EXTENDING THE COORDINATION OF COGNITIVE AND SOCIAL PERSPECTIVES

Martin A. Simon

Cognitive analyses are typically used to study individuals, whereas social analyses are typically used to study groups. In this article, I make a distinction between what one is looking with—one's theoretical lens—and what one is looking at—e.g., an individual or a group—. By emphasizing the former, I discuss social analyses of individuals and cognitive analyses of groups, additional analyses that can enhance mathematics education research. I give examples of each and raise questions about the appropriateness of such analyses.

Keywords: Constructivism; Emergent perspective; Learning theory; Research methodology; Social theory

Extensión de la coordinación de las perspectivas cognitiva y social

Los análisis cognitivos se usan típicamente para estudiar individuos mientras que los análisis sociales se usan normalmente para estudiar grupos. En este artículo, distingo entre lo que se usa para mirar —el lente teórico propio— y lo que se mira —por ejemplo, un individuo o un grupo—. Enfatizando lo primero, discuto los análisis sociales de individuos y los análisis cognitivos de grupos sociales, análisis adicionales que pueden enriquecer la investigación en educación matemática. Presento ejemplos de cada uno de ellos y planteo preguntas respecto de la conveniencia de estos análisis.

Términos clave: Constructivismo; Metodología de investigación; Perspectiva emergente; Teoría del aprendizaje; Teoría social

In recent years, there have been a number of learning theories available in mathematics education (Lerman & Tsatsaroni, 2004). The advantages of multiple theories in mathematics education research can be realized through a diverse set of research projects generated and structured using different theories. These advantages can also be realized through coordination of two or more theoretical perspectives in a single project when appropriate (Cobb & Yackel, 1996). In the

Simon, M. A. (2012). Extending the coordination of cognitive and social perspectives. *PNA*, *6*(2), 43-49.

case of cognitive[1] and social theories, researchers have pointed to the complementarity of the theories (e.g., Cole & Wertsch, 1996; Kieren, 2000; Niss, 2006) and argued that each theory has its affordances and limitations that make it the tool of choice for some kinds of work and less useful for others (Simon, 2009). In this article, I present a perspective on the domain of utility of cognitive and social perspectives that has emerged in my research projects and contrast it with a well-known existing theory. My hope is that this article will spark ongoing conversation about the issues raised and stimulate efforts to refine the ideas presented.

One of the better-developed examples of combining two theoretical approaches is the emergent perspective (cf., Cobb & Bauersfeld, 1995; Cobb & Yackel, 1996). The theory is based on the coordination of a cognitive and a social perspective. It was developed for the purpose of characterizing mathematics learning in classroom settings. A social perspective is used for characterizing learning when the unit of analysis is the class—including the teacher—. A cognitive perspective is used when the unit of analysis is individual students. The use of different theoretical tools for analysis of these different units of analysis gives the theory a certain elegance and clarity. The analysis of classroom observations is done by looking at emerging norms—social and sociomathematical—and the identification of a sequence of mathematical practices developed over time. These analyses are coordinated with analyses of data from interviews with individual students. The interview data are analysed using a constructivist perspective, identifying the students' conceptions at different points in the design experiments (Cobb, 2003).

At a certain point in the evolution of our research program, my colleagues and I attempted to use the emergent perspective for careful examination of learning in classrooms, but found that, without modification, it was inadequate for our purposes. The problem was the following. In the work of Cobb and colleagues (Bowers, Cobb, & McClain, 1999) the characterization of learning in a mathematics class results in postulating a set of mathematical practices. Typically, learning over the course of an academic year is characterized by a small set of mathematical practices. These practices, which may take weeks or months to develop, are too gross a tool for the detailed work we were attempting to do. We were interested in understanding the process that takes place as students move from one mathematical practice to the next. What theoretical tools could we use for that purpose?

[1] In earlier work researchers have used *psychological* where I use *cognitive* and *sociological* where I use *social*. My choice to use the terms cognitive and social reflects my understanding that both categories of analysis involve psychological phenomena and that sociological is too limited. The first category is meant to reflect a cognitive psychological perspective, whereas the second category encompasses social psychological, sociological, sociocultural, and anthropological perspectives.

In bringing theory to bear on this problem, we began to think about theories of learning using the following distinction. We distinguished what we are looking *with* from what we are looking *at.* This distinction allowed us to go beyond the use of social theories for studying classroom data and cognitive theories for studying individual data. Rather we thought about our work using the 2×2 matrix in Table 1.

Table 1
Analyses by Nature and Subject of the Analysis

Cognitive analysis	Social analysis
Individual	
Cognitive analysis of individual	Social analysis of individual
Group	
Cognitive analysis of group	Social analysis of group

The upper left quadrant (cognitive analysis of individual) and the lower right quadrant (social analysis of group) require little elaboration. They are characteristic of the emergent perspective as well as many other research programs that fall into one of these quadrants or both. That is, it is commonplace for researchers to conduct cognitive analyses—e.g., constructivism—of individuals' mathematical thinking and social analyses of mathematical communication in small groups and whole-class discussion—e.g., sociocultural theory, symbolic interactionism—. However, the upper right and lower left quadrants merit discussion. Do these quadrants represent valid types of analysis? Let us consider each in turn.

SOCIAL ANALYSIS OF AN INDIVIDUAL

This quadrant represents work that sociocultural theory has been doing for some time, social analysis of an individual—e.g., a student working on a problem in isolation—. In analyses characterized by this quadrant, the researcher considers that the activity of a child working by herself is influenced by the norms and practices of her mathematics class, the language that she speaks, the cultural practices of her family, etc. Such social explanations can be useful in understanding aspects of the activity and learning of the individual.

One might argue against such an analysis by claiming that no matter what influences the social environment exerts on the individual, the results of those influences are reflected in the individual's cognition. However, accepting this claim as valid does not negate the value of a social analysis of an individual engaged with a mathematical task. Bringing a social lens to bear on the data means that those data are likely to be considered, using a set of constructs that are characteristic of work done from that social theoretical perspective. As a result aspects of the data will receive attention that might not be focused on from a cognitive perspective. Again, this point is based on the distinction between looking with and looking at.

Let me consider an example. A student is working alone on the following question provided by the researchers: "What is the purpose of the multiplication step in the traditional long division algorithm?"[2] Posing the question might be intended to find out whether the student understands the steps of the algorithm. Evidence of such understanding might be an answer of "to determine how many of the original set of items have already been put into groups." Instead, the student responds, "to find out whether the number that you put up top [in the quotient] is too large." Rather than making a cognitive interpretation of these data—a conjecture about the student's understanding of division—, researchers might consider a social explanation. They might conjecture that the student's response reflects his participation in a mathematics class in which the procedural role of algorithmic steps is emphasized—valued—. As a result of that participation, the student interprets the question as pertaining to the role of the multiplication step in obtaining a correct answer. Without looking with a social perspective, this interpretation of the data might not be considered.

Cognitive Analysis of a Group

Returning to our 2×2 matrix (Table 1), the lower left quadrant, using a cognitive lens to focus on a group, is likely to be the most controversial. Whereas, for the upper right quadrant, readers might readily accept that there is always a larger social frame for individual thought and action, applying a cognitive lens when looking at a group may seem less appropriate. However, the argument for cognitive analysis of group activity, including discourse, is parallel to the one made for social analysis of individual action; it uses useful knowledge and constructs to expand what is noticed—what is identified as relevant data—and to generate useful explanations for the data. In attempting to do detailed analysis of learning in classrooms, we began to do analyses of this type.

As indicated earlier, social analysis of classroom learning using the theoretical construct of mathematical practices did not allow for the detailed distinctions that we needed to make in our work. However, a vast knowledge base of distinctions about student conceptions exists in the context of cognitively oriented theories. We continued to use the social aspect of the emergent perspective to analyze classroom norms, i.e., the conditions for learning in the classroom. However, we used constructivist analyses to understand the learning processes, making use of

[2] This refers to the algorithm most commonly used in the United States.

the rich empirical results of prior work on individuals' conceptions (cf., Simon & Blume, 1996; Simon, Tzur, Heinz, Kinzel, & Smith, 2000).

Whereas from a social perspective, a conversation might be seen as a negotiation of meaning or increasing participation in the practices of the group, it has proved advantageous to also view it as two—or more—students with different conceptions attempting to understand the ideas of the other(s). At other times, it has been helpful to characterize the current *conception of the class* in relation to the conceptual goal of the teacher. Let me consider an example.

In one of our classroom teaching experiments (Simon & Blume, 1994), students were asked to find the number of non-square, cardboard rectangles—of which they had a sample—that could fit on their rectangular table. They did so by using the rectangle to measure along the width and the length of the table and multiplying the two measurements. From an observer's perspective, the students were finding the area of the table measured in cardboard rectangles of the given size. The instructor then asked the students to consider a solution in which a hypothetical student measured the width and length of the table, each time using the long side of the non-square cardboard rectangle—a strategy that was inappropriate for the original task of finding the number of rectangles that could fit on the table—. The instructor asked about the meaning of the product of these measures. The consensus response of the students was that the number did not represent anything meaningful. The students' current conceptions and the possible learning process that the class might undergo were important to the researchers. Simon and Blume engaged in a cognitive analysis of the ensuing extended classroom discussion and Simon's subsequent interventions with the class. Using these and other related data, the research team proceeded to develop a hypothesis regarding the students' understanding and its implications with respect to the development of multiplicative units.[3] The use of prior research on students' conceptions and the constructivist-based analysis of individual contributions to the conversation led to this hypothesis.

For example, consider the following claim from the study which built on Thompson's (1994) work on quantitative reasoning: "Our analysis has resulted in a hypothesis that the [learner's] anticipation of the structure of the quantified area (a rectangular array of equivalent units) is a first step in the quantitative reasoning involved in evaluating the area of a rectangle" (Simon & Blume, 1994, p. 492). The cognitive analysis of the data from the whole-class discussion afforded the kind of detailed analysis that fit our objectives. The use of constructs derived from cognitive theoretical research allowed us to make fine distinctions in student conceptions that were important in understanding aspects of the class discussion and the learning that ensued.

The analyses that we conduct that fit into this quadrant, while generative, lack some of the advantages of those that fit into the first and fourth quadrants.

[3] See Simon & Blume (1994) for a detailed analysis.

Our cognitive analyses of a class do not have the elegance of the emergent perspective, which uses different theoretical constructs for different units of analysis. The developers of the emergent analysis were careful to make sure that there was a fit between the analytical unit and the nature of the claim—e.g., a class unit can have an established mathematical practice, but not a concept—. Does the lack of this type of clarity make our extension of the use of constructivist theoretical constructs unwarranted? Or does the distinction between looking with and looking at offer additional benefits from significant areas of prior theoretical and empirical work?

CONCLUSION

I used the 2×2 matrix to highlight two types of analyses that are not typically discussed in mathematics education research, cognitive analysis of data involving a group and social analysis of data involving an individual acting alone. The argument for such analyses is based on the notion that cognitive and social refer to the theoretical constructs that the researchers use to structure their observations—identification of relevant data—and to account for those observations. This emphasis on what the researcher is looking with, as opposed to what the researcher is looking at, is aimed at maximizing the constructs available for data collection and analysis. Labinowicz (1985) pointed out: "We see what we understand" (p. 23). The attempts to expand theory use described above are aimed at using as much of our understanding as possible in analyzing mathematics learning situations. In both individual and group situations, cognitive and social constructs can provide tools for research analysis.

Between the two types of analyses on which I have focused, the social analysis of the individual is likely the easier sell. After all, even an interview of an individual is a social interaction. Further, it is easy to argue that even individual work exists in a social environment. Language, social practices, and social tools are implicated in the work of the individual. On the other hand, using cognitive constructs to make sense of classroom (group) data is more dependent on the acceptance of the looking-with-looking-at distinction.

REFERENCES

Bowers, J. S., Cobb, P., & McClain, K. (1999). The evolution of mathematical practices: a case study. *Cognition and Instruction, 17*, 25-64.

Cobb, P. (2003). Investigating students' reasoning about linear measurement as a paradigm case of design research. In M. Stephan, J. Bowers, & P. Cobb (Eds.), *Supporting students' development of measuring conceptions: Analyzing students' learning in social context. Journal for Research in Mathematics Education Monograph No. 12* (pp. 1-16). Reston, VA: NCTM.

Cobb, P., & Bauersfeld, H. (Eds.). (1995). *The emergence of mathematical meaning: interaction in classroom cultures*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Journal of Educational Psychology, 31*(3/4), 175-190.

Cole, M., & Wertsch, J. V. (1996). Beyond the individual-social antimony in discussions of Piaget and Vygotsky. *Human Development, 39*, 250-256.

Kieren, T. (2000). Dichotomies or binoculars: Reflections on the papers by Steffe and Thompson and by Lerman. *Journal for Research in Mathematics Education, 31*(2), 228-233.

Labinowicz, E. (1985). *Learning from children: new beginnings for teaching numerical thinking*. Menlo Park, CA: Addison-Wesley.

Lerman, S., & Tsatsaroni, A. (2004). *Surveying the field of mathematics education research*. Paper presented at the International Congress on Mathematics Education 10, Copenhagen, Denmark.

Niss, M. (2006). *The concept and role of theory in mathematics education*. Paper presented at the Nordic Conference on Mathematics Education, Trondheim, Norway. Retrieved August 30, 2007, from http://www.icme10.dk

Simon, M. A. (2009). Amidst multiple theories of learning in mathematics education. *Journal for Research in Mathematics Education, 40*(5), 477-490.

Simon, M. A., & Blume, G. (1994). Building and understanding multiplicative relationships: a study of prospective elementary teachers. *Journal for Research in Mathematics Education, 25*(5), 472-494.

Simon, M. A., & Blume, G. (1996). Justification in the mathematics classroom: a study of prospective elementary teachers. *Journal of Mathematical Behavior, 15*(1), 3-31.

Simon, M. A., Tzur, R., Heinz, K., Kinzel, M., & Smith, M. (2000). Characterizing a perspective underlying the practice of mathematics teachers in transition. *Journal for Research in Mathematics Education, 31*(5), 579-601.

Thompson, P. W. (1994). The development of the concept of speed and its relationship to concepts of rate. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 181-236). Albany, NY: SUNY Press.

Martin A. Simon
Pennsylvania State University
msimon@nyu.edu