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DANCING SYMMETRIES: UNVEILING PATTERNS IN COUPLE DANCES

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Abstract: *From an ethnomathematical perspective, a dialogue of mutual interrogation is proposed to describe the relation between the mathematical concept of symmetry and displacement movements performed in some couple dances: chacarera, swing, salsa, and tango. A rotational symmetry arises in the analysis of the figure of the chacarera, while a specular symmetry is described in some step structures of swing and salsa. The case of tango makes it necessary to define a new pattern based on translated translations. Some connections between the type of symmetry and the type of link in the couple, the management of the space on the dance floor, and the origins of each dance are determined. This research provides insights for future educational proposals that weave an embodiment approach with a focus on dynamic geometry.*

Keywords: Ethnomathematics, mathematics education, couple dance, geometric symmetry.

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1 INTRODUCTION

The study of different practices that involve mathematical concepts and ideas is the subject of the Ethnomathematics Programme, which is interested in how particular cultural groups develop and use mathematics in their practices (D'Ambrosio, 2006) with different aims, including the revalorisation of local knowledge and the relationship of teaching and learning processes with the students' environment (Albanese et al., 2017).

The idea is that the encounter of knowledge from different cultures around the same mathematical object -in this case, symmetry- enriches the mutual understanding of the mathematical object and the practice that materializes it -in this case, different couple dances- (Parra-Sánchez, 2017). This encounter must take place based on mutual respect and in such a way that the knowledge of both cultures is considered on the same level and the enrichment should be mutual, which in some cases is referred to as cultural symmetry (Meaney et al., 2021) -what a redundancy!-.

In the framework of Ethnomathematics, the choreography of some Argentinean folkloric couple dances has been previously eviously analyzed, recognizing geometrical figures that are drawn on the floor by the dancers' changes of position (Albanese and Perales, 2014; Sardella, 2004). Other geometrical figures have been identified in the movement of dancers' legs or feet in some ornaments (Albanese, 2016; Di Paola et al., 2008).

Studies on the potential of dance for mathematics education have been carried out in the framework of European projects such as the "Maths in Motion" (MiM) (Nasiakou et al., 2019) from the approach of embodied cognition (Núñez et al., 1999) exploring symmetries and triangles through creative tasks. Symmetry in dance has also been analysed (Schaffer, 2019), insisting on the importance of defining symmetry in space and analysing, among other things, the reciprocal positions of dancers, but from a static point of view.

> These symmetries are assumed to operate on one dancer's body shape and orientation and give as a result a second dancer's position, all in the direction of a given line; in this way T, G, M, and R

> [translation, glide symmetry, mirror symmetry, rotation symmetry] actually represent classes of symmetry operations *rather than specific motions* (Schaffer, 2014, p. 255, our emphasis).

However, the focus of this research is *on displacement movements*, so the corresponding mathematical concept would be translation and the question is: What relationship do the displacement movements of the dancers in couple dances have with each other? Is it possible to describe them in terms of symmetries?

The objective of this research is to analyze the relationship between the mathematical concept of symmetry and the displacement movements performed by dancers in different basic structures or figures of couple dances.

One of the aims of this research is to provide a starting point for designing activities in an embodied education framework that presents a dynamic approach to geometry. The effectiveness of a dynamic approach in teaching geometry has been repeatedly proven by studies that have observed a beneficial impact on learning the subject through the use of dynamic geometry software (Chan and Leung, 2014). Interweaving these ideas with embodied education in mathematics, arguing that "conceptual learning [...] could emanate from, or be triggered by, experiences of enacting or witnessing particular movement forms, even before these movements are explicitly signified as illustrating target content" (Abrahamson et al., 2020, p.1), opens the way for future educational applications of the results of this study.

1.1 Symmetry in mathematics

From a mathematical viewpoint, we consider the formal definitions of symmetries in a plane. Symmetry is one of the isometric movements or transformations defined in the plane, together with translation and rotation. There are two types of symmetry in the plane. Axial symmetry is defined as the application that, given a line r, transforms a point P (and then any figure) into another point P' in the plane in such a way that line r is the bisector of segment PP'. Axial symmetry changes the orientation of the figures and is often associated with reflection in a mirror.

Central symmetry is defined as the application which, given a point O, transforms a point P (and then any figure) into another point P' in the plane in such a way that the point O is the midpoint of segment PP'. The central symmetry maintains the orientation of the figures and is equivalent to a rotation of 180 degrees with center at point O.

Symmetries in space are defined similarly, with respect to a plane (denoted as specular symmetry), with respect to a line (denoted as rotational symmetry), or with respect to a point (denoted as symmetry by a point). There is also the glide symmetry, defined by the combination of other isometric movements in space, a translation with a specular symmetry (Schaffer et al., 2001).

2 METHOD

Ethnomathematics employs qualitative methodologies with an ethnographic imprint because the focus is usually on the meanings and uses that certain cultural groups make of mathematical concepts. In particular, the methodology used here is inspired by mutual interrogation (Adam et al., 2010), which is used in Ethnomathematics when the researcher is both a research agent and a member of the group under investigation, thus one of the key informants. In addition, a dancer with a background in Argentinian culture and folklore studies (the second author) participated in the study as a key informant.

The strong participatory component of the mutual interrogation, such as other recent methodologies in Ethnomathematics, engages the actors of the researched community, in this case the informant, in all phases of the research process, including the drafting of reports.

Data collection techniques consist of long immersion in natural settings and open interviews, or rather dialogues, mutual exchanges of questions and information between the researcher and the key informant. The underlying idea is the symmetrical encounter of knowledge (Meaney et al., 2021; Parra-Sánchez, 2017). These dialogues are interspersed with short dance performances between the actors of the research, hands or puppets reproductions of these steps, and sketches of drawn diagrams of the positions and movements of the two dancers.

2.1 Analysis

In this first exploratory approach, the focus is on the basic structures of different traditional and modern couples dances that are practiced and familiar to the informants: swing, chacarera, salsa, and tango. Analysis of the more complex structures of each dance is left for another occasion. In each dance, the units of analysis are agreed upon

with the informant, and this joint choice is part of the co-construction of the analytical process. The unit of analysis could be a figure or a step, being usually a figure when the dance is structured in a choreography made up of short sequences of steps (a figure precisely) that are repeated in a determined order and without variations among performances while being a step or a few steps structure where there is no a standardized configuration of repeating figures.

The analysis is organized as follows: in each dance, the unit of analysis is first indicated, then briefly described and accompanied by an image with puppets[1] and a diagram inspired by Schaffer et al. (2001) where the letters p, q, d, and b are used to represent the dancers in the different positions, "if you imagine the letters as people seen from above" then p is about to move his right leg and q is about to move his left leg. Therefore, unlike the original use where the bar line represents the raised arms, here the bar line is considered the moving legs. In the diagram, the points and segments represent the axis and planes of symmetries seen from above, respectively, while the arrows indicate the displacements of the dancers whose reciprocal relationship is to be analyzed.

3 RESULTS

In the dances of loose couples, there is usually a pre-established choreography. While, in linked couple dances, generally each member of the couple has a role, the leader (traditionally a man) is the one who sets the steps, i.e., he decides and communicates to the partner, named the follower (traditionally a woman), with body movements and points of contact (hands or embrace) which is the sequence or the steps to be performed.

The first figure in the choreography of the chacarera, an Argentinean folkloric dance of loose couples, is analyzed below. This figure is called forward and backward (*Avance retroceso* is the original Spanish term). In it, both dancers draw rhombuses on the floor starting with their respective left legs and making a step forward to the respective right sides (figure 1). Then, they approach each other with a step of the right foot advancing to the left. After that, they both take a step backward to the left with the respective left legs and finally return to the initial position with a last step backward to the right with

[1]The images with the puppets have been created with the help of AI

the respective right legs (Albanese and Perales, 2014). This implies that both perform the same movements detected in the dialogue as in the *reverse camera* (a term used by the informant).

Therefore, a rotational symmetry arises not only in the position but also in the displacement movements: the couple dances around a central axis of symmetry, and their reciprocal positions as well as the displacement movements of the dancers' bodies, i.e., their translations, respect at every moment of the choreography a symmetry defined in mathematical terms by a vertical axis in the middle point of the dancers' distance. Hence, it is a rotational symmetry.

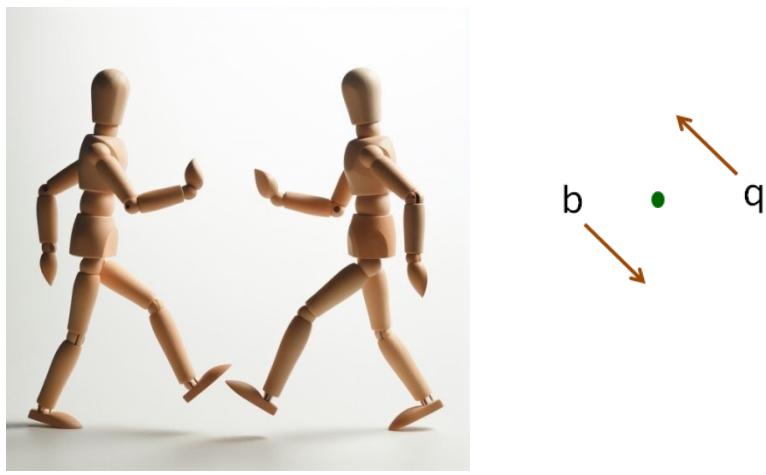

Figure 1: First step of the "forward and backward" figure in chacarera represented by puppets and by a diagram. Example of rotational symmetry.

Swing is a linked couple dance, where the dancers are generally holding at least by one hand (the leader's left hand and the follower's right hand). In terms of the informant "the dancers dance outwards".

As an example, we analyze one of the basic step sequences of swing. This is composed of four steps, indicated by the professional dancers as side-side-rock step, that is to say, the leader makes a small step with the left leg towards his left side, while the follower makes a small step with the right leg towards her right side, then the same happens on the right side for the leader and on the left for the follower, then the leader makes a small step backward with the left leg while the follower makes a step backward with the

left (Figure 2), and then the leader goes forwards with the left and the follower with the right.

Thus, in swing, it can be noticed that, in this basic step, the translation performed is in specular symmetry, i.e., the dancers facing each other move in space as if they were a mirror image of each other. In mathematical terms, the two dancers move in such a way that they are symmetrical with respect to a vertical plane (that is the mirror) that separates the couple and usually passes through the point of contact between the two, generally the left hand of the leader and the right hand of the follower.

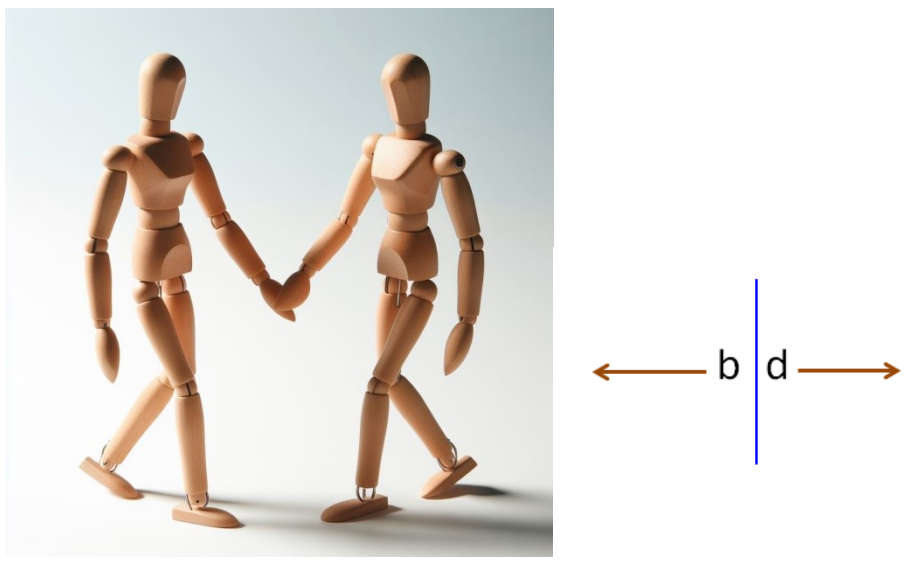

Figure 2: Rock step in Swing. Example of specular symmetry.

In salsa, a linked couple dance, the couple is held by both hands or alternatively by one hand, the left hand for the leader and the right hand for the follower.

There are different basic step sequences. Three of them are analyzed below. In the *lateral basic step* both dancers, facing each other, perform a step in the same lateral direction and then a step in the other direction, with the leader moving his left leg first and then the right while the follower moves the other way round. In this case, there is specular symmetry with respect to the plane that separates the dancers while they move together laterally. In *the open basic step*, the dancers move backwards, walking away

from each other and coming forward together, sometimes indicated as in an *elastic way*, starting the leader with the left leg and the follower with the right leg. Again, this is a mirror structure when the dancers move in specular symmetry with respect to the plane that separates them, in a similar way to swing.

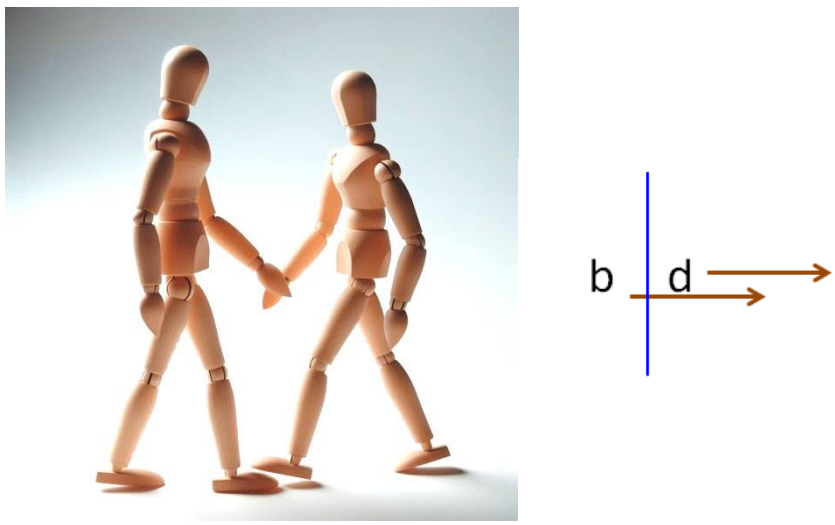

Figure 3: Line basic step in linea in Salsa. Example of translated translations

Instead, the analysis of the displacement of the couple together in the same sense and direction, like in the *line basic step*, is more complex. The leader marks a step that he performs forward with his left leg, and the follower performs it backward with his right leg (Figure 3). Hence, the legs that perform the movement are mirror-matched, but the movement itself does not respect this symmetry. The dancers move in the same direction, their movements being in the same line. Therefore, it can be considered that the displacements are translated with respect to each other but from a mirrored position. *Translated translations* is the term coined for this new pattern.

In tango, somewhere indicated as a ballroom dance, the informant insists on that the close embrace is not usually broken; therefore, neither specular symmetry nor rotational symmetry is generally present, otherwise the connection of the embrace would be lost. The informant underlines that one of the key elements of the dance is dissociation, which means that different relationships occur between the upper body of the dancers and the lower body. This allows for various movements of the dancers' hips

downwards, and so their legs, without affecting the connection of the embrace. This results in the upper part, the chest, of the dancers maintaining a position that respects the specular symmetry, while the movements of the legs are different. In order to identify the unit of analysis in tango, it must be considered that in this dance there are no figures or almost no structures, given the great freedom of movement. The dancer clarifies that tango is a crossed dance in X and that it is danced inwards. The importance of the X suggests to the researcher that the units of analysis should be the different *salidas*: the basic, the crossed, the inverted, and the crossed and inverted, usually the first step sequences taught in tango classes. It consists of an opening step, called *apertura* (a side step to the left of the leader and the right for the follower) and then a *salida* (a step forward for the leader and backward for the follower). There is no more to say about the side step in relation to the analysis already carried out for the side step in swing and salsa. Therefore, in the following, we will analyze the options for the *salida*. During these *salidas*, an X is formed because, from a side view, the front legs of both dancers overlap and form a cross, which generates a great variability in the options to continue dancing.

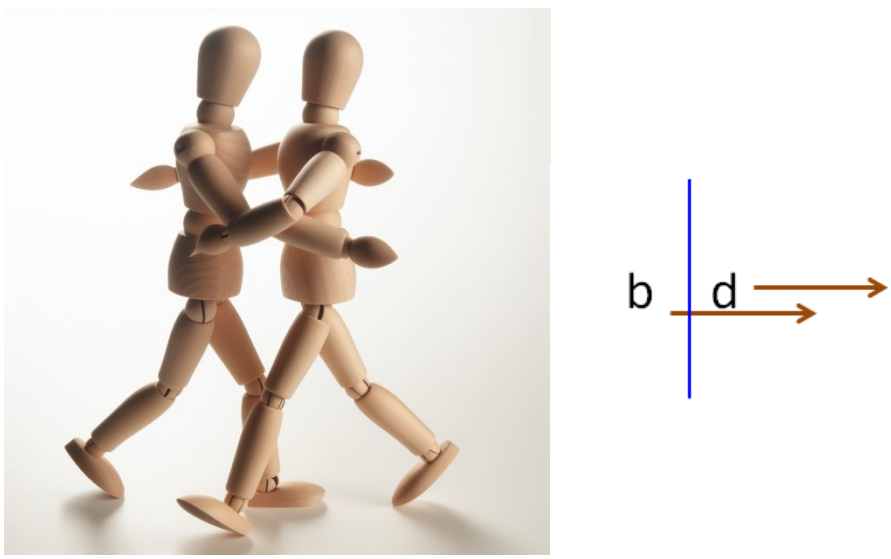

Figure 4: Basic *Salida* in tango. Example of translated translations from mirror positions.

In the basic *salida* of the tango, the leader performs a step forward with the right leg and the follower a step back with the left leg, clarifying that the leader's right leg is placed to the right of the follower's front leg, inwards, as if they were in a binary. In this case, we

recognize that both dancers move the leg on the same side, as at the beginning of the mirror movement, but the movement is a forward translation for the leader and a backward translation for the follower. Here, a first type of X is formed between the left legs of both dancers, which generates some possibilities to follow the dance.

In the case of the crossed *salida*, the leader performs a "weight shift" which means that he changes his leg; therefore, his step forward is with his left leg, exactly like the follower who goes backwards with his left leg, clarifying that the follower's leg performs his step inwards, towards the middle of the follower's legs (Figure 5). In this case, we recognize that the dancers both move their respective left leg as would happen in positions of rotational symmetry, but the movements lead to a translation of the couple as a whole. In this case, the relationship between the movements of the dancers is itself a translation. Here also an X is generated from the lateral view, but the fact that the crossed legs are different, left for the leader and right for the follower, generates another range of possibilities to follow the dance.

Figure 5: Crossed *Salida* in tango. Example of translated translations from rotational positions.

There is another nuance to this movement. If the leader makes his step passing outside the follower's step, the *salida* is indicated as inverted (when starting from the basic structure) or inverted crossed, in the case of starting from the crossed structure. These

will be other ways of creating the X, as already mentioned, which determine other dance options.

From a mathematical point of view, the interpretation of the relationship of these displacements is complex, just as it has been for the *line basic* step in Salsa. According to the analytical description made, these are compositions of symmetries and translations: in the case of the basic *salida*, it would be necessary to start from a position where the bodies are in specular symmetry but the translation of each dancer is in a direction perpendicular to the vertical plane that defines this symmetry translated one with respect to the other in the same direction of the displacement -that is why the mirror is somehow crossed, or it can be indicated that the plane of symmetry is translated-. The translated translation pattern is then detected from specular positions.

In the case of the crossed *salida*, we start from positions in rotational symmetry from which we generate some translations of the dancers that are themselves translated one with respect to the other and in the same direction of the displacement, it can be indicated that the axis of symmetry is translated, so again translated translations, but now from rotational positions.

It should be noted that these compositions of isometries that are applied to the translations of the dancers, rather than to their positions, make it possible to create new patterns, the translated translations, that are not ascribable to the composition of isometries studied by other authors and recognized in the Klein four-group (Belcastro and Schaffer, 2011; Schaffer, 2014).

In the case of the inverted *salida* -and inverted crossed- there is also a lateral translation of the leader to its left -or right- side, in a direction perpendicular to the displacement. This lateral translation is slight because it occurs only from the hip downwards, and therefore depends entirely on the dissociation, so that the connection of the embrace at the chest level is not lost.

Finally, it should be noted that no examples of symmetry by a point are presented, as this would imply that a dancer is head down. There are no cases of glide symmetry since in the couple dances analyzed, the dancers are always facing each other.

4 DISCUSSIONS

These results enrich the understanding of dance and the mathematical relationships between movements in space.

Focussing on the dance, this analysis would allow a mapping or classification of couple dances with respect to the type of symmetry that prevails in their movements or the types of symmetries that are found, if there are several of them. For example, the structure with rotational symmetry is typical of Andalusian folk dances (Southern Spain): "Some dance forms regularly use particular symmetries. For example, in the flamenco duet, two dancers circle each other closely with 180º rotational symmetry, creating a dynamic of intimate opposition" (Belcastro and Schaffer, 2011, p. 16).

Likewise, awareness of the type of symmetries present in each structure allows us to characterize some idiosyncratic elements of each dance that are closely related to the symmetries described.

An element highlighted by the informant is the type of connection that exists between the two dancers that allows or prevents the possibility of realizing certain movements. For example, if the connection is tight, the embrace is closed, and displacements that move the dancers away from each other (specular symmetry in movement that is perpendicular to the mirror plane) or that break the laterality (rotational symmetry) are not allowed, whereas if the connection is more open or lax, they are permitted.

In addition, the need to move around the dance floor, as in tango, implies that the couple as a whole must move in space and is therefore related to joint displacements that are in a translational relationship. In other dances, the couple has a certain amount of space available on the dance floor, and therefore the displacements are maintained around a fixed axis (as in the rotational symmetry of the chacarera) or mirrored with respect to a plane that is also fixed (as in the specular symmetry of swing and also prevalent in salsa).

Another idea that emerged in the dialogues is how these symmetries reflect the history of each dance and hint at ancient dances that may have originally influenced the dances as we conceive them today. For example, several Argentinian folk dances, including the chacarera, are derived from an ancient Renaissance dance called the Minué (Berruti, 1921), which has a choreography based on rotational symmetry. On the other hand, the tango is influenced by the waltz that spread in the romantic period in the European courts, and the later ballroom, from which it inherits the counterclockwise round

movement of the couples on the dance floor, and then the need for joint displacements of the couple.

Swing has various influences, such as the dances of the African slaves brought to North America and the Hornpipe, an English country dance brought to America by the British sailors, which are originally dances of a single dancer, or of a group of dancers in circles where one person, in turn, moves to the center to perform a solo. This gradually evolves into challenges of two dancers, who finally end up facing each other and are linked in pairs (Stevens and Stevens, 2011).

Salsa also draws on dances of Latin American origin which in turn have African and Spanish influences, in particular the Cuban son, a linked couple dance whose structure is predominantly with specular symmetry (Drake-Boyt, 2011).

However, the results also enrich mathematics, in particular mathematics education, first by emphasizing the relevance of the study of symmetries not only in the plane but also in space; then by offering a rich context for the experience of the corporeality of mathematical concepts that materialize in dance, providing new ideas for educational proposals (Nasiakou et al., 2019; Schaffer et al., 2001); and, finally, revealing a movement that intuitively contains a certain relation that is complex to define in formal mathematical terms with respect to the concept of symmetries (or even isometries), the translated translations, which is the characteristic movement of tango and also present in the line structure of salsa.

5 CONCLUSIONS

The exploratory analysis carried out through mutual interrogation has allowed the objective to be achieved, identifying different types of spatial or three-dimensional symmetry in the basic figures or steps of the four couple dances analyzed. A close relationship was found between the type of symmetry and the type of relationship the dancers maintain throughout the dance, whether there is contact and how close this contact is; the space management on the dance floor; and the origins of each dance.

Among the limitations we can point out, we have found those intrinsic to the mutual interrogation inherent in the challenge of translating certain practices into different languages, that of the art of dance, and that of formal mathematics. There is not always

a linear and simple way of reporting practice in the formal language (such as the translated translations of displacements in tango) or, on the other hand, the impossibility of embodying all possible symmetries in space with the dances of couples. This question is sometimes indicated as the incommensurability of different cultural mathematical practices (Knijnik, 2012), giving rise to new philosophical and methodological reflections for ethnomathematical research.

These results could be useful in future proposals for formal or non-formal education involving the spatial sense and, in particular, the isometries of the space focussing on a dynamic approach since it considers not only the reciprocal position of the dancers but also the displacement movements performed. The opportunity to produce not only positions but also movements that are specularly or rotationally symmetric could help students to materialize and embody the different types of symmetries.

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