella & Noël, 2004, for an alternative explanation). Campbell and colleagues (Campbell & Dowd, 2012; Campbell & Thompson, 2012) used an adaptation of the retrieval practice (RP) paradigm typically employed to demonstrate the inhibition of irrelevant information in adult population (Anderson, 2003; Anderson, Bjork, & Bjork, 1994). In these studies, participants performed a practice phase of simple multiplication problems (i.e., 2 x 3 = ?; 4 x 6 = ?) and, afterward, the same operands were used in a second test phase with simple addition problems (i.e., 2 + 3 = ?; 4 + 6 = ?). The overall finding was that practicing the multiplication problems slowed the response times to solve additions whose operands were presented in the practice phase relative to the response to addition problems whose operands were not presented previously. This retrieval induced forgetting (RIF) effect was interpreted in terms of inhibitory processes. When participants solved the multiplication problems in the practice phase, the addition problems which competed with the multiplications needed to be inhibited. Therefore, participants took more time to reactivate the additions when they were presented in the test phase.

Moreover, recent evidence suggests that this inhibitory mechanism acts in a continuous manner in order to reduce interference when competition between arithmetic facts takes place (Megías, Macizo, & Herrera, 2014). Megías et al. designed an adaptation of the negative priming paradigm (NP paradigm) (Macizo, Bajo, & Martín, 2010; Tipper & Driver, 1988) to address this issue. The original NP paradigm included two trials (Tipper, 1985; Tipper & Driver, 1988). In the first trial, two objects were presented simultaneously (prime stimuli) and participants had to ignore one of the objects while attending to the other. In the second trial, two probe stimuli were presented, the ignored object (the test stimulus) or a new object (the control stimulus) and participants had to categorize them. The results showed that with probe stimuli, the participants took more time to respond to the ignored object relative to the control object, suggesting that the ignored object was inhibited in the first trial and more time was needed to reactivate it when it appeared again in the second trial.

Several adapted versions of the NP paradigm have been used previously to evaluate the role of inhibitory processes when children use strategies to perform logical-mathematical tasks (Borst, Poirel, Pineau, Cassotti, & Houdé, 2012; Houdé & Borst, 2014; Houdé & Guichart, 2001; Lubin, Vidal, Lanoë, Houdé, & Borst, 2013; Perret, Paour, & Blaye, 2003). To illustrate, Houdé and Guichart (2001) adapted the NP paradigm to Piaget's conservation of number task. A set of 9 year-old children were presented with prime stimuli (two rows that contained the same number of items but differed in length) and they had to indicate whether the rows were numerically equivalent or not. To respond correctly to this task, children had to inhibit a strategy based on the length and realize that both rows had the same number of items. When probe stimuli were presented afterwards, children had to activate the inhibited strategy (the test stimulus) to resolve a problem where the length coincided with the number of items. In this situation, a negative priming effect was found, so that additional time was needed to reactivate the inhibited strategy again.

Megías et al. (2014) adapted the NP paradigm to evaluate the possible inhibitory mechanism responsible to resolve simple arithmetic facts. The authors used an arithmetic task in which additions were presented and adult participants decided whether they were correct or incorrect. The stimuli that participants had to ignore were the results associated to multiplying the addition operands. The task structure comprised blocks of two trials. In the first trial, in which prime stimuli were presented, the participants took more time to respond to incorrect additions whose result were those of multiplying the operands (i.e., 2 + 4 = 8) relative to a control condition with unrelated results (i.e., 2 + 4 = 10). This interference effect corroborated that participants activated multiplication facts when they verified addition problems. In the second trial, in which two types of probe stimuli were presented, the participants took more time to respond when a correct addition was presented and the result was the one of multiplying the operands of the previous trial (the test stimulus, i.e., 2 + 6 = 8 preceded by 2 + 4) relative to a neutral condition (the control stimulus, i.e., 4 + 6 = 10 preceded by 2 + 4). This interference effect obtained in the second trial was interpreted as the consequence of inhibiting the irrelevant multiplication result in the first trial. Hence, participants needed additional time to reactivate the inhibited result (i.e., 8) when it was presented again and it was the one needed to perform the task (i.e., 2 + 6 = 8).

This inhibitory mechanism to select and resolve simple arithmetic facts has not been evaluated in children to date, and there are reasons to think that it develops over the course of formal instruction in arithmetic. Firstly, the ability of inhibitory control emerges early in the third year of life (Gerstadt, Hong, & Diamond, 1994), reaching its greater development at the first stages of elementary school, when children are between 6-8 years of age (Davidson, Amso, Anderson, & Diamond, 2006; Gerstadt et al., 1994; Korkman, Kemp, & Kirk, 2001). Secondly, it has been observed the relationship between inhibitory control and mathematical skills in children (Adams & Hitch, 1997; Bull, Johnston, & Roy, 1999; Bull & Scerif, 2010; Fürst & Hitch, 2000; Geary, Hamson, & Hoard, 2000; McLean & Hitch, 1999; Van der Sluis, de Jong, & Van der Leij, 2004). Bull and Scerif (2010) evaluated 7 year-old children in both inhibitory control functions and mathematic ability, which included simple and multi-digit additions and subtractions. The authors showed that children with low vs. high mathematical ability had more difficulty suppressing the automatic activation of the irrelevant dimension in the Stroop task and they had more perseveration errors in the Wisconsin Card Sorting task (they maintained learned strategies which interfered with the demands of the test). In the same line, Van der Sluis et al. (2004) found that arithmetic-disabled children from 4th and 5th grade relative to a similar age-control group had difficulty in a number Stroop task to suppress the prepotent answer of naming a digit (e.g., to say 2 when 222 was presented) when they had to name the number of digits presented (e.g., to say 3). Moreover, recent research has shown that inhibitory control processes are involved in other aspects of children’s numerical cognition such as the resolution of arithmetic word problems (Lubin, Vidal, Lanoë, Houdé, & Borst, 2013) and the resolution of non-symbolic numerical tasks (Gilmore, Attridge, Clayton, Cragg, Johnson, Marlow, Simms, & Inglis, 2013).

The current study. The main aim of the present research was to establish the developmental course of inhibition as the underlying mechanism used by children to select arithmetic facts[1]. However, since inhibition acts when there is competition between arithmetic facts, in this study we also adressed the development of coactivation and competition of arithmetic facts with age. As we have argued before, there are reasons to believe that the establishment of an associate network to represent arithmetic facts might develop through formal learning. Similarly, the possible inhibitory mechanism involved in the selection of several arithmetic facts could develop with formal instruction in arithmetic at schooling.

In order to trace emerging inhibitory control as children become increasingly skilled in arithmetic, we selected a sample of children from three different educative cycles, namely 8-9 year-old children (second cycle of elementary school), 10-11 year-old children (third cycle of elementary school) and 12-13 year-old children (first cycle of high school). The rationale behind selecting these particular groups of children was based on research about the developmental trajectory of arithmetic cognition: When children are 8-9 year-olds, in second cycle of elementary school, the mental structure of addition facts and the processes responsible to select and retrieve the correct answer are being implemented gradually through formal instruction in arithmetic (Ashcraft, 1982; Ashcraft & Fierman, 1982; Ashcraft, 1992).When they are 10-11 year-olds, in third cycle of elementary school, children seem to have the associate network of arithmetic facts fully established relative to younger children (De Brauwer, Verguts, & Fias, 2006; Lemaire et al., 1991). Finally, we evaluated an additional group of 12-13 year-old children, who were in the first cycle of high school, to see the progress in the resolution of simple arithmetic problems after the acquisition of arithmetic facts is completed.

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[1]Inhibitory control is the mechanism widely accepted to account for the selection of arithmetic facts in adult population (Campbell & Dowd, 2012; Campbell & Thompson, 2012; Megías et al., 2014). There are non-inhibitory explanations also such as the cumulative activation of correct arithmetic facts with time to overcome the initial high activation of incorrect arithmetic facts (Censabella & Noël, 2004). However, since the inhibitory account has received the major amount of empirical evidence, we decided to focus on it here.

Furthermore, although the ability of inhibitory control progresses strongly between 6 and 8 years of age; it develop continues through educational experience (Davidson et al., 2006; Huizinga, Dolan, & Van der Molen, 2006; Leon-Carrion, García-Orza, & Perez-Santamaria, 2004). Therefore, the selection of arithmetic facts by means of an inhibitory mechanism would be fully observed in 12-13 year-old children. According to research discussed previously on the development of inhibition, we expected that the use of an inhibitory mechanism to select arithmetic facts would be functional when the network of arithmetic facts is established completely, at the age of 10-11 year-olds onwards.

In the current study, all children were tested in two experimental tasks. Firstly, they completed a production multiplication task in which they had to say the answer of one-digit multiplication problems. This task was selected with the aim of evaluating the acquisition of simple multiplication facts through educational experience. More important, children performed a verification task which was an adapted version of the NP paradigm (Tipper, 1985; Tipper & Driver, 1988) where they had to verify if simple addition problems were correct or incorrect. This task was used in order to evaluate the possible coactivation of several arithmetic facts associated to multiplication and addition problems by the manipulation of the first experimental trial, and the possible inhibitory mechanism responsible to select the correct answer and resolve the arithmetic task by the manipulation of the second experimental trial.

METHOD

Participants. One-hundred twenty children from different schools participated in the study (62 boys and 58 girls). Participants were divided into three groups depending on the age which corresponded to different educational cycles (see Table 1). The first group included forty 8-9 year-old children who were in the second cycle of elementary school (3^{rd} and 4^{th} grades). The next group included forty 10-11 year-old children who were in the third cycle of elementary school (5^{th} and 6^{th} grades). Finally, the last group included forty 12-13 year-old children who were in first cycle of high school (1st and 2nd grades). There were 7 left-handed children and 113 right-handed children. There were no differences in gender across the three groups of children, $X^2(2, N = 120) = 0.27$, $p = .88$. Both parents and teachers gave informed consent about the participation of children in the study. Children considered by their teachers to have learning difficulties were not included in the study. All children were native Spanish speakers (90% of them were born in Spain). Children of a cultural minority (10% of children) were equally distributed across the experimental groups, $X^2(2, N = 120) = 0.56$, $p = .76$. The cultural level of children and the mathematical grade was similar in the three age groups (all *p*s > .05). Demographic characteristics of children evaluated in the current study are reported in Table 1.

Table 1. Demographic characteristic of children evaluated in the study

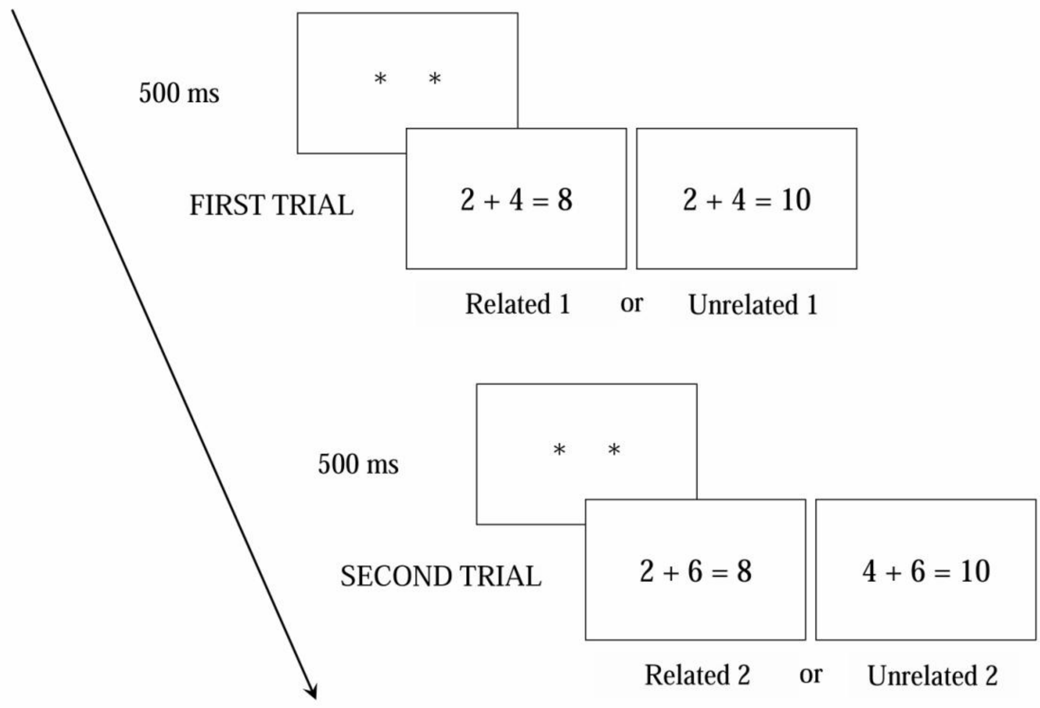
	8-9 year-old children	10-11 year-old children	12-13 year-old children
Mean age	8 years and 9 months ($SD = 0.54$)	10 years and 7 months ($SD = 0.67$)	12 years and 11 months ($SD = 0.92$)
Educational cycle	2nd cycle of elementary school (3rd and 4th grades)	3rd cycle of elementary school (5th and 6th grades)	1st cycle of high school (1st and 2nd grades)
Gender (female - male)	45% - 55%	50% - 50%	50% - 50%
Cultural minority	12.5%	10%	7.5%
Cultural level (0 - 4)	1.85 ($SD = 0.42$)	1.82 ($SD = 0.52$)	1.7 ($SD = 0.41$)
Mathematical grade (0 - 10)	8.21 ($SD = 1.28$)	8.41 ($SD = 1.4$)	7.72 ($SD = 1.86$)

Note. Cultural minority included children who belonged to a minority social group or her/his family was from foreign countries. *Cultural level* of each child was evaluated by parents according to the frequency of use of educational material such as books, papers, internet or encyclopedia at home; the visit of cultural places and exhibitions; and the reading and writing habits of children in a Likert scale (from 0 as never to 4 as always). *Mathematical grade* obtained by each child in the previous semester in a 10 point scale (0-insuficient, 10-excellent) where the passing grade was 5.

Design and Materials. We used a multiplication task to evaluate the children's knowledge about multiplications tables with the operands that were used in the main experiment (verification of arithmetic problems). Children received multiplication tables from 1 to 4 (i.e., 2 x 4 = ?) and they had to say aloud the correct result (i.e., 8). This task included twenty-three multiplication problems with two one-digit operands presented in ascending order. The problems were presented randomly.

To evaluate the possible effects of coactivation and inhibition of arithmetic facts, we used a verification task in which children received one-digit additions and they decided whether they were correct or incorrect (see Figure 1).

Figure 1. Adapted negative priming paradigm used in the current study.



Note. Adapted negative priming paradigm used in the current study. The verification task was presented in blocks of two trials. The first trial started with a fixation point followed by an addition problem. Two additions could be presented: Related 1 additions (i.e., 2 + 4 = 8) or Unrelated 1 additions (i.e., 2 + 4 = 10). After the participant's response, the second trial started with a fixation point followed by the second addition problem from the Related 2 condition (i.e., 2 + 6 = 8) or the Unrelated 2 condition (i.e., 4 + 6 = 10).

Participants received the verification task in blocks of two trials. In the first trial, two conditions were manipulated within-subject in order to evaluate the coactivation effect in simple arithmetic. The Related 1 condition was composed of an incorrect addition whose result was that of multiplying the operands (i.e., 2 + 4 = 8). The Unrelated 1 was a control condition that included an incorrect addition whose result was not the one of multiplying the operands (i.e., 2 + 4 = 10). In the second trial, two conditions were also manipulated within-subject in order to evaluate the possible inhibitory mechanism involved in the selection of simple arithmetic facts. The Related 2 trials were a test condition that included a correct addition whose result was the one of multiplying the operands of the previous trial (i.e., 2 + 6 = 8). The Unrelated 2 trials were a control condition that contained a correct addition with a result which was not the one of multiplying the operands of the previous trial (4 + 6 = 10). The complete set of experimental stimuli is presented in Appendix 1.

To make the experimental blocks of trials, 20 false additions were selected in the first trial (10 related 1 additions and 10 unrelated 1 additions), and 20 correct additions were used in the second trial (10 related 2 additions and 10 unrelated 2 additions). Across participants, each addition in each condition of trial 1 (related 1 and unrelated 1 additions) was presented half of the times followed by a related 2 addition and the other half they were followed by an unrelated 2 addition. Therefore, the related 2 and unrelated 2 additions were preceded an equal number of times by related 1 trials and unrelated 1 trials. Each participant received the experimental block of trials twice. Hence, for each participant there was a total number of 40 observations in each condition of trial 1 (related 1 and unrelated 1) and each condition of trial 2 (related 2 and unrelated 2).

The additions used in the verification task were carefully selected to equate them in several factors that might determine possible differences across conditions in the first and second trial of the task. All additions were composed of one-digit operands and the two operands of each problem were presented in ascending order (i.e., 2 + 6) and never in descending order (i.e., 6 + 2 was not used). The parity (even and odd digits) of operands and results was equally distributed across the conditions of the first trial and second trial of the experimental blocks. In each trial, the solution corresponded to multiplication tables from 1 to 4 and the solution was never one of the two operands presented in the addition (i.e., 2 + 1 = 2 was not presented).

In the first trial, the related 1 condition and the unrelated 1 condition were equated in problem size (the sum of the two operands in both conditions was exactly the same: 7.40). The size of the incorrect results presented in the related 1 condition and the unrelated 1 condition was also similar (11.80 and 11.60, respectively), $t(18) = 0.12$, $p = .90$. Also, the distance between the incorrect result presented to the participants and the correct result of the addition in the two conditions of the first trial was exactly the same (4.40). In the second trial, the problem size was equated in the related 2 condition (11.80) and the unrelated 2 condition (11.60), $t(18) = 0.12$, $p = .90$. In order to maintain the same problem size in the two conditions of trial 2, one addition problem in the related 2 condition (7 + 9 = 16) and one problem in the unrelated 2 condition (4 + 6 = 10) were repeated. Other problems could be repeated to maintain this criterion. Thus, the repeated problems were randomly selected.

We performed a pilot study in order to check that there were no differences in reaction times (RTs) and accuracy when children answered to the addition problems used in the related 2 and unrelated 2 conditions without any manipulation. We evaluated a sample of 54 children recruited from the same schools that in the experiment but that did not participate in the main study (eighteen 8-9 year-old children, eighteen 10-11 year-old children and eighteen 12-13 year-old children). The children had to perform a production task which contained the addition problems presented in the related 2 and unrelated 2 conditions. In this task, the order of presentation of additions was pseudorandom so we controlled that the result of one addition was different from the operands and the result of the previous addition. We analyzed the error percentages, the mean RT and the median RT on correct responses with Relation 2 (related 2 and unrelated 2) as a within-subject factor and age group (8-9 year-olds, 10-11 year-olds and 12-13 year-olds) as a between-subject factor. There were no differences in the percentage of errors associated to additions in the related 2 condition (9.47%) and the unrelated 2 condition (8.44%), $F < 1$. Moreover, the results on the mean RT did not show significant differences between related 2 additions (1685 ms) and unrelated 2 additions (1658 ms), $F < 1$. Similarly, the median RT was equated in the related 2 condition (1672 ms) and the unrelated 2 condition (1624 ms), $F < 1$. Furthermore, the Relation 2 x Age group interaction was not significant in any case (all *p*s > .05).

Furthermore, we controlled for the amount of similarities between the additions used in the first trial and those corresponding to the two conditions of the second trial (related 2 and unrelated 2). The numerical distance between the incorrect result presented in the first trial and the second trial was exactly the same in the related 2 condition and the unrelated 2 condition (1.40). The difference between the problem size in the first trial and the second trial was exactly the same in the related 2 condition and the unrelated 2 condition (4.40). The number of repetitions between digits presented in the first trial and the second trial (i.e., 2 was repeated in 2 + 3 = 6 followed by 2 + 4 = 6), was exactly the same in the related 2 condition and the unrelated 2 condition (8 repetitions).

To avoid the participants noticing the structure of the experimental blocks (a sequence of an incorrect operation in the first trial and a correct operation in the second trial), the experimental blocks were randomly intermixed with 10 filler blocks of trials which were repeated four times. The correct responses in the first and second trial of these blocks were ‘yes’-’yes’, ‘no’-’no’, and ‘yes’-’no’, respectively. Therefore, the sequence of responses within each block of two trials was unpredictable through the experiment. The filler blocks included 6 addition problems and 4 multiplication problems (see Appendix 2).

Before starting the verification task, the participants performed four blocks of practice trials (2 pairs of additions and 2 pairs of multiplications) with problems that were not used in the main experiment.

Procedure. The tasks of this study were designed and controlled by E-prime experimental software, 1.1 version. The stimuli were always presented in the middle of the screen in black color on a white background. Participants were tested individually outside their regular classroom, in a room provided by the school for this purpose. Children were seated at approximately 60 cm from the computer screen.

The multiplication task began with a fixation point in the middle of screen for 2000 ms; then the multiplication problem was presented until the participant's response. All problems were presented in Arabic digit format. Reaction times were collected using a microphone ATR 20 with low impedance connected to a PST serial Response Box. The participants' oral responses were recorded in audio files and afterwards, they were checked in order to eliminate incorrect responses in the RT analyses.

The verification of arithmetic problems task was presented in blocks of two trials. The problems were presented in Arabic digits. Participants had to decide if the result of each problem was correct or incorrect. The first trial began with a fixation point in the middle of screen for 500 ms; followed by the arithmetic problem until the participant's response. After giving the answer, the second trial appeared with the same sequence of events as that of the first trial: a fixation point for 500 ms and the arithmetic problem until the participant's response. After each block of two trials, the participants were instructed to press the space bar to continue with the following block. Participants were instructed to respond by pressing the keys labeled as 'correct' and 'incorrect'. The 'correct' and 'incorrect' key to left and right position assignment was counterbalanced across participants. The duration of the experiment was approximately 50-60 minutes depending on the participant.

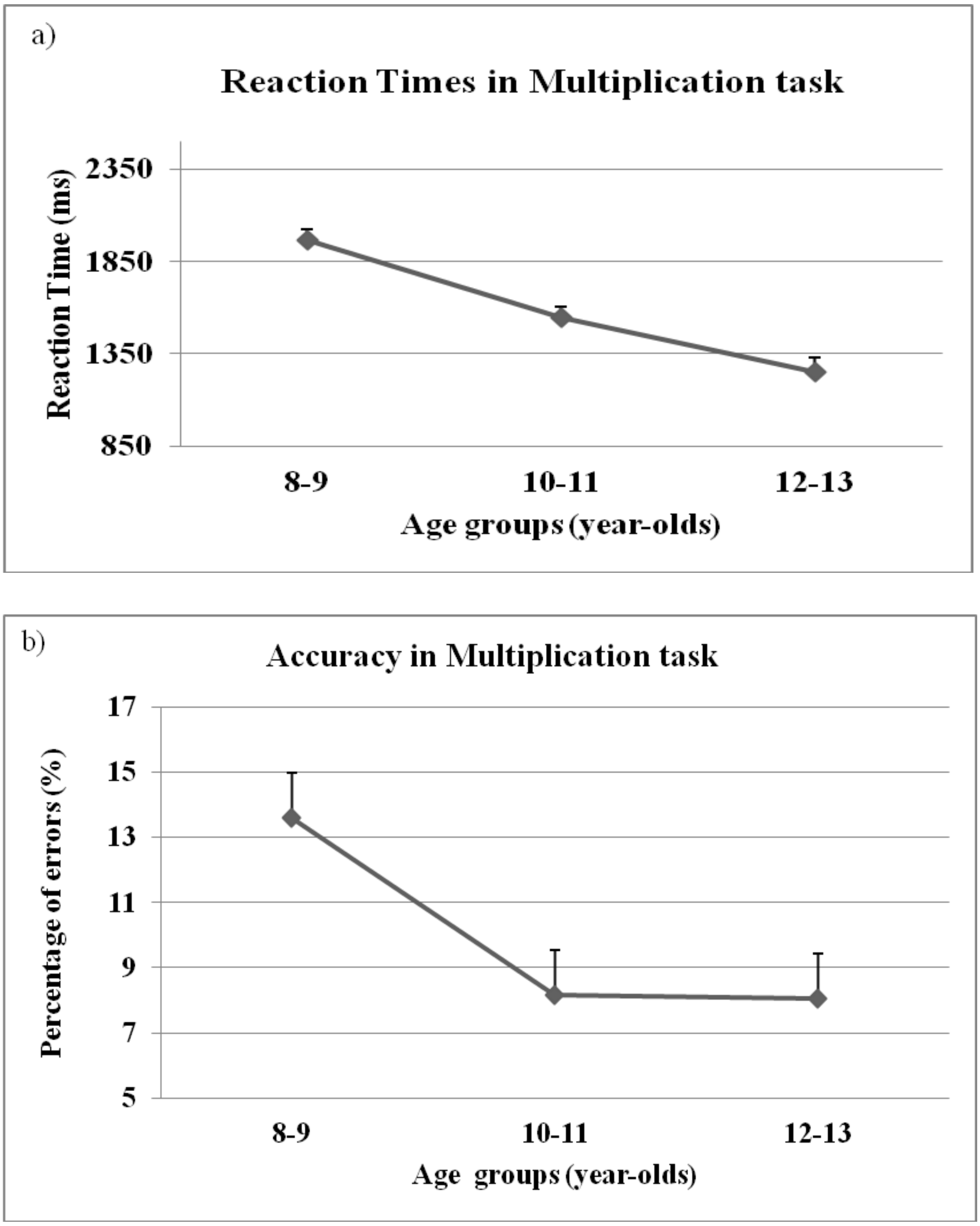
RESULTS

Multiplication task. Only correct responses were included in the RT analyses. Data points were excluded from the RT analyses if: (a) the participants produced nonverbal sounds that triggered the voice key, (b) the participants stuttered or hesitated in producing the result of the problem, (c) the participants produced something different than the result requested. The percentage of errors was 9.93% (13.59% in 8-9 year-old children, 8.15% in 10-11 year-old children and 8.04% in 12-13 year-old children). Afterward, the RTs associated to correct responses were trimmed following the procedure described by Tabachnick and Fidell (2001) to eliminate univariate outliers (data points that after standardization were 3 *SD* outside of the normal distribution for each age group). The percentage of outliers was 2.32% in 8-9 year-old children, 4.95% in 10-11 year-old children and 7.01% in 12-13 year-old children. The RT and error data were submitted to analyses of variance (ANOVA) with age group as a between-subject factor.

In the RT analysis, there were significant differences among age groups, $F(2, 105) = 29.12$, $p < .001$, $\eta^2 = 0.36$. The group of 8-9 year-old children were slower to respond to one-digit multiplications ($M = 1964$ ms, $SE = 61.14$) than 10-11 year-old children ($M = 1546$ ms, $SE = 61.14$), $F(1, 78) = 19.06$, $p < .001$, $\eta^2 = 0.20$ (418 ms difference). Similarly, 10-11 year-old children were slower compared to 12-13 year-old children ($M = 1253$, $SE = 73.07$), $F(1, 66) = 13.71$, $p < .001$, $\eta^2 = 0.17$ (292 ms difference). Likewise, 8-9 year-old children were slower in comparison to 12-13 year-old children (710 ms difference), $F(1, 66) = 53.16$, $p < .001$, $\eta^2 = 0.45$ (see Figure 2a).

In the accuracy analyses the main effect of age group was significant, $F(2, 117) = 5.25$, $p =.01$, $\eta^2 = 0.08$. Additional analyses showed that 8-9 year-old children committed a higher percentage of errors ($M = 13.59\%$, $SE = 1.38$) compared to10-11 year-old children ($M = 8.15\%$, $SE = 1.38$), $F(1, 78) = 6.40$, $p = .01$, $\eta^2 = 0.08$; but there were no differences in the percentage of errors committed by 10-11 year-old children and 12-13 year-old children ($M = 8.04\%$, $SE = 1.38$), $F < 1$. Finally, the difference between 8-9 year-old children and 12-13 year-old children was significant, $F(1, 78) = 6.83$, $p = .01$, $\eta^2 = 0.08$. As shown in Figure 2b, 8-9 year-old children committed more errors in simple multiplications relative to 10-11 year-olds and older children.

Figure 2. Reaction times (in milliseconds) (a), and accuracy results (percentage of errors) (b), in multiplication task as a function of age groups.



Note. a) Mean percentage of errors in the multiplication task as a function of age groups: 8-9, 10-11, 12-13 year-old children. b) Mean reaction times in milliseconds in the multiplication task as a function of age group: 8-9, 10-11, 12-13 year-old children. Standard errors are presented in error bars. *$p < .05$, $^{ns}p > .05$

Verification task. Trials in which participants committed an error were submitted to the accuracy analyses (6.9% in the first trial and 6.5% in the second trial: 8.16% and 7.72% in 8-9 year-old children, 6.22% and 6.94% in 10-11 year-old children, 6.31% and 4.84% in 12-13 year-old children). We filtered correct RT data following the same procedure used to analyze the data of the multiplication task. Trials outside 3 *SD* the normal distribution in 8-9 year-old children were 6.53% in the first trial and 3.83% in the second trial. In 10-11 year-old children, were eliminated 6.22% in the first trial and 6.94% in the second trial. Finally, in 12-13 year-old children were eliminated 4.09% in the first trial and 6.34% in the second trial. We analyzed the two conditions of the first trial and the second trial separately given that we were interested in possible differences between each condition within each trial[2]. Furthermore, a factorial design including the condition (related vs. unrelated) and the trial (first and second) could not be considered because the problem size of additions in the second trial was significantly larger (11.70) than that of the first trial (7.40), $t(38) = 5.09$, $p < .001$. This difference might produce a problem size effect (Ashcraft, 1992; Groen & Parkman, 1972) which consists in longer reaction times and more errors when solving additions with large problem size relative to problems with small problem size.

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[2]It is important to note that the type of second trial (related 2 vs. unrelated 2) could not be analyzed depending on the type of first trial (related 1 vs. unrelated 1) due to a repetition effect that might have a different impact on the two conditions of the second trial. For example, while the solution 8 is repeated in the related 2 condition: 2 + 6 = 8, after the related 1 condition 2 + 4 = 8; the solution 10 is repeated in the unrelated 2 condition 4 + 6 = 10 after the unrelated 1 condition 2 + 4 =10. Note, however, that this unbalanced repetition effect is avoided when related 2 and unrelated 2 conditions are directly compared since in both conditions, half of the solutions were explicitly presented in the previous trial.

Hence, we report first the results obtained in the first trial (related 1 condition vs. unrelated 1 condition) and then the results found in the analysis of the second trial (related 2 condition vs. unrelated 2 condition) (see Table 2) [3].

Table 2. Results in verification task in each age group

	First Trial	
	Related 1 condition *(2 + 4 = 8)*	Unrelated 1 condition *(2 + 4 = 10)*
8-9 year-old children	3324 ms (102.02)	3292 ms (101.36)
	12.63% (1.67)	3.69% (0.88)
10-11 year-old children	2407 ms (102.02)	2278 ms (101.36)
	8.81% (1.67)	3.63% (0.88)
12-13 year-old children	1653 ms (102.02)	1589 ms (101.36)
	9.88% (1.67)	2.75% (0.88)
	Second Trial	
	Related 2 condition *(2 + 6 = 8)*	Unrelated 2 condition *(4 + 6 = 10)*
8-9 year-old children	4186 ms (166.51)	4061 ms (161.23)
	8.31% (1.15)	7.13% (0.94)
10-11 year-old children	2953 ms (166.51)	2620 ms (161.23)
	7.88% (1.15)	6.00% (0.94)
12-13 year-old children	1824 ms (166.51)	1653 ms (161.23)
	4.88% (1.15)	4.81% (0.94)

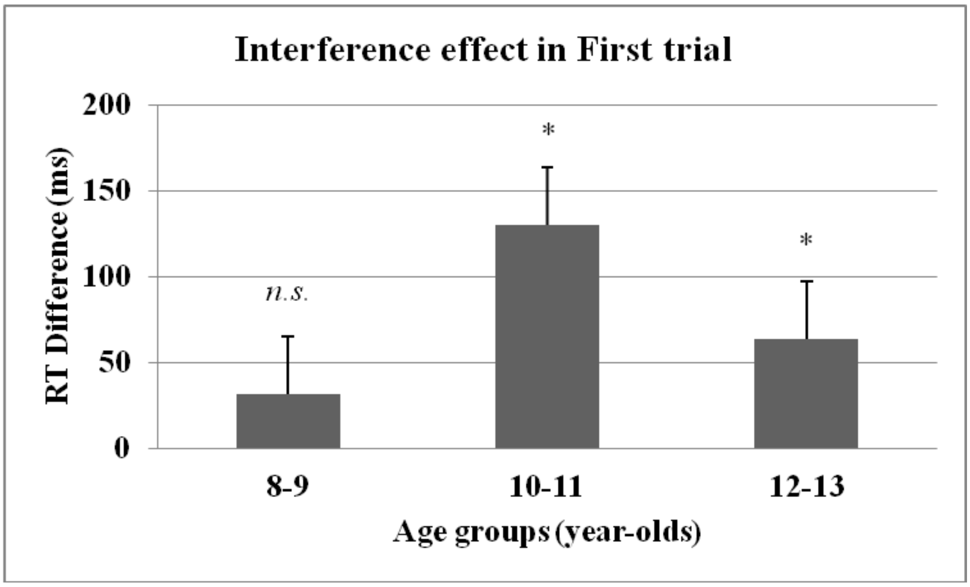
Note. Mean reaction times in milliseconds (ms) and percentage of errors for each condition in first and second trial. Standard errors are reported into brackets.

First Trial. We performed an ANOVAs on the RTs and percentage of errors with the variable Relation 1: related 1 (i.e., 2 +4 = 8) and unrelated 1 (i.e., 2 + 4 = 10) as a within-subject factor and the variable Age group (8-9 year-old children, 10-11 year-old children, and 12-13 year-old children) as a between-subject factor. The RT analysis showed a main effect of relation 1, $F(1, 117) = 15.08$, $p < .001$, $\eta_p^2 = 0.11$. The responses to related 1 trials were slower ($M = 2461$ ms, $SE = 58.90$) than the responses to unrelated 1 trials ($M = 2386$ ms, $SE = 58.52$) (75 ms difference). Furthermore, the difference across age groups was significant also, $F(2, 117) = 71.24$, $p < .001$, $\eta_p^2 = 0.55$. Bonferroni-corrected post-hoc comparisons showed that 8-9 year-old children were slower to respond ($M = 3308$ ms, $SE = 100.31$) than 10-11 year-old children ($M = 2342$ ms, $SE= 100.31$) (966 ms difference); they responded more slowly than 12-13 year-old children ($M = 1621$ ms, $SE = 100.31$) (722 ms difference); and 8-9 year-old children were slower than 12-13 year-old children (1687 ms difference), all ps < .001. However, the Relation 1 x Age interaction was not significant, $F(2, 117) = 2.23$, $p = .11$. Since we were interested in the development of the coactivation effect through formal instruction in arithmetic, we analyzed the performance in the first trial for each age group separately (see Figure 3). For 8-9 year-old children, we did not find significant the difference between the related 1 condition ($M = 3324$ ms, $SE = 139.60$) and the unrelated 1 condition ($M = 3292$ ms, $SE = 147.24$), $F < 1$. For 10-11 year-old children, the difference between the related 1 condition ($M = 2407$ ms, $SE = 90.17$) and the unrelated 1 condition ($M = 2278$ ms, $SE = 77.99$) was significant, $F(1, 39) = 20.01$, $p < .001$, $\eta_p^2 = 0.34$ (130 ms difference). Likewise, the difference between the related 1 condition ($M = 1653$ ms, $SE = 60.04$) and the unrelated 1 condition ($M = 1589$ ms, $SE = 55.33$) was significant for 12-13 year-old children, $F(1, 39) = 8.13$, $p = .007$, $\eta_p^2 = 0.17$

[3]The analyses reported in text were performed also after controlling for the variability explained by the arithmetic knowledge of children. The mean reaction times and percentage of errors in the multiplication task and the mathematical grade obtained by children were standardized and the mean of these three variables for each child was introduced as covariate in the analyses. The pattern of results was the same as that reported in the main text. The 8-9 year-old group did not show interference effects in either the first and second trials (ps > .21). However, the interference effects in the first and second trials were significant in the 10-11 year-old group and the 11-12 year-old group (ps < .02).

(64 ms difference). Moreover, analyses on the interference effect (RT in the related 1 condition minus RT in the unrelated 1 condition) showed a marginal difference between 10-11 year-old children (M = 130 ms, SE = 25.87) and 12-13 year-old children (M = 64 ms, SE = 25.87), $F(1, 78) = 3.26$, $p = .07$.

Figure 3. Interference effect in the first trial as a function of age groups.

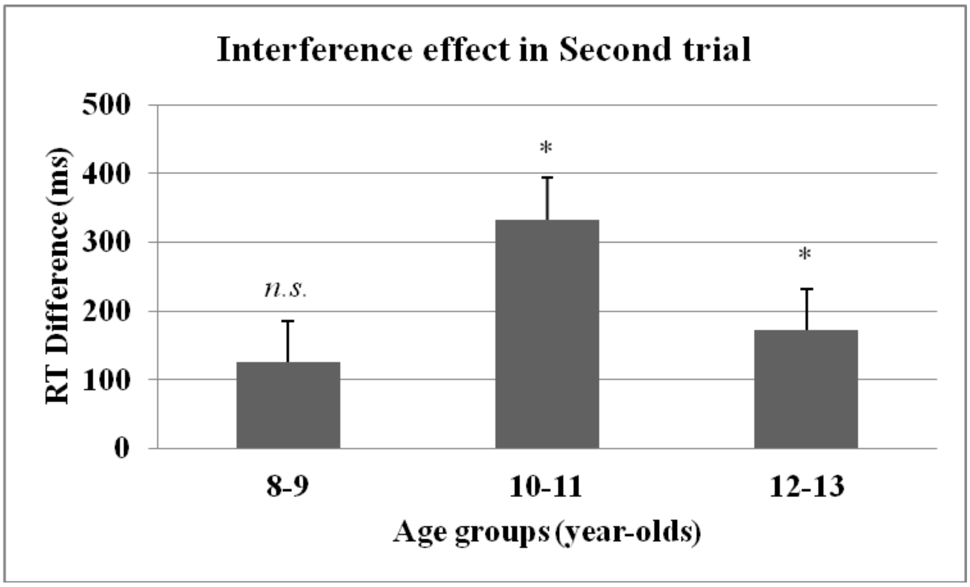


Note. Interference effect in the first trial as a function of age groups. Reaction time difference in milliseconds: Mean reaction time in related 1 condition (i.e., 2 + 4 = 8) minus mean reaction time in unrelated 1 condition (i.e., 2 + 4 = 10) obtained in the first trial of the verification task as a function of age group: 8-9, 10-11, 12-13 year-old children. Standard errors are presented in error bars. *$p < .05$, $^{ns}p > .05$.

Regarding the ANOVA on the percentage of errors, the main effect of relation 1 was significant, $F(1, 117) = 82.45$, $p < .001$, $\eta_p^2 = 0.41$. Participants committed more errors in related 1 trials (M = 10.44%, SE = 0.97) relative to unrelated 1 trials (M = 3.35%, SE = 0.51). However, the effect of age group was not significant, $F < 1$, so the percentage of errors was similar across age groups (8.16% in 8-9 year-old children, 6.22% in 10-11 year-old children and 6.31% in 12-13 year-old children). Finally, the Relation 1 x Age group interaction was not significant, $F(2, 117) = 1.93$, $p = .15$.

Second Trial. We performed an ANOVA on RTs with Relation 2 (related 2 condition and unrelated 2 condition) as a within-subject factor and the Age group (8-9 year-old children, 10-11 year-old children, and 12-13 year-old children) as a between-subject factor. The main effect of Relation 2 was significant, $F(1, 117) = 36.49$, $p < .001$, $\eta_p^2 = 0.24$ (210 ms difference), such that the responses to related 2 trials were slower ($M = 2988$ ms, $SE = 96.13$) than the responses to unrelated 2 trials ($M = 2778$ ms, $SE = 93.09$). Moreover, there were significant differences between the age groups, $F(2, 117) = 55.07$, $p < .001$, $\eta_p^2 = 0.48$. Bonferroni-corrected post-hoc comparisons showed that 8-9 year-old children were slower ($M = 4124$ ms, $SE = 161.10$) than 10-11 year-old children ($M = 2786$ ms, $SE = 161.10$) (1338 ms difference); 10-11 year-old children were slower relative to 12-13 year-old children ($M = 1738$ ms, $SE = 161.10$) (1048 ms difference); and 8-9 year-old children were slower to respond compared to 12-13 year-old children (2385 ms difference), all ps < .001. Importantly, the interaction between Relation 2 (related 2 vs. unrelated 2) and Age group (8-9, 10-11 and 12-13 year-old children) was significant, $F(2, 117) = 3.28$, $p = .04$, $\eta_p^2 = 0.05$. As in the first trial, we analyzed the performance in the second trial for each age group separately (see Figure 4). In 8-9 year-old children, the RTs were similar in the related 2 condition ($M = 4186$ ms, $SE = 220.16$) and the unrelated 2 condition ($M = 4061$ ms, $SE = 230.00$), $F(1, 39) = 2.40$, $p = .13$. However, in 10-11 year-old children, the difference between the related 2 condition ($M = 2953$ ms, $SE = 162.02$) and the unrelated 2 condition ($M = 2620$ ms, $SE = 133.84$) was significant, $F(1, 39) = 33.30$, $p < .001$, $\eta_p^2 = 0.46$ (333 ms difference). Similarly, the difference between the related 2 condition ($M = 1824$ ms, $SE = 91.95$) and the unrelated 2 condition ($M = 1653$ ms, $SE = 84.68$) was significant in 12-13 year-old children, $F(1, 39) = 30.03$, $p < .001$, $\eta_p^2 = 0.44$ (171 ms difference). Further analyses showed that the interference effect (RT in the related 2 condition minus RT in the unrelated 2 condition), was larger in 10-11 year-old children ($M = 333$ ms, $SE = 46.39$) relative to the interference found in 12-13 year-old children ($M = 171$ ms, $SE = 46.39$), $F(1, 78) = 6.06$, $p = .02$, $\eta_p^2 = 0.07$.

Figure 4. Interference effect in the second trial as a function of age groups.



Note. Reaction time difference in milliseconds: Mean reaction time in related 2 condition (i.e., 2 + 6 = 8 preceded by 2 + 4) minus mean reaction time in unrelated 2 condition (i.e., 4 + 6 = 10 preceded by 2 + 4) obtained in the second trial of the verification task as a function of age groups: 8-9, 10-11, and 12-13 year-old children. Standard errors are presented in error bars. $^*p < .05$, $^{ns}p > .05$.

Moreover, the accuracy analysis showed a main effect of relation 2, $F(1, 117) = 4.99$, $p = .03$, $\eta_p^2 = 0.04$. Participants committed more errors in related 2 trials (M = 7.02%, SE = 0.67) relative to unrelated 2 trials (M = 5.98%, SE = 0.54). However, the effect of age was not significant, $F(2, 117) = 2.34$, $p = .10$, so the percentage of errors was similar in all age groups: 8-9 year-old children (M = 7.72%, SE = 0.97), 10-11 year-old children (M = 6.94%, SE = 0.97) and 12-13 year-old children (M = 4.84%, SE = 0.97). Furthermore, the Relation 2 x Age group interaction was not significant, $F(2, 117) = 1.28$, $p = .28$.

Multiplication skills and verification of arithmetic problems. We performed additional analyses to evaluate whether the coactivation and selection of arithmetic facts were modulated by the proficiency of children in the resolution of multiplication problems. To this end, in each age group (8-9 year-old children, 10-11 year-old children, and 12-13 year-old children) we sorted the participants based on their accuracy in the multiplication task. Afterwards, we considered as variable the multiplication accuracy in each age group (low skilled children and high skilled children) depending on whether their accuracy in the multiplication task was above or below the median accuracy by group.

Regarding the performance in the first trial, the Relation 1 x Age group x Multiplication accuracy interaction was significant, $F(2,114) = 3.78$, $p = .03$, $\eta_p^2 = 0.06$. In 8-9 year-old children, there was a Relation 1 x Multiplication accuracy interaction, $F(1, 38) = 6.81$, $p = .01$, $\eta_p^2 = 0.15$. High skilled participants in the multiplication task showed significant differences between the related 1 condition ($M = 2937$ ms, $SE = 152.46$) and the unrelated 1 condition ($M = 2796$ ms, $SE = 144.89$) (141 ms difference), $F(1, 19) = 6.81$, $p = .002$, $\eta_p^2 = 0.40$; whereas participants with lower accuracy in the multiplication task did not show significant the difference between the related 1 ($M = 3711$ ms, $SE = 179.19$) and the unrelated 1 ($M = 3789$ ms, $SE = 177.52$) conditions, $F(1, 19) = 1.10$, $p = .31$. However, the Relation 1 x Multiplication accuracy interaction was not significant in 10-11 year-old children, $F < 1$; and 12-13 year-old children, $F < 1$. However, in the second trial the Relation 2 x Age group x Multiplication accuracy interaction was not significant, $F < 1$.

Furthermore, we examined whether the interference effects obtained in the first and second trials of the verification task were predicted by indexes of mathematical knowledge. To this end, we performed a multiple linear regression analyses with the multiplication reaction time, the multiplication accuracy (percentage of errors in the multiplication task), and the mathematical grade (grade obtained by each child in the previous semester) as predictors; and the interference effect in the first trial (RT in the related 1 condition minus RT in the unrelated 1 condition) and the second trial (RT in the related 2 condition minus RT in the unrelated 2 condition) as dependent variables. When the interference effect in the first trial was considered, the regression model was significant, $R^2 = .09$, $F(3,92) = 3.14$, $p = .03$. The only significant predictor was the multiplication accuracy, $t(92) = -3.06$, $p = .003$ (see Table 3). A decrease in the number of errors in the multiplication task was accompanied by an increase in the interference effect ($b = -.33$). However, when the interference effect in the second trial was examined, the multiple linear regression analyses was not significant, $F < 1$.

Table 3. Results of regression analyses with mathematical competences as predictors of the interference effects obtained in the study

	B	Standard error	Β
	Model of the interference in the First Trial (R^2 = .09)		
Constant	129.88	144.56	
Multiplication reaction time	0.06	0.05	.12
Multiplication accuracy	-7.78	2.55	-.33*
Mathematical grade	-8.20	15.25	-.05
	Model of the interference in the Second Trial (R^2 = .03)		
Constant	100.94	276.70	
Multiplication reaction time	0.11	0.10	.12
Multiplication accuracy	-6.67	4.87	-.15
Mathematical grade	0.96	29.18	.00

Note. Results of regression analyses with the interference effect in the first trial and the second trial of the experiment as dependent variable and the predictors: multiplication reaction time, multiplication accuracy (percentage of errors in the multiplication task), and mathematical grade (grade obtained by each child in the previous semester). The *R* square of the models in the first and second trials are reported into brackets.*p < .05.

DISCUSSION

The main goal of the current study was to evaluate the development of an inhibitory mechanism to select arithmetic facts in children through formal education in mathematical knowledge. We evaluated also the arithmetic performance and the coactivation of arithmetic facts in children. The results obtained in the multiplication task showed a lineal increase in arithmetic performance associated to the age of children in different educational cycles. The 8-9 year-old children were slower to give a correct answer to the multiplication problem relative to 10-11 and 12-13 year-old children. Similarly, 12-13 year-old children were faster in the multiplication task than 10-11 year-old children. This improvement in the reaction time of multiplication problems associated to the educational level has been previously found in the literature (Campbell & Graham, 1985; De Brauwer et al., 2006; De Brauwer & Fias, 2009) and it can be understood as an index of the increased automaticity with which arithmetic facts are retrieved from memory (Koshmider & Ashcraft, 1991). However, it could be argued that the pattern of response time results was not specifically associated to the development of arithmetic skills but that it was a consequence of a general improvement in the processing speed with age. In fact, previous research has shown that the speed of encoding and retrieving information emerges and constantly improves with formal education regardless of the specific task children perform (Kail, 1991; Kail, & Salthouse, 1994; Miller & Vernon, 1997). Hence, more interesting were the benefits associated to formal education in response accuracy when children resolved multiplication problems. The 8-9 year-old children had a higher percentage of errors in the multiplication task relative to 10-11 and 12-13 year-old children; while no differences were found between 10-11 and 12-13 year-old children. These results suggest that the acquisition of arithmetic facts associated to multiplication problems were formed when children took the third cycle of elementary school and they used them afterwards in the first cycle of high school. This interpretation agrees with the results found by De Brauwer et al. (2006), which explored how the network of multiplication facts took shape during formal learning. The authors compared the performance of 9, 10 and 11 year-old children, and adults in a production multiplication task. The results showed differences among the children depending on age. However, 11 year-old children behaved as adults suggesting that the network of multiplication facts in long-term-memory was fully established at this age.

The coactivation of arithmetic facts in school age children. Beyond the examination of how formal education influences the configuration of arithmetic facts network, we were interested in evaluating how children managed the coactivation and selection of arithmetic facts once the network was established. To evaluate this issue, it was necessary to determine the possible concurrent activation of facts associated to multiplications and additions when children verified the correctness of simple arithmetic problems. This was the goal of the first trial in the verification task. The results showed that children were slower to respond when the result of an incorrect addition problem was that of multiplying the operands (2 + 4 = 8) compared to an addition problem whose result was incorrect too, but it was unrelated with the multiplication counterpart (2 + 4 = 10).

We considered the interference effect found in the first trial of the current study as an index of the concurrent activation of arithmetic facts. This coactivation of arithmetic facts has been observed in previous studies with adult (Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986) and child population (Lemaire et al., 1991). When the network of simple arithmetic facts is established, several arithmetic facts seem to be activated in long-term-memory when participants evaluate the correctness of addition problems (i.e., 2 + 4 = 8). This concurrent activation produces competition between different arithmetic facts, since only one answer is required to solve the problem successfully.

However, when we analyzed each age group separately, we found that 8-9 year-old children did not present this concurrent activation of addition and multiplication facts while it was observed in 10-11 and 12-13 year-old children. In fact, interference effects in the first trial of the verification task were only present in these two last age groups. Furthermore, additional analyses in 8-9 year-old children showed that only high skilled children in the multiplication task presented the interference effect in the first trial, suggesting that when 8-9 year-old children had already acquired a good knowledge of simple multiplication facts, the coactivation of multiplications and additions is produced automatically. Therefore, a good knowledge of simple multiplication facts seems to be necessary to observe the coactivation effect (Ashcraft, 1982; Ashcraft & Fierman, 1982; Ashcraft, 1992). Moreover, the outcomes of the regression analyses showed that the accuracy in resolving multiplication problems predicted the interference effect found in the first trial. Higher proficiency in solving multiplications was associated to a large interference effect which seems to indicate that when multiplication skills increase, children have configured the network of arithmetic facts; and they coactivate addition and multiplication facts even when this simultaneous activation produces interference. These results highlighted the role of formal education in the coactivation of arithmetic facts. However, other indices of the children's mathematical knowledge were not significant predictors (i.e., mathematical grade of children gathered by the teacher). It might be possible that mathematical grade obtained by the children at school was a general measure of numerical knowledge (i.e., knowledge of numerical formats, magnitude representation, counting skills, arithmetic knowledge) so it was not sensitive enough to capture possible differences in the resolution of simple arithmetic facts.

The results obtained when 8-9 year-old children resolved the verification task in the first trial suggest that children in this age group were consolidating the network of arithmetic facts needed to perform simple problems by retrieving the results of the problems directly from long-term-memory. This conclusion agrees with previous studies in which the development of arithmetic skills was addressed. Ashcraft and Fierman (1982) showed that children in 4th grade of formal education showed a good performance in the verification of correct addition problems (3 + 4 = 7). However, the children had difficulties in verifying the correctness of false problems (3 + 4 = 8). The authors interpreted that children in 4th grade were consolidating the network of arithmetic facts so it could be used easily to verify correct addition problems. However, the retrieval from memory was still unstable at this educational level which was observed when children verified false addition problems. Hence, only when the network of arithmetic facts is fully established, the coactivation of several arithmetic problems show up. The current study suggests that this phenomenon occurs in 8-9 year-old children with good mathematical skills and it continues henceforth[4].

The selection by inhibition of arithmetic facts in school age. The main contribution of the current study was to evaluate the developmental course of inhibition as the underlying mechanism used by children to select arithmetic facts. Recent experimental evidence has been found to support the idea that inhibition underlies the resolution of conflict between coactivated arithmetic facts (Campbell & Dowd, 2012; Campbell & Thompson, 2012; Megías et al., 2014). This inhibitory mechanism has been corroborated in adult participants. However, to our knowledge, the current study is the first approach to this issue in child population.

The second trial of the adapted NP paradigm used in our study was intended to evaluate this inhibitory mechanism responsible to select simple arithmetic facts. The results obtained in this trial showed that children took more time to respond when the result of multiplying the operands of the previous trial (2 and 4) was the correct result of the second trial (2 + 6 = 8), relative to a control condition (4 + 6 = 10). This interference effect suggests that children inhibited the irrelevant result in the first trial (8). Therefore, when the inhibited result was presented again in the second trial (2 + 4 = 8) and it was the correct result to solve the problem, they needed additional time to retrieve it. Importantly, the interference effect in the second trial appeared in children who were 10-11 and 12-13 years of age, but it was not observed in 8-9 year-old children. The absence of interference is easily explained by assuming that inhibition is a reactive mechanism devoted to resolve competition among coactivated arithmetic facts. Since 8-9 year-old children did not coactivate competing multiplication facts (no interference effect was observed in the first trial), there was no need of applying inhibition to resolve competition. Furthermore, the results of the regression analyses showed that the interference effect found in the second trial was not predicted by the mathematical knowledge of children. Therefore, inhibition seems to be applied only when competition among arithmetic facts is found regardless of the children's competence in the resolution of arithmetic problems.

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[4]In the current study children performed the task with experimental trials (addition problems) intermixed with multiplication trials (filler trials). Previous research in adult population has shown the same coactivation effects when additions are blocked relative to a mixed condition with additions and multiplications (Megías et al., 2014). However, it might be possible that the presence of multiplication problems would foster the coactivation of arithmetic facts to a more extend in children. Hence, the pattern of coactivation effects obtained in this study might apply only to situation in which children have to perform addition and multiplication problems intermixed in the same session (i.e., experimental task, educational setting). However, it is important to note that this restriction does not have relevance to the finding obtained in this study about the developmental course of selection by inhibition once coactivation of arithmetic facts is observed in children.

Moreover, the interference effects in trial 1 and trial 2 obtained in 10-11 and 12-13 year-old children seem to indicate that from 10-11 years of age onwards children use the network of arithmetic facts, they coactivate related facts and they use an inhibitory mechanism to select the one needed to resolve mathematical problems. A closer look at the results obtained in the second trial of the verification task showed that the interference effect was larger in 10-11 year-old children relative to that found in 12-13 year-old children. These differences seem to indicate that the cost associated to the inhibition of irrelevant information reduces with age. This observation agrees with previous research in which the continuous improvement of inhibitory control with educational instruction is corroborated (Davidson et al., 2006; Huizinga et al., 2006; Leon-Carrion et al., 2004). To illustrate, Davidson et al. (2006) evaluated the performance of children from preschool to high school in a typical task to evaluate inhibitory control (the Simon task). The authors observed that the magnitude of the interference effect (slower responses in spatially incongruent trials where the stimulus and the response side were in different locations) decreased from first grade of elementary school (6 year-olds) onward. Further evidence about the development of this inhibitory mechanism comes from studies in which an adapted version of the NP paradigm is used (Borst et al., 2013; Perret et al., 2003). Borst et al. (2013) evaluated the involvement of inhibitory processes in the resolution of a Piaget-like class-inclusion task. In this task, colored circles were presented (i.e., 8 circles in red color and 4 circles in yellow color) while adults and 10 year-old children had to indicate if there were more circles than reds or more reds than circles. To perform the task correctly, participants had to suppress a strategy based on the perceptual comparison of red and yellow elements, and they had to realize that the red color circles were included inside the category of circles. The cost associated to inhibit the irrelevant strategy based on color was higher for 10 year-old children (291 ms) compared to adults (129 ms). Thus, the ability of inhibiting irrelevant information increased with age.

The evidence about the development of an inhibitory mechanism to select arithmetic facts is closely related to the developmental model proposed by Siegler (1996, 1999). In the strategy choice and discovery simulation model (SCADS model; Siegler & Araya, 2005), a multiple strategy approach is assumed in which several strategies are considered to resolve arithmetic problems (counting the operands to reach the result of the problem, the retrieval of the answer from long-term-memory, etc.). The model proposes that children choose adaptively among available alternatives to select the more efficient strategy. In this scenario, inhibitory control would be useful to choose among strategies so children would select one strategy (i.e., the retrieval from memory) by inhibiting alternative ways to solve the problem. This inhibitory control would underlie the functioning of the interruption of procedures mechanism proposed in the SCADS model (Siegler & Araya, 2005). This mechanism would be involved in the change from overlearned strategies to new strategies which are more efficient to resolve a problem.

In our opinion, inhibitory control might be considered a general mechanism involved in arithmetic cognition at several levels of processing. At a higher level, it would be involved in the selection of the strategy used to resolve the arithmetic problems (i.e., retrieval from memory) by rejecting other potential alternatives. At a lower level, once the children have selected one strategy to resolve the problem (i.e., retrieval from memory), inhibition would be also applied to select the correct arithmetic facts (i.e., addition facts) by suppressing the activation of related arithmetic facts (i.e., multiplication facts).

Pedagogical and clinical implications can be drawn from the results found in the current study. If inhibition is involved in the selection of arithmetic facts and inhibitory control develops with age, teachers might use tasks involving the suppression of irrelevant information in order to increase the children ability to select correct arithmetic facts to resolve arithmetic problems. Moreover, inhibition would be a general ability that might underlie deficits in mathematical processing such as developmental dyscalculia (i.e., Szucs, Devine, Soltesz, Nobes, & Gabriel, 2013). Thus, interference control might be evaluated systematically when children are screened for developmental dyscalculia and inhibitory control tasks would be used by teachers in the training protocol.

To conclude, the present study traces the development of inhibitory control as the mechanism responsible to select arithmetic facts in children from second cycle of elementary school to first cycle of high school. We have observed a developmental trend in resolving simple mathematical problems which is accompanied by the concurrent activation of several arithmetic facts and the use of an emerging inhibitory mechanism responsible to select the one needed to correctly perform simple arithmetic tasks.

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CHAPTER VIII

GENERAL DISCUSSION

The current doctoral dissertation was framed within the field of cognitive arithmetic. We focused on retrieval from memory as the way of solving simple arithmetic problems. Concretely, we evaluated two aspects associated to the functioning of the network of arithmetic facts stored in long-term memory: coactivation of arithmetic facts across operations (additions and multiplications), and selection of correct answers by the use of an inhibitory mechanism. In this chapter, we discuss the contribution of our research work to the understanding of coactivation and selection in simple arithmetic.

In our research work, we have observed an associative confusion effect which reflected coactivation of arithmetic facts. Moreover, we offered evidence about the use of an inhibitory mechanism to select the appropriate arithmetic fact needed to resolve simple addition problems (Chapter III). We analyzed in deep the associative confusion effect to confirm that it involved coactivation of arithmetic facts. To this end, we gathered electrophysiological evidence of this phenomenon. We also corroborated that electrophysiological measures were sensitive to the consequences of applying inhibition to select arithmetic facts (Chapter IV). We continued with the evaluation of whether numerical format, written number words (Chapter V) and oral numbers (Chapter VI) modulated the retrieval and selection of arithmetic facts. Finally, we showed that coactivation and inhibition of arithmetic facts develop with formal instruction in childhood (Chapter VII).

The structure of this Chapter is as follows. Firstly, we talk about our first goal: coactivation of simple arithmetic facts associated to additions and multiplications. Secondly, we focus on selection-by-inhibition of correct answers. Afterwards, all findings are taken together to give an integrative view of simple arithmetic processing.

COACTIVATION OF ARITHMETIC FACTS

The coactivation of arithmetic facts is one of the most important processes underlying the functioning of the arithmetic network (Campbell, 1987; Campbell & Graham, 1985). If we assume that arithmetic facts are represented in an associative network whose nodes are interconnected, when a simple problem is presented (e.g., 2 + 4), the correct answer would be activated in the network (6). Also, other answers related with the problem would be activated (e.g., the result of multiplying the operands, 8) due to spreading of activation between interconnected nodes in the network (Campbell, 1987). In sum, within this architecture, coactivation of arithmetic facts between operations (e.g., additions and multiplications) would take place. This concurrent activation would produce interference (as it was proposed in Campbell's network interference model, Campbell & Graham, 1985). Concretely, arithmetic facts associated to additions and multiplications would compete in the selection of the correct response. This competition would produce interference which is shown in the associative confusion effect (Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986).

In our experimental series, we showed the associative confusion effect in several occasions. In Chapter III, we showed that adult participants took more time to reject an incorrect addition problem whose result was that of multiplying the operands (e.g., 2 + 4 = 8) compared to an unrelated addition problem (e.g., 2 + 4 = 10). Furthermore, this effect was also found in Experiment 2, when participants executed the addition verification task without the presentation of filler multiplication problems. Together, this pattern of results suggests that activation spreads automatically through the network of arithmetic facts. Even when multiplications are no needed to perform the task, multiplication facts become activated when people have to resolve addition problems. Our pattern of results agrees with previous studies (Galfano, Rusconi, & Umiltà, 2003; García-Orza, Damas-López, Matas, & Rodríguez, 2009; LeFevre, Bisanz, & Mrkonjic, 1988; Rusconi, Galfano, Speriani, & Umiltà, 2004) in which activation of multiplication facts seems to take place automatically, even when there are no cues in the experimental task to promote the retrieval of these operations.

Therefore, in the first part of our experimental work, we have confirmed that the associative confusion effect is a robust phenomenon. Importantly, in all studies where the associate confusion effect has been shown, the underlying assumption is that it involves coactivation in the network of arithmetic facts (Lemaire, Fayol, & Abdi, 1991; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). This premise is corroborated in our experimental series (Chapter IV). Concretely, we considered the N400 component as an index of accessing to arithmetic information by spreading of activation in long-term memory (Niedeggen & Rösler, 1996, 1999; Niedeggen, Rösler, & Jost, 1999). We found an attenuation of the N400 component in related problems (e.g., 2 + 4 = 8) compared to unrelated problems (e.g., 2 + 4 = 10). This electrophysiological effect suggests that in fact, the associative confusion effect involves coactivation of additions and multiplications in long-term memory. Moreover, the N400 attenuation we observed with electrophysiological measures seems to suggest that firstly, coactivation of related multiplication facts facilitates the retrieval of addition facts. Afterwards, a competitive process would take place between coactivated arithmetic facts in order to select the correct answer, a process which was captured with behavioral measures in our experimental series (slower responses to related problems compared to unrelated problems).

Additional data found in our research work allows us to characterize the retrieval of arithmetic facts from long-term memory. An old question in numerical cognition is whether arithmetic facts are stored as abstract representations which do not depend on the format of the problem (Blankenberger & Vorberg, 1997; McCloskey, Macaruso, & Whetstone, 1992) or they are format-dependent representations (Campbell & Clark, 1988; 1992). In our research work, once we determined that the associate confusion effect really involved coactivation of arithmetic facts, we were ready to evaluate whether this effect was modulated by the format in which the problems were presented. An affirmative answer was found to this question. Concretely, the associative confusion effect was reduced when individuals resolved problems with written numbers relative to problems with Arabic digits, even when we made sure that all participants used retrieval from memory to resolve the problems (Chapter V, Experiment 2). Hence, our data indicate that activation of arithmetic facts depends on numerical format (see Bulthé, de

Smedt, & Op de Beeck, 2015, for a similar discussion in the field of magnitude representation in the intraparietal surcus with symbolic numbers and non-symbolic numerosities).

Moreover, the data we found in oral calculation (Chapter VI) also adds to the conclusion that numerical format determines the retrieval of arithmetic facts. In this case, we did not observed associative confusion effect when participants received addition problems orally. We interpreted this result as due to time constraints imposed by the auditory signal. Compared to the processing of problems with Arabic digits, oral problems can be processed before the participant receives the complete stimulus due to the temporal sequence in which operands and result are presented. As a consequence, individuals would have time to coactivate arithmetic facts and reach the correct answer before the addition result was listened. This interpretation was further corroborated in a subsequent experiment (Experiment 2, Chapter VI) in which Arabic digits were presented with the same temporal sequence as that of oral additions. No associative confusion effect was obtained which suggested that numerical format determines the retrieval of arithmetic facts.

Our research work also focused on the development of retrieval from memory in children from elementary school to high school (8-9, 10-11 and 12-13 years-old children). Firstly, we focused on coactivation of arithmetic facts through the study of the associative confusion effect (Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986), as we did with adults (Chapter III). In this regard, we assumed that the associative confusion effect reflected automatic access to arithmetic facts due to the spreading of activation among related nodes in the arithmetic network. If this assumption is right, the associative confusion effect would depend on the knowledge of simple arithmetic facts stored in memory and the effect would appear only when children have acquired a good knowledge of simple arithmetic. The results reported in Chapter VII agree with these predictions.

We observed that 8-9 years-old children did not show coactivation of arithmetic facts. However, when they were classified depending on their knowledge about simple multiplication problems, we found that high-skilled children showed the associative confusion effect. Thus, it seems that a good knowledge about simple multiplication facts is needed to automatically coactivate additions and multiplications. Furthermore, the accuracy of simple multiplication problems predicted the associative confusion effect in all children, so that improvements in multiplication skills fostered the coactivation of addition and multiplication facts even when this concurrent activation interfered in the selection process. When children are 10-11 years-old onwards, the associated network seems to be established fully in long-term memory compared to early ages (De Brauwer, Verguts, & Fias, 2006; Lemaire et al., 1991). According with this assumption, we found the associative confusion effect at this age, suggesting that coactivation of related multiplication information took place when resolving addition problems.

In short, we have considered the associative confusion effect as an index of retrieval and coactivation of arithmetic facts as supported with behavioral and electrophysiological measures (Winkelman & Schmidt, 1974; ºZbrodoff & Logan, 1986). Differences found between additions with Arabic digits, written numbers and oral numbers suggest that retrieval and coactivation are format-dependent. The absence and presence of the associative confusion effect with age indicate that retrieval and coactivation develop with formal education in arithmetic.

SELECTION-BY-INHIBITION OF ARITHMETIC FACTS

In previous pages we have discussed the results we found about the first goal of our research work: coactivation of arithmetic facts. The second goal of the current doctoral dissertation was to explore the mechanism responsible to select the answers needed to resolve arithmetic problems. We proposed the inhibitory nature of this mechanism, which would suppress irrelevant information when competition is observed in the arithmetic network. In this regard, both theoretical models (Whalen, 2000) and empirical research highlight the relevance of inhibition in arithmetic performance (Adams & Hitch, 1997; Bull, Johnston, & Roy, 1999; Bull & Scerif, 2001; Dooren & Inglis, 2015; Fürst & Hitch, 2000; Geary, Hamson, & Hoard, 2000; Lemaire et al., 1991). Concretely, Campbell et al. (Campbell, Chen, & Maslany, 2013; Campbell & Dowd, 2012; Campbell, Dufour, & Chen, 2015; Campbell & Thompson, 2012) proposed that the selection of arithmetic facts was mediated by inhibition. The authors found a RIF effect in arithmetic, so that the retrieval of multiplication facts in a training phase (e.g., 2 x 4 = ?) slows down the retrieval of addition counterparts (e.g., 2 + 4 = ?) in a second test phase. This suggests the existence of an inhibitory mechanism that suppresses irrelevant information associated to addition facts in the training phase.

In several experiments reported in our research work we obtained empirical evidence of this inhibitory mechanism during the resolution of simple arithmetic problems. The results showed that participants took more time to verify addition problems whose result was that of multiplying the operands of the previous trial (e.g., 2 + 6 = 8, preceded by 2 + 4) compared to control addition problems (e.g., 4 + 6 = 10, preceded by 2 + 4). Therefore, an inhibitory mechanism seems to be responsible to suppress irrelevant information in the first trial (8), so individuals would be able to resolve competition between several arithmetic facts (8, 6) and select the correct one (6) at the end of the process. As a consequence, in the second trial it is more difficult to retrieve the inhibited result (8) from memory again. The difficulty of retrieving arithmetic facts previously inhibited is reinforced by the pattern of results found with electrophysiological measures (Chapter IV). Differences were found between the related and unrelated condition after inhibition (second trial) as reflected by P200 modulations (more positive in the related condition); an ERP component associated to the difficulty of retrieving information from long-term memory (Dunn, Dunn, Languis, & Andrews, 1998; Raney, 1993; Smith, 1993).

As we have commented, previous studies showed inhibition of arithmetic facts during a training phase which was captured in a second test phase (Campbell et al., 2013; Campbell & Dowd, 2012; Campbell & Thompson, 2012). If we compare previous studies about inhibition in simple arithmetic and those reported in our research work, the main contribution of our studies is the demonstration that inhibition applies in a continuous manner every time competition between arithmetic facts takes place.

Across our experimental series, we have observed that coactivation of arithmetic facts is modulated by the format in which the problems are presented. Similarly, format effects impact the inhibitory mechanism used to verify additions problems (Chapter V). When problems with Arabic digits and written numbers are compared, format effects in coactivation go hand in hand with inhibition. Specifically, when an overall comparison is performed between the verification of problems presented with Arabic digits vs. number words, the associative confusion effect is only found in the first case. The same is observed when the consequences of applying inhibition were evaluated (the relation effect in the second trial was observed with problems in Arabic digit format only). Put it differently, inhibition seems to be applied when competition between arithmetic facts takes place. Moreover, there is a fine-grained adjustment between the degree of competition and the inhibition applied during the selection process. When we controlled for the strategy used to resolve addition problems, participants with a high use of retrieval from memory showed evidence of coactivation and inhibition in both formats (digit and written number word formats), but the magnitude of these effects was smaller when problems were presented with written-words. Similarly, participants with less use of retrieval from memory showed coactivation and inhibition only when problems were presented with Arabic digits. However, in the written-word format, less automatic spreading activation in the network reduced coactivation of several arithmetic facts and inhibition was not needed. Therefore, inhibition seems to be applied to the same extend as competition is observed.

Moreover, the results we observed in oral calculation (Chapter VI) seem to indicate that inhibition is quickly applied when participants receive an oral problem so they have time to resolve interference before the complete problem has been processed. Concretely, we did not observe associative confusion effect with oral problems in the first trial. However, analysis of the second trial supported inhibition of related multiplication facts in the first trial to select the correct answer associated to additions. In other words, when participants listened the operands (e.g., 2 + 4), the correct result (6) and the related result associated to multiplication (8) could be activated automatically. Afterwards, participants had time to inhibit the irrelevant multiplication fact (8) before the presentation of the proposed result (6). Thus, when the proposed result was presented, participants took similar time to respond to related and unrelated addition problems. In Experiment 2 of Chapter VI, we corroborated this interpretation. We found that the associative confusion effect disappeared when Arabic digit problems were presented sequentially, simulating the timing in which oral problems were listened in Experiment 1. Moreover, in the second trial, we again observed an interference effect suggesting that the related result was inhibited in the first trial. This pattern of results fits well with previous research with Arabic digits (Lemaire et al., 1991): the presentation of a delay (higher than 300 ms) between the operands and the result produces the absence of the associative confusion effect, suggesting that participants had enough time to resolve competition by inhibiting irrelevant information.

Finally, we aimed at examining the development of the selection-by-inhibition mechanism in school-age. There were reasons to predict that this mechanism is present in children population. Firstly, the ability of inhibitory control have a greater development when children are between 6-8 years-old and it continues to develop through school-age (Davidson, Amso, Anderson, & Diamond, 2006; Korkman, Kemp, & Kirk, 2001; Leon-Carrion, García-Orza, & Pérez-Santamaría, 2004). Secondly, previous research demonstrated a relationship between inhibitory control and arithmetic skills; so that better inhibitory control, better performance in arithmetic (Adams & Hitch, 1997; Bull et al., 1999; Bull & Scerif, 2001; Geary, et al., 2000).

To evaluate the existence of an inhibitory mechanism in children, in Chapter VII we analyzed the second trial of our experimental task to index the consequences of applying inhibition to resolve competition among coactivated arithmetic facts. The group of 8-9 years-old children did not show evidence of this inhibitory mechanism. It is important to note that coactivation did not take place in this group, so inhibition was not needed to resolve the task. We found signs of inhibition in 10-11 and 12-13 years-old children, so that they took more time to refuse addition problems whose results were those of multiplying the operands of the previous trial (e.g., 2 + 6 = 8, preceded by 2 + 4) compared to unrelated addition problems. Therefore, in Chapter VII, we observed the developmental course of inhibition in mental arithmetic. No inhibition seems to be applied in young children (8-9 years-old) since no retrieval seems to be used (no associative confusion effect); while 10-11 years-old children onwards seems to apply inhibition to resolve competition of arithmetic facts. A fine-grained analysis showed that the cost associated to inhibit irrelevant information seems to decrease between 10-11 years-old and 12-13 years-old children. This agrees with previous assumptions about the continuous develop of inhibitory control through school-age (Davidson, et al., 2006; Korkman, et al., 2001). Thus, 10-11 years-old children need more time to retrieve inhibited information than 12-13 years old children, showing that the inhibitory mechanism is more efficient in the last group.

To sum up, in our experimental series we have gathered evidence of selection-by inhibition as the mechanism used to select arithmetic facts. When inhibition applies, additional time is required to reactivate the inhibited information as shown by the interference effect found in the second trial of our experimental paradigm (Chapters III and IV). Inhibition seems to be somehow proportional to the degree of competition so the format of the problem determines coactivation of arithmetic facts and inhibition of irrelevant information (Chapters V and VI). Finally, we have observed that the use of inhibition in cognitive arithmetic follows a developmental trajectory (Chapter VII).

SIMPLE ARITHMETIC: AN INTEGRATIVE VIEW

In the introduction section we described several models of cognitive arithmetic. Some models are directly related to the resolution of arithmetic problems: Ashcraft's network retrieval model (Ashcraft, 1982), distribution of associations model proposed by Siegler and Jenkins (1989), Campbell's network interference model (Campbell, 1987; Campbell & Graham, 1985), Whalen's semantic network retrieval model (Whalen, 2000) and Baroody's schema-based model (Baroody, 1983; 1994). Other models commented later in the introduction section focus on possible format effects in mental arithmetic: McCloskey's abstract-modular model (McCloskey et al., 1992) and Campbell's encoding-complex model (Campbell & Clark, 1988). In this section, we would like to offer an integrative view about simple arithmetic in light of the results found in our experimental series. We will focus on critical aspects about representation and processing of arithmetic facts. We also addresses some sensitive questions (with difficult answers) to outline tentative explanations.

An integrated network represents cross-operation arithmetic facts (additions, multiplications).

The continuous practice in a specific field leads to automatic performance. This process is associated with structures of knowledge which are formed, stored and quickly retrieved from long-term-memory. The classical view to understand how arithmetic knowledge is represented in memory is within associative models (Ashcraft, 1982; Campbell, 1987; Campbell & Graham, 1985; Siegler & Jenkins, 1989; Whalen, 2000). All these models agree with the existence of a mental structure in which simple arithmetic facts are represented and stored in an associative network whose nodes are interrelated. Furthermore, in Ashcraft's network retrieval model (Ashcraft, 1982) and Campbell's network interference model (Campbell y Graham, 1985) cross-operation connections in the network are considered. In this regard, the associative confusion effects found in our research work supports these assumptions: arithmetic facts are stored in long-term memory and addition and multiplication facts are associated in the network (see Fabbri, 2015, for an alternative explanation).

The network of arithmetic facts is format-dependent.

In Whalen's semantic network retrieval model (Whalen, 2000) excitatory connections are proposed between nodes that represent the problem (e.g., 2 + 4), other related nodes (e.g., 2 x 4) and answer-nodes (e.g., 6, 8). This proposal is in line with previous associative models (Ashcraft, 1982; Campbell, 1987; Campbell & Graham, 1985) in which it is assumed that the access to the arithmetic network involves spreading of activation. Our research work supports the access to the arithmetic network due to spreading of activation, because when an addition problem is presented coactivation of addition and multiplication facts takes place. Additionally, our findings can be accommodated with Campbell's encoding-complex model (Campbell & Clark, 1988) because in this model it is assumed that the access to the network is modulated by numerical format. In this regard, our results about numerical format effects in mental arithmetic suggest that the access to arithmetic network depends on automaticity in which problems are processed: With well-practiced formats (Arabic digits or oral problems) the access to the network seems to be automatic due to spreading of activation, whereas this automatic access is somewhat disrupted with unfamiliar formats (written number words). It is important to note that the results found in our research work are against format-independent models that assume an abstract-modular representation of arithmetic facts (McCloskey et al., 1992). Numerical format determines the associative confusion effect and this effect is a consequence of the spreading of activation (coactivation) at a central level of processing. Thus, our data seem to support models in which numerical format modulates the representation and processing of arithmetic facts (Campbell's encoding-complex model, Campbell & Clark, 1988).

What is exactly inhibited?

As we have indicated, the results found in our study clearly indicate that arithmetic facts from different operations are coactivated (additions, multiplications). In Campbell's network interference model (Campbell, 1987; Campbell & Graham, 1985), it is proposed that this coactivation produces interference in the network. This interference reflects competition between several answers because only one is required to give the correct response. Under this model, the answer with higher activation is selected finally. However, the mechanism used to solve competition and select the answer with higher activation is not discussed in this model. In Whalen's semantic network retrieval model (Whalen, 2000) it is proposed a possible inhibitory mechanism responsible for resolving interference and selecting the correct answer to an arithmetic problem. Our research work supports the involvement of an inhibitory mechanism in the selection of simple arithmetic facts. In all studies, we found that after competition between related arithmetic facts (e.g., after presentation of 2 + 4, coactivation of the correct result 6 and related result 8 generates competition between 6 and 8), inhibition of irrelevant facts takes place (e.g., inhibition of the related answer 8, in order to select the correct answer 6).

After observing inhibition in the resolution of competition between arithmetic facts, a critical question is about what was exactly inhibited in our study. Several alternatives are possible and our experimental work cannot support completely any of them. It might be possible that participants inhibited the complete arithmetic fact (2 x 4 = 8) when they retrieved the correct one (2 + 4 = 6). This explanation seems to be not correct since the consequences of applying inhibition with additions in the first trial were observed in the second trial with additions too. A way of examining this point would be to compare the interference effect in the second trial with related additions and multiplications. If inhibition applies to the whole multiplication problem, participants would take more time to answer afterwards when exactly the same multiplication problem was presented again. Thus, large interference effect with related multiplications in the second trial relative to related additions would support the idea that inhibition was applied to the whole multiplication fact.

Another plausible argumentation is that participants inhibited competing results. In the example 2 + 4 = 8, participants might inhibit 8 in order to select 6. If we assume this explanation, further questions show up. It is possible that participants inhibited the number as an entity (8 represents a magnitude of eight elements) regardless of its role in the operation (the result of the problem). Alternatively, participants might inhibit the number labeled as a possible response (potential response-8). We are planning future experiments to dissociate between these accounts. If participants inhibited the number as an entity, interference effect would be found in the second trial when participants performed a non-arithmetic task with the inhibited number (e.g., a parity task, a magnitude task, etc.). If participants inhibited the number as a response (potential response-8), they would take more time to respond to a subsequent operation with the same result (10 - 2 = 8).

Development of simple arithmetic: From procedures to retrieval from memory.

Several models about cognitive arithmetic have considered that the way in which simple arithmetic problems are resolved follows a developmental trajectory. Two main views have been outlined to explain how arithmetic resolution changes in childhood. Firstly, the practice with additions, for example, would foster the efficient use of procedures as the way of solving these problems (Baroody's schema-based model, Baroody, 1983, 1994). On the contrary, other authors defend that practicing arithmetic problems would favor retrieval from memory. To illustrate, in Siegler's distribution of associations model (Siegler & Jenkins, 1989), it is assumed a balance between the use of procedures and retrieval from memory. The weight associated to retrieval as the way of solving the problem would increase with formal instruction in arithmetic. Moreover, in Ashcraft's network retrieval model (Ashcraft, 1982) and Campbell's network interference model (Campbell & Graham, 1985), it is assumed that related nodes in the network of arithmetic facts would strength their connections in childhood. If we consider that the associative confusion effect reflects the use of retrieval from memory, the results found in our work seems to indicate that arithmetic resolution in children would change from procedures to retrieval with age. Concretely, 8-9 years-old children did not show associative confusion effect while it was present in children with 10-11 years-old onwards. Hence, it seems that 10 years of age is the inflexion point to change from procedures to retrieval as the mechanism underlying arithmetic resolution. We acknowledge our research remains silent about the possible automaticity of procedural strategies with practice (Baroody, 1983, 1994); however, what is clear is that older children preferred retrieval from memory since the associative confusion effect found in our study cannot be accounted by the use of procedural strategies.

The use of procedures in simple arithmetic.

From the very beginning of cognitive arithmetic a debate was open between advocates of direct retrieval of arithmetic facts to resolve simple problems (Ashcraft's network retrieval model, Ashcraft, 1982), and those proposing the use of procedural strategies (Baroody's schema-based model, Baroody, 1983; 1994). Over the years, retrieval form memory was assumed to be the best way of resolving problems since it would be associated to automatic and faster performance (Campbell's network interference model, Campbell, 1987; Campbell & Graham, 1985). However, the idea of procedures always remained latent; for example, as the initial operations used by children to resolve simple problems until they acquired the network of arithmetic facts (Siegler's distribution of associations model; Siegler & Jenkins, 1989). Our data seems to suggest that the use of procedures is not an all or nothing question but they would be flexible adopted depending on some factors. The resolution of simple arithmetic problems with Arabic digits and written number words might involve both, retrieval from memory and procedural strategies (Geary & Wiley, 1991; Healy, Rickard, & Bourne, 1993). To illustrate, when we compared the processing of problems presented with Arabic digits and written numbers, there is a tendency to prefer procedures with problems in verbal format. Hence, numerical format might determine the use of procedures over retrieval. Furthermore, it is also true that even with problems presented with Arabic digits, participants reported the use of procedures in some occasions. This finding is in line with very recent research showing that simple arithmetic problems can be resolve with procedural strategies (Barrouillet & Thevenot, 2013; Della Puppa et al., 2015; LeFevre, Sadesky, & Bisanz, 1996; Roussel, Fayol, & Barrouillet, 2002; Thevenot, Barrouillet, Castel, & Uittenhove, 2016; Thevenot, Fanget, & Fayol, 2007). Hence, some simple problems received in a well-practiced format might be still resolved with procedures in some occasions.

CONCLUDING REMARKS

In the current doctoral dissertation, we contributed to understand simple arithmetic. When individuals resolve addition problems, they usually retrieve arithmetic facts from memory. Arithmetic facts are interconnected, they become activated and they might compete for selection. This competition seems to produce interference which is resolved with the involvement of an inhibitory mechanism. Both coactivation and selection-by-inhibition can be considered two steps of the retrieval process, and both seems to depend on the automaticity in which problems are processed according to the format of the problem and the development of the arithmetic network through formal instruction in childhood.

We might ask whether the pattern of results found in our study is restricted to cognitive arithmetic. The response seems to be negative. Coactivation and inhibition has been shown in other fields of cognitive psychology. For example, the two languages of bilingual speakers seem to be activated even when they need only one language to communicate. In some occasions, this non-selective coactivation produces interference and inhibition seems to be applied to resolve competition (Macizo, Bajo, & Martín, 2010). This pattern of results is analogous to the one presented in our research work, suggesting that representations of different types of information (linguistic, arithmetic, etc.) are retrieved in a similar way, through coactivation of related information in memory and selection-by-inhibition when competition arises.

Moreover, if we focus on the findings found about simple arithmetic in children, pedagogical and clinical implications can be drawn. We have demonstrated that the correct functioning of arithmetic resolution in adults involves inhibition. Also, we have corroborated that this inhibitory mechanism develops with instruction in arithmetic. Thus, cognitive programs based on the training in inhibition might be implemented in order to foster the acquisition of arithmetic skills in children. Furthermore, training in inhibition might be a tool to remediate or compensate deficits in number processing such as dyscalculia. Future research will shed light on these suggestions.

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CHAPTER IX

APPENDICES

APPENDIX 1

Experimental trials used in the study

First trial		Second trial	
Multiplication 1	Unrelated 1	Multiplication 2	Unrelated 2
2 + 3 = 6	2 + 3 = 4	2 + 4 = 6	1 + 3 = 4
2 + 4 = 8	2 + 4 = 10	2 + 6 = 8	4 + 6 = 10
3 + 4 = 12	3 + 4 = 10	5 + 7 = 12	4 + 6 = 10
2 + 5 = 10	2 + 5 = 14	3 + 7 = 10	6 + 8 = 14
2 + 6 = 12	2 + 6 = 10	4 + 8 = 12	3 + 7 = 10
2 + 7 = 14	2 + 7 = 12	6 + 8 = 14	5 + 7 = 12
4 + 4 = 16	4 + 4 = 18	7 + 9 = 16	9 + 9 = 18
2 + 8 = 16	2 + 8 = 14	7 + 9 = 16	5 + 9 = 14
3 + 3 = 9	3 + 3 = 11	3 + 6 = 9	7 + 4 = 11
3 + 5 = 15	3 + 5 = 13	7 + 8 = 15	6 + 7 = 13

APPENDIX 2

Filler trials used in the study

First trial	Second trial
Addition trials	
8 + 9 = 17	1 + 5 = 6
1 + 7 = 9	4 + 9 = 15
3 + 9 = 12	1 + 2 = 5
1 + 8 = 9	1 + 9 = 10
6 + 9 = 17	1 + 4 = 7
3 + 8 = 11	1 + 6 = 9
Multiplication trials	
8 x 9 = 72	1 x 4 = 4
1 x 6 = 5	4 x 9 = 36
3 x 8 = 26	1 x 7 = 6
1 x 7 = 6	3 x 9 = 25