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Optimization consensus modeling of a closed-loop carbon quota trading mechanism regarding revenue and fairness

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Abstract:	Consensus modeling aims to obtain collective agreement through group decision-making (GDM), generally by building mathematical programming models. This paper describes the use of optimization consensus modeling to explore theoretical innovations regarding flexible carbon quota trading mechanisms, with basic allocation schemes provided within a closed-loop trading system by simultaneously taking revenue and fairness into account. A series of optimization consensus models are constructed from the perspective of maximizing the corresponding revenue, resulting in optimal/fair carbon quota allocation schemes that include detailed trading information, e.g., participating individuals, transferred quantities, and unit transaction prices. To solve these models, a relaxation method based on particle swarm optimization is proposed. The inability to conduct real-life GDM usually stems from conflicts of interest based on the decision-makers' mutual competition, thus, two practical strategies are presented to deal with the resulting unfairness within the trading system. Finally, a numerical example incorporating five regions demonstrates the effectiveness of the proposed trading mechanisms. The results show that sufficient interactions among decision-makers are of great significance in achieving fairness within a trading system.

Dear editors and reviewers:

We'd like to express our honest thanks to your critical comments and suggestions concerning our manuscript entitled "Optimization consensus modeling of a closed-loop carbon quota trading mechanism regarding revenue and fairness". We have spared no efforts to revise and improve the paper according to your constructive advices.

Changes in the new version of our paper include but are not limited to:

* The section of Introduction has been improved with some updated references;
* Definition of variables are further clarified, so as to make the closed-loop trading mechanism more reasonable and feasible;
* Several hypotheses have been added in the new version to strengthen the logic of the proposed optimization consensus models;
* Regarding those issues raised by the four anonymous reviewers, we have revised some descriptions in the paper so as to be more accurate.

For your convenience, all changes are marked in blue in the revised manuscript and they won't damage the content and the framework of this paper. We earnestly appreciate Editor's/ Reviewers' constructive work, and hope all the corrections would meet with approval. Once again, thank you very much for your comments and suggestions!

Best regards,

Xiaoxia Xu, Zaiwu Gong, Weiwei Guo, Zhongming Wu, Enrique Herrera-Viedma and Francisco Javier Cabrerizo

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Optimization consensus modeling of a closed-loop carbon quota trading mechanism regarding revenue and fairness

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Highlights:

Carbon quota trading mechanism is explored by optimization consensus modeling.

Flexible allocation schemes are derived by maximizing overall or individual revenue.

Two strategies are raised to realize fairness within the carbon quota trading system.

Relaxation method based on PSO algorithm is used to solve the proposed models.

A numerical example is adopted to demonstrate the feasibility of the models.

Responses to Editor's and Reviewers' Comments

Thank you very much for the letter and the four anonymous reviewers' comments concerning our research paper entitled "Optimization consensus modeling of a closed-loop carbon quota trading mechanism regarding revenue and fairness" (ID: CAIE-D-21-00365). Your comments are of great significance to our research and have provided tremendous help for revising and improving our paper. A**ll changes are marked in blue** in the revised manuscript and detailed responses to editor's and the four reviewers' comments are as follows:

Response to Editor:

AE: The structure of the paper should be enhanced for more readability. Some minor issues (as reviewers' comments) should be addressed.

Response: Thank you very much for your time and effort in dealing with our paper. We have carefully revised our paper according to all critical comments and suggestions. In this version, our revisions are including, but not limited to, some corrections regarding the variables in the proposed models (see pages 7 and 17), the reorganization of our introduction for more readability (see pages 2-4), adding several assumptions to strengthen the logic of the paper (see page 8), and updating some references with the latest information (see pages 38-41). Hope all corrections could meet with approval.

Response to Reviewers:

Reviewer #1: This paper uses optimization-based consensus models to discuss the allocation schemes (including optimal quantities and transaction prices) in a closed-loop carbon quota trading system. Authors consider the principles of both revenue maximization and fairness, and put forward a PSO-based relaxation method to solve the newly proposed models. The work has certain theoretical innovations and practical values. But the following issues need to be focused on.

Response: We sincerely appreciate your positive comment. We carefully considered all the issues you mentioned, and we have made revisions according to your suggestions one by one. Our detailed responses are as follows.

1. The price variables of p_i and q_i are fixed in this paper, whether it is necessary to consider an unfixed case?

Response: Thank you very much for raising this interesting and important issue! The unit price variables of p_i and q_i are actually decision variables in this paper, which have been clarified in Models (3, 6, 7) (see pages 13, 17, 21), indicating that these variables are not really fixed. It just happens that the optimal values of the above variables in different consensus models (as shown in Section 5) are equal. However, combining with the comments from Reviewer #4, we did neglect one important hypothesis that prices fluctuate with time or the changes of supply and demand on the market. In fact, once taking the above hypothesis into consideration, the complexity of the proposed optimization-based consensus models will undoubtedly increase, and further increase the difficulty of solving these models. Currently, we mainly focus on the idea of incorporating consensus modeling into trading mechanisms regarding revenue and fairness, so we might have to ignore some influence factors (e.g., time) on price variables and simplify our problem to the greatest extent, however, we added a relevant assumption as “Variables of unit prices (i.e., p_i, q_i and T_{ij}) are static, meaning that they don't fluctuate with time, supply and demand, and etc.” on page 8 so as to strengthen the logic of our proposed models, also we considered this issue as one important direction of our subsequent research (see page 34).

2. It is better to revise "Table 2 provides the initial carbon quota allocated to each region (i.e., o_i)" on page 7 into "Table 2 provides the initial carbon quota (i.e., o_i) allocated to each region".

Response: Thank you so much! We revised our description as “Table 2 provides the initial carbon quota (i.e., o_i) allocated to each region along with its fixed unit revenue (i.e., r_i), from which the initial total revenue (i.e., $r_i o_i$) of each region can be obtained”, please see page 11.

3. On page 8, how to calculate I_12=2, I_23=1.

Response: Thanks a lot for this important comment! Actually, these values on page 12 are not obtained through calculation here. In other words, all the discussions over Fig. 1 on pages 11-12 are only served as a simple introduction to the modeling ideas proposed in this article. That’s why we made a statement as “Note that the elaborated example only corresponds to the aforementioned basic assumptions, and does not really involve the consensus modeling in the next section” (see the 2nd paragraph on page 11). However, these kinds of variables do be solved by the newly proposed optimization consensus models (see Section 5 on pages 23-33). Anyway, we replaced the phrase “are obtained as” by “are assumed to be obtained through mathematical modeling as” on page 12, so as to avoid confusion.

4. As authors state that "the 80/20 Rule (i.e., the Pareto principle) implies that 20% input is critical and enough for 80% output. Therefore, we may wish to adjust the endpoints of the expected carbon quota interval by 20% of their initial values" on page

19, please clarify what 80% and 20% correspond to in this paper?

Response: We really appreciate this valuable comment! We added more discussion on 80/20 rule as "Initially concluded from a phenomenon of 20% people possessing 80% of the wealth in the world, the 80/20 Rule (i.e., the Pareto principle) is now extended to a fact that an optimal ratio exists between the effort and gain. In other words, once we change 20% of the key factors, qualitative change will occur, implying that we can derive enough (like 80% of) expected results on that critical point", thus we use this rule as a reference to determine the change amount of the original expected intervals in this paper (see page 29). In addition, we also provided some detailed explanation on how to obtain the adjusted intervals from the original ones by combining with the comment from Reviewer #4, please see page 29.

5. Pay attention to the format of your references: "Chu, L. Y. & Shen, Z. - J. M. (2006). Agent competition double auction mechanism. Management science, 52 (8), 1215-1222"; and "CO2" should be CO_2 in "Duro, J. A. & Padilla, E. (2006). International inequalities in per capita CO2 emissions: a decomposition methodology by kaya factors. Energy Economics, 28(2), 170-187."

Response: Thank you, and we fixed these issues according to your comments (see pages 38-39). In addition, we updated some references with their latest information (see pages 38-41).

6. The following references are helpful: Joint decision of financing and ordering in an emission-dependent supply chain with yield uncertainty, CAIE, 2021; Optimal pricing strategy of competing manufacturers under carbon policy and consumer environmental awareness, CAIE, 2019; Integrated decisions for supplier selection and lot-sizing considering different carbon emission regulations in Big Data environment, CAIE, 2020; A production inventory model with interval-valued carbon emission parameters under price-sensitive demand, CAIE, 2021.

Response: We'd like to express our honest thanks to this comment! All the references mentioned above are of significant help for improving our paper, and we cited them mainly in the part of Introduction, please see pages 2-4 and 38-41.

Reviewer #2: This paper deals with the up-to-date issue of optimal and fair allocation on closed-loop carbon quota trading through optimization consensus models, and those methods are plausible to be applied into real-life carbon markets. As the manuscript is oriented mainly on the theoretical aspects, it requires some revisions to provide a convincing and easy way to understand all the elements embodied in those models.

Response: We appreciate a lot for your positive opinions, and we have carefully revised our manuscript, especially the definitions of the variables in the proposed models, so as to provide a more convincing and easier way for readers to understand. Hope you would satisfy all the corrections.

Here are my detailed remarks:

1. As for the keywords on page 1, I suggest to replace "Group decisions and negotiations" by "Group decision-making (GDM)".

Response: We totally agree with your suggestion, and we have replaced "Group decisions and negotiations" by "Group decision-making (GDM)", please see page 1.

2. The introduction on pages 2-4 is a bit lengthy, for example, the sentence seems irrelevant as "Namely, conventional trading mechanisms mainly rely on the interaction between multiple factors such as price, supply and demand, and competition, and eventually promote the harmonious development of a social economy through market players' automatic adjustments to production and operation activities", so I suggest to remove it.

Response: Thank you so much! We removed all the lengthy sentences, and in order to improve the readability of this paper, we also made some revisions in Introduction, which are marked in blue on pages 2-4.

3. Authors need to add some references about the influential factors on carbon trading markets, since they discussed them on page 2.

Response: Thanks a lot for this important comment! We added some references (e.g., Lamba et al., 2019; Ruidas et al., 2021; Zhou et al., 2020b; Zou et al., 2021) regarding the influential factors on carbon trading markets, please see pages 3 and 38-41.

4. On page 9, how about adding the constraint $\sum_{i=1}^{n} o_i^{'} = \sum_{i=1}^{n} o_i$ into Model (3)?

Response: Thank you so much for this valuable comment, but we insist not to add the constraint $\sum_{i=1}^{n} o_i^{'} = \sum_{i=1}^{n} o_i$ into Model (3). This constraint means that the final total amount of carbon quota for all decision-makers (DMs) equals to all DMs' initial total amount, which reflects that the total carbon quota amount in the closed-loop trading system is fixed. In fact, for all $i \in N$, if we respectively add up the two sides of the Eq. (3-1), then we obtain the constraint as $\sum_{i=1}^{n} o_i^{'} = \sum_{i=1}^{n} o_i$. Given our original statements may be not clear enough, we made some revisions on page 13.

5. In system (3) the conditions (3-2) and (3-3) are not needed. Without them, the system has the same solution (they are always satisfied in the optimal solution of the system without them).

Response: Thank you so much for this important comment! Through careful consideration, we believe our original statements regarding the constraints in Model (3) are confusing and misleading, so we made some corrections (see page 13). However, we insist on maintaining Model (3) as its original form due to the following three reasons: (1) The transferred quantities (i.e., the decision variable of I_{ij}) are actually constrained by price variables (see Theorem 6 on page 15), so if we remove constraints (3-2) and (3-3), decision variables of I_{ij} can't be solved; (2) Unit price variables (i.e., p_i and q_i) are decision variables, however, due to insufficient constraints (e.g., the absence of specific transaction prices building connections with these variables), only the ranges of p_i and q_i instead of their optimal values can be obtained, and that's the reason why we didn't give their optimal values in its corresponding model (i.e., Model (9) in Section 5.2 on page 25), and also the reason why we made the previous wrong statement; and (3) the existence of conditions (3-2) and (3-3) in the benchmark Model (3) provides the theoretical foundations and the modeling reasons for those subsequent models. In conclusion, the conditions (3-2) and (3-3) are necessary in system (3).

6. When explaining model (5), why uniform distribution is assumed to denote T_{ij}?

Response: Thank you for raising this question! The reason why we chose uniform distribution to denote T_{ij} is that each point within the interval can be selected with equal possibility, which makes it easy to calculate, understand and be applied into real-life GDM. However, as we pointed out in Section 6, we may incorporate game theory to more accurately determine these variables in our subsequent research (see page 34). To avoid confusion, we added relevant descriptions on page 16.

7. In the process of carbon quota trading, there is a transaction cost. Why we need to take the transaction cost into consideration?

Response: Thank you for this valuable comment! In this paper, we aim to achieve the optimal or fair allocation schemes through consensus modeling within a closed-loop carbon quota trading market. As far as the market is concerned, it is profit-oriented (i.e., simultaneously pursuing the maximization of revenue and the minimization of costs) (see the 1st paragraph on page 3), so the transaction cost is important and cannot be ignored. In other words, if we conduct analysis only from the revenue maximization perspective, it will make no sense. However, we do neglect other costs in real-life trading process, so we added one assumption concerning the cost perspective in the new manuscript, so as to be more logical (see page 8).

Reviewer #3: 1. the authors should explain how have they set the value of δ in Eq.(8-4)? how does the value of δ change the results of solution?

Response: Thank you so much for this valuable comment! δ in Eq. (8-4) is the non-Archimedean infinitesimal, viz. a sufficiently small positive value approaching zero (see Theorem 6 on page 15), normally it's pre-determined, and we set this parameter as $\delta=10^{-6}$ (see page 25), which is small enough compared with other parameters in Section 5. And the value of δ doesn't change the results of solution.

2. Given that, the proposed model had been solved by using PSO algorithm. How do the authors will be sure about the optimality of solutions? it means to check the quality of the PSO algorithm which is an algorithm tailored to optimize the proposed model.

Response: We really appreciate for this helpful comment! It should be noted that the proposed model in our paper is nonconvex, which means it is difficult to obtain the global optimal solution. Due to this, we adopt the PSO algorithm that is a classic and promising approach. Based on the initial point and stopping criteria, it must find a local optimal solution. Furthermore, if the selection of parameters (e.g., initial points) is appropriate, a global optimal solution can be found. We added some references in the paper which can guarantee the optimality of the solution obtained from PSO algorithm, please see (Campana et al., 2010; Sun et al., 2012) on page 22.

3. How did the authors control the feasibility of constraints regarding PSO algorithm? it works based on random search. (Handling constraints)

Response: Thank you so much for this important question! Although PSO algorithm is a random search method, it works and searches the solution within the feasible domain at each iteration, where the feasible domain is constructed by the constraints in our proposed models.

4. The assumptions of model need to reinforcement.

Response: Thank you so much for this valuable suggestion! Currently, this paper mainly focuses on how to use consensus modeling to derive the optimal or fair allocation schemes within a closed-loop carbon quota trading system, and we simplify the problem to the greatest extent, so as to reduce the computational complexity of the proposed models. However, it is really necessary for us to add some assumptions to strengthen the logic of the constructed models considering all reviewers' comments, thus, on page 8, we made some clarifications as:

"To be noted, this paper aims to describe the most essential trading behavior within a carbon quota market by consensus modeling. Meanwhile, in order to reduce the computational complexity of the subsequent models, we currently simplify the problem to the greatest extent. Therefore, several basic assumptions need to be clarified as:

1. The carbon quota market discussed remains stable during a certain period, and DMs can freely participate in the trading system;

2. Price variables (i.e., p_i, q_i and T_{ij}) are static, meaning that they don't fluctuate with time, supply and demand, and etc;

3. Unit revenue of d_i's carbon quota (i.e., r_i) is a constant, which is only determined by d_i's own inherent characteristics rather than o_i, indicating the standard law of diminishing returns assumption is not considered;

4. Factors regarding costs within the profit-oriented trading system are implicit in d_i's initial unit revenue, which means we only need to conduct analysis from the perspective of revenue maximization.

Actually, assumptions listed above are all to reduce the complexity of our GDM problem, and every point could be an interesting topic in our subsequent research."

5. How have the authors set the parameters of PSO algorithm?

Response: Sorry for missing the information on the parameters' setting of our algorithm. We added some relevant descriptions in Section 5.3 on page 27.

Reviewer #4: This paper attempts to explore the theoretical innovation of a flexible carbon quota trading mechanism within a closed-loop trading system. A series of optimization consensus models are constructed to ensure both overall revenue maximization and fairness. The problem considered in the paper is interesting and useful. However, I have several concerns as follows:

Response: Thank you so much for your interest on our paper, and we hope all the revisions could resolve your concerns.

1. (P. 5) In Table 1, symbol p_i represents selling price of d_i's CQ. It should be clearly defined that it represents the unit price or the total price. Similarly, the symbols of q_i and $T_{i,j}$ should be clarified.

Response: Thank you for this helpful suggestion, and we feel sorry for the previous imprecise statements. We made new clarifications on these variables (see Table 1 on page 7), and we also made revisions regarding this issue throughout the manuscript.

2. (p. 6) In the proof of Theorem 1, the authors described "To sum up, once DM d_i buys (sells) carbon quota from (to) d_j, he/she will no longer sell (buy) carbon quota to (from) d_j.".

What if the time or price changes? This statement is inappropriate.

Response: We really appreciate this important comment! In our previous manuscript, we did neglect these important features. As you know, this paper aims to incorporate consensus modeling into obtaining the optimal or fair allocation schemes within a trading system, meanwhile, we want to keep the proposed models as essential as possible, so we simplify the problem to the greatest extent. However, it is really necessary for us to emphasize some features attached to these variables, thus, we added several hypotheses in Section 3 to strengthen the logic of our paper (see page 8), also to make Theorem 1 more appropriate (see page 9).

3. (P. 9) In Model3, the authors mentioned that "Note that no matter with or without conditions (3-2) and (3-3), the above system may have the same solution mathematically, but these constraints must be retained, so as to fully show the trading behaviors within the closed-loop system." However, placing irrelevant conditions in the model increases complexity and can easily confuse readers. It is recommended to separate practical model from descriptions showing the trading behaviors within the closed-loop system. Same for subsequent models.

Response: We appreciate this significant comment! Through careful consideration, we had to admit that our original statements regarding the constraints in Model (3) are confusing and misleading, so we made some corrections (see page 13). However, we insist to keep Model (3) as its original form due to the following three reasons: (1) The transferred quantities (i.e., the decision variable of I_{ij}) are actually constrained by price variables (see Theorem 6 on page 15), so if we remove constraints (3-2) and (3-3), decision variables of I_{ij} can't be solved; (2) Unit price variables (i.e., p_i and q_i) are decision variables, but due to insufficient constraints (e.g., the absence of specific transaction prices building connections with these variables), only the ranges of p_i and q_i instead of their optimal values can be obtained, and that's the reason why we didn't give their optimal values in its corresponding model (i.e., Model (9) in Section 5.2 on page 25), and also the reason why we made the previous wrong statement; and (3) the existence of conditions (3-2) and (3-3) in the benchmark Model (3) provides the theoretical foundations and the modeling reasons for those subsequent models. Thus, the conditions (3-2) and (3-3) are actually necessary in Model (3).

4. (P. 9) In the proof of Theorem 4, "…Theorem 2 implies that there is no Iij >= 0 that would further increase the objective function. Thus, the solution at this time is exactly the optimal solution, and the objective function becomes …"

This proof should be described and verified more clearly and in detail.

Response: Thank you so much for this important suggestion! To make the proof of Theorem 4 clear enough, we added some detailed clarifications as "there exists no $I_{ij} > 0$ to further increase the objective function. That is, except d_m, all DMs have reached their critical points of their expected carbon quota intervals (i.e., $[o_i^-,\ o_i^+]$), either the lower limit of d_i ($1\leq i\leq m$-1) or the upper limit of d_i (m+1$\leq i \leq n$), making DMs with a location index smaller than m cannot further sell carbon quota while DMs with a location index larger than m cannot further buy carbon quota, based on the given condition as $r_1 \leq r_2 \leq \cdots \leq r_n$. In a nutshell, if $I_{ij}> 0$, the objective function of Model (3) increases to $f^*= f +(r_j-r_i)* I_{ij}$, where f is the total revenue before the transaction. Due to $r_j \geq r_i$, ($i < j$), we get $f^*\geq f$, indicating that if and only if I_{ij}=0, the value of the objective function no longer increases and becomes the optimal value" on page 14, and we hope our revisions can resolve your concerns.

5. (P. 12) In Model 3, the decision variables should be clearly defended. Is variable m a given value? If so, how do we get the value of m?

Response: Thank you for this helpful comment! Actually, *m* is a decision variable rather than a given parameter, so we made some clarifications concerning the decision variables in Model (6) (see page 17).

6. (P. 15) In Model 8, a relaxation model was developed to replace Model 6 because Model 6 is a non-convex optimization problem with many decision variables to be determined. Then, PSO algorithm was applied to solve the model. The effect of using the original model and the relaxation model on the results should be further analyzed or discussed.

Response: We really appreciate you for this important suggestion! Actually, Model (8) is the sub-problem of Model (6), thus the solution of the original Model (6) can be directly obtained after solving Model (8). From Algorithm 1, we can see that PSO algorithm is applied to solve the sub-problem of Model (6) (i.e., Model (8)), and Algorithm 1 is designed to solve the original Model (6). Meanwhile, the relaxation model can be seen as a sub-model of Model (6), so it does not affect the solution of the original model. Some relevant discussions were added on page 22.

7. (P. 15) The title of Section 5.1 shows "Flowchart of the research on carbon quota trading mechanism". However, no flowchart can be found in this section.

Response: We are sorry for this mistake, and we have changed it into "Steps of the research on carbon quota trading mechanism", please see page 24.

8. (P. 18) In Table 6, why only the value of q_1 and p_5 is a range instead of an exact number? How do we get these range values directly from Model 10?

Response: We use the relaxation model (i.e., Model (8)) and Algorithm 1 (see pages 21-

23) to solve Model (10). In a nutshell, we first get the optimal values of T_{ij}, then by joint constraints of (10-1) and (10-5), we obtain the optimal values (i.e., the exact number) of p_i and q_i. In fact, p_i and q_i are obtained as intervals when only (10-1) considered, where some variables derive the same values for its two critical points of the interval. Then, the optimal values for p_i and q_i are finally determined after taking the expression of I_{ij} (i.e., the constraint (10-5)) into account. As for the special cases of q_1 (i.e., d_1's unit buying price) and p_5 (i.e., d_5's unit selling price), due to the hypothesis $r_1 \leq r_2 \leq \cdots \leq r_5$, d_1 cannot be a buyer, while d_5 cannot be a seller in the discussed trading market, thus the values of these two variables are obtained as a range instead of an exact number (viz. only subjected to unilateral constraints of T_{ij}), which also make sense in real-life GDM.

9. (P. 20) "…Because d2 sold too much of his/her quota, the quota interval is adjusted from [16; 24] to [20; 24]; and as d5 purchased too much carbon quota, his/her expected range is adjusted from [10; 26] to [10; 23:6]." How to derive "20" and "23.6" should be clearly described.

Response: Thank you for this valuable suggestion! We added some detailed explanation on how to obtain these values, please see page 29.

My general conclusion is that the paper in its present form is not ready for publication in Computers & Industrial Engineering.

Response: We deeply appreciate all your valuable comments and suggestions concerning our manuscript, and hope you would satisfy with all our responses.

To sum up, we have spared no efforts to revise and improve the manuscript. All changes have been marked in blue in the revised version and they won't damage the content and the framework of this paper. We earnestly appreciate Editor's/ Reviewers' constructive work, and hope all the corrections could meet with approval. Once again, thank you very much for your comments and suggestions!

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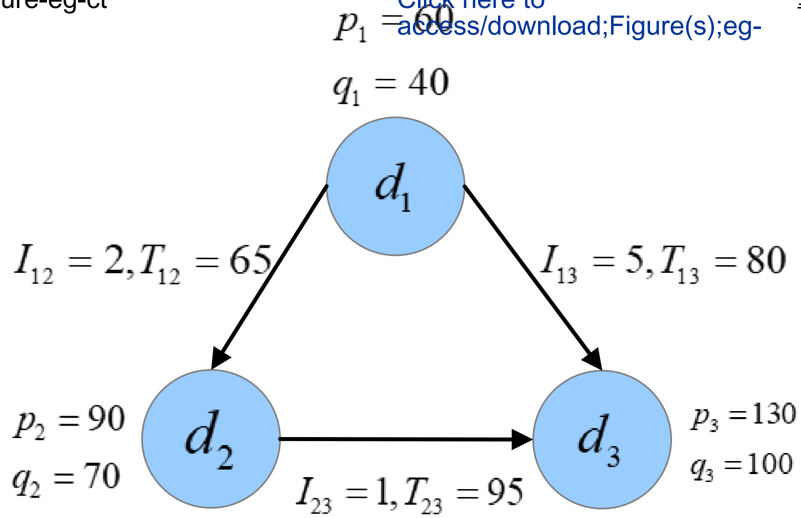
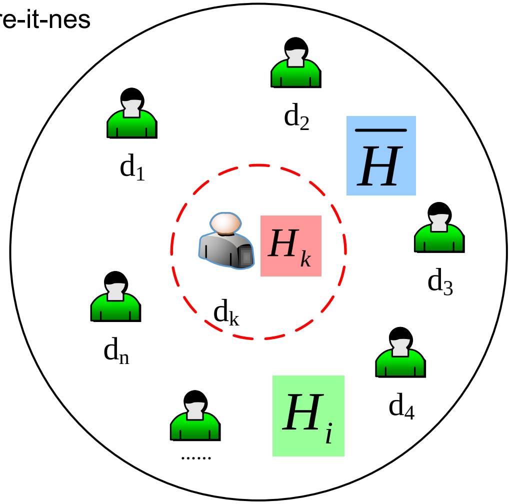
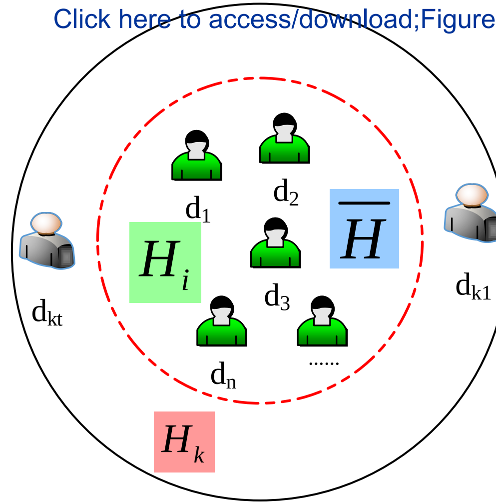
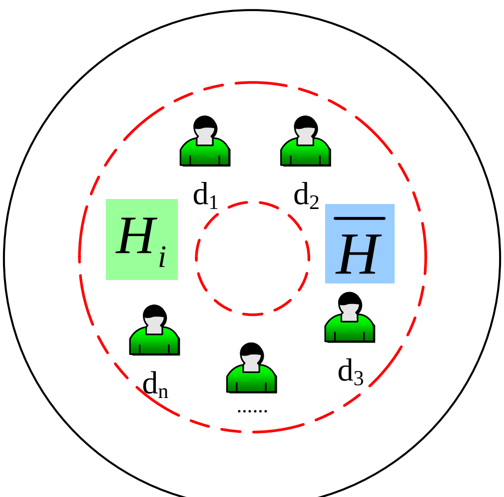




(a) Non-equilibrium state with too small value of H_k for d_k



(b) Non-equilibrium state with too big value of H_k for d_k



(c) Equilibrium state without d_k

Meanings of notations:

$\overline{H}$: Group development index

H_i : Individual development index for DM d_i

H_k : Individual development index for discordant DM d_k

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Optimization consensus modeling of a closed-loop carbon quota trading mechanism regarding revenue and fairness

Abstract

Consensus modeling aims to obtain collective agreement through group decision-making (GDM), generally by building mathematical programming models. This paper describes the use of optimization consensus modeling to explore theoretical innovations regarding flexible carbon quota trading mechanisms, with basic allocation schemes provided within a closed-loop trading system by simultaneously taking revenue and fairness into account. A series of optimization consensus models are constructed from the perspective of maximizing the corresponding revenue, resulting in optimal/fair carbon quota allocation schemes that include detailed trading information, e.g., participating individuals, transferred quantities, and unit transaction prices. To solve these models, a relaxation method based on particle swarm optimization is proposed. The inability to conduct real-life GDM usually stems from conflicts of interest based on the decision-makers' mutual competition, thus, two practical strategies are presented to deal with the resulting unfairness within the trading system. Finally, a numerical example incorporating five regions demonstrates the effectiveness of the proposed trading mechanisms. The results show that sufficient interactions among decision-makers are of great significance in achieving fairness within a trading system.

Keywords: Group decision-making (GDM); Consensus; Revenue and fairness; Carbon quota trading mechanism; Allocation scheme

1. Introduction

Group decision-making (GDM) refers to a process in which multiple individuals participate in decision-making analysis and make a final choice based on their collective wisdom: Clark & Stephenson (1995) have pointed out that GDM represents a collective recall of information. Generally, communication and negotiation effectively promote the interactions among decision-makers (DMs) (Hirokawa & Poole, 1996) and the flow of information within the group. Moreover, technological innovations have significantly updated the means of group communication and decision-making (Kiesler & Sproull, 1992). Without loss of generality, three stable states of fragmentation, polarization, or consensus may finally be achieved by rational DMs considering their own interests (Hegselmann & Krause, 2002; Liang et al., 2020; Zhao et al., 2016). Among them, consensus usually requires multiple rounds of communication, coordination, preference modification, and even concessions or compromises within the group. Only in this way can a relatively consistent collective agreement be obtained (Cabrerizo et al., 2014; Liu et al., 2019; Wu & Chiclana, 2014; Wu et al., 2018; Zhang et al., 2020a,b). For example, if a new allocation scheme of resources is obtained through GDM within a trading system, which is widely accepted by the whole group, then a consensus is reached. Liang et al. (2020) clarified that the consensus-reaching process (CRP) does not mean that an optimal solution must be achieved. Instead, CRP is more like a decision tool or a synthesizing process that assists DMs in building connections and communicating with each other, thereby providing a more effective way for the group to find unity on how to proceed (Susskind et al., 1999).

Considering that cost, which may be embodied as human, material, financial, time and other resources, is an important influencing factor in GDM, Ben-Arieh & Easton (2007) first proposed the concept of minimum cost consensus, and acquired the optimal collective opinion with a linear/quadratic cost function (Ben-Arieh et al., 2009). Since then, other scholars have made further extensions to their minimum cost consensus models (MCCMs) by taking various factors into account, such as uncertain preference structures (Gong et al., 2021; Guo et al., 2021), aggregation rules (Zhang et al., 2011), measurement of consensus effectiveness (Labella et al., 2020) or parameter improvements of initial models (Cheng et al., 2018; Lu et al., 2021; Zhang et al., 2020a). Since unit costs are difficult to objectively determine in advance, and DMs' opinions are hard to modify during GDM, Dong et al. (2010) proposed minimum adjustment consensus models (MACMs) with an ordered weighted average operator, which preserve the DMs' initial preference information as much as possible. Similarly, their modeling idea has also been widely explored (del Moral et al., 2018; Dong et al., 2016; Gong et al., 2020; Yu et al., 2021; Zhang et al., 2018), especially under social networks (Cheng et al., 2020; Wu et al., 2018) or opinion evolution contexts (Chen et al., 2021; Liang et al., 2020). Moreover, Zhang et al. (2020b) summarized the original and basic consensus models based on feedback mechanisms with a minimum cost/adjustment and reviewed diverse consensus modeling under some complicated GDM scenarios.

Different from the above consensus modeling with a minimum cost/adjustment, this paper was partially inspired by the construction of consensus models that aim to maximize the total revenue. By introducing linear primal-dual theory, various MCCMs (including hard and soft consensus (Herrera-Viedma et al., 2014; Zhang et al., 2011)) with specific preference structures (e.g., DM's opinion denoted by crisp numbers or interval values) were adopted as the primal models, and then their corresponding dual forms (i.e., the optimization maximum compensation consensus models) along with their economic significance were deeply explored by Gong et al. (2015a,b) and Zhang et al. (2019). Subsequently, taking the essential architecture of Stackelberg's game into account, Zhang et al. (2020a) presented a bi-level optimization consensus model that depicts the interaction between DMs and the moderator, and divided the DM's total return into a modification component (also known as external compensation) provided by the moderator for the DM's initial preference adjustment and a recognition component based on the similarity between the DM's original opinion and the final consensus. It is well known that the market is profit-oriented (i.e., simultaneously pursuing the maximization of revenue and the minimization of costs) and its operating mechanism is mostly affected by pricing strategy, participants' competition, supply and demand, and etc. (Lamba et al., 2019; Ruidas et al., 2021; Zhou et al., 2020b; Zou et al., 2021). Therefore, in discussing closed-loop trading mechanisms, the revenue maximization of either the whole group or a single DM is set as our objective function in this paper, and constraints such as supply and demand or prices are introduced. A series of optimization consensus models are then constructed as a means of deriving the optimal resource allocation schemes within a trading system.

Rapid industrialization and economic growth have led to significant increases in emissions of carbon dioxide and other greenhouse gases, and have rendered environmental pollution and extreme weather events increasingly serious and frequent, resulting in severe negative impacts on economic development and human health (Wang et al., 2017). Therefore, mitigating the impact of human activities on the environment through reductions in carbon emissions has gradually become a global consensus. Diaz-Rainey & Tulloch (2018) conducted the first empirical analysis of New Zealand's carbon trading scheme using allowance importation and exportation data, and found that the imports of offsets are the major carbon price determinant, with small trading systems able to reap benefits from imposing quantitative import restrictions. Aiming at developing sustainable supply chain, joint decisions were made under various carbon emission regulatory policies, with respect to different influence factors, such as inventory, pricing, financing and ordering (Ruidas et al., 2021; Zhou et al., 2020b; Zou et al., 2021). Furthermore, carbon issues combined with decision-making technology has also been investigated (Gong et al., 2021; Huang & Xu, 2020; Lamba et al., 2019). For instance, Lamba et al. (2019) proposed a mixed-integer nonlinear program for supplier selection and the right lot-sizes determination under a dynamic background with multiple periods, products and suppliers, and evaluated different costs of carbon emissions under three regulating policies (viz. cap-and-trade, strict cap on emissions and carbon tax) using big data technology. Huang & Xu (2020) constructed a bi-level multi-objective programming model to solve the carbon emission quota allocation problem with co-combustion of coal and sewage sludge, and formulated the interaction between authorities and coal-fired power plants before examining a real case demonstrating the trade-off between economic development, energy conservation, and renewable energy utilization.

Setting targets for carbon emissions in different countries/regions (i.e., operating collective schemes for optimal carbon quota allocation) is one of the main obstacles to reaching a comprehensive agreement on global warming. This is exacerbated by long-term tensions between industrialized and developing countries regarding unfairness issues on burden-sharing, with industrialized countries pleading special circumstances and seeking differentiation in their obligations (Rose et al., 1998). Fairness concerns, gained widespread attention in the supply chain management (Liu et al., 2021; Zheng et al., 2019), are also critical for GDM (Du et al., 2021), because participants are motivated by not only the final results, but also the fairness they feel compared with others (Adams, 1963). Under a fixed total carbon quota, the scientific allocation of binding carbon allowances for different regions is a complex and arduous task, because it directly involves the economic development rights of each region. In general, the fairness of carbon emissions quotas is measured using the Atkinson index (Hedenus & Azar, 2005), Theil index (Duro & Padilla, 2006), and Gini coefficient (Chen et al., 2017). The traceability method, which uses historical carbon emissions as the relevant feature of the initial carbon quota allocation (i.e., the free distribution principle), has been criticized by Fromm & Hansjürgens (1996) and Sijm et al. (2007) for being inconsistent with the "polluter pays" principle and lacking fairness from the perspective of society as a whole. In addition, Van Steenberghe (2004) found that the so-called fair rule to allocate greenhouse gas emission permits is not beneficial for all nations, with some countries being worse off under global agreement than under non-cooperative contexts. Under the framework of the Kyoto Protocol, Gomes & Lins (2008) adopted the zero-sum gains data envelopment analysis method to provide a fair carbon emissions allocation plan for various countries, which not only stabilizes the concentration of greenhouse gases in the atmosphere, but also achieves carbon quota trading with no impact on global emissions. The above studies have mostly considered the fairness of carbon quota allocations at the global level, ignoring the interest-driven issues of individual/regional perspectives. Therefore, the analysis of carbon trading mechanisms through consensus modeling with all participators' interests taken into account is of great significance.

Although many studies have investigated carbon issues, there has been few research on carbon quota trading mechanisms, and consensus decision-making theory has not been adopted to deal with the design of carbon trading mechanisms and their resulting unfairness issues. That is, using optimization consensus models to assist DMs in exchanging carbon quotas and the development of fair connections among them within a closed-loop trading system are neglected. Hence, the main contributions of this study are as follows: (i) By referring to conventional market trading mechanisms, a benchmark consensus model with the aim of overall revenue maximization is presented to derive the optimal carbon quota allocation scheme. (ii) By building a two-stage programming model, new allocation schemes are acquired that focus on different single DM's revenue maximization, allowing detailed trading information such as the transferred quantities, DM's unit selling and buying prices, and unit transaction prices to be acquired. (iii) Two strategies based on individual/group development indices are proposed to deal with the unfairness issue within the trading system. (iv) A relaxation method based on particle swarm optimization (PSO) (Kennedy & Eberhart, 1995) is proposed to solve the above consensus models. And (v) numerical analysis of a trading system composed of five regions is conducted to verify the effectiveness of the proposed models.

The rest of this paper is organized as follows. Section 2 briefly reviews the optimization consensus models, then Section 3 presents some assumptions of the trading mechanisms, and justifies the rationality of the hypothesis through theoretical deduction. Section 4 constructs a series of new consensus models from which optimal/fair allocation schemes are obtained within the closed-loop trading system, and further proposes an optimization algorithm to solve these models. A numerical example is reported in Section 5 to demonstrate the feasibility of the proposed mechanisms. Finally, Section 6 gives some concluding remarks and identifies future research directions.

2. Preliminaries on optimization consensus modeling

To better understand the subsequent construction of optimization closed-loop carbon trading consensus models, this section briefly reviews theoretical GDM models for obtaining the optimal consensus. However, before introducing the basic consensus models, we define some related notation. Let $D = \{d_1, d_2, \cdots, d_n\}$ be the set of all DMs, where d_i denotes the i-th DM and $i \in N = \{1, 2, \cdots, n\}$. Let $O = \{o_1, o_2, \cdots, o_n\}$ and $O^{'} = \{o_1^{'}, o_2^{'}, \cdots, o_n^{'}\}$ be the sets of initial and final preferences (i.e., opinions, judgements) of the group, where $o_i, o_i^{'}$ denote d_i's initial and final opinions, respectively. The existing forms of expressions for DMs include, but are not limited to, linear uncertainty preferences (Gong et al., 2020, 2021), linguistic preferences (Cabrerizo et al., 2013; Wu et al., 2018; Yu et al., 2021), fuzzy preference (Herrera-Viedma et al., 2014; Wu & Chiclana, 2014; Zhang et al., 2018). Nevertheless, aiming to solve real-life GDM problems, we adopt traditional forms, i.e., positive and real numbers, to denote DM's opinions in this paper. Let ω_i denote the unit cost provided by the moderator for d_i adjusting his opinions, $i \in N$. In fact, the modeling mechanisms are similar for both MCCM (Ben-Arieh & Easton, 2007; Ben-Arieh et al., 2009) and MACM (Dong et al., 2016, 2010). If all DMs' unit costs satisfy $w_i = w_j, \forall i, j \in N, i \neq j$, then the former reduces to the latter (Zhang et al., 2020b). A general framework of the minimum cost/adjustment consensus model provided by Zhang et al. (2011) can be introduced as:

$$\min \sum_{i=1}^{n} w_i * d(o_i^{'}, o_i)$$
$$\text{s.t.} \begin{cases} o^c = F(o_1^{'}, o_2^{'}, \cdots, o_n^{'}) & (1-1) \\ CD(o_i^{'}, o^c) \leq \alpha, \forall i \in N & (1-2) \end{cases} \tag{1}$$

In Model (1), $d(o_i^{'}, o_i)$ represents the distance or deviation between d_i's initial and final (or adjusted) opinions (del Moral et al., 2018), which is generally given by the Manhattan distance (Ben-Arieh & Easton, 2007) or Euclidean distance (Ben-Arieh et al., 2009). Constraint (1-1) means that the collective opinion (i.e., consensus) o^c should be obtained by the aggregation function F over all DMs' final opinions $\{o_1^{'}, o_2^{'}, \cdots, o_n^{'}\}$, which corresponds to various social selections; and constraint (1-2) measures the consensus level CD attached to d_i's adjusted opinion $o_i^{'}$ and the consensus o^c, where α is a pre-defined threshold that is usually employed when solving soft consensus problems (Herrera-Viedma et al., 2014; Zhang et al., 2011, 2019).

The above model is an optimization consensus model with a minimum cost/adjustment from the moderator's perspective. However, individuals in GDM always expect some compensation for adjusting their opinion, the more the better. Hence, introducing linear primal-dual theory, Gong et al. (2015a,b) and Zhang et al. (2019) explored the dual forms of Model (1) in specific contexts so as to obtain the maximum compensation for all DMs. In particular, Zhang et al. (2019) provided a concise form of the maximum compensation consensus models (i.e., Model (2)), where R means the set of real numbers, and y_i is the unit compensation expected by d_i. As discussed earlier, Zhang et al. (2020a) divided the objective function of Model (2) into a modification return provided by the moderator for the DM's opinion adjustment and a recognition return based on the similarity between the DM's initial opinion and the final consensus. However, their model is omitted here due of space limitations.

$$\begin{aligned} \max \quad & \sum_{i=1}^{n} y_i * (o_i - o^c) \\ \text{s.t.} \quad & y_i \in R, \ i \in N \end{aligned} \tag{2}$$

The optimal collective opinion o^c can always be obtained, regardless from the minimum cost perspective (i.e., Model (1)) or the maximum compensation perspective (i.e., Model (2)). Therefore, the idea of discussing the closed-loop carbon quota trading mechanism with an objective function that maximizes the overall revenue is feasible. In addition, the above two models obtain the optimal collective opinion o^c, whereas this paper aims to derive all DMs' optimal adjusted opinions (i.e., the set of $O^{'}$) during the trading process. Thus, in the following discussion, we introduce some influential factors into the conventional market trading mechanisms and build a series of optimization consensus models that provide optimal or fair carbon quota allocations within a closed-loop trading system.

3. Assumptions for carbon quota trading mechanisms

This paper explores how to develop a satisfactory carbon quota allocation scheme under the goal of maximizing the revenue for either the whole group or a single DM through market trading mechanisms. To facilitate a better understanding, Table 1 presents the main notation used in this paper. Suppose that multiple DMs (e.g., companies, regions, nations) form a closed-loop trading system with a fixed total carbon quota. Let r_i be d_i's initial fixed unit revenue and $r_1 \leq r_2 \leq ... \leq r_n$, where r_i is determined by each DM's unique qualities, such as social and economic development, natural conditions, resource endowments, industrial structures, and energy usage rates.

Table 1 Summary of notation used in this paper

Notation	Meaning	Notation	Meaning
d_i	The i-th DM	I_{ij}	Quantity transferred from d_i to d_j
r_i	Initial fixed unit revenue of d_i's CQ	T_{ij}	Unit transaction price between d_i and d_j
p_i	Unit selling price of d_i's CQ	δ	Non-archimedean infinitesimal
q_i	Unit buying price of d_i's CQ	γ	Fairness threshold
o_i	d_i's initial CQ	α	Fairness measure variable
$o_i^{'}$	d_i's final CQ	Z_1	Obj to maximize overall revenue
o_i^-	Lower limit of d_i's IECQI	Z_2	Obj to maximize a specific DM's revenue
o_i^+	Upper limit of d_i's IECQI	Z_3	Obj regarding revenue and fairness
H_i	Individual development index	$\bar{H}$	Group development index

Note: CQ, IECQI and Obj are short for carbon quota, initially expected carbon quota interval and the objective function, respectively.

To be noted, this paper aims to depict the most essential trading behavior within a carbon quota market by consensus modeling. Meanwhile, in order to reduce the computational complexity of the subsequent models, we currently simplify the problem to the greatest extent. Therefore, several basic assumptions need to be clarified as:

1. The carbon quota market discussed remains stable during a certain period, and DMs can freely participate in the trading system;
2. Variables of unit prices (i.e., p_i, q_i, T_{ij}) are static, meaning that they don't fluctuate with time, supply and demand, and etc.;
3. Unit revenue of d_i's carbon quota (i.e., r_i) is a constant, which is only determined by d_i's own inherent characteristics rather than o_i, meaning that the standard law of diminishing returns assumption is not considered;
4. Factors regarding costs within the profit-oriented trading system are implicit in d_i's initial unit revenue, which means we only need to conduct analysis from the perspective of revenue maximization.

Actually, assumptions listed above are all to reduce the complexity of our GDM problem, and each point could be an interesting topic in our subsequent research. Anyway, the final results obtained from the closed-loop trading system through consensus modeling should satisfy two main objectives:

- Goal 1: Each DM's total revenue derived from the trading is no less than his initial fixed total revenue;
- Goal 2: The sum of all DMs' revenue acquired from the closed-loop trading system should be maximized.

Goal 1 is set from the DM's perspective, and aims to maximize each DM's economic benefits. All DMs are assumed to be rational (that is, once the carbon quota trading is conducted, they must benefit themselves); otherwise, the transactions are invalid. This corresponds to real-life market trading and can be understood as the effectiveness of the trading mechanisms. On the contrary, Goal 2 is set from the collective angle. In general, the representative for the collective benefit is the participant who determines the initial carbon quota for all DMs, and also the one who plays the role as a moderator in GDM problems (Ben-Arieh & Easton, 2007; Gong et al., 2021), such as local governments or world organizations. For those representatives, the primary goal is to maximize the overall revenue.

To realize Goal 1, we have the following constraints: (1) $p_i \geq r_i$, (2) $q_i \leq r_i$, where p_i denotes the unit selling price, q_i represents the unit buying price, and r_i is the original fixed revenue for one unit of d_i's carbon quota. Let the quantity transferred from d_i to d_j be I_{ij}, and their final unit transaction price be T_{ij}. Then, the following statement holds: If $p_i \leq q_j$, then the one-way carbon quota transaction from d_i to d_j can be realized, that is, d_i can sell a carbon quota to d_j, and so $I_{ij} \geq 0$ and the unit transaction price $T_{ij} \in [p_i, q_j]$, which indicates there is a negotiable space in the trading process between d_i and d_j. At the same time, we derive $I_{ji} = 0$, since $I_{ij} * I_{ji} = 0$ holds under the premise of one-way trading.

The above constraint indicates that there is a directionality in the carbon quota trading between any two DMs. Specifically, once a carbon transaction occurs between d_i and d_j, the transferred quantity sold by d_i to d_j is I_{ij}, and we get $r_i \leq p_i \leq q_j \leq r_j$. Moreover, because the unit transaction price satisfies $p_i \leq T_{ij} \leq q_j$, we have that $r_i \leq p_i \leq T_{ij} \leq q_j \leq r_j$. Thus, d_i's revenue is $T_{ij}I_{ij} - r_iI_{ij} \geq 0$, whereas d_j's revenue is $r_jI_{ij} - T_{ij}I_{ij} \geq 0$. This trading mechanism guarantees that every carbon transaction that occurs is profitable for both parties, implying that each DM's final revenue after the carbon trading is no less than their initial total fixed revenue. Thereby, Goal 1 is always met.

Theorem 1. *$I_{ij} * I_{ji} = 0$ and $I_{ij} \geq 0$, $I_{ji} \geq 0$ $(i \neq j, i, j \in N)$, if and only if $p_i = q_i = p_j = q_j = r_i = r_j$, $I_{ij} \geq 0$, and $I_{ji} \geq 0$ hold simultaneously. At this time, the unit selling and buying prices, as well as the initial fixed unit revenue for both d_i and d_j, are equal. In this case, the transaction does not bring about a change in revenue, so it has no economic significance.*

Proof. As $p_i \geq r_i$ and $q_i \leq r_i$, we have $p_i \geq r_i \geq q_i$. When $I_{ij} \geq 0$ and $I_{ji} \geq 0$ hold simultaneously, $p_i \leq q_j$ and $p_j \leq q_i$ are obtained, that is, $r_i \leq p_i \leq q_j \leq r_j \leq p_j \leq q_i \leq r_i$. So when $p_i = q_i = p_j = q_j = r_i = r_j$, both $I_{ij} \geq 0$ and $I_{ji} \geq 0$ hold. Under other situations, if $I_{ij} \geq 0$, we have $I_{ji} = 0$; on the contrary, if $I_{ji} \geq 0$, we get $I_{ij} = 0$. To sum up, based on the aforementioned four assumptions, once DM d_i buys (sells) carbon quota from (to) d_j, he/she will no longer sell (buy) carbon quota to (from) d_j. □

Theorem 1 guarantees that the transactions between any two DMs in the closed-loop carbon quota trading system are one-way. When the initial parameters provided by the two DMs (including unit buying and selling prices as well as their initial fixed unit revenue) are all equal, their transaction has no direction constraint. However, any transaction realized under these conditions cannot increase the DMs' revenue, so it has no economic value.

Theorem 2. *Suppose r_i is d_i's initial fixed unit revenue and $r_1 \leq r_2 \leq ... \leq r_n$, if $i \leq j$, $I_{ij} \geq 0$ holds; if $i > j$ and $r_i \neq r_j$, $I_{ij} = 0$ holds; and if $i > j$ and $r_i = r_j$, $I_{ij} \geq 0$ holds.*

Proof. If $i \leq j$, we have $r_i \leq r_j$, and because $p_i \geq r_i, q_j \leq r_j$, there must exist p_i, q_j such that $r_i \leq p_i \leq q_j \leq r_j$, then $I_{ij} \geq 0$. Besides, if $i > j$ and $r_i \neq r_j$, then $r_i > r_j$, and since $p_i \geq r_i, q_j \leq r_j$, that is, $p_i \geq r_i > r_j \geq q_j$, thus there exist no p_i, q_j such that $p_i \leq q_j$, thereby we have $I_{ij} = 0$. Similarly, if $i > j$ and $r_i = r_j$, then $p_i \geq r_i = r_j \geq q_j$. Clearly, only if $p_i = r_i = r_j = q_j$, $I_{ij} \geq 0$ holds, otherwise, we have $I_{ij} = 0$. □

Theorem 2 takes the basic hypothesis of this paper into consideration: all DMs are arranged in order based on the relationships among their original fixed unit revenues, that is, $r_1 \leq r_2 \leq ... \leq r_n$. The quantity of the carbon quota that is transferred is not only affected by the DM's location index, but also by the size of the DM's fixed unit revenue. This theorem implies that carbon quota trading can only be carried out from one DM with a smaller fixed unit revenue to another with a larger unit revenue. Therefore, DMs with small fixed unit revenues have to sell their carbon quota to increase their total revenue, because $p_i \geq r_i$. On the contrary, DMs with large unit revenues can only improve their revenue by purchasing carbon quotas, because $q_i \leq r_i$.

Theorem 3. *Let d_i's final carbon quota be $o_i^{'}$. Considering that some uncertainty exists during the trading process, the above final carbon quota is represented by an interval value, denoted as $[o_i^-, o_i^+]$, whose endpoints satisfy:*

$$\sum_{i=1}^{n} o_i^- \leq \sum_{i=1}^{n} o_i \leq \sum_{i=1}^{n} o_i^+$$

Proof. Since $o_i^- \leq o_i^{'} \leq o_i^+$, we have $\sum_{i=1}^n o_i^- \leq \sum_{i=1}^n o_i^{'} \leq \sum_{i=1}^n o_i^+$. Meanwhile, because the total carbon quota in the closed-loop trading system is fixed, namely $\sum_{i=1}^n o_i^{'} = \sum_{i=1}^n o_i$, then $\sum_{i=1}^n o_i^- \leq \sum_{i=1}^n o_i \leq \sum_{i=1}^n o_i^+$. □

Theorem 3 is based on the assumption that the total carbon quota in the closed-loop trading system is fixed, which complies with the provisions of the clean development mechanism. That is, under the premise of fixed global carbon emission levels, high-emission countries can finance some projects in low-emission countries to reach their established limit (i.e., compensatory reduction) (Gomes & Lins, 2008). In short, the so-called "carbon market" can reduce the economic impact on high-emission countries and achieve the overall goal of reducing carbon emissions. In addition, for rational DMs, threshold constraints attached to their final carbon quota can better exhibit the uncertainties during the trading process (Ruidas et al., 2021); for the moderator, there is no need to grasp all transaction details, namely, the moderator only needs to have overall control of the total amount, that is, the lower limit of the final total carbon quota is no greater than the initial total amount, while the upper limit should be no less than the sum of all DMs' original carbon quotas.

An example of carbon quota trading conducted by three regions is presented below to preliminarily clarify our modeling ideas. Initial information is listed in Table 2, while the trading results, including the final carbon quota, and the corresponding revenue, are shown in Table 3. Meanwhile, the specific trading process is exhibited in Fig. 1. Note that the elaborated example only corresponds to the aforementioned basic assumptions, and does not really involve the consensus modeling in the next section.

Table 2 Example of the initial information provided by three regions

	d_1	d_2	d_3
o_i	10	10	10
r_i	50	80	120
$r_i o_i$	500	800	1200

Table 3 Example of the final carbon quotas through the trading conducted by three regions

	d_1	d_2	d_3
$o_i^{'}$	3	11	16
r_i	50	80	120
$r_i o_i^{'}$	150	880	1920
Trading revenue	530	-35	-495
Total revenue	680	845	1425

Table 2 provides the initial carbon quota (i.e., o_i) allocated to each region along with its fixed unit revenue (i.e., r_i), from which the initial total revenue (i.e., $r_i o_i$) of each region can be obtained. As d_1 has the smallest unit revenue r_1, this DM can only increase his revenue by selling a carbon quota; as d_3 has the largest unit revenue r_3, this DM can only increase his total revenue by purchasing a carbon quota. For d_2, revenue may be increased by selling, purchasing, or combining both trading behavior (see Fig. 1).

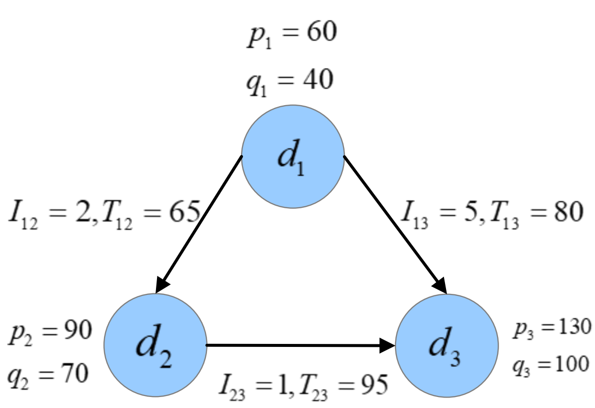

Fig. 1 Schematic diagram of carbon quota trading among three regions

To make the trading mechanism effective and feasible, DM's unit selling price should be no less than his initial unit revenue (i.e., $p_i \geq r_i$), while the unit buying price should be no larger than the fixed unit revenue (i.e., $q_i \leq r_i$). Take d_2 as an example for detailed analysis: the total revenue for d_2's initial carbon quota is $10 * 80 = 800$, and suppose through optimization consensus modeling, d_2's unit selling and buying prices are derived as $p_2 = 90$ and $q_2 = 70$, respectively. The parameters for other regions see Fig. 1. Since $T_{ij} \in [p_i, q_j]$, here we might as well let the unit transaction price be $T_{ij} = \frac{p_i+q_j}{2}$, then we derive $T_{12} = 65, T_{23} = 95$, and the transferred carbon quota quantities related to d_2 are assumed to be obtained through mathematical modeling as $I_{12} = 2, I_{23} = 1$. As a result, d_2's total carbon quota is $10 + 2 - 1 = 11$, and the new fixed revenue for holding his carbon quota is $11 * 80 = 880$, while the transaction revenue (i.e., the difference between the income from selling carbon quotas and the cost of buying quotas) is $1 * 95 - 2 * 65 = -35$, making d_2's final total revenue of $880 - 35 = 845$ be larger than the initial total revenue of 800. Results in Tables 2 and 3 demonstrate that the final revenue of every region in the closed-loop trading system has increased with respect to their initial total revenue, indicating that the proposed trading mechanism is feasible.

4. Optimization consensus modeling concerning carbon quota trading mechanism

Chu & Shen (2006) indicated that the purpose of designing a trading mechanism is to provide a method for ensuring that the allocation decisions and pricing decisions in decision-making processes result in the desired outcomes. They also found that, once the allocation principle is set in a truthful mechanism, the prices are determined; similarly, once the pricing rule is determined, the allocation is settled. Different from the extant research on the carbon market (Diaz-Rainey & Tulloch, 2018; Gomes & Lins, 2008; Lamba et al., 2019; Ruidas et al., 2021; Van Steenberghe, 2004; Zhou et al., 2020b; Zou et al., 2021), this section takes the maximization of the overall revenue or a single DM's revenue as the objective function, and uses optimization consensus modeling to determine the allocation scheme (i.e., determination of variables $o_i^{'}, I_{ij}$) and the pricing scheme (i.e., determination of variables p_i, q_i, T_{ij}) in the carbon quota trading system.

4.1. Benchmark carbon trading consensus model with overall revenue maximization

To realize Goal 2 (as defined in Section 3), we build the following optimization consensus model (i.e., Model (3)) to maximize the sum of the revenues of all DMs within the closed-loop trading system as:

$$\max Z_1 = \sum_{i=1}^{n} r_i o_i^{'}$$
$$\text{s.t.}\begin{cases} o_i^{'} = o_i - \sum\limits_{j=1,j\neq i}^{n} I_{ij} + \sum\limits_{j=1,j\neq i}^{n} I_{ji}, i \in N & (3-1) \\ q_i \le r_i \le p_i, i \in N & (3-2) \\ \begin{cases} I_{ij} \ge 0, \text{if } p_i \le q_j \text{ and } i < j,\ i,j \in N \\ I_{ij} = 0, \text{otherwise} \end{cases} & (3-3) \\ o_i^{-} \le o_i^{'} \le o_i^{+}, i \in N & (3-4) \\ p_i \ge 0, q_i \ge 0, I_{ij} \ge 0, i,j \in N & (3-5) \end{cases} \tag{3}$$

The objective function Z_1 in Model (3) attempts to maximize the final total revenue for all DMs within the carbon quota trading system. Constraint (3-1) is the expression of d_i's final quota, which is equal to the initial quantity minus all the sold quantities $\sum\limits_{j=1,j\neq i}^{n} I_{ij}$ and plus all the purchased quantities $\sum\limits_{j=1,j\neq i}^{n} I_{ji}$, where I_{ij} denotes the carbon quota quantity transferred from d_i to d_j. Since the sum of all transfer-out quantities equals to the sum of all transfer-in quantities, we can easily obtain $\sum_{i=1}^{n} o_i^{'} = \sum_{i=1}^{n} o_i$ through constraint (3-1), corresponding to the fact that the total carbon quota amount in the closed-loop trading system is fixed. Constraint (3-2) is the threshold constraint attached to the unit selling price p_i and the unit buying price q_i based on the pre-defined initial fixed unit revenue r_i. Constraint (3-3) specifies the achievable conditions of the carbon trading between any two DMs. Namely, only when the seller's location index is smaller than the purchaser's index, and the unit selling price p_i is no greater than the unit buying price q_j, will the transaction from d_i to d_j be achieved (i.e., $I_{ij} \ge 0$). Constraint (3-4) assumes that d_i's final quota is located in his own expected interval provided initially. Constraint (3-5) indicates that all variables are nonnegative. Hence, Model (3) explores the optimal carbon quota allocation problem under the maximization of the overall revenue of the trading system, where $Z_1, o_i^{'}, I_{ij}, p_i, q_i, (i \in N)$ are decision variables and $r_i, o_i, o_i^{-}, o_i^{+}$ are known parameters. In fact, due to insufficient constraints (e.g., the absence of specific transaction prices building connections with the unit price variables), only the ranges of p_i and q_i instead of their optimal values can be obtained through Model (3).

***Theorem* 4.** *There must exist an m-th DM such that* $\sum_{i=1}^{m-1} o_i^- + o_m^{'} + \sum_{i=m+1}^{n} o_i^+ = \sum_{i=1}^{n} o_i$ *and* $o_m^- \leq o_m^{'} \leq o_m^+$*. By then, the optimal value for the objective function of Model (3) is* $\sum_{i=1}^{m-1} r_i o_i^- + r_m o_m^{'} + \sum_{i=m+1}^{n} r_i o_i^+$ *and the optimal solution is* $o_i^{'} = o_i^-(1 \leq i \leq m-1), o_m^{'} = \sum_{i=1}^{n} o_i - \sum_{i=1}^{m-1} o_i^- - \sum_{i=m+1}^{n} o_i^+, o_i^{'} = o_i^+(m+1 \leq i \leq n)$.

Proof. First, when $o_i^{'} = o_i^-(1 \leq i \leq m-1), o_m^{'} = \sum_{i=1}^{n} o_i - \sum_{i=1}^{m-1} o_i^- - \sum_{i=m+1}^{n} o_i^+, o_i^{'} = o_i^+(m+1 \leq i \leq n)$, there exists no $I_{ij} > 0$ to further increase the objective function. That is, except d_m, all DMs have reached their critical points of their expected carbon quota intervals (i.e., $[o_i^-, o_i^+]$), either the lower limit of d_i $(1 \leq i \leq m-1)$ or the upper limit of d_i $(m+1 \leq i \leq n)$, making DMs with a location index smaller than m cannot further sell carbon quota while DMs with a location index larger than m cannot further buy carbon quota, based on the given condition as $r_1 \leq r_2 \leq ... \leq r_n$. In a nutshell, if $I_{ij} > 0$, the objective function of Model (3) increases to $f^* = f + (r_j - r_i) * I_{ij}$, where f is the total revenue before the transaction. Due to $r_j \geq r_i, (i < j)$, we get $f^* \geq f$, indicating that if and only if $I_{ij} = 0$, the value of the objective function no longer increases and becomes the optimal value. Thus, the solution at this point is exactly the optimal solution, and the objective function becomes $\sum_{i=1}^{m-1} r_i o_i^- + r_m o_m^{'} + \sum_{i=m+1}^{n} r_i o_i^+$.

Next, we prove that this critical DM with the m-th location index always exists. Because $o_i^- \leq o_i^{'} \leq o_i^+$, we have $\sum_{i=1}^{n} r_i o_i^- \leq \sum_{i=1}^{n} r_i o_i^{'} \leq \sum_{i=1}^{n} r_i o_i^+$. If $m = 1$, then $r_1 o_1^- + \sum_{i=2}^{n} r_i o_i^+ \leq \sum_{i=1}^{n} r_i o_i^{'} \leq \sum_{i=1}^{n} r_i o_i^+$. If $m = 2$, then $\sum_{i=1}^{2} r_i o_i^- + \sum_{i=3}^{n} r_i o_i^+ \leq \sum_{i=1}^{n} r_i o_i^{'} \leq r_1 o_1^- + \sum_{i=2}^{n} r_i o_i^+$. In the same way, if $m = n$, then $\sum_{i=1}^{n} r_i o_i^- \leq \sum_{i=1}^{n} r_i o_i^{'} \leq \sum_{i=1}^{n-1} r_i o_i^- + r_n o_n^+$. Therefore, once m takes a specific value within the set N, $\sum_{i=1}^{n} r_i o_i^{'}$ can take any value from the interval $[\sum_{i=1}^{n} r_i o_i^-, \sum_{i=1}^{n} r_i o_i^+]$, and so the known constraint $\sum_{i=1}^{n} r_i o_i^- \leq \sum_{i=1}^{n} r_i o_i^{'} \leq \sum_{i=1}^{n} r_i o_i^+$ means that d_m must exist such that $o_i^{'} = o_i^-(1 \leq i \leq m-1), o_m^{'} = \sum_{i=1}^{n} o_i - \sum_{i=1}^{m-1} o_i^- - \sum_{i=m+1}^{n} o_i^+, o_i^{'} = o_i^+(m+1 \leq i \leq n)$ hold. □

***Theorem* 5.** *When Model (3) reaches its maximum value, we obtain* $\sum_{j=1,j\neq i}^{n} I_{ij} - \sum_{j=1,j\neq i}^{n} I_{ji} = o_i - o_i^-$ $(1 \leq i \leq m-1)$, $\sum_{j=1,j\neq m}^{n} I_{mj} - \sum_{j=1,j\neq m}^{n} I_{jm} = o_m - \sum_{i=1}^{n} o_i + \sum_{i=1}^{m-1} o_i^- + \sum_{i=m+1}^{n} o_i^+$, $\sum_{j=1,j\neq i}^{n} I_{ij} - \sum_{j=1,j\neq i}^{n} I_{ji} = o_i - o_i^+$ $(m+1 \leq i \leq n)$.

Proof. Theorem 4 implies that once Model (3) reaches its maximum value, and if $1 \leq i \leq m-1$, then $o_i^{'} = o_i^-$ holds, meantime, due to $o_i^{'} = o_i - \sum_{j=1,j\neq i}^{n} I_{ij} + \sum_{j=1,j\neq i}^{n} I_{ji}$, we have $\sum_{j=1,j\neq i}^{n} I_{ij} - \sum_{j=1,j\neq i}^{n} I_{ji} = o_i - o_i^-(1 \leq i \leq m-1)$. Similar analysis can be conducted for the rest situations. □

Theorems 4 and 5 indicate that the optimal solution of Model (3) and the maximum value of the objective function exist and are unique. Therefore, the optimal allocation for all DMs' carbon quotas is determined. In other words, by solving Model (3), we obtain all information about carbon quota transfers within the trading system. However, note that only the feasible regions can be obtained by Model (3), rather than the optimal values of the decision variables p_i, q_i.

***Theorem* 6.** *The achievable constraints of the carbon quota trading mechanism are determined by d_i's unit selling price p_i and d_j's unit buying price q_j as:*

$$\begin{cases} I_{ij} \geq 0, & \text{if } p_i \leq q_j \text{ and } i < j,\ i, j \in N \\ I_{ij} = 0, & \text{otherwise} \end{cases}$$

which is equivalent to

$$\begin{cases} I_{ij} \leq \dfrac{|q_j - p_i| + q_j - p_i}{\delta}, & i < j, i, j \in N \\ I_{ij} = 0, & \text{otherwise} \end{cases} \tag{4}$$

where δ is the non-Archimedean infinitesimal, viz. a sufficiently small positive value approaching zero (Charnes et al., 1994; Mehrabian et al., 2000).

Proof. If $i < j, i, j \in N$, then carbon quota trading between the seller d_i and the purchaser d_j is achievable, so $I_{ij} \geq 0$ holds. Next, we discuss the effect of prices on the transferred quantity: when $p_i < q_j$, according to Eq. (4), we have $I_{ij} < \dfrac{2(q_j - p_i)}{\delta}$, and because δ is the non-Archimedean infinitesimal, $I_{ij} < +\infty$, that is, $I_{ij} \geq 0$ holds; when $p_i \geq q_j$, based on Eq. (4), we have $I_{ij} = 0$. In addition, if $i \geq j, i, j \in N$, the one-way transaction from d_i to d_j cannot be achieved, so we have $I_{ij} = 0$. This completes the proof of Theorem 6. □

Theorem 6 states the achievable conditions for a closed-loop trading system. Specifically, carbon quota trading can only be achieved when the unit selling price of one DM with a small location index is no greater than the unit buying price of another DM with a large location index; otherwise, their carbon quota transaction fails.

4.2. Carbon trading consensus models with single DM's revenue maximization

The competition mechanism refers to the struggle among market practitioners to maximize their own economic benefits, so it focuses more on individual standpoints than the collective perspective. The model developed in

Section 4.1 only maximizes the overall revenue of the trading process, and ignores the individual DM's interests and the resulting unfairness issues. This section considers individual DMs as the research object, and uses optimization consensus models to derive detailed information about the trading process, including the participating DMs, transferred quantities, and the final unit transaction prices. That is, when the group realizes its optimal allocation by considering every DM's revenue maximization, this section attempts to determine not only d_i's final carbon quota $o_i^{'}$ from its expected interval $[o_i^-, o_i^+]$, but also each DM's psychological expected unit selling and buying prices (i.e., p_i, q_i) and the transferred quantity I_{ij} along with the best achievable unit transaction price T_{ij}. Based on the above principles, a two-stage programming model is built as:

$$
\max Z_2 = r_i o_i^{'} + \sum_{j=1,j\neq i}^{n} T_{ij} I_{ij} - \sum_{j=1,j\neq i}^{n} T_{ji} I_{ji}
$$
$$
\text{s.t.} \begin{cases} \begin{cases} p_i \leq T_{ij} \leq q_j, & \text{if } p_i \leq q_j,\ i<j, i,j \in N \\ T_{ij} = 0, & \text{otherwise} \end{cases} & (5-1) \\ \begin{cases} Max \ \sum_{i=1}^{n} r_i o_i^{'} \\ o_i^{'} = o_i - \sum_{j=1,j\neq i}^{n} I_{ij} + \sum_{j=1,j\neq i}^{n} I_{ji}, i \in N \\ q_i \leq r_i \leq p_i, i \in N \\ \begin{cases} I_{ij} \leq \dfrac{|q_j - p_i| + q_j - p_i}{\delta}, & i<j, i,j \in N \\ I_{ij} = 0, & \text{otherwise} \end{cases} \\ o_i^- \leq o_i^{'} \leq o_i^+,\ p_i \geq 0, q_i \geq 0, I_{ij} \geq 0, T_{ij} \geq 0, \delta > 0, i,j \in N \end{cases} & (5-2) \end{cases} \tag{5}
$$

Model (5) introduces constraint (5-1) into Model (3), that is, adding the expression of the unit transaction price T_{ij} between DMs d_i and d_j, which is a range bounded by d_i's unit selling price p_i and d_j's unit buying price q_j. As stated in Section 3, only the location indices satisfy $i < j, i, j \in N$, and $p_i \leq q_j$ holds, can the unit transaction price between d_i and d_j be denoted as $T_{ij} \in [p_i, q_j]$. Here, the unit transaction price T_{ij} obeys a uniform distribution by default, as each point within the interval $[p_i, q_j]$ can be selected with equal possibility, which makes it easy to calculate, understand and be applied into real-life GDM. The objective function in Model (5) is the sum of d_i's final carbon quota holding revenue (i.e., $r_i o_i^{'}$) and the transaction revenue for selling or buying carbon quotas (i.e., $\sum_{j=1,j\neq i}^{n} T_{ij} I_{ij} - \sum_{j=1,j\neq i}^{n} T_{ji} I_{ji}$), and this value the larger the better. Model (5) indicates that maximizing a single DM's revenue is not unconstrained; instead, it should be carried out within the context of maximizing the overall revenue for the whole group (i.e., constraint (5-2)). Referring to Theorem 4, Model (5) can be further transformed into Model (6), where constraints (6-2)–(6-9) provide the analytical formula of constraint (5-2). The definitions of other variables and constraints in Model (6) are consistent with those in Models (3) and (5).

Theorem 4 states that the optimal solution of Model (3) exists and is unique. Thus, there must exist feasible solutions for Model (6). Actually, constraints (6-6)–(6-8) in Model (6) provide the analytical formula for the DM's final carbon quota $o_i^{'}$, and are acquired by solving Model (3). Hence, variables $Z_2, I_{ij}, p_i, q_i, T_{ij}$ and m in Model (6) are decision variables, while $r_i, o_i, o_i^-, o_i^+, \delta$ are known parameters. In short, under the premise of maximizing the overall revenue, and by further adding the expression of the unit transaction prices, Model (6) determines the optimal values for d_i's unit selling and buying prices (i.e., p_i and q_i), and further obtains detailed trading information including the quantity I_{ij} transferred from d_i to d_j and their corresponding unit transaction price T_{ij}.

$$\max Z_2 = r_i o_i^{'} + \sum_{j=1, j\neq i}^{n} T_{ij} I_{ij} - \sum_{j=1, j\neq i}^{n} T_{ji} I_{ji}$$

$$\text{s.t.} \begin{cases} \begin{cases} p_i \leq T_{ij} \leq q_j, & \text{if } p_i \leq q_j,\ i<j, i,j \in N \\ T_{ij} = 0, & \text{otherwise} \end{cases} & (6-1) \\ \sum_{i=1}^{n} r_i o_i^{'} = \sum_{i=1}^{m-1} r_i o_i^- + r_m o_m^{'} + \sum_{i=m+1}^{n} r_i o_i^+ & (6-2) \\ o_i^{'} = o_i - \sum_{j=1, j\neq i}^{n} I_{ij} + \sum_{j=1, j\neq i}^{n} I_{ji}, i \in N & (6-3) \\ q_i \leq r_i \leq p_i, i \in N & (6-4) \\ \begin{cases} I_{ij} \leq \dfrac{|q_j - p_i| + q_j - p_i}{\delta}, & i<j, i,j \in N \\ I_{ij} = 0, & \text{otherwise} \end{cases} & (6-5) \\ o_i^{'} = o_i^-, 1 \leq i \leq m-1 & (6-6) \\ o_m^{'} = \sum_{i=1}^{n} o_i - \sum_{i=1}^{m-1} o_i^- - \sum_{i=m+1}^{n} o_i^+,\ o_m^- \leq o_m^{'} \leq o_m^+ & (6-7) \\ o_i^{'} = o_i^+, m+1 \leq i \leq n & (6-8) \\ p_i \geq 0,\ q_i \geq 0, I_{ij} \geq 0, T_{ij} \geq 0, \delta > 0, i, j, m \in N & (6-9) \end{cases} \quad (6)$$

4.3. Identification and adjustment rules for discordant DMs

In Section 4.2, we considered the case in which every single DM pursues the maximization of his own revenue, which inevitably results in unfairness (e.g., the unbalanced growth of the DMs' revenue). Therefore, this section examines the potential to achieve a relatively balanced state within the closed-loop carbon quota trading system by adjusting some DMs' initial parameters. Once fairness is achieved, DMs with too much revenue growth or too little revenue growth should no longer exist in the final stage of carbon trading. Any such DMs are collectively referred to as **discordant DMs** in the trading system. During CRP, if the DMs' improper initial parameters can be modified as early as possible, systemic losses (e.g., cost, time) will be significantly reduced. In short, an earlier intervention during GDM is more advantageous (Liang et al., 2020). Compared with extant research adopting utility function (Du et al., 2021) or fuzzy theory (Liu et al., 2021) to characterize the fairness concerns, this paper defines two indicators to directly judge whether the GDM results are fair, so as to further identify discordant DMs and make some corresponding adjustments.

***Definition* 1.** An individual development index is defined as a relative proportion of the DM's final revenue obtained through the carbon quota trading process with respect to their initial fixed revenue, that is,

$$H_i = \frac{r_i o_i^{'} + \sum\limits_{j=1, j\neq i}^{n} T_{ij} I_{ij} - \sum\limits_{j=1, j\neq i}^{n} T_{ji} I_{ji}}{r_i o_i}, i \in N$$

***Definition* 2.** The group development index is defined as a relative proportion of the final total revenue obtained through the carbon quota trading process with respect to the initial fixed total revenue of the group, that is,

$$\bar{H} = \frac{\sum\limits_{i=1}^{n} r_i o_i^{'}}{\sum\limits_{i=1}^{n} r_i o_i}$$

This section follows the idea of fair development of all DMs in the trading system. By default, the difference between the individual development index H_i and the group development index $\bar{H}$ should be within a certain range, otherwise DMs will be identified as discordant DMs with too much or too little revenue growth. These two development indices mainly depend on the DM's final carbon quota $o_i^{'}$, which further depends on the endpoints of the expected interval $[o_i^-, o_i^+]$ provided by DM d_i. Here, we choose interval values instead of crisp numbers to denote d_i's expected carbon quota quantity due to various uncertainties (Ruidas et al., 2021). Hence, by adjusting the expected carbon quota range $[o_i^-, o_i^+]$ of discordant DMs, an equilibrium state with a minimum loss can be achieved within the trading system (see Fig. 2(c)). Let a discordant DM be $d_k, k \in \{0, 1, \cdots, n\}$, and his expected final carbon quota be adjusted from $[o_k^-, o_k^+]$ to $[o_k^{'-}, o_k^{'+}]$ through the following adjustment rules.

- When $H_k << \bar{H}$ and $|H_k - \bar{H}| > \gamma$, where γ is a pre-determined threshold and $<<$ denotes far less than, d_k is identified as a discordant DM with too little revenue growth. This DM is located in the unbalanced state shown in Fig. 2(a), and his adjustment rules are:
  - If $k > m$, then the amount purchased is too little, and so o_k^+ needs to be increased;
  - If $k < m$, then the amount sold is too little, and so o_k^- needs to be further decreased;
  - If $k = m$, then the current expected interval is improperly set, and we need to simultaneously reduce o_k^- and increase o_k^+.
- When $H_k >> \bar{H}$ and $|H_k - \bar{H}| > \gamma$, where γ is a pre-determined threshold and $>>$ means far more than, d_k is identified as a discordant DM with too much revenue growth. This DM is located in the unbalanced state shown in Fig. 2(b), and his adjustment rules are:
  - If $k > m$, then the quantity purchased is too great, and so o_k^+ needs to be decreased;
  - If $k < m$, then the amount sold is too great, and so o_k^- should be increased;
  - If $k = m$, then the current interval of the DM's expected carbon quota is inappropriate, and we need to increase o_k^- and decrease o_k^+ at the same time.

Through the above adjustment rules, a set of updated trading information for all DMs can always be acquired. Based on the individual/group development indices, we obtain the values of all $|H_i - \bar{H}|$ based on Model (6) so as to determine the threshold for the variable γ, as well as the difference value $|H_i - H_j|$ between any two DMs. By repeating the calculations of Models (3) and (6), it is then possible to verify whether the above adjustments are effective or not. The above rules are used to identify discordant DMs and provide the corresponding direction of adjustments. However, the identification parameter γ needs to be manually set, and the specific adjustment range for each DM cannot be accurately specified, that is, we cannot determine by how much each discordant DM needs to adjust the upper and lower limits of their initial expected carbon quota intervals. To overcome these deficiencies, a fairness measure variable α is introduced in the next section, and the optimal carbon quota allocation scheme considering fairness is directly acquired through consensus modeling. Furthermore, by applying a sensitivity analysis to the variable α, we can obtain flexible allocation schemes according to specific GDM scenarios.

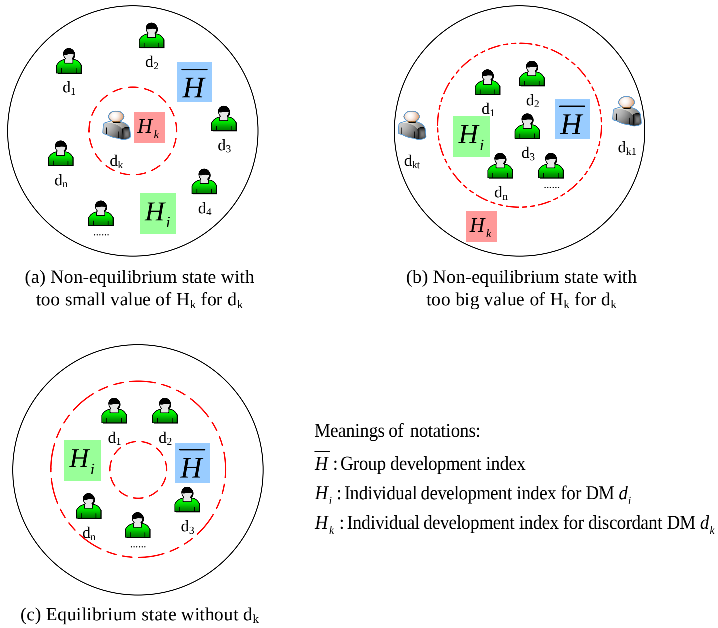

Fig. 2 Identification of non-equilibrium states in closed-loop carbon trading system

4.4. Carbon trading consensus model regarding fairness and revenue

When only a single DM's revenue is considered, the overall revenue cannot be maximized; moreover, when only the overall revenue is taken into account, there can be large gaps between the total revenue of different DMs, highlighting the unfairness issues. Thus, this section introduces a fairness constraint (that is, the difference between any two individual development indices should be within a certain acceptable threshold) under the premise of ensuring the maximization of the overall revenue. Specifically, the fairness constraint is expressed as $|H_i - H_j| \leq \alpha(\alpha \geq 0, i < j, i, j \in N)$, and the optimization carbon quota consensus model considering both revenue and fairness is built as follows:

$$
\max\ Z_3 = \sum_{i=1}^{n} r_i o_i^{'}
$$

$$
\text{s.t.}\begin{cases}
o_i^{'} = o_i - \sum\limits_{j=1, j\neq i}^{n} I_{ij} + \sum\limits_{j=1, j\neq i}^{n} I_{ji}, i \in N & (7-1)\\
q_i \leq r_i \leq p_i, i \in N & (7-2)\\
\begin{cases} p_i \leq T_{ij} \leq q_j, & \text{if } p_i \leq q_j,\ i<j, i,j \in N \\ T_{ij} = 0, & \text{otherwise} \end{cases} & (7-3)\\
\begin{cases} I_{ij} \leq \dfrac{|q_j - p_i| + q_j - p_i}{\delta}, & i<j, i,j \in N \\ I_{ij} = 0, & \text{otherwise} \end{cases} & (7-4)\\
H_i = \dfrac{r_i o_i^{'} + \sum\limits_{j=1, j\neq i}^{n} T_{ij} I_{ij} - \sum\limits_{j=1, j\neq i}^{n} T_{ji} I_{ji}}{r_i o_i}, i \in N & (7-5)\\
|H_i - H_j| \leq \alpha, i<j, i,j \in N & (7-6)\\
o_i^{-} \leq o_i^{'} \leq o_i^{+},\ q_i \geq 0,\ p_i \geq 0, I_{ij} \geq 0, T_{ij} \geq 0, \delta > 0, \alpha \geq 0, i,j \in N & (7-7)
\end{cases} \tag{7}
$$

Z_3 in Model (7) aims to maximize the overall revenue after carbon quota trading under the premise that each DMs' revenue has been fairly developed. Constraint (7-1) is the expression of d_i's final carbon quota, which guarantees $\sum_{i=1}^{n} o_i^{'} = \sum_{i=1}^{n} o_i$. Constraint (7-2) sets d_i's optimal psychological expected unit selling price p_i and unit buying price q_i based on his own initial fixed unit revenue r_i. Constraint (7-3) denotes the unit transaction price between any two DMs, and (7-4) provides the achievable conditions for carbon quota trading considering both the DMs' location indices (i.e., i, j) and the relationships between p_i and q_j. Constraint (7-5) defines the individual development index (i.e., Definition 1), and (7-6) specifies the fairness constraints attached to different DMs, where $\alpha \geq 0$ is the fairness measure variable that is pre-determined from the differences among individual development indices (see Section 4.3). Finally, (7-7) provides the thresholds for all variables. Variables $Z_3, o_i^{'}, I_{ij}, p_i, q_i, T_{ij}, H_i, (i \neq j, i, j \in N)$ in Model (7) are to be solved, while $r_i, o_i, o_i^{-}, o_i^{+}, \alpha, \delta, (i \in N)$ are determined in advance.

4.5. Solution method to solve carbon trading consensus models

Clearly, Model (6) is a non-convex optimization problem with many decision variables to be determined. As the pricing decisions (i.e., variables p_i, q_i) have no direct effect on the objective function Z_2, we remove these two variables using constraints (6-1), (6-4), and (6-5), thus obtaining the following relaxation model:

$$
\max\ Z_2 = r_i o_i^{'} + \sum_{j=1, j \neq i}^{n} T_{ij} I_{ij} - \sum_{j=1, j \neq i}^{n} T_{ji} I_{ji}
$$
$$
\text{s.t.} \begin{cases}
\begin{cases} T_{ij} = 0, & \text{if } i \geq j,\ i, j \in N \\ r_i \leq T_{ij} \leq r_j, & \text{if } i < j,\ i, j \in N \end{cases} & (8-1) \\
\sum_{i=1}^{n} r_i o_i^{'} = \max\ Z_1 & (8-2) \\
o_i^{'} = o_i - \sum_{j=1, j \neq i}^{n} I_{ij} + \sum_{j=1, j \neq i}^{n} I_{ji}, i \in N & (8-3) \\
\begin{cases} I_{ij} = 0, & \text{if } i \geq j, i, j \in N \\ I_{ij} \leq \dfrac{2(r_j - r_i)}{\delta}, & \text{if } i < j, i, j \in N \end{cases} & (8-4) \\
o_i^{-} \leq o_i^{'} \leq o_i^{+}, I_{ij} \geq 0, T_{ij} \geq 0, \delta > 0, i, j \in N & (8-5)
\end{cases} \tag{8}
$$

where Z_1 is the maximum value obtained from Model (3), and definitions of other variables and constraints refer to Model (6). Without loss of generality, Model (6) is the original problem, and the relaxation model (i.e., Model (8)) is its sub-problem, thus the solution of Model (6) can be directly obtained after solving Model (8). In other words, as the submodel of Model (6), the solution of the relaxation Model (8) doesn't affect the results of Model (6).

To our knowledge, PSO algorithm was put forward to optimize nonlinear functions based on the initial point and stopping criteria (Kennedy & Eberhart, 1995), and has been proven to be an effective tool for streamlining decision making (Cabrerizo et al., 2013; Zhou et al., 2020a). In this paper, a relaxation method based on the PSO algorithm (i.e., Algorithm 1) is proposed for determining the optimal solution of Model (6). In specific, Algorithm 1 is proposed to solve the original problem (i.e., Model (6)), while PSO algorithm is used to solve its sub-problem (i.e., Model (8)). Note that, if the selection of parameters (e.g., initial points) is appropriate, a global optimal solution can be found (Campana et al., 2010; Sun et al., 2012). Generally, the above-mentioned non-convex models can be linearized and solved using standard exact solvers, but linearization only obtains an approximate solution, while our proposed relaxation method can derive the equivalent form of the original problem. By adopting similar principles, a fine-tuning algorithm can be used to solve Model (7), but it is omitted here due to space limitations.

Algorithm 1 Relaxation method based on PSO algorithm for solving Model (6).

Input: Number of DMs, N; d_i's initial carbon quota, o_i; d_i's initial fixed unit revenue, r_i; d_i's expected carbon quota interval, $[o_i^-, o_i^+]$; the maximum overall revenue obtained from Model (3), Z_1; the maximal number of iterations, *limit*; population size, M.

Output: d_i's final carbon quota, $o_i^{'}$; d_i's unit selling and buying prices, p_i, q_i; the transferred quantity, I_{ij}; the unit transaction price, T_{ij}; the specific DM's maximum total revenue, Z_2.

Step 1: Remove decision variables p_i, q_i to obtain a relaxation optimization model (see Model (8)), based on constraints (6-1), (6-4) and (6-5);

Step 2: Use PSO algorithm to solve Model (8);

Set current iteration as $t = 0$;
for each particle i **do**
    Initialize velocity V_i and position X_i for particle i;
    Evaluate particle i by the defined fitness function and set $pBesti = X_i$;
end for
$gBest$=min $\{pBesti\}$;
while t<$limit$ **do**
    for $i = 1$ to M **do**
        Update the velocity and position of particle i;
        Evaluate particle i by the defined fitness function;
        if $fit(X_i) < fit(pBesti)$ **then return** $pBesti$=X_i;
        end if
        if $fit(pBesti) < fit(gBest)$ **then return** $gBest = pBesti$;
        end if
    end for
end while
Z_2= $-fit(gBest)$;
return Best solution of $o_i^{'}, I_{ij}, T_{ij}, Z_2$.

Step 3: Derive the optimal values of p_i, q_i based on relaxation constraints (6-1) and (6-5) .

5. Numerical analysis

To verify the feasibility of the optimization consensus models proposed in this paper, this section presents a numerical case study. Suppose there are five regions (d_1, d_2, d_3, d_4, d_5) in a closed-loop carbon quota trading system (i.e., $N = \{1, \cdots, 5\}$). The initial information provided by each region is summarized in Table 4.

Table 4 Summary of the initial trading information provided by five regions

Regions	r_i	o_i	o_i^-	o_i^+	$r_i o_i$
d_1	12	16	13	19	192
d_2	15	20	16	24	300
d_3	23	34	27	41	782
d_4	34	18	14	22	612
d_5	40	12	10	26	480
Total	—	100	80	132	2366

Note: d_i is the i-th region; r_i denotes the initial fixed unit revenue; o_i is d_i's initial carbon quota; o_i^- and o_i^+ are the lower and upper limit of d_i's initially expected interval, respectively; and $r_i o_i$ is d_i's initial carbon quota holding revenue.

5.1. *Steps of the research on carbon quota trading mechanism*

To clarify the construction mechanism described in this paper, five steps are presented below.

Step 1: Referring to Model (3), an optimization carbon trading model is built to achieve overall revenue maximization, i.e., to obtain the optimal carbon quota allocation scheme for different regions from the collective perspective. Specifically, the carbon quota quantities transferred among regions and the maximum value of the final total revenue of the system are acquired.

Step 2: Using the maximum overall revenue obtained in Step 1, and by adding the constraint of the unit transaction price, a series of optimization consensus models are built based on Model (6). Hence, a total of n allocation schemes are derived by maximizing each region's revenue, and detailed information such as d_i's unit buying and selling prices, transferred quantities, and unit transaction prices is obtained.

Step 3: Through a comparison of the individual/group development indices, it can be determined whether regions have developed fairly or not. If not, some discordant regions are identified by a pre-defined threshold γ, then their initial parameters are adjusted accordingly. Next, the calculations in Steps 1 and 2 are repeated until the allocation scheme satisfies the fairness requirement.

Step 4: Introduce the fairness measure variable α to build consensus models based on Model (7), so as to directly obtain fair carbon quota allocation schemes for the five regions in terms of the maximum overall revenue, quantities of carbon quota transferred, and the unit transaction prices. Additionally, a sensitivity analysis is applied to α to provide flexible suggestions for authorities involved in the trading system.

Step 5: Conduct a comparison and discussion based on the results obtained in each step.

5.2. Analysis of the overall revenue maximization model

Based on Model (3), we obtain a closed-loop carbon quota trading system involving the five regions listed in Table 4. Aiming to maximize the overall revenue, an optimization consensus model is constructed:

$$\max Z_1 = 12 * o_1^{'} + 15 * o_2^{'} + 23 * o_3^{'} + 34 * o_4^{'} + 40 * o_5^{'}$$

$$\text{s.t.} \begin{cases} \begin{cases} o_1^{'} = 16 - \sum\limits_{j=2}^{5} I_{1j} + \sum\limits_{j=2}^{5} I_{j1};\ o_2^{'} = 20 - \sum\limits_{j=1, j\neq 2}^{5} I_{2j} + \sum\limits_{j=1, j\neq 2}^{5} I_{j2} \\ o_3^{'} = 34 - \sum\limits_{j=1, j\neq 3}^{5} I_{3j} + \sum\limits_{j=1, j\neq 3}^{5} I_{j3};\ o_4^{'} = 18 - \sum\limits_{j=1, j\neq 4}^{5} I_{4j} + \sum\limits_{j=1, j\neq 4}^{5} I_{j4} \\ o_5^{'} = 12 - \sum\limits_{j=1}^{4} I_{5j} + \sum\limits_{j=1}^{4} I_{j5} \end{cases} & (9-1) \\ q_1 \leq 12 \leq p_1,\ q_2 \leq 15 \leq p_2, q_3 \leq 23 \leq p_3,\ q_4 \leq 34 \leq p_4,\ q_5 \leq 40 \leq p_5 & (9-2) \\ \begin{cases} I_{ij} \leq \dfrac{|q_j - p_i| + q_j - p_i}{\delta}, & i < j, i, j \in N \\ I_{ij} = 0, & \text{otherwise} \end{cases} & (9-3) \\ 13 \leq o_1^{'} \leq 19,\ 16 \leq o_2^{'} \leq 24,\ 27 \leq o_3^{'} \leq 41,\ 14 \leq o_4^{'} \leq 22,\ 10 \leq o_5^{'} \leq 26 & (9-4) \\ p_i \geq 0, q_i \geq 0, I_{ij} \geq 0, \delta > 0, i, j \in N & (9-5) \end{cases} \tag{9}$$

The objective function in Model (9) aims to maximize the total holding revenue for all five regions through the carbon quota trading process, where $o_i^{'}, i \in N$ is the final quota for the i-th region, which is restricted by both (9-1) and (9-4). Constraints (9-2)–(9-3) concern the unit selling and buying prices, and the transferred quantity for each region. δ in constraint (9-3) is a non-Archimedean infinitesimal, and hereafter it is set as $\delta = 10^{-6}$. The optimal solution of Model (9) is presented in Table 5.

Table 5 Optimal solution of Model (9) with overall revenue maximization

Regions	r_i	$o_i^{'}$	$r_i o_i^{'}$	I_{ij}	Value-I
d_1	12	13	156	(1,5)	3
d_2	15	16	240	(2,5)	4
d_3	23	27	621	(3,5)	7
d_4	34	18	612		
d_5	40	26	1040		
Total	—	100	2669	—	—

Note: d_i denotes the i-th region; r_i denotes the initial fixed unit revenue of carbon quota; $o_i^{'}$ is d_i's final carbon quota; $r_i o_i^{'}$ is d_i's final carbon quota holding revenue; and I_{ij} denotes the quantity transferred from d_i to d_j.

The results in Table 5 show that the maximum value of the objective function in Model (9) is 2669. According to Theorem 4, the critical region within the trading system is d_4, namely, $m = 4$. When $i < m$, the regions with small original fixed unit revenues are d_1, d_2, d_3. These regions can increase their revenue by selling carbon quotas, and their final quotas are the lower limit of their original expected intervals, namely 13, 16, and 27, respectively (i.e., o_i^- in Table 4). When $i > m$, i.e., for d_5, the only way to increase revenue is to purchase carbon quotas, and the final quota for this region is the upper limit of the original interval, namely 26 (i.e., o_i^+ in Table 4). Moreover, the final quota for d_4 is located in the initial range, and the data of I_{ij} show that region d_4 does not become involved in the trading. In summary, Theorem 4 has been verified. Region d_1 sold three carbon quota units to d_5; region d_5 bought three, four, and seven carbon quota units from regions d_1, d_2, d_3, respectively, making its total buying quantity $3 + 4 + 7 = o_5^{'} - o_5 = 26 - 12 = 14$. The transferred quantities for the remaining regions can be obtained in the same way. Thus, Theorem 5 has also been verified. Note that the revenue for each region in Table 5 only involves the fixed revenue for holding a certain carbon quota, while the transaction revenue from the trading of carbon quotas is not included.

5.3. Analysis of the single-region revenue maximization model

Model (9) can only provide feasible regions for variables $p_i, q_i, (i \in N)$, rather than their optimal values (see Section 4.1). Therefore, we construct Model (10) to acquire these optimal values under the objective of maximizing the revenue of individual regions, which follows the research ideas of Models (5) and (6). Obviously, we obtain five allocation schemes, one for each of the five regions taking part in the carbon quota trading process. For brevity, only the model that maximizes revenue for d_4 is illustrated here.

$$\max Z_2 = 34 * o_4^{'} + \sum_{j=1, j\neq 4}^{5} T_{4j} I_{4j} - \sum_{j=1, j\neq 4}^{5} T_{j4} I_{j4}$$

$$\text{s.t.} \begin{cases} \begin{cases} p_i \leq T_{ij} \leq q_j, & \text{if } p_i \leq q_j,\ i<j, i,j \in N \\ T_{ij} = 0, & \text{otherwise} \end{cases} & (10-1) \\ 12 * o_1^{'} + 15 * o_2^{'} + 23 * o_3^{'} + 34 * o_4^{'} + 40 * o_5^{'} = 2669 & (10-2) \\ \begin{cases} o_1^{'} = 16 - \sum_{j=2}^{5} I_{1j} + \sum_{j=2}^{5} I_{j1};\ o_2^{'} = 20 - \sum_{j=1,j\neq 2}^{5} I_{2j} + \sum_{j=1,j\neq 2}^{5} I_{j2} \\ o_3^{'} = 34 - \sum_{j=1,j\neq 3}^{5} I_{3j} + \sum_{j=1,j\neq 3}^{5} I_{j3};\ o_4^{'} = 18 - \sum_{j=1,j\neq 4}^{5} I_{4j} + \sum_{j=1,j\neq 4}^{5} I_{j4} \\ o_5^{'} = 12 - \sum_{j=1}^{4} I_{5j} + \sum_{j=1}^{4} I_{j5} \end{cases} & (10-3) \\ q_1 \leq 12 \leq p_1,\ q_2 \leq 15 \leq p_2, q_3 \leq 23 \leq p_3,\ q_4 \leq 34 \leq p_4,\ q_5 \leq 40 \leq p_5 & (10-4) \\ \begin{cases} I_{ij} \leq \dfrac{|q_j - p_i| + q_j - p_i}{\delta}, & i<j, i,j \in N \\ I_{ij} = 0, & \text{otherwise} \end{cases} & (10-5) \\ 13 \leq o_1^{'} \leq 19,\ 16 \leq o_2^{'} \leq 24,\ 27 \leq o_3^{'} \leq 41,\ 14 \leq o_4^{'} \leq 22,\ 10 \leq o_5^{'} \leq 26 & (10-6) \\ p_i \geq 0, q_i \geq 0, I_{ij} \geq 0, T_{ij} \geq 0, \delta > 0, i \in N & (10-7) \end{cases} \quad (10)$$

The objective function Z_2 in Model (10) is the maximum total revenue that can be achieved by region d_4 through carbon trading. This is composed of fixed revenue for holding carbon quotas (i.e., $34 * o_4^{'}$) and transaction revenue for trading behavior (i.e., $\sum_{j=1,j\neq 4}^{5} T_{4j} I_{4j} - \sum_{j=1,j\neq 4}^{5} T_{j4} I_{j4}$). Constraint (10-2) ensures that the above trading is carried out under the premise of maximizing the overall revenue, where 2669 is the maximum value obtained by solving Model (9). Other definitions see Model (9). Using Algorithm 1, the optimal solution of Model (10) is presented in Table 6, while the results of maximizing the revenue for other regions see Table A1. Here, all the demand parameters in Algorithm 1 are set as $N = 5, Z_1 = 2669, limit = 5000$ and $M = 50$. In addition, the values for o_i, r_i, o_i^-, o_i^+ refer to Table 4 and the parameters regarding the PSO algorithm are set in Matlab R2016a by default.

The decision variable $o_i^{'}$ in Model (6) is directly given by constraints (6-6)–(6-8), but needs to be solved under constraints (10-3) and (10-6) in Model (10). The results in Tables 6 and A1 indicate that, regardless of which region's revenue is maximized, the optimal allocation scheme is fixed and consistent with the results obtained in Section 5.2, that is, $o_1^{'} = 13, o_2^{'} = 16, o_3^{'} = 27, o_4^{'} = 18, o_5^{'} = 26$. Moreover, the unit selling and buying prices of each region

Table 6 Optimal solution of Model (10) with d_4's revenue maximization

Regions	$o_i^{'}$	p_i	q_i	I_{ij}	Value-I	Value-T	H_i	$\|H_i - \bar{H}\|$	Z_2
d_1	13	12	[0,12]	(1,4)	3	12	1.0000	0.1281	
d_2	16	15	15	(2,4)	4	15	1.0000	0.1281	
d_3	27	23	23	(3,4)	7	23	1.0000	0.1281	915
d_4	18	34	34	(4,5)	14	40	1.4951	0.3670	
d_5	26	[40,+∞)	40				1.0000	0.1281	

Note: $o_i^{'}$ is d_i's final carbon quota; p_i, q_i are d_i's unit selling and buying prices; I_{ij} is the quantity transferred from d_i to d_j with Value-I as its specific value and Value-T as its corresponding unit transaction price; $H_i, \bar{H}$ are individual/group development index; and Z_2 is the optimal value of the objective function regarding single DM's revenue maximization.

are also consistent, although the transferred quantities, corresponding unit transaction prices, and the individual development indices differ in each model. Note that, the values of p_5, q_1 are intervals due to the reason that they are subjected to unilateral constraints of corresponding T_{ij}. In fact, these are auxiliary variables for realizing the trading process, because d_1 cannot purchase carbon quotas and d_5 cannot sell carbon quotas considering their fixed order of unit revenues. Optimal values of all $T_{ij}, (i, j \in N)$ are provided during the calculation, but most are omitted here because they don't affect our analysis on the results.

The relationship between the individual development index H_i and the group development index $\bar{H}$ is now analyzed to identify whether there exist some discordant regions with too much or too little revenue growth. First, based on $\sum_{i=1}^{5} r_i o_i = 2366$ in Table 4 and $\sum_{i=1}^{5} r_i o_i^{'} = 2669$ in Table 5, we derive the group development index as $\bar{H} = \dfrac{2669}{2366} = 1.128064$. Based on the data in Tables 4, 5, and 6, individual development indices for each region can then be computed. Taking d_4 as an example, $H_4 = \dfrac{r_4 o_4^{'} + T_{45} I_{45} - T_{14} I_{14} - T_{24} I_{24} - T_{34} I_{34}}{r_4 o_4} = \dfrac{34 * 18 + 14 * 40 - 3 * 12 - 4 * 15 - 7 * 23}{34 * 18} = 1.4951$. The individual development indices in Tables 6 and A1 can be derived using a similar calculation method.

5.4. Identification and parameter adjustment of discordant regions

Using the individual development indices H_i in Table A1, we obtain the absolute values of the differences in development indices between each region and the group (i.e., $|H_i - \bar{H}|$) or the absolute difference between any two regions (i.e., $|H_i - H_j|$). Generally, in actual GDM problems, we can always judge whether the development of different regions is balanced, namely, we can always pre-determine a threshold γ to identify discordant regions. To determine the value of the parameter γ, Table 7 summarizes various development indices based on Table A1, including the maximum, minimum, and mean for the abovementioned difference values. Numbers in bold font indicate relatively large values in each column, which require more attention.

Table 7 Summary of the development indices under the region's revenue maximization

Difference between individual and group $\lvert H_i - \bar{H}\rvert$			**Difference between individuals** $\lvert H_i - H_j\rvert$		
Maximum	Minimum	Average	Maximum	Minimum	Average
0.3094	0	0.1461	0.4375	0	0.2336
0.4853	0.0396	0.1692	**0.6133**	0	**0.2789**
0.2594	0.1281	0.1543	0.3875	0	0.1550
0.3670	0.1281	0.1759	0.4951	0	0.1980
0.5032	0.1281	**0.2031**	**0.6313**	0	**0.2525**

Referring to the adjustment rules designed in Section 4.3, the initial parameters provided by discordant regions, namely, their predetermined expected carbon quota interval $[o_i^-, o_i^+]$, will be adjusted accordingly. If we set $\gamma = 0.5$, then only d_5 is identified as a discordant region with too much revenue growth. However, if the difference between the development indices of any two regions is considered, the corresponding maximum values for d_2, d_5 should be considered, as they are both greater than 0.6. Thus, the threshold for the parameter is adjusted to $\gamma = 0.45$. Liang et al. (2020) concluded that the shorter the time required to reach a consensus, the more necessary it is to make greater adjustments to the initial opinions. Initially concluded from a phenomenon of 20% people possessing 80% of the wealth in the world, the 80/20 Rule (i.e., the Pareto principle) is now extended to a fact that an optimal ratio exists between the effort and gain. In other words, once we change 20% of the key factors, qualitative change will occur, implying that we can derive enough (like 80% of) expected results on that critical point. Therefore, we may wish to adjust the endpoints of the expected carbon quota interval by 20% of their initial values. Because d_2 sold too much of his quota, the quota interval is adjusted from $[16, 24]$ to $\underline{[20, 24]}$; and as d_5 purchased too much carbon quota, his expected range is adjusted from $[10, 26]$ to $\underline{[10, 23.6]}$. Here, taking d_2 as an instance for specific explanation. Acted as a seller, d_2 needs to decrease its sales volum to reduce its revenue growth, so d_2 increases its lower limit by adding 20% of its initial carbon quota (i.e., o_2), thus we derive the adjusted lower limit of d_2's expected interval as $16 + 20\% * 20 = 20$. Distinguished from d_2, the buyer d_4 should decrease its upper limit so as to possess less carbon quota at the end.

After repeating the calculations of Models (3) and (6), new allocation schemes are obtained. For brevity, the specific calculation models are omitted here. Using updated information, the new optimal allocation scheme for overall revenue maximization is as presented in Table 8; the schemes maximizing different region's revenue are presented in Table A2. Table 9 provides an updated summary of the development indices after adjusting the initial parameters of d_2, d_5 by 20% of their initial carbon quotas.

Table 8 Optimal solution for maximizing overall revenue after adjusting the initial parameters of d_2, d_5

Regions	r_i	$o_i^{'}$	$r_i o_i^{'}$	I_{ij}	Value-I
d_1	12	13	156	(1,2)	3
d_2	15	20	300	(2,5)	3
d_3	23	27	621	(3,5)	7
d_4	34	16.4	557.6	(4,5)	1.6
d_5	40	23.6	944		
Total	—	100	2578.6	—	—

Note: Definitions of notation see Table 5.

Table 9 Summary of the development indices under the region's revenue maximization after adjusting the initial parameters of d_2, d_5

Difference between individual and group $\lvert H_i - \bar{H}\rvert$			**Difference between individuals $\lvert H_i - H_j\rvert$**		
Maximum	Minimum	Average	Maximum	Minimum	Average
0.3476	0.0064	0.1211	**0.4375**	0	**0.2073**
0.1901	0.0064	0.0896	0.2800	0	0.1443
0.1697	0.0699	0.1018	0.2596	0	0.1078
0.2575	0.0899	0.1234	0.3474	0	0.1390
0.3531	0.0899	**0.1425**	**0.4429**	0	0.1772

Results in Tables 7 and 9 show that the unfairness in the system is ameliorated by adjusting the initial parameters of d_2, d_5. Specifically, the maximum difference between the individual and group development indices drops from 0.5032 to 0.3531, while the maximum difference between any two regions drops from 0.6313 to 0.4429. In fact, if policy-makers are not satisfied with the results in Table 9, they may repeat the above calculations. The maximum value of each region's revenue declines in most scenarios because the total transaction amount decreases as the overall revenue drops from 2669 to 2578.6 (see column Z_2 in Tables A1 and A2). Note that the identification of discordant regions, adjustment of their parameters, and fairness of the final result all depend on the experience of the policy-makers. In addition, the adjustment range of the initial parameters for those discordant regions has a significant influence on the number of adjustments and the final allocation scheme of the trading system. Obviously, the "fairness" reached through the above strategy is effective, but subjective and rather complicated.

5.5. Analysis regarding both fairness and revenue

Based on Table 4 and Model (7), this section considers the optimization consensus model (i.e., Model (11)) for obtaining a relatively fair carbon quota allocation scheme with the goal of maximizing the final overall revenue within the closed-loop trading system.

$$
\max\ Z_3 = 12 * o_1^{'} + 15 * o_2^{'} + 23 * o_3^{'} + 34 * o_4^{'} + 40 * o_5^{'}
$$

$$
\text{s.t.}\begin{cases}
\begin{cases}
o_1^{'} = 16 - \sum_{j=2}^{5} I_{1j} + \sum_{j=2}^{5} I_{j1};\ o_2^{'} = 20 - \sum_{j=1,j\neq 2}^{5} I_{2j} + \sum_{j=1,j\neq 2}^{5} I_{j2} \\
o_3^{'} = 34 - \sum_{j=1,j\neq 3}^{5} I_{3j} + \sum_{j=1,j\neq 3}^{5} I_{j3};\ o_4^{'} = 18 - \sum_{j=1,j\neq 4}^{5} I_{4j} + \sum_{j=1,j\neq 4}^{5} I_{j4} \\
o_5^{'} = 12 - \sum_{j=1}^{4} I_{5j} + \sum_{j=1}^{4} I_{j5}
\end{cases} & (11-1) \\
q_1 \leq 12 \leq p_1,\ q_2 \leq 15 \leq p_2, q_3 \leq 23 \leq p_3,\ q_4 \leq 34 \leq p_4,\ q_5 \leq 40 \leq p_5 & (11-2) \\
\begin{cases}
p_i \leq T_{ij} \leq q_j, \quad \text{if } p_i \leq q_j,\ i<j, i,j \in N \\
T_{ij} = 0, \qquad \text{otherwise}
\end{cases} & (11-3) \\
\begin{cases}
I_{ij} \leq \frac{|q_j - p_i| + q_j - p_i}{\delta}, \quad i<j, i,j \in N \\
I_{ij} = 0, \qquad \text{otherwise}
\end{cases} & (11-4) \\
13 \leq o_1^{'} \leq 19,\ 16 \leq o_2^{'} \leq 24,\ 27 \leq o_3^{'} \leq 41,\ 14 \leq o_4^{'} \leq 22,\ 10 \leq o_5^{'} \leq 26 & (11-5) \\
H_i = \frac{r_i o_i^{'} + \sum_{j=1,j\neq i}^{n} T_{ij} I_{ij} - \sum_{j=1,j\neq i}^{n} T_{ji} I_{ji}}{r_i o_i}, i \in N & (11-6) \\
|H_i - H_j| \leq \alpha, i<j, i,j \in N & (11-7) \\
p_i \geq 0, q_i \geq 0, I_{ij} \geq 0, T_{ij} \geq 0, H_i \geq 0, \delta > 0, \alpha \geq 0, i,j \in N & (11-8)
\end{cases}
\tag{11}
$$

Z_3 in Model (11) maximizes the overall revenue of the carbon quota trading system. Constraint (11-1) describes the relationship between the final quotas and the carbon quotas transferred by each region, and $\sum_{i=1}^{n} o_i^{'} = 100$. Constraints (11-2)–(11-4) concern the unit buying and selling prices, the unit transaction prices and transferred quantities, where δ is a pre-determined non-Archimedean infinitesimal. Constraint (11-5) is the threshold for decision variable $o_i^{'}$, and (11-6) defines the individual development index. Constraint (11-7) is the fairness restriction, where α is the pre-determined fairness measure variable. Other variables are consistent with those in Model (7).

Table 10 presents the solution set for Model (11) when the fairness measure variable $\alpha = 0$. At this time, the trading system achieves an absolutely fair state, that is, all individual development indices are equal to the group development index of 1.1281. The results of a sensitivity analysis of α are given in Table B1, and show that any value of α in the interval [0,0.5] gives an optimal value of the objective function of 2669. The final carbon quotas for all regions are also fixed to $o_1^{'} = 13, o_2^{'} = 16, o_3^{'} = 27, o_4^{'} = 18, o_5^{'} = 26$.

Table 10 Solutions to Model (11) when $\alpha = 0$

	p_i	q_i	I_{ij}	Value-I	Value-T	I_{ij}	Value-I	Value-T	H_i
d_1	15	[0,12]	(1,2)	0.09	15	(2,5)	1.82	36.15	1.1281
d_2	15	15	(1,3)	1.54	23	(3,4)	5.36	34	1.1281
d_3	34	23	(1,4)	1.37	17.36	(3,5)	3.51	34	1.1281
d_4	36.15	34	(2,3)	0.32	15	(4,5)	8.68	36.15	1.1281
d_5	[40,+∞)	36.15	(2,4)	1.95	15				1.1281

Note: Definitions of notation see Table 6.

Tables 10 and B1 show that, as the fairness measure variable α gradually decreases, although the final carbon quota of each region $o_i^{'}$ is fixed, the transaction frequency significantly increases, implying that carbon quotas are fully traded within the system. Besides, when α is greater than 0.1, the variables p_i, q_i for each region are fixed, but when $\alpha \leq 0.1$, those pricing decisions change. Overall, the introduction of the fairness measure changes the allocation schemes by increasing the number of trading paths in the system. Clearly, as the closed-loop carbon quota trading mechanism gradually complicates the transaction process, a state of absolute fairness is finally reached, namely, sufficient interactions among regions are achieved as the fairness measure variable decreases to zero.

5.6. Discussion

To verify the rationality and effectiveness of the proposed models in the paper, this section has considered the example of carbon quota trading among five regions. Our optimization consensus models can derive the optimal allocation scheme from the global perspective (i.e., the moderator's perspective in GDM), and can also obtain allocation schemes from different DM's perspectives, in which the maximization of each region's revenue is the modeling goal. The following findings can be elicited from our results:

- Consensus modeling to maximize the overall revenue can obtain the optimal allocation scheme for the whole group, but cannot identify specific pricing decisions. Moreover, the final carbon quotas of different regions obtained from the models that maximize each region's revenue are the same as those obtained from the former modeling mechanism. That is, the optimal values of $o_i^{'}$ are fixed. However, detailed trading information, such as the trading regions involved and the unit transaction prices, change with the specific region being studied (see Tables 5, A1, 8, and A2).
- The unit selling and buying prices of each region (i.e., variables p_i, q_i) derived from the proposed optimization consensus models do not change according to which region's revenue is being maximized (see Tables A1 and

A2) and do not depend on the value of the fairness measure variable (see Table B1). This indirectly implies that the carbon quota trading mechanism discussed in this paper is robust to some extent.

- For the two strategies proposed to deal with the unfairness issue within the trading system, adjusting the initial parameters of discordant regions is effective (see Tables 7 and 9), but complicated in practice. In addition, the parameter γ for identifying discordant regions, the adjustment range for each region, and whether the final allocation scheme meets the GDM requirements are all subjective (see Section 5.4). In contrast, the strategy of directly introducing the fairness measure variable α is convenient and effective, and further sensitivity analysis enables feasible allocation schemes to be obtained (see Tables 10 and B1).

- The introduction of the fairness measure variable increases the number of trading paths among different regions (see Tables 10 and B1), meaning that absolute fairness within the closed-loop system is realized only when carbon quotas are fully traded among different regions. Thus, sufficient interactions among participators are highly significant in achieving consensus or the pursuit of DMs' balanced development during a GDM process.

6. Conclusion

This paper has described the use of optimization consensus modeling theory to explore theoretical innovations regarding flexible carbon trading mechanisms. Specifically, we have investigated essential carbon quota allocation schemes within a closed-loop trading system with the aim of ensuring both revenue maximization and fairness. First, the optimal carbon quota allocation scheme was derived by maximizing the overall revenue through Model (3). Then, its analytical formula and the achievable conditions for successful trading were provided through theoretical deduction. Next, simultaneously taking the group revenue maximization and the competition mechanism into account, models for deriving the optimal allocation schemes by maximizing individual's revenues were constructed as Models (5) and (6). Since conflicts of interest are the main reasons for the failure of GDM in the real world, individual/group development indices were defined as Definitions (1) and (2), and two fairness strategies were further presented. The former is based on calculating the difference between the development indices, with fairness achieved through the identification of discordant DMs and the adjustment of their initial parameters. The latter introduces a fairness measure variable, allowing fair allocation schemes to be directly obtained from Model (7). Finally, a numerical example was conducted to demonstrate the performance of the proposed models.

The results show that the final carbon quotas of all regions can be determined through the proposed consensus models, but detailed trading information (including the participating regions and the unit transaction prices) can only be acquired through the models that focus on single-region revenue maximization. In addition, the strategies for dealing with the unfairness issue are both practical and effective, but the second strategy of directly introducing a fairness measure variable is more objective and easier to operate. Finally, the results of a sensitivity analysis of the fairness measure variable show that, as the variable decreases to zero, that is, when the group approaches the state of absolute fairness, the frequency of DMs' transactions within the group increases significantly, corresponding to the fact that reaching fairness within a group requires sufficient interactions among DMs.

In the future, some varaibles in our proposed models will be comprehensively determined to be more in line with real-life, for example, price variables are no longer static and could be accurately positioned by combining with game theory (Liu et al., 2021; Zheng et al., 2019). In addition, trading mechanisms should also focus on some critical factors, such as risk or utility (Zheng & Chang, 2021) in practical markets, rather than only considering the allocation and pricing decisions from the revenue maximization perspective. Moreover, with large-scale GDM problems (Dong et al., 2018; Zhang et al., 2017), especially under social network contexts (Liu et al., 2019; Wu et al., 2019), attracting increased attention, the use of artificial intelligence methods (Ding et al., 2020) to solve large-scale trading issues will also be a focus of our subsequent research.

Appendix A. Results with single region's revenue maximization

Based on Sections 4.2 and 4.3, Table A1 lists the optimal solutions (including $o_i^{'}, p_i, q_i, I_{ij}, T_{ij}$, and Z_2) to Model (6) and the values of the development indices (including H_i and $|H_i - \bar{H}|$) in the case of each region maximizing its revenue (note: the specific region discussed in Model (6) is marked with $\star$ in the first column in Table A1). Moreover, Table A2 exhibits the corresponding results after the initial parameters of d_2, d_5 have been adjusted by 20% of their initial carbon quotas.

Appendix B. Sensitivity analysis of the fairness measure variable

If the fairness measure variable α in Model (11) is decreased from 0.5 at intervals of 0.1, then the optimal solutions of the above optimization consensus model are as listed in Table B1.

Table A1 Optimal solution of Model (6) with different region's revenue maximization

Regions	$o_i^{'}$	p_i	q_i	I_{ij}	Value-I	Value-T	H_i	$\|H_i - \bar{H}\|$	Z_2
$\star d_1$	13	12	[0,12]	(1,5)	3.00	40	1.4375	0.3094	
d_2	16	15	15	(2,4)	2.10	15	1.0000	0.1281	
d_3	27	23	23	(2,5)	1.90	15	1.0000	0.1281	276
d_4	18	34	34	(3,4)	3.50	23	1.1280	0	
d_5	26	[40,+∞)	40	(3,5)	3.50	23	1.2930	0.1650	
–	–	–	–	(4,5)	5.60	34	–	–	
d_1	13	12	[0,12]	(1,2)	3	12	1.0000	0.1281	
$\star d_2$	16	15	15	(2,5)	7	40	1.6133	0.4853	
d_3	27	23	23	(3,4)	3.50	23	1.0000	0.1281	484
d_4	18	34	34	(3,5)	3.50	23	1.0629	0.0652	
d_5	26	[40,+∞)	40	(4,5)	3.50	34	1.1677	0.0396	
d_1	13	12	[0,12]	(1,3)	3	12	1.0000	0.1281	
d_2	16	15	15	(2,3)	4	15	1.0000	0.1281	
$\star d_3$	27	23	23	(3,5)	14	40	1.3875	0.2594	1085
d_4	18	34	34				1.0000	0.1281	
d_5	26	[40,+∞)	40				1.0000	0.1281	
d_1	13	12	[0,12]	(1,4)	3	12	1.0000	0.1281	
d_2	16	15	15	(2,4)	4	15	1.0000	0.1281	
d_3	27	23	23	(3,4)	7	23	1.0000	0.1281	915
$\star d_4$	18	34	34	(4,5)	14	40	1.4951	0.3670	
d_5	26	[40,+∞)	40				1.0000	0.1281	
d_1	13	12	[0,12]	(1,5)	3	12	1.0000	0.1281	
d_2	16	15	15	(2,5)	4	15	1.0000	0.1281	
d_3	27	23	23	(3,5)	7	23	1.0000	0.1281	783
d_4	18	34	34				1.0000	0.1281	
$\star d_5$	26	[40,+∞)	40				1.6313	0.5032	

Note: $o_i^{'}$ is d_i's final carbon quota; p_i, q_i are d_i's unit selling and buying prices; I_{ij} is the quantity transferred from d_i to d_j with Value-I as its specific value and Value-T as its corresponding unit transaction price; $H_i, \bar{H}$ are individual/group development index; and Z_2 is the optimal value of the objective function regarding single DM's revenue maximization.

Table A2 Optimal solution of Model (6) with different region's revenue maximization after adjusting the initial parameters of d_2, d_5

Regions	$o_i^{'}$	p_i	q_i	I_{ij}	Value-I	Value-T	H_i	$\lvert H_i - \bar{H} \rvert$	Z_2
$\star d_1$	13	12	[0,12]	(1,5)	3.00	40	1.4375	0.3476	
d_2	20	15	15	(3,4)	4.64	23	1.0000	0.0899	
d_3	27	23	23	(3,5)	2.36	23	1.0000	0.0899	276
d_4	16.4	34	34	(4,5)	6.24	34	1.0835	0.0064	
d_5	23.6	[40,+∞)	40				1.1615	0.0716	
d_1	13	12	[0,12]	(1,2)	3.00	12	1.0000	0.0899	
$\star d_2$	20	15	15	(2,5)	3.00	40	1.2800	0.1901	
d_3	27	23	23	(3,4)	4.64	23	1.0000	0.0899	384
d_4	16.4	34	34	(3,5)	2.36	23	1.0835	0.0064	
d_5	23.6	[40,+∞)	40	(4,5)	6.24	34	1.1615	0.0716	
d_1	13	12	[0,12]	(1,3)	3	12	1.0000	0.0899	
d_2	20	15	15	(3,5)	10	40	1.0000	0.0899	
$\star d_3$	27	23	23	(4,5)	1.6	34	1.2596	0.1697	985
d_4	16.4	34	34				1.0000	0.0899	
d_5	23.6	[40,+∞)	40				1.0200	0.0699	
d_1	13	12	[0,12]	(1,4)	3	12	1.0000	0.0899	
d_2	20	15	15	(3,4)	7	23	1.0000	0.0899	
d_3	27	23	23	(4,5)	11.6	40	1.0000	0.0899	824.6
$\star d_4$	16.4	34	34				1.3474	0.2575	
d_5	23.6	[40,+∞)	40				1.0000	0.0899	
d_1	13	12	[0,12]	(1,5)	3	12	1.0000	0.0899	
d_2	20	15	15	(3,5)	7	23	1.0000	0.0899	
d_3	27	23	23	(4,5)	1.6	34	1.0000	0.0899	692.6
d_4	16.4	34	34				1.0000	0.0899	
$\star d_5$	23.6	[40,+∞)	40				1.4429	0.3531	

Note: $o_i^{'}$ is d_i's final carbon quota; p_i, q_i are d_i's unit selling and buying prices; I_{ij} is the quantity transferred from d_i to d_j with Value-I as its specific value and Value-T as its corresponding unit transaction price; $H_i, \bar{H}$ are individual/group development index; and Z_2 is the optimal value of the objective function regarding single DM's revenue maximization.

Table B1 Sensitivity of the results to the fairness measure variable α

Regions	p_i	q_i	I_{ij}	Value-I	Value-T	H_i	α
d_1	12	[0,12]	(1,5)	3	40	1.4375	
d_2	15	15	(2,5)	4	40	1.3333	
d_3	23	23	(3,5)	7	40	1.1522	0.5
d_4	34	34				1.0000	
d_5	[40,+∞)	40				1.0000	
d_1	12	[0,12]	(1,3)	0.42	23	1.4000	
d_2	15	15	(1,5)	2.58	40	1.3333	
d_3	23	23	(2,5)	4	40	1.1614	0.4
d_4	34	34	(3,5)	7.42	40	1.0000	
d_5	[40,+∞)	40				1.0000	
d_1	12	[0,12]	(1,3)	1.55	23	1.3000	
d_2	15	15	(1,5)	1.45	40	1.3000	
d_3	23	23	(2,5)	4	37.5	1.1859	0.3
d_4	34	34	(3,5)	8.55	40	1.0000	
d_5	[40,+∞)	40				1.0208	
d_1	12	[0,12]	(1,3)	2.68	23	1.2000	
d_2	15	15	(1,5)	0.32	40	1.2000	
d_3	23	23	(2,5)	4	30	1.2000	0.2
d_4	34	34	(3,5)	9.68	39.15	1.0000	
d_5	[40,+∞)	40				1.1004	
d_1	15	[0,12]	(1,3)	1.15	23	1.0946	
d_2	23	15	(1,4)	1.85	15	1.1358	
d_3	34	23	(2,5)	4	25.19	1.1186	0.1
d_4	40	34	(3,4)	1.93	34	1.0946	
d_5	[40,+∞)	40	(3,5)	6.22	34.51	1.1946	
—	—	—	(4,5)	3.78	40	—	

Note: d_i denotes the i-th region; p_i, q_i are d_i's unit selling and buying prices; I_{ij} is the quantity transferred from d_i to d_j with Value-I as its specific value and Value-T as the corresponding unit transaction price; H_i is the individual development index; and α is the fairness measure variable.

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Optimization consensus modeling of a closed-loop carbon quota trading mechanism regarding revenue and fairness

Abstract

Consensus modeling aims to obtain collective agreement through group decision-making (GDM), generally by building mathematical programming models. This paper describes the use of optimization consensus modeling to explore theoretical innovations regarding flexible carbon quota trading mechanisms, with basic allocation schemes provided within a closed-loop trading system by simultaneously taking revenue and fairness into account. A series of optimization consensus models are constructed from the perspective of maximizing the corresponding revenue, resulting in optimal/fair carbon quota allocation schemes that include detailed trading information, e.g., participating individuals, transferred quantities, and unit transaction prices. To solve these models, a relaxation method based on particle swarm optimization is proposed. The inability to conduct real-life GDM usually stems from conflicts of interest based on the decision-makers' mutual competition, thus, two practical strategies are presented to deal with the resulting unfairness within the trading system. Finally, a numerical example incorporating five regions demonstrates the effectiveness of the proposed trading mechanisms. The results show that sufficient interactions among decision-makers are of great significance in achieving fairness within a trading system.

Keywords: Group decision-making (GDM); Consensus; Revenue and fairness; Carbon quota trading mechanism; Allocation scheme

1. Introduction

Group decision-making (GDM) refers to a process in which multiple individuals participate in decision-making analysis and make a final choice based on their collective wisdom: Clark & Stephenson (1995) have pointed out that GDM represents a collective recall of information. Generally, communication and negotiation effectively promote the interactions among decision-makers (DMs) (Hirokawa & Poole, 1996) and the flow of information within the group. Moreover, technological innovations have significantly updated the means of group communication and decision-making (Kiesler & Sproull, 1992). Without loss of generality, three stable states of fragmentation, polarization, or consensus may finally be achieved by rational DMs considering their own interests (Hegselmann & Krause, 2002; Liang et al., 2020; Zhao et al., 2016). Among them, consensus usually requires multiple rounds of communication, coordination, preference modification, and even concessions or compromises within the group. Only in this way can a relatively consistent collective agreement be obtained (Cabrerizo et al., 2014; Liu et al., 2019; Wu & Chiclana, 2014; Wu et al., 2018; Zhang et al., 2020a,b). For example, if a new allocation scheme of resources is obtained through GDM within a trading system, which is widely accepted by the whole group, then a consensus is reached. Liang et al. (2020) clarified that the consensus-reaching process (CRP) does not mean that an optimal solution must be achieved. Instead, CRP is more like a decision tool or a synthesizing process that assists DMs in building connections and communicating with each other, thereby providing a more effective way for the group to find unity on how to proceed (Susskind et al., 1999).

Considering that cost, which may be embodied as human, material, financial, time and other resources, is an important influencing factor in GDM, Ben-Arieh & Easton (2007) first proposed the concept of minimum cost consensus, and acquired the optimal collective opinion with a linear/quadratic cost function (Ben-Arieh et al., 2009). Since then, other scholars have made further extensions to their minimum cost consensus models (MCCMs) by taking various factors into account, such as uncertain preference structures (Gong et al., 2021; Guo et al., 2021), aggregation rules (Zhang et al., 2011), measurement of consensus effectiveness (Labella et al., 2020) or parameter improvements of initial models (Cheng et al., 2018; Lu et al., 2021; Zhang et al., 2020a). Since unit costs are difficult to objectively determine in advance, and DMs' opinions are hard to modify during GDM, Dong et al. (2010) proposed minimum adjustment consensus models (MACMs) with an ordered weighted average operator, which preserve the DMs' initial preference information as much as possible. Similarly, their modeling idea has also been widely explored (del Moral et al., 2018; Dong et al., 2016; Gong et al., 2020; Yu et al., 2021; Zhang et al., 2018), especially under social networks (Cheng et al., 2020; Wu et al., 2018) or opinion evolution contexts (Chen et al., 2021; Liang et al., 2020). Moreover, Zhang et al. (2020b) summarized the original and basic consensus models based on feedback mechanisms with a minimum cost/adjustment and reviewed diverse consensus modeling under some complicated GDM scenarios.

Different from the above consensus modeling with a minimum cost/adjustment, this paper was partially inspired by the construction of consensus models that aim to maximize the total revenue. By introducing linear primal-dual theory, various MCCMs (including hard and soft consensus (Herrera-Viedma et al., 2014; Zhang et al., 2011)) with specific preference structures (e.g., DM's opinion denoted by crisp numbers or interval values) were adopted as the primal models, and then their corresponding dual forms (i.e., the optimization maximum compensation consensus models) along with their economic significance were deeply explored by Gong et al. (2015a,b) and Zhang et al. (2019). Subsequently, taking the essential architecture of Stackelberg's game into account, Zhang et al. (2020a) presented a bi-level optimization consensus model that depicts the interaction between DMs and the moderator, and divided the DM's total return into a modification component (also known as external compensation) provided by the moderator for the DM's initial preference adjustment and a recognition component based on the similarity between the DM's original opinion and the final consensus. It is well known that the market is profit-oriented (i.e., simultaneously pursuing the maximization of revenue and the minimization of costs) and its operating mechanism is mostly affected by pricing strategy, participants' competition, supply and demand, and etc. (Lamba et al., 2019; Ruidas et al., 2021; Zhou et al., 2020b; Zou et al., 2021). Therefore, in discussing closed-loop trading mechanisms, the revenue maximization of either the whole group or a single DM is set as our objective function in this paper, and constraints such as supply and demand or prices are introduced. A series of optimization consensus models are then constructed as a means of deriving the optimal resource allocation schemes within a trading system.

Rapid industrialization and economic growth have led to significant increases in emissions of carbon dioxide and other greenhouse gases, and have rendered environmental pollution and extreme weather events increasingly serious and frequent, resulting in severe negative impacts on economic development and human health (Wang et al., 2017). Therefore, mitigating the impact of human activities on the environment through reductions in carbon emissions has gradually become a global consensus. Diaz-Rainey & Tulloch (2018) conducted the first empirical analysis of New Zealand's carbon trading scheme using allowance importation and exportation data, and found that the imports of offsets are the major carbon price determinant, with small trading systems able to reap benefits from imposing quantitative import restrictions. Aiming at developing sustainable supply chain, joint decisions were made under various carbon emission regulatory policies, with respect to different influence factors, such as inventory, pricing, financing and ordering (Ruidas et al., 2021; Zhou et al., 2020b; Zou et al., 2021). Furthermore, carbon issues combined with decision-making technology has also been investigated (Gong et al., 2021; Huang & Xu, 2020; Lamba et al., 2019). For instance, Lamba et al. (2019) proposed a mixed-integer nonlinear program for supplier selection and the right lot-sizes determination under a dynamic background with multiple periods, products and suppliers, and evaluated different costs of carbon emissions under three regulating policies (viz. cap-and-trade, strict cap on emissions and carbon tax) using big data technology. Huang & Xu (2020) constructed a bi-level multi-objective programming model to solve the carbon emission quota allocation problem with co-combustion of coal and sewage sludge, and formulated the interaction between authorities and coal-fired power plants before examining a real case demonstrating the trade-off between economic development, energy conservation, and renewable energy utilization.

Setting targets for carbon emissions in different countries/regions (i.e., operating collective schemes for optimal carbon quota allocation) is one of the main obstacles to reaching a comprehensive agreement on global warming. This is exacerbated by long-term tensions between industrialized and developing countries regarding unfairness issues on burden-sharing, with industrialized countries pleading special circumstances and seeking differentiation in their obligations (Rose et al., 1998). Fairness concerns, gained widespread attention in the supply chain management (Liu et al., 2021; Zheng et al., 2019), are also critical for GDM (Du et al., 2021), because participants are motivated by not only the final results, but also the fairness they feel compared with others (Adams, 1963). Under a fixed total carbon quota, the scientific allocation of binding carbon allowances for different regions is a complex and arduous task, because it directly involves the economic development rights of each region. In general, the fairness of carbon emissions quotas is measured using the Atkinson index (Hedenus & Azar, 2005), Theil index (Duro & Padilla, 2006), and Gini coefficient (Chen et al., 2017). The traceability method, which uses historical carbon emissions as the relevant feature of the initial carbon quota allocation (i.e., the free distribution principle), has been criticized by Fromm & Hansjürgens (1996) and Sijm et al. (2007) for being inconsistent with the "polluter pays" principle and lacking fairness from the perspective of society as a whole. In addition, Van Steenberghe (2004) found that the so-called fair rule to allocate greenhouse gas emission permits is not beneficial for all nations, with some countries being worse off under global agreement than under non-cooperative contexts. Under the framework of the Kyoto Protocol, Gomes & Lins (2008) adopted the zero-sum gains data envelopment analysis method to provide a fair carbon emissions allocation plan for various countries, which not only stabilizes the concentration of greenhouse gases in the atmosphere, but also achieves carbon quota trading with no impact on global emissions. The above studies have mostly considered the fairness of carbon quota allocations at the global level, ignoring the interest-driven issues of individual/regional perspectives. Therefore, the analysis of carbon trading mechanisms through consensus modeling with all participators' interests taken into account is of great significance.

Although many studies have investigated carbon issues, there has been few research on carbon quota trading mechanisms, and consensus decision-making theory has not been adopted to deal with the design of carbon trading mechanisms and their resulting unfairness issues. That is, using optimization consensus models to assist DMs in exchanging carbon quotas and the development of fair connections among them within a closed-loop trading system are neglected. Hence, the main contributions of this study are as follows: (i) By referring to conventional market trading mechanisms, a benchmark consensus model with the aim of overall revenue maximization is presented to derive the optimal carbon quota allocation scheme. (ii) By building a two-stage programming model, new allocation schemes are acquired that focus on different single DM's revenue maximization, allowing detailed trading information such as the transferred quantities, DM's unit selling and buying prices, and unit transaction prices to be acquired. (iii) Two strategies based on individual/group development indices are proposed to deal with the unfairness issue within the trading system. (iv) A relaxation method based on particle swarm optimization (PSO) (Kennedy & Eberhart, 1995) is proposed to solve the above consensus models. And (v) numerical analysis of a trading system composed of five regions is conducted to verify the effectiveness of the proposed models.

The rest of this paper is organized as follows. Section 2 briefly reviews the optimization consensus models, then Section 3 presents some assumptions of the trading mechanisms, and justifies the rationality of the hypothesis through theoretical deduction. Section 4 constructs a series of new consensus models from which optimal/fair allocation schemes are obtained within the closed-loop trading system, and further proposes an optimization algorithm to solve these models. A numerical example is reported in Section 5 to demonstrate the feasibility of the proposed mechanisms. Finally, Section 6 gives some concluding remarks and identifies future research directions.

2. Preliminaries on optimization consensus modeling

To better understand the subsequent construction of optimization closed-loop carbon trading consensus models, this section briefly reviews theoretical GDM models for obtaining the optimal consensus. However, before introducing the basic consensus models, we define some related notation. Let $D = \{d_1, d_2, \cdots, d_n\}$ be the set of all DMs, where d_i denotes the i-th DM and $i \in N = \{1, 2, \cdots, n\}$. Let $O = \{o_1, o_2, \cdots, o_n\}$ and $O^{'} = \{o_1^{'}, o_2^{'}, \cdots, o_n^{'}\}$ be the sets of initial and final preferences (i.e., opinions, judgements) of the group, where $o_i, o_i^{'}$ denote d_i's initial and final opinions, respectively. The existing forms of expressions for DMs include, but are not limited to, linear uncertainty preferences (Gong et al., 2020, 2021), linguistic preferences (Cabrerizo et al., 2013; Wu et al., 2018; Yu et al., 2021), fuzzy preference (Herrera-Viedma et al., 2014; Wu & Chiclana, 2014; Zhang et al., 2018). Nevertheless, aiming to solve real-life GDM problems, we adopt traditional forms, i.e., positive and real numbers, to denote DM's opinions in this paper. Let ω_i denote the unit cost provided by the moderator for d_i adjusting his opinions, $i \in N$. In fact, the modeling mechanisms are similar for both MCCM (Ben-Arieh & Easton, 2007; Ben-Arieh et al., 2009) and MACM (Dong et al., 2016, 2010). If all DMs' unit costs satisfy $w_i = w_j, \forall i, j \in N, i \neq j$, then the former reduces to the latter (Zhang et al., 2020b). A general framework of the minimum cost/adjustment consensus model provided by Zhang et al. (2011) can be introduced as:

$$\begin{aligned} &\min \sum_{i=1}^{n} w_i * d(o_i^{'}, o_i) \\ &\text{s.t.} \begin{cases} o^c = F(o_1^{'}, o_2^{'}, \cdots, o_n^{'}) & (1-1) \\ CD(o_i^{'}, o^c) \leq \alpha, \forall i \in N & (1-2) \end{cases} \end{aligned} \tag{1}$$

In Model (1), $d(o_i^{'}, o_i)$ represents the distance or deviation between d_i's initial and final (or adjusted) opinions (del Moral et al., 2018), which is generally given by the Manhattan distance (Ben-Arieh & Easton, 2007) or Euclidean distance (Ben-Arieh et al., 2009). Constraint (1-1) means that the collective opinion (i.e., consensus) o^c should be obtained by the aggregation function F over all DMs' final opinions $\{o_1^{'}, o_2^{'}, \cdots, o_n^{'}\}$, which corresponds to various social selections; and constraint (1-2) measures the consensus level CD attached to d_i's adjusted opinion $o_i^{'}$ and the consensus o^c, where α is a pre-defined threshold that is usually employed when solving soft consensus problems (Herrera-Viedma et al., 2014; Zhang et al., 2011, 2019).

The above model is an optimization consensus model with a minimum cost/adjustment from the moderator's perspective. However, individuals in GDM always expect some compensation for adjusting their opinion, the more the better. Hence, introducing linear primal-dual theory, Gong et al. (2015a,b) and Zhang et al. (2019) explored the dual forms of Model (1) in specific contexts so as to obtain the maximum compensation for all DMs. In particular, Zhang et al. (2019) provided a concise form of the maximum compensation consensus models (i.e., Model (2)), where R means the set of real numbers, and y_i is the unit compensation expected by d_i. As discussed earlier, Zhang et al. (2020a) divided the objective function of Model (2) into a modification return provided by the moderator for the DM's opinion adjustment and a recognition return based on the similarity between the DM's initial opinion and the final consensus. However, their model is omitted here due of space limitations.

$$\begin{aligned} \max \quad & \sum_{i=1}^{n} y_i * (o_i - o^c) \\ \text{s.t.} \quad & y_i \in R,\ i \in N \end{aligned} \tag{2}$$

The optimal collective opinion o^c can always be obtained, regardless from the minimum cost perspective (i.e., Model (1)) or the maximum compensation perspective (i.e., Model (2)). Therefore, the idea of discussing the closed-loop carbon quota trading mechanism with an objective function that maximizes the overall revenue is feasible. In addition, the above two models obtain the optimal collective opinion o^c, whereas this paper aims to derive all DMs' optimal adjusted opinions (i.e., the set of $O^{'}$) during the trading process. Thus, in the following discussion, we introduce some influential factors into the conventional market trading mechanisms and build a series of optimization consensus models that provide optimal or fair carbon quota allocations within a closed-loop trading system.

3. Assumptions for carbon quota trading mechanisms

This paper explores how to develop a satisfactory carbon quota allocation scheme under the goal of maximizing the revenue for either the whole group or a single DM through market trading mechanisms. To facilitate a better understanding, Table 1 presents the main notation used in this paper. Suppose that multiple DMs (e.g., companies, regions, nations) form a closed-loop trading system with a fixed total carbon quota. Let r_i be d_i's initial fixed unit revenue and $r_1 \leq r_2 \leq ... \leq r_n$, where r_i is determined by each DM's unique qualities, such as social and economic development, natural conditions, resource endowments, industrial structures, and energy usage rates.

Table 1 Summary of notation used in this paper

Notation	Meaning	Notation	Meaning
d_i	The i-th DM	I_{ij}	Quantity transferred from d_i to d_j
r_i	Initial fixed unit revenue of d_i's CQ	T_{ij}	Unit transaction price between d_i and d_j
p_i	Unit selling price of d_i's CQ	δ	Non-archimedean infinitesimal
q_i	Unit buying price of d_i's CQ	γ	Fairness threshold
o_i	d_i's initial CQ	α	Fairness measure variable
$o_i^{'}$	d_i's final CQ	Z_1	Obj to maximize overall revenue
o_i^-	Lower limit of d_i's IECQI	Z_2	Obj to maximize a specific DM's revenue
o_i^+	Upper limit of d_i's IECQI	Z_3	Obj regarding revenue and fairness
H_i	Individual development index	$\bar{H}$	Group development index

Note: CQ, IECQI and Obj are short for carbon quota, initially expected carbon quota interval and the objective function, respectively.

To be noted, this paper aims to depict the most essential trading behavior within a carbon quota market by consensus modeling. Meanwhile, in order to reduce the computational complexity of the subsequent models, we currently simplify the problem to the greatest extent. Therefore, several basic assumptions need to be clarified as:

1. The carbon quota market discussed remains stable during a certain period, and DMs can freely participate in the trading system;
2. Variables of unit prices (i.e., p_i, q_i, T_{ij}) are static, meaning that they don't fluctuate with time, supply and demand, and etc.;
3. Unit revenue of d_i's carbon quota (i.e., r_i) is a constant, which is only determined by d_i's own inherent characteristics rather than o_i, meaning that the standard law of diminishing returns assumption is not considered;
4. Factors regarding costs within the profit-oriented trading system are implicit in d_i's initial unit revenue, which means we only need to conduct analysis from the perspective of revenue maximization.

Actually, assumptions listed above are all to reduce the complexity of our GDM problem, and each point could be an interesting topic in our subsequent research. Anyway, the final results obtained from the closed-loop trading system through consensus modeling should satisfy two main objectives:

- Goal 1: Each DM's total revenue derived from the trading is no less than his initial fixed total revenue;
- Goal 2: The sum of all DMs' revenue acquired from the closed-loop trading system should be maximized.

Goal 1 is set from the DM's perspective, and aims to maximize each DM's economic benefits. All DMs are assumed to be rational (that is, once the carbon quota trading is conducted, they must benefit themselves); otherwise, the transactions are invalid. This corresponds to real-life market trading and can be understood as the effectiveness of the trading mechanisms. On the contrary, Goal 2 is set from the collective angle. In general, the representative for the collective benefit is the participant who determines the initial carbon quota for all DMs, and also the one who plays the role as a moderator in GDM problems (Ben-Arieh & Easton, 2007; Gong et al., 2021), such as local governments or world organizations. For those representatives, the primary goal is to maximize the overall revenue.

To realize Goal 1, we have the following constraints: (1) $p_i \geq r_i$, (2) $q_i \leq r_i$, where p_i denotes the unit selling price, q_i represents the unit buying price, and r_i is the original fixed revenue for one unit of d_i's carbon quota. Let the quantity transferred from d_i to d_j be I_{ij}, and their final unit transaction price be T_{ij}. Then, the following statement holds: If $p_i \le q_j$, then the one-way carbon quota transaction from d_i to d_j can be realized, that is, d_i can sell a carbon quota to d_j, and so $I_{ij} \ge 0$ and the unit transaction price $T_{ij} \in [p_i, q_j]$, which indicates there is a negotiable space in the trading process between d_i and d_j. At the same time, we derive $I_{ji} = 0$, since $I_{ij} * I_{ji} = 0$ holds under the premise of one-way trading.

The above constraint indicates that there is a directionality in the carbon quota trading between any two DMs. Specifically, once a carbon transaction occurs between d_i and d_j, the transferred quantity sold by d_i to d_j is I_{ij}, and we get $r_i \le p_i \le q_j \le r_j$. Moreover, because the unit transaction price satisfies $p_i \le T_{ij} \le q_j$, we have that $r_i \le p_i \le T_{ij} \le q_j \le r_j$. Thus, d_i's revenue is $T_{ij}I_{ij} - r_iI_{ij} \ge 0$, whereas d_j's revenue is $r_jI_{ij} - T_{ij}I_{ij} \ge 0$. This trading mechanism guarantees that every carbon transaction that occurs is profitable for both parties, implying that each DM's final revenue after the carbon trading is no less than their initial total fixed revenue. Thereby, Goal 1 is always met.

Theorem 1. *$I_{ij} * I_{ji} = 0$ and $I_{ij} \ge 0$, $I_{ji} \ge 0$ $(i \ne j, i, j \in N)$, if and only if $p_i = q_i = p_j = q_j = r_i = r_j$, $I_{ij} \ge 0$, and $I_{ji} \ge 0$ hold simultaneously. At this time, the unit selling and buying prices, as well as the initial fixed unit revenue for both d_i and d_j, are equal. In this case, the transaction does not bring about a change in revenue, so it has no economic significance.*

Proof. As $p_i \ge r_i$ and $q_i \le r_i$, we have $p_i \ge r_i \ge q_i$. When $I_{ij} \ge 0$ and $I_{ji} \ge 0$ hold simultaneously, $p_i \le q_j$ and $p_j \le q_i$ are obtained, that is, $r_i \le p_i \le q_j \le r_j \le p_j \le q_i \le r_i$. So when $p_i = q_i = p_j = q_j = r_i = r_j$, both $I_{ij} \ge 0$ and $I_{ji} \ge 0$ hold. Under other situations, if $I_{ij} \ge 0$, we have $I_{ji} = 0$; on the contrary, if $I_{ji} \ge 0$, we get $I_{ij} = 0$. To sum up, based on the aforementioned four assumptions, once DM d_i buys (sells) carbon quota from (to) d_j, he/she will no longer sell (buy) carbon quota to (from) d_j. □

Theorem 1 guarantees that the transactions between any two DMs in the closed-loop carbon quota trading system are one-way. When the initial parameters provided by the two DMs (including unit buying and selling prices as well as their initial fixed unit revenue) are all equal, their transaction has no direction constraint. However, any transaction realized under these conditions cannot increase the DMs' revenue, so it has no economic value.

Theorem 2. *Suppose r_i is d_i's initial fixed unit revenue and $r_1 \le r_2 \le ... \le r_n$, if $i \le j$, $I_{ij} \ge 0$ holds; if $i > j$ and $r_i \ne r_j$, $I_{ij} = 0$ holds; and if $i > j$ and $r_i = r_j$, $I_{ij} \ge 0$ holds.*

Proof. If $i \leq j$, we have $r_i \leq r_j$, and because $p_i \geq r_i, q_j \leq r_j$, there must exist p_i, q_j such that $r_i \leq p_i \leq q_j \leq r_j$, then $I_{ij} \geq 0$. Besides, if $i > j$ and $r_i \neq r_j$, then $r_i > r_j$, and since $p_i \geq r_i, q_j \leq r_j$, that is, $p_i \geq r_i > r_j \geq q_j$, thus there exist no p_i, q_j such that $p_i \leq q_j$, thereby we have $I_{ij} = 0$. Similarly, if $i > j$ and $r_i = r_j$, then $p_i \geq r_i = r_j \geq q_j$. Clearly, only if $p_i = r_i = r_j = q_j$, $I_{ij} \geq 0$ holds, otherwise, we have $I_{ij} = 0$. □

Theorem 2 takes the basic hypothesis of this paper into consideration: all DMs are arranged in order based on the relationships among their original fixed unit revenues, that is, $r_1 \leq r_2 \leq ... \leq r_n$. The quantity of the carbon quota that is transferred is not only affected by the DM's location index, but also by the size of the DM's fixed unit revenue. This theorem implies that carbon quota trading can only be carried out from one DM with a smaller fixed unit revenue to another with a larger unit revenue. Therefore, DMs with small fixed unit revenues have to sell their carbon quota to increase their total revenue, because $p_i \geq r_i$. On the contrary, DMs with large unit revenues can only improve their revenue by purchasing carbon quotas, because $q_i \leq r_i$.

***Theorem* 3.** *Let d_i's final carbon quota be $o_i^{'}$. Considering that some uncertainty exists during the trading process, the above final carbon quota is represented by an interval value, denoted as $[o_i^-, o_i^+]$, whose endpoints satisfy:*

$$\sum_{i=1}^{n} o_i^- \leq \sum_{i=1}^{n} o_i \leq \sum_{i=1}^{n} o_i^+$$

Proof. Since $o_i^- \leq o_i^{'} \leq o_i^+$, we have $\sum_{i=1}^{n} o_i^- \leq \sum_{i=1}^{n} o_i^{'} \leq \sum_{i=1}^{n} o_i^+$. Meanwhile, because the total carbon quota in the closed-loop trading system is fixed, namely $\sum_{i=1}^{n} o_i^{'} = \sum_{i=1}^{n} o_i$, then $\sum_{i=1}^{n} o_i^- \leq \sum_{i=1}^{n} o_i \leq \sum_{i=1}^{n} o_i^+$. □

Theorem 3 is based on the assumption that the total carbon quota in the closed-loop trading system is fixed, which complies with the provisions of the clean development mechanism. That is, under the premise of fixed global carbon emission levels, high-emission countries can finance some projects in low-emission countries to reach their established limit (i.e., compensatory reduction) (Gomes & Lins, 2008). In short, the so-called "carbon market" can reduce the economic impact on high-emission countries and achieve the overall goal of reducing carbon emissions. In addition, for rational DMs, threshold constraints attached to their final carbon quota can better exhibit the uncertainties during the trading process (Ruidas et al., 2021); for the moderator, there is no need to grasp all transaction details, namely, the moderator only needs to have overall control of the total amount, that is, the lower limit of the final total carbon quota is no greater than the initial total amount, while the upper limit should be no less than the sum of all DMs' original carbon quotas.

An example of carbon quota trading conducted by three regions is presented below to preliminarily clarify our modeling ideas. Initial information is listed in Table 2, while the trading results, including the final carbon quota, and the corresponding revenue, are shown in Table 3. Meanwhile, the specific trading process is exhibited in Fig. 1. Note that the elaborated example only corresponds to the aforementioned basic assumptions, and does not really involve the consensus modeling in the next section.

Table 2 Example of the initial information provided by three regions

	d_1	d_2	d_3
o_i	10	10	10
r_i	50	80	120
$r_i o_i$	500	800	1200

Table 3 Example of the final carbon quotas through the trading conducted by three regions

	d_1	d_2	d_3
$o_i^{'}$	3	11	16
r_i	50	80	120
$r_i o_i^{'}$	150	880	1920
Trading revenue	530	-35	-495
Total revenue	680	845	1425

Table 2 provides the initial carbon quota (i.e., o_i) allocated to each region along with its fixed unit revenue (i.e., r_i), from which the initial total revenue (i.e., $r_i o_i$) of each region can be obtained. As d_1 has the smallest unit revenue r_1, this DM can only increase his revenue by selling a carbon quota; as d_3 has the largest unit revenue r_3, this DM can only increase his total revenue by purchasing a carbon quota. For d_2, revenue may be increased by selling, purchasing, or combining both trading behavior (see Fig. 1).

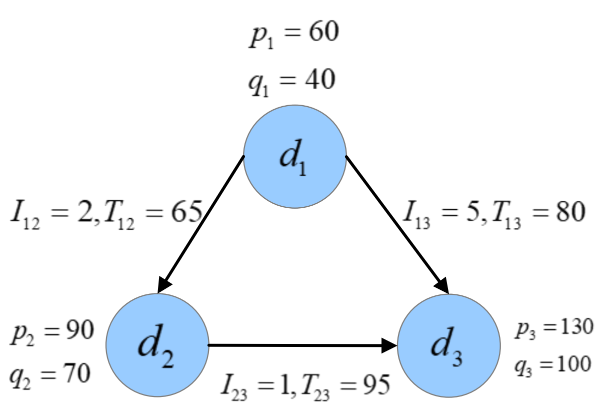

Fig. 1 Schematic diagram of carbon quota trading among three regions

To make the trading mechanism effective and feasible, DM's unit selling price should be no less than his initial unit revenue (i.e., $p_i \geq r_i$), while the unit buying price should be no larger than the fixed unit revenue (i.e., $q_i \leq r_i$). Take d_2 as an example for detailed analysis: the total revenue for d_2's initial carbon quota is $10 * 80 = 800$, and suppose through optimization consensus modeling, d_2's unit selling and buying prices are derived as $p_2 = 90$ and $q_2 = 70$, respectively. The parameters for other regions see Fig. 1. Since $T_{ij} \in [p_i, q_j]$, here we might as well let the unit transaction price be $T_{ij} = \frac{p_i+q_j}{2}$, then we derive $T_{12} = 65, T_{23} = 95$, and the transferred carbon quota quantities related to d_2 are assumed to be obtained through mathematical modeling as $I_{12} = 2, I_{23} = 1$. As a result, d_2's total carbon quota is $10 + 2 - 1 = 11$, and the new fixed revenue for holding his carbon quota is $11 * 80 = 880$, while the transaction revenue (i.e., the difference between the income from selling carbon quotas and the cost of buying quotas) is $1 * 95 - 2 * 65 = -35$, making d_2's final total revenue of $880 - 35 = 845$ be larger than the initial total revenue of 800. Results in Tables 2 and 3 demonstrate that the final revenue of every region in the closed-loop trading system has increased with respect to their initial total revenue, indicating that the proposed trading mechanism is feasible.

4. Optimization consensus modeling concerning carbon quota trading mechanism

Chu & Shen (2006) indicated that the purpose of designing a trading mechanism is to provide a method for ensuring that the allocation decisions and pricing decisions in decision-making processes result in the desired outcomes. They also found that, once the allocation principle is set in a truthful mechanism, the prices are determined; similarly, once the pricing rule is determined, the allocation is settled. Different from the extant research on the carbon market (Diaz-Rainey & Tulloch, 2018; Gomes & Lins, 2008; Lamba et al., 2019; Ruidas et al., 2021; Van Steenberghe, 2004; Zhou et al., 2020b; Zou et al., 2021), this section takes the maximization of the overall revenue or a single DM's revenue as the objective function, and uses optimization consensus modeling to determine the allocation scheme (i.e., determination of variables $o_i^{'}, I_{ij}$) and the pricing scheme (i.e., determination of variables p_i, q_i, T_{ij}) in the carbon quota trading system.

4.1. Benchmark carbon trading consensus model with overall revenue maximization

To realize Goal 2 (as defined in Section 3), we build the following optimization consensus model (i.e., Model (3)) to maximize the sum of the revenues of all DMs within the closed-loop trading system as:

$$
\begin{aligned}
&\max\ Z_1 = \sum_{i=1}^{n} r_i o_i^{'} \\
&\text{s.t.} \begin{cases}
o_i^{'} = o_i - \sum\limits_{j=1,j\neq i}^{n} I_{ij} + \sum\limits_{j=1,j\neq i}^{n} I_{ji}, i \in N & (3-1) \\
q_i \leq r_i \leq p_i, i \in N & (3-2) \\
\begin{cases} I_{ij} \geq 0, \text{if } p_i \leq q_j \text{ and } i < j,\ i,j \in N \\ I_{ij} = 0, \text{otherwise} \end{cases} & (3-3) \\
o_i^{-} \leq o_i^{'} \leq o_i^{+}, i \in N & (3-4) \\
p_i \geq 0, q_i \geq 0, I_{ij} \geq 0, i,j \in N & (3-5)
\end{cases}
\end{aligned}
\tag{3}
$$

The objective function Z_1 in Model (3) attempts to maximize the final total revenue for all DMs within the carbon quota trading system. Constraint (3-1) is the expression of d_i's final quota, which is equal to the initial quantity minus all the sold quantities $\sum\limits_{j=1,j\neq i}^{n} I_{ij}$ and plus all the purchased quantities $\sum\limits_{j=1,j\neq i}^{n} I_{ji}$, where I_{ij} denotes the carbon quota quantity transferred from d_i to d_j. Since the sum of all transfer-out quantities equals to the sum of all transfer-in quantities, we can easily obtain $\sum_{i=1}^{n} o_i^{'} = \sum_{i=1}^{n} o_i$ through constraint (3-1), corresponding to the fact that the total carbon quota amount in the closed-loop trading system is fixed. Constraint (3-2) is the threshold constraint attached to the unit selling price p_i and the unit buying price q_i based on the pre-defined initial fixed unit revenue r_i. Constraint (3-3) specifies the achievable conditions of the carbon trading between any two DMs. Namely, only when the seller's location index is smaller than the purchaser's index, and the unit selling price p_i is no greater than the unit buying price q_j, will the transaction from d_i to d_j be achieved (i.e., $I_{ij} \geq 0$). Constraint (3-4) assumes that d_i's final quota is located in his own expected interval provided initially. Constraint (3-5) indicates that all variables are nonnegative. Hence, Model (3) explores the optimal carbon quota allocation problem under the maximization of the overall revenue of the trading system, where $Z_1, o_i^{'}, I_{ij}, p_i, q_i, (i \in N)$ are decision variables and $r_i, o_i, o_i^{-}, o_i^{+}$ are known parameters. In fact, due to insufficient constraints (e.g., the absence of specific transaction prices building connections with the unit price variables), only the ranges of p_i and q_i instead of their optimal values can be obtained through Model (3).

***Theorem* 4.** *There must exist an m-th DM such that* $\sum_{i=1}^{m-1} o_i^- + o_m^{'} + \sum_{i=m+1}^{n} o_i^+ = \sum_{i=1}^{n} o_i$ *and* $o_m^- \leq o_m^{'} \leq o_m^+$*. By then, the optimal value for the objective function of Model (3) is* $\sum_{i=1}^{m-1} r_i o_i^- + r_m o_m^{'} + \sum_{i=m+1}^{n} r_i o_i^+$ *and the optimal solution is* $o_i^{'} = o_i^- (1 \leq i \leq m-1), o_m^{'} = \sum_{i=1}^{n} o_i - \sum_{i=1}^{m-1} o_i^- - \sum_{i=m+1}^{n} o_i^+, o_i^{'} = o_i^+ (m+1 \leq i \leq n)$*.*

Proof. First, when $o_i^{'} = o_i^- (1 \leq i \leq m-1), o_m^{'} = \sum_{i=1}^{n} o_i - \sum_{i=1}^{m-1} o_i^- - \sum_{i=m+1}^{n} o_i^+, o_i^{'} = o_i^+ (m+1 \leq i \leq n)$, there exists no $I_{ij} > 0$ to further increase the objective function. That is, except d_m, all DMs have reached their critical points of their expected carbon quota intervals (i.e., $[o_i^-, o_i^+]$), either the lower limit of d_i $(1 \leq i \leq m-1)$ or the upper limit of d_i $(m+1 \leq i \leq n)$, making DMs with a location index smaller than m cannot further sell carbon quota while DMs with a location index larger than m cannot further buy carbon quota, based on the given condition as $r_1 \leq r_2 \leq ... \leq r_n$. In a nutshell, if $I_{ij} > 0$, the objective function of Model (3) increases to $f^* = f + (r_j - r_i) * I_{ij}$, where f is the total revenue before the transaction. Due to $r_j \geq r_i, (i < j)$, we get $f^* \geq f$, indicating that if and only if $I_{ij} = 0$, the value of the objective function no longer increases and becomes the optimal value. Thus, the solution at this point is exactly the optimal solution, and the objective function becomes $\sum_{i=1}^{m-1} r_i o_i^- + r_m o_m^{'} + \sum_{i=m+1}^{n} r_i o_i^+$.

Next, we prove that this critical DM with the m-th location index always exists. Because $o_i^- \leq o_i^{'} \leq o_i^+$, we have $\sum_{i=1}^{n} r_i o_i^- \leq \sum_{i=1}^{n} r_i o_i^{'} \leq \sum_{i=1}^{n} r_i o_i^+$. If $m = 1$, then $r_1 o_1^- + \sum_{i=2}^{n} r_i o_i^+ \leq \sum_{i=1}^{n} r_i o_i^{'} \leq \sum_{i=1}^{n} r_i o_i^+$. If $m = 2$, then $\sum_{i=1}^{2} r_i o_i^- + \sum_{i=3}^{n} r_i o_i^+ \leq \sum_{i=1}^{n} r_i o_i^{'} \leq r_1 o_1^- + \sum_{i=2}^{n} r_i o_i^+$. In the same way, if $m = n$, then $\sum_{i=1}^{n} r_i o_i^- \leq \sum_{i=1}^{n} r_i o_i^{'} \leq \sum_{i=1}^{n-1} r_i o_i^- + r_n o_n^+$. Therefore, once m takes a specific value within the set N, $\sum_{i=1}^{n} r_i o_i^{'}$ can take any value from the interval $[\sum_{i=1}^{n} r_i o_i^-, \sum_{i=1}^{n} r_i o_i^+]$, and so the known constraint $\sum_{i=1}^{n} r_i o_i^- \leq \sum_{i=1}^{n} r_i o_i^{'} \leq \sum_{i=1}^{n} r_i o_i^+$ means that d_m must exist such that $o_i^{'} = o_i^- (1 \leq i \leq m-1), o_m^{'} = \sum_{i=1}^{n} o_i - \sum_{i=1}^{m-1} o_i^- - \sum_{i=m+1}^{n} o_i^+, o_i^{'} = o_i^+ (m+1 \leq i \leq n)$ hold. □

***Theorem* 5.** *When Model (3) reaches its maximum value, we obtain* $\sum_{j=1, j \neq i}^{n} I_{ij} - \sum_{j=1, j \neq i}^{n} I_{ji} = o_i - o_i^-$ $(1 \leq i \leq m-1)$*,* $\sum_{j=1, j \neq m}^{n} I_{mj} - \sum_{j=1, j \neq m}^{n} I_{jm} = o_m - \sum_{i=1}^{n} o_i + \sum_{i=1}^{m-1} o_i^- + \sum_{i=m+1}^{n} o_i^+$*,* $\sum_{j=1, j \neq i}^{n} I_{ij} - \sum_{j=1, j \neq i}^{n} I_{ji} = o_i - o_i^+$ $(m+1 \leq i \leq n)$*.*

Proof. Theorem 4 implies that once Model (3) reaches its maximum value, and if $1 \leq i \leq m-1$, then $o_i^{'} = o_i^-$ holds, meantime, due to $o_i^{'} = o_i - \sum_{j=1, j \neq i}^{n} I_{ij} + \sum_{j=1, j \neq i}^{n} I_{ji}$, we have $\sum_{j=1, j \neq i}^{n} I_{ij} - \sum_{j=1, j \neq i}^{n} I_{ji} = o_i - o_i^- (1 \leq i \leq m-1)$. Similar analysis can be conducted for the rest situations. □

Theorems 4 and 5 indicate that the optimal solution of Model (3) and the maximum value of the objective function exist and are unique. Therefore, the optimal allocation for all DMs' carbon quotas is determined. In other words, by solving Model (3), we obtain all information about carbon quota transfers within the trading system. However, note that only the feasible regions can be obtained by Model (3), rather than the optimal values of the decision variables p_i, q_i.

***Theorem* 6.** *The achievable constraints of the carbon quota trading mechanism are determined by d_i's unit selling price p_i and d_j's unit buying price q_j as:*

$$\begin{cases} I_{ij} \geq 0, & \text{if } p_i \leq q_j \text{ and } i < j,\ i,j \in N \\ I_{ij} = 0, & \text{otherwise} \end{cases}$$

which is equivalent to

$$\begin{cases} I_{ij} \leq \dfrac{|q_j - p_i| + q_j - p_i}{\delta}, & i < j, i, j \in N \\ I_{ij} = 0, & \text{otherwise} \end{cases} \tag{4}$$

where δ is the non-Archimedean infinitesimal, viz. a sufficiently small positive value approaching zero (Charnes et al., 1994; Mehrabian et al., 2000).

Proof. If $i < j, i, j \in N$, then carbon quota trading between the seller d_i and the purchaser d_j is achievable, so $I_{ij} \geq 0$ holds. Next, we discuss the effect of prices on the transferred quantity: when $p_i < q_j$, according to Eq. (4), we have $I_{ij} < \dfrac{2(q_j - p_i)}{\delta}$, and because δ is the non-Archimedean infinitesimal, $I_{ij} < +\infty$, that is, $I_{ij} \geq 0$ holds; when $p_i \geq q_j$, based on Eq. (4), we have $I_{ij} = 0$. In addition, if $i \geq j, i, j \in N$, the one-way transaction from d_i to d_j cannot be achieved, so we have $I_{ij} = 0$. This completes the proof of Theorem 6. □

Theorem 6 states the achievable conditions for a closed-loop trading system. Specifically, carbon quota trading can only be achieved when the unit selling price of one DM with a small location index is no greater than the unit buying price of another DM with a large location index; otherwise, their carbon quota transaction fails.

4.2. Carbon trading consensus models with single DM's revenue maximization

The competition mechanism refers to the struggle among market practitioners to maximize their own economic benefits, so it focuses more on individual standpoints than the collective perspective. The model developed in

Section 4.1 only maximizes the overall revenue of the trading process, and ignores the individual DM's interests and the resulting unfairness issues. This section considers individual DMs as the research object, and uses optimization consensus models to derive detailed information about the trading process, including the participating DMs, transferred quantities, and the final unit transaction prices. That is, when the group realizes its optimal allocation by considering every DM's revenue maximization, this section attempts to determine not only d_i's final carbon quota $o_i^{'}$ from its expected interval $[o_i^-, o_i^+]$, but also each DM's psychological expected unit selling and buying prices (i.e., p_i, q_i) and the transferred quantity I_{ij} along with the best achievable unit transaction price T_{ij}. Based on the above principles, a two-stage programming model is built as:

$$
\max\ Z_2 = r_i o_i^{'} + \sum_{j=1, j\neq i}^{n} T_{ij} I_{ij} - \sum_{j=1, j\neq i}^{n} T_{ji} I_{ji}
$$
$$
\text{s.t.} \begin{cases}
\begin{cases} p_i \leq T_{ij} \leq q_j, & \text{if } p_i \leq q_j,\ i<j, i,j \in N \\ T_{ij} = 0, & \text{otherwise} \end{cases} & (5-1) \\
\begin{cases} Max\ \sum_{i=1}^{n} r_i o_i^{'} \\ o_i^{'} = o_i - \sum_{j=1, j\neq i}^{n} I_{ij} + \sum_{j=1, j\neq i}^{n} I_{ji}, i \in N \\ q_i \leq r_i \leq p_i, i \in N \\ \begin{cases} I_{ij} \leq \dfrac{|q_j - p_i| + q_j - p_i}{\delta}, & i<j, i,j \in N \\ I_{ij} = 0, & \text{otherwise} \end{cases} \\ o_i^- \leq o_i^{'} \leq o_i^+,\ p_i \geq 0, q_i \geq 0, I_{ij} \geq 0, T_{ij} \geq 0, \delta > 0, i,j \in N \end{cases} & (5-2)
\end{cases}
\tag{5}
$$

Model (5) introduces constraint (5-1) into Model (3), that is, adding the expression of the unit transaction price T_{ij} between DMs d_i and d_j, which is a range bounded by d_i's unit selling price p_i and d_j's unit buying price q_j. As stated in Section 3, only the location indices satisfy $i < j, i, j \in N$, and $p_i \leq q_j$ holds, can the unit transaction price between d_i and d_j be denoted as $T_{ij} \in [p_i, q_j]$. Here, the unit transaction price T_{ij} obeys a uniform distribution by default, as each point within the interval $[p_i, q_j]$ can be selected with equal possibility, which makes it easy to calculate, understand and be applied into real-life GDM. The objective function in Model (5) is the sum of d_i's final carbon quota holding revenue (i.e., $r_i o_i^{'}$) and the transaction revenue for selling or buying carbon quotas (i.e., $\sum_{j=1, j\neq i}^{n} T_{ij} I_{ij} - \sum_{j=1, j\neq i}^{n} T_{ji} I_{ji}$), and this value the larger the better. Model (5) indicates that maximizing a single DM's revenue is not unconstrained; instead, it should be carried out within the context of maximizing the overall revenue for the whole group (i.e., constraint (5-2)). Referring to Theorem 4, Model (5) can be further transformed into Model (6), where constraints (6-2)–(6-9) provide the analytical formula of constraint (5-2). The definitions of other variables and constraints in Model (6) are consistent with those in Models (3) and (5).

Theorem 4 states that the optimal solution of Model (3) exists and is unique. Thus, there must exist feasible solutions for Model (6). Actually, constraints (6-6)–(6-8) in Model (6) provide the analytical formula for the DM's final carbon quota $o_i^{'}$, and are acquired by solving Model (3). Hence, variables $Z_2, I_{ij}, p_i, q_i, T_{ij}$ and m in Model (6) are decision variables, while $r_i, o_i, o_i^-, o_i^+, \delta$ are known parameters. In short, under the premise of maximizing the overall revenue, and by further adding the expression of the unit transaction prices, Model (6) determines the optimal values for d_i's unit selling and buying prices (i.e., p_i and q_i), and further obtains detailed trading information including the quantity I_{ij} transferred from d_i to d_j and their corresponding unit transaction price T_{ij}.

$$
\max Z_2 = r_i o_i^{'} + \sum_{j=1, j\neq i}^{n} T_{ij} I_{ij} - \sum_{j=1, j\neq i}^{n} T_{ji} I_{ji}
$$

$$
\text{s.t.}\begin{cases}
\begin{cases} p_i \le T_{ij} \le q_j, & \text{if } p_i \le q_j,\ i<j, i, j \in N \\ T_{ij} = 0, & \text{otherwise} \end{cases} & (6-1) \\
\sum_{i=1}^{n} r_i o_i^{'} = \sum_{i=1}^{m-1} r_i o_i^- + r_m o_m^{'} + \sum_{i=m+1}^{n} r_i o_i^+ & (6-2) \\
o_i^{'} = o_i - \sum_{j=1, j\neq i}^{n} I_{ij} + \sum_{j=1, j\neq i}^{n} I_{ji}, i \in N & (6-3) \\
q_i \le r_i \le p_i, i \in N & (6-4) \\
\begin{cases} I_{ij} \le \dfrac{|q_j - p_i| + q_j - p_i}{\delta}, & i<j, i, j \in N \\ I_{ij} = 0, & \text{otherwise} \end{cases} & (6-5) \\
o_i^{'} = o_i^-, 1 \le i \le m-1 & (6-6) \\
o_m^{'} = \sum_{i=1}^{n} o_i - \sum_{i=1}^{m-1} o_i^- - \sum_{i=m+1}^{n} o_i^+,\ o_m^- \le o_m^{'} \le o_m^+ & (6-7) \\
o_i^{'} = o_i^+, m+1 \le i \le n & (6-8) \\
p_i \ge 0,\ q_i \ge 0, I_{ij} \ge 0, T_{ij} \ge 0, \delta > 0, i, j, m \in N & (6-9)
\end{cases} \qquad (6)
$$

4.3. Identification and adjustment rules for discordant DMs

In Section 4.2, we considered the case in which every single DM pursues the maximization of his own revenue, which inevitably results in unfairness (e.g., the unbalanced growth of the DMs' revenue). Therefore, this section examines the potential to achieve a relatively balanced state within the closed-loop carbon quota trading system by adjusting some DMs' initial parameters. Once fairness is achieved, DMs with too much revenue growth or too little revenue growth should no longer exist in the final stage of carbon trading. Any such DMs are collectively referred to as **discordant DMs** in the trading system. During CRP, if the DMs' improper initial parameters can be modified as early as possible, systemic losses (e.g., cost, time) will be significantly reduced. In short, an earlier intervention during GDM is more advantageous (Liang et al., 2020). Compared with extant research adopting utility function (Du et al., 2021) or fuzzy theory (Liu et al., 2021) to characterize the fairness concerns, this paper defines two indicators to directly judge whether the GDM results are fair, so as to further identify discordant DMs and make some corresponding adjustments.

***Definition* 1.** An individual development index is defined as a relative proportion of the DM's final revenue obtained through the carbon quota trading process with respect to their initial fixed revenue, that is,

$$H_i = \frac{r_i o_i^{'} + \sum\limits_{j=1, j \neq i}^{n} T_{ij} I_{ij} - \sum\limits_{j=1, j \neq i}^{n} T_{ji} I_{ji}}{r_i o_i}, i \in N$$

***Definition* 2.** The group development index is defined as a relative proportion of the final total revenue obtained through the carbon quota trading process with respect to the initial fixed total revenue of the group, that is,

$$\bar{H} = \frac{\sum\limits_{i=1}^{n} r_i o_i^{'}}{\sum\limits_{i=1}^{n} r_i o_i}$$

This section follows the idea of fair development of all DMs in the trading system. By default, the difference between the individual development index H_i and the group development index $\bar{H}$ should be within a certain range, otherwise DMs will be identified as discordant DMs with too much or too little revenue growth. These two development indices mainly depend on the DM's final carbon quota $o_i^{'}$, which further depends on the endpoints of the expected interval $[o_i^-, o_i^+]$ provided by DM d_i. Here, we choose interval values instead of crisp numbers to denote d_i's expected carbon quota quantity due to various uncertainties (Ruidas et al., 2021). Hence, by adjusting the expected carbon quota range $[o_i^-, o_i^+]$ of discordant DMs, an equilibrium state with a minimum loss can be achieved within the trading system (see Fig. 2(c)). Let a discordant DM be $d_k, k \in \{0, 1, \cdots, n\}$, and his expected final carbon quota be adjusted from $[o_k^-, o_k^+]$ to $[o_k^{'-}, o_k^{'+}]$ through the following adjustment rules.

- When $H_k << \bar{H}$ and $|H_k - \bar{H}| > \gamma$, where γ is a pre-determined threshold and $<<$ denotes far less than, d_k is identified as a discordant DM with too little revenue growth. This DM is located in the unbalanced state shown in Fig. 2(a), and his adjustment rules are:
  - If $k > m$, then the amount purchased is too little, and so o_k^+ needs to be increased;
  - If $k < m$, then the amount sold is too little, and so o_k^- needs to be further decreased;
  - If $k = m$, then the current expected interval is improperly set, and we need to simultaneously reduce o_k^- and increase o_k^+.
- When $H_k >> \bar{H}$ and $|H_k - \bar{H}| > \gamma$, where γ is a pre-determined threshold and $>>$ means far more than, d_k is identified as a discordant DM with too much revenue growth. This DM is located in the unbalanced state shown in Fig. 2(b), and his adjustment rules are:
  - If $k > m$, then the quantity purchased is too great, and so o_k^+ needs to be decreased;
  - If $k < m$, then the amount sold is too great, and so o_k^- should be increased;
  - If $k = m$, then the current interval of the DM's expected carbon quota is inappropriate, and we need to increase o_k^- and decrease o_k^+ at the same time.

Through the above adjustment rules, a set of updated trading information for all DMs can always be acquired. Based on the individual/group development indices, we obtain the values of all $|H_i - \bar{H}|$ based on Model (6) so as to determine the threshold for the variable γ, as well as the difference value $|H_i - H_j|$ between any two DMs. By repeating the calculations of Models (3) and (6), it is then possible to verify whether the above adjustments are effective or not. The above rules are used to identify discordant DMs and provide the corresponding direction of adjustments. However, the identification parameter γ needs to be manually set, and the specific adjustment range for each DM cannot be accurately specified, that is, we cannot determine by how much each discordant DM needs to adjust the upper and lower limits of their initial expected carbon quota intervals. To overcome these deficiencies, a fairness measure variable α is introduced in the next section, and the optimal carbon quota allocation scheme considering fairness is directly acquired through consensus modeling. Furthermore, by applying a sensitivity analysis to the variable α, we can obtain flexible allocation schemes according to specific GDM scenarios.

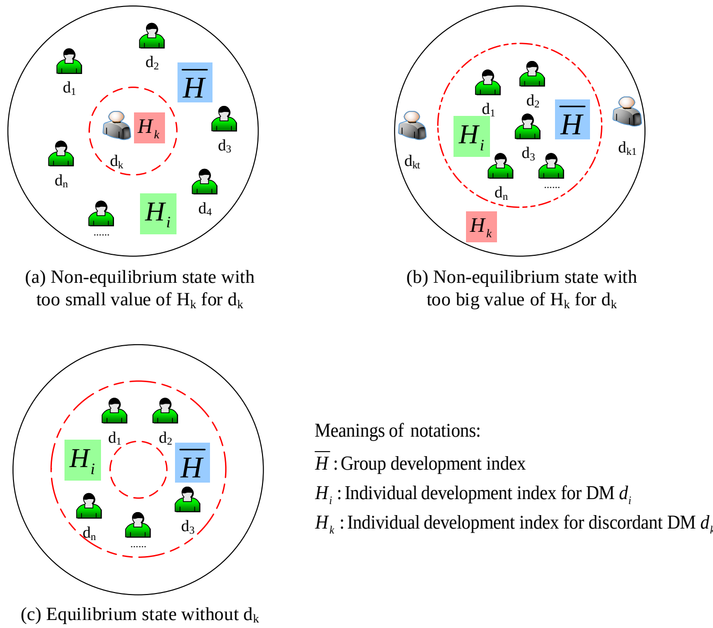

Fig. 2 Identification of non-equilibrium states in closed-loop carbon trading system

4.4. Carbon trading consensus model regarding fairness and revenue

When only a single DM's revenue is considered, the overall revenue cannot be maximized; moreover, when only the overall revenue is taken into account, there can be large gaps between the total revenue of different DMs, highlighting the unfairness issues. Thus, this section introduces a fairness constraint (that is, the difference between any two individual development indices should be within a certain acceptable threshold) under the premise of ensuring the maximization of the overall revenue. Specifically, the fairness constraint is expressed as $|H_i - H_j| \leq \alpha(\alpha \geq 0, i < j, i, j \in N)$, and the optimization carbon quota consensus model considering both revenue and fairness is built as follows:

$$
\max\ Z_3=\sum_{i=1}^{n} r_i o_i^{'}
$$
$$
\text{s.t.}\begin{cases}
o_i^{'}=o_i-\sum\limits_{j=1,j\neq i}^{n} I_{ij}+\sum\limits_{j=1,j\neq i}^{n} I_{ji}, i\in N & (7-1)\\
q_i\le r_i\le p_i, i\in N & (7-2)\\
\begin{cases} p_i\le T_{ij}\le q_j, & \text{if } p_i\le q_j,\ i<j, i,j\in N\\ T_{ij}=0, & \text{otherwise}\end{cases} & (7-3)\\
\begin{cases} I_{ij}\le \dfrac{|q_j-p_i|+q_j-p_i}{\delta}, & i<j, i,j\in N\\ I_{ij}=0, & \text{otherwise}\end{cases} & (7-4)\\
H_i=\dfrac{r_i o_i^{'}+\sum\limits_{j=1,j\neq i}^{n} T_{ij}I_{ij}-\sum\limits_{j=1,j\neq i}^{n} T_{ji}I_{ji}}{r_i o_i}, i\in N & (7-5)\\
|H_i-H_j|\le\alpha, i<j, i,j\in N & (7-6)\\
o_i^{-}\le o_i^{'}\le o_i^{+},\ q_i\ge 0,\ p_i\ge 0, I_{ij}\ge 0, T_{ij}\ge 0, \delta>0, \alpha\ge 0, i,j\in N & (7-7)
\end{cases}
\quad (7)
$$

Z_3 in Model (7) aims to maximize the overall revenue after carbon quota trading under the premise that each DMs' revenue has been fairly developed. Constraint (7-1) is the expression of d_i's final carbon quota, which guarantees $\sum_{i=1}^{n} o_i^{'}=\sum_{i=1}^{n} o_i$. Constraint (7-2) sets d_i's optimal psychological expected unit selling price p_i and unit buying price q_i based on his own initial fixed unit revenue r_i. Constraint (7-3) denotes the unit transaction price between any two DMs, and (7-4) provides the achievable conditions for carbon quota trading considering both the DMs' location indices (i.e., i, j) and the relationships between p_i and q_j. Constraint (7-5) defines the individual development index (i.e., Definition 1), and (7-6) specifies the fairness constraints attached to different DMs, where $\alpha\ge 0$ is the fairness measure variable that is pre-determined from the differences among individual development indices (see Section 4.3). Finally, (7-7) provides the thresholds for all variables. Variables $Z_3, o_i^{'}, I_{ij}, p_i, q_i, T_{ij}, H_i, (i\neq j, i,j\in N)$ in Model (7) are to be solved, while $r_i, o_i, o_i^{-}, o_i^{+}, \alpha, \delta, (i\in N)$ are determined in advance.

4.5. Solution method to solve carbon trading consensus models

Clearly, Model (6) is a non-convex optimization problem with many decision variables to be determined. As the pricing decisions (i.e., variables p_i, q_i) have no direct effect on the objective function Z_2, we remove these two variables using constraints (6-1), (6-4), and (6-5), thus obtaining the following relaxation model:

$$\max Z_2 = r_i o_i^{'} + \sum_{j=1,j\neq i}^{n} T_{ij} I_{ij} - \sum_{j=1,j\neq i}^{n} T_{ji} I_{ji}$$
$$\text{s.t.} \begin{cases} \begin{cases} T_{ij} = 0, & \text{if } i \geq j,\ i,j \in N \\ r_i \leq T_{ij} \leq r_j, & \text{if } i < j,\ i,j \in N \end{cases} & (8-1) \\ \sum_{i=1}^{n} r_i o_i^{'} = \max Z_1 & (8-2) \\ o_i^{'} = o_i - \sum_{j=1,j\neq i}^{n} I_{ij} + \sum_{j=1,j\neq i}^{n} I_{ji}, i \in N & (8-3) \\ \begin{cases} I_{ij} = 0, & \text{if } i \geq j, i, j \in N \\ I_{ij} \leq \dfrac{2(r_j - r_i)}{\delta}, & \text{if } i < j, i, j \in N \end{cases} & (8-4) \\ o_i^{-} \leq o_i^{'} \leq o_i^{+}, I_{ij} \geq 0, T_{ij} \geq 0, \delta > 0, i, j \in N & (8-5) \end{cases} \tag{8}$$

where Z_1 is the maximum value obtained from Model (3), and definitions of other variables and constraints refer to Model (6). Without loss of generality, Model (6) is the original problem, and the relaxation model (i.e., Model (8)) is its sub-problem, thus the solution of Model (6) can be directly obtained after solving Model (8). In other words, as the submodel of Model (6), the solution of the relaxation Model (8) doesn't affect the results of Model (6).

To our knowledge, PSO algorithm was put forward to optimize nonlinear functions based on the initial point and stopping criteria (Kennedy & Eberhart, 1995), and has been proven to be an effective tool for streamlining decision making (Cabrerizo et al., 2013; Zhou et al., 2020a). In this paper, a relaxation method based on the PSO algorithm (i.e., Algorithm 1) is proposed for determining the optimal solution of Model (6). In specific, Algorithm 1 is proposed to solve the original problem (i.e., Model (6)), while PSO algorithm is used to solve its sub-problem (i.e., Model (8)). Note that, if the selection of parameters (e.g., initial points) is appropriate, a global optimal solution can be found (Campana et al., 2010; Sun et al., 2012). Generally, the above-mentioned non-convex models can be linearized and solved using standard exact solvers, but linearization only obtains an approximate solution, while our proposed relaxation method can derive the equivalent form of the original problem. By adopting similar principles, a fine-tuning algorithm can be used to solve Model (7), but it is omitted here due to space limitations.

Algorithm 1 Relaxation method based on PSO algorithm for solving Model (6).

Input: Number of DMs, N; d_i's initial carbon quota, o_i; d_i's initial fixed unit revenue, r_i; d_i's expected carbon quota interval, $[o_i^-, o_i^+]$; the maximum overall revenue obtained from Model (3), Z_1; the maximal number of iterations, *limit*; population size, M.

Output: d_i's final carbon quota, $o_i^{'}$; d_i's unit selling and buying prices, p_i, q_i; the transferred quantity, I_{ij}; the unit transaction price, T_{ij}; the specific DM's maximum total revenue, Z_2.

Step 1: Remove decision variables p_i, q_i to obtain a relaxation optimization model (see Model (8)), based on constraints (6-1), (6-4) and (6-5);

Step 2: Use PSO algorithm to solve Model (8);

Set current iteration as $t = 0$;
for each particle i **do**
    Initialize velocity V_i and position X_i for particle i;
    Evaluate particle i by the defined fitness function and set $pBesti = X_i$;
end for
$gBest$=min $\{pBesti\}$;
while t<*limit* **do**
    for $i = 1$ to M **do**
        Update the velocity and position of particle i;
        Evaluate particle i by the defined fitness function;
        if $fit(X_i) < fit(pBesti)$ **then return** $pBesti$=X_i;
        end if
        if $fit(pBesti) < fit(gBest)$ **then return** $gBest = pBesti$;
        end if
    end for
end while
Z_2= $-fit(gBest)$;
return Best solution of $o_i^{'}, I_{ij}, T_{ij}, Z_2$.

Step 3: Derive the optimal values of p_i, q_i based on relaxation constraints (6-1) and (6-5) .

5. Numerical analysis

To verify the feasibility of the optimization consensus models proposed in this paper, this section presents a numerical case study. Suppose there are five regions (d_1, d_2, d_3, d_4, d_5) in a closed-loop carbon quota trading system (i.e., $N = \{1, \cdots, 5\}$). The initial information provided by each region is summarized in Table 4.

Table 4 Summary of the initial trading information provided by five regions

Regions	r_i	o_i	o_i^-	o_i^+	$r_i o_i$
d_1	12	16	13	19	192
d_2	15	20	16	24	300
d_3	23	34	27	41	782
d_4	34	18	14	22	612
d_5	40	12	10	26	480
Total	—	100	80	132	2366

Note: d_i is the i-th region; r_i denotes the initial fixed unit revenue; o_i is d_i's initial carbon quota; o_i^- and o_i^+ are the lower and upper limit of d_i's initially expected interval, respectively; and $r_i o_i$ is d_i's initial carbon quota holding revenue.

5.1. Steps of the research on carbon quota trading mechanism

To clarify the construction mechanism described in this paper, five steps are presented below.

Step 1: Referring to Model (3), an optimization carbon trading model is built to achieve overall revenue maximization, i.e., to obtain the optimal carbon quota allocation scheme for different regions from the collective perspective. Specifically, the carbon quota quantities transferred among regions and the maximum value of the final total revenue of the system are acquired.

Step 2: Using the maximum overall revenue obtained in Step 1, and by adding the constraint of the unit transaction price, a series of optimization consensus models are built based on Model (6). Hence, a total of n allocation schemes are derived by maximizing each region's revenue, and detailed information such as d_i's unit buying and selling prices, transferred quantities, and unit transaction prices is obtained.

Step 3: Through a comparison of the individual/group development indices, it can be determined whether regions have developed fairly or not. If not, some discordant regions are identified by a pre-defined threshold γ, then their initial parameters are adjusted accordingly. Next, the calculations in Steps 1 and 2 are repeated until the allocation scheme satisfies the fairness requirement.

Step 4: Introduce the fairness measure variable α to build consensus models based on Model (7), so as to directly obtain fair carbon quota allocation schemes for the five regions in terms of the maximum overall revenue, quantities of carbon quota transferred, and the unit transaction prices. Additionally, a sensitivity analysis is applied to α to provide flexible suggestions for authorities involved in the trading system.

Step 5: Conduct a comparison and discussion based on the results obtained in each step.

5.2. Analysis of the overall revenue maximization model

Based on Model (3), we obtain a closed-loop carbon quota trading system involving the five regions listed in Table 4. Aiming to maximize the overall revenue, an optimization consensus model is constructed:

$$
\max Z_1 = 12 * o_1^{'} + 15 * o_2^{'} + 23 * o_3^{'} + 34 * o_4^{'} + 40 * o_5^{'}
$$

$$
\text{s.t.} \begin{cases}
\begin{cases}
o_1^{'} = 16 - \sum_{j=2}^{5} I_{1j} + \sum_{j=2}^{5} I_{j1};\ o_2^{'} = 20 - \sum_{j=1,j\neq 2}^{5} I_{2j} + \sum_{j=1,j\neq 2}^{5} I_{j2} \\
o_3^{'} = 34 - \sum_{j=1,j\neq 3}^{5} I_{3j} + \sum_{j=1,j\neq 3}^{5} I_{j3};\ o_4^{'} = 18 - \sum_{j=1,j\neq 4}^{5} I_{4j} + \sum_{j=1,j\neq 4}^{5} I_{j4} \\
o_5^{'} = 12 - \sum_{j=1}^{4} I_{5j} + \sum_{j=1}^{4} I_{j5}
\end{cases} & (9-1) \\
q_1 \leq 12 \leq p_1,\ q_2 \leq 15 \leq p_2, q_3 \leq 23 \leq p_3,\ q_4 \leq 34 \leq p_4,\ q_5 \leq 40 \leq p_5 & (9-2) \\
\begin{cases}
I_{ij} \leq \dfrac{|q_j - p_i| + q_j - p_i}{\delta}, & i < j, i, j \in N \\
I_{ij} = 0, & \text{otherwise}
\end{cases} & (9-3) \\
13 \leq o_1^{'} \leq 19,\ 16 \leq o_2^{'} \leq 24,\ 27 \leq o_3^{'} \leq 41,\ 14 \leq o_4^{'} \leq 22,\ 10 \leq o_5^{'} \leq 26 & (9-4) \\
p_i \geq 0, q_i \geq 0, I_{ij} \geq 0, \delta > 0, i, j \in N & (9-5)
\end{cases} \quad (9)
$$

The objective function in Model (9) aims to maximize the total holding revenue for all five regions through the carbon quota trading process, where $o_i^{'}, i \in N$ is the final quota for the i-th region, which is restricted by both (9-1) and (9-4). Constraints (9-2)–(9-3) concern the unit selling and buying prices, and the transferred quantity for each region. δ in constraint (9-3) is a non-Archimedean infinitesimal, and hereafter it is set as $\delta = 10^{-6}$. The optimal solution of Model (9) is presented in Table 5.

Table 5 Optimal solution of Model (9) with overall revenue maximization

Regions	r_i	$o_i^{'}$	$r_i o_i^{'}$	I_{ij}	Value-I
d_1	12	13	156	(1,5)	3
d_2	15	16	240	(2,5)	4
d_3	23	27	621	(3,5)	7
d_4	34	18	612		
d_5	40	26	1040		
Total	—	100	2669	—	—

Note: d_i denotes the i-th region; r_i denotes the initial fixed unit revenue of carbon quota; $o_i^{'}$ is d_i's final carbon quota; $r_i o_i^{'}$ is d_i's final carbon quota holding revenue; and I_{ij} denotes the quantity transferred from d_i to d_j.

The results in Table 5 show that the maximum value of the objective function in Model (9) is 2669. According to Theorem 4, the critical region within the trading system is d_4, namely, $m = 4$. When $i < m$, the regions with small original fixed unit revenues are d_1, d_2, d_3. These regions can increase their revenue by selling carbon quotas, and their final quotas are the lower limit of their original expected intervals, namely 13, 16, and 27, respectively (i.e., o_i^- in Table 4). When $i > m$, i.e., for d_5, the only way to increase revenue is to purchase carbon quotas, and the final quota for this region is the upper limit of the original interval, namely 26 (i.e., o_i^+ in Table 4). Moreover, the final quota for d_4 is located in the initial range, and the data of I_{ij} show that region d_4 does not become involved in the trading. In summary, Theorem 4 has been verified. Region d_1 sold three carbon quota units to d_5; region d_5 bought three, four, and seven carbon quota units from regions d_1, d_2, d_3, respectively, making its total buying quantity $3 + 4 + 7 = o_5^{'} - o_5 = 26 - 12 = 14$. The transferred quantities for the remaining regions can be obtained in the same way. Thus, Theorem 5 has also been verified. Note that the revenue for each region in Table 5 only involves the fixed revenue for holding a certain carbon quota, while the transaction revenue from the trading of carbon quotas is not included.

5.3. Analysis of the single-region revenue maximization model

Model (9) can only provide feasible regions for variables $p_i, q_i, (i \in N)$, rather than their optimal values (see Section 4.1). Therefore, we construct Model (10) to acquire these optimal values under the objective of maximizing the revenue of individual regions, which follows the research ideas of Models (5) and (6). Obviously, we obtain five allocation schemes, one for each of the five regions taking part in the carbon quota trading process. For brevity, only the model that maximizes revenue for d_4 is illustrated here.

$$
\max Z_2 = 34 * o_4^{'} + \sum_{j=1,j\neq 4}^{5} T_{4j}I_{4j} - \sum_{j=1,j\neq 4}^{5} T_{j4}I_{j4}
$$

$$
\text{s.t.}\begin{cases}
\begin{cases} p_i \leq T_{ij} \leq q_j, & \text{if } p_i \leq q_j,\ i<j, i,j\in N \\ T_{ij}=0, & \text{otherwise} \end{cases} & (10-1)\\
12*o_1^{'}+15*o_2^{'}+23*o_3^{'}+34*o_4^{'}+40*o_5^{'}=2669 & (10-2)\\
\begin{cases} o_1^{'}=16-\sum_{j=2}^{5} I_{1j}+\sum_{j=2}^{5} I_{j1};\ o_2^{'}=20-\sum_{j=1,j\neq 2}^{5} I_{2j}+\sum_{j=1,j\neq 2}^{5} I_{j2} \\ o_3^{'}=34-\sum_{j=1,j\neq 3}^{5} I_{3j}+\sum_{j=1,j\neq 3}^{5} I_{j3};\ o_4^{'}=18-\sum_{j=1,j\neq 4}^{5} I_{4j}+\sum_{j=1,j\neq 4}^{5} I_{j4} \\ o_5^{'}=12-\sum_{j=1}^{4} I_{5j}+\sum_{j=1}^{4} I_{j5} \end{cases} & (10-3)\\
q_1\leq 12\leq p_1,\ q_2\leq 15\leq p_2, q_3\leq 23\leq p_3,\ q_4\leq 34\leq p_4,\ q_5\leq 40\leq p_5 & (10-4)\\
\begin{cases} I_{ij}\leq \dfrac{|q_j-p_i|+q_j-p_i}{\delta}, & i<j, i,j\in N \\ I_{ij}=0, & \text{otherwise} \end{cases} & (10-5)\\
13\leq o_1^{'}\leq 19,\ 16\leq o_2^{'}\leq 24,\ 27\leq o_3^{'}\leq 41,\ 14\leq o_4^{'}\leq 22,\ 10\leq o_5^{'}\leq 26 & (10-6)\\
p_i\geq 0, q_i\geq 0, I_{ij}\geq 0, T_{ij}\geq 0, \delta>0, i\in N & (10-7)
\end{cases}
\quad (10)
$$

The objective function Z_2 in Model (10) is the maximum total revenue that can be achieved by region d_4 through carbon trading. This is composed of fixed revenue for holding carbon quotas (i.e., $34 * o_4^{'}$) and transaction revenue for trading behavior (i.e., $\sum_{j=1,j\neq 4}^{5} T_{4j}I_{4j} - \sum_{j=1,j\neq 4}^{5} T_{j4}I_{j4}$). Constraint (10-2) ensures that the above trading is carried out under the premise of maximizing the overall revenue, where 2669 is the maximum value obtained by solving Model (9). Other definitions see Model (9). Using Algorithm 1, the optimal solution of Model (10) is presented in Table 6, while the results of maximizing the revenue for other regions see Table A1. Here, all the demand parameters in Algorithm 1 are set as $N = 5, Z_1 = 2669, limit = 5000$ and $M = 50$. In addition, the values for o_i, r_i, o_i^-, o_i^+ refer to Table 4 and the parameters regarding the PSO algorithm are set in Matlab R2016a by default.

The decision variable $o_i^{'}$ in Model (6) is directly given by constraints (6-6)–(6-8), but needs to be solved under constraints (10-3) and (10-6) in Model (10). The results in Tables 6 and A1 indicate that, regardless of which region's revenue is maximized, the optimal allocation scheme is fixed and consistent with the results obtained in Section 5.2, that is, $o_1^{'} = 13, o_2^{'} = 16, o_3^{'} = 27, o_4^{'} = 18, o_5^{'} = 26$. Moreover, the unit selling and buying prices of each region

Table 6 Optimal solution of Model (10) with d_4's revenue maximization

Regions	$o_i^{'}$	p_i	q_i	I_{ij}	Value-I	Value-T	H_i	$\lvert H_i-\bar{H}\rvert$	Z_2
d_1	13	12	[0,12]	(1,4)	3	12	1.0000	0.1281	
d_2	16	15	15	(2,4)	4	15	1.0000	0.1281	
d_3	27	23	23	(3,4)	7	23	1.0000	0.1281	915
d_4	18	34	34	(4,5)	14	40	1.4951	0.3670	
d_5	26	[40,+∞)	40				1.0000	0.1281	

Note: $o_i^{'}$ is d_i's final carbon quota; p_i, q_i are d_i's unit selling and buying prices; I_{ij} is the quantity transferred from d_i to d_j with Value-I as its specific value and Value-T as its corresponding unit transaction price; $H_i, \bar{H}$ are individual/group development index; and Z_2 is the optimal value of the objective function regarding single DM's revenue maximization.

are also consistent, although the transferred quantities, corresponding unit transaction prices, and the individual development indices differ in each model. Note that, the values of p_5, q_1 are intervals due to the reason that they are subjected to unilateral constraints of corresponding T_{ij}. In fact, these are auxiliary variables for realizing the trading process, because d_1 cannot purchase carbon quotas and d_5 cannot sell carbon quotas considering their fixed order of unit revenues. Optimal values of all $T_{ij}, (i, j \in N)$ are provided during the calculation, but most are omitted here because they don't affect our analysis on the results.

The relationship between the individual development index H_i and the group development index $\bar{H}$ is now analyzed to identify whether there exist some discordant regions with too much or too little revenue growth. First, based on $\sum_{i=1}^{5} r_i o_i = 2366$ in Table 4 and $\sum_{i=1}^{5} r_i o_i^{'} = 2669$ in Table 5, we derive the group development index as $\bar{H} = \frac{2669}{2366} = 1.128064$. Based on the data in Tables 4, 5, and 6, individual development indices for each region can then be computed. Taking d_4 as an example, $H_4 = \frac{r_4 o_4^{'} + T_{45} I_{45} - T_{14} I_{14} - T_{24} I_{24} - T_{34} I_{34}}{r_4 o_4} = \frac{34*18+14*40-3*12-4*15-7*23}{34*18} = 1.4951$. The individual development indices in Tables 6 and A1 can be derived using a similar calculation method.

5.4. Identification and parameter adjustment of discordant regions

Using the individual development indices H_i in Table A1, we obtain the absolute values of the differences in development indices between each region and the group (i.e., $|H_i - \bar{H}|$) or the absolute difference between any two regions (i.e., $|H_i - H_j|$). Generally, in actual GDM problems, we can always judge whether the development of different regions is balanced, namely, we can always pre-determine a threshold γ to identify discordant regions. To determine the value of the parameter γ, Table 7 summarizes various development indices based on Table A1, including the maximum, minimum, and mean for the abovementioned difference values. Numbers in bold font indicate relatively large values in each column, which require more attention.

Table 7 Summary of the development indices under the region's revenue maximization

Difference between individual and group $\vert H_i - \bar{H}\vert$			**Difference between individuals** $\vert H_i - H_j\vert$		
Maximum	Minimum	Average	Maximum	Minimum	Average
0.3094	0	0.1461	0.4375	0	0.2336
0.4853	0.0396	0.1692	**0.6133**	0	**0.2789**
0.2594	0.1281	0.1543	0.3875	0	0.1550
0.3670	0.1281	0.1759	0.4951	0	0.1980
0.5032	0.1281	**0.2031**	**0.6313**	0	**0.2525**

Referring to the adjustment rules designed in Section 4.3, the initial parameters provided by discordant regions, namely, their predetermined expected carbon quota interval $[o_i^-, o_i^+]$, will be adjusted accordingly. If we set $\gamma = 0.5$, then only d_5 is identified as a discordant region with too much revenue growth. However, if the difference between the development indices of any two regions is considered, the corresponding maximum values for d_2, d_5 should be considered, as they are both greater than 0.6. Thus, the threshold for the parameter is adjusted to $\gamma = 0.45$. Liang et al. (2020) concluded that the shorter the time required to reach a consensus, the more necessary it is to make greater adjustments to the initial opinions. Initially concluded from a phenomenon of 20% people possessing 80% of the wealth in the world, the 80/20 Rule (i.e., the Pareto principle) is now extended to a fact that an optimal ratio exists between the effort and gain. In other words, once we change 20% of the key factors, qualitative change will occur, implying that we can derive enough (like 80% of) expected results on that critical point. Therefore, we may wish to adjust the endpoints of the expected carbon quota interval by 20% of their initial values. Because d_2 sold too much of his quota, the quota interval is adjusted from $[16, 24]$ to $\underline{[20, 24]}$; and as d_5 purchased too much carbon quota, his expected range is adjusted from $[10, 26]$ to $\underline{[10, 23.6]}$. Here, taking d_2 as an instance for specific explanation. Acted as a seller, d_2 needs to decrease its sales volum to reduce its revenue growth, so d_2 increases its lower limit by adding 20% of its initial carbon quota (i.e., o_2), thus we derive the adjusted lower limit of d_2's expected interval as $16 + 20\% * 20 = 20$. Distinguished from d_2, the buyer d_4 should decrease its upper limit so as to possess less carbon quota at the end.

After repeating the calculations of Models (3) and (6), new allocation schemes are obtained. For brevity, the specific calculation models are omitted here. Using updated information, the new optimal allocation scheme for overall revenue maximization is as presented in Table 8; the schemes maximizing different region's revenue are presented in Table A2. Table 9 provides an updated summary of the development indices after adjusting the initial parameters of d_2, d_5 by 20% of their initial carbon quotas.

Table 8 Optimal solution for maximizing overall revenue after adjusting the initial parameters of d_2, d_5

Regions	r_i	$o_i^{'}$	$r_i o_i^{'}$	I_{ij}	Value-I
d_1	12	13	156	(1,2)	3
d_2	15	20	300	(2,5)	3
d_3	23	27	621	(3,5)	7
d_4	34	16.4	557.6	(4,5)	1.6
d_5	40	23.6	944		
Total	—	100	2578.6	—	—

Note: Definitions of notation see Table 5.

Table 9 Summary of the development indices under the region's revenue maximization after adjusting the initial parameters of d_2, d_5

Difference between individual and group $\|H_i - \bar{H}\|$			**Difference between individuals $\|H_i - H_j\|$**		
Maximum	Minimum	Average	Maximum	Minimum	Average
0.3476	0.0064	0.1211	**0.4375**	0	**0.2073**
0.1901	0.0064	0.0896	0.2800	0	0.1443
0.1697	0.0699	0.1018	0.2596	0	0.1078
0.2575	0.0899	0.1234	0.3474	0	0.1390
0.3531	0.0899	**0.1425**	**0.4429**	0	0.1772

Results in Tables 7 and 9 show that the unfairness in the system is ameliorated by adjusting the initial parameters of d_2, d_5. Specifically, the maximum difference between the individual and group development indices drops from 0.5032 to 0.3531, while the maximum difference between any two regions drops from 0.6313 to 0.4429. In fact, if policy-makers are not satisfied with the results in Table 9, they may repeat the above calculations. The maximum value of each region's revenue declines in most scenarios because the total transaction amount decreases as the overall revenue drops from 2669 to 2578.6 (see column Z_2 in Tables A1 and A2). Note that the identification of discordant regions, adjustment of their parameters, and fairness of the final result all depend on the experience of the policy-makers. In addition, the adjustment range of the initial parameters for those discordant regions has a significant influence on the number of adjustments and the final allocation scheme of the trading system. Obviously, the "fairness" reached through the above strategy is effective, but subjective and rather complicated.

5.5. Analysis regarding both fairness and revenue

Based on Table 4 and Model (7), this section considers the optimization consensus model (i.e., Model (11)) for obtaining a relatively fair carbon quota allocation scheme with the goal of maximizing the final overall revenue within the closed-loop trading system.

$$
\max Z_3 = 12 * o_1^{'} + 15 * o_2^{'} + 23 * o_3^{'} + 34 * o_4^{'} + 40 * o_5^{'}
$$

$$
\text{s.t.}\begin{cases}
\begin{cases}
o_1^{'} = 16 - \sum\limits_{j=2}^{5} I_{1j} + \sum\limits_{j=2}^{5} I_{j1};\ o_2^{'} = 20 - \sum\limits_{j=1,j\neq 2}^{5} I_{2j} + \sum\limits_{j=1,j\neq 2}^{5} I_{j2} \\
o_3^{'} = 34 - \sum\limits_{j=1,j\neq 3}^{5} I_{3j} + \sum\limits_{j=1,j\neq 3}^{5} I_{j3};\ o_4^{'} = 18 - \sum\limits_{j=1,j\neq 4}^{5} I_{4j} + \sum\limits_{j=1,j\neq 4}^{5} I_{j4} \\
o_5^{'} = 12 - \sum\limits_{j=1}^{4} I_{5j} + \sum\limits_{j=1}^{4} I_{j5}
\end{cases} & (11-1) \\
q_1 \leq 12 \leq p_1,\ q_2 \leq 15 \leq p_2, q_3 \leq 23 \leq p_3,\ q_4 \leq 34 \leq p_4,\ q_5 \leq 40 \leq p_5 & (11-2) \\
\begin{cases}
p_i \leq T_{ij} \leq q_j, \quad \text{if } p_i \leq q_j,\ i < j, i, j \in N \\
T_{ij} = 0, \qquad \text{otherwise}
\end{cases} & (11-3) \\
\begin{cases}
I_{ij} \leq \dfrac{|q_j - p_i| + q_j - p_i}{\delta}, \quad i < j, i, j \in N \\
I_{ij} = 0, \qquad \text{otherwise}
\end{cases} & (11-4) \\
13 \leq o_1^{'} \leq 19,\ 16 \leq o_2^{'} \leq 24,\ 27 \leq o_3^{'} \leq 41,\ 14 \leq o_4^{'} \leq 22,\ 10 \leq o_5^{'} \leq 26 & (11-5) \\
H_i = \dfrac{r_i o_i^{'} + \sum\limits_{j=1,j\neq i}^{n} T_{ij} I_{ij} - \sum\limits_{j=1,j\neq i}^{n} T_{ji} I_{ji}}{r_i o_i}, i \in N & (11-6) \\
|H_i - H_j| \leq \alpha, i < j, i, j \in N & (11-7) \\
p_i \geq 0, q_i \geq 0, I_{ij} \geq 0, T_{ij} \geq 0, H_i \geq 0, \delta > 0, \alpha \geq 0, i, j \in N & (11-8)
\end{cases} \tag{11}
$$

Z_3 in Model (11) maximizes the overall revenue of the carbon quota trading system. Constraint (11-1) describes the relationship between the final quotas and the carbon quotas transferred by each region, and $\sum\limits_{i=1}^{n} o_i^{'} = 100$. Constraints (11-2)–(11-4) concern the unit buying and selling prices, the unit transaction prices and transferred quantities, where δ is a pre-determined non-Archimedean infinitesimal. Constraint (11-5) is the threshold for decision variable $o_i^{'}$, and (11-6) defines the individual development index. Constraint (11-7) is the fairness restriction, where α is the pre-determined fairness measure variable. Other variables are consistent with those in Model (7).

Table 10 presents the solution set for Model (11) when the fairness measure variable $\alpha = 0$. At this time, the trading system achieves an absolutely fair state, that is, all individual development indices are equal to the group development index of 1.1281. The results of a sensitivity analysis of α are given in Table B1, and show that any value of α in the interval [0,0.5] gives an optimal value of the objective function of 2669. The final carbon quotas for all regions are also fixed to $o_1^{'} = 13, o_2^{'} = 16, o_3^{'} = 27, o_4^{'} = 18, o_5^{'} = 26$.

Table 10 Solutions to Model (11) when $\alpha = 0$

	p_i	q_i	I_{ij}	Value-I	Value-T	I_{ij}	Value-I	Value-T	H_i
d_1	15	[0,12]	(1,2)	0.09	15	(2,5)	1.82	36.15	1.1281
d_2	15	15	(1,3)	1.54	23	(3,4)	5.36	34	1.1281
d_3	34	23	(1,4)	1.37	17.36	(3,5)	3.51	34	1.1281
d_4	36.15	34	(2,3)	0.32	15	(4,5)	8.68	36.15	1.1281
d_5	$[40,+\infty)$	36.15	(2,4)	1.95	15				1.1281

Note: Definitions of notation see Table 6.

Tables 10 and B1 show that, as the fairness measure variable α gradually decreases, although the final carbon quota of each region $o_i^{'}$ is fixed, the transaction frequency significantly increases, implying that carbon quotas are fully traded within the system. Besides, when α is greater than 0.1, the variables p_i, q_i for each region are fixed, but when $\alpha \leq 0.1$, those pricing decisions change. Overall, the introduction of the fairness measure changes the allocation schemes by increasing the number of trading paths in the system. Clearly, as the closed-loop carbon quota trading mechanism gradually complicates the transaction process, a state of absolute fairness is finally reached, namely, sufficient interactions among regions are achieved as the fairness measure variable decreases to zero.

5.6. Discussion

To verify the rationality and effectiveness of the proposed models in the paper, this section has considered the example of carbon quota trading among five regions. Our optimization consensus models can derive the optimal allocation scheme from the global perspective (i.e., the moderator's perspective in GDM), and can also obtain allocation schemes from different DM's perspectives, in which the maximization of each region's revenue is the modeling goal. The following findings can be elicited from our results:

- Consensus modeling to maximize the overall revenue can obtain the optimal allocation scheme for the whole group, but cannot identify specific pricing decisions. Moreover, the final carbon quotas of different regions obtained from the models that maximize each region's revenue are the same as those obtained from the former modeling mechanism. That is, the optimal values of $o_i^{'}$ are fixed. However, detailed trading information, such as the trading regions involved and the unit transaction prices, change with the specific region being studied (see Tables 5, A1, 8, and A2).

- The unit selling and buying prices of each region (i.e., variables p_i, q_i) derived from the proposed optimization consensus models do not change according to which region's revenue is being maximized (see Tables A1 and

A2) and do not depend on the value of the fairness measure variable (see Table B1). This indirectly implies that the carbon quota trading mechanism discussed in this paper is robust to some extent.

- For the two strategies proposed to deal with the unfairness issue within the trading system, adjusting the initial parameters of discordant regions is effective (see Tables 7 and 9), but complicated in practice. In addition, the parameter γ for identifying discordant regions, the adjustment range for each region, and whether the final allocation scheme meets the GDM requirements are all subjective (see Section 5.4). In contrast, the strategy of directly introducing the fairness measure variable α is convenient and effective, and further sensitivity analysis enables feasible allocation schemes to be obtained (see Tables 10 and B1).
- The introduction of the fairness measure variable increases the number of trading paths among different regions (see Tables 10 and B1), meaning that absolute fairness within the closed-loop system is realized only when carbon quotas are fully traded among different regions. Thus, sufficient interactions among participators are highly significant in achieving consensus or the pursuit of DMs' balanced development during a GDM process.

6. Conclusion

This paper has described the use of optimization consensus modeling theory to explore theoretical innovations regarding flexible carbon trading mechanisms. Specifically, we have investigated essential carbon quota allocation schemes within a closed-loop trading system with the aim of ensuring both revenue maximization and fairness. First, the optimal carbon quota allocation scheme was derived by maximizing the overall revenue through Model (3). Then, its analytical formula and the achievable conditions for successful trading were provided through theoretical deduction. Next, simultaneously taking the group revenue maximization and the competition mechanism into account, models for deriving the optimal allocation schemes by maximizing individual's revenues were constructed as Models (5) and (6). Since conflicts of interest are the main reasons for the failure of GDM in the real world, individual/group development indices were defined as Definitions (1) and (2), and two fairness strategies were further presented. The former is based on calculating the difference between the development indices, with fairness achieved through the identification of discordant DMs and the adjustment of their initial parameters. The latter introduces a fairness measure variable, allowing fair allocation schemes to be directly obtained from Model (7). Finally, a numerical example was conducted to demonstrate the performance of the proposed models.

The results show that the final carbon quotas of all regions can be determined through the proposed consensus models, but detailed trading information (including the participating regions and the unit transaction prices) can only be acquired through the models that focus on single-region revenue maximization. In addition, the strategies for dealing with the unfairness issue are both practical and effective, but the second strategy of directly introducing a fairness measure variable is more objective and easier to operate. Finally, the results of a sensitivity analysis of the fairness measure variable show that, as the variable decreases to zero, that is, when the group approaches the state of absolute fairness, the frequency of DMs' transactions within the group increases significantly, corresponding to the fact that reaching fairness within a group requires sufficient interactions among DMs.

In the future, some varaibles in our proposed models will be comprehensively determined to be more in line with real-life, for example, price variables are no longer static and could be accurately positioned by combining with game theory (Liu et al., 2021; Zheng et al., 2019). In addition, trading mechanisms should also focus on some critical factors, such as risk or utility (Zheng & Chang, 2021) in practical markets, rather than only considering the allocation and pricing decisions from the revenue maximization perspective. Moreover, with large-scale GDM problems (Dong et al., 2018; Zhang et al., 2017), especially under social network contexts (Liu et al., 2019; Wu et al., 2019), attracting increased attention, the use of artificial intelligence methods (Ding et al., 2020) to solve large-scale trading issues will also be a focus of our subsequent research.

Appendix A. Results with single region's revenue maximization

Based on Sections 4.2 and 4.3, Table A1 lists the optimal solutions (including $o_i^{'}, p_i, q_i, I_{ij}, T_{ij}$, and Z_2) to Model (6) and the values of the development indices (including H_i and $|H_i - \bar{H}|$) in the case of each region maximizing its revenue (note: the specific region discussed in Model (6) is marked with $\star$ in the first column in Table A1). Moreover, Table A2 exhibits the corresponding results after the initial parameters of d_2, d_5 have been adjusted by 20% of their initial carbon quotas.

Appendix B. Sensitivity analysis of the fairness measure variable

If the fairness measure variable α in Model (11) is decreased from 0.5 at intervals of 0.1, then the optimal solutions of the above optimization consensus model are as listed in Table B1.

Table A1 Optimal solution of Model (6) with different region's revenue maximization

Regions	$o_i^{'}$	p_i	q_i	I_{ij}	Value-I	Value-T	H_i	$\lvert H_i - \bar{H} \rvert$	Z_2
$\star d_1$	13	12	[0,12]	(1,5)	3.00	40	1.4375	0.3094	
d_2	16	15	15	(2,4)	2.10	15	1.0000	0.1281	
d_3	27	23	23	(2,5)	1.90	15	1.0000	0.1281	276
d_4	18	34	34	(3,4)	3.50	23	1.1280	0	
d_5	26	[40,+∞)	40	(3,5)	3.50	23	1.2930	0.1650	
–	–	–	–	(4,5)	5.60	34	–	–	
d_1	13	12	[0,12]	(1,2)	3	12	1.0000	0.1281	
$\star d_2$	16	15	15	(2,5)	7	40	1.6133	0.4853	
d_3	27	23	23	(3,4)	3.50	23	1.0000	0.1281	484
d_4	18	34	34	(3,5)	3.50	23	1.0629	0.0652	
d_5	26	[40,+∞)	40	(4,5)	3.50	34	1.1677	0.0396	
d_1	13	12	[0,12]	(1,3)	3	12	1.0000	0.1281	
d_2	16	15	15	(2,3)	4	15	1.0000	0.1281	
$\star d_3$	27	23	23	(3,5)	14	40	1.3875	0.2594	1085
d_4	18	34	34				1.0000	0.1281	
d_5	26	[40,+∞)	40				1.0000	0.1281	
d_1	13	12	[0,12]	(1,4)	3	12	1.0000	0.1281	
d_2	16	15	15	(2,4)	4	15	1.0000	0.1281	
d_3	27	23	23	(3,4)	7	23	1.0000	0.1281	915
$\star d_4$	18	34	34	(4,5)	14	40	1.4951	0.3670	
d_5	26	[40,+∞)	40				1.0000	0.1281	
d_1	13	12	[0,12]	(1,5)	3	12	1.0000	0.1281	
d_2	16	15	15	(2,5)	4	15	1.0000	0.1281	
d_3	27	23	23	(3,5)	7	23	1.0000	0.1281	783
d_4	18	34	34				1.0000	0.1281	
$\star d_5$	26	[40,+∞)	40				1.6313	0.5032	

Note: $o_i^{'}$ is d_i's final carbon quota; p_i, q_i are d_i's unit selling and buying prices; I_{ij} is the quantity transferred from d_i to d_j with Value-I as its specific value and Value-T as its corresponding unit transaction price; $H_i, \bar{H}$ are individual/group development index; and Z_2 is the optimal value of the objective function regarding single DM's revenue maximization.

Table A2 Optimal solution of Model (6) with different region's revenue maximization after adjusting the initial parameters of d_2, d_5

Regions	$o_i^{'}$	p_i	q_i	I_{ij}	Value-I	Value-T	H_i	$\|H_i - \bar{H}\|$	Z_2
$\star d_1$	13	12	[0,12]	(1,5)	3.00	40	1.4375	0.3476	
d_2	20	15	15	(3,4)	4.64	23	1.0000	0.0899	
d_3	27	23	23	(3,5)	2.36	23	1.0000	0.0899	276
d_4	16.4	34	34	(4,5)	6.24	34	1.0835	0.0064	
d_5	23.6	[40,+∞)	40				1.1615	0.0716	
d_1	13	12	[0,12]	(1,2)	3.00	12	1.0000	0.0899	
$\star d_2$	20	15	15	(2,5)	3.00	40	1.2800	0.1901	
d_3	27	23	23	(3,4)	4.64	23	1.0000	0.0899	384
d_4	16.4	34	34	(3,5)	2.36	23	1.0835	0.0064	
d_5	23.6	[40,+∞)	40	(4,5)	6.24	34	1.1615	0.0716	
d_1	13	12	[0,12]	(1,3)	3	12	1.0000	0.0899	
d_2	20	15	15	(3,5)	10	40	1.0000	0.0899	
$\star d_3$	27	23	23	(4,5)	1.6	34	1.2596	0.1697	985
d_4	16.4	34	34				1.0000	0.0899	
d_5	23.6	[40,+∞)	40				1.0200	0.0699	
d_1	13	12	[0,12]	(1,4)	3	12	1.0000	0.0899	
d_2	20	15	15	(3,4)	7	23	1.0000	0.0899	
d_3	27	23	23	(4,5)	11.6	40	1.0000	0.0899	824.6
$\star d_4$	16.4	34	34				1.3474	0.2575	
d_5	23.6	[40,+∞)	40				1.0000	0.0899	
d_1	13	12	[0,12]	(1,5)	3	12	1.0000	0.0899	
d_2	20	15	15	(3,5)	7	23	1.0000	0.0899	
d_3	27	23	23	(4,5)	1.6	34	1.0000	0.0899	692.6
d_4	16.4	34	34				1.0000	0.0899	
$\star d_5$	23.6	[40,+∞)	40				1.4429	0.3531	

Note: $o_i^{'}$ is d_i's final carbon quota; p_i, q_i are d_i's unit selling and buying prices; I_{ij} is the quantity transferred from d_i to d_j with Value-I as its specific value and Value-T as its corresponding unit transaction price; $H_i, \bar{H}$ are individual/group development index; and Z_2 is the optimal value of the objective function regarding single DM's revenue maximization.

Table B1 Sensitivity of the results to the fairness measure variable α

Regions	p_i	q_i	I_{ij}	Value-I	Value-T	H_i	α
d_1	12	[0,12]	(1,5)	3	40	1.4375	
d_2	15	15	(2,5)	4	40	1.3333	
d_3	23	23	(3,5)	7	40	1.1522	0.5
d_4	34	34				1.0000	
d_5	[40,+∞)	40				1.0000	
d_1	12	[0,12]	(1,3)	0.42	23	1.4000	
d_2	15	15	(1,5)	2.58	40	1.3333	
d_3	23	23	(2,5)	4	40	1.1614	0.4
d_4	34	34	(3,5)	7.42	40	1.0000	
d_5	[40,+∞)	40				1.0000	
d_1	12	[0,12]	(1,3)	1.55	23	1.3000	
d_2	15	15	(1,5)	1.45	40	1.3000	
d_3	23	23	(2,5)	4	37.5	1.1859	0.3
d_4	34	34	(3,5)	8.55	40	1.0000	
d_5	[40,+∞)	40				1.0208	
d_1	12	[0,12]	(1,3)	2.68	23	1.2000	
d_2	15	15	(1,5)	0.32	40	1.2000	
d_3	23	23	(2,5)	4	30	1.2000	0.2
d_4	34	34	(3,5)	9.68	39.15	1.0000	
d_5	[40,+∞)	40				1.1004	
d_1	15	[0,12]	(1,3)	1.15	23	1.0946	
d_2	23	15	(1,4)	1.85	15	1.1358	
d_3	34	23	(2,5)	4	25.19	1.1186	0.1
d_4	40	34	(3,4)	1.93	34	1.0946	
d_5	[40,+∞)	40	(3,5)	6.22	34.51	1.1946	
—	—	—	(4,5)	3.78	40	—	

Note: d_i denotes the i-th region; p_i, q_i are d_i's unit selling and buying prices; I_{ij} is the quantity transferred from d_i to d_j with Value-I as its specific value and Value-T as the corresponding unit transaction price; H_i is the individual development index; and α is the fairness measure variable.

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