Implementing direct democracy via representation

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ABSTRACT

Today, several social movements in western democracies argue that traditional representative democracy has failed to adequately represent the will of the "people", and instead support direct democracy as the only political system to restore the will of the majority. We analyze under what conditions the policy – a vector of decisions on every issue – implemented by the winner of a bipartisan electoral competition coincides with the policy that citizens would choose by means of direct democracy. We find necessary and sufficient conditions for this equivalence to hold, implying that, as long as at least one of them is not fulfilled, a divergence of outcomes between direct and representative democracy arises. The first condition requires that the outcome of majority voting issue-by-issue is the Condorcet winner relative to the voters' preference profile over the set of policies. The second requires that either that outcome is the preferred policy for at least one of the candidates, or that candidates' preferred policies differ on every single issue. We reinterpret some findings in the literature in the light of our model and present them as potential reasons why the equivalence between direct and representative democracy may fail.

1. Introduction

A significant phenomenon in current society is the emergence of social movements with a rhetoric that is against representative democracy. These groups criticize systems based on the delegation of power, largely because of the risk that politicians could be self-seeking and behave against people's interest. They claim the need for the direct participation of citizens in the decision-making process to guarantee the representativeness of society's preferences in the electoral outcome. The recent success of this discourse means the emergence of political parties in favor of direct democracy has become a generalized fact in several European countries: *Freedom and Direct Democracy* in the Czech Republic, the *Five Star Movement* in Italy, *Podemos* in Spain, *Direktdemokraterna* in Sweden, and *Something New* in the United Kingdom are just some examples of parties demanding the right of people to use the instruments of direct democracy, such as referendums and popular consultations. These organizations defend the idea that citizens should be able to decide on each and every issue that comes up for discussion. When decisions about several issues must be made, these political parties demand that, for each of these issues, the desires of the majority should be carried out.

The claims of these movements about the need to shift towards a system of direct democracy to achieve such an outcome could be interpreted as a forewarning about the inability of representative democracy to implement what the majority of people desires for each specific issue. The purpose of this paper is to study under what conditions a system of representative democracy implements a policy – which in the model is a vector of decisions on every issue – such that it coincides with the majority preferences on each issue. Our analysis therefore simply echoes current public pressure in favor of the results from citizen participation and analyzes whether traditional representative democracy, the prevailing government system in most of the world, could lead to similar outcomes.[1]

Formally, we investigate whether there are necessary and sufficient conditions such that the decisions made on each issue in a representative democracy, which bundles issues together, coincide with the decisions that would have been chosen in a direct democracy by majority voting issue-by-issue. We propose a model in which there are a finite number of issues on which a binary decision must be made. We assume that there are no complementarities among issues, so the order in which decisions on different issues are made has no effect on the choice made for each issue. If the decisions among issues are linked, the comparison between the two systems would depend on the order

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[1] The question of whether or not a deviation between direct and representative democracy is socially desirable (on this see Madison, 1787; Sieyès, 1789; Kartik and McAfee, 2007; Morelli and Van Weelden, 2013, for opposing views) needs to take into account other factors, such as the cost of voting (see Leininger, 2015, for a reference on how referendums could lead to voter exhaustion and thus not really improve representation).

according to which they are chosen, so the comparison between both systems would become blurred.

In direct democracy, voters vote truthfully on each of the issues. Logrolling or coalition building among voters is not allowed in the model, so majority voting issue-by-issue is the outcome in this system. In representative democracy, we introduce a two-party system competition where the political platforms of these parties are known. It is common knowledge that candidates representing each of the two parties will have these political platforms as the most preferred set of decisions for the bundle of issues. Candidates care about both the implemented policy and being in office. Even though candidates' preferences are monotonically increasing in both policy and office, the model allows for candidates who differ in the importance each attaches to each of these aspects. This could inevitably lead to the existence of politicians whose motivations are very different to those of citizens, which is one of the main criticisms that systems of political representation receive from groups in favor of direct democracy. Considering this wide-ranging domain of candidates' preferences in our analysis, our findings for the coincidence of implemented policies in both systems hold even in the presence of politicians whose preferences are highly misaligned with those of the population. Candidates' preferences are explained in more detail later in the paper.

We find conditions guaranteeing that the equilibrium outcome of the electoral game induced by a representative democracy is unique and coincides with the policy yielded by majority voting issue-by-issue irrespective of candidates' preferences, that is, regardless of whether candidates are more *office-prioritizer* or more *policy-prioritizer*. These necessary and sufficient conditions impose restrictions over both the preference profile of voters and the political platforms of candidates. The first condition requires that the outcome of majority voting issue-by-issue is the Condorcet winner relative to the voters' preference profile over the set of policies. The second requires that, either that outcome is the preferred policy of at least one of the two candidates, or that candidates' preferred policies differ on every single issue.[2] The two conditions above taken together guarantee the existence and uniqueness of an electoral game equilibrium according to which both candidates propose the outcome of majority voting issue-by-issue. With commitment to the announced policy, this equilibrium implies that such a policy will be implemented by whoever wins the electoral competition. The reason why this equilibrium is unique, and coincides with the policy chosen in direct democracy, is that under these conditions any other policy – preferred by both candidates and against the interests of the majority – cannot be sustained as an equilibrium of the electoral game.

Knowing when the outcomes derived from both systems coincide, will ultimately give us an understanding of the reasons for the emergence of discontent among populations with representative democracies. The non-fulfillment of the conditions guaranteeing the equivalence between direct and representative democracy can be interpreted as a threat to the stability of representative democracies. Failure to obtain the majority preferred result for each issue through representative democracy is the perfect argument for those groups in favor of direct democracy to raise their voices against the representative system. In this paper, we also review findings in the literature and reinterpret them in the light of our model as potential reasons why the equivalence between direct and representative democracy may fail.

1.1. Related literature

An essential paper in understanding the addition of this work to the literature is Besley and Coate (2008). This is one of the main theoretical contributions to display the risk posed by representative democracy

of failing to represent the preferences of the majority. The authors show that the issue-bundling usually carried out in representative democracies is the reason why representative systems yield different outcomes compared to direct democratic institutions. In representative democracies, many policy issues are bundled together. Consequently, it is possible that, for each issue, the chosen policy is not the one preferred by the majority. The current work limits the importance of issue-bundling on divergence of policy outcomes between direct democracy and representative democracy. In particular, we identify several conditions that allow equivalence in terms of outcomes between representative democracy and direct democracy to exist, even when representative democratic institutions operates under issue-bundling. Our model thus provides evidence for the existence of factors that would contribute to moderating the role of issue-bundling in triggering divergence from majority desires.

Faced with the problem of lack of representativeness of citizen preferences posed by representative democracy, Besley and Coate (2008) explain how direct democracy can contribute to dealing with this obstacle. They show that allowing citizen-initiated referendums prevents policy outcomes on specific issues to differ significantly from what the majority desires. On a similar vein, Kessler (2005) proposes a model comparing direct and representative democracy to study the advantages of each of these regimes. Among her findings, the author highlights the greater alignment of policies with the preferences of the median voter when policies were chosen through direct democracy. Gerber (1996) offers empirical evidence to show that by providing citizens with the ability to participate in lawmaking, in particular through the popular initiative, reduces the discretionary performance of legislators. The result is a higher prevalence of the median voter's preferences when elected representatives choose the policies to be implemented. In this paper, we identify societies in which the lack of representativeness of citizen preferences is no longer a problem affecting the outcomes of representative democracy. Specifically, we find conditions under which representative democracy gives the same result as unbundling the issues and provides the outcome that the majority of voters desire. If these conditions hold, it would therefore not be necessary to resort to public participation instruments to overcome the problems of poor citizen representation that until now have been attributed to representative democracies.

There is in fact a strand in the literature that examines how the presence of the institutions of direct democracy may affect the behavior of politicians in representative democracies. Still, a consensus on the predominance of positive or negative effects has not been reached among the different authors. Le Bihan (2018) finds that access to citizen-initiated referendums induces a greater alignment between the appointed policies and voters' preferences on issues that can be subjected to referendum as well as issues that cannot. In a framework in which elected officials have a preference for both policies and being in office, allowing voters to call for a referendum reduces the political benefits of going against the public interest, since the elected official could find her policy decision annulled and her re-election chances compromised. While Le Bihan (2018) claims for the preponderance of benefits that instruments of direct democracy bring to representative democracies, Prato and Strulovici (2017) find that negative effects could outweigh the initial welfare derived from the alignment between policies and voters' preferences. Since elected officials know that voters can amend a wrong decision, they feel exempt from any responsibility, reducing the effort they exert, thus entailing the risk of elected officials taking wrong decisions on those matters for which a referendum cannot be called. The beauty of the findings in this paper is that, under specific conditions, representative democracy would achieve full alignment between policies and voters' preferences, without the need to have instruments of direct democracy altering the behavior of elected politicians.

As far as we know, Coffman (2016) is the only paper in the literature to develop a theoretical analysis investigating the conditions

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[2] Note that the second condition's requirements are not mutually exclusive so both may hold at the same time.

under which representative democracy implements the choices made by people in direct democracy. The author considers the existence of a single issue for which there are a finite number of alternatives. She focuses on a case where the decision made in direct democracy leads to a strict ordering of these alternatives and looks for conditions under which the candidate with this ordering is elected. Contrary to this, we consider that there are a finite number of issues, for each of which a binary choice must be made. We aim to find conditions under which decisions for each issue that people would have made, one at a time, in independent referendums, are implemented by the elected candidate when she decides for the bundle of issues, regardless of what that candidate's preferences are. To this extent, this paper addresses the question of the equivalence between direct and representative democracy under a framework that better captures the true essence of representative democracies. First, because elected representatives do not usually decide only on a single issue but on several ones. Second, because we are interested in the equivalence between systems even when a candidate with preferences perfectly aligned with those of the citizens might not exist.

A matter that has also aroused interest among scholars has been the suitability of each of these systems. Maskin and Tirole (2004) study whether decisions should be made by the public directly, by politicians subject to re-election, or by independent judges to maximize social welfare. Correa-Lopera (2019) investigates whether direct or representative democracy would be preferred by a society as a form of government. The author considers a framework in which voters have an informational disadvantage about the economically efficient policy, but both voters and politicians can be driven by either individual or social interests. In this way, the present work can be understood as a complement to these studies, insofar as it offers a framework under which the debate on the appropriateness of one or the other system would have no place, since both would be equivalent in terms of outcomes.

Conditions guaranteeing the outcome equivalence between direct and representative democracy allow this paper to relate to two additional branches of the literature. Several papers expose the benefits of polarization of political candidates. Bernhardt et al. (2009) propose a model with purely office-motivated parties and show that divergence in the parties' proposed policies improves voter welfare. Fauli-Oller et al. (2003) introduce a model with purely ideology-motivated parties and conclude that, to maximize the party chances of winning the election, parties have incentives to nominate candidates with a more radical ideology, which leads to a greater polarization of the policies proposed by parties. Dodlova and Zudenkova (2021) provide both theoretical and empirical evidence that the better an incumbent's performance the more it leads to higher political polarization. To the extent that the polarization of candidates' preferences is found to be one of the conditions that contributes to the equivalence between the systems of direct and representative democracy, this paper enlarges the range of benefits recognized by a politicians' polarization. The other branch of the literature to which this paper contributes relates to the advantages of voters' preferences convergence. Casella (2005) proposes a model in which members of a committee that periodically meets to make decisions are allowed to store their votes to use them in some future voting. The author finds that, in such a context, welfare improvements happen if the preferences of those members are not too polarized. The requirement of this present study, of having the outcome of majority voting issue-by-issue as the Condorcet winner relative to the voters' preference profile, can be understood as the need for a certain degree of homogeneity among voters' preferences. In this regard, this paper incorporates the alignment of outcomes between direct and representative democracy to the set of benefits from the convergence of voters' preferences.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 presents the results as well as an interpretation of them. Section 4 offers a discussion about the emergence of feelings against representative democracy when equivalence conditions are not met. Finally, Section 5 concludes. The Appendix contains the proofs of the results presented in Section 3.

2. The model

There are $q \geq 1$ issues and for each of them a binary decision must be made. Let $K = \{1, \ldots, q\}$ be the set of issues and $k \in K$ an arbitrary issue. Let $x = (x^1, \ldots, x^q)$ be a vector of decisions on q issues where $x^k \in \{-1, 1\}$ denotes the decision for issue k. We call a vector x a policy and $A \equiv \{-1, 1\}^q$ the set of policies.

Let $N = \{1, \ldots, n\}$ be an odd finite set of voters. Each voter $i \in N$ has strict separable preferences defined over the set of policies, which means that for each issue k a voter i has either -1 or 1 as her most preferred decision, and this remains invariant regardless of the decisions for all the other issues.[3] For voter $i \in N$, let $\mathcal{P}_i$ be the set of all strict separable preference relations defined on A, with typical element P_i. Let $P_N = (P_1, \ldots, P_n)$ be a voters' preference profile, which in our model is common knowledge.[4] In direct democracy, voters cast their ballots truthfully on each of the issues.[5] Logrolling or coalition building among voters are not allowed, so majority voting issue-by-issue is the outcome in this system. Given a preference profile of voters, let $x_{maj}^k(P_N) \in \{-1, 1\}$ be the decision preferred by a majority of voters for issue k and $x_{maj}(P_N) = (x_{maj}^1(P_N), \ldots, x_{maj}^q(P_N)) \in A$ the policy for which the decision on each issue k is made by majority voting. We say that a policy is the Condorcet winner relative to the voters' preference profile when no other policy defeats it in pairwise majority comparisons. In our setting with strict preferences and an odd number of voters there is at most one Condorcet winner. In particular, Buechel (2014) shows that if a Condorcet winner exists, then it is equal to $x_{maj}(P_N)$. Formally:

Definition 1. The policy $x_{maj}(P_N) \in A$ is the **Condorcet winner at** P_N if there is no $z \in A$ such that $\# \{i \in N \ / \ z \ P_i \ x_{maj}(P_N)\} > \frac{n}{2}$.

We investigate under which conditions $x_{maj}(P_N)$, the policy outcome in direct democracy, is also the policy chosen under representative democracy. To study policy choice in representative democracy, we analyze an electoral game between two candidates, each representing a different political party. Let $C = \{L, R\}$ be the set of candidates, with typical element c. Candidates simultaneously announce a policy and voters cast their vote for one of them. Let m_L and m_R be the announced policies by candidates L and R respectively where $m_L, m_R \in A$. The candidate who receives more votes wins the election and implements the policy that she has announced. For any voter, to vote for the candidate who announces a more preferred policy is a dominant strategy.[6] We then simplify the analysis and study the simultaneous move game between the candidates in which the winning candidate is the one who announces the policy which is preferred by the majority of voters. Since we assume that there is an odd number of voters and preferences are strict, candidates are equally likely to win the election only if they announce the same policy. We refer to this situation as a "tie" between candidates.[7] We use w_L or w_R to denote that candidate L or candidate

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[3] See Breton and Sen (1999) for a deep understanding of how every strict separable preference relation over the set of policies induces a unique strict preference relation over each issue.

[4] Note that the frequent surveys and opinion polls conducted among the population allow us to have accurate knowledge of the preferences of a society. The worldwide efforts of organizations such as the Pew Research Center and Gallup are especially noteworthy in this context. Public opinion collected at the subnational level is also quite significant thanks to the work carried out by institutions such as the Institut Français d'Opinion Publique in France, the Allensbach Institute in Germany, and the Centro de Investigaciones Sociológicas in Spain, among others.

[5] Note that truth-telling is a dominant strategy for any voter under majority voting when there are only two decisions for each issue.

[6] Note that truth-telling is a dominant strategy for any voter under majority voting when there are only two policies.

[7] We assume that for the case in which both candidates announce the same policy, the votes are allocated equally.

R wins the election and *tie* for the coinciding announcements case. Let $O = \{w_L, tie, w_R\}$ be the set of office-outcomes. Note that these office-outcomes could be interpreted as a probability of being in office for each of the candidates. We define an electoral-outcome as a pair of office-outcome and implemented policy. Let $\mathcal{E} \equiv O \times A$ be the set of electoral-outcomes, with typical element $(o; x)$ where $o \in O$ and $x \in A$. We make the following assumptions about candidates' preferences:

1. Candidates have a preference for both being in office and the policy that will ultimately be implemented.[8]
2. Candidates have strict separable preferences over the set of electoral-outcomes.
3. Candidates' preferences are strictly increasing in both office and policy. For any candidate, an electoral-outcome is more preferred:

    (i) the greater the probability of being in office and
    (ii) the closer the implemented policy to her most preferred policy, where candidates' preferences over the set of policies are strict separable.

Let $\mathcal{P}_c$ be the set of all strict separable preference relations for candidate $c \in C$ defined on $\mathcal{E}$, with typical element P_c. Let $P_C = (P_L, P_R)$ be a candidates' preference profile.

By imposing no further restrictions on candidates' preferences, our model allows for the marginal rate of substitution between preference for being in office and preference for the implemented policy to be different between candidates. In our framework, candidates may differ in the importance they attach to each of the two dimensions. Candidates' preferences range from *extreme office-prioritization*, where winning the election becomes the candidate's first concern regardless of the policy implemented, to *extreme policy-prioritization*, where getting the implementation of a policy as close as possible to her top is pre-eminent for the candidate, even if it means losing the election.[9] Formally:

Definition 2. We say that candidate $c \in C$ is an **extreme office-prioritizer** if for each $x, y \in A$, we have that $(w_c; x) \, P_c \, (tie; y)$.

Definition 3. We say that candidate $c \in C$ is an **extreme policy-prioritizer** if for each $x, y \in A$ with $(w_c; x) \, P_c \, (w_c; y)$, we have that $(w_{-c}; x) \, P_c \, (w_c; y)$.

Note that candidates' preferences can be, at the same time, monotonically increasing in both office and policy, and prioritize either office or policy. Also, preferences described in Definitions 2 and 3 are just the two extreme cases of all the preference relations belonging to $\mathcal{P}_c$: preferences according to which a candidate, without being extreme, might be more inclined towards either the office dimension or the policy dimension are also admissible.

We refer to the most preferred policy on A of each candidate as her "top". Given a preference profile of candidates, let τ_L and τ_R be the tops of candidates L and R respectively, where $\tau_L, \tau_R \in A$. For each candidate $c \in C$, let $\tau_c = (\tau_c^1, \dots, \tau_c^q)$ where $\tau_c^k \in \{-1, 1\}$ is the decision preferred by candidate c for issue $k \in K$. Note that, for any candidate, there will be different preference relations that share the same top. As a result, different preference profiles of candidates can generate the same pair of tops (τ_L, τ_R). For instance, consider a bi-dimensional policy: a policy consists of the decision to remain or leave the European Union and a decision about drug liberalization. Suppose candidate L's top

is to remain and to liberalize. Candidate L's most preferred electoral-outcome is winning the election and this policy being implemented, but her second preferred electoral-outcome depends on her "type": candidate L's second preferred electoral-outcome might be winning the election, remaining in the European Union and giving up drug liberalization, or may be winning the election and implementing drug liberalization but leaving the EU; or could even be winning the election with only one-half probability but being her top implemented, that is, having remained in the EU and having had drugs liberalized.

Definition 4. Given a pair of tops (τ_L, τ_R), we say that a preference profile of candidates P_C **is consistent with** (τ_L, τ_R) if $\tau_L(P_L) = \tau_L$ and $\tau_R(P_R) = \tau_R$.

We highlight the situation in which, issue by issue, the most preferred decision of one of the candidates is the opposite to the most preferred decision of the other candidate. We refer to this condition as maximal top-differentiation between candidates.

Definition 5. We say that candidates are **maximally top-differentiated** when, for each issue, the most preferred decision of candidate L is the opposite to the most preferred decision of candidate R. Formally, $\tau_L = -\tau_R$.

Given a pair of candidates' tops (τ_L, τ_R), our aim is to study when the policy announced by the winning candidate of the electoral game is $x_{maj}(P_N)$, whatever the candidates' preferences are consistent with the given pair of candidates' tops. This implementation approach allows us to echo the concerns about representative democracies that have recently been expressed by a part of society. These are related to the risk voters face when they must choose a candidate knowing only the preferred policy of each of the candidates and not their *full* preferences. We define a society by a preference profile of voters and a pair of tops of candidates. Let the triplet (P_N, τ_L, τ_R) define a society. We focus on the pure strategy Nash equilibrium concept of the electoral game, henceforth referred to as PSNE.

Definition 6. Given a society (P_N, τ_L, τ_R) and a preference profile of candidates P_C consistent with (τ_L, τ_R), we say that announcements m_L and m_R **constitute a PSNE of the electoral game** if no candidate $c \in C$ has incentives to deviate and announce some $m'_c \in A \setminus \{m_c\}$.

3. When representative democracy equals direct democracy

For every possible society, our purpose is to find under what conditions there is a unique PSNE of the electoral game in which the policy announced by the winning candidate is $x_{maj}(P_N)$ for each preference profile of candidates which is consistent with (τ_L, τ_R). We refer to this situation as having $x_{maj}(P_N)$ as the unique *PSNE outcome* of the electoral game.

Lemma 1 states that, if there is a PSNE of the electoral game, then both candidates are announcing the same policy.

Lemma 1. *If announcements m_L and m_R constitute a PSNE of the electoral game, then $m_L = m_R$.*

All proofs can be found in Appendix. Intuitively, it is easy to see why a situation in which different policies are announced cannot be sustained as equilibrium. If this were the case, we know that the candidate announcing a more preferred policy by a majority of voters wins the election and carries out her announced policy. By preference for being in office, the losing candidate has incentives to deviate and announce the same policy as the winning candidate since, given the implementation of such policy, this candidate prefers to tie rather than to lose.

We now identify a necessary condition for having $x_{maj}(P_N)$ as the unique PSNE outcome of the electoral game. Proposition 1 states that only if $x_{maj}(P_N)$ is the Condorcet winner, is there room for the achievement of such purpose.

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[8] Let us highlight here that a candidate would not derive utility from the policy announcement that she makes in the candidates' game but instead from the policy that ends up being implemented by the politician in office (either herself or her opponent).

[9] The term *to lose* refers to the scenario in which the opposing candidate wins.

Proposition 1. *Given any society (P_N, τ_L, τ_R), suppose that $x_{maj}(P_N)$ is the unique PSNE outcome of the electoral game for every P_C consistent with (τ_L, τ_R). Then $x_{maj}(P_N)$ is the Condorcet winner at P_N.*

To prove Proposition 1, we show that when the outcome of majority voting issue-by-issue is not the Condorcet winner, both candidates announcing such policy may not be sustained as equilibrium; this would create room for profitable deviations to exist. In particular, the realization of the preference profile of candidates, consistent with the given pair of tops, can be such that at least one of the candidates is an extreme office-prioritizer. Such candidate will have incentives to deviate and announce the policy that defeats $x_{maj}(P_N)$ in pairwise comparisons since by doing so she would win the election.

The following proposition claims that having $x_{maj}(P_N)$ as a PSNE outcome of the electoral game is guaranteed when the issue-by-issue majority voting outcome is the Condorcet winner.

Proposition 2. *Let (P_N, τ_L, τ_R) be a society such that $x_{maj}(P_N)$ is the Condorcet winner at P_N. Then, $x_{maj}(P_N)$ is a PSNE outcome of the electoral game for every P_C consistent with (τ_L, τ_R).*

The intuition is that no candidate will have incentives to deviate and announce a policy that is less preferred than the outcome of majority voting issue-by-issue by voters, since in that case she would have no options in the election. Proposition 2 provides a sufficient condition for the existence of the PSNE outcome we are interested in. However, we are not only interested in its existence but in its uniqueness. Theorem 1 states necessary and sufficient conditions for the issue-by-issue majority voting outcome, that is, $x_{maj}(P_N)$, to be the unique PSNE outcome of the electoral game.

Theorem 1. *Given any society (P_N, τ_L, τ_R), $x_{maj}(P_N)$ is the unique PSNE outcome of the electoral game for every P_C consistent with (τ_L, τ_R) if and only if $x_{maj}(P_N)$ is the Condorcet winner at P_N and:*

(i) $x_{maj}(P_N) = \tau_c$ for some $c \in C$, or

(ii) candidates are maximally top-differentiated.

Theorem 1 identifies necessary and sufficient conditions for the two considered decision-making procedures, that is, by direct vote of voters over each single issue or allowing that decisions are made in an electoral game, to be equivalent in terms of the decision made for each issue when tops of candidates is all that is known about their preferences.[10] Before analyzing the robustness of the result in Theorem 1 when any of its conditions is not met, it is worthwhile making an observation for the case of one-dimensional policy space, $q = 1$.

Remark 1. *Let $q = 1$. Given any society (P_N, τ_L, τ_R), $x_{maj}(P_N)$ is the unique PSNE outcome of the electoral game for every P_C consistent with (τ_L, τ_R) if and only if candidates are maximally top-differentiated.*

With only a single issue, the existence of the Condorcet winner at P_N is guaranteed – it is $x_{maj}(P_N)$ – and condition (ii) in Theorem 1 implies condition (i). For a single dimension, maximal top-differentiation between candidates translates into one candidate preferring -1 and the other 1, which implies that one of the candidates necessarily has $x_{maj}(P_N)$ as top. Conditions (i) and (ii) in Theorem 1 are therefore not independent for the case of a single issue. However, when we move to a multi-dimensional policy space, $q > 1$, the conditions for the

equivalence of direct and representative democracy become much more demanding, they become independent of each other, which suggests that multi-dimensionality of the policy space is really what drives the comparison of the two systems in this model.

We now explain why conditions in Theorem 1 are necessary and sufficient for the outcome of direct democracy being the unique PSNE outcome of the electoral game in representative democracy. We start by discussing necessity. When $x_{maj}(P_N)$ is not the Condorcet winner, the existence of a PSNE with $x_{maj}(P_N)$ as outcome is not guaranteed.[11] Suppose that there is a candidate who is an extreme office-prioritizer. Since $x_{maj}(P_N)$ is not the Condorcet winner, there is at least one policy that defeats $x_{maj}(P_N)$ in pairwise majority comparisons. Thus, the extreme office-prioritizer candidate has incentives to deviate and announce some policy which is preferred to $x_{maj}(P_N)$ by a majority of voters. This would allow her to win the election, which is the most important aspect for such a candidate regardless of the policy to be implemented. When there is neither a candidate with $x_{maj}(P_N)$ as top, nor are candidates maximally top-differentiated, the uniqueness of a PSNE with $x_{maj}(P_N)$ as outcome is not guaranteed. Assume that the tops of both candidates are equal and different from $x_{maj}(P_N)$. Suppose that both candidates are extreme policy-prioritizers. Then, the top of these candidates, which is different from $x_{maj}(P_N)$, can be sustained as a PSNE outcome of the electoral game.

We now focus on the sufficiency of the conditions. Assume first that $x_{maj}(P_N)$ is the Condorcet winner and there is at least one candidate with $x_{maj}(P_N)$ as top. Existence of the equilibrium is guaranteed since both candidates announcing $x_{maj}(P_N)$ is a PSNE of the electoral game. By Lemma 1, candidates announce the same policy in equilibrium. Note that a tie between candidates and the implementation of $x_{maj}(P_N)$ is the electoral-outcome in this case. Since $x_{maj}(P_N)$ is the Condorcet winner, no candidate has incentives to deviate and announce a different policy. If a candidate does so, the deviant candidate loses the election while her opponent wins and carries out the policy $x_{maj}(P_N)$. But, by preference for being in office, such a candidate prefers to tie and implement $x_{maj}(P_N)$ rather than lose having that policy implemented, so no candidate has a profitable deviation. To show the uniqueness of the equilibrium, suppose that both candidates are announcing the same policy but different from $x_{maj}(P_N)$. Announcing $x_{maj}(P_N)$ is a profitable deviation for the candidate with such a policy as top: if she announces $x_{maj}(P_N)$, she wins the election and carries out her top, which is the best possible scenario for such a candidate. Therefore, no policy other than $x_{maj}(P_N)$ can be sustained as a PSNE outcome of the electoral game. Assume now that $x_{maj}(P_N)$ is the Condorcet winner and candidates are maximally top-differentiated. Existence of the equilibrium is similar to the previous case. For the uniqueness of the equilibrium, suppose again that both candidates are announcing the same policy but different from $x_{maj}(P_N)$. Consider one issue for which the decision announced by candidates is different from the decision included in $x_{maj}(P_N)$. By maximal top-differentiation, there is necessarily a candidate who, for such an issue, has the decision contained by $x_{maj}(P_N)$ on that issue as her most preferred decision. Consider a variant of the policy initially announced by the candidates, in which decisions announced for all the issues remain the same as at the beginning except for the issue at hand, which would now become the decision specified by $x_{maj}(P_N)$. Note that, by announcing this modified policy, the aforementioned candidate wins the election and such a policy is carried out. By separability of preferences, and preference for being in office, this electoral-outcome is preferred by this candidate to the initial electoral-outcome in which she tied and a policy more distant from her top was carried out. Thus, there is a candidate with a profitable deviation so no policy other than $x_{maj}(P_N)$ can be sustained as a PSNE outcome of the electoral game.

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[10] Were voters' preferences restricted to a subdomain in which the existence of a strong Condorcet winner is guaranteed, as in the pioneering work of Plott (1967), Theorem 1 would be restated as follows: Given any society (P_N, τ_L, τ_R), $x_{maj}(P_N)$ is the unique PSNE outcome of the electoral game for every P_C consistent with (τ_L, τ_R) if and only if either (i) $x_{maj}(P_N) = \tau_c$ for some $c \in C$, or (ii) candidates are maximally top-differentiated.

[11] For a discussion of the existence of a Nash equilibrium in pure and mixed strategies, see Concluding Remarks.

4. The roots of opposition towards representative democracy: A discussion

In this section, we review some of the findings in the literature and offer a reinterpretation of them in the light of our model. Specifically, we think of them as potential reasons for which the equivalence between direct and representative democracy may fail.

Esteban and Ray (2008) consider an index of fractionalization that captures the degree to which a society is split into distinct groups; they conclude that groups defying the existing political institution are more likely to flourish in highly fractionalized societies. In this sense, we could identify a threat to the stability of the traditional system of political representation when the outcome of majority voting issue-by-issue is not the Condorcet winner relative to the voters' preference profile. Recall that, on the domain of separable preferences, when $x_{maj}(P_N)$ is not the Condorcet winner a Condorcet winner does not exist. It is therefore a society so *fractionalized* as to preclude the existence of a majority agreeing on which policy should be implemented. Note that, in a sense, this fractionalization can be interpreted as a very high level of heterogeneity among voters' preferences. This makes it difficult for politicians in office to satisfy the bulk of the citizenry through the implemented policy, thus dissatisfaction among the electorate with representative democracy is likely to increase.

There is a body of literature providing empirical evidence showing that, when deciding on legislative proposals, the weight assigned by representatives to the preferences of special interest groups is higher than the weight assigned to the preferences of their voters (see Gilens and Page, 2014; Giger and Klüver, 2016; Balles et al., 2018; Stadelmann and Torrens, 2020). This poses a threat to the representativeness of society's preferences under representative democracy to the extent that the aspirations of special interest groups and the population at large are not usually aligned. Having no candidate with $x_{maj}(P_N)$ as most preferred policy could be due to the increased activity of special interest groups, which prevent the existence of a candidate with the preferences of the median voter. Social requests for the abolition of representative democracies could therefore be a natural reaction to the mistrust that has arisen about the motivations of the political class.

As stated in Tullock et al. (2002), logrolling or vote trading is a very common phenomenon in any democratic political system. Logrolling is the process by which politicians trade support for one issue in exchange for another politician's support (Holcombe, 2006). Interestingly, Tullock et al. (2002) point out that logrolling can take place in contexts where pressure groups exist. The activity of these pressure groups may encourage logrolling between candidates, thus forming coalitions for certain issues, which would prevent maximal top-differentiation between candidates. Casella and Macé (2021) develop an insightful review of the literature on the welfare effects of vote trading. The paper distinguishes between the exchange of votes for votes (i.e., logrolling) and the exchange of votes for a numeraire. When discussing logrolling, the authors state: "... we find that the problem of delivering an outcome that is a sincere mirror of the will of the majority is not solved by vote trading". Casella and Macé (2021, p. 66). To the extent that the outcome of logrolling may represent the preferences of interest groups rather than the preferences of the voters, social disapproval of representative democracies could emerge.

Our research therefore contributes to a deeper explanation of the instability of current representative democratic institutions. From Plott (1967) we know that $x_{maj}(P_N)$ is a Condorcet winner relative to the voters' preference profile if and only if it is a median in all directions. Still, given the implausibility of the existence of a median in all directions, this might already explain the crisis of modern representative democracies, as already argued by Besley and Coate (2008). Now Theorem 1 adds that even the existence of a median in all directions is not sufficient to ensure citizen acceptance of the representative system. On top, either (i) one candidate's preferred policy must be the Condorcet winner, or (ii) candidates' preferences must be maximally top-differentiated.

5. Concluding remarks

More and more social movements and political parties are claiming that true respect for the will of a society lies in carrying out, for each single issue, what the majority desires. Given the risk of having self-seeking politicians in representative democracy, these groups defend the better suitability of direct democracy when respecting the interests of society. We have proposed a model to study when a system of representative democracy would be equivalent to a system of direct democracy in terms of the implemented policy, regardless of whether candidates are either more policy-oriented or more office-oriented. We find necessary and sufficient conditions such that: decisions made on each issue in representative democracies, operating under issue-bundling, coincide with the decisions that would have been chosen in direct democracies by majority voting issue-by-issue. The first condition can be interpreted as the need for some degree of homogeneity among voters' preferences about the suitability of the implementation of decisions yielded by majority voting issue-by-issue. The second condition can be understood as the requirement that at least one of the following circumstances occurs: either there is at least one candidate whose preferences are in line with preferences of society, or candidates have opposing preferences on each of the issues to be addressed. The concurrent fulfillment of both conditions guarantees that the equilibrium outcome of the electoral game induced by a representative democracy is unique and coincides with the policy chosen in direct democracy. Under these conditions, any other policy – preferred by both candidates and against the interests of the majority – cannot be sustained as an equilibrium of the electoral game. This study has also allowed us to identify the class of societies in which sentiments that are against representative democracy could be expected to emerge. When any of these conditions are not being met, the breakdown of the equivalence between the systems of direct and representative democracy occurs. It is then that social demands claiming the need to remove the traditional system of political representation may be likely to arise, as a response to the fear that the will of the majority becomes violated under this system. Deeply divided societies, the activity of special interest groups, and the inappropriate vote trading by the political class could stand as some of the reasons explaining the emergence of mistrust in the systems of representative democracy.

We acknowledge that the present paper has not dealt with the interesting question of the endogenous selection of candidates in the electoral competition. From our results we have learned that not having candidates whose policy preferences are aligned with those of the general public could become, among other things, a source of mistrust in the traditional system of political representation. Understanding why the political process cannot generate such candidates, and why candidates with such preferences would not be elected, are worthy considerations for further research. Another intriguing aspect would be to examine how different the outcomes under direct and representative democracy would be if any of the conditions stipulated in Theorem 1 were not satisfied. Of particular interest is the case where there is no Condorcet winner, a possibility that is enhanced by the presence of issue-bundling. If this were the case, there would be no Nash equilibrium in pure strategies. However, it is important to note that there is always a Nash equilibrium in mixed strategies because of the finite number of players and the finite strategy space (Nash, 1950). Xefteris (2014) shows that mixed-strategy policy proposals constitute a Nash equilibrium in representative systems when candidates are uncertain about voters' policy preferences. In our model, the voters' preference profile is common knowledge. This means that politicians can anticipate the electoral support they will receive based on their policy announcements. For this reason, in a first cut to the problem of finding equivalence between policies implemented in representative democracy and direct democracy, we focused on the Nash equilibrium in pure strategies. Nevertheless, in the absence of the latter, we recognize the existence of a Nash equilibrium in mixed strategies where the outcome

of the electoral game could be, with some positive less-than-one probability, equal to $x_{maj}(P_N)$. Finding out when this might happen, and assessing how voters would value a policy other than $x_{maj}(P_N)$ – which could also be a mixed strategy Nash equilibrium outcome with positive probability – are questions that deserve to be addressed in forthcoming studies. What would happen if there were more than two decisions for each issue, or if the issues were interdependent, or if logrolling or coalition building among voters were allowed, are other topics that could also be the subject of future work.

CRediT authorship contribution statement

Guadalupe Correa-Lopera: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization.

Data availability

No data was used for the research described in the article.

Appendix

Proof of Lemma 1. Suppose not, i.e., let $m_L = y$ and $m_R = z$ where $y \neq z$ and $y, z \in A$ such that m_L and m_R are a PSNE of the electoral game. Suppose, without loss of generality, that $(w_L; y)$ is the obtained electoral-outcome after voting by voters. Consider candidate R. Let $m'_R \in A$ such that $m'_R = y$. Note that, if candidate R deviates and announces $m'_R = y$, then $(tie; y)$ is the resulting electoral-outcome. By preference for being in office, $(tie; y) \, P_R \, (w_L; y)$, so announcing $m'_R = y$ is a profitable deviation for candidate R when $m_L = y$. Therefore, m_L and m_R are not a PSNE of the electoral game. ∎

Proof of Proposition 1. By way of contradiction, suppose that for some society $x_{maj}(P_N)$ is not the Condorcet winner at P_N. Then, there exists $y \in A$ such that $\# \{i \in N \, / \, y \, P_i \, x_{maj}(P_N)\} > \frac{n}{2}$. Let $P_C = (P_L, P_R)$ be a candidates' preference profile consistent with (τ_L, τ_R) such that, for at least one of the candidates, say L, $P_L \in \mathcal{P}_L$ and, for each $h, s \in A$, we have $(w_L; h) \, P_L \, (tie; s)$. In words, candidate L is an extreme office-prioritizer. By Lemma 1, we know that, $m_L = m_R = x_{maj}(P_N)$ is the only PSNE of the electoral game. Let $m'_L \in A$ be such that $m'_L = y$. Since $\# \{i \in N \, / \, y \, P_i \, x_{maj}(P_N)\} > \frac{n}{2}$, $(w_L; y)$ is the resulting electoral-outcome when m'_L and m_R are the candidates' announcements. By assumption, $(w_L; y) \, P_L \, (tie; x_{maj}(P_N))$, so announcing $m'_L = y$ is a profitable deviation for candidate L when $m_R = x_{maj}(P_N)$, which contradicts that m_L and m_R is a PSNE of the electoral game. ∎

Proof of Proposition 2. Let $m_L = m_R = x_{maj}(P_N)$. Note that $(tie; x_{maj}(P_N))$ is the resulting electoral-outcome for these announcements. Since $x_{maj}(P_N)$ is the Condorcet winner at P_N, for each $m'_L \in A \setminus \{x_{maj}(P_N)\}$, $(w_R; x_{maj}(P_N))$ is the resulting electoral-outcome when m'_L and m_R are the candidates' announcements. By preference for being in office, $(tie; x_{maj}(P_N)) \, P_L \, (w_R; x_{maj}(P_N))$, so announcing m'_L is not a profitable deviation for candidate L when $m_R = x_{maj}(P_N)$. The analysis for candidate R follows an analogous reasoning. Therefore, m_L and m_R are a PSNE of the electoral game. ∎

Proof of Theorem 1. We prove this theorem by showing the following two claims.

Claim 1 (*Necessity*). *If $x_{maj}(P_N)$ is the unique PSNE outcome of the electoral game for every P_C consistent with (τ_L, τ_R), then $x_{maj}(P_N)$ is the Condorcet winner at P_N and (i) $x_{maj}(P_N) = \tau_c$ for some $c \in C$, or (ii) candidates are maximally top-differentiated.*

That $x_{maj}(P_N)$ is the Condorcet winner at P_N follows from Proposition 1. By way of contradiction, suppose that neither (i) $x_{maj}(P_N) = \tau_c$ for some $c \in C$, nor (ii) candidates are maximally top-differentiated. Then, (1) $\tau_L^r = -x_{maj}^r(P_N)$ for some $r \in K$, (2) $\tau_R^s = -x_{maj}^s(P_N)$ for some $s \in K$, and (3) $\tau_L^t = \tau_R^t$ for some $t \in K$. Next, we show that there exists some $y \in A \setminus \{x_{maj}(P_N)\}$ such that $m_L = m_R = y$ is a PSNE of the electoral game for some P_C consistent with (τ_L, τ_R). We distinguish two cases:

- Case 1. $\tau_L^t = \tau_R^t = -x_{maj}^t(P_N)$ for some $t \in K$.
  Suppose, without loss of generality, $\tau_L^t = \tau_R^t = 1$ and $x_{maj}^t(P_N) = -1$. Let $Y = \{Y \in A \, : \, y^t = -x_{maj}^t(P_N) = 1\}$. By construction, $x_{maj}(P_N) \notin Y$. Let $P_C = (P_L, P_R)$ be a candidates' preference profile consistent with (τ_L, τ_R) such that, for each $y \in Y$ and $z \in A \setminus Y$, we have $(w_R; y) \, P_L \, (w_L; z)$ and $(w_L; y) \, P_R \, (w_R; z)$. Let $h \in Y$ be such that for each $k \in K \setminus \{t\}$, $h^k = x_{maj}^k(P_N)$. Since $h \in Y$ we know that $h^t = -x_{maj}^t(P_N) = 1$, so clearly $h \neq x_{maj}(P_N)$. We now show that $m_L = m_R = h$ is a PSNE of the electoral game. Note that $(tie; h)$ is the resulting electoral-outcome. Observe that candidate L has no incentives to deviate to some $z \in A \setminus Y$ since by assumption $(w_R; h) \, P_L \, (w_L; z)$ and by preference for being in office $(tie; h) \, P_L \, (w_R; h)$, so by transitivity of preferences $(tie; h) \, P_L \, (w_L; z)$. Thus, candidate L would only consider deviations to policies that belong to the set Y. Let $m'_L \in Y$ such that $m'_L = g$ where $g \in Y \setminus \{h\}$. Since $g \neq h$, there exists at least $k \in K \setminus \{t\}$ such that $g^k \neq h^k$. By construction, $h^k = x_{maj}^k(P_N)$ while $g^k = -x_{maj}^k(P_N)$. Since $x_{maj}(P_N)$ is the Condorcet winner, by separability of voters' preferences we have $\# \{i \in N \, / \, h \, P_i \, g\} > \frac{n}{2}$. Hence, $(w_R; h)$ is the resulting electoral-outcome when m'_L and m_R are the candidates' announcements. By preference for being in office $(tie; h) \, P_L \, (w_R; h)$, so announcing $m'_L = g$ is not a profitable deviation for candidate L when $m_R = h$. The analysis for candidate R follows an analogous reasoning. Therefore, m_L and m_R are a PSNE of the electoral game.
- Case 2. For every $t \in K$ with $\tau_L^t = \tau_R^t$ we have $\tau_L^t = \tau_R^t = x_{maj}^t(P_N)$.[12]
  Then, from points (1), (2), and (3), $\tau_L^r = -x_{maj}^r(P_N) = -\tau_R^r$ for some $r \in K$, and $\tau_R^s = -x_{maj}^s(P_N) = -\tau_L^s$ for some $s \in K$. Suppose, without loss of generality, $\tau_L^r = 1$, $x_{maj}^r(P_N) = \tau_R^r = -1$, $\tau_R^s = 1$, and $x_{maj}^s(P_N) = \tau_L^s = -1$. Let $Q = \{Q \in A \, : \, q^r = \tau_L^r = -x_{maj}^r(P_N) = 1 \text{ and } q^s = \tau_R^s = -x_{maj}^s(P_N) = 1\}$. By construction, $x_{maj}(P_N) \notin Q$. Let $P_C = (P_L, P_R)$ be a candidates' preference profile consistent with (τ_L, τ_R) such that, for each $q \in Q$ and $z \in A \setminus Q$, we have

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[12] We would like to thank an anonymous referee who pointed out that this excludes the case in which $x_{maj}(P_N)$ would be the unique PSNE outcome of the electoral game under a more general condition than ours. In particular, if for each issue there is at least one candidate who prefers the policy that is preferred by the majority of voters on that issue.

$(w_R; q) \, P_L \, (w_L; z)$ and $(w_L; q) \, P_R \, (w_R; z)$. Let $h \in Q$ be such that for each $k \in K \setminus \{r, s\}$, $h^k = x^k_{maj}(P_N)$. Since $h \in Q$ we know that $h^r = \tau^r_L = -x^r_{maj}(P_N) = 1$ and $h^s = \tau^s_R = -x^s_{maj}(P_N) = 1$, so clearly $h \neq x_{maj}(P_N)$. We now show that $m_L = m_R = h$ is a PSNE of the electoral game. Note that $(tie; h)$ is the resulting electoral-outcome. Observe that candidate L has no incentives to deviate to some $z \in A \setminus Q$ since by assumption $(w_R; h) \, P_L \, (w_L; z)$ and by preference for being in office $(tie; h) \, P_L \, (w_R; h)$, so by transitivity of preferences $(tie; h) \, P_L \, (w_L; z)$. Thus, candidate L would only consider deviations to policies that belong to the set Q. Let $m'_L \in Q$ such that $m'_L = g$ where $g \in Q \setminus \{h\}$. Since $g \neq h$, there exists at least $k \in K \setminus \{r, s\}$ such that $g^k \neq h^k$. By construction, $h^k = x^k_{maj}(P_N)$ while $g^k = -x^k_{maj}(P_N)$. Since $x_{maj}(P_N)$ is the Condorcet winner, by separability of voters' preferences we have $\#\{i \in N \, / \, h \, P_i \, g\} > \frac{n}{2}$. Hence, $(w_R; h)$ is the resulting electoral-outcome when m'_L and m_R are the candidates' announcements. By preference for being in office $(tie; h) \, P_L \, (w_R; h)$, so announcing $m'_L = g$ is not a profitable deviation for candidate L when $m_R = h$. The analysis for candidate R follows an analogous reasoning. Therefore, m_L and m_R are a PSNE of the electoral game.

Claim 2 (*Sufficiency*). *If $x_{maj}(P_N)$ is the Condorcet winner at P_N and (i) $x_{maj}(P_N) = \tau_c$ for some $c \in C$, or (ii) candidates are maximally top-differentiated, then $x_{maj}(P_N)$ is the unique PSNE outcome of the electoral game for every P_C consistent with (τ_L, τ_R).*

That $x_{maj}(P_N)$ is a PSNE outcome of the electoral game for each preference profile of candidates consistent with (τ_L, τ_R) follows from Proposition 2. It remains to be shown that it is unique. By way of contradiction, suppose that there exists a PSNE of the electoral game such that $x_{maj}(P_N)$ is not the resulting outcome. From Lemma 1 we have that there are m_L and m_R such that $m_L = m_R = y$ where $y \in A \setminus \{x_{maj}(P_N)\}$ which are a PSNE of the electoral game with $(tie; y)$ as the resulting electoral-outcome. We distinguish two cases:

- *Case 1.* $x_{maj}(P_N) = \tau_c$ for some $c \in C$.
  Assume, without loss of generality, that $\tau_L = x_{maj}(P_N)$. Let $m'_L = x_{maj}(P_N)$. Since $x_{maj}(P_N)$ is the Condorcet winner, $(w_L; x_{maj}(P_N))$ is the resulting electoral-outcome when m'_L and m_R are the candidates' announcements. By preference for being in office and separability of candidates' preferences, $(w_L; x_{maj}(P_N)) \, P_L \, (tie; y)$, so announcing $m'_L = x_{maj}(P_N)$ is a profitable deviation for candidate L when $m_R = y$. Therefore, $m_L = y$ and $m_R = y$ are not a PSNE of the electoral game.

- *Case 2.* Candidates are maximally top-differentiated.
  We distinguish two subcases:

  - *Subcase 2.1. For some $c \in C$, $\tau_c = x_{maj}(P_N)$.*
    We come back to Case 1.
  - *Subcase 2.2. For each $c \in C$, $\tau_c \neq x_{maj}(P_N)$.*
    We distinguish two sub-subcases:

    - *Sub-subcase 2.2.1. For some $c \in C$, $\tau_c = m_c = y$.*
      Assume, without loss of generality, that $\tau_L = m_L = y$. By maximal top-differentiation, $y = -\tau_R$. Let $m'_R = x_{maj}(P_N)$. Since $x_{maj}(P_N)$ is the Condorcet winner, $(w_R; x_{maj}(P_N))$ is the resulting electoral-outcome when m_L and m'_R are the candidates' announcements. By preference for being in office and separability of candidates' preferences, $(w_R; x_{maj}(P_N)) \, P_R \, (tie; -\tau_R)$, so announcing $m'_R = x_{maj}(P_N)$ is a profitable deviation for candidate R when $m_L = y$. Therefore, $m_L = y$ and $m_R = y$ are not a PSNE of the electoral game.
    - *Sub-subcase 2.2.2. For each $c \in C$, $\tau_c \neq m_c = y$.*
      Since $y \neq x_{maj}(P_N)$ by construction, there exists at least $h \in K$ such that $y^h \neq x^h_{maj}(P_N)$. Assume, without

loss of generality, that $y^h = 1$ and $x^h_{maj}(P_N) = -1$. Furthermore, by maximal top-differentiation, we know that $\tau^h_L = -\tau^h_R$. Assume, without loss of generality, that $\tau^h_L = y^h = 1$ and $\tau^h_R = x^h_{maj}(P_N) = -1$. Let $m'_R = \tilde{m}_R$ where for each $k \in K \setminus \{h\}$, $\tilde{m}^k_R = y^k$ and for $h \in K$, $\tilde{m}^h_R = -y^h = x^h_{maj}(P_N) = -1$. Since $x_{maj}(P_N)$ is the Condorcet winner, by separability of voters' preferences we have $\#\{i \in N \, / \, \tilde{m}_R \, P_i \, y\} > \frac{n}{2}$. Thus, $(w_R; \tilde{m}_R)$ is the resulting electoral-outcome when $m_L = y$ and $m'_R = \tilde{m}_R$ are the candidates' announcements. Note that $\tilde{m}^h_R = \tau^h_R$. By preference for being in office and separability of candidates' preferences, $(w_R; \tilde{m}_R) \, P_R \, (tie; y)$, so announcing $m'_R = \tilde{m}_R$ is a profitable deviation for candidate R when $m_L = y$. Therefore, $m_L = y$ and $m_R = y$ are not a PSNE of the electoral game. ■

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