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Departamento de Teoría e Historia Económica

Three Essays On Informatics Decision Support Systems In Product Selection

Tesis Doctoral

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To

the memory of the martyrs of Syria

El doctorando Fadi Amroush y los directores de la tesis Nikolaos Georgantzís, Antonio Gabriel López Herrera, y, Josean Garrues Irurzun

Garantizamos, al firmar esta tesis doctoral, que el trabajo ha sido realizado por el doctorando bajo la dirección de los directores de la tesis y hasta donde nuestro conocimiento alcanza, en la realización del trabajo, se han respetado los derechos de otros autores a ser citados, cuando se han utilizado sus resultados o publicaciones.

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Abstract

Product selection has become more important since online consumers are given more product choices than they can find in traditional shops. Expert systems play a key role in providing an intelligent decision support system in the phases of commerce, including the pre-sale, sale, and post-sale phases.

There are many techniques for building a product selection expert system. This thesis is interested in case-based reasoning (CBR), the analytic hierarchy process (AHP), and the fuzzy AHP (FAHP). The purpose of this thesis is to understand and investigate how people select based on two different decision support system models, CBR and FAHP.

This thesis is comprised of three studies related to product selection. The first study discusses the effects gender has on product selection based on two conducted experiments, and the effect of the informatics advice. The second study experimentally and statistically studies the difference between conventional AHP and FAHP and evaluates gender-specific product selection based on both models. The third study is motivated by the results of the first two studies and proposes an expert system for product selection using FAHP and CBR-based approaches.

Resumen

La importancia de la selección de productos ha aumentado desde que los consumidores en línea pueden elegir entre muchos más productos de los que encuentran en negocios tradicionales. Los sistemas expertos son fundamentales para proveer un sistema inteligente de apoyo a la decisión en las etapas de comercialización, incluidas la preventa, la venta y la posventa.

Hay diversas técnicas para construir un sistema experto de selección de productos. Esta tesis se interesa en case-based reasoning (CBR), el analytic hierarchy process (AHP) y el fuzzy AHP (FAHP). El objetivo de esta tesis es entender e investigar la manera en que la gente elige a partir de dos modelos diferentes de sistemas de apoyo a la decisión: CBR and FAHP.

Esta tesis presenta tres investigaciones en la selección de productos y profundiza en la manera en que la gente elige un producto y los motivos que tiene para cambiar su selección. Se realizaron dos experimentos para lograr el objetivo y se utilizaron dos modelos diferentes durante las investigaciones: case-based reasoning (CBR) y el fuzzy analytic hierarchy process (FAHP).

La primera investigación se realizó para investigar el efecto que el género tiene en la selección de productos a partir de dos modelos. En el experimento también se evaluó el efecto que el asesoramiento informático de los diferentes sistemas de soporte de decisiones, CBR and FAHP, ejerció en la selección del consumidor.

En la segunda investigación se estudió estadísticamente la diferencia entre AHP convencional y FAHP haciendo uso de un punto de vista experimental. La

investigación se basó en el segundo experimento que proponía un modelo FAHP para la evaluación de productos.

En la tercera investigación se examinó la fusión de CBR y FAHP en situación de uso de un sistema experto en selección de productos. Lo fundamental de esta propuesta consistió en la fusión de los beneficios de CBR y AHP para presentar un nuevo sistema informático de apoyo a la decisión.

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Introduction

Product selection has become more important nowadays. Online selling has given consumers a better choice of products than can be found in traditional shops. Selecting what products to buy is one of the most common decisions a consumer makes. Mostly, the consumer selects products based on his or her personal preferences. Let us assume that you want to buy a laptop, and there are many types of laptops with different prices and features. The main question then is how to select the best laptop that meets your preferences, taking into account the budget you allocated for purchasing this item and without paying more for some features you do not really need. One of the most conventional ways is to get a list of product specifications (RFI[1]) and the corresponding prices and then compare the information you obtained from different suppliers or sellers to help you identify the best product with the least price and best characteristics, without considering its suitability to your needs and preferences.

A decision support system (DSS) started from management decision systems in the early seventies. Then, it was defined by Keen and Scott-Morton (1978, pp. 58 and 59) who stated the following (Turban, 1995):

> *'Decision Support Systems couple the intellectual resources of individuals with the capabilities of the computer to improve the quality of decisions. It is a computer-based support system for management decision makers who deal with semi-structured problems.'*

[1] A request for information (RFI) is a standard business process whose purpose is to collect written information about the capabilities of various suppliers. Normally, it follows a format that can be used for comparative purposes.

In general, there are three key sub-areas under e-commerce, namely, pre-sales, sales, and after-sales. Pre-sales is defined as the provision of information about services or products to consumers. Sales is defined as the negotiation process of these products and services along with the actual process of selling. After-sales is defined as the support offered to consumers regarding problems encountered while using the products.

Expert systems play a key role in the provision of intelligent support in all of the abovementioned three phases. In the pre-sales phase, expert systems can be induced to provide an intelligent user interface so that it can incorporate consumer needs. In practical terms, this means that if an e-commerce application involves a limited number of products, the products can be placed under different categories by utilizing different structures to display each product on a web page under different groups. However, if a large number of products are present, a specific query-based interface would be required to aid the consumers to find their preferred product according to their needs and the best available offer (Schmitt and Bergmann ,1999).

The problem of finding the best products is referred to as supplier evaluation and selection problem in the literature review. This problem has been studied extensively. Various decision-making approaches have been proposed to tackle the problem[2].

Choosing the right suppliers involves much more than scanning a series of price lists, and choices depend on a wide range of factors both quantitative and qualitative. Extensive multicriteria decision-making approaches have been proposed for supplier selection, such as the analytic hierarchy process (AHP), Fuzzy AHP (FAHP), analytic network process, case-based reasoning (CBR), data envelopment analysis, fuzzy set

[2] Chapter 8 presents a review of literature of case-based reasoning (CBR), analytic hierarchy process (AHP), and fuzzy AHP (FAHP) in supplier and product selection.

theory, genetic algorithm, mathematical programming, simple multi-attribute rating approach, and their hybridizations.

The aim of this thesis is to understand and investigate experimentally how people select products and how they change their choices according to different models of decision support systems, namely, CBR and FAHP. It is important to propose a proper decision support system based on the understanding obtained from the experimental results.

To achieve our goal, we conducted two experiments using two different models. The first experiment proposes and tests a similarity metric for the classic CBR cycle. It reports results from an experimental case study that investigated how subjects select products according to given preferences.

The first study aims to shed light on the gender differences in product selection according to the two conducted experiments, as well as on the effect of the informatics advice given by different decision support systems, such as CBR and FAHP. Questions that need to be addressed are as follows: Is there a difference in the selection process between women and men? For example, who changes his or her selection more, and who believes that the other participants will select another option than the one he or she selected. Do subjects follow the informatics advice regardless of the advice given? The most important findings from our experiment show that using the AHP approach helps subjects to understand their needs deeply, compared with the CBR approach whose fixed advice depended on similarity alone.

The second study experimentally compared the results between conventional AHP and FAHP approaches. The objectives of the second study are to determine if both models have approximately the same results and to find out how the percentage of matching is different from one participant to another. Some questions on whether there are gender differences and if there is a significant difference between the methods of the two models also arise.

Our investigation in the previous studies led us to propose an expert system for product selection using FAHP- and CBR-based approaches. We combined the results of the previous studies on an expert system for product selection. Therefore, the proposed system has the merged strength of both approaches, taking into account the results of the experimental studies.

The thesis is divided into three main sections discussing the background of the study, the research methodology, and the results. The Background section gives an overview of the used approaches, namely, CBR, AHP, and FAHP. The Research Methodology section describes the two conducted experiments. The Results section presents the three studies based on the background section and the experiments.

The remainder of the thesis is structured as follows. Chapter 1 introduces a well-known and widely used artificial intelligence approach of problem solving based on past experience, that is, CBR. Chapter 1 discusses the CBR cycle and stages and then defines the similarity measure. Chapter 2 introduces the AHP, which is a well-known and widely used structured method for organizing and analyzing complex decisions. It was originally developed by Saaty (1980) in the early 1980s. Chapter 2 describes the AHP method and stages and provides an example. Chapter 3 introduces merging fuzzy sets into AHP, that is FAHP. Chapter 3 describes the fuzzy set essentials and definitions and then discusses FAHP based on the extent analysis method introduced by Chang (1996). Chapter 4 describes experiment A and introduces the proposed local similarity measure, the experimental design, and the experimental procedure. Chapter 5 describes experiment B and presents the proposed FAHP approach, experimental design, and experimental procedure. Chapter 6 presents the results of the first study. It describes the gender differences in product selection and the effects of the informatics system. It presents the results of experiments A and B. Chapter 7 covers the second study that compared AHP and FAHP. It discusses the motivation of the study, the method employed, the general comparison of the two models, the statistical analyses, the gender differences, and the classification analysis between the weights of the two models. Chapter 8 is on the proposed expert system for product selection using FAHP and CBR approaches. It presents the literature review, the motivation, the model, and an example. Finally, the conclusions are presented.

CHAPTER 1

Case Based Reasoning (CBR)

1.1 INTRODUCTION

Case-based reasoning (CBR) is an artificial intelligence approach that involves problem solution and learning based on past experience. CBR combines knowledge-based systems with the machine learning field. CBR is based on a simple idea: 'Do not solve problems from scratch but remember how you (or someone else) solved a similar problem and apply this knowledge to solve your current problem.' (Althoff, 2001).

CBR is based on psychological theories of human cognition. It perceives that the human experience is not based on prescribed structures or rules but on experiences. Human experts are distinguished by their learning abilities to understand the relation between current and previous problems, the reasons of analogy between the old and new problems, and their ability to apply solutions learned from old experiences for new solutions and the recognition and avoidance of old failures and errors. In this well-known passage, Hume (1748, Section IV) states:

> *'In reality, all arguments from experience are founded on the similarity which we discover among natural objects, and by which we are induced to expect effects similar to those which we have found to follow from such objects. And though none but a fool or madman will ever pretend to dispute the authority of experience, or to reject that great guide of human life, it may surely be allowed a philosopher to have so much curiosity at least as to examine the principle of human nature, which gives this mighty authority to experience, and makes us draw advantage from that similarity which nature has placed among different objects. From causes which appear similar we expect similar effects.'*

Hume (1748) further emphasized that ‘This is the sum of all our experimental conclusions’. Hume’s ideas were formalized by the modern decision-making formulation by Gilboa and Schmeidler (1995, 2001). They further proposed that the induction of identical action to identical problems yielded the same results, according to the theory of case-based decision. Furthermore, action was evaluated on the basis of the weighted calculation of payoff similarity which they capitulated in identical problems.

CBR is considered to be one of the artificial intelligence techniques that can be used to build an expert system in which the knowledge base induced by experiments and previous experiences (called cases) is used. New problems will be solved by identifying a case from the knowledge base that is the most similar to the new one; then, the old solution will be reused to solve the adopted case. The new case will then be stored in the knowledge base and in turn be used to solve other similar problems in the future.

Kolodner (1984) and Schank (1982) were the first researchers to create prototypes for research and models based on cognitive science research and involving the reusability of scripts from old cases to adequately resolve new, identical situations. The term case-based reasoning was first used by these researchers. In 1977, Schank and Abelson’s work brought CBR from research into cognitive science (Watson, 1997). They proposed that our general knowledge about situations be recorded as scripts that allow us to set up expectations and perform inferences (Schank & Abelcon, 1977). Schank then investigated the role that the memory of previous situations and situation patterns (such as scripts and memory organization packets (MOPS)) play in problem solving and learning (Schank, 1982).

1.2 APPLICATIONS OF CBR

CBR has been used to create a wide range of applications across many domains. This section sheds light on the foundation of CBR domains, and the first application of CBR in several domains, including knowledge-based systems, learning from experience, human problem solving and learning, and experience management.

CBR is an approach for developing knowledge-based systems (e.g. Aamodt, 1991 and Althoff & Wess, 1992). Through CBR, knowledge can be formally represented in the form of cases and used for automated problem solving, such as to aid individuals in decision making (Kolodner, 1991), to solve problems in diagnostics as well as in planning or design (Althoff & Wess, 1991; Bergmann et al., 1998; Börner, 1998; Cunningham & Smyth, 1994; Lenz et al., 1998; Maher et al., 1995; Veloso, 1994). Apart from case-specific knowledge, CBR systems also exploit general domain knowledge (e.g. a set of rules) to support retrieval, similarity judgment, case adaptation, as well as learning. Thus, CBR can be viewed as an approach for developing knowledge-based systems (Motta, 2000).

CBR is an approach for learning from experience (Aha, 1991; Althoff & Wess, 1992; Globig & Lange, 1994; Kamp et al., 1998). Learning from cases can be implemented using CBR. The learning result then is the whole CBR system including its case base. The CBR system is gradually improved and enhanced with the addition of new cases.

CBR is used for human problem solving and learning (Kolodner, 1983a; Kolodner, 1983b; Leake, 1998; Schank, 1982; Schank, 1989). It is a natural approach for the development of knowledge, especially in the context of teaching and tutoring (Papagni et al., 1997; Schank, 1998; Seitz, 1999; Weber, 1996; Weber & Schult, 1998).

CBR is a knowledge-based extension of the nearest-neighbour classification paradigm known from pattern recognition (Aha et al., 1991; Aha, 1997; Brandt, 2000; Jarmulak, 1999). It is an organizational approach for experience management (Althoff & Wilke, 1997; Minor & Hanft, 2000; Tautz & Althoff, 1997; Watson, 1998)[3].

1.3 CBR CYCLE

CBR is based on the concept that past problem solving behaviour is the best predictor of future problems and solutions. It solves new problems by using or adapting solutions that were used to solve old problems, and it offers a reasoning paradigm similar to that routinely used by many people to solve problems. The key assumption is that if two problems are similar, their solutions are probably also similar.

For solving a new problem, a query is submitted to a CBR system to retrieve the solutions of the most similar problems/cases in the case database. The query could also be considered as a potentially new case. The classical view of a case imposes that it consists of a problem description and a described solution to this problem; however, this view is too restrictive (Burkhard, 1998). A more general view is that a case consists of a characterization (a more or less structured set of information entities) (Althoff et al., 1998; Althoff et al., 2000). These problem-solving steps are also known as the CBR cycle (Aamodt and Plaza, 1994).

[3] CBR is deployed on a large scale in product selection (Schmitt and Bergmann, 1999). Further details will be provided in the section 8.2.

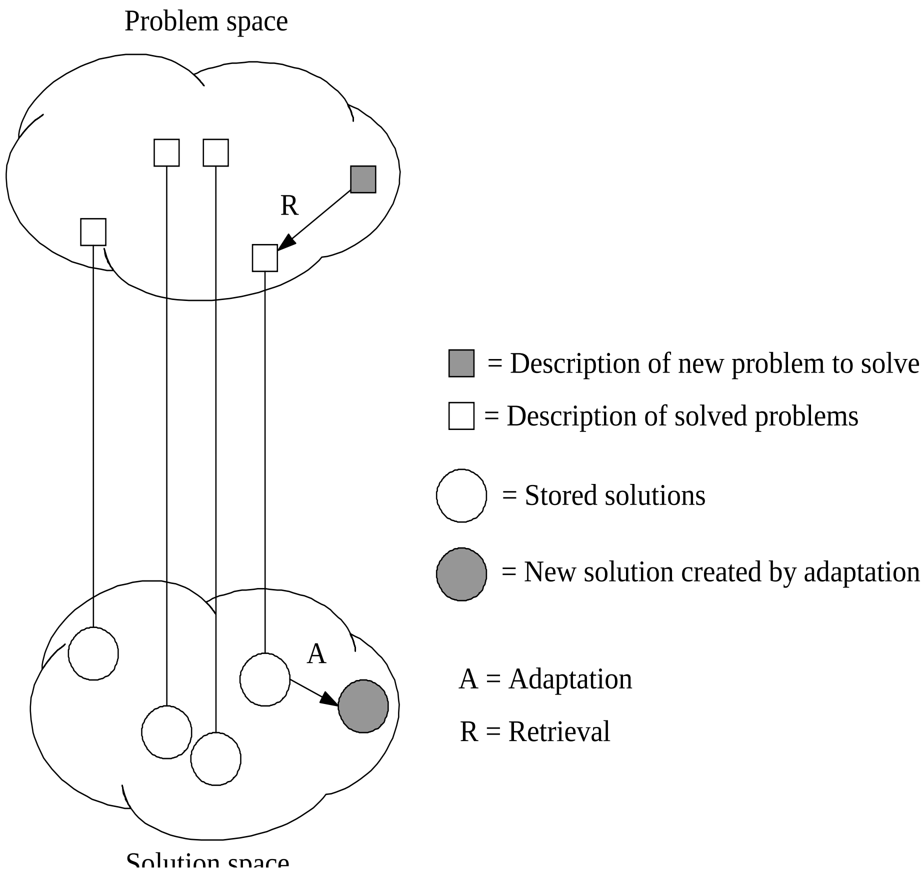

Figure 1.1. *Problem and solution spaces [Watson, 1997]*

1.4 CBR STAGES

Case retrieval

At this stage, the specified preferences are utilised to determine a case in the knowledge base that is most identical to the given case. This stage comprises of the following sub-stages:

- Description of the requisite characteristics along with their degree of support.
- Calculation of the similarities between the current and new cases along with the search of cases identical to the new case.
- Selection of the most identical case.

Case reuse

This is the process where the information and knowledge from the old case is used for solving the new one by keeping the differences between the new and old cases under consideration, which aids the transformation of some old parts into new ones. The process of reuse consists of two ways:

- *Copy the solution:* With this process, the solution from the old case is copied for use in the new case.
- *Adapt the solution:* The solution of the old case is modified to adapt to the new case.

Case Revision

This stage involves a process of testing and further revising the derived solution for the new case. This stage consists of two steps:

- *Solution evaluation:* Experiments are carried out to evaluate solutions or the solution is evaluated by an expert.
- *Error correction:* If errors are discovered, the solution is moulded to eliminate them.

New case retention

The retention process is carried out for the new case in the knowledge base. The CBR process consists of many processes such as induction and deduction to find the solution of a problem by reusing the solutions from previous problems (Figure 1.2; Aamodt and Plaza, 1994).

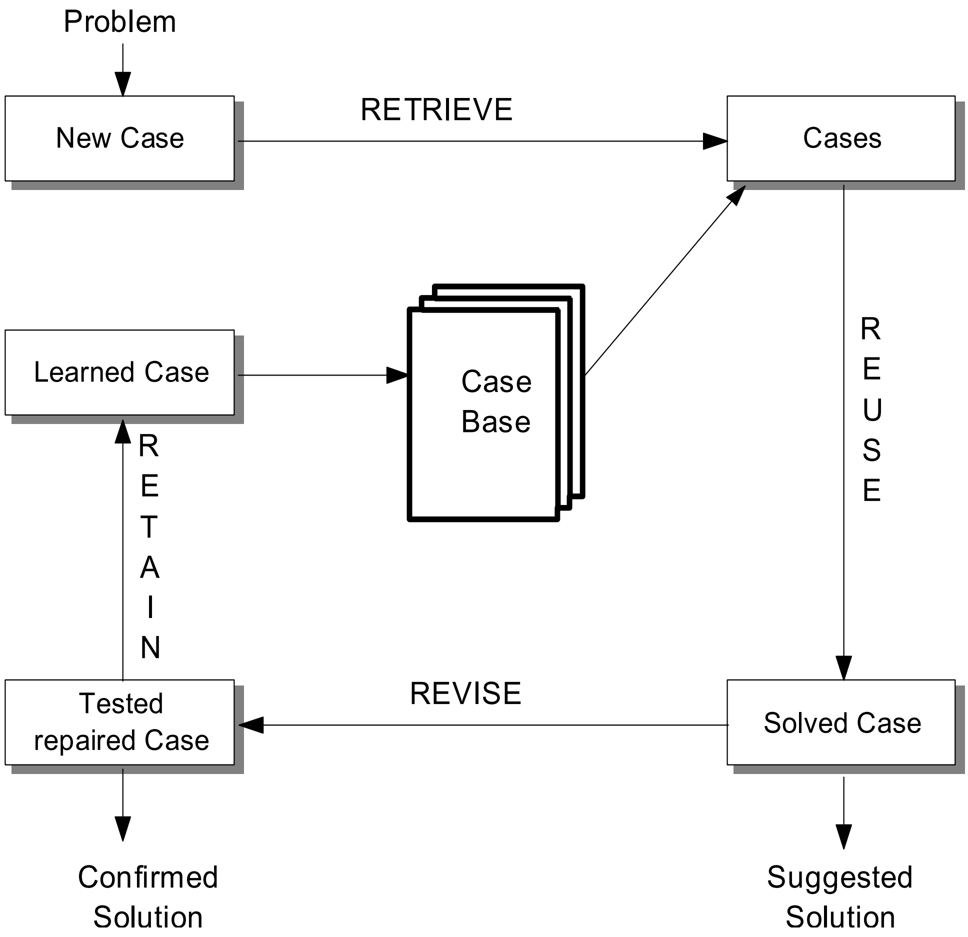

Figure 1.2. *The CBR Cycle.*

1.5 THE SIMILARITY MEASURE

As mentioned in Section 1.3, calculation of the similarities between the current and new cases along with the search for cases identical to the new one is the main step in case retrieval; therefore, the case similarity metric is the most important step in the CBR cycle. Thus, it is important that the system finds and retrieves a case similar to the target case, calculates the similarity between the source and target cases, and finally selects the case with the greatest similarity value with the new case.

There are three key steps involved for finding the case with the highest level of similarity.

- Local similarity: This step involves the calculation of similarity locally for the characteristics that are relevant between the cases.
- Full similarity: This process involves the calculation of full similarity between the past and present characteristics of identical cases to acquire similarity.
- Sorting: This process involves the process of sorting all cases.

1.5.1 Local similarity

Normally, the similarity among cases is estimated using metrics and by considering that cases are represented as attribute-value pairs. [4]To understand how such a similarity metric is used to find the best choice for case selection, consider two cases represented as a fixed length vector of n characteristics. The local similarity function is obtained by calculating the similarity between two specific characteristics of the case, and the full similarity is obtained by merging all the local similarities to determine the similarity between the two cases. There are many functions could be used as local similarity Sim, and the proper function depends on the type of problem.

1.5.2 Full similarity

There is a wide range of similarities that can be utilised for the process of full similarity evaluation. The most famous one is the nearest-neighbour retrieval approach (Kolodner, 1991). It is a simple approach used to compute the similarity between new and stored

[4] The similarity values are often normalized to the interval [0, 1], where 0 indicates that the cases do not match at all and 1 indicates a complete match.

cases on the basis of features of weight. Usually, the similarity between $(Case_I, Case_R)$ is defined as the sum of similarities along with constituent features which are further multiplied on the basis of their relevant weights, as shown in the near-neighbour algorithm in Equation 1:

$$similarity(Case_I, Case_R) = \frac{\sum_{i=1}^{n} w_i \times sim(f_i^I, f_i^R)}{\sum_{i=1}^{n} w_i} \tag{1}$$

where W_i represents the importance weighting for characteristic *i*, which is depicted in numerical values from 0 to 1. The distant neighbours possess values nearer to 0 while the nearest neighbours have values tending to 1. f_i^R and f_i^I are feature values of *i* in the new and retrieved cases, for primitives, *sim* is the local similarity.

CHAPTER 2

Analytic Hierarchy Process (AHP)

2.1 INTRODUCTION

This chapter introduces analytic hierarchy process (AHP), which is a well-known and widely used structured method for organizing and analyzing complex decisions. AHP was originally developed by Saaty (1980) in the early 80s.

As a technique based on mathematics and psychology, AHP is used for multi-criteria decision-making. It is designed to incorporate tangible as well as non-tangible factors, which is the most effective when the decision-making process mainly involves personal opinions in choosing from different alternatives (Saaty, 1980). A special hierarchy based on mathematical structure and judgment matrices as well as unique consistency test gives AHP the ability to generate weights that approximate the real importance degree for each objective in the decision-making process (Mirkin, 1979; Saaty, 1980, 1994).

AHP is gaining importance as its widespread use brings more transparency in management decision-making processes (Ossadnik & Lange, 1999). AHP simulates the natural method by which human beings resolve problems that require decision making. It proposes a procedure-oriented way to solve problems while simultaneously simulating the intuitive results a human brain can generate.

AHP is based on the assumption that human beings naturally group decision elements according to their common characteristics and compare alternatives on the basis of each characteristic before they arrive at a decision by summing up all the comparison results. It gives satisfactory results for proposing solutions to problems that require decision making and comparison among alternatives that cannot be easily quantified, according to Saaty (1994).

The primary application of AHP is to aid in selecting the ideal solution for multi-criteria problems. The objectives and alternatives are compared in a natural, pairwise manner and the importance of each objective is quantified to yield its degree of recommendation. AHP is generally regarded as valid as it is not only capable of resolving situations that require comparison between sets of objectives but it also has thousands of diverse applications in which the AHP results are recognized and accepted (Saaty, 1994).

The number and diversity of AHP applications are growing every day because of its simplicity and ease of use and because it is based on well-established and structured mathematical and psychological hierarchies. It reduces the complex judgments into a series of pairwise comparisons without losing accuracy, assesses the participant consistency by testing on redundant judgments, and derives weights for the final results from the eigenvector of the computed judgment matrix. Since it claims to simulate intuitive thinking while retaining the easy-to-handle features, AHP is widely employed in management routines in fields where computer-aided decision making is required, such as business, government, and education.

AHP is a discrete choice technique. AHP has (1) the ability to handle uncertain, imprecise, and subjective data; (2) the robustness for solving practical ranking problems; and (3) the methodological clarity and mathematical simplicity (Tong & Bonissone, 1984; Zimmermann, 1987; Chen & Hwang, 1992; Deng, 1999).

2.2 APPLICATIONS OF AHP

AHP has a wide range of applications such as recommending the optimal salary for business (Troutt & Tadisina, 1992), assessing highway utilization (Weiwu & Jun, 1994), evaluating for performance (Suwignjo et al, 2000), evaluating the influence on environment (Ramanathan, 2001), evaluating the quality of indoor environments (Chiang & Lai, 2002), finding essential business functions for companies (Hafeez et al., 2002), proposing the credit level for manual labour corporations (Yurdakula & Tansel, 2003), selecting the optimal solution for transportation (Yedla & Shrestha, 2003), and evaluating an AHP software itself (Ossadnik & Lange, 1999). AHP is used widely in product selection, as described in detail in the literature review.

2.3 METHOD

2.3.1 Overview

AHP can be divided into five main steps:

- ***Step 1:*** *Constructing a decision hierarchy*

  The problem is structured into a hierarchy of decision elements, in order to obtain objectives for further comparison.

- ***Step 2:*** *Collecting input*

  A judgment matrix is computed by pairwise comparisons of objectives and alternatives.

- ***Step 3:*** *Testing the consistency*

It is determined whether the input data satisfies certain consistency criteria. If not, Step 2 is repeated.

- ***Step 4:*** *Calculating the weights*

  The weights for each objective are calculated.

- ***Step 5:*** *Ranking the alternatives*

  The weights for each alternative are computed, and a rank is generated accordingly.

The AHP method depends on determining the relative importance of a given set of preferences and then alternatives in relation to a predetermined goal, taking into account the criteria and sub-criteria. AHP tries to introduce analytical thinking into a decision-making process, and is based on three basic principles (Saaty, 1995, p. 17): (1) constructing hierarchies, (2) establishing priorities, and (3) achieving logical consistency. The first phase would involve designing the decision-making process by establishing a proper hierarchical structure of the elements involved. In the second phase, the priorities (weights) would be calculated via pairwise comparison, taking into account the logical consistency of the process for each hierarchical level.

2.3.2 Phase one: Decision hierarchy

In the first phase, a hierarchical structure is designed in a multi-criteria context. The objective of this phase is to determine an achievable goal. Thereafter, the hierarchy would be subjected to some criteria, and this involves identifying the available alternatives. This also involves getting the right procedure involving the sub-criteria. Finally, after defining the criteria and possible sub-criteria, the next level would consist of the decision alternatives that the decision maker can choose from.

Figure 2.1 shows a schematic decision problem with a one-level hierarchical structure, which accounts for a goal and a set of four criteria. A choice can be made from a set of three alternatives.

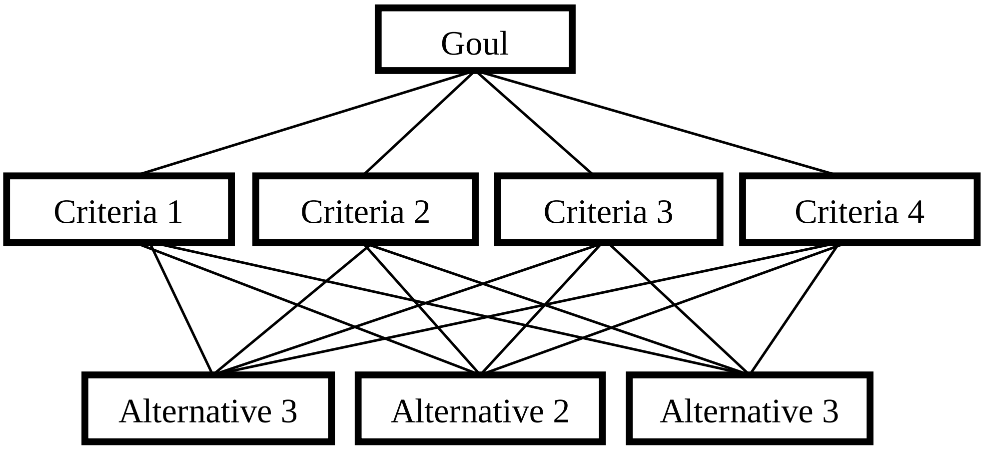

Figure 2.1. *An example of decision hierarchy*

2.3.3 Phase Two: Estimating the weights

The second phase of the AHP involves estimating the weights, which is an important part of the decision-making process. Once the hierarchy problem is designed (goal, criteria, and alternatives), the weights will consists a series of matrices composed of different assessments that, in turn, allow us to obtain priorities for each hierarchical level in regard to the next highest level. Finally, the global priorities for the alternatives that will enable decision-making are obtained.

Pairwise comparison

Weight assessment is based on pairwise comparison[5]. This pairwise comparison occurs between the criteria, followed by pairwise comparison of the alternatives according to each criterion. For example, Table 2.1 presents the pairwise comparison for the three criteria C1, C2, and C3 and the three alternatives A1, A2, and A3.

Table 2.1. *Importance comparison*

Criterion	C1 & C2	C1 & C3	C2 & C3
According to C1	A1 & A2	A1 & A3	A2 & A3
According to C1	A1 & A2	A1 & A3	A2 & A3
According to C3	A1 & A2	A1 & A3	A2 & A3

This involves pairwise comparisons by using a 9-point scale recommended by Saaty.

2.3.4 Ratio scale for pairwise comparison

The ratio-scale form is used to provide a reference for input in the AHP method, which states individual perceptions in a decision-making situation. It rates the degree of importance using digital expression on a scale of 1–9. Since the human brain has limited recognition power, the ratio-scale is meant to be limited accordingly. For measuring the AHP, a range of 1–9 is used, which should adequately represent human perception (Miller, 1956). In Miller’s psychological experiment, human brains were shown to be unable to simultaneously compare more than seven objects without losing consistency in their decision. The threshold increases or decreases by no more than two from seven

[5] The pairwise comparison method was first introduced by Fechner (1860), and developed by Thurstone (1927). However, it is correct to say that the AHP method based on pairwise comparison has been developed, refined and widely popularised by Saaty (1977, 1980).

for different identities. Thus, 9 was selected as the upper limit of the scale. Table 2.2 shows the scale adopted by Saaty.

Table 2.2. *Fundamental Scale for Making Judgments*

Preference level	**Numerical value**
Equal importance	1
Equal to moderate	2
Moderate	3
Moderate to strong	4
Essential or strong importance	5
Strong to very strong	6
Very strong	7
Very strong to extreme	8
Extreme	9
Reciprocals	If activity *i* has one of the above numbers assigned to it when compared with activity *j*, then *j* has the reciprocal value when compared with *i*
Rational	If consistency were to be forced by obtaining n numerical values to span the matrix

The ratio scale is (1) reciprocals if activity *i* has one of the above numbers assigned to it when compared with activity *j*, then j has the reciprocal value when compared with *i*. (2) rational if consistency were to be forced by obtaining n numerical values to span the matrix.

2.3.5 Generating the matrices

Criterion assessment

Through the pairwise comparison, the first matrix is obtained from a sequence of judgments of N × (N – 1)/2 by comparing the relative importance of criteria with the goal. Saaty's recommended scale is used for this purpose.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1N} \\ a_{21} & a_{22} & \dots & a_{2N} \\ \dots & \dots & \dots & \dots \\ a_{N1} & a_{N2} & \dots & a_{NM} \end{bmatrix} \tag{2}$$

The major properties of the square matrix (eq 2) are as follows:

1 The elements of the diagonal level take the value ($a_{ii} = 1 \ \forall \ i$),

2 The remaining elements assume that the existing pair-wise comparisons take a reciprocal position as follows; if $a_{ij} = x$ then $a_{ji} = 1/x$.

Calculating the eigenvector EM

The weights are calculated by calculating the EM as follows:

- ***Step 1:*** Square the matrix $A^2 = A.A$
- ***Step 2:*** Add the rows as eq 3

$$\begin{bmatrix} a_{11} & + & a_{12} & + & \dots & + & a_{1N} \\ a_{21} & + & a_{22} & + & \dots & + & a_{2N} \\ \dots & + & \dots & + & \dots & + & \dots \\ a_{M1} & + & \dots & + & \dots & + & a_{MN} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} a_{1i} \\ \sum_{i=1}^{N} a_{2i} \\ \dots \\ \sum_{i=1}^{N} a_{Mi} \end{bmatrix} \tag{3}$$

- ***Step 3:*** Summation of all the rows (eq 4)

$$T = \sum_{i=1}^{N} a_{1i} + \sum_{i=1}^{N} a_{2i} + ... + \sum_{i=1}^{N} a_{Mi} \tag{4}$$

- ***Step 4:*** Normalization

Normalization is achieved by dividing the row sum by the row totals (eq5)

$$\sum_{i=1}^{N} a_{1i} / T, \sum_{i=1}^{N} a_{2i} / T, ..., \sum_{i=1}^{N} a_{Mi} / T \tag{5}$$

The result is the first eigenvector

$$EM_1 = \begin{bmatrix} \sum_{i=1}^{N} a_{1i} / T \\ \sum_{i=1}^{N} a_{2i} / T \\ ..... \\ \sum_{i=1}^{N} a_{Mi} / T \end{bmatrix} \tag{6}$$

- *Step 5:* The previous steps are repeated with the obtained eigenvector (EM 1) in order to obtain the new EM (EM2). This process is repeated until the difference between the new EM and the old one is very small.

$$if\,(EM_2 - EM_1 < \varepsilon) \qquad Stop \tag{7}$$

ε could be (0.1, 0.01, 0.001, 0.00001,..., etc). The final EM is the final weight W_i of the criterion.

Assessment of alternatives

In the absence of sub-criteria, the next step in the AHP is to evaluate the alternatives through pairwise comparisons (usually by a group of experts) to hierarchical level 2 in regard to level 1 criteria. A set of N matrices of size M × M is then created. In the same way as before, the corresponding vector of priorities is obtained for each matrix by using one of the methods described above.

$$X_i = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1M} \\ x_{21} & x_{22} & \dots & x_{2M} \\ \dots & \dots & \dots & \dots \\ x_{M1} & x_{M2} & \dots & a_{MM} \end{bmatrix} \quad i = 1....N \tag{8}$$

Each matrix X_i would associate a vector of priorities after the EM is calculated as described in **3.4.1**.

$$W_j^i = (w_1^1, w_2^1, ..., w_M^1), (w_1^2, w_2^2, ..., w_M^2), ..., (w_1^N, w_2^N, ..., w_M^N); i = 1, .., N, \; j = 1, .., M \tag{9}$$

Finally, the global priorities ($Zj = Z_1, Z_2, ..., Z_M$) of the alternatives upon which the decision will be based are derived from the sum of the products of the relative priorities obtained from the assessment of the alternatives multiplied by the relative priorities obtained from the assessment of the associated criteria.

$$Z_j = (\sum_1^{N,M} w_i . w_j^i) = \begin{bmatrix} w_1.w_1^1 & + & w_2.w_2^1 & + & \dots & + & w_N.w_M^1 \\ w_1.w_1^2 & + & w_2.w_2^2 & + & \dots & + & w_N.w_M^2 \\ (.......) & + & (.......) & + & \dots & + & (.......) \\ w_1.w_1^N & + & w_2.w_1^N & + & \dots & + & w_N.w_M^N \end{bmatrix} = \begin{bmatrix} Z_1 \\ Z_2 \\ \dots \\ Z_M \end{bmatrix} i = 1, .., N; \; j = 1, .., M \tag{10}$$

2.4 CONSISTENCY

While estimating the priorities for each hierarchical level, it is important to ensure logical consistency in the collection of assessments by the experts. This involves fulfilling two requirements, namely, transitivity and proportionality. First, the order relationship between the assessments must be respected. For instance, if A is preferred over B and B is preferred over C, then A must be preferred over C. Second, the proportions between the orders of magnitude of assessments, with a little margin of error or inaccuracy, must be respected. For instance, if the preference for A is 3× higher than that for B, and the preference for B is 2× higher than that for C, the preference for A should be 6× greater than that for C. The above example would be a 100% consistent judgment, as it satisfies the requirements of transitivity and proportionality. However, we must consider that in a normal situation, decision makers do not make entirely consistent judgments. However, perfect consistency in the assessments of decision makers rarely occurs in reality. There are many methods for approximating these priorities to overcome this obstacle, such as the right eigenvector (EM) (Saaty, 1980) and the geometric mean vector (GMM) (Crawford & Williams, 1985; Barzilai et al., 1987; Barzilai, 1997).

2.5 EXAMPLE

Here, we present an example to see how the AHP method is used. Let us consider the selection of a tablet. There are three criteria for this selection: style, screen size, and cost, and there are four alternatives: A, B, C, and D. Figure 2.2 shows the decision hierarchy of this problem.

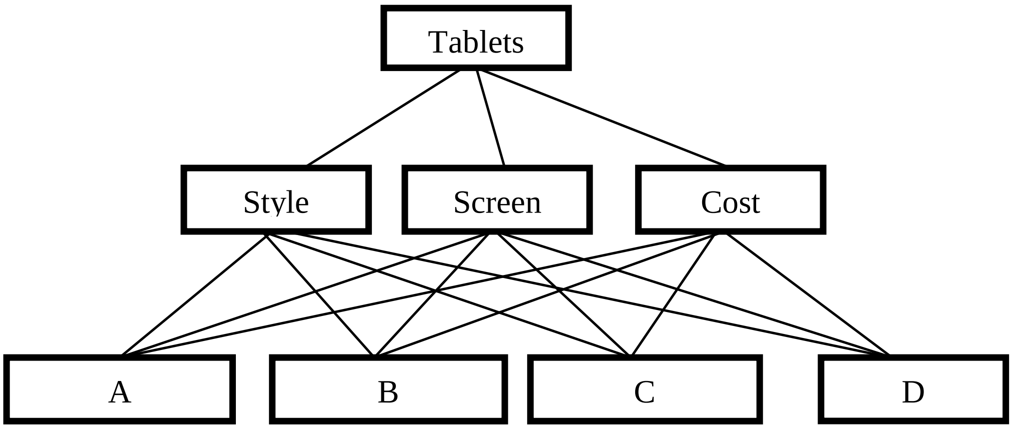

Figure 2.2. *Decision hierarchy of tablet selection*

Let us consider that the user preferences as Table 2.3 which are as follows:

- Style is 2× more important than Screen size
- Screen size is 3× more important than Cost
- Style is 4× more important than Cost

Table 2.3. *Pairwise comparison of Block I*

Style								vs.								Screen size
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	●	○	○	○	○	○	○	○

Screen size								vs.								Cost
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	●	○	○	○	○	○	○

Style								vs.								Cost
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	●	○	○	○	○	○

2.5.1 Assessment of criteria

$$W_R = \left[\begin{array}{cccc} & Screen & Style & Cost \\ Screen & 1 & 1/2 & 3/1 \\ Style & 2/1 & 1 & 4/1 \\ Cost & 1/3 & 1/4 & 1 \end{array}\right] \tag{11}$$

$$W_{R=} \begin{bmatrix} 1.00 & 0.50 & 3.00 \\ 2.00 & 1.00 & 4.00 \\ 0.30 & 0.25 & 1.00 \end{bmatrix} \tag{12}$$

- ***Step 1:*** Square the Matrix $A^2 = A.A$

$$W_R{}^2{}_= \begin{bmatrix} 1.0000 & 0.5000 & 3.0000 \\ 2.0000 & 1.000 & 4.0000 \\ 0.3000 & 0.2500 & 1.0000 \end{bmatrix} * \begin{bmatrix} 1.0000 & 0.5000 & 3.0000 \\ 2.0000 & 1.000 & 4.0000 \\ 0.3000 & 0.2500 & 1.0000 \end{bmatrix} = \begin{bmatrix} 3.0000 & 1.7500 & 8.0000 \\ 5.3332 & 3.0000 & 14.0000 \\ 1.1666 & 0.6667 & 3.0000 \end{bmatrix} (12) \tag{13}$$

- **Step 2:** Add the rows

$$\begin{bmatrix} 3.0000 & + & 1.7500 & + & 8.0000 \\ 5.3332 & + & 3.0000 & + & 14.0000 \\ 1.1666 & + & 0.6667 & + & 3.0000 \end{bmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{bmatrix} 12.7500 \\ 22.3332 \\ 4.8333 \end{bmatrix} \tag{14}$$

- ***Step3:*** Addition to obtain the row total

$$12.7500 + 22.3332 + 4.8333 = 39.9165$$

- ***Step 4:*** **N**ormalization

  The row sum is divided by the row total to obtain the first eigenvector (i.e. 12.7500/39.9165 = 0.3194).

$$\mathbf{EM}_1 = \begin{bmatrix} 12.7500/39.9165 \\ 22.3332/39.9165 \\ 4.8333/39.9165 \end{bmatrix} = \begin{bmatrix} 0.3194 \\ 0.5595 \\ 0.211 \end{bmatrix} \tag{15}$$

- ***Step 5:*** The previous steps are repeated

This process must be repeated until the eigenvector does not change from the previous one (four decimal places in our case). Thus, all the steps are repeated. Step 1: Square the matrix ${W_R}^2$

$$\begin{bmatrix} 3.0000 & 1.7500 & 8.0000 \\ 5.3332 & 3.0000 & 14.0000 \\ 1.1666 & 0.6667 & 3.0000 \end{bmatrix} * \begin{bmatrix} 3.0000 & 1.7500 & 8.0000 \\ 5.3332 & 3.0000 & 14.0000 \\ 1.1666 & 0.6667 & 3.0000 \end{bmatrix} = \begin{bmatrix} 27.6653 & 15.8330 & 72.4984 \\ 48.3311 & 27.6662 & 126.6642 \\ 10.5547 & 6.0414 & 27.6653 \end{bmatrix} \quad (16)$$

- ***Step 6:*** Compute the second eigenvector EM2

$$\begin{bmatrix} 27.6653 & + & 15.8330 & + & 72.4984 \\ 48.3311 & + & 27.6662 & + & 126.6642 \\ 10.5547 & + & 6.04140 & + & 27.6653 \end{bmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{bmatrix} 115.9967 \\ 202.6615 \\ 44.2614 \end{bmatrix} \quad (17)$$

- ***Step 7:*** Add the row total

$$115.9967 + 202.6615 + 44.2614 = 362.9196$$

- ***Step 4:*** Normalize by dividing the row sum by the row totals to obtain the second eigenvector (EM2)

$$\mathbf{EM}_2 = \begin{bmatrix} 115.9967/362.9196 \\ 202.6615/362.9196 \\ 44.2614/362.9196 \end{bmatrix} = \begin{bmatrix} 0.3196 \\ 0.5584 \\ 0.1220 \end{bmatrix} \quad (18)$$

Now the difference between the present and previously computed eigenvector is obtained to check whether $(EM_1 - EM_2) < \varepsilon$

$$\begin{bmatrix} 0.3194 \\ 0.5595 \\ 0.1211 \end{bmatrix} - \begin{bmatrix} 0.3196 \\ 0.5584 \\ 0.1220 \end{bmatrix} = \begin{bmatrix} -0.0002 \\ 0.0011 \\ -0.0009 \end{bmatrix} \quad (19)$$

Since the difference is small, we can stop iteration at this stage.

The final weights of the preferences are as follows:

$$W_R = \begin{bmatrix} 0.3196 \\ 0.5584 \\ 0.1220 \end{bmatrix} \begin{matrix} Screen \\ Style \\ Cost \end{matrix} \tag{20}$$

- Thus, the most important criterion is style with *55.84%*.
- Screen size was the second most important criterion at *31.96%*.
- Cost was the least important criterion at *12.20%*.

2.5.2 Alternative assessment

This section presents the pairwise comparison between the alternatives according to each criterion. The following alternatives were compared for screen size: A and B, A and C, A and D, B and C, B and D, and C and D, and the following matrix was generated.

$$W_{(P,Screen)=} \begin{bmatrix} W_{(A,Screen)} \\ W_{(B,Screen)} \\ W_{(C,Screen)} \\ W_{(D,Screen)} \end{bmatrix} \tag{21}$$

For pairwise comparison of style the following alternatives were compared: A and B, A and C, A and D, B and C, B and D, and C and D. The following matrix was generated.

$$W_{(P,Style)=} \begin{bmatrix} W_{(A,Style)} \\ W_{(B,Style)} \\ W_{(C,Style)} \\ W_{(D,Style)} \end{bmatrix} \tag{22}$$

The following alternatives were compared for style: A and B, A and C, A and D, B and C, B and D, and C and D. The following matrix was generated.

$$W_{(P,Cost)=}\begin{bmatrix} W_{(A,Cost)} \\ W_{(B,Cost)} \\ W_{(C,Cost)} \\ W_{(D,Cost)} \end{bmatrix} \tag{23}$$

Similarly, we can calculate each vector to obtain the whole matrix.

$$\begin{bmatrix} & Screen & Style & Cost \\ A & 0.1160 & 0.3790 & 0.3010 \\ B & 0.2470 & 0.2900 & 0.2390 \\ C & 0.600 & 0.740 & 0.2120 \\ D & 0.5770 & 0.2570 & 0.2480 \end{bmatrix} * \begin{bmatrix} 0.3196 \\ 0.5584 \\ 0.1220 \end{bmatrix} = \begin{bmatrix} 0.3060 \\ 0.2720 \\ 0.0940 \\ 0.3280 \end{bmatrix} \tag{24}$$

Thus, in this example, the best alternative is D with 32.8%.

CHAPTER 3

Fuzzy Analytic Hierarchy Process (FAHP)

3.1 FUZZY SETS

The representation of human-originated information and the formalization of common-sense reasoning has motivated different researchers in artificial intelligence in the second half of the 20th century. The history of fuzzy logic starts with the foundational 1965 paper by Lotfi Zadeh, entitled "Fuzzy Sets" (Zadeh, 1965). This paper was motivated by problems in pattern classification and information processing for capturing graded imprecision in information representation and reasoning devices. Zadeh proposes the idea of fuzzy sets as generalized sets having elements with intermediary membership grades. According to Zadeh (1965), a fuzzy set is characterized by its membership function, allocating a membership grade to any element of the referential domain, and the unit interval is usually taken as the range of these membership grades (Dubois et al., 2007).

The ability to represent vague data is considered the major contribution of fuzzy set theory to science and technology. Fuzzy set theory can effectively describe imprecise knowledge or human subjective judgement in linguistic terms. The linguistic terms that people use to express their feelings and judgements are vague. Because linguistic terms merely approximate the subjective judgement of decision-makers, the widely adopted triangular fuzzy number technique is used to represent the vagueness of these linguistic terms (Chan, Kao, Ng, & Wu, 1999). In the area of multi-criteria decision making, fuzzy set theory has made a significant contribution by accepting uncertainty and inconsistent judgement as part of the nature of human decision-making (Buckley, 1985_a).

The theory of fuzzy sets (Zadeh, 1965) defines a fuzzy set A by degree of membership $\mu_A(x)$ over a universe of discourse X in the following way: $\mu_A(x) \quad x:[0.1]$. Operations on fuzzy sets use connectives known as triangular norms T and S. T norms model the intersection operator in set theory, and S norms model the union operator. Although the family of T and S norms is large, the MIN and MAX operators, as defined by Zadeh, are the most frequently used. For connecting fuzzy sets, the most commonly used operators are the composition operators *sup* and *inf*. The former is the supremum of its membership function over the universe of discourse, and the latter is the infimum. Fuzzy operations are the combinations of norms and composition operators that enable operations on fuzzy sets.

Fuzzy arithmetic is made possible by Zadeh's extension principle, which states that if $f : X \to Y$ is a function and A is a fuzzy set in X, then $f(A)$ is defined as eq 25:

$$\mu_{f(A)}(y) = \sup_{x \in X,\, f(x)=y} \mu_A(x) \tag{25}$$

where $f : X \to Y,\ y \in Y$. Based on the extension principle, it is possible to describe fuzzy arithmetic operations such as addition, subtraction, multiplication, division, inversion, logarithmisation, and exponentiation (Triantaphyllou and Lin, 1996; Bender and Simonovic, 2000).

3.1.1 Positive Triangular Fuzzy Numbers

Positive triangular fuzzy numbers A are a special class of fuzzy number often expressed as $A = (a_1, a_2, a_3)$, where a_1, a_2, and a_3 are three real numbers that satisfy

$a_1 \geq 0$ and $a_1 \leq a_2 \leq a_3$. Any real number in interval $[a_1, a_3]$ is characterized with a grade of membership between 0 and 1. Its membership function $\mu_A(x)$ is piecewise continuous and linear (see Figure 3.1) and satisfies the following conditions:

$$\mu_A(x) = 0, \forall x \in (-\infty, a_1) \cup (a_3, \infty)$$

$$\mu_A(x) = 1, x = a_2$$

$$\mu_A(x) = (x - a_1)/(a_2 - a_1), \forall x \in [a_1, a_2]$$

$$\mu_A(x) = (a_3 - x)/(a_3 - a_2), \forall x \in [a_2, a_3]$$

The most probable value of fuzzy number A is modal value a_2. The lower and upper bounds, a_1 and a_3 respectively, support the modal value and illustrate the degree to which it is not a fuzzy number; if $a_3 - a_2 = a_2 - a_1$, the triangular fuzzy number A is symmetrical.

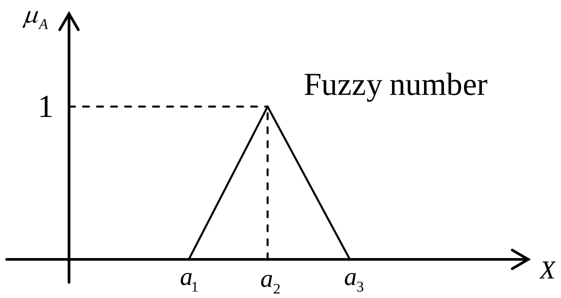

Figure 3.1. *Positive triangular fuzzy number*

Definition 1. $M \in F(R)$ is called a fuzzy number if:

1 there exists $x_0 \in R$ such that $\mu_M(x_0) = 1$, and

2 for any $\alpha \in [0,1]$, $A_\alpha = \lfloor x, \mu_{A_\alpha}(x) \geq \alpha \rfloor$,

is a closed interval. Here $F(R)$ represents all fuzzy sets, and R is the set of real numbers.

Definition 2. A fuzzy number M is defined on R to be a triangular fuzzy number if its membership function $\mu_M(x): R \to [0,1]$ is equal to eq 26:

$$\mu_M(x) = \begin{cases} \frac{x}{m-l} - \frac{l}{m-l}, & x \in [l,m], \\ \frac{x}{m-u} - \frac{u}{m-u}, & x \in [m,u], \\ 0, & otherwise, \end{cases} \tag{26}$$

where $l \leq m \leq u$, l and u stand respectively for the lower and upper values of the support of M, and m stands for the modal value. The triangular fuzzy number can be denoted by (l,m,u). The support of M is the set of elements $\{x \in R \mid l < x < u\}$. When $l = n = u$, M is a non-fuzzy number by convention.

3.1.2 Fuzzy Operational

Let us consider two triangular fuzzy numbers, M_1 and M_2, where $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$. Their operational laws are as follows:

$$(l_1,m_1,u_1)\oplus(l_2,m_2,u_2)=(l_1+l_2,m_1+m_2,u_1+u_2) \tag{27}$$

$$(l_1,m_1,u_1)\odot(l_2,m_2,u_2)=(l_1l_2,m_1m_2,u_1u_2) \tag{28}$$

$$(\lambda,\lambda,\lambda)\odot(l_1,m_1,u_1)=(\lambda l_1,\lambda m_1,\lambda u_1),\ \ \lambda>0,\lambda\in R \tag{29}$$

$$(l_1,m_1,u_1)^{-1}\approx(1/u_1,1/m_1,1/l_1) \tag{30}$$

3.2 MERGING FUZZY WITH AHP

The Fuzzy Analytic Hierarchy Process (FAHP) has the same core as the conventional AHP. Both are based on a hierarchical structure and use pairwise judgements to estimate the preferences of criteria and their alternatives. Similar to AHP, FAHP uses pairwise comparisons of criteria and alternatives in order to form a reciprocal decision matrix, but it transforms data to fuzzy ratios. The eigenvector (EM) method is also used to solve the reciprocal matrix and to determine the importance of criteria and the performance of alternatives with respect to each criterion.

The FAHP method is a systematic approach to the alternative selection and justification problem, using the concepts of fuzzy set theory and hierarchical structure analysis. The decision-maker can specify preferences, in the form of natural language or numerical values, about the importance of each performance attribute (Güngör et al., 2009). In the case of FAHP, defuzzification is necessary at the final stage to obtain conventional weights and rank alternatives.

Thus, while AHP is designed to handle the knowledge of a decision-maker, conventional AHP does not fully reflect a human thinking style (Buyukozkan, 2004). It is well recognized that human perceptions and judgements are represented by linguistic and imprecise patterns for complex problems. These linguistic and imprecise descriptions were difficult to solve using AHP, until the recent developments in fuzzy decision-making (Buckley, 1985_a). Fuzzy set theory resembles human reasoning in its use of approximate information and uncertainty in decision generation. A major contribution of fuzzy set theory is its capability of representing vagueness. AHP has been developed to solve the multiple-attribute decision-making problem, so by incorporating fuzzy set theory with AHP, FAHP enables a more accurate description of the multiple-attribute decision-making process (Bozbura et al., 2007).

AHP has the ability to be merged with fuzzy logic and fuzzy set theory (Tong and Bonissone, 1984; Zimmermann, 1987; Chen and Hwang, 1992; Deng, 1999). Laarhoven and Pedrycz (1983), Buckley (1985_a), and Chang (1996) extended Saaty's AHP in order to deal with the subjectivity of decision-makers' judgements by embedding the process into a fuzzy set. This fuzzy version of the original method is based on the use of triangular fuzzy numbers in pairwise comparisons that allow users to intimate criteria weights and the overall weights of alternatives. In order to arrive at the final stage within which alternatives are prioritized, fuzzy utilities must be defuzzified and ranked.

There are other fuzzy methods for prioritization in AHP that are also worth mentioning. These include methods based on polyoptimisation, as proposed by Wagenknecht and Hartmann (1983), fuzzy least squares, by Xu (2000), and a pseudo-inverse generalization by Kwiesielewicz (1998). Although these three methods have gained a certain level of attention and are considered, theoretically, to be the best methods of applying fuzzy methods to prioritization in AHP, the fuzzy extent analysis method, as proposed in Laarhoven and Pedrycz (1983), Buckley (1985_b), and Chang (1996) is more widely accepted in practice. This is because it is transparent and simple when handling uncertainties that are embedded in decision-making. These uncertainties include quantitative, qualitative, and 'grey' decision variables.

There are different fuzzy-based, multi-criteria analysis models, all of which more or less follow the AHP philosophy (see, for instance, Triantaphyllou and Lin, 1996; Raju and Pillai, 1999; and Arslan and Khisty, 2006). The most common models are those that completely imitate standard AHP and its principles for manipulating the priority vectors that are taken from judgement matrices and that consequently apply fuzzy arithmetic throughout the process. Here, we will introduce the fuzzy extent analysis (Chang 1996), which is widely used.[6]

3.3 FUZZIFY SAATY'S SCALE

Fuzzy numbers are intuitively easy to use when expressing the decision-maker's qualitative assessments. In order to facilitate the making of pairwise comparisons in the application of Fuzzy AHP, Saaty's original 9-point scale may be fuzzified as shown in the last column of Table 1. Membership functions for $\tilde{1} \leq \tilde{x} \leq \tilde{9}$ are assumed to be symmetrically triangular, different for an internal pair and odd integers and adjusted for edge values along the scale. Note that pair fuzzy numbers $\tilde{2}$, $\tilde{4}$, $\tilde{6}$ and $\tilde{8}$ are fuzzified with $\delta = 1$, due to their intermediate judgement positions within the scale, and that edge fuzzy numbers $\tilde{1}$ and $\tilde{9}$ are defined to reflect a real decision situation. According to the judgement definitions given in the third column of Table 3.1, the fuzzy distance for internal odd integers should be only within the interval.

6 Chang's paper has been cited 749 times till July2 012, according to Google Scholar.

Table 3.1. *Original and fuzzified Saaty's scale for pairwise comparisons*

Saaty's crisp values (x)	Judgement definition	Fuzzied Saaty's values
1	Equal importance	$(1,1,1+\delta)$
3	Weak dominance	$(3-\delta,3,3+\delta)$
5	Strong dominance	$(5-\delta,5,5+\delta)$
7	Demonstrated dominance	$(7-\delta,7,7+\delta)$
9	Absolute dominance	$(9-\delta,9,9)$
2, 4, 6, 8	Intermediate values	$(x-1,x,x+1), x=2,4,6,8$

δ is fuzzy distance $(0.5 \le \delta \le 2)$.

$0.25 \le \delta \le 2$. For example, Table 3.2 shows the full scale using δ =0.25 (Chen et al., 2011).

Table 3.2. *Triangle fuzzy scale*

Linguistic scale of importance	Normal number	Triangle fuzzy scale
Equal importance	1	(1,1,1)
Between equal and moderate importance	2	(1,1.25,1,1.5)
Moderate importance	3	(1.25,1.5,1.75)
Between moderate and strong importance	4	(1.5,1.75,2)
Essential or strong importance	5	(1.75,2,2.25)
Between strong and very strong importance	6	(2,2.25,2.5)
Very strong importance	7	(2.25,2.5,2.75)
Between very strong and extreme importance	8	(2.75,3,3.25)
Extreme importance	9	(3,3,3)

3.4 EXTENT ANALYSIS METHOD ON FUZZY AHP

In his paper, Da-Yong Chang (1996) introduced an approach for handling Fuzzy AHP that involves the use of triangular fuzzy numbers for a pairwise comparison scale of Fuzzy AHP, and then the use of the extent analysis technique for the synthetic extent value S_i of the pairwise comparison.

3.4.1 Value of Fuzzy Synthetic Extent

Let $X = \{x_1, x_x, \ldots, x_n\}$ be an object set, and $U = \{u_1, u_2, \ldots, u_m\}$ be a goal set. According to the method of extent analysis, we now take each object and perform extent analysis for each goal in turn. Therefore, we can get m extent analysis values for each object with the following signs:

$$M_{g_i}^1, M_{g_i}^2, \ldots, M_{g_i}^m, \; i = 1, 2, \ldots, n, \tag{31}$$

where all $M_{g_i}^j \; (j = 1, 2, \ldots, m)$ are triangular fuzzy numbers.

Definition 3. Let $M_{g_i}^1, M_{g_i}^2, \ldots, M_{g_i}^m$ be values of extent analysis of ith object for m goals. Then the value of fuzzy synthetic extent with respect to i-th object is defined as Eq. 32:

$$S_i = \sum_{j=1}^{m} M_{g_i}^j \odot \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^j \right]^{-1}. \tag{32}$$

To obtain $\sum_{j=1}^{m} M_{gi}^j$, perform the fuzzy addition operation of m extent analysis values for a particular matrix such that

$$\sum_{j=1}^{m} M_{gi}^{j} = \left(\sum_{j=1}^{m} l_i , \sum_{j=1}^{m} m_i , \sum_{j=1}^{m} u_i \right) \tag{33}$$

To obtain $\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j} \right]^{-1}$, perform the fuzzy addition operation of $M_{gi}^{j} \left(j = 1, 2, \ldots, m \right)$ values such that

$$\sum_{i=1}^{n} \sum_{j=1}^{m} = \left(\sum_{i=1}^{n} l_i , \sum_{i=1}^{n} m_i , \sum_{i=1}^{n} u_i \right) \tag{34}$$

and then compute the inverse of the vector in (9) such that

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j} \right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i} , \frac{1}{\sum_{i=1}^{n} m_i} , \frac{1}{\sum_{i=1}^{n} l_i} \right) \tag{35}$$

3.4.2 Presentation Method of Fuzzy Numbers for the Pairwise Comparison Scale

As mentioned before, the first task of the FAHP method is to decide on the relative importance of each pair of criteria in the same hierarchy. By using triangular fuzzy numbers via a pairwise comparison, the fuzzy evaluation matrix $A = \left(a_{ij} \right)_{n \times m}$ is constructed. For example, if an essential or strong importance of element i over element j exists under a certain criterion, then $a_{ij} = \left(l, 5, u \right)$, where l and u represent a fuzzy degree of judgement. The greater $u - l$, the fuzzier the degree; when $u - l = 0$, the judgement is a non-fuzzy number. This stays the same to scale 5 under general meaning. If the strong importance of element j over element i holds, then the pairwise comparison scale can be represented by the fuzzy number $a_{ij}^{-1} = \left(1/u, 1/m, 1/l \right)$.

3.4.3 Calculation of the Priority Vectors of Fuzzy AHP

Let $A = \left(a_{ij}\right)_{n \times m}$ be a fuzzy pairwise comparison matrix, where $a_{ij} = \left(l_{ij}, m_{ij}, u_{ij}\right)$, which are satisfied with $l_{ij} = \frac{1}{l_{ji}}$, $m_{ij} = \frac{1}{m_{ji}}$, $u_{ij} = \frac{1}{u_{ji}}$.

To obtain the estimates for the vectors of weights under each criterion, we need to consider a principle of the comparison of fuzzy numbers. We must evaluate the degree of possibility for $x \in R$ fuzzily restricted to belong to M. Thus, we give the definition as follows:

Definition 4. The degree of possibility of $M_1 \geq M_2$ is defined as

$$V(M_1 \geq M_2) = \sup_{x \geq y} \left\lfloor \min\left(\mu_{M_1}(x), \mu_{M_2}(y)\right) \right\rfloor \tag{36}$$

If a pair (x, y) exists such that $x \geq y$ and $\mu_{M_1}(x) = \mu_{M_2}(y) = 1$, then we have $V\left(M_1 \geq M_2\right) = 1$. Since M_1 and M_2 are convex fuzzy numbers, we have

$$V\left(M_1 \geq M_2\right) = 1 \;\; iff \;\; m_1 \geq m_2$$

$$V\left(M_2 \geq M_1\right) = hgt\left(M_1 \cap M_2\right) = \mu_{M_1}(d) \tag{37}$$

where d is the ordinate of the highest intersection point D between μ_{M_1} and μ_{M_2} (see Figure 3.2).

When $M_1 = \left(l_1, m_1, u_1\right)$ and $M_2 = \left(l_2, m_2, u_2\right)$, the ordinate of D is given by Eq. 13:

$$V\left(M_2 \geq M_1\right) = hgt\left(M_1 \cap M_2\right) = \frac{l_1 - u_2}{\left(m_2 - u_2\right) - \left(m_1 - l_1\right)} \tag{38}$$

To compare M_1 and M_2, we need the values of both $V\left(M_1 \geq M_2\right)$ and $V\left(M_2 \geq M_1\right)$.

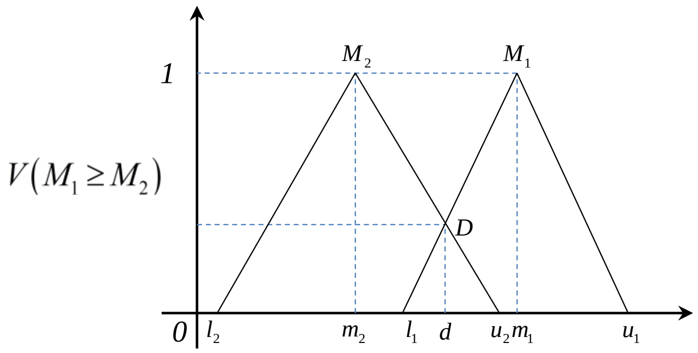

Figure 3.2. *Two fuzzy numbers,* M_1 *and* M_2

Definition 5. The degree possibility that a convex fuzzy number is greater than k convex fuzzy numbers $M_i \left(i = 1, 2, \ldots, k\right)$ can be defined by

$$\begin{aligned} &V\left(M \geq M_1, M_2, \ldots, M_k\right) \\ &\quad = V\left[\left(M \geq M_1\right) and \left(M \geq M_2\right) and \cdots and \left(M \geq M_k\right)\right] \\ &\quad = \min V\left(M \geq M_i\right), \ \ i = 1, 2, \ldots, k. \end{aligned} \tag{39}$$

Assume that

$$d'\left(A_i\right) = \min V\left(S_i \geq S_k\right) \tag{40}$$

for $k = 1, 2, \ldots n; k \neq i$. Then the weight vector is given by

$$W' = \left(d'\left(A_1\right), d'\left(A_2\right), \ldots, d'\left(A_n\right)\right)^T \tag{41}$$

where $A_i (i = 1, 2, \ldots, n)$ are n elements. Via normalization, we get the normalized weight vectors

$$W = \left(d\left(A_1\right), d\left(A_2\right), \ldots, d\left(A_n\right)\right)^T \tag{42}$$

where W is a non-fuzzy number.

3.5 USING FUZZY EXTENT ANALYSIS IN FAHP

3.5.1 Evaluating Criteria

The ranking procedure starts with the determination of the importance of criteria with respect to the goal. By using a fuzzified scale, a fuzzy reciprocal judgement matrix for criteria is determined as:

$$A = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1M} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2M} \\ \cdots & \cdots & \cdots & \cdots \\ \tilde{a}_{M1} & \tilde{a}_{M2} & \cdots & \tilde{a}_{MM} \end{bmatrix} \tag{43}$$

where $\tilde{a}_{ij} = 1$ for all $i = j\left(i, j = 1, 2, \ldots, M\right)$, and $\tilde{a}_{ij} = 1 / \tilde{a}_{ji}$.

By applying the fuzzy synthetic extent (Eq. 3), the corresponding weights of criteria can be determined as:

$$S_i = \sum_{j=1}^{M} \tilde{a}_{ij} \otimes \left[\sum_{k=1}^{M} \sum_{l=1}^{M} \tilde{a}_{kl} \right]^{-1}, \quad i = 1, \ldots, M. \tag{44}$$

All $S_i, i = 1, \ldots, M$ are normalized fuzzy numbers with medium values equalling 1. It should be noted that the fuzzy extent (Eq. 20) could be defined as the result of fuzzy arithmetic or by using the extension principle. The second is slightly more difficult, but it would lead to reduced uncertainty.

3.5.2 Evaluating Sub-Criteria

For the given criterion C_j, which splits into k_j sub-criteria, it is necessary to determine the relative importance of the sub-criteria with respect to this criterion. After doing so, the fuzzy judgement matrix can be determined as Eq. 45:

$$A_j = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \ldots & \tilde{a}_{1k_j} \\ \tilde{a}_{21} & \tilde{a}_{22} & \ldots & \tilde{a}_{2k_j} \\ \ldots & \ldots & \ldots & \ldots \\ \tilde{a}_{k_j 1} & \tilde{a}_{k_j 2} & \ldots & \tilde{a}_{k_j k_j} \end{bmatrix} \tag{45}$$

The weights of sub-criteria with respect to the given criterion are obtained again as fuzzy extents. Final sub-criteria weights are derived through the aggregation of the weights at two consecutive levels. Multiplying sub-criteria weighs by respective criterion weight (Eq. 46) gives:

$$S_j^p = \left(\sum_{l=1}^{k_j} \tilde{a}_{il} \otimes \left[\sum_{i=1}^{k_j} \sum_{l=1}^{k_j} \tilde{a}_{il} \right]^{-1} \right) \otimes w_j, \quad j = 1, \ldots, M, \, p = 1, \ldots, k_j \tag{46}$$

where S_j^p are the aggregated fuzzy weights of the sub-criteria. They are entries of the weight vector (48) with the total length K (cf. Eq. 47).

$$S = \left(S_1^1, S_1^2, \ldots S_1^{k_1}, S_2^1, S_2^2, \ldots, S_2^{k_2}, \ldots, S_j^1, S_j^2, \ldots S_j^{k_j}, \ldots, S_M^1, S_M^2, \ldots S_M^{k_M}\right) \tag{47}$$

For simplicity, entries of vector Eq. 22 can be rewritten

$$S = \left(s_1, s_2, \ldots, s_K\right). \tag{48}$$

3.5.3 Evaluating Alternatives

The provided N alternatives are pairwise compared with respect to each of the K sub-criteria. After obtaining K fuzzy judgement matrices of type, the fuzzy extent produces the decision matrix.

$$S_k = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \ldots & \tilde{a}_{1N} \\ \tilde{a}_{21} & \tilde{a}_{22} & \ldots & \tilde{a}_{2N} \\ \ldots & \ldots & \ldots & \ldots \\ \tilde{a}_{N1} & \tilde{a}_{N2} & \ldots & \tilde{a}_{NN} \end{bmatrix} \tag{49}$$

$$s_{ij} = \left(\sum_{k=1}^{K} \tilde{a}_{ik} \otimes \left[\sum_{l=1}^{N} \sum_{m=1}^{K} \tilde{a}_{lm}\right]^{-1}\right), \quad i = 1, \ldots, N, \; j = 1, \ldots, K \tag{50}$$

$$S = \begin{bmatrix} x_{11} & x_{12} & \ldots & x_{1K} \\ x_{21} & x_{22} & \ldots & x_{2K} \\ \ldots & \ldots & \ldots & \ldots \\ x_{N1} & x_{N2} & \ldots & x_{NK} \end{bmatrix} \tag{51}$$

In the decision matrix S, s_{ij} represents the resultant fuzzy performance assessment of the alternative $A_i \left(i = 1, 2, \ldots, N\right)$ with respect to the jth sub-criterion $\left(j = 1, 2, \ldots, K\right)$.

After this, the calculation of priority vectors of the Fuzzy AHP (Chang, 1996) is applied by applying the principle of the comparison of fuzzy numbers. The final weights of the alternatives are obtained from $W=\left(d\left(A_1\right),d\left(A_2\right),\ldots,d\left(A_n\right)\right)^T$; the transpose of W is used just to express them as a row, where W is a non-fuzzy number. We obtain W after the normalisation of W', where $W'=\left(d'\left(A_1\right),d'\left(A_2\right),\ldots,d'\left(A_n\right)\right)^T$. $d\left(A_i\right)=\min V\left(S_i\geq S_k\right),\ k=1,2,\ldots,n;\ k\neq i$. Here, $A_i(i=1,2,\ldots,n)$ are n elements. $V\left(M_1\geq M_2\right)=1\ \ iff\ \ m_1\geq m_2$, $V\left(M_1\geq M_2\right)=hgt\left(M_1\cap M_2\right)=\mu_{M_1}\left(d\right)$.

CHAPTER 4

The Experiment A

4.1 INTRODUCTION

The main objective of this experiment was to determine subject selection behaviour on product preferences. The experiment also sought to investigate the effects of the informatics system advice so as to report their behaviour and gender differences on product selection.

To achieve the goal of this experiment, we proposed a similarity measure that could yield different results and hence we followed a case-based reasoning (CBR) approach to investigate the product selection behaviour of the study subjects. We used CBR because changing the similarity measure would lead to different results. In order to establish accurate findings, we proposed a different similarity measure that would lead to different results from the traditional similarity measure.

In the next section, we sought to provide our proposed similarity measure, experiment design, and the working experiment.

4.2 PROPOSED LOCAL SIMILARITY MEASURE

Similarity measures play a critical role in the CBR cycle. It is considered as the being most important factor to consider in the model. Due to the limitations of the traditional similarity measure, we develop a modified version of the similarity measure to study product selection intended to behaviour. Further, we chose to develop a new measure so as to get different results from the traditional similarity measure, with the aid of which we could give our subjects non-traditional advice.

We started with the traditional similarity measure, which is a typical nearest-neighbour algorithm as shown in Eq. 52

$$similarity(Case_I, Case_R) = \frac{\sum_{i=1}^{n} w_i \times sim(f_i^I, f_i^R)}{\sum_{i=1}^{n} w_i} \tag{52}$$

where w_i is the importance weight of character i, represented as a numerical value between 0 and 1. Nearer neighbours have values closer to 1, while more distant neighbours have values closer to 0. f_i^I and f_i^R represent the values of character i in the input and retrieved cases, respectively, and $sim(f_i^I, f_i^R)$ is the local similarity function for primitives (traditional). The traditional local similarity $sim(f_i^I, f_i^R)$ is calculated as illustrated in Eq. 53 (Xiao-tai et al., 2004).

$$Local\_sim(f_i^I, f_i^R) = 1 - \frac{\left| f_i^I - f_i^R \right|}{k_i} \tag{53}$$

where k_i is the scale value of the degree of support of character i.

From our point of view, the local similarity was not suitable for product selection because when a product has a greater degree of support for a specific characteristic than required, it was considered as not being supported. For example, assume that there are two products P1 and P2 with different degrees of support for different characteristics (Table 4.1)

Table 4.1. *A case where traditional local similarity is not suitable*

	A	B
P_1	10	7
P_2	10	3
C	10	5

Then, by applying Equation 2 where k_i = 10 for characteristic B, it can be determined that:

$sim(P_1, C)$ = 1 – |7 – 5|/10 = 08.

$sim(P_2, C)$ 1 – |3 – 5|/10 = 0.8

It can be noted that P_2 is better than P_1 because it was supported to a greater degree. The previous example further supported this local similarity function implying, that it was more realistic for product selection behaviours. We suggested that if during the product selection process, a given product was found to have a greater degree of support for a specific characteristic, then it was considered good and could not therefore affect full similarity.

Assuming that:

1 P is the degree of support that a given product has for a particular characteristic.

2 C is the minimum degree of support for the characteristic that the consumer desires to see in the product.

Thus, we proposed the following definition for the local similarity between two characteristics, our proposed local similarity $P\_sim(C, P)$ is calculated as illustrated in Equation 3:

$$P\_sim(C,P) = \begin{cases} 0 & if \quad C = 0 \\ 1 & if \quad P > C \\ P/C & if \quad P \leq C \end{cases} \tag{54}$$

4.3 EXPERIMENTAL DESIGN

This experiment sought to investigate how subjects selected products on the basis of their preferences of the minimum level of preference they wished to contain in their products, and how they were influenced by the advice provided by the informatics system. In the experiment, we used three products, namely, P_1, P_2, and P_3, each having three similar characteristics A, B, and C, and which were to be selected by subjects who had provided their preferences. The subjects were informed about the preferences as indicated below. According to the subjects, the minimum level of a product's characteristic was as follows:

1 Characteristic A was essential; therefore, it was assigned to be of 100% importance.

2 Characteristic B was desirable; thus, it was assigned to have 50% importance.

3 Characteristic C was not important; thus, it had 0% importance.

The three products shared the same three characteristics A, B, and C. None of these products completely matched the subjects' preferences of the level of any of the characteristics. We selected three products as follows:

1 P_3 had full support only for the essential characteristic ‘A' and was the best product in horizontal differentiation.

2 P_2 had a higher level of support for characteristics that were not considered important by the subject. This product represented subjects who were interested in selecting those products with more support for unnecessary characteristics; therefore, it was the best product in general for vertical differentiation without preferences.

3 P_1 had mixed support for different characteristics; thus, it was in between the horizontal and vertical differentiations.

We presented this information at the beginning of the experiment, without explaining the horizontal and vertical differentiations. Table 4.2 illustrates the level of support for characteristics A, B, and C in each product.

Table 4.2. *Degree of support for characteristics in each product*

	A	B	C
P1	7	5	0
P2	5	10	5
P3	10	1	0
R	10	5	0

4.4 COMPARISON OF THE PROPOSED AND TRADITIONAL SIMILARITY MEASURES

As outlined in Chapter 1, three steps were involved in determining the total similarity between consumers’ preferences on characteristics and the degree of support for each characteristic in the three products. These steps included: calculating local similarity, calculating full similarity, and sorting. In the following example, we compare both the traditional and proposed local similarity measures.

4.4.1 Applying our proposed local similarity metric

Calculating local similarity

The weight of each characteristic (w_i) was calculated according to the importance of each feature to the total sum. For example: $w_i = (\mathrm{A})/(\mathrm{A}+\mathrm{B}+\mathrm{C})$. The total sum of weights equal to 1, $\sum_{i=1}^{n} w_i = 1$

In the course of applying our approach, we first had to move to the weighted table of preferences: w_A=10/15=0.66, w_B=5/15=0.33, w_C=0/15=0

Table 4.3 presents the results of the application of our local similarity P_Sim in Equation 4:

Table 4.3. *Weighted table of local similarity*

	A	B	C
Local Sim (R, P_1)	0.7	1	0
Local Sim (R, P_2)	0.5	1	1
Local Sim (R, P_3)	1	0.2	0
W_R	0.66	0.33	0

Calculating full similarity

These findings were obtained by applying Equation 2 to calculate full similarity:

Sim (P_1, R) = (0.7) (0.66) + (1) (0.33) + (0) (0) = 0.792.

Sim (P_2, R) = (0.5) (0.66) + (1) (0.33) + (1) (0) = 0.66.

Sim (P_3, R) = (1) (0.66) + (0.2) (0.33) + (0) (0) = 0.726.

Sorting

The sorting involves P_1, P_3, P_2. so in our approach, P_1 was the best option.

4.4.2 Applying the traditional approach

In order to compare the results between the two approaches, we used the traditional approach using the three steps adopted in our approach.

Calculating local similarity

The weight of each characteristic (w_i) was calculated in the same way as in our approach, and the calculation was based on the importance of each feature to the sum. By applying Eq 3, we obtained the values of traditional local similarity (Table 4.4).

Table 4.4. *Weighted table of preferences and traditional local similarity for A, B, and C.*

	A	B	C
Local Sim(R, P_1)	0.7	1	1
Local Sim(R,P_2)	0.5	0.5	1
Local Sim(R,P_3)	1	0.6	1
W_R	0.66	0.33	0

Calculating full similarity

Equation 2 was used to calculate full similarity:

Sim $(P_1, R) = (0.7)(0.66) + (1)(0.33) + (0)(0) = 0.792$.

Sim $(P_2, R) = (0.5)(0.66) + (0.5)(0.33) + (1)(0) = 0.495$.

Sim $(P_3, R) = (1)(0.66) + (0.6)(0.33) + (0)(0) = 0.858$.

Sorting

Sorting involves P_3, P_2, P_1. The traditional approach results revealed that P_3 was the best among the three products. Figure 4.1 shows a comparison between our approach and the traditional one.

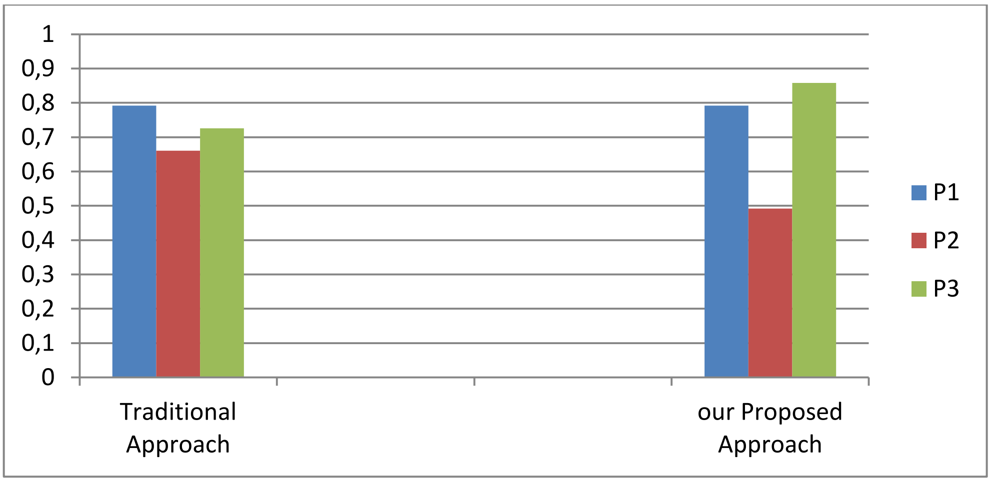

Figure 4.1. *Traditional approach and proposed approach*

4.5 EXPERIMENTAL DESIGN

This experiment consisted of four stages, and in each stage we asked the subjects to select one product as if they were to buy it according to their given preferences. The subjects were asked to answer the following questions: own opinion, opinion about others' selection, opinion after proposing our similarity measure, and final decision after a prize of 10–20 Euros was proposed. If the selection was common among the others, this prize was meant to assure that the subject was able to seriously select what he/she thought was the best product, and not with the aim of selecting an option for winning a prize money. The following were the four stages, and their respective questions to the subjects.

Stage 1 (S_1): What is the product that you will choose to buy according to your given preferences in the instructions?

Stage 2 (S_2): What do you think others will choose to buy according to the same given preferences in the instructions?

Stage 3 (S_3): There is an intelligent system that uses mathematical similarity function to identify the ideal product to calculate the similarity between your requirements and the products. This intelligent informatics system revealed the following results.

- Product P_1 matched the subject preferences to 79% and P1 was the optimal choice for you.
- Product P_2 showed a 66% of matching.
- Product P_3 showed 73% of matching.

Which of these products would you select to buy after becoming aware of this information?

Stage4 (S_4): If you have the same information given in S3, what would be your final decision given a prize amount of 10–20 Euros if you choice among all the subjects performing the experiment. It was important to use the prize in this experiment in order to avoid random selections as this experiment sought to find out if subjects followed our proposed similarity metric, and it also aided in studying the behaviour of subjects through the stages.

The experiment was computerized using a web-based program developed by me using PHP programming language. Each participant had his/her identity, which was unknown by him/her. All inputs were registered in the database, and at the end of the experiment, at the winning step, which was the most common among the participants, was calculated instantly, and appeared only to the experimenter.

There were two treatments. The first three questions were the same in both treatments however the only difference was in the amount of prize money for the second session was half (10 Euros). This difference was important in establishing whether the prize amount was affecting the behaviour to follow the informatics advice or not, and if not, then subjects followed it because they believed in that.

The experiment is interested in investigating the subject's behaviour from one stage to another by using the significance of our proposed similarity metric.

4.5.1 Participants

A total of 46 subjects from the University of Granada participated in this study. They were categorized as either undergraduate or graduate. Since there were no missing data, in this report the results of 46 participants were present (23 men and 23 women). Regarding the recruiting process, Subjects were selected after recruiting them via ORSEE (Greiner, 2004). We assigned similar numbers of men and women and distributed the subject pool, which comprised the following phases: (1) Students from the faculty of economics at different years of their course received an invitation to participate in the experiment; (2) Interested students registered in the proposed session schedule. Table 4.5 shows the summery of each session.

Table 4.5. *Summary of each session*

	First Session	Second Session
Prize	20 Euro	10 Euro
Number of men	12	11
Number of women	11	12
Average age of subjects	22.34 years	21.3 years
Average age of men	22.34 years	21.3 years
Average age of subjects	22.34 years	21.3 years
Average age of men	21.75	21.54
Average age of women	20.9	23
Average age of subjects who selected P_1	21.58	22.55
Average age of men who selected P_1	22.1	21.5
Average age of women who selected P_1	21	23.3
Percentage of men who selected P_1	75%	81%
Percentage of women who selected P_1	72%	92%
Percentage of subjects who selected P_1	74%	87%

4.5.2 Procedures Common to All Treatments

All the sessions were conducted at Laboratory de Experimental Economic ¨EGEO¨ in the Faculty of Economics and Business, University of Granada, Spain. Spanish was used as the main language of the experiment. The experiment consisted of two sessions, each session contained twenty-three participants. The participants completed the computerized tasks individually and were monitored by the two experimenters

(Researchers). No subject was allowed to participate in more than one session. On an average, a session lasted 40 minutes, including the initial instruction and subjects payment. There were two treatments which differed from the designated prize money. Table 5 contains some summary information about each session. Each treatment consisted of four phases as well as a questionnaire about subjects. Research items for the subjects were categorized as: age, gender, academic year, study, comment, and percentage of matching for their selection. Each subject earned 5 Euros as show up fees and additional prize (10 or 20 Euros). On an average, the amount earned was 16.95 Euros. Sessions took place in sequences and were carried out on the same day in order to minimize the possibility that participants in a different experiment session might share relevant information that could affect their decisions.

Each experiment session started with participants receiving a sheet of instructions. It included the experimental design described in Table 4 (see Appendix 1 for instructions). Apart from the instructions, the experiment was explained to them in neutral language and no mentoring was offered to subjects on what to select. No communication was allowed between the participants. Subjects were to indicate whether they had any questions about the process and the experimenter (Researcher), who would then answer them in private. Subjects were given five minutes to read and internalize the instructions. They were not informed about the stages; however they discovered the stages during the experiment process. The experiment was carried out in a stepwise manner to ensure that all participants (subjects) completed the questionnaires together. The experimenter urged the subjects to start by reading instructions at each stage despite having displayed the instructions on the screen. The subjects answered the questions stage by stage until all questionnaires had been completed. More details about the experiment, instructions, and screenshots are available in the Appendix section.

4.5.3 Dataset

All inputs were stored by the program in the databases, which included answers of each participant at each stage.

CHAPTER 5

The Experiment B

5.1 INTRODUCTION

The main intent of our experiment was to determine through the AHP approach how subjects assign their preferences. The experiment also sought to investigate the effects of informatics system advice using a different model, specifically Fuzzy AHP instead of CBR which was used in the previous experiment, to compare the results between conventional AHP and Fuzzy AHP and finally report the behaviours and gender difference which emerged. We followed the same structure as in experiment A in order to study the effect of changing the informatics model and the different advice.

To achieve the experiment's goal, we used the Fuzzy AHP approach as developed by Chang (1996), which was described in Chapter 3. We chose Fuzzy AHP for the following two reasons: firstly, to give the subjects the ability to assign their preferences according to the same given information and to see whether or not they would reflect this information in the same way or not; and secondly, to investigate the effect of different informatics advice, especially in terms of whether using Fuzzy AHP would give different results from the first experiment. The results will lead to a deeper understand of how people select products, and demonstrate whether or not they follow the informatics advice, regardless of its content.

5.2 THE PROPOSED FUZZY AHP APPROACH

5.2.1 Overview

We followed the Fuzzy AHP model shown in Chapter 3. Each participant was asked to perform a pairwise comparison between his or her preferences, and then between the products according to each preference, in sequence. In the mentioned model, participants were asked to perform the pairwise comparison after a full example had been introduced; they then needed to assign their preferences according to table 5.1.

Table 5.1. *Degree of support for characteristics in each product*

	A	B	C
P_1	7	5	0
P_2	5	10	5
P_3	10	1	0
R	10	5	0

There were four blocks, each of which included three questions. The first block related to the pairwise comparison of preferences and the last three involved the pairwise comparison of products according to each preference. Each question included an explanation of each number, the explanation is based on Table 5.2.

Table 5.2. *Linguistic scale of importance of preferences*

Preference Level	Numerical Value
Equal importance	1
Moderate importance	3
Essential or strong importance	5
Between very strong and extreme importance	7
Extreme importance	9
Intermediate between the two values	2, 4, 6 and 8

5.2.2 Example

Table 5.3 shows a comparison of the relative importance between characteristic A and characteristic B. The table illustrates a case wherein characteristic B is moderately important (3) in comparison with characteristic A.

Table 5.3. *Characteristic B is moderately important compared to characteristic A*

			A					vs.					B			
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	●	○	○	○	○	○	○

Table 5.4 gives a comparison of the relative importance between characteristic A and characteristic B. In this case, characteristic A is strongly important (5) in relation to characteristic B.

Table 5.4. *Characteristic A is strongly important **5** in relation to characteristic B*

			A					vs.					B			
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	●	○	○	○	○	○	○	○	○	○	○	○	○

5.2.3 Implementation

Similar to the example, the implementation of the pairwise comparison needs four blocks, each of which included three comparisons.

Block I

This block was used to compare the preferences in a pairwise manner, table 5.5 contained the questions of block 1 (questions 1, 2 and 3). The output of this block after applying Fuzzy AHP would be the W_R matrix Eq. 55:

$$W_R \begin{bmatrix} W_{(R_A)} \\ W_{(R_B)} \\ W_{(R_C)} \end{bmatrix} \tag{55}$$

Table 5.5. *Pairwise comparison of Block I*

C								vs.								B
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

C								vs.								A
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

B								vs.								A
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

Block II (according to preference A):

This block was used to compare products with each other which related to preference A. Tables 5.6 contained the questions of block II (questions 4, 5 and 6), the output of this block after applying Fuzzy AHP was the first column of the matrix, specifically

$$W_{(P,R)} \begin{bmatrix} W_{(P_1,A)} \\ W_{(P_2,A)} \\ W_{(P_2,A)} \end{bmatrix} \tag{56}$$

Table 5.6. *Pairwise comparison of Block II*

P_2								vs.								P_1
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

P_3								vs.								P_1
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

P_3								vs.								P_2
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

Block III (according to preference B)

This block was used to compare products with each other which related to preference B. Tables 5.7 contained the questions of block III (questions 7, 8 and 9), the output of this block after applying Fuzzy AHP was the second column of the matrix $W_{(P,R)}$, specifically

$$\begin{bmatrix} W_{(P_1,B)} \\ W_{(P_2,B)} \\ W_{(P_2,B)} \end{bmatrix} \tag{57}$$

Table 5.7. *Pairwise comparison of Block III*

P3								vs.								P2
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

P3								vs.								P1
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

P2								vs.								P1
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

Block IV (according to preference C

This block was used to compare products with each other relating to preference B. Tables 5.8 contained the questions of block IV (questions 10, 11 and 12), the output of this block after applying Fuzzy AHP was the third column of matrix $W_{(P,R)}$, specifically

$$\begin{bmatrix} W_{(P_1,C)} \\ W_{(P_2,C)} \\ W_{(P_2,C)} \end{bmatrix} \tag{58}$$

Table 5.8. *Pairwise comparison of Bolack IV*

P2								vs.								P1
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

P3								vs.								P1
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

P3								vs.								P2
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

5.2.4 The Mathematical Approach of Fuzzy AHP

The final weights of the products can be generally calculated by Eq. 59:

$$W_P = W_{(P,R)} * W_R \tag{59}$$

This can also be expressed as Eq. 60:

$$W_P \begin{bmatrix} W_{P_1} \\ W_{P_2} \\ W_{P_3} \end{bmatrix}, w_{(P,R)} = \begin{bmatrix} W_{(P_1,A)} & W_{(P_1,B)} & W_{(P_1,C)} \\ W_{(P_2,A)} & W_{(P_2,B)} & W_{(P_2,C)} \\ W_{(P_3,A)} & W_{(P_3,B)} & W_{(P_3,C)} \end{bmatrix}, W_R \begin{bmatrix} W_{(R_A)} \\ W_{(R_B)} \\ W_{(R_C)} \end{bmatrix} \tag{60}$$

Calculation of the weights is explained in the following. Let $A = \left(a_{ij}\right)_{n\times m}$ be a fuzzy pairwise comparison matrix, where $a_{ij} = \left(l_{ij}, m_{ij}, u_{ij}\right)$, which are satisfied with $l_{ij} = \frac{1}{l_{ji}}, m_{ij} = \frac{1}{m_{ji}}, u_{ij} = \frac{1}{u_{ji}}$.

We have applied Chang's (1996) proposal, as described in detail in Chapter 3. The fuzzy scale regarding relative importance used to measure the relative weights is given in Table 5.9. Here, δ =0.25 because we use the full scale.

Table 5.9. *Triangle fuzzy scale*

Linguistic scale of importance	Normal number	Triangle fuzzy scale
Equal importance	1	(1,1,1)
Between equal and moderate importance	2	(1,1.25,1,1.5)
Moderate importance	3	(1.25,1.5,1.75)
Between moderate and strong importance	4	(1.5,1.75,2)
Essential or strong importance	5	(1.75,2,2.25)
Between strong and very strong importance	6	(2,2.25,2.5)
Very strong importance	7	(2.25,2.5,2.75)
Between very strong and extreme importance	8	(2.75,3,3.25)
Extreme importance	9	(3,3,3)

The final weights of the products are obtained from $W = \left(d\left(A_1\right), d\left(A_2\right), \ldots, d\left(A_n\right)\right)^T$; the transpose of W is used just to get them as a row, where W is a nonfuzzy number. We obtain W after the normalisation of W', where $W' = \left(d'\left(A_1\right), d'\left(A_2\right), \ldots, d'\left(A_n\right)\right)^T$. Here, $A_i (i = 1, 2, \ldots, n)$ are n elements. Furthermore, $d'\left(A_i\right)$ is calculated as the following: $d'\left(A_i\right) = \min V\left(S_i \geq S_k\right)$, for $k = 1, 2, \ldots n; k \neq i$. Moreover, $\min V\left(S_i \geq S_k\right)$ is calculated as shown in Eq. (7):

$$\begin{aligned} V\left(M \geq M_1, M_2, \ldots, M_k\right) &= V\left[\left(M \geq M_1\right) and \left(M \geq M_2\right) and \cdots and \left(M \geq M_k\right)\right] \\ &= \min V\left(M \geq M_i\right), \ i = 1, 2, \ldots, k. \end{aligned}$$

$$\begin{aligned} &V\left(M_1 \geq M_2\right) = 1 \ \ iff \ \ m_1 \geq m_2 \\ &V\left(M_2 \geq M_1\right) = hgt\left(M_1 \cap M_2\right) = \frac{l_1 - u_2}{\left(m_2 - u_2\right) - \left(m_1 - l_1\right)} \end{aligned} \tag{61}$$

S_i is calculated as follows:

$$S_i = \sum_{j=1}^{m} M_{g_i}^{j} \odot \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^{j} \right]^{-1}. \tag{62}$$

To obtain $\sum_{j=1}^{m} M_{gi}^{j}$, we have to perform the fuzzy addition operation of m extent analysis values for a particular matrix, such that:

$$\sum_{j=1}^{m} M_{gi}^{j} = \left(\sum_{j=1}^{m} l_i , \sum_{j=1}^{m} m_i , \sum_{j=1}^{m} u_i \right) \tag{63}$$

Furthermore, to obtain $\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j} \right]^{-1}$, we have to perform the fuzzy addition operation of $M_{gi}^{j} \left(j = 1, 2, \ldots, m \right)$ values such that:

$$\sum_{i=1}^{n} \sum_{j=1}^{m} = \left(\sum_{i=1}^{n} l_i , \sum_{i=1}^{n} m_i , \sum_{i=1}^{n} u_i \right) \tag{64}$$

and then compute the inverse of the vector in Eq. 65, such that:

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j} \right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i} \right) \tag{65}$$

This can be calculated directly from the fuzzy triangle numbers, for example, $a_{ij} = \left(l_{ij}, m_{ij}, u_{ij} \right)$.

All of these steps were programmed via PHP programming language, so the final answer was calculated instantly at the end of stage3 and proposed as advice in stage4. Furthermore, the traditional AHP was calculated to compare both methods.

5.3 THE EXPERIMENTAL DESIGNE

5.3.1 Experiment Overview

This experiment investigated how subjects select products according to their preferences at the minimum level in relation to the product and whether they follow the informatics system advice, as well as how they assign their preferences according to given numerical information. The experiment followed a similar structure to the previous one. Thus, we had the same three products(P_1, P_2 and P_3) and each had the same three same characteristics A, B and C. One of the products was selected by the subject who gave his or her preferences. The subjects were informed about the preferences as indicated below. According to the subjects, the minimum level of a product's characteristic was same as the previous experiment as follows:

1. Characteristic A was essential; therefore, it was assigned to be of 100% importance.
2. Characteristic B was desirable; thus, it was assigned to have 50% importance.
3. Characteristic C was not important; thus, it had 0% importance.

The three products, P_1, P_2, and P_3, had the same characteristics, A, B and C. There was no product which totally matched the previous given preferences. We selected the same products as in the first experiment, and presented them to the participants at the beginning of the experiment, without any explanation about horizontal or vertical differentiation. As a result, the level of support that the products included for characteristics A, B and C was the same as in table 2:

Four stages were followed in this experiment. The first two proceeded exactly as in the first experiment, where the subjects were asked to make a decision in the selection of one product, assuming that they were to buy a product according to their given preferences related to their own opinion, as well as their opinion about the other selections. In the third stage, we proposed the Fuzzy AHP approach and asked the participants to assign a pairwise comparison; then, we iterated the results as informatics advice and asked whether or not they would follow it. The third stages is introduced exactly as described 2.3 and 2.4 sections. First of all an example is given to as 2.3 section, after filling the example and being sure of understanding the method, the 12 questions were asked as in 2.4 section. The fourth stage was exactly the same as in the previous experiment, and involved asking the participants for their final decision after offering a prize of 10 Euros if the selection was selected by most subjects; this prize was meant to ensure the subject would seriously select what he/she think thought was best rather than making a random choice. This is one difference between this and the previous experiment was that, in the previous experiment, there were two treatments of 10 and 20 Euros, whereas here only the 10 Euro prize was offered.

The following are the questions asked of the subjects in each of the four stages.

- **Stage1 'S_1':** What product will you choose to buy according to your given preferences in the instructions?
- **Stage2 'S_2':** What do you think others will choose to buy according to the same given preferences?
- **Stage3 'S_3':** Answer the questions which were exactly the same as the 2.3 and 2.4 sections.

- **Stage4 ‘S_4’:** There is an intelligent System which finds out the optimal product by using mathematical Similarity function to calculate the similarity between your requirements and the products. This intelligent informatics System finds out the following results[7]:

  1 The Product P_1 had the Percentage of the matching X

  2 The Product P_2 had the Percentage of the matching Y

  3 The Product P_3 had the Percentage of the matching Y

What is the product you will select to buy after becoming aware of this information?

- **Stage5 ‘S_5’:** If you had the same information given in S_3, what would your final decision be given that a prize of 10 Euros will be provided if your choice is common among all of the subjects doing the experiment?

5.3.2 Participants

A total of forty six subjects from the University of Granada participated in our experiments. They were categorised as undergraduates, studying economics and had not done any experiment before. There were no missing data; hence, this report’s results relate to 46 participants (23 males and 23 females). There were some inconsistent results, as described below. Regarding the recruiting process, subjects were selected after being recruited via ORSEE (Greiner, 2004). We included similar numbers of males and females, and distributed the subject pool according to the two following phases: 1) Students from the Faculty of Economics in different years received an invitation to participate in the experiment; 2) interested students were registered in the proposed session schedule.

[7] The results were calculated through the program and were introduced as X,Y,Z.

5.3.3 Procedures Common to All Treatments

All of the sessions were conducted at Laboratory of Experimental Economics (EGEO) in the Faculty of Economics and Business at the University of Granada, Spain. Spanish was used as the main language of the experiment. The experiment consisted of two sessions, each including 23 participants. Participants completed the computerised tasks individually and were monitored by the two experimenters (researchers). No subject was allowed to participate in more than one session. On average, a session lasted 40 minutes, including initial instruction and payment of subjects. There was only one treatment, and the two sessions were similar. Each treatment consisted of the four phases plus the final questionnaire for subjects to fill out. Each subject earned 5 Euros as an attendance fee and an additional prize (10 Euros) if his or her answer was selected. On average, the amount earned was 13.95 Euros. Sessions took place in sequence and were carried out on the same day in order to minimise the possibility that participants in different experimental session might share relevant information that could affect their decisions.

Each experimental session started with participants receiving a sheet of instructions. This included Table 1, as given in the experimental design (see Appendix 2 for the instructions). Apart from the instructions, the experiment was explained to participants in neutral language and no mentoring was offered to subjects on what to select. No communication was allowed between the participants. Subjects were asked to indicate whether they had any questions about the process and the experimenter would answer them in private. Subjects were given 5 minutes to read and internalise the instructions. They were not informed about the stages; however, they discovered the stages during the experimental process. The experiment was done step by step so as to ensure that all participants completed the questionnaires together. The experimenter urged the subjects to start by reading the instructions at each stage, although the instructions had been displayed on the screen. The subjects answered the questions stage by stage until all the questionnaires were filled out. More details about the experiment, instructions and screenshots are available in Appendix 2.

5.3.4 Dataset

All input was stored in a database using a program which included the answers from each participant at each stage; the results for both the AHP and Fuzzy AHP models were calculated instantly and stored in the database.

CHAPTER 6

Gender difference and the effect of informatics advice on product selection

In this Chapter[8], we will shed light on gender differences in product selection according to two experiments, which have been described in detail previously. Further, we will investigate the effect of the informatics advice given by different decision support systems.

6.1 RESULTS OF EXPERIMENT A

6.1.1 Effect of the prize amount

There were two treatments in experiment A. Although the first three questions were the same for both treatments, the prize amount differed: 20 and 10 Euros for the first and second treatments, respectively. This difference was used to investigate whether the prize amount influenced the participants' selection behaviour and whether or not they followed the informatics advice. If the results do not differ, it would mean that they followed the advice because they believed in it, and not because of the prize amount. There were two sessions, with one treatment per session. Figure 6.1 shows the statistics of the two sessions.

[8] An earlier version of this manuscript has been presented at Sixth EBIM Doctoral Workshop on Economic Theory (EBIM 2011), the Sixth Alhambra Experimental Workshop 2011. Financial support by the Junta de Andalucía (P07-SEJ-03155) is gratefully acknowledged.

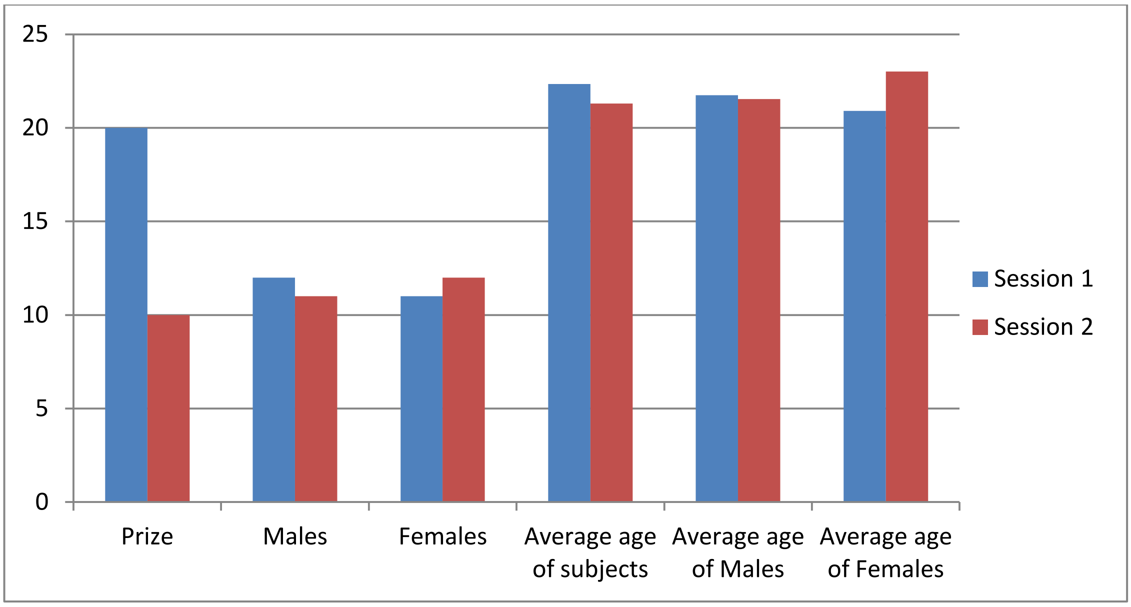

Figure 6.1. *Summary of the two treatments.*

A similar number of subjects from each treatment selected P1 in the last stage (stage 4), although this figure was higher in the 10 Euro treatment. The other factors were similar, so there were no differences between the two sessions in each treatment, and since the effect of the prize amount on selection was not significant, we analyzed the entire group as one unit to observe the effects of gender.

6.1.2 Stage selection

Experiment A had 4 Stages, and the percentage of subjects who selected each product in each stage is shown in Figure 6.2.

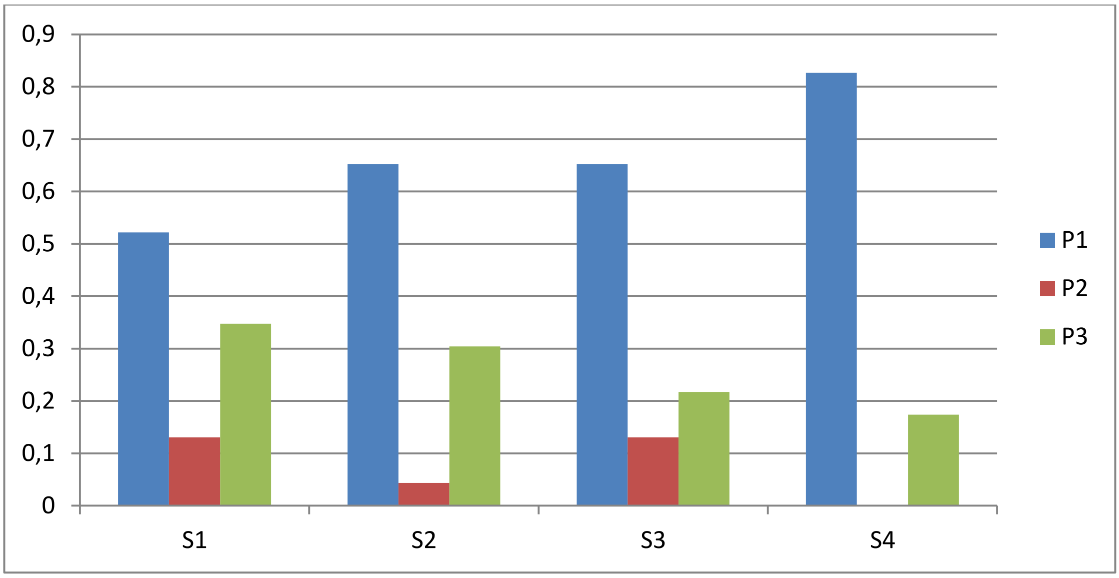

Figure 6.2. *Percentage of product selection by the subjects at each stage (Experiment A.)*

From Figure 6.3, it is clear that an increasing percentage of subjects selected P1 from the first stage to the fourth stage.

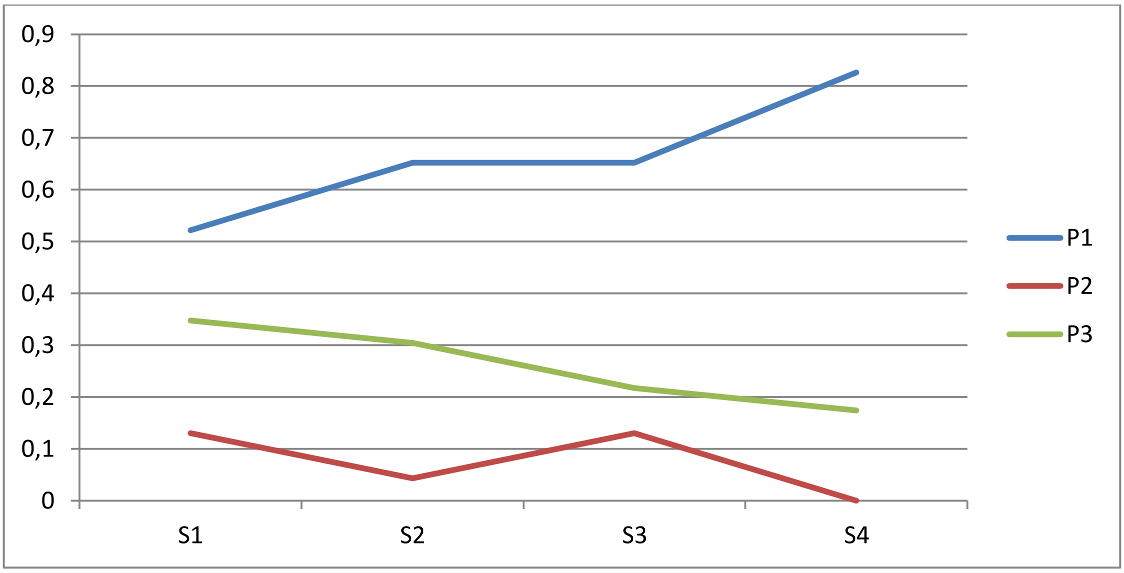

Figure 6.3. *Changes in the percentage of subjects that selected each product across the stages (Experiment A.)*

6.1.3 Selection at each stage according to gender

Men

Figure 6.4 shows the selection tendencies of men at each stage. As shown in the figure, the percentage of P1 selection by men tended to increase at every stage (except for stage 2, after following the proposed advice.

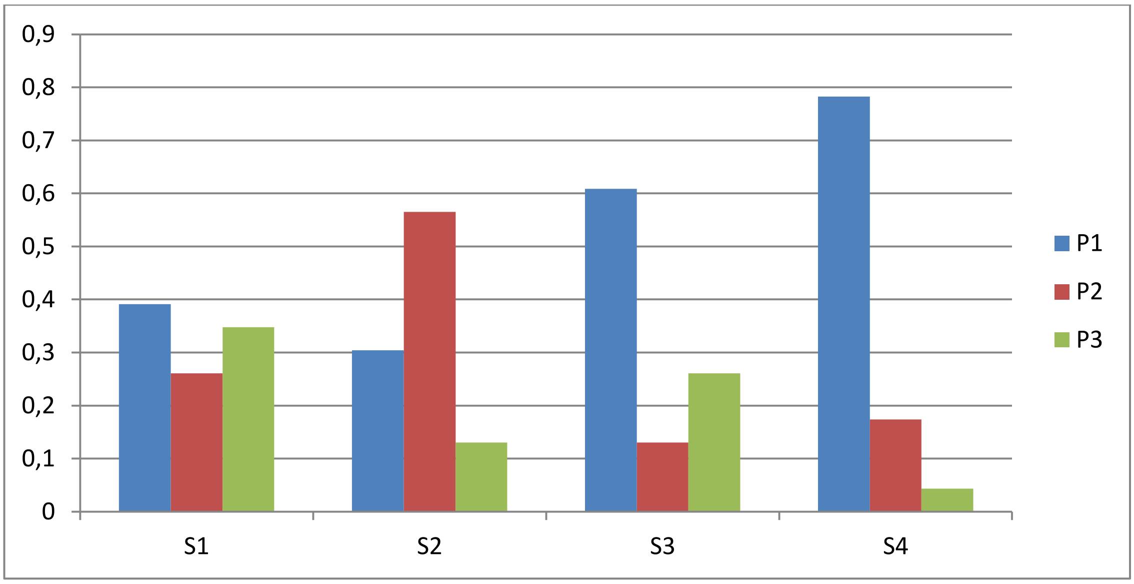

Figure 6.4. *Percentage of product selection by men at each stage (Experiment A.)*

Figure 6.5 shows changes in product selection behaviour of men at each stage. This figure suggests that the men believe that the other participants will select another option than the one they will select.

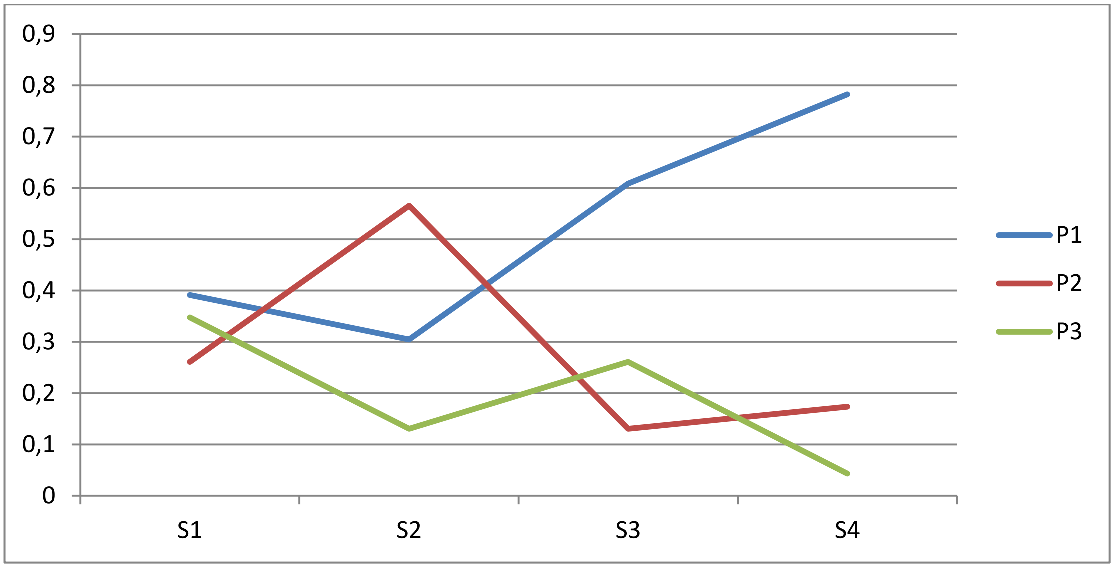

Figure 6.5. *Changes in the percentage of men who selected each product at every stage (Experiment A.)*

Women

Figure 6.6 shows how women selected products at each stage. Similar to the case observed for men, the percentage of P1 was found to increase at each stage.

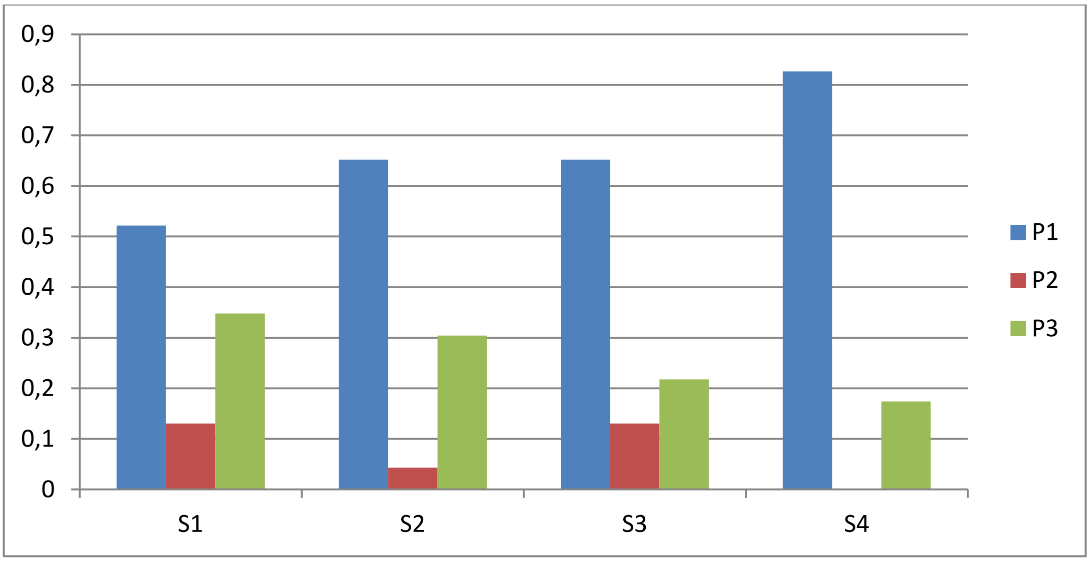

Figure 6.6. *Percentage of product selection by women at each stage (Experiment A.)*

Figure 6.7 shows changes in product selection behaviour by women at each stage. As it can be gleaned from the figure, the product selection behaviour is considerably different from that of men.

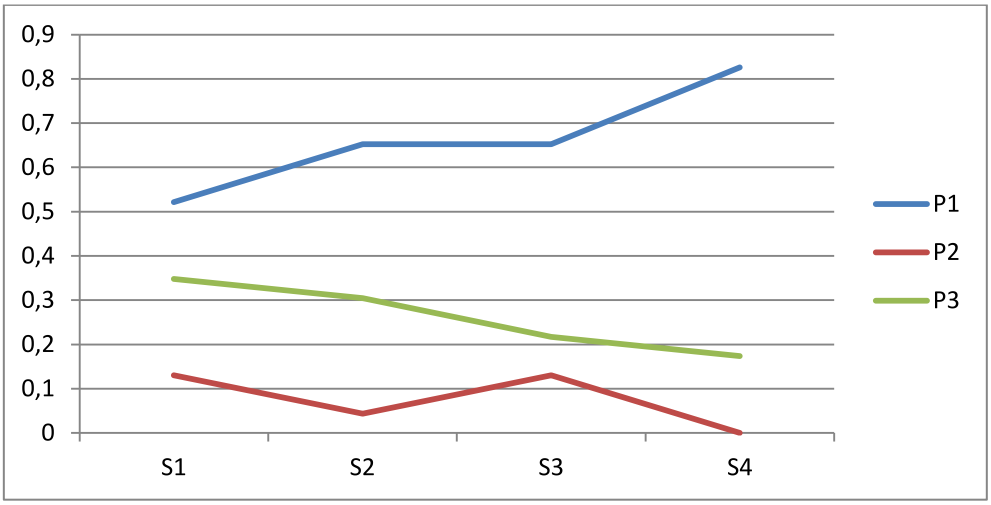

Figure 6.7. *Changes in the percentage of women who selected each product at each stage (Experiment A.)*

Figures 6.5 and 6.7 illustrate a clear difference in the selection behaviour between men and women. While men think that the others will select an option that is different from the one they select, women do not think so. Further, P1 selection by women was higher than that by men.

6.1.4 Changing selection across stages

Figure 6.8 shows that the highest percentage of participants (overall and men) changed their selection from S_2 to S_3, indicating that after following the informatics advice, more men than women changed their selection. Thus, men showed the tendency to change their selection more than women.

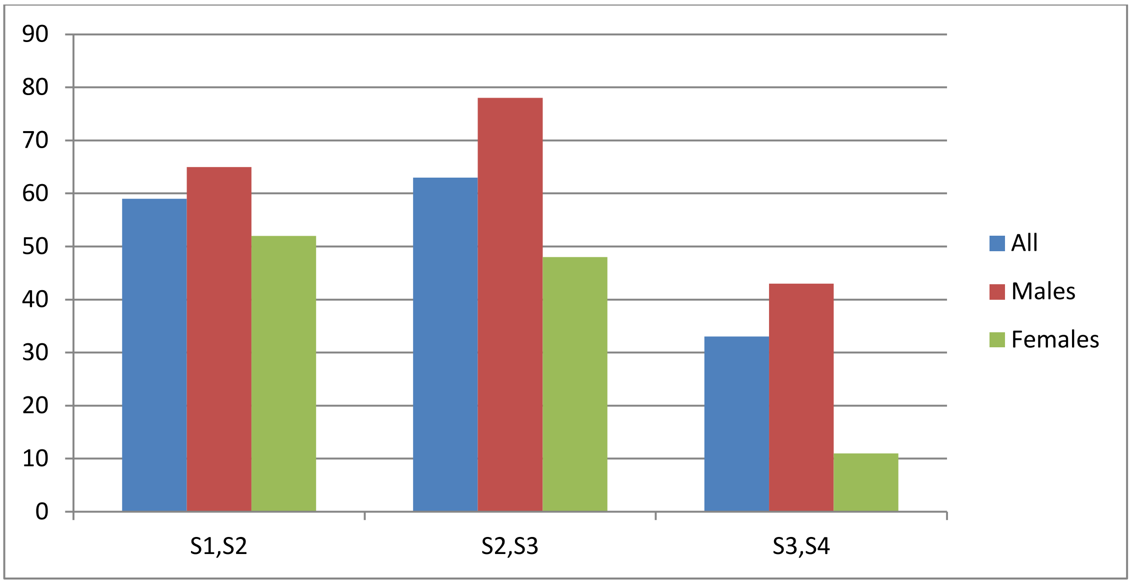

Figure 6.8. *Changing selection across stages (Experiment A.)*

6.1.5 Final decision

Here is how the informatics system information affected the subjects' decision in paid stage S_4. Figure 6.9 shows the number of both men and women who selected P1 at the final stage.

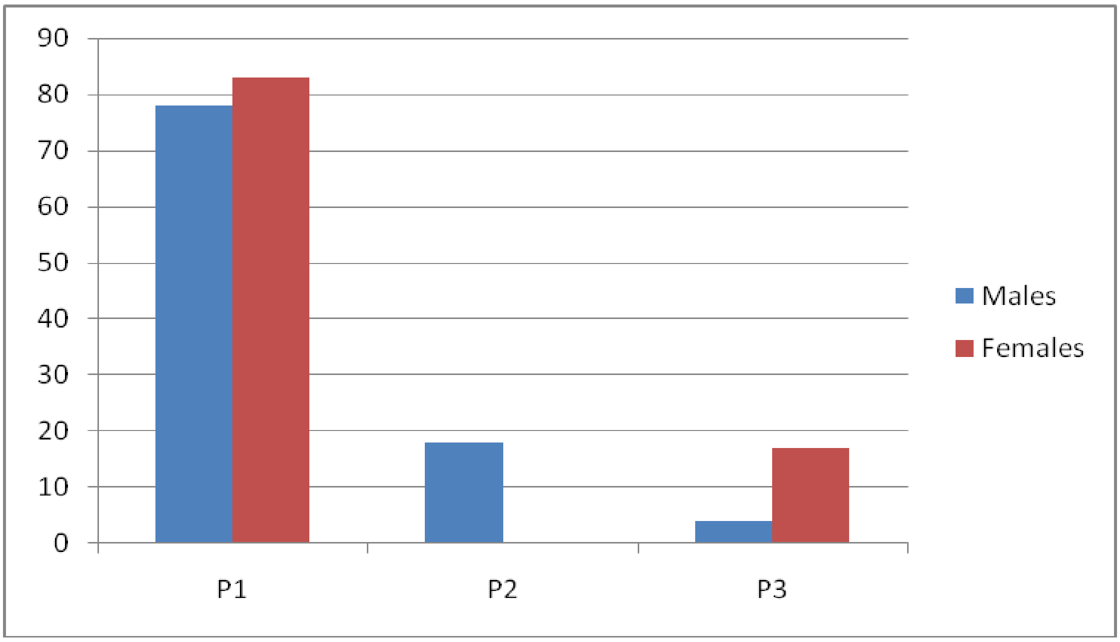

Figure 6.9. *Number of subjects who selected each product at the final stage (Experiment A.)*

Figure 6.9 shows that most of the subjects followed the informatics advice.

6.1.6 Discussion

The results provide experimental evidence of the degree to which subjects follow informatics advice. The results of the first experiment reveal that approximately 80% of all subjects agreed with the informatics advice (and 63% of subjects followed it regardless of the prize). This is not to say that the informatics advice is right; rather, it indicates that the subjects preferred to follow the advice. To obtain a deeper understanding of this finding, further investigation with another model is necessary.

There is a no gender effect on the final decision at S4, but there is a clear effect on the selection behaviour before the informatics advice is provided. The men tended to select P1 and P2 whereas women tended to select P1 and P3. These results show that in general, men prefer to select products with greater degree of support than required or similar to what they want, so they may not always care about their highest priority preference. On the other hand, women prefer to select products with the highest priority preference and a product similar to their first preference.

There was no age effect in this analysis. Age was not an important factor affecting selection by men. The average age of men who selected P1 and those who did not was 21.3 years, whereas the average age of all women in the study was 21.87 years, and the average age of women who selected P1 was 22.05 years. Lastly, the prize amount did not affect the selection of P1 by subjects.

6.2 RESULTS OF EXPERIMENT B

This experiment aimed to better understand the role of the informatics advice and to investigate how the subjects handled it. In this experiment, another model is used, which leads to informatics advice different from that provided in the first experiment. In this experiment, the winning product was not always the same, but generally using Fuzzy Analytic Hierarchy Process (FAHP) model as decission support system led that P3 is the winning product. Table 6.1 shows a brief comparison between the two experiments.

Table 6.1. *Comparison between experiments A & B.*

	Experiment A	Experiment B
Informatics decision model	CBR	FAHP
Percentage of the winning product	Constant	Variable according to the subjects' inputs

6.2.1 Model results

In this model, unlike the first experiment, the final winning product is not fixed. Figure 6.10 shows the averages results of the FAHP model. As shown in Figure 6.10, the order of the products in this experiment is P3, P2, P1 whereas the order of products in the previous experiment was P1, P3, P2.

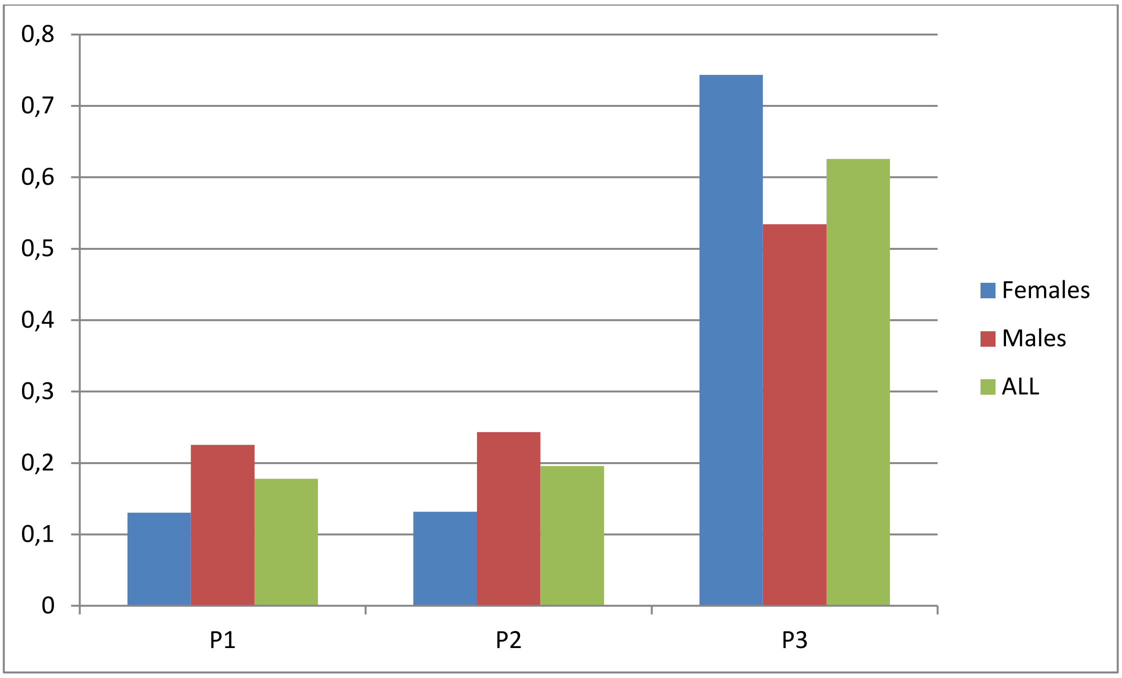

Figure 6.10. *Average results of the model (Experiment B.)*

It shows that the results between men and women were similar, although a higher percentage of women than men selected P3.

6.2.2 Stage selection

Experiment B also had 4 stages. Figure 6.11 shows the percentage of subjects who selected each product at each stage.

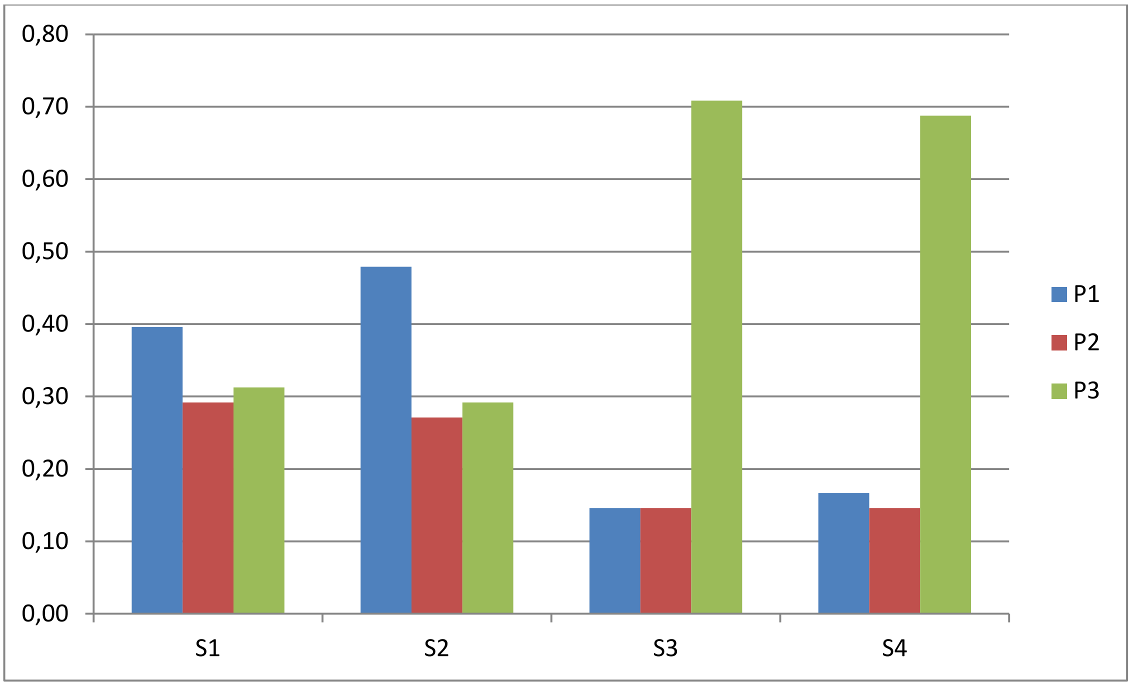

Figure 6.11. *Percentage of subjects who selected each product at each stage (Experiment B.)*

It is evident that the number of subjects who selected P3 increased over the last two stages, as shown in Figure 6.12; these results differ from those of the first experiment in which P1 selection increased.

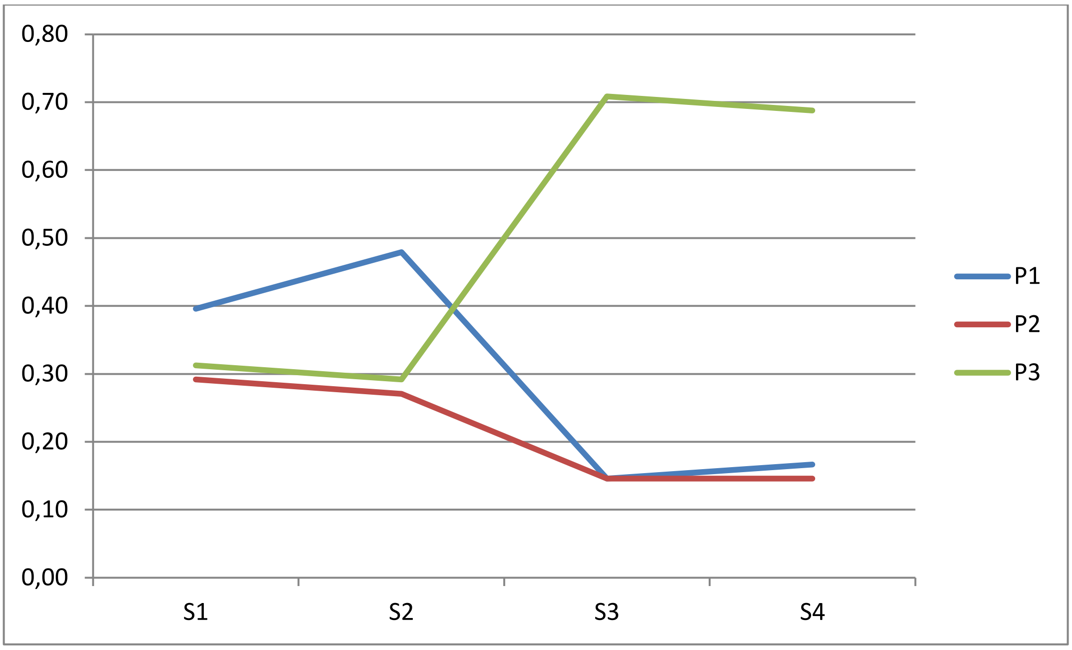

Figure 6.12. *Changes in the percentage of women selecting each product (Experiment B.)*

6.2.3 Men

Figure 6.13 shows the selection tendencies of men at each stage. Note that the number of subjects who selected P3 increased dramatically after following the informatics advice.

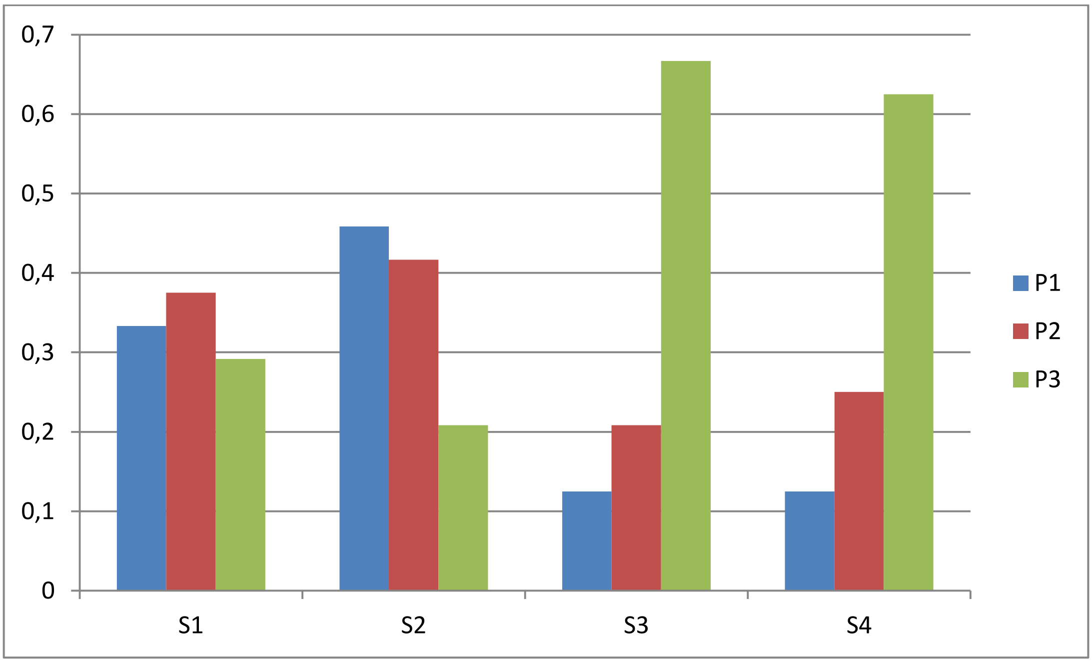

Figure 6.13. *Percentage of men who selected each product at each stage (Experiment B.)*

Figure 6.14 shows changes in the selection of each product across each stage by men.

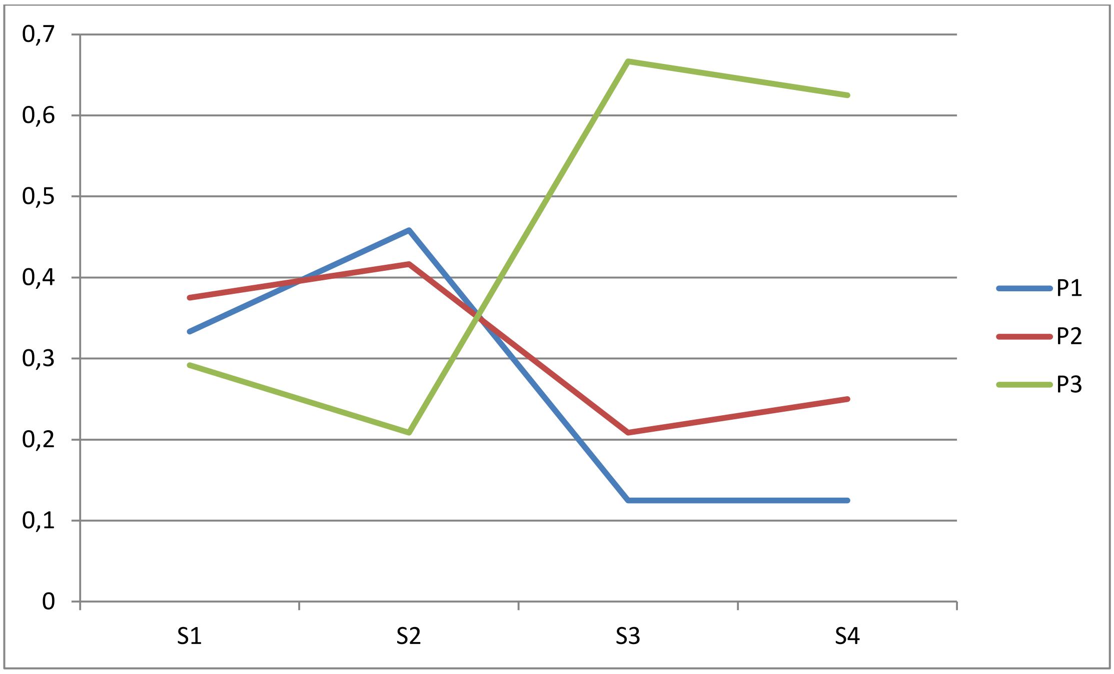

Figure 6.14. *Changes in the percentage of men who selected each product (Experiment B)*

6.2.4 Women

Figure 6.15 shows the selection behaviour of women at each stage.

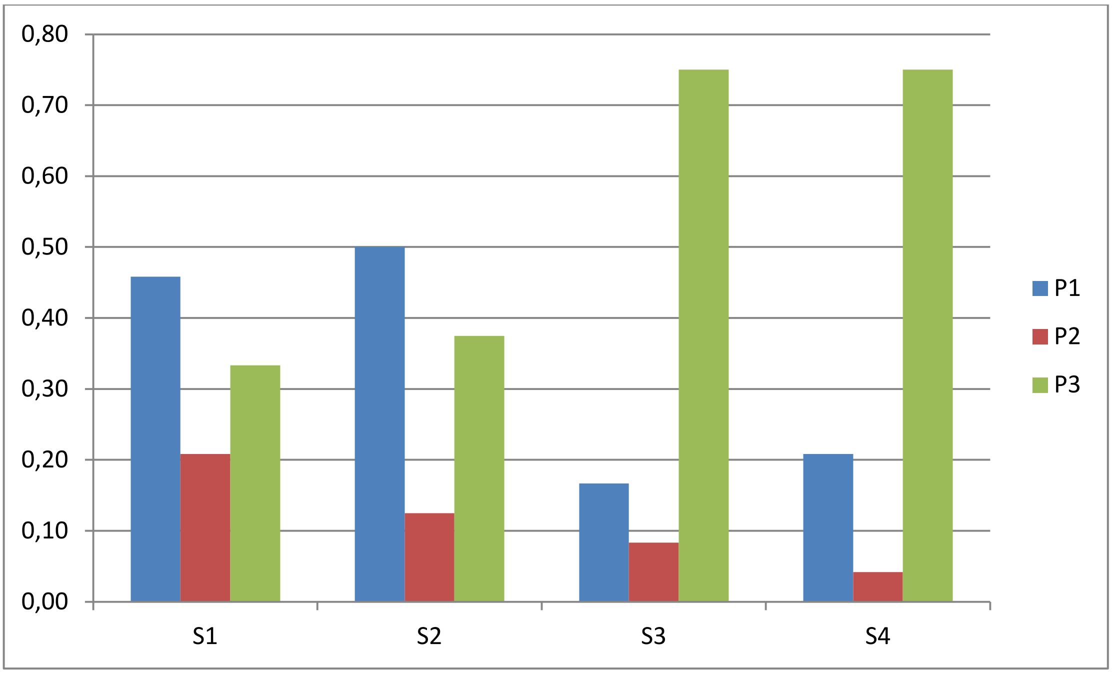

Figure 6.15. *Percentage of women who selected each product at each stage (Experiment B.)*

Figure 6.16 shows changes in the percentage of women who selected each product at each stage.

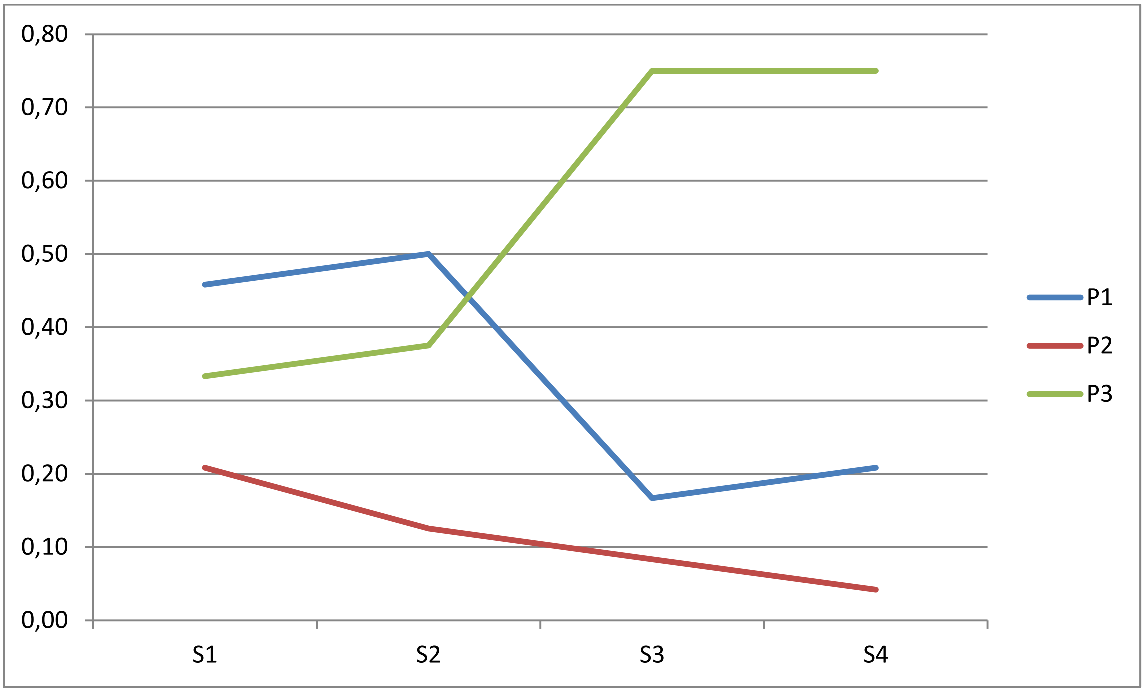

Figure 6.16. *Changes in the percentage of women who selected each product at each stage*

Figures 6.14 and 6.16 illustrate that the difference in selection behaviour follows a similar trend as in the first experiment. In this experiment, P3 selection by women was higher than that by men.

6.2.5 Changing selection across stages:

Figure 6.17 shows how subjects changed their selection across stages.

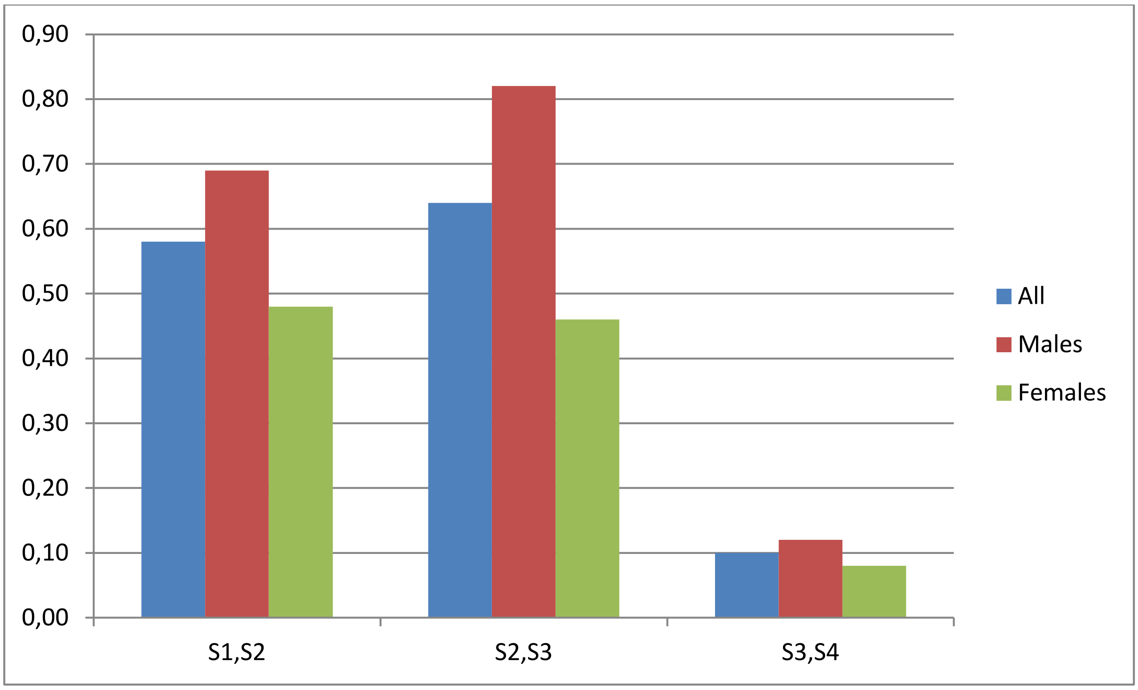

Figure 6.17. *Changing selection across stages (Experiment B.)*

In Experiment B, a high percentage of participants (overall and men) changed their selection from S_2 to S_3, indicating that after proposing the AHP approach and the informatics advice thereafter, most subjects changed their selection. More men changed their selection than women. On the other hand, the percentage of subjects who changed their selection from S_3 to S_4 is small, this could be an evidence that the AHP Approach facilitates trust the advice proposed by the informatics decision support system, resulting in participants who did not change their selection for the prize amount.

6.2.6 Gender difference in pairwise assignment

There is a clear gender-based difference in assigning the values of the comparing stage. The linguistic scale (Table 6.2) was given at the start of the instructions.

Table 6.2. *Linguistic scale of importance of preferences*

Preference Level	Numerical Value
Equal importance	1
Moderate importance	3
Essential or strong importance	5
Between very strong and extreme importance	7
Extreme importance	9
Intermediate between the two values	2, 4, 6 and 8

For example, the subject had to compare A and B as shown in Table 6.3.

Table 6.3. *Scale of importance of preferences*

A						vs.						B				
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
Extreme importance		Extreme importance		Extreme importance		Moderate importance		Moderate importance		Moderate importance		Strong importance		Strong importance		Strong importance

Each subject carried out twelve pairwise comparisons in the experiment, generating a total of 576 values for 48 subjects. Only 3% of the comparisons (9 times) of men were ‘intermediate’ values, which were even values (2, 4, 6, 8), whereas 10% (29 times) of the comparisons by women, i.e. three times the number of comparisons by men. were ‘intermediate’ values. Thus, the women were more precise in assigning their preferences than men.

6.2.7 Gender difference in inconsistency

Inconsistency is a general issue in AHP, which occurred if there were one or more wrong comparisons through the twelve pairwise comparison from the participants. We did not alert the participants about their mistakes because we were interested in reporting these errors. A total of 17 inconsistencies were reported (10 men and 7 women). Figure 6.18 shows the difference in inconsistencies between men and women.

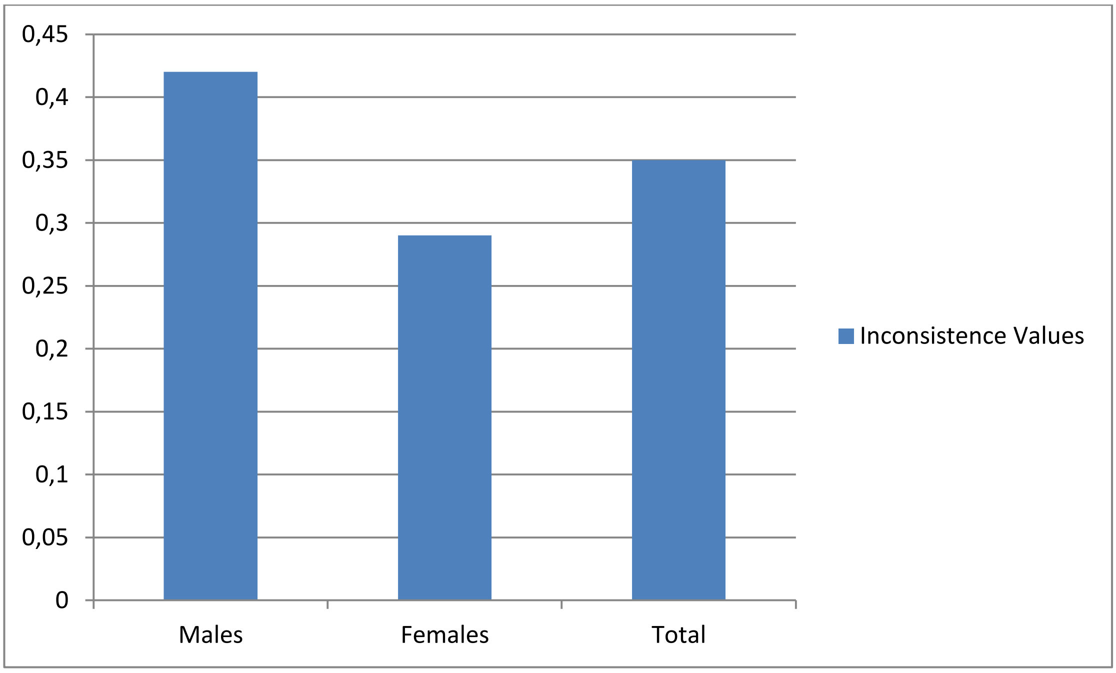

Figure 6.18. *The percentages of inconsistency values for men and women (Experiment B.)*

The figure shows that the inconsistency values were considerably higher for men than women, illustrating more evidence that women were more precise than men.

6.2.8 Final decision

Figure 6.19 shows that the final decision was similar to the model results in Figure 6.8, which illustrates that men were interested in P2 products.

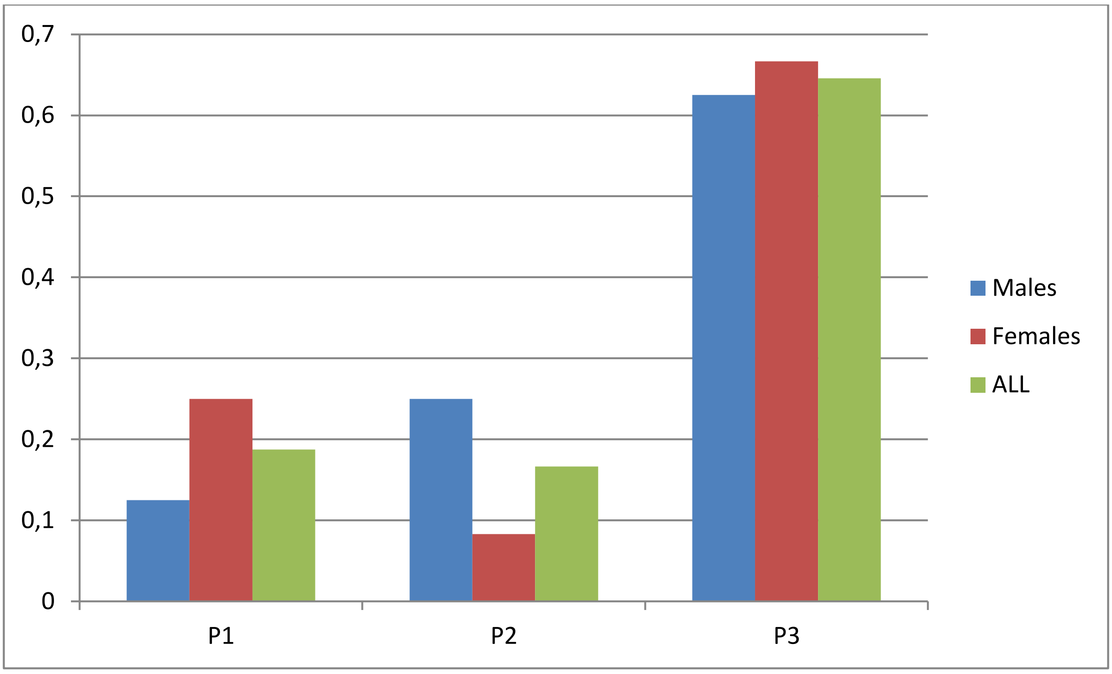

Figure 6.19. *Percentages of inconsistency values for men and women (Experiment B.)*

6.3 CONCLUSION

Both experiments showed the same trend in the first two stages: in both cases, men thought that the other respondents will select an option other than the one they chose. However, women do not hold such beliefs. Experiment B showed that the AHP-based pairwise approach helps the subjects to determine their preferences carefully, and to understand their need. Subjects in this experiment followed the new informatics advice; moreover, they did not change their decision after the prize was proposed. This means that the AHP approach is better than the fixed advice based on similarity measure.

It was evident that that the difference between S_3 and S_4 is low; this means that the AHP approach of pairwise comparison aids in improving the decision process. The informatics advice differed from one subject to another, and in most cases the participants followed it.

Both experiments demonstrated that before the advice, men were interested in P2 (more support of not-needed preferences, i.e. vertical differentiation) than the women. Women were more interested in their most important preferences (horizontal differentiation). Both men and women followed the informatics advice. Women were found to be more precise than men in assigning their comparative selections in the pairwise comparison, e.g. women tended to select values such as 4, 6, and 8 more than men, who preferred the values 5, 7, and 9.

We can summarize the results as follows:

- Unlike women, men believed that the other participants will select another option than the one they have opted for.
- Men tended to change their selection more than the women.
- The subjects followed the informatics advice, regardless of the advice given, and changed their selection in accordance with it.
- The AHP approach helped subjects to understand their needs, and it was found to be better than the fixed advice, which depended on similarity alone.
- Women were more precise than men in both assigning their preferences and avoiding inconsistency.

CHAPTER 7

A Comparison between AHP and FAHP

7.1 MOTIVATION

This chapter aimed to study the differences between traditional AHP and fuzzy AHP (FAHP) from an experimental point of view. FAHP is defined as a variant of the traditional AHP model, which differs from the original in the construction of the judgment matrix. Both models need to obtain a vector of weights from the judgment matrix and use these weights to compute the importance of each alternative. The output of this model generates a new vector. While other conditions remain constant (i.e. both the inputs and the composition of the analytic hierarchy remain constant), the two processes differ in the computed weight and output vectors.

Few papers have attempted to compare traditional AHP with fuzzy AHP. Actually, many researchers have tried to apply both these approaches on a specific problem and then compared the result. Ozdagoglu and Ozdagoglu (2007) compared between the traditional and fuzzy AHP approaches for multi-criteria decision-making processes with linguistic evaluations. Liyuan (2010) achieved a similar comparison of the classical and fuzzy AHP approaches in multi-criteria decision making for the commercial vehicle information systems and networks (CVISN) project. Recently, researchers have tried to investigate the general errors of the extent analysis method on fuzzy AHP proposed by Chang (1996). Zhu (2009) improved the formulation of comparing the size of the triangular fuzzy number in the extent analysis method and the application of fuzzy AHP and recently, Zhu (2012) mathematically justified the invalidity of triangular fuzzy AH. Zhu et al. (2012) called attention in their research to the errors and of the extent analysis method and found that its results are unreliable upon reapplication. Wang et al. (2008) pointed out the zero-weight problem numerically. Actually this paper did not stop using

Chang (1996). For example Chang's paper earns additional 445 citations ever since March 2007 where Wang's paper was available online. Zhu et al. (2012)[9].

7.2 METHOD

In this chapter, we present an investigation of the difference between fuzzy and traditional AHP experimentally and numerically. This chapter is based on experiment B, in which the fuzzy AHP model was used to evaluate product selection. We will focus on stage 3 of the judgment experiment, where we calculated the traditional as well as fuzzy AHP matrices and results for the same input. Thus, we obtained all the results of both these approaches for the same input values assigned by the subjects.

Let us refer once again to Stage 3 of the abovementioned experiment, experiment B. In this experiment, three products, namely, P1, P2, and P3, each containing the same three same characteristics A, B, and C, were used. Each subject selected one of the three products on the basis of his or her preferences. The instructions gave the subjects the minimum level of each characteristic they desired the product to possess, and assigned the levels as given as follows:

- Characteristic A was essential; therefore, it was assigned to be of 100% importance.
- Characteristic B was desirable; thus, it was assigned to have 50% importance.
- Characteristic C was not important; thus, it had 0% importance.

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[9] Chang's paper had 758 citations in Google Scholar until July 2012, and 669 citations until February 2012, which is when Zhu et al. published their paper.

We presented Table 7.1 to the participants at the beginning of the experiment. R represents the preferred level of each characteristic in a product, and the degree of support of each characteristic for the three products is also given.

Table 7.1. *Degree of preferred support for each characteristic and the actual support available in the three products.*

	A	B	B
P1	7	5	0
P2	5	10	5
P3	10	1	0
R	10	5	0

The participants followed the AHP approach as described previously in detail in Chapter 5. The participants carried out pairwise comparisons of their preferences R of their ideal product 'R' with the actual products P1, P2, and P3 in terms of characteristics A, B, and C in that sequence. We calculated both these models mathematically to obtain the results of the traditional and fuzzy AHP approaches.

The following output of weights of subject preferences were obtained by both the approaches:

- The preference weights generated by traditional AHP—W_A (traditional), W_B (traditional), and W_C (traditional)—were denoted using the notations AWT, BWT, and CWT, respectively
- The preference weights generated by fuzzy AHP—W_A (fuzzy), W_B (fuzzy), and W_C (fuzzy)—were denoted using the notations AWF, BWF, and CWF, respectively.

- The final product weights generated by traditional AHP—P1 (traditional), P2 (traditional), and P3 (traditional)—were denoted using the notations P1T, P2T, and P3T, respectively.
- The final product weights generated by fuzzy AHP—P1 (fuzzy), P2 (fuzzy), and P3 (fuzzy)—were denoted using the notations P1F, P2F, and P3F, respectively.

Some inconsistencies were present because of one or more wrong comparisons that occurred when the participants carried out the twelve pairwise comparisons. We excluded any inconsistent data from the dataset. We did not alert the participants through the experiment about their errors because we were interested in reporting these discrepancies. There were a total of 31 inputs in this experiment (from 14 men and 17 women).

7.3 GENERAL COMPARISON OF BOTH MODELS

7.3.1 Overview

Of the final results, 97% of responses were the same as that of the winning product in this experiment, namely, P3. Only one input, equivalent to 3% of the responses, were for P2. Figure 7.1 shows the values of P1, P2, and P3 according to both approaches. This figure provides a holistic picture of distribution of the results in both the models. The range of fuzzy AHP is 0-1, whereas the range of traditional AHP is 0-69. Smaller weights tended to zero and the higher weights tended to 1 in fuzzy AHP.

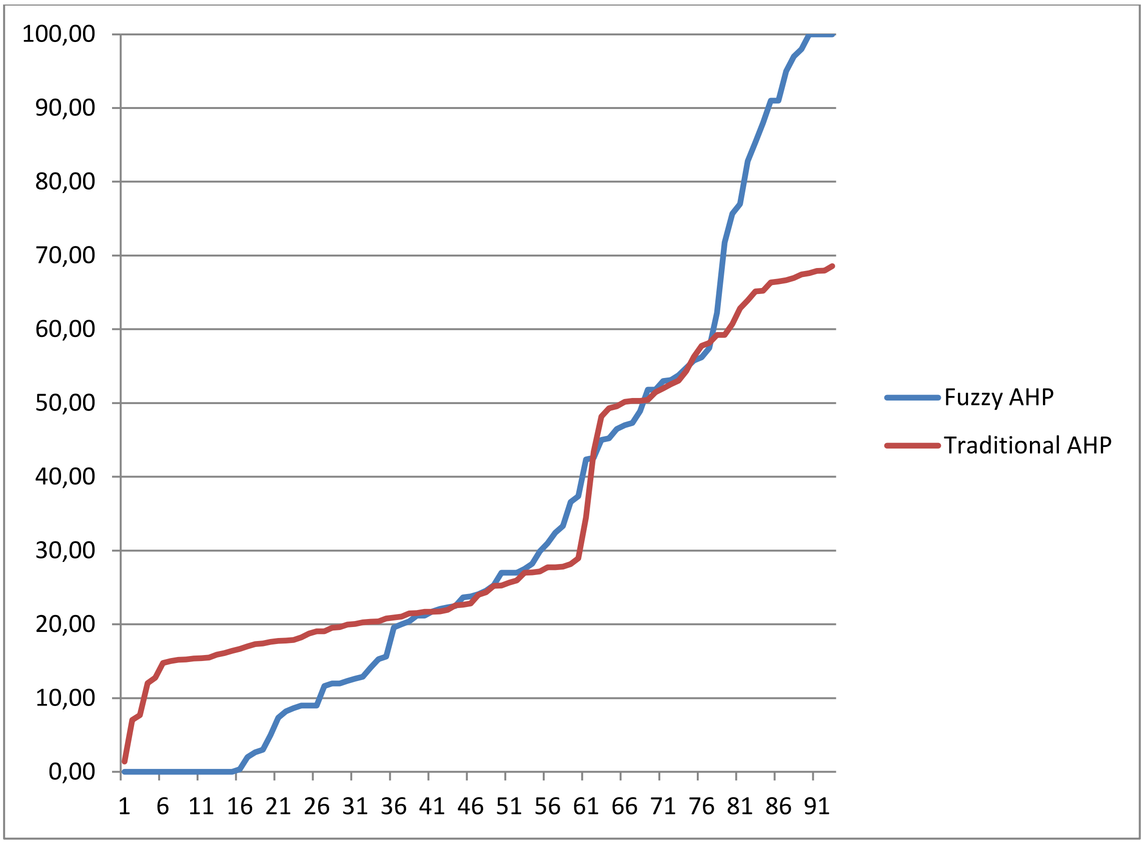

Figure 7.1. *All values obtained by both approaches.*

Figure 7.2 shows the average results of both models. The winning product according to both models is the same, i.e. P3; however, the percentage of matching differed across individual participants and gender groups.

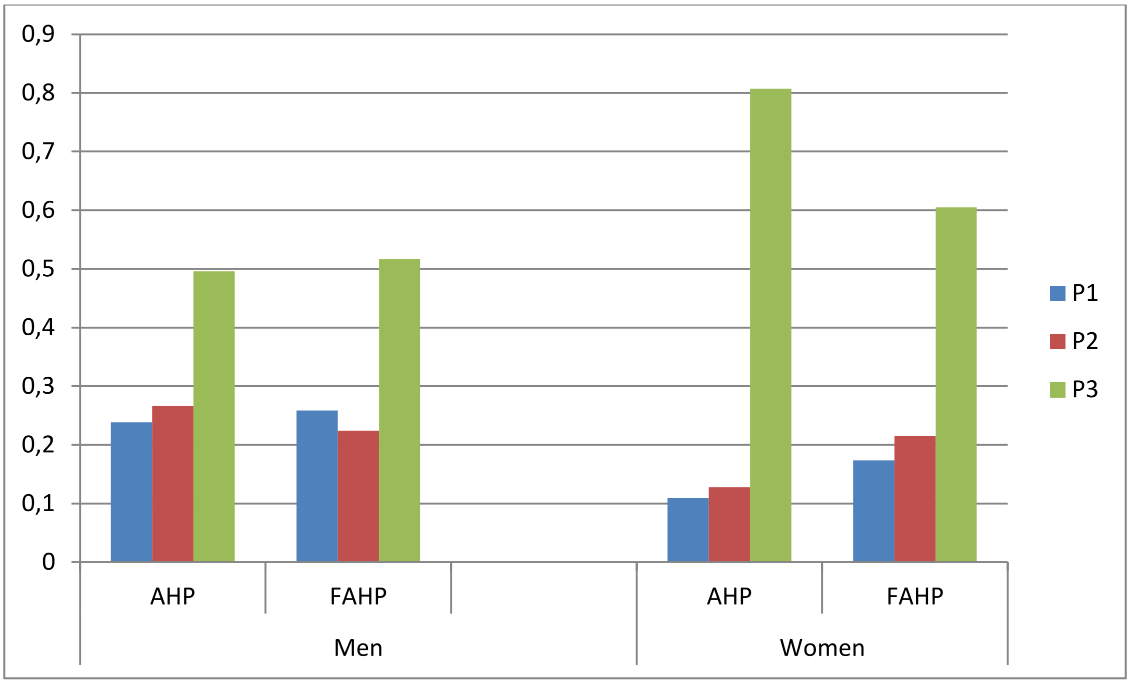

Figure 7.2. *Average results of the AHP and FAHP*

7.3.2 Visualization of all results

Figure 7.3 illustrates the visualization of all results for all products in both models.

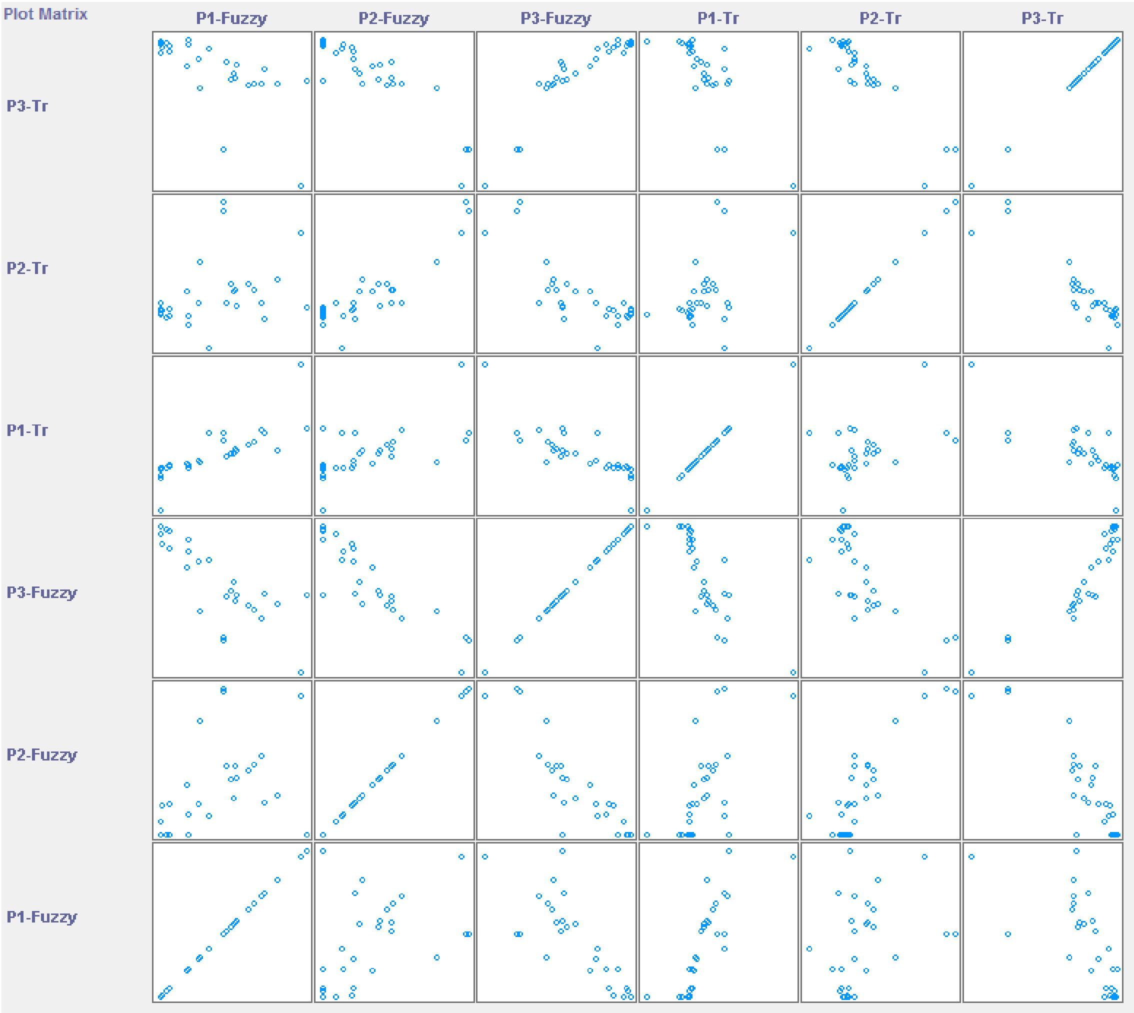

Figure 7.3. *Visualization of all results*

Linear regression, Gaussian processes, pace regression, multilayer perception, isotonic regression, and least median of squares linear regression functions were used to simulate the correlation between the results of the two models. No satisfying result was found in any of the two different items, nor were there any obvious correlations between the outputs of the two models. In Figure 3 we can only observe clustering along an anonymous curve such as that in the (P3 (fuzzy), P3 (traditional) plot, suggesting that the results are correlated but cannot be represented by a simple function. Statistical analysis is required to analyse this in further detail.

7.4 STATISTICAL ANALYSES

We tested the percentages of the three products (P1, P2, and P3) on both models. The resultant six conditions were tested across men and women, thus gender was added to the analyses as a factor. All the statistical analyses were computed using SPSS software version 20, and included means and standard deviations, Pearson's correlation coefficients, and Student's *t* test for mean differences. The p value was set at 0.05 for all tests.

7.4.1 Comparison between AHP and FAHP

Statistical analysis of P1

Pearson correlations were calculated to test the relationship between the two AHP models among men, women, and both genders combined, when applied to P1. The results revealed that both models were strongly correlated in all three groups, with women showing the weakest correlation ($r = 0.72$), followed by the men ($r = 0.76$), and the full sample ($r = 0.79$). To test the mean difference in accuracy between the two models, independent sample *t* tests were computed for the three groups. A highly reliable significant difference was found for women ($t = 3.73$, $p < 0.01$), who scored higher on the traditional AHP ($M = 17.37$, $SD = 5.70$) than on the fuzzy AHP ($M = 10.92$, $SD = 10.53$). Further, a marginally significant difference was found for the combined sample ($t = 1.80$, $p = 0.08$), who also scored higher on the traditional ($M = 20.11$, $SD = 7.68$) than the fuzzy ($M = 17.26$, $SD = 13.89$) model. The converse was seen in men, who scored higher on the fuzzy AHP model ($M = 25.88$, $SD = 13.52$)

compared to the traditional model (M = 23.84, SD = 8.62), although this difference was not significant (p = 0.41; Table 7.2)

Table 7.2. *Statistical analysis of P1*

Gender	Model	Pearson correlation	P value	*t* test of mean	Standard deviation	Mean
Men	Traditional AHP	0.76	0.41	0.85	8.62	23.84
	Fuzzy AHP				13.52	25.88
Women	Traditional AHP	0.72	0.002	3.73	5.70	17.37
	Fuzzy AHP				10.53	10.92
All	Traditional AHP	0.79	0.08	1.80	7.68	20.11
	Fuzzy AHP				13.89	17.26

Statistical analysis of P2

When the same tests were computed for P2, a similar pattern of correlation emerged, with the two models showing high correlation in the female group (r = 0.84), the combined group (r = 0.86) and the male group (r = 0.87). Further, the *t* tests revealed highly significant differences in the mean values provided by women (t = 3.78, p < 0.01), with higher scores in the traditional model (M = 21.48, SD = 8.56) than in the fuzzy model (M = 12.75, SD = 16.17), and by participants of both genders (t = 3.86, p < 0.01), with higher scores in the traditional (M = 23.65, SD = 9.51) than the fuzzy (M = 16.84, SD = 17.17) model. Similar to the results from P1, the mean difference in the values by men between the models was not significant (p = 0.14; Table 7.3).

Table 7.3. *Statistical analysis of P2*

Gender	Model	Pearson correlation	P Value	*t* test of mean	Standard deviation	Mean
Men	Traditional AHP	0.87	0.14	1.58	10.26	26.59
	Fuzzy AHP				17.49	22.40
Women	Traditional AHP	0.84	0.001	3.78	8.56	21.48
	Fuzzy AHP				16.17	12.75
All	Traditional AHP	0.86	0.001	3.86	9.51	23.65
	Fuzzy AHP				17.17	16.84

Statistical analysis of P3

The correlation coefficients were somewhat more varied for P3, with strong correlations shown in the responses by men (r = 0.90) and by all participants (combined group; r = 0.73), with only medium correlation for the responses by women (r = 0.65). Further, *t* tests revealed no differences in the mean values of any group for P3 ($p > 0.21$), and a reliable pattern of model accuracy could not be discerned (Table 7.4).

Table 7.4. *Statistical analysis of P3*

Gender	Model	Pearson correlation	P Value	*t* test of mean	Standard deviation	Mean
Men	Traditional AHP	0.90	0.54	0.64	16.23	49.58
	Fuzzy AHP				25.00	51.72
Women	Traditional AHP	0.65	0.28	1.11	10.58	60.49
	Fuzzy AHP				35.15	67.95
All	Traditional AHP	0.73	0.21	1.27	14.14	55.86
	Fuzzy AHP				31.86	61.07

Statistical analysis of all products

The correlation coefficients for all the products were stronger than those of each individual product, with similar correlations shown in the responses by the male (r = 0.90), female (0.87), and combined (r = 0.88) groups. Further, the *t* tests revealed no differences in the mean values for any group (p values for all groups > 0.32; Table 7.5).

Table 7.5. *Statistical analysis of all products*

Gender	Model	Pearson Correlation	P Value	*t* test of mean	Standard deviation	Mean
Men	Traditional AHP	0.90	0.99	0.0001	16.64	33.33
	Fuzzy AHP				22.98	33.33
Women	Traditional AHP	0.87	0.32	0.998	21.31	33.12
	Fuzzy AHP				35.07	30.54
All	Traditional AHP	0.88	0.37	0.90	19.38	33.21
	Fuzzy AHP				30.43	31.72

7.4.2 Gender differences

Statistical analysis of P1, P2, and P3 revealed that at least in some situations, the fuzzy model generates more discriminating results and therefore may be a better tool in the decision-making process. However, such results were mostly observed in the female and combined-gender groups, with men showing no evidence of difference between the two models. Thus, we conducted follow-up *t* tests to quantify the gender difference to aid in clarifying the applicability of the two models. Gender differences were only calculated for P1 and P2 because the values for P3 in the male group showed no difference as mentioned above. For P1, both models showed highly reliable mean differences between men and women for the traditional AHP ($t = 2.60$, $p < 0.01$) and (fuzzy AHP: t = 3.58, p < 0.01) models. However, these differences were not observed in P2, where the mean differences across genders were not significant in either model (p > 0.11; Table 7.6).

Table 7.6. Gender differences.

	Model	Model	P Value	*T* test of mean
P1	Traditional AHP	Men	0.01	2.60
		Women		
	Fuzzy AHP	Men	0.001	3.58
		Women		
P2	Traditional AHP	Men	0.13	1.56
		Women		
	Fuzzy AHP	Men	0.11	1.64
		Women		
P3	Traditional AHP	Men	0.03	2.34
		Women		
	Fuzzy AHP	Men	0.15	1.47
		Women		
ALL	Traditional AHP	Men	0.63	0.48
		Women		
	Fuzzy AHP	Men	0.96	0.06
		Women		

7.5 WEIGHT COMPARISON

We decided to go further to determine the difference between the weights of both the models investigated in this study.

- The preference weights generated by traditional AHP were AWT, BWT, and CWT.
- The preference weights generated by fuzzy AHP were AWF, BWF, and CWF.

7.5.1 Visualization of all weights

Figure 7.4 presents a visualization of all the weights together. it represents a combination of all the plots, with which we can obtain an intuitive impression of the similar pattern generated by both models. Figure 7.5 is an enlargement of the square marked in red in Figure 7.4. As shown in the figure, the 3 × 3 square highlighted in yellow plots any two combinations of the three traditional weights, and the 3 × 3 square marked in blue plots any two combinations of three fuzzy weights. By comparing these two squares, it is evident that they show considerable similarity, indicating that the correlations among the weights are similar between the two models.

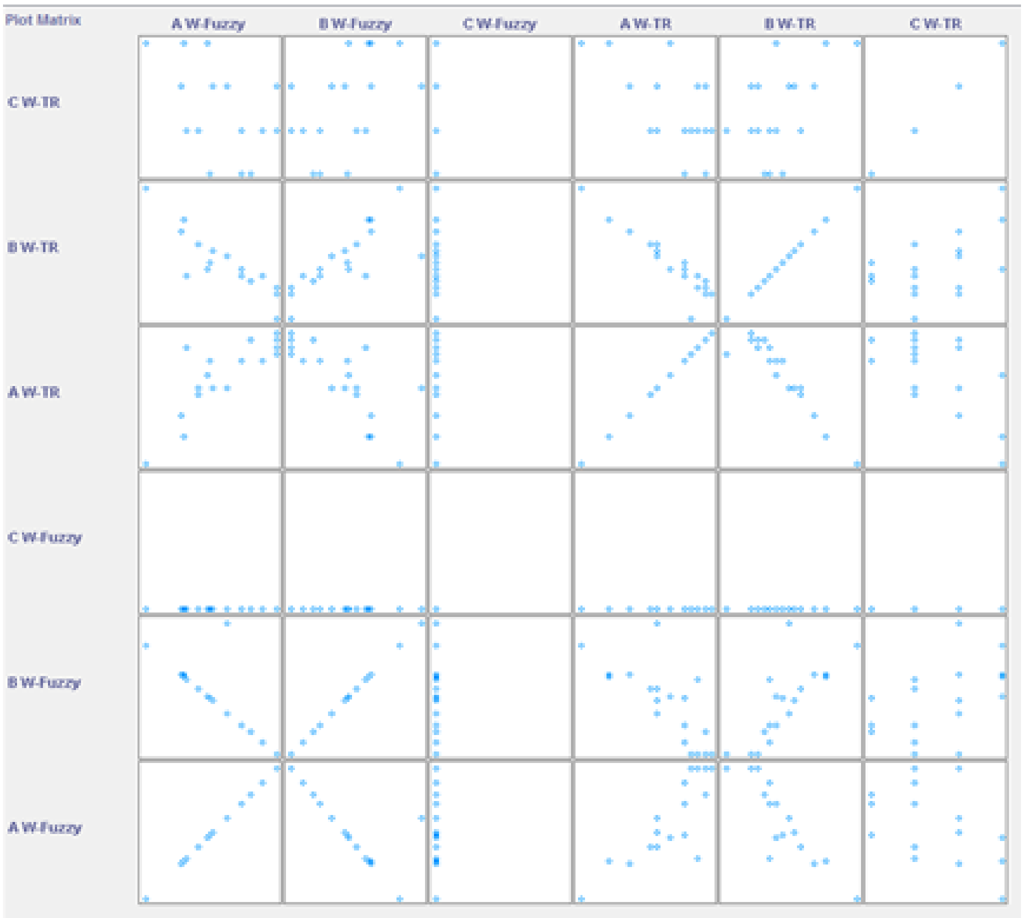

Figure 7.4. *Visualization of all weights*

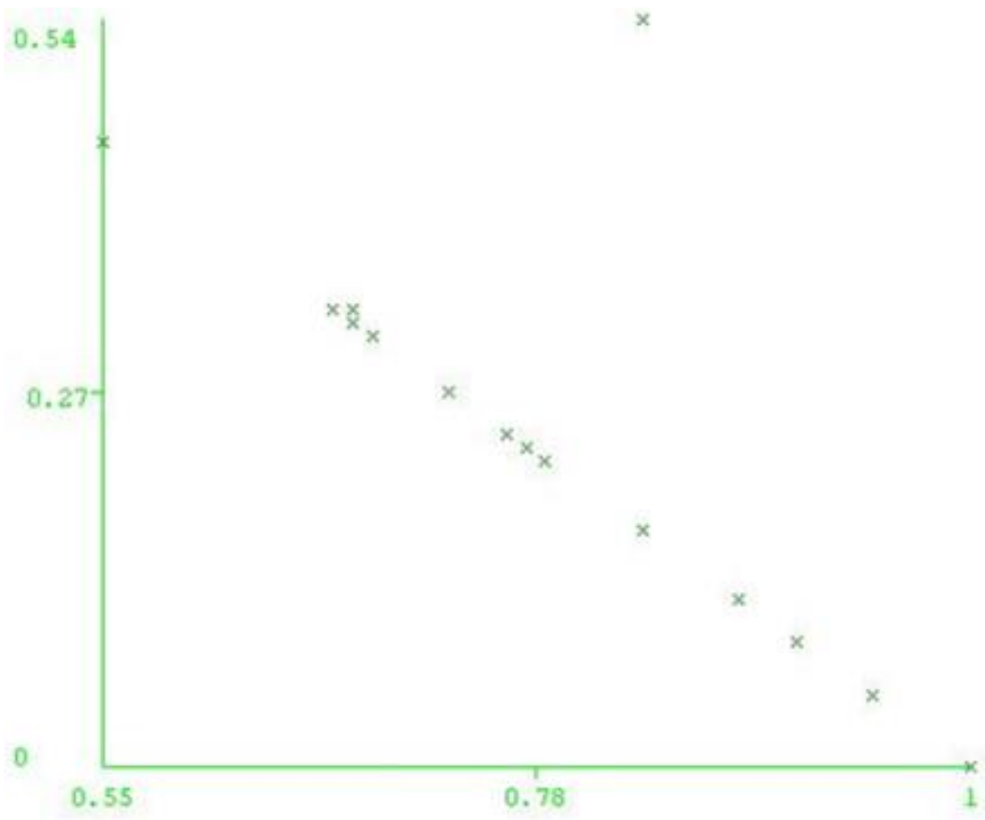

Figure 7.5. *X: W_A (fuzzy), Y: W_B (fuzzy)*

The Pearson correlation coefficient (r) between the fuzzy and traditional AHP models was found to be 0.77. No good result was obtained to map an accurate conversion between the two sets of weights. AWF and BWF were found to have negative linear correlation, and CWF was nearly zero. The average values of AWF, BWF, and CWF were 0.84, 0.17, and 0.00, respectively, and the average values of AWT, BWT, and CWT were 0.76, 0.19, and 0.04, respectively. If we divide both vectors by their largest element, they will be transformed to (1, 0.2, 0) and (1, 0.25, 0.05). From these results, it is obvious that the relative weights of the three characteristics were similar since they differed at the percentile level.

The 3 × 3 square on the upper right (for traditional weights) and the one on the lower left (for fuzzy weights) showed a similar pattern. We found that W_B (traditional) = –0.85 ×

W_A(traditional) + 0.8354 with a relative absolute error of 25.9272% (Figure 7.6), and W_B (fuzzy) = –1.0144 W_A (fuzzy) + 1.0244 with a relative absolute error of 19.7759% (Figure 7.7). These findings suggest that W_A and W_B show more linearity in the fuzzy model.

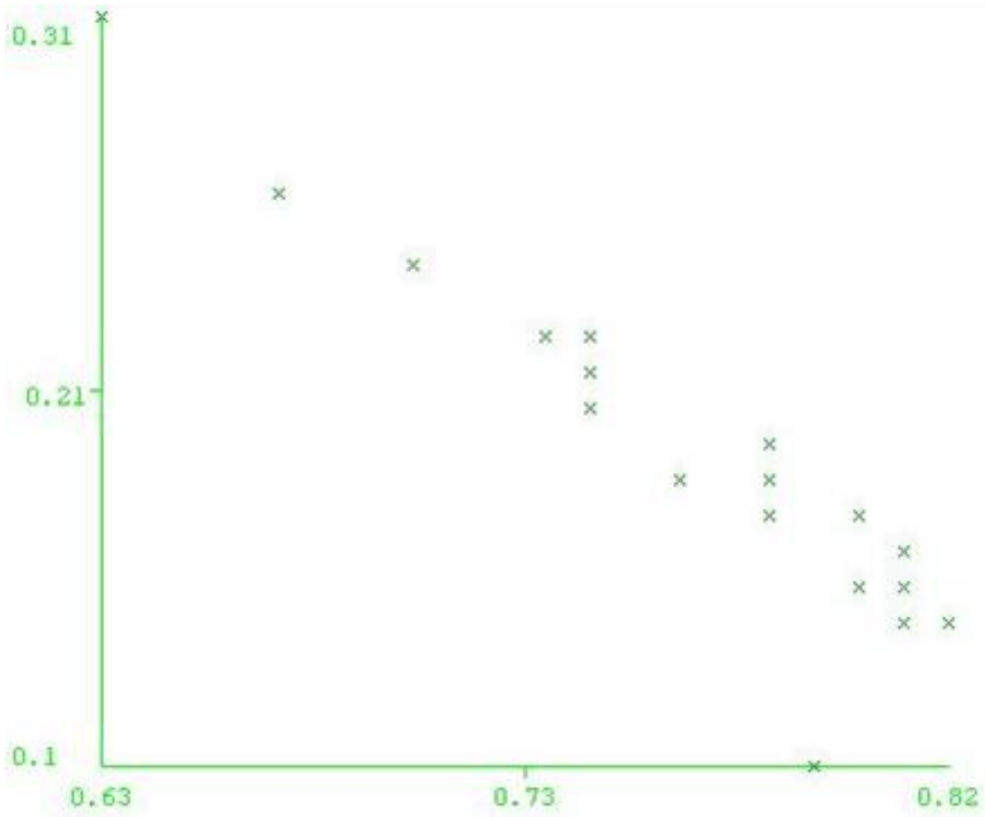

Figure 7.6. *X: AWT, Y: BWT*

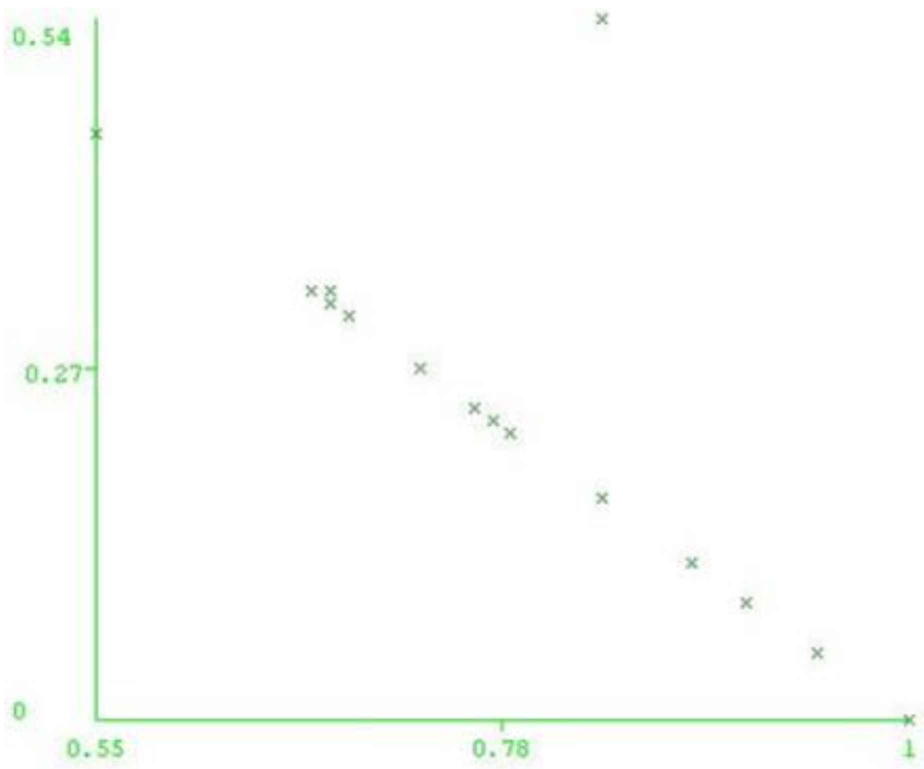

Figure 7.7. *X: AWF, Y: BWF*

7.5.2 Visualization of weights and output

Figure 7.8 and 7.9 illustrate the visualization of the weights according to the output in both the models.

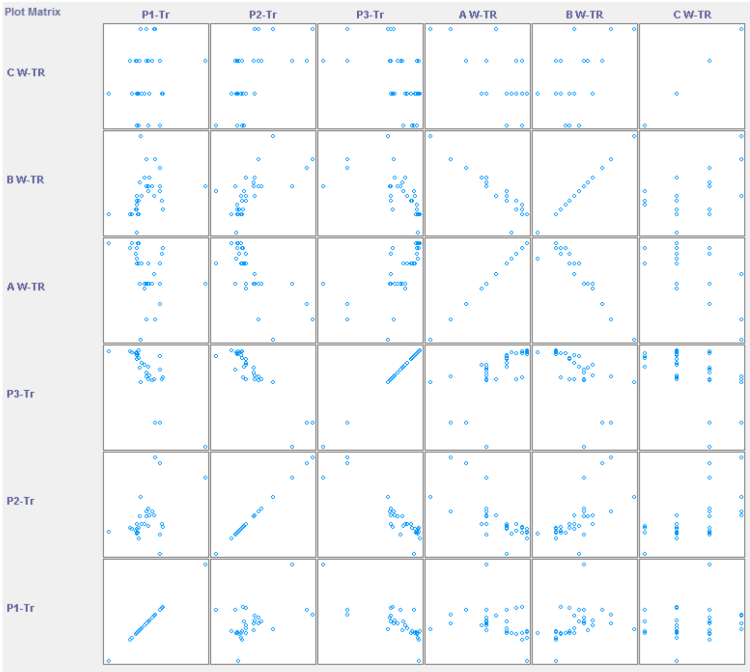

Figure 7.8. *Visualization of weights and output of the traditional model*

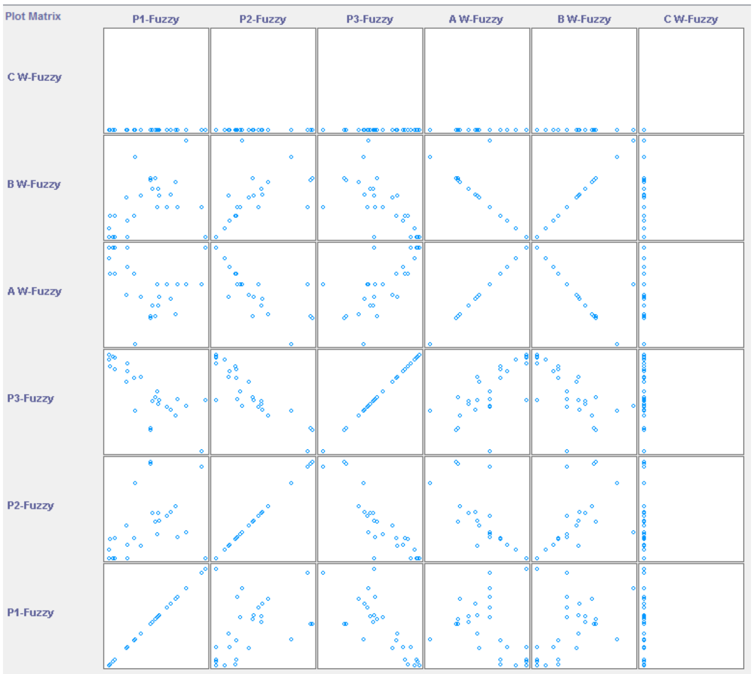

Figure 7.9 *Visualization of weights and output of the fuzzy model*

It is evident that by introducing the fuzzy method, more sparse results are obtained, resulting in better discrimination between products. The average value obtained by the fuzzy AHP model was 32.67 ± 30.60 (mean ± standard deviation (SD)) as compared to that obtained by the traditional model, which was 33.09 ± 19.44. The benefit is achieved through the construction of fuzzy consistent judgment matrix during the fuzzy AHP process. At the same time, the fuzzy method also simplified the decision-making process by setting the very small and confounding values to zero. In our case, the CWT ranged from 0.03–0.06 with a mean of 0.045 ± 0.009 (percentage of SD: 20%), which is very small with a relatively large percentage of deviation as compared to the W_A (traditional) value of 0.63–0.82 with a mean of 0.764 ± 0.049 (6.4%). These findings show that although characteristic C was of little importance, it could result in great variation in the decision-making process. By eliminating CWF to zero, the model avoids disturbance from C and generates more sparse results.

7.5.3 Classification analysis between the weights of the two models

By building classifiers using one model's weight as an attribute and one of the other model's weights as a target, we could examine the extent to which the target is related to the attributes. The classifiers chosen are the most widely used, covering three categories of classifying methods: rule-based, function, and tree classification. The combined methodology is used to ensure that the potential correlation is examined through different approaches.

The attribute selection process computes which attribute(s) are most important in deciding the value of the target value[10]. The following five classifiers were used:

- Rules: M5 rules, conjunctive rule.
- Functions: linear regression, Gaussian processes.
- Tree: M5P.

Attribute selection was carried out through these methods:

- CFS subset evaluation method
- Exhaustive search

7.5.4 Fuzzy weights as a target

When fuzzy weights were used as the target, the traditional weights AWT, BWT, and CWT were used as the classification attributes. The results indicated that the values ware highly determined by the traditional weights values by the rule and tree classification methods. Performances of function classifiers are not very desirable. This reflects the non-linear and tree-like nature of the AHP model.

7.5.5 Traditional weights as a target

The results of attribute selection showed that the traditional weights were mostly determined by AWF values when AWT and CWT were computed, and were not correlated with all the fuzzy weights.

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[10] All methods used in this study were obtained from the WEKA library (see http://www.cs.waikato.ac.nz/ml/weka/).

7.5.6 Final findings

On the basis of our results, it can be said that AWF, BWF, and CWF depended on AWT, BWT, and CWT but not vice versa. This means that we can get approximate AWF, BWF, and CWF values if AWT, BWT, and CWT are known.

7.6 DISCUSSION

Our study revealed that there were no significant differences between the mean values generated by both models; therefore, using group decision making will result in similar results regardless of the model employed. The difference is clear for individual decision making. The results of the fuzzy AHP model were correlated with those of the AHP model, but not vice versa. Further research will help identify the correlation of the traditional AHP model with the fuzzy AHP model. Small weights tend to zero and higher weights tend to one in the fuzzy AHP model.[11]

The pairwise weight values of fuzzy AHP increased when those of AHP increased; but the incremental rate in the former is greater, indicating that the larger the pairwise weight values, the larger the differences between the two approaches will be.

For fuzzy AHP, the pairwise weight values decrease when the pairwise weight values of AHP decrease; but the rate of decrease is greater, indicating that the smaller the pairwise weight values, the larger the differences between the two approaches will be. In conclusion, the fuzzy AHP approach produces the weight for each pair of criteria with the same tendency as the classic AHP approach, but with wide differences in the

[11] Same finding is found by Wang et al. (2008)

values. This outcome is attributed to the uncertainty of evaluators' preferences on the criteria. The uncertainty results in the span of fuzzy pairwise comparison numbers which balances the weights of each criterion. Our results of zero weights are similar to those of Wang et al. (2008) and Zhu et al. (2012).

CHAPTER 8

Proposing an Expert System for Product Selection Using Fuzzy AHP and CBR-based Approach

8.1 INTRODUCTION

Due to the worldwide accessibility of the Internet, the importance of electronic commerce is increasing (Schmitt and Bergmann ,1999). In general, there are three key sub-areas under e-commerce, namely, pre-sales, sales, and after-sales. Pre-sales is defined as the provision of information about services or products to consumers. Sales is defined as the negotiation process of these products and services along with the actual process of selling. After-sales is defined as the support offered to consumers regarding problems encountered while using the products.

Expert systems plays a key role in the provision of intelligent support in all of the abovementioned three phases. In the pre-sales phase, expert systems can be induced to provide an intelligent user interface so that it can incorporate consumer needs. In practical terms, this means that if an e-commerce application involves a limited number of products, the products can be placed under different categories by utilizing different structures to display each product on a web page under different groups. However, if a large number of products are present, a specific query-based interface would be required to aid the consumers to find their preferred product according to their needs and the best available offer. This situation triggers three kinds of principle situations:

In the first situation, the consumer might be aware about the product and know the product reference in the database well, subject to its availability. It would be the consumer’s luck to if he or she directly found the product. Second, the consumer may be aware about the product but is not aware about the precise name in the database, or the product might be out of stock. In the third situation, consumers are not aware of the products they are seeking. Their search is based on their needs, and they search for a complete or partial solution. Standard databases undergo specific problems for the two abovementioned situations. The consumer is either flooded with bulk offers or he/she leaves without arriving at any solution in the end (Schmitt and Bergmann ,1999)

It is obvious that none of these situations are satisfactory. A knowledge-based system is strongly required to inform the consumers about the alternatives along with information indicating the appropriateness of a product to consumer needs. Further, this knowledge needs to be compacted into the product database.

8.2 LITERATURE REVIEW

The problem of product selection is referred generally as a supplier evaluation and selection problem. This chapter presents a review of literature of case-based reasoning (CBR), analytic hierarchy process (AHP), and fuzzy AHP (FAHP) in supplier and product selection. The problem of supplier evaluation and selection has been studied extensively, and various decision-making approaches have been proposed to tackle it. In contemporary supply chain management, the performance of potential suppliers is evaluated against multiple criteria rather than considering cost as the sole factor.

Choosing the right supplier involves much more than scanning a price list, and the choices depend on a wide range of quantitative and qualitative factors. Extensive multi-criteria decision-making approaches have been proposed for supplier selection, such as AHP, FAHP, analytic network process (ANP), CBR, data envelopment analysis (DEA), fuzzy set theory, genetic algorithm (GA), mathematical programming, simple multi-attribute rating technique (SMART), and their hybrids. In this study, we are interested in the CBR, AHP, and FAHP approaches of decision making.

This chapter is based on the survey conducted by Ho et al. (2010). Ho et al. (2010) presented a literature review of the multi-criteria decision-making approaches for supplier evaluation and selection based on articles appearing in international journals between 2000 and 2008. The review covered the abovementioned approaches.

8.2.1 Case-Based Reasoning

Many papers have introduced the CBR technique for supplier selection. Choy and Lee (2002) presented a generic model using the CBR technique for supplier selection. The various evaluating criteria were grouped into three categories: technical capability, quality system, and organizational profile. The model was implemented in a consumer product manufacturing company, which had stored the performance of past suppliers and their attributes in a database. The proposed model would then retrieve or select a supplier who met the specification predefined by the company most. The CBR-based model was applied in studies by Choy et al. (2002), Choy and Lee (2003), and Choy et al. (2003a, 2003b, 2004, 2005) to aid decision makers in supplier selection. The approach, was very similar to that proposed in Choy and Lee (2002). In addition, the model was deployed to the same company.

Schmitt and Bergmann (1999) proposed the application of CBR advancement to customisation and product selection in environments based on electronic commerce. Lin et al. (2010) proposed a selection strategy for the system of product services. Ricci and Werthner (2002) adapted the case-based query system to apply recommendations for travel planning, and further utilised the CBR system to serve as a modified recommendation system based on the experience of previous systems. In addition to this, the query enhancement technique is used to further aid the moulding of queries based on available data for specific product catalogues. Kumar and Viswanadham (2007) proposed a specific framework for DSS based on CBR advancement, which offers support for decision makers in interceptive as well as preventive constructive risk management for supply chains.

8.2.2 AHP

Many papers have proposed the use of AHP to deal with the supplier selection problem. Nydick and Hill (1992) and Barbarosoglu and Yazgacß (1997) used it to structure vendor selection according to Narasimhan (1983). They formalised the tradeoffs between the conflicting selection criteria, which are weighed according to the importance attached to them by several specialists from different sub-fields in the company.

Akarte et al. (2001) developed a web-based AHP system to evaluate the casting suppliers on the basis of 18 criteria. Suppliers had to register in the system, and then input their casting specifications. To evaluate the suppliers, buyers had to determine the relative importance of each criterion on the basis of the casting specifications, and then assign a performance rating for each criterion using a pairwise comparison.

Muralidharan et al. (2002) proposed a five-step AHP-based model to aid decision makers in rating and selecting suppliers on nine criteria. People from different functions of the company, such as purchasing, stores, and quality control, were involved in the selection process.

Chan (2003) developed an interactive selection model using AHP to facilitate decision makers in selecting suppliers. The model was called ‘interactive’ because it incorporated a method termed ‘chain of interaction’, which was deployed to determine the relative importance of evaluating criteria without subjective human judgment. AHP was only applied to generate the overall score for alternative suppliers based on the relative importance ratings.

Chan and Chan (2004) applied AHP for supplier evaluation and selection. The AHP hierarchy consists of six evaluating criteria and 2 sub-factors, of which the relative importance ratings were computed on the basis of customer requirements.

Liu and Hai (2005) applied AHP to evaluate and select suppliers. Similar to Chan (2003), the authors did not apply the pairwise comparison of AHP to determine the relative importance ratings among the criteria and sub-factors. Instead, the authors used Noguchi’s voting and ranking method, which allowed every manager to vote or determine the order of criteria instead of the weights.

Chan et al. (2007) developed an AHP-based decision making approach to solve the supplier selection problem. Potential suppliers were evaluated on the basis of 14 criteria. A sensitivity analysis was performed using ‘Expert Choice’ software to examine the response of alternatives when the relative importance rating of each criterion was changed.

Hou and Su (2007) developed an AHP-based decision support system for the supplier selection problem in a mass customization environment. Factors from external and internal influences were considered to meet the needs of markets within the global changing environment.

8.2.3 Fuzzy Set

Some studies utilized the fuzzy set theory in the supplier selection process. Chen et al. (2006) presented a hierarchy model based on the fuzzy set theory to deal with the supplier selection problem. The linguistic values were used to assess the ratings and weights for the supplier evaluating factors. These linguistic ratings could be expressed in trapezoidal or triangular fuzzy numbers. The proposed model was capable of dealing with both quantitative and qualitative criteria.

Sarkar and Mohapatra (2006) suggested that performance and capability were two major measures in the supplier evaluation and selection problem. The authors used the fuzzy set approach to account for the imprecision involved in numerous subjective characteristics of suppliers. A hypothetical case was adopted to illustrate how the two best suppliers were selected with respect to four performance-based and ten capability-based factors.

Florez-Lopez (2007) picked up the 14 most important evaluating factors among a set of 84 potential added-value attributes, which were based on questionnaire responses from US purchasing managers. To better represent suppliers’ ability to create value for the customers, a two-tuple fuzzy linguistic model was employed to combine both numerical and linguistic information. Further, the proposed model could generate a graphical view to illustrate the relative suitability of suppliers and to identify strategic groups of suppliers.

8.2.4 Integrated fuzzy sets and AHP

Many papers proposed integrated fuzzy approaches to deal with the supplier evaluation and selection problem.

Kahraman et al. (2003) applied a fuzzy AHP approach to select the best supplier in a Turkish white goods manufacturing company. Decision makers used linguistic variables to specify preferences about the importance of each criterion.

Chan and Kumar (2007) also used a fuzzy AHP for supplier selection, similar to the study of Kahraman et al. (2003). In this approach, triangular fuzzy numbers and fuzzy synthetic extent analysis was used to represent decision makers' comparison judgment and decide the final priority of different criteria.

8.3 MOTIVATION

As described in previous literature review, there have been many papers proposing CBR in decision support systems as well as a lot of papers that introduce AHP and FAHP for solving multi-criteria problems. On the other hand, only a limited amount of papers have suggested combining them together in one system; for example, Zhenhui *et al.* (2010) presented a model for case retrieving based on AHP after the introduction of the basic principles and processes of CBR and AHP.

The main motivation behind our proposal is based on the results of the previous experiments. Experiment B showed that participants don't reflect the given table in the same ways using the AHP approach. For example, Table 8.1, which is given at the start of Experiment B as described before, is transformed by participants in many ways.

Table 8.1. *Preferences and the degree of support of each product*

	A	B	B
P1	7	5	0
P2	5	10	5
P3	10	1	0
R	10	5	0

Applying the AHP approach in a pairwise comparison and FAHP model generated many different results. Figure 8.1 shows that the results of the model are not equal for all participants even if the table is the same for all of them.

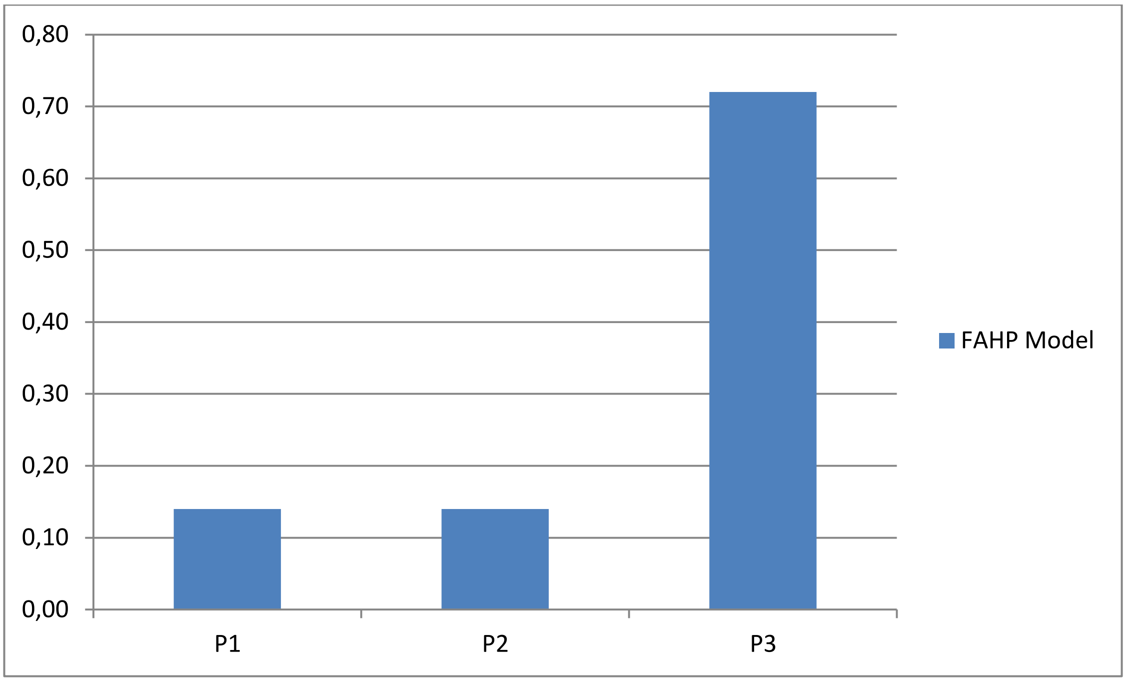

Figure 8.1. *FAHP results (Experiment B)*

This difference is because the comparison process is not the same for all participants, and the priority is relatively different from one participant to another. Pairwise comparison required each participant to compare his/her preference as well as each of the products according to each preference as shown in Table 8.2. For example, Table

8.2 shows a comparison of the relative importance between characteristic A and characteristic B.

Table 8.2. *The relative importance between characteristic A and characteristic B*

A								vs.								B
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

For example, the same given preferences in Experiment B are as follows:

1. The characteristic A was essential; thus, it had 100% importance.
2. The characteristic B was desirable; thus, it had 50% importance.
3. The characteristic C was not important; thus, it had 0% importance.

The mentioned preferences have been transformed into table form as seen in Table 8.3.

Table 8.3. *The given preferences of the participants*

	A	B	B
R	10	5	0

Through the experiment, we received different inputs as follows:

- Characteristic B is moderately important (9) in comparison with characteristic A.
- Characteristic B is moderately important (7) in comparison with characteristic A.
- Characteristic B is moderately important (5) in comparison with characteristic A.

It is evident that each participant has his/her own priority generator, so it is not realistic to use existing similarity measures, which would result in the same results for all users. The AHP approach helps users in the decision process. On the other hand, it is quite impossible for users to apply the AHP approach for a large number of products. For example, a user should do twelve comparisons for three products. The key core of our proposal is merging the benefits of the CBR and the AHP approach in proposing a new informatics decision support system.

8.4 THE MODEL

The model is based on our previous model which was Applying Case-Based Reasoning in decision support systems for Software product selection (Amroush,2012, Amroush and Alkhoder, 2011)[12]. The key principle of our new proposal is dividing the decision process into two main stages: the filtering stage, which filters product pools according to extreme importance preferences (CBR techniques are used in this step), and a pairwise stage using the AHP approach and applying FAHP to the three highest scoring products, which were found during the first stage. In the second stage, the user follows the AHP approach in pairwise between his/her preferences and the product, and FAHP is applied.

8.4.1 The Filtering Stage

In this stage, a knowledge base should be built that involves all of the products. This stage involves using the CBR technique to calculate the similarities between products and the user's preferences. The proposed model determine the user's preferences using

[12] The previous papers are: proposing a similarity measure in Case Based Reasoning for products selection, Experimental evidence, puplished in The Proceedings of the 4th International Conference on Agents and Artificial Intelligence,(ICAART2012) and Applying Case-Based Reasoning in Decision support systems for Software products Selection. The Proceedings of the 1st International Conference on Information Systems and Technologies (ICIST'2011).

a series of questions similar to the expert system. This stage contains two phases: building the knowledge base and the evaluation process through an expert system.

This stage also aims to help the user to find a product that meets his/her preferences; therefore, the user, at a certain question, can select only the answers existing in the knowledge base. Association rules can be applied to find the relationship between the questions and the answers; for example, using FP-Tree with confidence equal to 100% would be helpful to avoid asking unnecessary question. Therefore, the knowledge base represents only the available features of products, and the system also eliminates a question in case all of the products in the knowledge base at a certain question have the same features. In such a case, the question would not be asked.

The decision-making process in the first stage is done by asking the user to answer the same questions. This sets the importance of each question, and the products can be evaluated depending on the importance of each question for the user and the availability of the product. This process continues, step by step, until the end of the questions.

The Knowledge Base

The knowledge base is built by answering a number of questions. These questions regard product features. The questions involve scenarios in the decision process for a product. We can obtain suggested questions through the help of an expert or by collecting data through questionnaires. These questions could be the same questions that participants will answer in order to add a product to the knowledge base.

Example

Let us assume the problem of selecting a tablet according to a specific set of preferences; therefore, the user needs to select a tablet among many available tablets. The knowledge base could contain the following points[13]:

Screen size

- Operating system
- Brand name
- The price
- The hard drive size

When adding a product, a number of options are set as suggestions for the user. These options include answers from the previous points. Referring back to our example regarding the decision support system for selecting a tablet, the series of questions could be as follows:

- What is the screen size of the product?
- What operating system do you prefer for the tablet?
- What is the cost you are willing pay?

The Evaluation Process

The user will answer and assign importance to each question indicating how important that question is for him/her. The more important a question is for the user, the higher the evaluation of the product. Consequently, products are evaluated according to their support for this feature and how important it is for the user; therefore, the degree of

[13] The system should have the ability for users to add new features later to update the knowledge base.

matching will vary from one question to another until the final question. So in the case that the percentage of importance is 100%, this means that the user wants the product, which includes this feature precisely. On the other hand, if the percentage was less than this, then this means that there is no objection concerning the presence of another product that partially supports this feature. Products are evaluated according to their support for this feature, and consequently, the degree of evaluation will vary from one question to another until the final question. Back to our example, the series of the questions could be as follows:

Question 1

What screen size do you prefer?

- 7 inches
- 8 inches
- 10 inches

Identify the importance of the question:

Important Desirable Not Important

Question 2

What tablet operating system do you prefer?

- Android
- Windows
- Other

Identify the importance of the question:

Important Desirable Not Important

Question 3

What is the cost you are willing to pay?

- Less than 100 Euros
- Between 100-250 Euros
- Between 250-500 Euros
- More than 500 Euros

Identify the importance of the question:

Important Desirable Not Important

8.4.2 FAHP Stage

The first stage is not enough to find the best product. In the FAHP stage, the user determines his/her most important preferences (for example, three), and then follows the AHP approach for the best three products which were found in the first stage. This stage is important because as we have seen in Experiment B, individuals understand the same information in different ways; for example, if we have tablets A and B, and tablet A has the Android 2.3 operating system and tablet B has the Android 4.0 system, the comparison between A and B will vary from one user to another. In such case, someone may say tablet B is extremely better (9 times) than tablet A, and someone else may disagree and say that tablet B is better (3 times) than tablet A.

This stage is important in case there is not a product that completely matches the user's preferences. If there is a 100% product match, then there is no need for this stage. Referring back to our previous example, let us assume the highest scores in the first stage results were for tablets A, B, and C and the most desirable preferences are price and screen size. The system would present the data for the specific products and their degree of support for each of the characteristics. Then, Table 8.4 would be presented to the user.

Table 8.4. *The data sheet of the best three tablets*

	Price	screen size	Brand
Tablet A	400 Euro	10.1	X
Tablet B	350 Euro	10.1	Y
Tablet C	300 Euro	7	Z

The user would now do a pairwise comparison as previously described in detail. Finally, multiplying the matrixes of the two stages will result in the final score and the best product.

8.5 FUTURE WORK

A prototype will be experimentally implemented and tested and investigated whether or not the subjects will follow the expert system or not. In addition, to achieve some other measures, such as risk aversion and cognitive abilities tests.

CHAPTER 9

Conclusion

This thesis presents three studies on product selection and investigates how people select products and the reasons they change their product choices. Two experiments were conducted to achieve this goal, and two different models were used during the investigations: case-based reasoning (CBR) and the fuzzy analytic hierarchy process (FAHP).

The first study was conducted to identify the effect gender has on product selection based on the two models. This experiment also evaluated the effect informatics advice from different decision support systems, CBR and FAHP, had on consumer choice. It found out that unlike women, men believed that other study participants will select an option other than the one they have opted for, and men had a tendency to change their product choices more often than the women. Also, the study participants followed the informatics advice regardless of the advice that was given. The AHP approach gave the study participants a deeper understanding of their needs, and it was more effective than the fixed advice used in the CBR approach, which depended on similarity measure alone. Finally, women were more precise than men in both assigning their preferences through AHP approach and applying AHP steps in rational and consistent way.

The second study statistically studied the difference between conventional AHP and FAHP using an experimental point of view. The study was based on the second experiment, which proposed an FAHP model for product evaluation. The study showed that both AHP and FAHP had similar results, but each participant's matching percentage of the products was different, and there was a difference in the matching percentage of the products between men and women. The range of the matching percentage of the FAHP results is [0.1] where of traditional AHP is [0.69]. Small matching percentage go to zero and high ones go to one in FAHP. There was no significant difference between means of both models; therefore, using group decision making will give the same results from both models.

The difference between AHP and FAHP was clear for individual decision making. The FAHP model results are related to the AHP model but not vice verse.

The third study discussed merging CBR and FAHP when an expert system for product selection is proposed. The motivation behind this model merging was that there was a clear difference in individual decision making based on the same information that was given to the participants. The second experiment showed that each participant has his/her own priority generator, so it is not realistic to use existing similarity measures, as in CBR, because this method would result in the same results for all users. The AHP approach helps users in their decision-making processes. The core of the proposal merged the benefits of the CBR and the AHP approaches to propose a new informatics decision support system. The proposal divided the decision process into two main stages: the filtering stage and the pairwise stage. The first stage filters product pools according to extreme importance preferences (CBR techniques are used in this step), and the second stage follows the AHP approach and applies FAHP to the three highest scoring products from the first stage.

CHAPTER 10

Appendix 1

Each experiment session started with participants receiving a sheet of instructions. Apart from the instructions, the experiment was explained to them in neutral language. The experiment was conducted in Spanish.

10.1 THE INSTRUCTION SHEET

Por favor, lee las instrucciones atentamente, y cuando finalices levanta la mano para empezar a responder en el ordenador. Tu participación se compensará con 5 Euros por acudir a tiempo y otros 20 Euros si aciertas en una determinada pregunta de las que te haremos más adelante. Por favor, no se lleve esta hoja tras finalizar el experimento.

Imagina que deseas comprar un producto con tres (3) características principales: A, B y C (por ejemplo: si se tratara de un pc, las características podrían ser la Memoria Ram, el Disco Duro y los Altavoces).

Tus preferencias sobre el nivel mínimo de cada característica que te gustaría que el producto tuviera, se pueden presentar de la siguiente manera:

Table 10.1. *Nivel Mínimo deseado para el producto*

Nivel Mínimo	Verbalmente
10/10	Imprescindible
5/10	Deseable
0/10	No es importante para mí

Ahora, la situación es:

Quieres comprar un producto con tres (3) características principales: A, B y C. Hay tres productos posibles: P1, P2, P3, de los cuales debes elegir solo uno.

Consideraremos que el Nivel Mínimo de cada característica que desearías que tu producto tuviera es el siguiente.

- La característica A es: Imprescindible para ti (10/10).
- La característica B es: Deseable para ti (5/10).
- La característica C es: No importante para ti (0/10).

Resumiendo, el Nivel Mínimo que desearías que tu producto tuviera de A, B y C podría ser resumido como:

Table 10.2. *el Nivel Mínimo de cada característica*

A	B	C
10	5	10

Ahora tienes que elegir el producto óptimo de acuerdo a las preferencias indicadas previamente. Tenemos los siguientes productos: P1, P2 y P3. Los cuales tienen los siguientes niveles de A, B y C:

Table 10.3. *el Nivel Mínimo de los productos*

	A	B	C
P1	7	5	0
P2	5	10	5
P3	10	1	0

10.2 RUNNING THE EXPERIMENT

The experiment was computerized using a web-based program, I have developed the program using PHP programming language.

Here is the screens appeared to the participants. All identities were anonymous.

Pantalla 1

Bienvenido ID: a1

Aviso importante: No debes hacer clic en ¨ir a la página interior¨

¿Cuál es el producto que seleccionarás para comprar, con las preferencias de la Tabla 1?

- ○ El producto P1.
- ○ El producto P2.
- ○ El producto P3.

Ha finalizado, por favor haga clic Siguiente.

Siguiente

Pantalla 2

Su selección anterior es

Aviso importante: No debes hacer clic en ¨ir a la página interior¨

¿Cuál crees que es el producto que **otras personas** -con las preferencias de la Tabla 1- seleccionarán para comprar?

- ○ El producto P1.
- ○ El producto P2.
- ○ El producto P3.

Ha finalizado, por favor haga clic en Siguiente.

Siguiente

Pantalla3

Aviso importante: No debes hacer clic en ¨ir a la página interior¨

Ahora observa la siguiente información.

Un Sistema Inteligente ha encontrado cuál es el producto óptimo para ti, utilizando una Función de Similitud Matemática para calcular la similitud entre tus preferencias -Tabla 1- y las características de los productos.

El Sistema informático ha encontrado los siguientes resultados:

- El Producto P1 tiene un porcentaje de correspondencia del **79%** y es la opción óptima para ti.
- El Producto P2 tiene un porcentaje de correspondencia del **66%**.
- El Producto P3 tiene un porcentaje de correspondencia del **73%**.

Tras conocer esta información adicional. ¿Cuál es ahora su decisión? (***su selección inicial no será tomada en cuenta a partir de ahora).***

- ○ El producto P1.
- ○ El producto P2.
- ○ El producto P3.

Ha finalizado, por favor haga clic en Siguiente.

Siguiente

Pantalla4

Aviso importante: No debes hacer clic en ¨ir a la página interior¨

Con la misma información proporcionada por el Sistema Inteligente, es decir:

- El Producto P1 tiene un porcentaje de correspondencia del **79**% y es la opción óptima para ti.
- El Producto P2 tiene un porcentaje de correspondencia del **66**%.
- El Producto P3 tiene un porcentaje de correspondencia del **73**%.

Si su elección final coincide con la elección mas común entre el resto de los participantes de éste experimento, obtendrá un premio de 10 Euros.

¿Cuál es su decisión final?

- ○ El producto P1.
- ○ El producto P2.
- ○ El producto P3.

Las selecciones que haya tomado previamente no se tendrán en cuenta a partir de éste momento.

Ha finalizado, por favor haga clic en Siguiente.

10.3 THEBOTTOM OF FORM

questionnaire

si no sigue el consejo del sistema informático, ¿Cuál es el porcentaje que da a su elección?

Por favor responda el siguiente cuestionario:

- Sexo : Hombre, Mujer.
- Edad.
- Tipo de estudio.

- Comienzo de pregrado estudios.
- Haga un comentario sobre tus decisiones.

CHAPTER 11

Appendix 2

Each experiment session started with participants receiving a sheet of instructions. Apart from the instructions, the experiment was explained to them in neutral language. The experiment was conducted in Spanish.

11.1 THE INSTRUCTION SHEET

Por favor, lee las instrucciones atentamente, y cuando finalices levanta la mano para empezar a responder en el ordenador. Tu participación se compensará con 5 Euros por acudir a tiempo y otros 20 Euros si aciertas en una determinada pregunta de las que te haremos más adelante. Por favor, no se lleve esta hoja tras finalizar el experimento.

Imagina que deseas comprar un producto con tres (3) características principales: A, B y C (por ejemplo: si se tratara de un pc, las características podrían ser la Memoria Ram, el Disco Duro y los Altavoces).

Tus preferencias sobre el nivel mínimo de cada característica que te gustaría que el producto tuviera, se pueden presentar de la siguiente manera:

Table 11.1. *Nivel Mínimo deseado para el producto*

Nivel Mínimo	Verbalmente
10/10	Imprescindible
5/10	Deseable
0/10	No es importante para mí

Ahora, la situación es:

Quieres comprar un producto con tres (3) características principales: A, B y C. Hay tres productos posibles: P1, P2, P3, de los cuales debes elegir solo uno.

Consideraremos que el Nivel Mínimo de cada característica que desearías que tu producto tuviera es el siguiente.

- La característica A es: Imprescindible para ti (10/10).
- La característica B es: Deseable para ti (5/10).
- La característica C es: No importante para ti (0/10).

Resumiendo, el Nivel Mínimo que desearías que tu producto tuviera de A, B y C podría ser resumido como:

Table 11.2. *el Nivel Mínimo de cada característica*

A	B	C
10	5	10

Ahora tienes que elegir el producto óptimo de acuerdo a las preferencias indicadas previamente. Tenemos los siguientes productos: P1, P2 y P3. Los cuales tienen los siguientes niveles de A, B y C:

Table 11.3. *el Nivel Mínimo de los productos*

	A	B	C
P1	7	5	0
P2	5	10	5
P3	10	1	0

11.2 RUNNING THE EXPERIMENT

The experiment was computerized using a web-based program, I have developed the program using PHP programming language.

Here is the screens appeared to the participants. All identities were anonymous.

Pantalla 1

Bienvenido ID:

Aviso importante: No debes hacer clic en ¨ir a la página interior¨

¿Cuál es el producto que seleccionarás para comprar, con las preferencias de la Tabla 1?

- El producto P1.
- El producto P2.
- El producto P3.

Ha finalizado, por favor haga clic Siguiente.

Siguiente

Pantalla 2

Su selección anterior es P2

Aviso importante: No debes hacer clic en ¨ir a la página interior¨

¿Cuál crees que es el producto que **otras personas** -con las preferencias de la Tabla 1- seleccionarán para comprar?

- El producto P1.
- El producto P2.
- El producto P3.

Ha finalizado, por favor haga clic en Siguiente.

Siguiente

Pantalla 3

Aviso importante: No debes hacer clic en -ir a la pagina aanterior-

Por favor, lea cuidadosamente:

Téngase en cuenta la siguiente escala: 1: igual importancia; 3: Importancia moderada; 5: Más importancia; 7: Mucha más importancia; 9: Extremadamente más importancia. 2,4,6 y 8 son valores intermedios entre cada una de las opciones anteriores.

→ <u>Ejemplo 1</u> de aplicación de la escala: Comparación de importancia relativa entre una característica cualquiera A vs. una característica cualquiera B

Característica A								vs.								Característica B
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	●	○	○	○	○	○	○

Según la preferencia indicada, la característica B es moderadamente más importante (3) que la característica A de cara a conseguir el objetivo indicado.

→ <u>Ejemplo 2</u> de aplicación de la escala: Comparación de importancia relativa de la característica A vs. característica B

Característica A								vs.								Característica B
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	●	○	○	○	○	○	○	○	○	○	○	○	○

Según la preferencia indicada, la característica A es más importante (5) que la característica B de cara a conseguir el objetivo indicado.

→ Ejemplo 3 de aplicación de la escala: Comparación de importancia relativa de la característica A vs. característica B

Característica A								vs.								Característica B
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	●	○	○	○	○	○	○	○	○	○	○	○	○	○

También puede seleccionar un valor intermedio, la característica A es más importante que la característica B en una cantidad superior a 5 e inferior a 7 de cara a conseguir el objetivo indicado.

→ Ejemplo 4 de aplicación de la escala: Comparación de importancia relativa de la característica A vs. característica B

Característica A								vs.								Característica B
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	●	○	○	○	○	○	○	○	○

Según la preferencia indicada, la característica A tiene la misma importancia (1) que la característica B de cara a conseguir el objetivo indicado.

Si tiene alguna pregunta, por favor, levante su mano, si no haga clic en Siguiente

Siguiente

Pantalla 4

Aviso importante: No debes hacer clic en -ir a la pagina anterior-

Ahora, por favor responda a estas preguntas.

Considera que las tres (3) características expuestas tienen la misma importancia relativa de cara a la elección del mejor producto.

<u>BLOQUE I.</u>

Si tuviera que comparar las características expuestas entre sí, en relación a su importancia relativa de cara a la consecución del objetivo indicado.

En su opinión ¿qué característica es más importante?

Téngase en cuenta la siguiente escala: 1: igual importancia; 3: Importancia moderada; 5:más importancia; 7: Mucha más importancia; 9: Extremadamente más importancia. 2,4,6 y 8 son valores intermedios entre cada una de las opciones anteriores.

La característica C						vs.						La característica B				
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

La característica C						vs.						La característica A				
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

La característica B						vs.						La característica A				
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

BLOQUE II:

En este bloque se trata de comparar productos entre sí. Téngase en cuenta la siguiente escala: 1: igual; 3: moderadamente mejor; 5: mejor; 7: Mucho mejor; 9: Extremadamente mejor.2,4,6 y 8 son valores intermedios entre cada una de las opciones anteriores.

→ Ejemplo de aplicación de la escala: Comparación relativa entre el producto A y el producto B, con respecto a una característica X.

Producto B					vs.						Producto A					
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	●

Según la preferencia indicada, el producto B es extremadamente mejor (9) que el producto A, de acuerdo a la característica X.

BLOQUE III (Con respecto a la característica A)

En su opinión, ¿qué producto es el mejor teniendo en cuenta la característica A?

			P_3					vs.						P_1		
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

			P_3					vs.						P_1		
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

			P_3					vs.						P_2		
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

BLOQUE III (Con respecto a la característica B)

En este bloque se trata, de nuevo, de comparar productos entre sí. En su opinión, ¿qué producto es el mejor teniendo en cuenta la característica B?

P_3								vs.						P_2		
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

P_3								vs.						P_1		
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

P_2								vs.						P_1		
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

BLOQUE IV (Con respecto a la característica C)

En este bloque se trata, de nuevo, de comparar productos entre sí. En su opinión, ¿qué producto es el mejor teniendo en cuenta la característica C?

P_2								vs.						P_1		
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

P_3								vs.						P_1		
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

P_3								vs.						P_2		
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

Pantalla 5

Aviso importante: No debes hacer clic en ¨ir a la página interior¨

Ahora observa la siguiente información.

Un Sistema Inteligente ha encontrado cuál es el producto óptimo para ti, utilizando una Función de Similitud Matemática para calcular la similitud entre tus preferencias -Tabla 1- y las características de los productos.

El Sistema informático ha encontrado los siguientes resultados:

- El Producto P1 tiene un porcentaje de correspondencia del % .
- El Producto P2 tiene un porcentaje de correspondencia del %.
- El Producto P3 tiene un porcentaje de correspondencia del %.

Tras conocer esta información adicional. ¿Cuál es ahora su decisión? (su selección inicial no será tomada en cuenta a partir de ahora).

○ El producto P1.
○ El producto P2.
○ El producto P3.

Ha finalizado, por favor haga clic en Siguiente.

Siguiente

Pantalla 6

Aviso importante: No debes hacer clic en -ir a la página anterior-

Con la misma información proporcionada por el Sistema Inteligente, es decir:

- El Producto P1 tiene un porcentaje de correspondencia del % .
- El Producto P2 tiene un porcentaje de correspondencia del %.
- El Producto P3 tiene un porcentaje de correspondencia del %.

Si su elección final coincide con la selección más común entre el resto de los participantes de este experimento, obtendrá un premio de 10 Euros.

¿Cuál es su decisión final?

- ○ El producto P1.
- ○ El producto P2.
- ○ El producto P3.

Las selecciones que haya tomado previamente no se tendrán en cuenta a partir de éste momento.

Ha finalizado, por favor haga clic en Siguiente.

Final

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