# TOTAL CROSS-SECTIONS AND BLOCH-NORDSIECK GLUON RESUMMATION* 

G. Pancheri<br>INFN, Frascati National Laboratories, I00044 Frascati, Italy<br>R.M. Godbole<br>Centre for High Energy Physics, Indian Institute of Science Bangalore, 560 012, India<br>A. Grau<br>Departamento de Física Teórica y del Cosmos, Universidad de Granada, Spain

and Y.N. SRIVAStava

Physics Department and INFN, University of Perugia, Perugia, Italy
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The physics underlying the fall and eventual rise in various total crosssections at high energies has been investigated over a decade using a model based on the Bloch-Nordsieck resummation in QCD. Here a brief review of our latest results is presented and comparison made with experimental data on $p p, \gamma$ proton and $\gamma \gamma$ total cross-sections.

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## 1. Introduction

Total cross-sections at high energies provide significant information about the distribution of the constituents and the nature of their interaction at very short distances. Even though QCD is the fundamental theory for strong interactions, our lack of knowledge about the confinement of quarks and

[^0]glue has not allowed a first principle determination of hadronic total crosssections and hence one has to resort to phenomenological models. Over several years we have developed and refined a model based on a BlochNordsieck (BN) resummation of soft partons. Through it, we have been improving our understanding of the observed variations in the cross-sections, in a quantitative way. Details of our work and its evolution, can be followed through references [1-8]. A short summary of data for various processes and their comparison with our model predictions are discussed in the subsequent sections. Given the paucity of space, we shall only focus on the energy dependences in $p p, \gamma p$ and $\gamma \gamma$ reactions and the uncertainities therein present.

## 2. QCD and the energy dependence of total cross-sections

In Fig. 1, we show a comparison of the energy dependence in different processes. The data show a clear initial fall and eventual rise in the total cross-sections for all processes.


Fig. 1. $p p / \bar{p} p, \gamma p$ and $\gamma \gamma$ total cross-sections. The scaling factor to compare photons on the same scale is obtained from quark counting rules and VMD.

The uncertainties in the data for $\gamma p[9-12]$ and $\gamma \gamma[13,14]$ are shown in Fig. 2 and Fig. 3.

Theoretically, perturbative QCD provides a natural mechanism to explain the rise with energy of total cross-sections. As the hadronic c.m. energy increases from 5 to $10^{4} \mathrm{GeV}$ the number of parton collisions increases. In this approach. it is the rise with energy of the jet cross-section

$$
\begin{equation*}
\sigma_{\text {jet }}=\int_{p_{\mathrm{tmin}}}^{\sqrt{s} / 2} d p_{\mathrm{t}} \int_{4 p_{\mathrm{t}}^{2} / s}^{1} d x_{1} \int_{4 p_{\mathrm{t}}^{2} /\left(x_{1} s\right)}^{1} d x_{2} \sum_{i, j, k, l} f_{i \mid a}\left(x_{1}\right) f_{j \mid b}\left(x_{2}\right) \frac{d \hat{\sigma}_{i j \rightarrow k l}(\hat{s})}{d p_{\mathrm{t}}}, \tag{2.1}
\end{equation*}
$$

which drives the rise of the total cross-section. This quantity depends strongly on $p_{\text {tmin }}$, the minimum transverse momentum of the produced jets


Fig. 2. Photoproduction data compared with predictions from the Aspen model and the EMM.


Fig. 3. At left we show $\gamma \gamma$ cross-section data compared with predictions from different models. At right the corresponding predictions for $e^{+} e^{-}$hadronic crosssections in the EMM, Aspen and BKKS models.
and can be calculated by convoluting the parton densities for protons and photons. To satisfy unitarity, the jet cross-sections are embedded into the eikonal formalism. In this Eikonal Minijet Model (EMM) the total crosssection is given by

$$
\begin{equation*}
\sigma_{\mathrm{tot}}=2 \int d^{2} \vec{b}\left[1-e^{-n(b, s) / 2}\right] \tag{2.2}
\end{equation*}
$$

where $n(b, s)$ is the average number of inelastic collisions at impact parameter $b$. Introducing a separation between the soft and hard contributions and
assuming factorization of the impact parameter and energy dependence we can write

$$
\begin{equation*}
n(b, s)=n_{\mathrm{soft}}+n_{\mathrm{hard}}=A_{\mathrm{soft}}(b) \sigma_{\mathrm{soft}}+A_{\mathrm{jet}}(b) \sigma_{\mathrm{jet}} \tag{2.3}
\end{equation*}
$$

where $\sigma_{\text {jet }}$ drives the rise and the function $A(b)$ represents the impact parameter distribution of partons in the collision.

In the simplest EMM formulation $A(b)$ is obtained through convolution of the electromagnetic form factors of the colliding particles, i.e.

$$
\begin{equation*}
A_{a b}(b) \equiv A\left(b ; k_{a}, k_{b}\right)=\frac{1}{(2 \pi)^{2}} \int d^{2} \vec{q} e^{i q \cdot b} \mathcal{F}_{a}\left(q, k_{a}\right) \mathcal{F}_{b}\left(q, k_{b}\right) \tag{2.4}
\end{equation*}
$$

This model is unable to describe - without further adjustments - the experimental data for total cross-sections in all the energy range. This can be seen in Fig. 2 where data for $\gamma p$ cross-section are compared with a band corresponding to different sets of parameters in the EMM. Similarly, we show in Fig. $3 \gamma \gamma$ cross section data and its comparison with various models, EMM [3], Regge-Pomeron [15], Aspen [16], BSW [17], GLMN [18], Cuddell et al. [19], BKKS [20]. The predictions for $e^{+} e^{-} \rightarrow$ hadrons cross-section appear in Fig. 3 for the Aspen [16], EMM [3] and BKKS [20] models.

### 2.1. Energy dependence of soft gluon emission

A more realistic EMM is obtained taking into account soft gluon emission from initial state valence quarks. In this model, the impact parameter distribution of partons is obtained as the Fourier transform of the transverse momentum distribution of the colliding partons computed through soft gluon resummation techniques. The resulting expression is

$$
\begin{equation*}
A_{\mathrm{BN}}=\frac{e^{-h(b, s)}}{\int d^{2} \vec{b} e^{-h(b, s)}} \tag{2.5}
\end{equation*}
$$

where

$$
\begin{equation*}
h(b, s)=\frac{8}{3 \pi} \int_{0}^{q_{\max }} \frac{d k}{k} \alpha_{\mathrm{s}}\left(k^{2}\right) \ln \left(\frac{q_{\max }+\sqrt{q_{\max }^{2}-k^{2}}}{q_{\max }-\sqrt{q_{\max }^{2}-k^{2}}}\right)\left[1-J_{0}(k b)\right] \tag{2.6}
\end{equation*}
$$

The upper limit $q_{\max }$ is the maximum energy allowed to each single soft gluon emitted in the collision and can be calculated for hard processes (those with
$\left.p_{\mathrm{t}}^{\mathrm{parton}} \geq p_{\mathrm{tmin}}\right)$ by averaging over the valence parton densities, i.e.

$$
\begin{equation*}
M \equiv\left\langle q_{\max }(s)\right\rangle=\frac{\sqrt{s}}{2} \frac{\sum_{i, j} \int \frac{d x_{1}}{x_{1}} f_{i / a}\left(x_{1}\right) \int \frac{d x_{2}}{x_{2}} f_{j / b}\left(x_{2}\right) \sqrt{x_{1} x_{2}} \int_{z_{\min }}^{1} d z(1-z)}{\sum_{i, j} \int \frac{d x_{1}}{x_{1}} f_{i / a}\left(x_{1}\right) \int \frac{d x_{2}}{x_{2}} f_{j / b}\left(x_{2}\right) \int_{z_{\min }}^{1}(d z)} \tag{2.7}
\end{equation*}
$$

with $z_{\min }=4 p_{\text {tmin }}^{2} /\left(s x_{1} x_{2}\right)$. As $q_{\max }$ depends on the energy of the colliding partons, the impact parameter distribution Eq.(2.5) will be energy dependent. The behaviour of $q_{\max }$ with energy is shown in Fig. 4 where the upper line is the one obtained with Eq.(2.7) and the lower curve are the $q_{\max }$ values through which the soft part $n_{\text {soft }}(b, s)$ has been calculated phenomenologically to describe $p p$ scattering at low energy. Using these values of $q_{\text {max }}$, in Fig. 4 we show the predictions of the model for $p p$ and $\bar{p} p$ total cross-sections with GRV [21] densities.


Fig. 4. At left we show the energy dependence of the maximum energy allowed to single gluon emission for hard or soft processes. At right we show $p p$ and $\bar{p} p$ total cross-sections data compared with predictions from EMM with Bloch-Nordsieck soft gluon resummation.

The EMM model with Bloch-Nordsieck soft gluon resummation have also been applied to $\gamma p$ and $\gamma \gamma$ collisions. Theoretical results are compared with experimental data and shown in Fig. 5 for $\gamma p$ and in Fig. 6, for $\gamma \gamma$, using two different partonic densities for the photon, GRS [22] and CJKL [23].


Fig. 5. We show photoproduction data compared with the soft gluon improved EMM for different values of $p_{\text {tmin }}$ using GRS densities for the photon (at left) and CJKL densities (at right).


Fig. 6. We show $\gamma \gamma$ cross-section data compared with the soft gluon improved EMM using GRS photon densities (at left) and CJKL densities (at right).

## 3. Conclusion

In this brief survey, we have presented a comparison of our model predictions with available data for various processes. The BN resummed gluon distributions appear to describe quite adequately the rise and fall visible in the data. Experimentally, there are still significant uncertainites. Theoretically, we need a better understanding of the $q_{\text {max }}$ parameter for the soft part.

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## REFERENCES

[1] G. Pancheri, Y.N. Srivastava, Phys. Lett. B158, 402 (1986).
[2] A. Corsetti, A. Grau, G. Pancheri, Y.N. Srivastava, Phys. Lett. B382, 282 (1996).
[3] A. Corsetti, R.M. Godbole, G. Pancheri, Phys. Lett. B435, 441 (1998).
[4] A. Grau, G. Pancheri, Y.N. Srivastava, Phys. Rev. D60, 114020 (1999);
[5] R.M. Godbole, A. Grau, G. Pancheri, Y.N. Srivastava, Invited talk at the International Workshop on QCD, Martina Franca, Italy, June, 2001 [arXiv: hep-ph/0205196]; [hep-ph/0408355].
[6] R.M. Godbole, A. de Roeck, A. Grau, G. Pancheri, J. High Energy Phys. 0306, 061 (2003) [arXiv: hep-ph/0305071].
[7] A. Grau, S. Pacetti, G. Pancheri, Y.N. Srivastava, Nucl. Phys. B (Proc. Suppl.) 126, 84 (2004).
[8] R.M. Godbole, A. Grau, G. Pancheri, Y.N. Srivastava, Nucl. Phys. B (Proc. Suppl.) 126, 94 (2004).
[9] ZEUS Collaboration, Phys. Lett. B293, 465 (1992); Z. Phys. C63, 391 (1994); Nucl. Phys. B627, 3 (2002) [hep-ex/0202034].
[10] H1 Collaboration, Z. Phys. C69, 27 (1995).
[11] ZEUS Collaboration, J. Breitweg et al., Phys. Lett. B487, 53 (2000) [arXiv:hep-ex/0005018].
[12] ZEUS Collaboration (C. Ginsburg et al. ), Proc. 8th International Workshop on Deep Inelastic Scattering, April 2000, Liverpool, editors: J.A. Gracey and T. Greenshaw, World Scientific, 2001.
[13] L3 Collaboration, M. Acciarri et al., Phys. Lett. B408, 450 (1997); Phys. Lett. B519, 33 (2001) [hep-ex/0102025]; L3 Collaboration, A. Csilling, Nucl. Phys. Proc. Suppl. B82, 239 (2000).
[14] OPAL Collaboration. G. Abbiendi et al., Eur. Phys. J. C14, 199 (2000).
[15] G. Schuler, T. Sjöstrand, Z. Phys. C68, 607 (1995); Phys. Lett. B376, 193 (1996); Z. Phys. C73, 677 (1997).
[16] M.M. Block, E.M. Gregores, F. Halzen, G. Pancheri, Phys. Rev. D58, 17503 (1998); M. Block, E.M. Gregores, F. Halzen, G. Pancheri, Phys. Rev. D60, 54024 (1999).
[17] C. Bourelly, J. Soffer, T.T. Wu, Mod. Phys. Lett. A15, 9 (2000).
[18] E. Gotsman, E. Levin, U. Maor, E. Naftali, Eur. Phys. J. C14, 511 (2000) [hep-ph/0001080].
[19] J.R. Cudell et al., arXiv:hep-ph/0212101; Phys. Rev. Lett. 89, 201801 (2002) [arXiv:hep-ph/0206172].
[20] B. Badelek, M. Krawczyk, J. Kwiecinski, A.M. Stasto, Phys. Rev. D62, 074021 (2000); arXiv:hep-ph/0001161.
[21] M. Glück, E. Reya, A. Vogt, Phys. Rev. D46, 921973.
[22] M. Glueck, E. Reya, I. Scheinbein, Phys. Rev. D60, 054019 (1999).
[23] F. Cornet, P. Jankowski, M. Krawczyk, A. Lorca, Phys. Rev. D68, 014010 (2003) [arXiv:hep-ph/0212160].


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