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A cyclic dynamic trust-based consensus model for large-scale group decision making with probabilistic linguistic information

--Manuscript Draft--

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Abstract:	This paper investigates a consensus reaching process (CRP) considering dynamic trust in large-scale group decision making (LSGDM). In the traditional trust-based consensus model, it is assumed that the trust relationship generated by decision makers (DMs)' previous knowledge remain unchanged during the whole decision process. However, this relationship will be dynamic rather than static especially in a social network with a new decision problem. This study explores the dynamic nature of trust through two stages. In the first stage, the trust degree will be functionally reformed by the conflict caused by DM's opposite preferences. In the second stage, it will be effected by surroundings according to the ``assimilation effect'' in network. To handle the CRP with large-scale decision settings, a clustering technique is used to classify DMs with similar preference and preference accuracy. Based on the classifications, an optimization model is constructed to obtain the trust degrees between subgroups. The consensus measurements are investigated from similarity network within subgroups and min-max programming model between subgroups, respectively. Moreover, preference modification will effect trust in the aggregation and next iteration, the cyclic dynamic trust mechanism is established. The feasibility of the proposed model is verified by a numerical example. Comparisons declare the constructed consensus model's universality without any essential conditions, as well as superiority with fully consideration of DM's utility and centrality in network.

Declaration of Interest Statement

Xiao Tan, Jianjun Zhu, Francisco Javier Cabrerizo and Enrique Herrera-Viedma declared that they have no conflicts of interest to this work.

We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

Highlights:

- Propose the concept of individuals' conflict with three levels based on the defined preference attitudes.
- Design a cyclic dynamic trust mechanism throughout the consensus reaching process.
- Construct local and global feedback mechanisms by similarity network and min-max goal programming model, respectively.
- Introduce trust centrality and conflict constraint into aggregation process can guarantee individuals' utilities.

Dear Editor:

I wish to submit the revised research paper for publication in *Applied Soft Computing*, titled “A cyclic dynamic trust-based consensus model for large-scale group decision making with probabilistic linguistic information.”

This study proposes a consensus reaching process with dynamic social network. A model is proposed to simulate the unsteady trust social network. To solve the consensus mechanisms in probabilistic linguistic large scale group decision making, an optimization model is proposed to obtain the trust degrees between sub-groups after clustering. And modification rules have been set based on local similarity network and group min-max programming model respectively.

Further, we believe that this paper will be of interest to the readership of your journal, because it fits the topic of the journal and has some novel aspects.

This manuscript has not been published or presented elsewhere in part or in entirety and is not under consideration by another journal. We have read and understood your journal’s policies, and we believe that neither the manuscript nor the study violates any of these. There are no conflicts of interest to declare.

Thank you for your consideration. I look forward to hearing from you.

Sincerely,
Xiao Tan, Jianjun Zhu, Francisco Javier Cabrerizo, Enrique Herrera-Viedma

Dear editors and reviewers:

Thank you for giving us the opportunity to revise paper "ASOC-D-20-02148" titled "A cyclic dynamic trust-based consensus model for large-scale group decision making with probabilistic linguistic information". Your valuable comments on our manuscript are of important guiding significance to our paper writing and future study.

We have studied comments carefully and revised paper according to your insightful advice, we hope the improved manuscript can meet your requirements and be closer to your approval earnestly. The point to point responds to the reviewer's comments are listed as following:

Reviewer #1:

The manuscript target a very important topic and provides meaningful insights to solve the problem in the field. However, the writing style is not very impressive. The abstract and highlights are not concrete therefore requires thorough revision. Besides following are some specific comments:

Restructuring of the content is strictly suggested. Introduction must be shortened with proper referencing for the intended users. Details of the section 2 may be clipped out from the manuscript and better fit as appendix at the end.

Technically the manuscript sounds ok and might be accepted after restructuring.

Firstly, we want to express our gratitude for this comment. Then we will give the point-by-point response to comments.

Response:

Thank you for your advice.

(1) Abstract has been written. The original version contains some long and meaningless sentences, it cannot stress the key point.

The modified version firstly summarizes the aim of this paper: *This paper investigates a consensus reaching process (CRP) considering dynamic trust in large-scale group decision making (LSGDM).*

Then we state the limitation of the traditional trust method for CRP: *In the traditional trust-based consensus model, it is assumed that the trust relationship generated by decision makers (DMs)' previous knowledge remain unchanged during the whole decision process. However, this relationship will be dynamic rather than static especially in a social network with a new decision problem.* **Aiming at this topic, we provide the innovations:** *This study explores the dynamic nature of trust through two stages. In the first stage, the trust degree will be functionally reformed by the conflict caused by DM's opposite preferences. In the second stage, it will be effected by surroundings according to the "assimilation effect" in network.*

After showing the basic concept, we state the proposed model including methods: *To handle the CRP with large-scale decision settings, a clustering technique is used to classify DMs with similar preference and preference accuracy. Based on the classifications, an optimization model is constructed to obtain the trust degrees between*

subgroups. The consensus measurements are investigated from similarity network within subgroups and min-max programming model between subgroups, respectively.
And the main feature of the consensus model has been given: *Moreover, preference modification will effect trust in the aggregation and next iteration, the cyclic dynamic trust mechanism is established.*
At last, the example and comparison are concluded: *The feasibility of the proposed model is verified by a numerical example. Comparisons declare the constructed consensus model's universality without any essential conditions, as well as superiority with fully consideration of DM's utility and centrality in network.*

(2)**The original highlights is incomplete. We have rewritten highlights including innovations and conclusions:**
• Propose the concept of individuals' conflict with three levels based on the defined preference attitudes.
• Design a cyclic dynamic trust mechanism throughout the consensus reaching process.
• Construct local and global feedback mechanisms by similarity network and min-max goal programming model, respectively.
• Introduce trust centrality and conflict constraint into aggregation process can guarantee individuals' utilities.

(3)When it comes to Introduction, we have deleted some meaningless sentences in introduction, such as: "The development of information technology like social media has allowed the participation of thousands of users in decision making processes [1-4]. The models developed from conventional group decision making (GDM) problems, in which a small number of decision makers (DMs) are involved to solve a problem, have been replaced for new models allowing to involve a larger number of DMs [5–8]. A problem involving several tens and thousands…."
And we have shorted the first paragraph and combine it with the part of second paragraph to show "consensus reaching process in group decision making".
Moreover, we remove the part about notions like follows in Introduction into model section.
For instance, if we assume that a DM dm^k does not feel sure about some alternative in a certain decision problem, even though the DM dm^l has a totally trust to dm^k, dm^l will change his or her trust attitude toward dm^k aiming at this specific decision problem. Let us suppose a trust network [25], if dm^k is the only one in the network that trusts dm^l, his/her trust will decrease as no other DM trusts dm^l. This behavior is called "assimilation effect" [49]
We have shorted the limits and given the point to point contributions like follows in Introduction, which can provide proper referencing for the intended users.
Several mechanisms and techniques have been constructed in trust-based CRP with large scale DMs. However, there are still some limitations to be solved:
•When it comes to detect the conflict behavior between DMs, it has been assumed that the discretionary selected conflict threshold can measure whether there is conflict among DMs [1, 2]. However, the determination of threshold is subjective, hence it is

necessary to explore a method to analyze the existence of conflict objectively.

•The trust relationships in the above consensus models stay steady across the whole process. However, it is worth noting that the trust relationships are provided by DMs' previous knowledge, it will be likely effected by the current decision problem and surroundings in a network. Therefore, the static trusts between DMs is unreasonable.

•The previous consensus models mainly focused on the consensus index before aggregating, such as the similarity between individual preferences[31] and the similarity between individual and group preferences[23]. However, these models ignored whether the final collective preference can be accepted by all DMs.

•The feedback parameter, which controls the accepting degree of recommendation advices, is discretionary selected and its value is equal for all DMs in the previous work[12]. However, individual has respective willingness of making modification. Therefore, its value should depend on the DMs' behavior characteristics, being different for individual one.

With the above hypothesis, the main purpose of this study is to construct the consensus models with a cyclic dynamic trust mechanism in LSGDM. In particular, the contributions are listed as follows:

• We propose that there will be conflict between DMs who hold opposite preferences. Three levels of conflict degree are provided: conflict degree between DMs about one certain alternative; conflict degree between DMs about all the alternatives; conflict degree of one identified individual with all the other DMs about certain alternative.

• Based on the initial trusts, we propose that the trust degree will change depending on the conflict between DMs in first stage. In the second stage, it will make a modification actively according to the surroundings. Moreover, the preference adjustment will effect DMs' trust by the two stages, then the renewed trust will go into aggregation and next iteration. Therefore, the cyclic dynamic trust mechanism is established.

• It carries out the consensus models from local and group perspectives, respectively. First, a mechanism to improve the consensus within each subgroup based on similarity network is carried out to guarantee the compactedness of cluster (local consensus). Second, a min-max goal programming model is applied to guarantee the collective preference acceptable (global consensus).

• The feedback parameter can be obtained by means of two behavioral criteria: the respective conflict degree and the comparison with other DMs' preferences. This operation fully considers the behavior characteristics and utilities of DMs.

(4) We have reconstructed the content under your guidance. Now the updated outline is shown below. We split the original chapter 4 into two chapters (Section 4 and Section 5), describing local and global consensus models, respectively. Compared with the former version, the models presented are shown more clearly. Moreover, we have deleted the subsections: 4.1. Communication of preferences and 4.2. Update of trust degrees, which are not appropriate to mention in the modeling section.

In detail, the updated Section 4 tries to improve the agreement within each subgroup. It is composed of four steps: (1) clustering, which divides the DMs into subgroups with similar preference and preference accuracy; (2) local consensus reaching process, which includes consensus index based on local similarity network and local feedback mechanism; (3) update of trust degrees after individual preference modification and (4) acquisition of local collective preferences.

The updated Section 5 includes four steps: (1) acquiring the trust degrees between subgroups, which can be realized through an optimization model; (2) conducting the global CRP, which includes consensus index based on group min-max programming model and global feedback mechanism; (3) update of trust degrees after sub-group preference modification and (4) acquisition of global collective preference.

Updated outline:

(5) Section 2 (Preliminaries) has been shorted. We have moved the details of clustering shown by original Section 2.3 into Section 4.3, making the paper more coherent and compact. In addition, some original sentences have been modified, this operation deletes redundant details, such as:

Original version: Since Zadeh introduced the concept of a linguistic variable[35–37], which can simulate humans' thinking effectively and flexibly when making judgements. Different approaches to deal with linguistic information allowing the DMs to provide a single linguistic value[38] or several linguistic values[39–41] have been proposed. Recently, Pang et al.[32] summarized the differences of the methods allowing to express several linguistic values and developed a new general concept to extend the conventional linguistic term sets: the probabilistic linguistic term sets (PLTSs). Compared to other approaches dealing with linguistic information, PLTSs allows the DMs to express several linguistic values along with probabilistic information over an alternative and also can deal with partially incomplete evaluations.

Modified version: Pang et al.[32] developed a new general concept to extend the conventional linguistic term sets: the probabilistic linguistic term sets (PLTSs). Compared to other approaches[35–41] dealing with linguistic information, PLTSs allows DMs to express several linguistic values along with probabilistic information over an alternative, and it also can deal with partially incomplete evaluations.

In conclusion, we have removed some of the unimportant parts from Section 2, making this section clear, simplified and concise instead of arranging them into appendix.

We hope the current version meets your requirements.

Lastly, special thanks to you for your good comments.

Reviewer #2:

Firstly, we want to express our gratitude for this comment. Then we will give the point-by-point response to comments.

1. Section Abstract - Authors are suggested to rephrase or rewrite this sentence as it is too long with too many comma (,) in the sentence. This sentence should be separated into at least two sentences.

"In the trust based consensus models developed for group decision making, it is assumed that the trust relationship established between the decision makers, as a consequence of the prior information that they provided, remains unchanged during the whole decision process."

Response:
Thank you for your advice. This sentence has been rewritten as follows. We have deleted some redundant attributive sentences and reorganized this sentence.
"In the traditional trust-based consensus model, it is assumed that the trust relationships generated by decision makers (DMs)' previous knowledge remain unchanged during the whole decision process."

2. Section Abstract - Authors are suggested to rephrase or rewrite this sentence as it is not appropriate for an abstract. It doesn't carry any meaning. Authors are suggested to write a proper sentence if they want to give example on something.

"(think, for example, in a social network)"

Response:
Thank you for your advice. Considering that it is not appropriate to include examples in the abstract. Therefore, the following sentence has replaced the original sentence to briefly show that the trust will be dynamic as the existence of network.
"However, this relationship will be dynamic rather than static especially in a social network with a new decision problem."

3. Section Abstract - Authors are suggested to include the results for their proposed model in abstract.

Response:
Thank you for your valuable advice. Considering that the proposed model is simulated by a numerical example, it does not involve the actual data. We use the summarized sentence to describe it, including the advantages of the proposed model through comparisons briefly in abstract.

The feasibility of the proposed model is verified by a numerical example. Comparisons *declare the constructed consensus model's universality without any essential conditions as well as superiority with fully consideration of DM's utility and centrality in network.*

4. Section 5 - Authors are suggested to include more explanation and discussion based on Table 1, Table 2, Table 3, Table 4 and Figure 3 to better discuss the example of application on their proposed model. Authors are suggested to discuss more on the results analysis of their example of application.

Response:

Thank you for your valuable advice.

Original Table 1, Table 2, Table 3, Table 4 and Figure 3 are the Table 3, Table 4, Table 5, Table 6 and Figure 5, respectively now.

(1) Table 3 is DMs' original preferences. The discussion about Table 3 are shown around line 555 in blue. We discuss it from two dimensions: expected values and preference accuracy. Expected values can show the distributions of the 30 DMs' original preferences on alternatives, preference accuracy can help to analyze the behavior characteristics of DMs. Moreover, the relevant Figure.3 is provided to visually show the preferences. The details are below with Figure.3:

"We can find that the expected values for x_1 are in the interval (1,4.5), for x_2 are in the interval (3,6) and for x_3 are in the interval (1,5.5). It means there is the smallest difference between preference attitudes of 30 DMs for x_2, and preferences for x_3 show the maximum span. Therefore, we can conclude that it will be easier for x_2 to reach consensus than x_1 and x_3. Moreover, there is no totally accurate preference with preference accuracy as "1" for all alternatives."

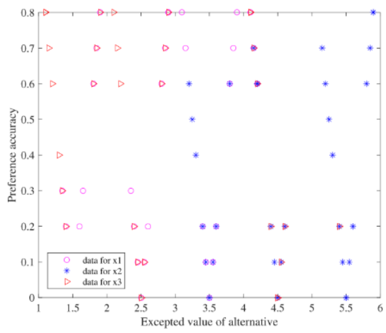

Figure 3: Preferences for alternatives of 30 DMs.

(2) Table 4 and Table 5 show initial trust relationships between 30 DMs. The discussion about Table 4 and Table 5 are shown around line 570 in blue. The relevant Figure.4 is provided to show the average initial trust in-degree of DMs, the DM with the darker color means he/she is more important in the network. From Figure.4, we can obtain the general status of individuals in the network, as well as the overall trust characteristics of network. The details are below with Figure.4:

"Fig.4 depicts the average initial trust in-degree of each DM by means of a heatmap. The darker the colour, the higher the trust centrality associated with the DM. It is clear that dm^{29} is the core of this social network, while dm^4, dm^9, $dm^{26}$and dm^{27}, are those who have achieved a lower trust. Moreover, we can observe the maximum trust in-

degree of DMs is about 0.65 and the minimum trust in-degree of DMs is about 0.4. It means there is no evident huge difference between DMs' centralities, and no DM is absolutely trusted or distrusted."

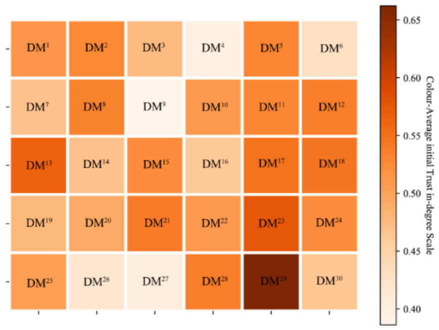

Figure 4: Heatmap related to average initial trust in-degree of DMs.

(3) Table 6 show the trust degrees between subgroups. The discussion about Table 6 is shown in Section 6.1.5 in blue. The details are below:

"The subgroups' average trust in-degrees for x_3 can be computed from Tab.6: 0.446, 0.470, 0.513, 0.471, 0.390, 0.485, 0.422 and 0.433, respectively. It can be easily found that there is no evident difference between them, which means these subgroups are nearly of equal importance in this trust network."

(4) Figure 5 is the heatmap related to average final updated trust in-degree of DMs. We analyze it from the scale and chromatism respectively. From Figure.5, we can obtain the general status of individuals in the network, as well as the overall trust characteristics of network after trust modification. Moreover, we analyze the difference between Figure.4 and Figure.5. The details are around line 585 in blue:

"Fig.5 depicts the average final updated trust in-degree of each DM by means of a heatmap (because of space limitations, we have omitted the computations to obtain the final updated trust degree). It is clear that dm^{29} is the core of this social network, while dm^4, $dm^9$and dm^{27} are those who have achieved a lower trust. Moreover, we can observe this social network does not show a high level of trust between DMs, as the maximum trust degree is about 0.55. Compared Fig.4 and Fig.5, we can find that the trust from Fig.5 is less than it from Fig.4 in general, this is because the conflicts derived from preferences cause the decrease of trusts. Moreover, the chromatism of different DMs in Fig.5 is less than it in Fig.4, this is because the trust will be concentrated after "assimilation effect"."

(5) Result analysis has been supplemented by Section 6.2. Considering that the aim of the proposed model is to obtain the collective preference, therefore, we supplement the descriptive analysis for the characteristics of the collective preference. Firstly, we make the sensitivity analysis of conflict degree, and we conclude that the setting of the conflict threshold can balance the collective preference and individual preferences, also it should fully consider the feasibility of model. Moreover, we describe the acceptance of preferences and the accuracy of the collective one: (1) We find that the numerical results are consistent with the initial intuitive analysis. Therefore, we can conclude that the simplified analysis of the original preference is needed, it can measure whether it is necessary to conduct CRP if the differences of individuals' opinions are too evident or

the overall accuracy of preference is too low. (2) We find that some collective preferences for different alternatives have improved accuracy level while some do not. Therefore, we conclude that the constraint of accuracy can be introduced into the consensus models to guarantee the accuracy of opinions.

5. Section Conclusion - Authors are suggested to include the result of their proposed model in comparison towards other models to highlight and justify the advantages of their proposed model.

Response:
Thank you for your valuable advice. Inspired by your advice, we have made supplement and improvements, now the comparison is composed of two parts: we firstly make a descriptive analysis of the proposed trust model and other trust models. Then, we compare our proposal with the existing consensus reaching approaches for LSGDM according to four aspects: (i) the update of the trust relationships; (ii) the way in which the collective preference is obtained by considering trust; (iii) the way in which the collective preference is obtained by considering conflict and (iv) the way in which the collective preference is obtained by goal programming model. Originally, we only make comparisons from the update of the trust relationships and the way in which the collective preference is obtained by considering trust, which is incomplete. The last three comparisons include the result of the proposed model in comparison towards other models, which are shown from line 715 to line 760 in Section 6.3.
In Section Conclusion, we have emphasized the advantages of proposed model marked in blue:
"This proposed trust model is universal, as to utilize the proposed dynamic trust model, there is no need of any certain environment. Moreover, comparisons find that the introduction of conflict between preferences can guarantees DM's utility. And the in-degree centrality-based distance between DMs considers individual's importance degree in the CRP, unlike treating DMs equally in the traditional models, which is necessary in SNA. Hence our model not only constitutes an extension of conventional methods, but also it shows evident advantages."

6. General comments - Authors are suggested to send the paper to proofread. There are many long sentences that should be separated into at least two sentences and some grammatical errors in the paper.

Response:
Thank you for your advice. We have made modifications through the whole paper. We have shorted the long sentences by reorganizing sentence structure, simplifying redundant sentences. Some examples are shown below:
(1)**Original:** On one hand, some works have focused on recognizing subgroups whose preferences is far from the collective preference in order to persuade the DMs located in them to modify their preferences.
Modified: On one hand, some works focused on recognizing subgroups whose

preferences is far from the collective preference, and then persuaded the involved DMs to modify their preferences.
(2)**Original:** On the other, some researchers have focused on supporting DMs within the same subgroup to improve the consensus and then build a consensus reaching process between the different subgroups.
Modified: On the other, some works aimed at supporting DMs within the same subgroup to improve the consensus firstly, and then built the CRP between different subgroups.
(3)**Original:** The existing approaches uses a parameter establishing a trade-off between the original preference provided by the DM and the referenced (expected) one to obtain the new preference that should be given by the DM to improve the consensus. This parameter is usually predefined However, its value should depend on the DMs' behavior characteristics, being different for each one of them.
Modified: The feedback parameter, which controls the accepting degree of recommendation advices, is discretionary selected and its value is equal for all DMs in the previous works. However, its value should depend on the DMs' behavior characteristics, being different for individual one.
(4)**Original:** We propose to calculate the conflict degree by means of the attitude toward the preference so that the conflict appears between DMs who hold opposing preferences.
Modified: We propose that there will be conflict between DMs who hold opposite preferences.
(5)**Original:** It uses a modification mechanism obtaining the trade-off between the original preference and the referenced (expected) one by means of two behavioral criteria: the conflict degree and the preferences provided by other DMs.
Modified: The feedback parameter can be obtained by means of two behavioral criteria: the respective conflict degree and the comparison with other DMs' preferences.
(6)**Original:** It is clear that DMs have to be ready for adjusting their preferences to improve consensus. Because DMs have been classified into subgroups according to their preferences, the consensus achieved in each subgroup should be high enough. As we have mentioned, both the consensus and the selection process depend on the expected value. Therefore, the consensus index of each subgroup is analyzed from the point of view of the expected value.
Modified: According to Section 4.1, DMs have been classified into subgroups. The preferences in each cluster should be concentrated enough. When a unified opinion cannot be obtained, DMs contributing less to consensus need to modify preference in order to improve consensus. The consensus index of each subgroup is analyzed from the point of view of the expected value.
(7)**Original:** To adjust the preferences in order to improve the consensus achieved within a subgroup, a method composed of an identification rule and a modification rule from the local point of view is carried out.
Modified: The local feedback mechanism composed of an identification rule and a modification rule is carried out.
(8)**Original:** Here, we assume that even in a trust network environment, individuals

tend to express preferences similar to those provided by DMs located in the same subgroup, due to these DMs have similar knowledge and cognition about the problem.

Modified: Here, we assume that even in a trust network environment, individuals tend to refer to the DMs located in the same subgroup, due to they have similar knowledge and cognition about the problem.

(9)**Original:** As described in Section 1, a parameter establishing a trade-off between the original preference given by the DM and the collective one is usually determined according to the particular decision making environment. However, in this study we determine the value of this parameter according to two criteria.

Modified: In this study, we determine the value of this parameter according to two criteria rather than select it discretionarily.

(10)**Original:** Given the fact that the greater the adjustment done to the preference provided by the DM dm^k, the lower her or his satisfaction.

Modified: Given the fact that the more adjustment, the lower individual's utility

(11)**Original:** That is, the existing approaches considers equally important the preferences provided by the DMs when aggregating them to obtain the collective preference.

Modified: That is, the existing approaches considered equally DMs' preferences when making an aggregation.

(12) **Original:** Considering the trust centrality priority and conflict between individuals (subgroups) and collective opinion about one certain alternative in the meanwhile can balance network feature and final preference well.

Modified: The aggregation model considers the individuals' (subgroups') in-degree trust centrality as well as the conflict between individuals (subgroups) and collective opinion.

……

Moreover, we have checked some other grammar errors, such as (the red part is where there is an error, and the part in ( ) is the modified version):

"On the other, some researchers have focused on supporting DMs within the same subgroup to improve the consensus and then build (built) a consensus reaching process between the different subgroups."

"The total conflict degree between dm^k and dm^l covering all alternatives denoted as CD^{kl} are (is)computed as follows."

"the final updated trust degree will be(delete) equal to…"

"Arrange the DMs by the number of pairs, in which they are located, in (a) descending order."

However, the proposed approach also takes into account the trust (takes trust into account)."

"Because in real-world situations, in which the structure of the decision group is decentralized, there is a (an) increasing need for software making easy distributed LSGDM and CRP."

We hope the current version meets your requirements.

Lastly, special thanks to you for your good comments.

Reviewer #3:

Firstly, we want to express our gratitude for this comment. Then we will give the point-by-point response to comments.

The paper has many concerns that should be considered before publication as follow:

- In the introduction section, the authors have supported some methodologies or ideas by their corresponding references. However, the authors have to briefly discuss each separately to make easy to distinguish the difference between the contribution of each cited paper not to mention them as a bulk (three or seven concatenated references together).

Response:
Thank you for your valuable advice.
The first paragraph in Introduction briefly states the importance of consensus reaching process in large-scale group decision making. It does not refer to the difference between the previous work and our proposed model. Hence, it is appropriate to mention reference as a bulk. And the second paragraph in Introduction briefly states the two operations of consensus modeling large-scale group decision making and gives the reason of our choice. It also does not refer to the difference between the previous work and our proposed model. Hence, it is appropriate to mention reference as a bulk.
The third paragraph in Introduction has been modified as your requirement, we have briefly discussed each separately to make easy to reflect the issues about trust in consensus reaching process.
In addition, we have rewritten limits and contribution parts to distinguish the difference between the contribution of each cited paper easily. The modified limits and contributions are shown below:

Several mechanisms and techniques have been constructed in trust-based CRP with large scale DMs. However, there are still some limitations to be solved:
•When it comes to detect the conflict behavior between DMs, it has been assumed that the discretionary selected conflict threshold can measure whether there is conflict among DMs [1, 2]. However, the determination of threshold is subjective, hence it is necessary to explore a method to analyze the existence of conflict objectively.
•The trust relationships in the above consensus models stay steady across the whole process. However, it is worth noting that the trust relationships are provided by DMs' previous knowledge, it will be likely effected by the current decision problem and surroundings in a network. Therefore, the static trusts between DMs is unreasonable.
•The previous consensus models mainly focused on the consensus index before aggregating, such as the similarity between individual preferences[31] and the similarity between individual and group preferences[23]. However, these models ignored whether the final collective preference can be accepted by all DMs.
•The feedback parameter, which controls the accepting degree of recommendation

advices, is discretionary selected and its value is equal for all DMs in the previous work[12]. However, individual has respective willingness of making modification. Therefore, its value should depend on the DMs' behavior characteristics, being different for individual one.

With the above hypothesis, the main purpose of this study is to construct the consensus models with a cyclic dynamic trust mechanism in LSGDM. In particular, the contributions are listed as follows:

- *We propose that there will be conflict between DMs who hold opposite preferences. Three levels of conflict degree are provided: conflict degree between DMs about one certain alternative; conflict degree between DMs about all the alternatives; conflict degree of one identified individual with all the other DMs about certain alternative.*
- *Based on the initial trusts, we propose that the trust degree will change depending on the conflict between DMs in first stage. In the second stage, it will make a modification actively according to the surroundings. Moreover, the preference adjustment will effect DMs' trust by the two stages, then the renewed trust will go into aggregation and next iteration. Therefore, the cyclic dynamic trust mechanism is established.*
- *It carries out the consensus models from local and group perspectives, respectively. First, a mechanism to improve the consensus within each subgroup based on similarity network is carried out to guarantee the compactedness of cluster (local consensus). Second, a min-max goal programming model is applied to guarantee the collective preference acceptable (global consensus).*
- *The feedback parameter can be obtained by means of two behavioral criteria: the respective conflict degree and the comparison with other DMs' preferences. This operation fully considers the behavior characteristics and utilities of DMs.*

- To facilitate the reading flow of the manuscript and formulas, a table of abbreviations and a table of notations are desired.

Response:

Thank you for your valuable advice. A table of abbreviations and a table of notations are shown in Section 2. as Table 1 and Table 2.

Table 1: The abbreviations of special nouns

abbreviations	special noun
CRP	consensus reaching process
LSGDM	large-scale group decision making
DM	decision maker
GDM	group decision making
PLTSs	probabilistic linguistic term sets
SNA	social network analysis

Table 2: The notations in proposed consensus model

notations	meanings	notations	meanings
dm^k	DM k	CI_i^a	consensus index of SG_i^a
x_i	alternative i	δ	local consensus threshold
TD^{kl}	trust degree from dm^k to dm^l	$APD_i^{a(k)}$	average preference distance between dm^k and all the others in SG_i^a
TD^k	in-degree centrality index of dm^k	$\overline{RDM}_i^{a(k)}$	set of referenced DMs for dm^k in SG_i^a
TC^k	importance degree of dm^k	w_i^{kh}	dm^k's referenced weight for dm^h on x_i
S	linguistic term set	ρ_i^k	local feedback parameter of dm^k on x_i
$L(p)$	PLTSs	λ_i^k	local feedback parameter of dm^k on x_i under comparison situation
$\sharp(*)$	the number of $*$	ϕ^{SG}	conflict threshold within subgroup
$EV(*)$	excepted value of $*$	SG_i	set of all subgroups on x_i
$I(*)$	subscript value of $*$	RSG_i^a	set of subgroups in SG_i except SG_i^a
r_i^k	dm^k's preference on x_i	$CD^{SG_i^aG}$	conflict degree between SG_i^a's preference and the collective preference
AV_i^k	attitude vector of dm^k on x_i	ϕ^G	global conflict threshold
CD_i^{kl}	conflict degree between dm^k and dm^l on x_i	η^G	global consensus threshold
CD^{kl}	conflict degree between dm^k and dm^l	$\overline{RSG}_i^a$	set of SG_i^a's referenced subgroups
CD_i^k	conflict degree of dm^k on x_i	$TD^{SG_i^al}$	trust degree from SG_i^a to dm^l
$\widehat{r}_i^k$	dm^k's referenced preference on x_i	$TD^{SG_i^aSG_i^b}$	trust degree from SG_i^a to SG_i^b
$\overline{r}_i^k$	dm^k's modified preference on x_i	$TD^{SG_i^a}$	in-degree centrality index of SG_i^a
$\widetilde{TD}^{kl}$	intermediate updated trust degree of TD^{kl}	$TC^{SG_i^a}$	importance degree of SG_i^a
RR^{kl}	retention ratio of TD^{kl}	$w^{SG_i^aSG_i^h}$	SG_i^a's referenced weight for SG_i^h
$TC^{l(\neg k)}$	dm^l's average in-degree centrality index except dm^k	$\widehat{r}^{SG_i^a}$	SG_i^a's referenced preference
$\overline{TD}^{kl}$	final updated trust degree of TD^{kl}	$\overline{r}^{SG_i^a}$	SG_i^a's modified preference
SG_i^a	the ath subgroup on x_i	$\rho^{SG_i^a}$	global feedback parameter of SG_i^a
d_i^{kl}	preference distance between dm^k and dm^l on x_i	$\lambda^{SG_i^a}$	global feedback parameter of SG_i^a under comparison situation
ζ	distance threshold for d_i^{kl}	$CD^{SG_i^a}$	conflict degree of SG_i^a
CD_i^{kc}	conflict degree between dm^k and collective preference within the same subgroup on x_i		

- The abbreviation of DMs should be mentioned in the abstract.

Response:

Thank you for your valuable advice. The abbreviation of DMs have been mentioned in the abstract.

- Why do the authors choose this kind of trust model? In other words, what are the main features of the proposed model as compared to the corresponding ones in the literature.

Response:

The main features of the proposed model include the dynamic characteristic of trust, the trust centrality in aggregation, the conflict in aggregation and no need of any essential conditions. The details are shown below:

Firstly, the literatures listed in References about trust in group decision making all assumed that trust relationships are static, but our proposed paper analyzes the possibility that the trust is dynamic. This is more in line with the actual decision-making environment.

Secondly, we consider the trust centrality and conflict constraint into preference aggregating process, which is more reasonable and comprehensive to obtain the solution than traditional model. Related comparisons can be seen in section 6.3 from line 715 to line 730.

Lastly, we have made enough survey about the others types of trust models. Details can be seen in the next response. The proposed model does not need relevant essential conditions unlike the most trust models, which means our model is universal.

In the original version, we have not expounded these features clearly. In the revised version, we have concluded it in Section Conclusion around line 790 in blue shown as:

"The proposed trust model is universal, as to utilize the proposed dynamic trust model,

there is no need of any certain environment. And the in-degree centrality-based distance between DMs considers individual's importance degree in the CRP, unlike treating DMs equally in the traditional models, which is necessary in SNA. Hence our model not only constitutes an extension of conventional methods, but also it shows evident advantages."

- The authors have to make enough survey about the others types of trust models and show the outperforming benefits of the proposed one among the others such as fuzzy trust model, entropy based trust model, game theory trust model, Bayesian trust model, and clustering trust. [THE MANAGING EDITOR REMOVED ALL THE REFERENCES SUGGESTED BY THE REVIEWER, TO PRESERVE BLINDNESS]

Response:

Thank you for your valuable advice. Indeed, it is necessary to make comparison with other trust models to show the advantages of this proposed model. Therefore, in Section 6.3: Comparative analysis, we have supplemented the related survey about fuzzy trust model, entropy based trust model, game theory trust model, Bayesian trust model, and clustering trust model. A comparison table composed of the function of the trust model, whether it contains dynamic trust (the feature of this paper), and the essential condition is shown as follows (Table 9), we choose the following researches to make a survey. I hope the selected literatures could make you satisfied.

Table 9: Comparative analysis about various trust models

type	function	dynamic trust	essential condition
Bayesian-based[28, 56]	trust evaluation	-	the distribution of nodes
entropy-based[29, 57]	trust evaluation	-	incomplete trust relations
game theory-based[58]	trust updating	√	trust behaviors and utility
clustering trust[55, 60]	prompt consensus	-	the relations between nodes
fuzzy trust[59]	preference estimation	-	incomplete information
fuzzy trust[23]	prompt consensus	-	-
this paper	prompt consensus	√	-

Our research aims to solve the consensus reaching process in large-scale group decision making. We need the trust relationships between decision makers to generate the suggestions feedback.

In this paper, we propose the dynamic characteristics of trust caused by individuals' opposite preferences and the surrounding trust attitudes. Hence, we can analyze the dynamic characteristics of trust just based on the preference information. Next, we will summarize the characteristics of other trust models:

(1) Bayesian trust model is utilized to derive an identified node's trust value when the distribution or probability of its related nodes are known.

(2) Entropy theory can help to evaluate the missing trust value like Bayesian theory. It works based on the incomplete trust network.

(3) Game theory trust model can generate updated trust when nodes' strategy is known.

When it comes to trust applied in group decision making, many researches about clustering trust models and fuzzy trust models have been conducted.

(4) **Fuzzy trust model refers to individual's fuzzy trust relationship or a fuzzy**

decision making environment. Our paper focus on the second one, we analyze the consensus reaching process with trust network in probabilistic linguistic group decision making.

(5) **Clustering trust model tries to clustering decision makers by trust linkages. It works well with incomplete network.** Our paper uses clustering technology according to preference similarity and accuracy level. We have not considered the trust relationships into clustering, as the trust network in our paper is complete and the individual trust centrality is similar.

It is obviously found that the most trust models require relevant essential conditions while the proposed model does not need it. Our trust model is universal. It is worth noting that there are fuzzy trust models which do not need essential conditions, however, it does not reflect dynamic trust. In conclusion, our model can solve the general situation as well as reflect the dynamic characteristics.

The related detailed survey is conducted in section 6.3 from line 685 to line 710.

The selected literatures:

Bayesian trust model:

[28] W. Meng, K. K. R. Choo, S. Furnell, A. V. Vasilakos, C. W. Probst, Towards Bayesian-based Trust Management for Insider Attacks in Healthcare Software-Defined Networks, IEEE Transactions on Network & Service Management.

[56] B. Jin, Y. Wang, Z. Liu, X. Jingfeng, A Trust Model Based on Cloud Model and Bayesian Networks, Procedia Environmental Sciences 11 (2011) 452–459.

Entropy based trust model:

[29] S. Ahmed, K. Tepe, Entropy-Based Recommendation Trust Model for Machine to Machine Communications, Springer International Publishing, 2017.

[57] J. Zhao, J. Huang, N. Xiong, An Effective Exponential-Based Trust and Reputation Evaluation System in Wireless Sensor Networks, IEEE Access 7 (2019) 33859–33869.

Game theory trust model:

[58] H. Fang, L. Xu, X. Huang, Self-adaptive trust management based on game theory in fuzzy large-scale networks, Soft Computing 21 (4) (2017) 907–921.

Clustering trust model:

[55] Y. C. Dong, Z. G. Ding, L. Martinez, F. Herrera, Managing consensus based on leadership in opinion dynamics, Information Sciences 397–398 (2017) 187–205.

[60] X. Xu, Q. Zhang, X. Chen, Consensus-based non-cooperative behaviors management in large-group emergency decision-making considering experts' trust relations and preference risks, Knowledge-Based Systems.

Fuzzy trust model:

[59] N. Capuano, F. Chiclana, H. Fujita, E. Herrera-Viedma, V. Loia, Fuzzy Group Decision Making With Incomplete Information Guided by Social Influence, IEEE Transactions on Fuzzy Systems (99).

[23] J. Wu, L. Dai, F. Chiclana, H. Fujita, E. Herrera-Viedma, A minimum adjustment cost feedback mechanism based consensus model for group decision making under social network with distributed linguistic trust, Computers & Industrial Engineering 41 (2018) 232–242

- What are the kind of networks that can be treated by the proposed model?

Response:
We focus on the complete directional social network in this paper. That is, the network should satisfy: (1) there exist directional trust relationships between each DMs, and (2) the relationship between dm^k and dm^l is different.
In the revised version, we stated it in Introduction and Section 2.1 marked in purple red around line 70 and line 125. In Introduction, we propose one of the hypothesis: there exist directional trust relationships between each DM. And in Preliminaries, after introduction the graph theory, we have emphasized the type of the utilized network.

- What are the kind of attacks that can be confronted by the proposed model?

Response:
Our research aims to achieve a solution of a decision making problem.
The proposed model can solve the problem when (1) the scale of decision makers is large: we propose the local and global feedback mechanism based on clustering, respectively; (2) when decision makers cannot reach consensus: we propose the consensus reaching process based on similarity network and min-max goal programming model; (3) when the preferences are expressed by fuzzy set: we define the preference attitude based on probabilistic linguistic term sets to reflect preference conflict between DMs, and the constructed model can aggregate probabilistic linguistic preferences logically; (4) when there is a need to analyzing the dynamic trust relationships between individuals: we propose the trust modification mechanism based on conflict effect and assimilation effect.

- Normally, in the introduction, no need to mention notions, just discuss the main ideas and the correlated ones. The notations can be mentioned in the system model section.

Response:
Thank you for your valuable advice. Inspired by your advice, we rewritten the Introduction. In the modified introduction, we list some limitations and contributions briefly from line 45 and line 75 in blue.
Moreover, we have moved the “notions” in the original version to the model section. Such as the blue part around line 205. This example can show the reason that we propose the concept of dynamic trust effected by preference conflict.
And such as the blue part around line 240. This explains the notion of assimilation effect.

- The definition 1 is written as italic but only the notations should be italic not the text too. Also, the mentioned reference [41] should not be included at the beginning of the definition. Similarly, the rest of definitions have to follow the same concept.

Response:
Thank you for your valuable advice. All the definitions have been modified according

to your suggestion. They are not in italic now and the mentioned reference have been moved after “Definition” instead of at the beginning of the definition.

- What do the authors mean by #?

Response:
#(*) means the number of *. Moreover, it has been emphasized around line 350 in red. Also, it is included in the table of notations in Section 2.

- More descriptive results are desired to show the enhanced performance using the proposed model.

Response:
Thank you for your advice.
Result analysis has been supplemented by Section 6.2. Considering that the aim of the proposed model is to obtain the collective preference, therefore, we supplement the descriptive analysis for the characteristics of the collective preference. Firstly, we make the sensitivity analysis of conflict degree, and we conclude that the setting of the conflict threshold can balance the collective preference and individual preferences.
Moreover, we describe the acceptance of preferences and the accuracy of the collective one: (1) We find that the numerical results are consistent with the initial intuitive analysis. Therefore, we can conclude that the simplified analysis of the original preference is needed, it can measure whether it is necessary to conduct CRP if the differences of individuals’ opinions are too evident or the overall accuracy of preference is too low. (2) We find that some collective preferences for different alternatives have improved accuracy level while some do not. Therefore, we conclude that the constraint of accuracy can be introduced into the consensus models to guarantee the accuracy of opinions.

We hope the current version meets your requirements.

Lastly, special thanks to you for your good comments.

Reviewer #4:

The technical content is well organized as well as the reference topic, the article is not compact. I do not think it can be compact when testing results were included. When I viewed the math, I realized I had seen some of the equations and almost identical content in other scientific articles. Double check to see whether there is plagiarism in your article. The graphics coincided with what was written and shown in the math. There were few errors, but I found them to be minor.

Firstly, we want to express our gratitude for this comment. Then we will give the point-by-point response to comments.

Response:

About plagiarism:

We have read a large number of literatures related with our topic, we are sure that this proposed model has no plagiarism. Our research aims to achieve a solution of a decision making problem. When the unified agreement cannot be reached among the large-scale decision makers, we explore a consensus reaching process. In this process, we construct local and global feedback mechanisms to obtain the final solution based on analyzing trust relationships.

The following literatures are highly correlated with our papers, and they all focused on consensus reaching process, some of them analyzed consensus reaching process with trust relationships, some of them focused on large-scale group decision making, some of them explored conflict relationships in consensus reaching process. The above research points are contained in this paper. We will conduct relevant surveys about them to show our innovations.

About consensus reaching process with trust relationships (We main analyze two papers)**:**

[1] J. Wu, L. Dai, F. Chiclana, H. Fujita, E. Herrera-Viedma, A minimum adjustment cost feedback mechanism based consensus model for group decision making under social network with distributed linguistic trust, Computers & Industrial Engineering 41 (2018) 232–242

[2] X. Liu, Y. Xu, R. Montes, F. Herrera, Social network group decision making: Managing self-confidence-based consensus model with the dynamic importance degree of experts and trust-based feedback mechanism, Information Sciences 505 (2019) 215–232.

Wu et al. [1] designed a minimum adjustment cost feedback mechanism under social network with distributed linguistic trust. Liu et al. [2] explored a dynamic importance degree of experts which combines the external trust and internal self-confidence is proposed to determine their weights. The rule of the feedback mechanism is that experts dynamically adjust their self-confidence levels while revising the preferences.

Our innovations: The trust in-degree centrality indexes defined in [1] is cited in our proposed model to assign an importance degree to the associated decision maker.

However, the trust is static in [1,2] and many other related literatures. The trust in this paper is dynamic caused by conflict and assimilation effect, also it is cyclic caused by the feedback mechanism while revising the preferences. This is one of our innovations.

About consensus reaching process with conflict detection (We main analyze two papers)**:**
[3] R. X. Ding, X. Wang, K. Shang, F. Herrera, Social network analysis-based conflict relationship investigation and conflict degree-based consensus reaching process for large scale decision making using sparse representation, Information Fusion 50 (2019) 251–272.
[4] B. Liu, Q. Zhou, R. X. Ding, I. Palomares, F. Herrera, Large-scale group decision making model based on social network analysis: Trust relationship-based conflict detection and elimination, European Journal of Operational Research 275 (2) (2019) 737–754.

Ding et al. [3] insisted that the conflict relationships can be divided into two parts: the opinion conflict and the behavior conflict. And they adopted a threshold to measure whether the conflict is acceptable. Liu et al. [4] defined the concept of conflict degree and quantify the collective conflict degree by combining the assessment information and trust relationships among decision makers in the large group, also they adopted a threshold to measure whether the conflict is acceptable.
Our innovations: However, we proposed three levels of conflict degrees based on the defined preference attitude, which avoids the discretionary selected conflict threshold. The method proposed in this paper is more objective compared with the subjective operation about conflict in [3,4]. This is one of our innovations

About large-scale group decision making (We main focus on six papers)**:**
[7] T X. Liu, Y. Xu, R. Montes, R. X. Ding, F. Herrera, Alternative ranking-based clustering and reliability index-based consensus reaching process for hesitant fuzzy large scale group decision making, IEEE Transactions on Fuzzy Systems 27 (1) (2018) 159–171.
[8] Z. Wu, J. Xu, A consensus model for large-scale group decision making with hesitant fuzzy information and changeable clusters, Information Fusion 41 (2018) 217–231.
[9] Z. P. Tian, R. X. Nie, J. Q. Wang, Social network analysis-based consensus-supporting framework for large-scale group decision-making with incomplete interval type-2 fuzzy information, Information Science 502 (2019) 446–471.
[10] M. Tang, X. Zhou, H. Liao, J. Xu, F. Fujita, F. Herrera, Ordinal consensus measure with objective threshold for heterogeneous large-scale group decision making, Knowledge-Based Systems 180 (2019) 62–74.
[11] J. Xiao, X. Wang, H. Zhang, Managing personalized individual sematnics and consensus in linguistic distribution large-scale group decision making, Information Fusion 53 (2020) 20–34.
[12] Y. Xu, X. Wen, W. Zhang, A two-stage consensus method for large-scale multi-

attribute group decision making with an application to earthquake shelter selection, Computers & Industrial Engineering 116 (2018) 113–129.

To improve the efficiency of managing large-scale group decision making, clustering techniques have been commonly applied so that each subgroup (cluster) of DMs is treated as a basic unit. Therefore, the decision problem can be simplified. There are two rules of conducting feedback mechanism: one is focused on recognizing subgroups whose preferences is far from the collective preference, and then persuaded the involved DMs to modify their preferences [7-9]; and another aimed to assist decision makers in achieving a consensus within each obtained cluster in the first stage, and the second stage is devoted to facilitating the consensus building among the different clusters [10-12].

Our innovations: In this study, we adopt the second approach because the collective opinion aggregated from individuals in the same classification should represent this cluster well and properly, to do so, a local CRP must be carried out first within each subgroup. However, the consensus models in the literatures mainly focused on the consensus index before aggregating, such as the similarity between individual preferences and the similarity between individual and group preferences. However, these models ignored whether the final collective preference can be accepted by all DMs. Therefore, although our consensus measure is based on the similarity like the previous literatures, we propose a min-max programming model to guarantee the collective preference acceptable. This is one of our innovations.

About the determination of feedback parameter:

[13] X.J, Gou, Z.S, Xu, Francisco, H. Consensus reaching process for large-scale group decision making with double hierarchy hesitant fuzzy linguistic preference relations. Knowledge Based Systems. (2018).

We cite the common feedback rule like previous literatures, however, we provide different methods to obtain its value. When it comes to its determination, Wu et al. [1] constructed the minimum programming model to obtain its minimum value to reach consensus. Liu et al. [2] and Gou et al. [13] gave the discretionary selected feedback parameter.

Our innovations: It is worth noting that there exits active decision maker who is willing to make modification, which is ignored in [1]. Different from the objective method in [2,3], we propose the novel angle to obtain the feedback parameter, considering decision makers' utilities while improving consensus. This is one of our innovations.

In conclusion, although we cite some basic rules in consensus reaching process like previous literatures, but we have the specialized innovations in the process of conducting them, including:

(1) We propose that there will be conflict between DMs who hold opposite preferences and three levels of conflict degree are given. Compared with the subjective previous works, the method proposed is objective.
(2) Different from the traditional static trust-based consensus models, we propose that the trust relationship is dynamic, which is more in line with the actual decision-

making environment. Moreover, apart from the constructed two-stage trust modification mechanism, we propose that the renewed trust will go into aggregation and next iteration. Therefore, the cyclic dynamic trust mechanism is established.

(3) We carry out the consensus models from local and group perspectives, respectively. The conflict between individual and collective preferences and individual importance degree are considered into modeling. This operation can guarantee individual utility.

(4) Instead of the discretionary selected feedback parameter, we provide two behavioral criteria to determine its value. This operation fully considers individual behavior characteristics.

The Introduction and Section 6.3 Comparative analysis show the advantages of the proposed model compared with the traditional ones in detail.

For some equations, we mainly adopt four methods to avoid misunderstanding:

(1) Some basic formulas in this paper are built similar as the previous works, we have **added cites** for some mathematical formulas, such as [23] has been added for Definition 2, [12,25] for Eq.(15).

(2) We **arrange some universal formulas into main text** instead of emphasizing them by marked number (such as Eq. 13, 16, 17, 30, 31in the original version), which are shown around line 390 and line 520 marked in purple red in the revised version.

(3) The **supplementary explanations** about the formulas in Definition 7,8,9 are provided in red; the supplementary explanations about Eq.(13) are provided in red, which can show our original considerations.

(4) Also, we add some **inspiration sources** marked in red, such as "Inspired by the previous work[23] about individual in-degree centrality index computation…" Eq.(11) has been introduced; "…, according to the consensus measurements[23, 31],..." Eq.(14) has been introduced, which shows that although the relevant formulas may be similar in forms, but they reflect the different basic principles.

In addition, we emphasize the innovative designs of the important models and formulas, highlighting the originality of this paper. Such as (1) the blue part around line 340 about local consensus measure: Considering that the characteristic of cluster is that the elements in it should be compact: the local consensus measure is provided by Eq.(13) based on similarity; (2) the blue part around line 400 about feedback parameter setting: although the feedback rule is the same as the previous works, we provide two novel criterion to determine the feedback parameter; (3) the blue part around line 435 and 440 about the acquisition of local collective preference: apart from citing some constraints about preference logic into modeling, we introduce individual importance degree into objective function and conflict requirement into constraints. The proposed model is verified to be superior to the previous works; (4) the blue part around line 490 about global consensus measurement, also it can help to obtain global collective preference:

the proposed min-max programming model can measure whether the collective preference is acceptable instead of just measuring consensus index before aggregating preference. The proposed model is verified to be superior to the previous works in comparison section.

We hope the reply can dispel your doubts.

-There is a grammar error on line 35.

Response:
Thank you for your advice.
Original sentence on line 35: Even though different properties related to trust relationships (for instance, trust propagation, trust level, trust policy, among others) have been considered.
Modified: Different properties related to trust relationships have been explored, such as trust propagation operator [24, 25]; trust numerical expression [23, 26]; trust policy [27]; trust evaluation [28, 29] and so on.

-Equation 21, 22, 28,29 very tightly written it is very hard to read

Response:
Thank you for your advice. The original Model (21) (22) (28) (29) have been rewritten by widening the space between the formulas as the modified Model (18) (19) (25) (26), respectively. Some explanations have been added to make the models read easily, like the blue part around line 435. Moreover, Model (20) has been rewritten by widening the space between the formulas.

-Line 46 there is an error

Response:
Thank you for your advice.
Original sentence on line 46: For instance, if we assume that a DM dm^k does not feel sure about some alternative in a certain decision problem.
The "alternative" should be modified as "alternatives". However, this sentence is found to be inappropriate in the process of revision and has been removed from Introduction, we have replaced it by the new version:
Modified: "That is, even though dm^l once has a total trust to dm^k, it is likely that dm^l will change the trust toward dm^k in the current specific decision making situation." around line 205 in blue.

-Recheck equation 14 and 15

Response:
Thank you for your advice. We have checked Eq.14 and Eq.15, which are Eq.13 and

Eq.14 in the modified version. We modified the subscript of the summation symbol as follows, in addition, we provide the extra illustrations below the equation. Now the equations are simplified and can be understood easily.

$$CI_i^a = \frac{\#(\sum_{k \neq l} d_i^{kl} \leq \zeta)}{\#(dm^k) \cdot (\#(dm^k) - 1)/2} \tag{13}$$

where $dm^k, dm^l \in SG_i^a$ and $\#(*)$ extracts the number of $*$. In detail, $\#(\sum_{k \neq l} d_i^{kl} \leq \zeta)$ means the number of edges with $d_i^{kl} \leq \zeta$, $\#(dm^k)$ means the number of DMs in subgroup SG_i^a, and there is $\#(dm^k) \cdot (\#(dm^k) - 1)/2$ edges in the similarity network.

$$APD_i^{a(k)} = \frac{\sum_{l \neq k} d_i^{kl}}{\#(dm^l)} \tag{14}$$

where $dm^l \in SG_i^a$ and $\#(dm^l)$ means the number of DMs in SG_i^a except dm^k.

-Line 378 some variables are not defined.

Response:
Thank you for your advice. We have supplemented the definitions about the variables around line 395 now in blue as follows. The definitions of EV (*) has been shown around line 145 in blue.

The expected value of the modified preference $\overline{r}_i^k$ can be obtained by linearly aggregating the original preference r_i^k and the referenced preference $\hat{r}_i^k$[12, 25]:

$$EV(\overline{r}_i^k) = (1 - \rho_i^k) \cdot EV(r_i^k) + \rho_i^k \cdot EV(\hat{r}_i^k) \tag{15}$$

In this study, we determine the value of dm^k's local feedback parameter ρ_i^k on x_i according to two criteria rather than selecting it discretionarily.

Moreover, we have checked some other **grammar errors**, such as (the red part is where there is an error, and the part in ( ) is the modified version):
"On the other, some researchers have focused on supporting DMs within the same subgroup to improve the consensus and then build (built) a consensus reaching process between the different subgroups."
"The total conflict degree between dm^k and dm^l covering all alternatives denoted as CD^{kl} are (is)computed as follows."
"the final updated trust degree will be(delete) equal to…"
"Arrange the DMs by the number of pairs, in which they are located, in (a) descending order."
However, the proposed approach also takes into account the trust (takes trust into

account).”

“Because in real-world situations, in which the structure of the decision group is decentralized, there is a (an) increasing need for software making easy distributed LSGDM and CRP.”

“Then, after obtain(ing) the trust degrees between subgroups……”

We hope the current version meets your requirements.

Lastly, special thanks to you for your good comments.

Finally, we appreciate very much for your time in editing our manuscript and the reviewers for their valuable suggestions and comments.

Kind regards,

Sincerely

Xiao Tan, Jianjun Zhu, Francisco Javier Cabreriz, Enrique Herrera-Viedma

A cyclic dynamic trust-based consensus model for large-scale group decision making with probabilistic linguistic information

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Abstract

This paper investigates a consensus reaching process (CRP) considering dynamic trust in large-scale group decision making (LSGDM). In the traditional trust-based consensus model, it is assumed that the trust relationship generated by decision makers (DMs)' previous knowledge remain unchanged during the whole decision process. However, this relationship will be dynamic rather than static especially in a social network with a new decision problem. This study explores the dynamic nature of trust through two stages. In the first stage, the trust degree will be functionally reformed by the conflict caused by DM's opposite preferences. In the second stage, it will be effected by surroundings according to the "assimilation effect" in network. To handle the CRP with large-scale decision settings, a clustering technique is used to classify DMs with similar preference and preference accuracy. Based on the classifications, an optimization model is constructed to obtain the trust degrees between subgroups. The consensus measurements are investigated from similarity network within subgroups and min-max programming model between subgroups, respectively. Moreover, preference modification will effect trust in the aggregation and next iteration, the cyclic dynamic trust

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mechanism is established. The feasibility of the proposed model is verified by a numerical example. Comparisons declare the constructed consensus model's universality without any essential conditions, as well as superiority with fully consideration of DM's utility and centrality in network.

Keywords: Consensus reaching process, Preference attitude, Conflict detection, Assimilation effect, Dynamic trust

1. Introduction

Because of the complexity of decision problem, and a large number of decision makers (DMs) with diversity in attitudes, behaviors, knowledge, backgrounds, and conflicting interests involved, it is necessary to guarantee the final solution acceptable and collective[1–4]. This issue has been called the large-scale group decision making (LSGDM)[5]. When it comes to develop a model for a LSGDM problem, we must focus on the consensus reaching process (CRP)[6–10], which attempts to bring the DMs' preferences closer and closer through some rounds of discussions, negotiations and communications. It's key issue is to design an effective feedback mechanism to produce recommendation advices. CRP uses "soft consensus" to show the possibility of working with different partial agreement levels rather than the impractical unanimous agreement[11].

To improve the efficiency of managing CRP in LSGDM, clustering techniques have been commonly applied so that each subgroup (cluster) of DMs is treated as a basic unit[12, 13]. On one hand, some works focused on recognizing subgroups whose preferences is far from the collective preference, and then persuaded the involved DMs to modify their preferences[14–16]. On the other, some works aimed at supporting DMs within the same subgroup to improve the consensus firstly, and then built the CRP between different subgroups[17–19]. In this study, we adopt the second approach because the collective opinion aggregated from individuals in the same classification should represent this cluster well and properly, to do so, a local CRP must be carried out first within each subgroup.

In addition, in heterogeneous and dynamic decision making environments like those based on Web 2.0[20], DMs are not completely independent as there are various kinds of connections between them: friendship relationship, similarity relationship, trust relationship, antagonistic relationship, and so on. Among them, trust relationship is the basis for interactions among DMs[2, 21].

Social network analysis (SNA), a theoretical tool studying the linkages between individuals, groups, organizations and societies[22], has been widely used in trust based-group decision making (GDM)[1, 2, 23]. Different properties related to trust relationships have been explored, such as trust propagation operator[24, 25]; trust numerical expression[23, 26]; trust policy[27]; trust evaluation[28, 29] and so on. Trust relationships can help to improve consensus in GDM: Wu et al.[27] introduced the recommendation mechanism induced by the attitudinal trust; Liu et al.[2] obtained the modified opinion through analyzing conflict composed of trust and preference similarity; Wu et al.[30] provided the personalized advice by trust network and collaborative filtering; and so on. In conclusion, trust relationship can be utilized to deal with and reflect many issues in GDM.

Several mechanisms and techniques have been constructed in trust-based CRP with large scale DMs. However, there are still some limitations to be solved:

- When it comes to detect the conflict behavior between DMs, it has been assumed that the discretionary selected conflict threshold can measure whether there is conflict among DMs[1, 2]. However, the determination of threshold is subjective, hence it is necessary to explore a method to analyze the existence of conflict objectively.
- The trust relationships in the above consensus models stay steady across the whole process. However, it is worth noting that the trust relationships are provided by DMs' previous knowledge, it will be likely effected by the current decision problem and surroundings in a network. Therefore, the static trusts between DMs is unreasonable.
- The previous consensus models mainly focused on the consensus index before aggregating, such as the similarity between individual preferences[31] and the similarity between individual and group preferences[23]. However, theses models ignored whether the final collective preference can be accepted by all DMs.
- The feedback parameter, which controls the accepting degree of recommendation advices, is discretionary selected and its value is equal for all DMs in the previous work[12]. However, individual has respective willingness of making modification. Therefore, its value should depend on the DMs' behavior characteristics, being different for individual one.

In order to overcome these limitations, we propose a consensus model based on SNA for LSGDM in which probabilistic linguistic information is used to represent the DMs' preferences. Probabilistic linguistic information can properly deal with hesitancy and uncertainty[32, 33]. The proposed consensus model is based on the following hypothesis: (i) there exist directional trust relationships between each DM; (ii) the preference adjustment, whose objective is to increase the consensus, brings about the dynamic trust degrees; and (iii) there is no DM rejecting adjustment and DMs are willing to promote consensus.

With the above hypothesis, the main purpose of this study is to construct the consensus models with a cyclic dynamic trust mechanism in LSGDM. In particular, the contributions are listed as follows:

- We propose that there will be conflict between DMs who hold opposite preferences. Three kinds of conflict degree are provided: conflict degree between DMs about one certain alternative; conflict degree between DMs about all the alternatives; conflict degree of one identified individual with all the other DMs about certain alternative.

- Based on the initial trusts, we propose that the trust degree will change depending on the conflict between DMs in first stage. In the second stage, it will make a modification actively according to the surroundings. Moreover, the preference adjustment will effect DMs' trust, then the renewed trust will go into aggregation and next iteration. Therefore, the cyclic dynamic trust mechanism is established.

- It carries out the consensus models from local and group perspectives, respectively. First, a mechanism to improve the consensus within each subgroup based on similarity network is carried out to guarantee the compactedness of cluster (local consensus). Second, a min-max goal programming model is applied to guarantee the collective preference acceptable (global consensus).

- The feedback parameter can be obtained by means of two behavioral criteria: the respective conflict degree and the comparison with other DMs' preferences. This operation fully considers the behavior characteristics of DMs.

The study is structured into seven main sections. Section 2 briefly recalls some concepts related to social networks and probabilistic linguistic information. Section 3 introduces both the concept of conflict degree and the trust modification mechanism based on both conflict degree and assimilation effect. Section 4 and Section 5 are devoted to the design and development of the proposed consensus models locally and globally, respectively. Section 6 reports an example of application to show the performance of this consensus model. In addition, the result analysis and comparisons with other models are provided. Finally, some conclusions and future studies are given in Section 7.

2. Preliminaries

Let us suppose $M = \{1, 2, \dots, m\}$ and $N = \{1, 2, \dots, n\}$. In a GDM problem, there are m DMs denoted as dm^k $(k \in M)$ expressing their preferences about n alternatives denoted as x_i $(i \in N)$. In this section, we recall the concepts of Social network analysis and Probabilistic linguistic information briefly.

2.1. Social network analysis

In a social network, a trust relationship is made when a pair of DMs trust each other, and the collection of trust relationships of all DMs in a social network leads to a graph. Actually, the trust SNA consists in the application of the graph theory.

Definition 1[34]. A simple weighted graph $G(V, E, W)$ consists of a non-empty finite set $V = \{v^k \mid k \in M\}$ of m vertices, a finite set $E = \{e^{kl} \mid k \neq l \wedge k, l \in M\}$ of edges, and a finite set $W = \{w^{kl} \mid k \neq l \wedge k, l \in M\}$ of weights. The edge e^{kl} indicates the connection between the vertices v^k and v^l with weight w^{kl}.

In our setting, V represents the set of DMs, E represents the trust relationships between DMs, and W represents the trust degrees between DMs. In addition, we focus on a directional social network[1], that is, the link from dm^k to dm^l is different to the link from dm^l to dm^k. Hence, the DM's importance degree can be computed: the higher the average trust degree aiming at a DM, he/she is more likely being the core of the network[23].

Definition 2[23]. The importance degree associated with dm^k represented by TC^k in a social network can be computed as:

$$TC^k = \frac{TD^k}{\sum_{k=1}^{m} TD^k} \tag{1}$$

where $TD^k = \frac{1}{m-1}\sum_{l=1,l\neq k}^{m} TD^{lk}$, which is called the in-degree centrality index of dm^k.

2.2. Probabilistic linguistic information

Pang et al.[32] developed a new general concept to extend the conventional linguistic term sets: the probabilistic linguistic term sets (PLTSs). Compared to other approaches[35–41] dealing with linguistic information, PLTSs allows DMs to express several linguistic values along with probabilistic information, and it also can deal with partially incomplete evaluations.

Definition 3[32]. Let $S = \{S_0, S_1, \ldots, S_g\}$ be a linguistic term set, the PLTSs can be defined as:

$$L(p) = \left\{ L^{(\kappa)}(p^{(\kappa)}) \mid L^{(\kappa)} \in S,\ p^{(\kappa)} \geq 0,\ \kappa = 1, \ldots, \#L(p), \sum_{\kappa=1}^{\#L(p)} p^{(\kappa)} \leq 1 \right\} \quad (2)$$

where $L^{(\kappa)}(p^{(\kappa)})$ is the linguistic term $L^{(\kappa)}$ associated with the probability $p^{(\kappa)}$ and $\#L(p)$ is the number of all different linguistic terms in $L(p)$.

In this study, we set $\sum_{\kappa=1}^{\#L(p)} p^{(\kappa)} = 1$, which means we have the complete information of probabilistic distribution of all possible linguistic terms. In addition, we assume $\#L(p) = 1$ or 2 in this study, because the PLTSs with many linguistic terms is inaccurate to some extent[42, 43].

Definition 4[44]. The expected value associated with the PLTSs $L(p)$ can be defined as follows:

$$EV(L(p)) = \sum_{k=1}^{\#L(p)} p^{(k)} \cdot I(L^{(k)}) \quad (3)$$

where $I(*)$ is a function extracting the subscript of the linguistic term $L^{(k)}$. $EV(*)$ represents the excepted value of $*$. In this study, the expected value will be used both to carry out the clustering operation and to assist ranking the alternatives[23, 45].

The abbreviations of special nouns and the list of notations in proposed consensus model are shown in Tab.1 and Tab.2, respectively.

3. A trust modification mechanism based on conflict effect and assimilation effect

This section is devoted to develop a dynamic trust mechanism that is based on conflict effect and assimilation effect. First, the definition of the

Table 1: The abbreviations of special nouns

abbreviations	special noun
CRP	consensus reaching process
LSGDM	large-scale group decision making
DM	decision maker
GDM	group decision making
PLTSs	probabilistic linguistic term sets
SNA	social network analysis

Table 2: The notations in proposed consensus model

notations	meanings	notations	meanings
dm^k	DM k	CI_i^a	consensus index of SG_i^a
x_i	alternative i	δ	local consensus threshold
TD^{kl}	trust degree from dm^k to dm^l	$APD_i^{a(k)}$	average preference distance between dm^k and all the others in SG_i^a
TD^k	in-degree centrality index of dm^k	$\overline{RDM}_i^{a(k)}$	set of referenced DMs for dm^k in SG_i^a
TC^k	importance degree of dm^k	w_i^{kh}	dm^k's referenced weight for dm^h on x_i
S	linguistic term set	ρ_i^k	local feedback parameter of dm^k on x_i
$L(p)$	PLTSs	λ_i^k	local feedback parameter of dm^k on x_i under comparison situation
$\sharp(*)$	the number of $*$	ϕ^{SG}	conflict threshold within subgroup
$EV(*)$	excepted value of $*$	SG_i	set of all subgroups on x_i
$I(*)$	subscript value of $*$	RSG_i^a	set of subgroups in SG_i except SG_i^a
r_i^k	dm^k's preference on x_i	$CD^{SG_i^aG}$	conflict degree between SG_i^a's preference and the collective preference
AV_i^k	attitude vector of dm^k on x_i	ϕ^G	global conflict threshold
CD_i^{kl}	conflict degree between dm^k and dm^l on x_i	η^G	global consensus threshold
CD^{kl}	conflict degree between dm^k and dm^l	$\overline{RSG}_i^a$	set of SG_i^a's referenced subgroups
CD_i^k	conflict degree of dm^k on x_i	$TD^{SG_i^al}$	trust degree from SG_i^a to dm^l
$\widehat{r}_i^k$	dm^k's referenced preference on x_i	$TD^{SG_i^aSG_i^b}$	trust degree from SG_i^a to SG_i^b
$\overline{r}_i^k$	dm^k's modified preference on x_i	$TD^{SG_i^a}$	in-degree centrality index of SG_i^a
$\widetilde{TD}^{kl}$	intermediate updated trust degree of TD^{kl}	$TC^{SG_i^a}$	importance degree of SG_i^a
RR^{kl}	retention ratio of TD^{kl}	$w^{SG_i^aSG_i^h}$	SG_i^a's referenced weight for SG_i^h
$TC^{l(\neg k)}$	dm^l's average in-degree centrality index except dm^k	$\widehat{r}^{SG_i^a}$	SG_i^a's referenced preference
$\overline{TD}^{kl}$	final updated trust degree of TD^{kl}	$\overline{r}^{SG_i^a}$	SG_i^a's modified preference
SG_i^a	the ath subgroup on x_i	$\rho^{SG_i^a}$	global feedback parameter of SG_i^a
d_i^{kl}	preference distance between dm^k and dm^l on x_i	$\lambda^{SG_i^a}$	global feedback parameter of SG_i^a under comparison situation
ζ	distance threshold for d_i^{kl}	$CD^{SG_i^a}$	conflict degree of SG_i^a
CD_i^{kc}	conflict degree between dm^k and collective preference within the same subgroup on x_i		

conflict degree is introduced because it is used by this modification mechanism.

3.1. Conflict degree based on the attitude toward the preference

Let us suppose that a DM use a linguistic term set $S = \{S_0, S_1, \ldots, S_g\}$ with odd cardinality (it means g is an even number) to express preferences about the alternatives[38]. Depending on the linguistic term used, the attitude of the DM can be classified into one of the following:

- If S_α with $\alpha < g/2$, then the attitude is negative.

- If S_α with $\alpha = g/2$, then the attitude is neutral.

- If S_α with $\alpha > g/2$, then the attitude is positive.

With the purpose of quantifying the problem, we use $-1, 0$, and 1, to represent the negative, neutral, and positive attitude, respectively. In particular, the attitude vector for dm^k is defined.

Definition 5. Let $R^k = (r_i^k)_{1\times n}$ be the preference vector given by dm^k on the alternatives x_i using the linguistic term set S, that is, $r_i^k \in S$. The attitude vector $AV^k = (AV_1^k, \ldots, AV_n^k)$ associated with dm^k is determined as follows:

$$AV_i^k = \begin{cases} -1, & \text{if} \quad I(r_i^k) < g/2 \\ 0, & \text{if} \quad I(r_i^k) = g/2 \\ 1, & \text{if} \quad I(r_i^k) > g/2 \end{cases} \tag{4}$$

In this study, as PLTSs is used to represent the preferences provided by DMs, the analysis of the attitude is also required under such setting. In this case, the probability distribution is used to determine the attitude vector associated with dm^k.

Definition 6 Let $R^k = (r_i^k)_{1\times n}$ be the preference vector given by dm^k on x_i using the PLTSs composed of two values, that is, $r_i^k = L_i^k(p) = \{L_{i(1)}^k(p_{i(1)}^k), L_{i(2)}^k(p_{i(2)}^k)\}$. The attitude vector $AV^k = (AV_1^k, \ldots, AV_n^k)$ associated with dm^k is determined as follows:

$$AV_i^k = \begin{cases} -1, & \text{if} \quad I(L_{i(2)}^k) < g/2 \\ -p_{i(1)}^k, & \text{if} \quad I(L_{i(1)}^k) = g/2 - 1,\ I(L_{i(2)}^k) = g/2 \\ 0, & \text{if} \quad I(L_{i(1)}^k) = I(L_{i(2)}^k) = g/2 \\ p_{i(2)}^k, & \text{if} \quad I(L_{i(1)}^k) = g/2,\ I(L_{i(2)}^k) = g/2 + 1 \\ 1, & \text{if} \quad I(L_{i(1)}^k) > g/2 \end{cases} \tag{5}$$

The attitude of DM can be classified into one of the following in probabilistic linguistic group decision making:

- If $I(L_{i(2)}^k) < g/2$, then the attitude is strictly negative.

- If $I(L_{i(1)}^k) = g/2 - 1,\ I(L_{i(2)}^k) = g/2$, then the attitude is weakly negative.

- If $I(L_{i(1)}^k) = I(L_{i(2)}^k) = g/2$, then the attitude is neutral.

- If $I(L^k_{i(1)}) = g/2,\ I(L^k_{i(2)}) = g/2+1$, then the attitude is weakly positive.

- If $I(L^k_{i(1)}) > g/2$, then the attitude is strictly positive.

The preference attitude can reflect the conflict relationship between DMs. In detail, if the attitude of dm^k and dm^l on each alternative is the same, there is no conflict between them. If the attitude of dm^k and dm^l on each alternative is strictly contrary, there is a total conflict between them. Three kinds of conflicts are listed as below:

Definition 7. The conflict degree between dm^k and dm^l on the alternative x_i denoted as CD^{kl}_i is computed as follows:

$$CD^{kl}_i = \frac{|AV^k_i - AV^l_i|}{2} \tag{6}$$

Obviously, the maximum value of $|AV^k_i - AV^l_i|$ is 2 from Eq.(5). Therefore, Eq.(6) can be controlled in the interval [0,1].

Definition 8. The average conflict degree between dm^k and dm^l covering all alternatives denoted as CD^{kl} is computed as follows:

$$CD^{kl} = \frac{\sum_{i=1}^{n} |AV^k_i - AV^l_i|}{2n} \tag{7}$$

As the number of alternatives is n and the maximum value of $|AV^k_i - AV^l_i|$ is 2, Eq.(7) can be controlled in the interval [0,1].

Definition 9. The average conflict degree between dm^k and the remaining DMs on the alternative x_i denoted as CD^k_i, is computed as follows:

$$CD^k_i = \frac{\sum_{l=1;l\neq k}^{m} |AV^k_i - AV^l_i|}{2(m-1)} \tag{8}$$

As the number of DMs except dm^k in system is $m-1$ and the maximum value of $|AV^k_i - AV^l_i|$ is 2, Eq.(8) can be controlled in the interval [0,1].

3.2. Trust modification mechanism

The trust modification mechanism updates the initial trust relationships between DMs with two stages. The first one is based on conflict and the second one is based on "assimilation effect"[46]. Both steps are described in the following.

3.2.1. Trust modification based on conflict degree

In real-world environments, it is logical that the trust relationships between individuals will not stay steady over time. That is, even though dm^l once has a total trust to dm^k, it is likely that dm^l will change the trust toward dm^k in the current specific decision making situation.

In this study, we assume that the conflict between DMs can reflect DMs' preference differences on the current issue, which can effect on the trust between them. If $CD^{kl} = 0$, it means there is no conflict between dm^k and dm^l, namely the trust degree will stay the same. Otherwise, it means dm^k and dm^l provide partial or total opposing preferences and, as a consequence, the trust degrees TD^{kl} and TD^{lk} will be effected and modified. Therefore, we construct a functional relationship between trust and conflict. This modification mechanism is defined as follows.

Definition 10. The intermediate updated trust degree of dm^k toward dm^l, denoted as $\widetilde{TD}^{kl}$, is obtained according to the initial trust degree TD^{kl}. Let RR^{kl} be the retention ratio of TD^{kl}, then the intermediate updated trust degree is computed as follows:

$$\widetilde{TD}^{kl} = RR^{kl} \cdot TD^{kl} \tag{9}$$

Definition 11. The retention ratio RR^{kl} is based on the conflict degree CD^{kl} and it is computed as follows:

$$RR^{kl} = -\sigma^k (CD^{kl})^2 + 1 \tag{10}$$

where $\sigma^k \in [0, 1]$, the smaller value of σ^k, the more proportion of initial trust retained.

Fig.1 depicts the functional relationship between RR^{kl} and CD^{kl} with $\sigma^k = 0.5$. The intermediate updated trust degree $\widetilde{TD}^{kl}$ is determined by TD^{kl} and CD^{kl}. It is obvious that $TD^{kl} \in [0, 1]$ and $RR^{kl} \in [-\sigma^k+1, 1]$. According to interval mathematics, $\widetilde{TD}^{kl} \in [\min(0, -\sigma^k + 1, 1), \max(0, -\sigma^k + 1, 1)] = [0, 1]$. Therefore $\widetilde{TD}^{kl} \in [0, 1]$.

Fig. 1 can reflect how the trust degree is affected by the conflict degree CD^{kl}. First, it is a monotone decreasing function, which means there is a negative relation between CD^{kl} and RR^{kl}, that is, the higher the CD^{kl}, the lower the RR^{kl}. Second, the marginal utility of the function increases progressively, which means the decrement of RR^{kl} accentuates as CD^{kl} increases. If $CD^{kl} = 0$, then $RR^{kl} = 1$ and therefore the trust degree stays

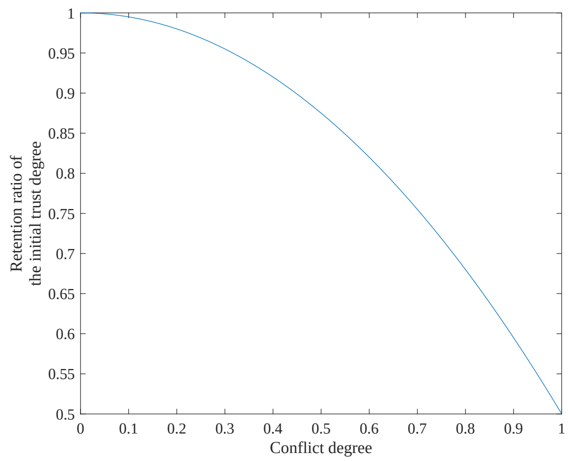

Figure 1: Functional relationship between the conflict degree and the retention ratio of the initial trust degree.

unchanged. If $CD^{kl} = 1$, even though there exists a total conflict, the trust degree between the DMs will not disappear totally because of the initial trust degree. Here, $RR^{kl} = -\sigma^k + 1$, it means the original trust is retained with a proportion of $(-\sigma^k + 1)$. If $CD^{kl} \in (0, 1)$, it is easy to see that the original trust degree is retained with a proportion located within the interval $(-\sigma^k + 1, 1)$.

3.2.2. Trust modification based on assimilation effect

After the first stage of trust modification based on conflict degree, the renewed trust is formed. In the following, we analyze the unintentional adjustment influenced by the surroundings. Let us suppose a trust network [25], if dm^k is the only one in the network that trusts dm^l, his/her trust will decrease as no other DM trusts dm^l. This behavior is called "assimilation effect" [46]. To simulate this behavior, the concept of trust attitude is introduced:

The linguistic labels "low" = $[0, 0.25]$, "medium" = $(0.25, 0.75]$ and "high" = $(0.75, 1]$, could be adopted to describe a low trust (negative attitude), a medium trust (neutral attitude), and a high trust (positive attitude), respectively (see Fig 2(a)). However, this distribution is evidently uneven. Therefore, if we consider the linguistic labels "low" = $[0, 0.125]$, "very low" = $(0.125, 0.375]$, "medium" = $(0.375, 0.625]$, "high" = $(0.625, 0.875]$ and

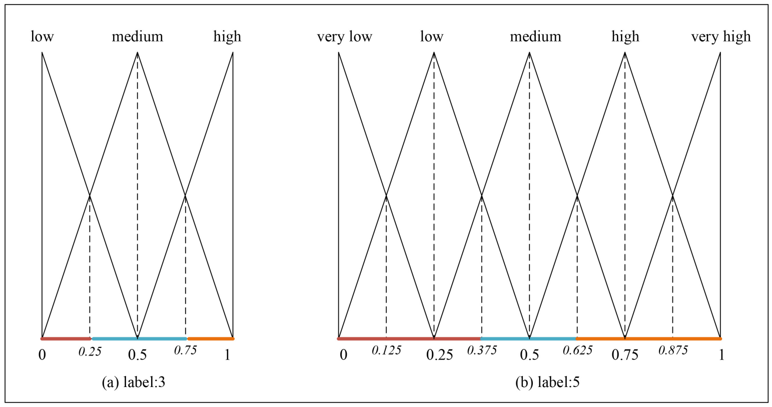

Figure 2: Linguistic labels of the attitude toward trust of the DMs.

"very high" $= (0.875, 1]$, and translate "very low" into "low" and "very high" into "high", we can get "low" $= [0, 0.375]$, "medium" $= (0.375, 0.625]$ and "high" $= (0.625, 1]$, whose distribution is almost the same as the average distribution (see Fig 2(b)). From it, if $\widetilde{TD}^{kl} \in [0, 0.375]$, the trust attitude from dm^k to dm^l is negative; if $\widetilde{TD}^{kl} \in (0.375, 0.625]$, the trust attitude from dm^k to dm^l is neutral; and if $\widetilde{TD}^{kl} \in (0.625, 1]$, the trust attitude from dm^k to dm^l is positive.

Inspired by the previous work[23] about individual in-degree centrality index computation, we can obtain the in-degree centrality index of dm^l apart from dm^k, which is denoted as $TC^{l(\neg k)}$ and computed as follows:

$$TC^{l(\neg k)} = \frac{\sum_{h=1;h\neq l,k}^{m} \widetilde{TD}^{hl}}{m-2} \tag{11}$$

If the value of $\widetilde{TD}^{kl}$ is evidently different to the value of $TC^{l(\neg k)}$, it is more likely that dm^k will adjust his/her trust degree to dm^l. It means that influenced by the surroundings, the DM's attitude and behavior will change, making his/her trust closer to the global one gradually. This is an unintentional adjustment influenced by the external environment.

The final updated trust degree $\overline{TD}^{kl}$ should be located in the interval $[\min(\widetilde{TD}^{kl}, TC^{l(\neg k)}), \max(\widetilde{TD}^{kl}, TC^{l(\neg k)})]$. However, we should consider that $\overline{TD}^{kl}$ and $\widetilde{TD}^{kl}$ have to belong to the same trust attitude to guarantee DM's own initial preference. In summary, the method to obtain the final updated trust degree $\overline{TD}^{kl}$ is:

- If $\widetilde{TD}^{kl}$ and $TC^{l(\neg k)}$ belong to the same trust attitude, $\widetilde{TD}^{kl}$ keeps steady, that is, $\overline{TD}^{kl} = \widetilde{TD}^{kl}$.
- If $\widetilde{TD}^{kl}$ and $TC^{l(\neg k)}$ do not belong to the same trust attitude and $TC^{l(\neg k)} > \widetilde{TD}^{kl}$, the final updated trust degree $\overline{TD}^{kl}$ will equal to the right bound of the attitude interval $\widetilde{TD}^{kl}$ locating in.
- If $\widetilde{TD}^{kl}$ and $TC^{l(\neg k)}$ do not belong to the same trust attitude and $TC^{l(\neg k)} < \widetilde{TD}^{kl}$, the final updated trust degree $\overline{TD}^{kl}$ will equal to the left bound of the attitude interval $\widetilde{TD}^{kl}$ locating in.

Once the final updated trust degrees have been computed, the importance degree of each DM can be obtained according to Eq.(1).

4. The consensus reaching process based on local adjustments

In this section, we present an approach for CRP from the local perspective after the trust updates according to Section 3, which tries to improve the agreement within each subgroup. It is composed of four steps: (1) clustering, which divides the DMs into subgroups with similar preference and preference accuracy; (2) local consensus reaching process, which includes consensus index based on local similarity network and local feedback mechanism; (3) update of trust degrees after individual preference modification and (4) acquisition of local collective preferences. In the next four subsections we describe these steps in detail.

4.1. Clustering

To deal with a large number of DMs, clustering techniques is an effective tool as they can divide the DMs having similar characteristics into small-scale subgroups, which simplifies the decision process[15, 20]. As there is no significantly better clustering techniques between those used in LSGDM[15, 47–49], in this study we adopt hierarchical clustering[50]. In addition, we use the silhouette coefficient proposed by Peter J. Rousseeuw[51] to determine the optimal number of clusters. Its advantage lies in combining cohesion and separation to evaluate the clustering validity.

Preference similarity can be a criterion in the clustering process, that is, a subgroup is composed of several DMs whose preferences are similar [4, 12, 18, 52]. In this study, as we assume PLTSs to represent the preferences given by DMs, the expected value could be used to form the subgroups. However, the probability distribution could be also considered. As an example, suppose the preference of dm^1 over x_1 is $r_1^1 = \{S_4(0.5), S_5(0.5)\}$ and the preference of dm^2 over x_1 is $r_1^2 = \{S_4(0.7), S_5(0.3)\}$. They have the similar expected values 4.5 and 4.3 in the interval [0,6] when $g = 6$, it is likely to classify them into the same cluster. Even so, there is some possibilities to assign them into different clusters because their preference accuracy of the decision problem is different: dm^1 is equally hesitant to S_4 and S_5, while dm^2 prefers S_4 as the proportion of 0.7. In particular, if one of the two probabilities is close to 1, the probabilistic linguistic term could be replaced by the associated linguistic term. In this case, dm^2 presents a higher accuracy in his/her preference than dm^1. From this analysis, we can conclude that the greater the difference between the probabilities, the more accurate the preference. Therefore, in addition to the expected value, the preference accuracy can be used to form the subgroups.

Definition 12. The preference accuracy of r_i^k (expressed by PLTSs) denoted as CL_i^k can be obtained as:

$$CL_i^k = |p_{i(1)}^k - p_{i(2)}^k| \tag{12}$$

In summary, using both the expected value and the preference accuracy, the DMs are divided into subgroups by hierarchical clustering [50]. In particular, different subgroups SG_i^a $(a = 1, \dots, s)$ are obtained for each alternative x_i, being s the optimal number of subgroups according to the silhouette coefficient[51].

4.2. Local consensus reaching process

According to Section 4.1, DMs have been classified into subgroups. The preferences in each cluster should be concentrated enough. When a unified opinion cannot be obtained, DMs contributing less to consensus need to modify preference in order to improve consensus. The consensus index of each subgroup is analyzed as follows:

4.2.1. Consensus index based on local similarity network

Building the similarity network of each subgroup helps to analyze the agreement between the DMs' preferences[12]. The similarity network has the

DMs as vertices and the distance between their preferences as edges. The distance between the preferences provided by dm^k and dm^l on the alternative x_i is calculated as: $d_i^{kl} = |EV(r_i^k) - EV(r_i^l)|$. The absolute distance is used here, but other distance measures like the euclidean distance or the cosine distance could be also used[53]. Obviously, $d_i^{kl} = d_i^{lk}$ and therefore the similarity network is undirectional.

Considering that the characteristic of cluster is that the elements in it should be compact, a distance threshold ζ is preset to measure whether the similarity network is concentrated. When all d_i^{kl} is no more than ζ, it means this cluster is completely concentrated; when there exits d_i^{kl} which is more than ζ, it means the cluster is unconcentrated and incompact to some extent. The definition of consensus index for SG_i^a is given:

Definition 13. The consensus index denoted as CI_i^a of SG_i^a can be computed as Eq.(13):

$$CI_i^a = \frac{\#(\sum_{k \neq l} d_i^{kl} \leq \zeta)}{\#(dm^k) \cdot (\#(dm^k) - 1)/2} \tag{13}$$

where $dm^k, dm^l \in SG_i^a$ and $\#(*)$ extracts the number of $*$. In detail, $\#(\sum_{k \neq l} d_i^{kl} \leq \zeta)$ means the number of edges with $d_i^{kl} \leq \zeta$, $\#(dm^k)$ means the number of DMs in subgroup SG_i^a, and there is $\#(dm^k) \cdot (\#(dm^k) - 1)/2$ edges in the similarity network.

If CI_i^a is no less than a predefined local consensus threshold δ, the consensus is reached. Otherwise, there are DMs needing adjustment. The local feedback mechanism is given below.

4.2.2. Local feedback mechanism

The local feedback mechanism composed of the local identification rule, local direction rule and local modification rule is carried out. This method is applied in an iterative way until the consensus index CI_i^a satisfies the threshold value δ.

Local identification rule. This rule helps identifying the DMs who should modify their preferences. For all the pairs of DMs (dm^k, dm^l) whose $d_i^{kl} > \zeta$ in the subgroup SG_i^a, there exist two cases:

(1) All the DMs are identified in only one pair of DMs. Then, according to the consensus measurements[23, 31], the average preference distance between each DM dm^k and all the others in the same subgroup SG_i^a, denoted as

$APD_i^{a(k)}$, is computed:

$$APD_i^{a(k)} = \frac{\sum_{l \neq k} d_i^{kl}}{\#(dm^l)} \tag{14}$$

where $dm^l \in SG_i^a$ and $\#(dm^l)$ means the number of DMs in SG_i^a except dm^k.

Arrange $APD_i^{a(k)}$ in a descending order. This is the order in which the DMs will be suggested to make modification. Only one DM in an identified pair needs adjustment (the other DM should be deleted from the ordering). If there exist different DMs having the same value of the average preference distance, choose one to make modification randomly.

(2) There exist DMs identified in more than one pair. Arrange the DMs by the number of pairs, in which they are located, in a descending order. This is the order in which the DMs need modification. This operation can reduce the distance associated with all the DMs connected to the adjusted individual. If there are two or more DMs having the same number of appearances, we select one of them according to the first case.

Local direction rule. Once we have identified dm^k who needs modification, the direction of the adjustment should be determined. As the objective is to reduce the distance between dm^k and dm^l, the preference provided by dm^l guides the direction rule. Due to this direction rule, the order established in the local identification rule will be reconsidered in the modification process: the DM with farther distance to dm^l will be suggested to modify preference.

Local modification rule. This rule is vital to improve consensus, as it guides the identified individuals to make adjustments in order to get them closer to the group preference. Here, we assume that even in a trust network environment, individuals tend to refer to the DMs located in the same subgroup, due to they have similar knowledge and cognition about the problem. However, the identified individuals should not refer to all DMs in the same cluster, because referring to the opinions that are opposite to the adjustment direction may cause greater distance from the group opinion.

In the subgroup SG_i^a, the set of referenced DMs for identified dm^k is denoted as $\overline{RDM}_i^{a(k)} = \{dm^h \mid dm^h \in SG_i^a\}$, the referenced weight of dm^k for each DM in $\overline{RDM}_i^{a(k)}$ can be obtained by $w_i^{kh} = \frac{\overline{TD}_i^{kh}}{\sum_{dm^h \in \overline{RDM}_i^{a(k)}} \overline{TD}_i^{kh}}$. Then, the expected value of the aggregated referenced information $\hat{r}_i^k$ can be computed by $EV(\hat{r}_i^k) = \sum_{dm^h} w_i^{kh} \cdot EV(r_i^h)$. The modified preference $\overline{r}_i^k$ can be obtained by linearly aggregating the original preference r_i^k and the referenced preference $\hat{r}_i^k$[12, 25]:

$$EV(\overline{r}_i^k) = (1-\rho_i^k)\cdot EV(r_i^k) + \rho_i^k \cdot EV(\hat{r}_i^k) \tag{15}$$

In this study, we determine the value of dm^k's local feedback parameter ρ_i^k on x_i according to two criteria rather than selecting it discretionarily.

The first one is the conflict degree. The higher the conflict between dm^k and the other DMs, the more his/her preference is far from the group preference. Therefore the more acceptance of referenced information is needed to improve consensus, based on the hypothesis that DMs are willing to improve consensus, we assume ρ_i^k takes the value as CD_i^k.

The second one depends on the comparisons with other preferences. Given the fact that the more adjustment, the lower individual utility[54]. The feedback mechanism will be allowed to stop when the adjusted opinion equals to one of the referenced DM's preference. Because in such a case, dm^k will think his/her own preference is not the farthest from the group preference or the only one contributing less to the consensus, it is likely that dm^k will not make modification more.

Hence, the feedback parameter denoted as λ_i^k in this situation can be defined: If the $EV(r_i^k)$ is lower than all the referenced DMs' expected values, λ^k can be derived through Eq.(16); and if $EV(r_i^k)$ is higher than others, λ^k can be derived through Eq.(17).

$$\lambda_i^k = \frac{\min_{dm^h \in \overline{RDM}_i^{a(k)}}\{EV(r_i^h)\} - EV(r_i^k)}{EV(\hat{r}_i^k) - EV(r_i^k)} \tag{16}$$

$$\lambda_i^k = \frac{\max_{dm^h \in \overline{RDM}_i^{a(k)}}\{EV(r_i^h)\} - EV(r_i^k)}{EV(\hat{r}_i^k) - EV(r_i^k)} \tag{17}$$

Considering the utility of DMs, the parameter ρ_i^k is obtained as $\rho_i^k = \min\{CD_i^k, \lambda_i^k\}$. Then, the distance between the preference given by dm^k and the group one is reduced by using Eq.(15).

4.3. Update of trust degrees after individual preference modification

If the adjustment causes the change of the identified DM's preference attitude, the trusts related to him/her will update according to the above analysis. The updated process is still measured on the basis of initial trusts.

And the renewed trust relationship will be applied in the next iteration and aggregation of individual information.

4.4. Acquisition of local collective preferences

Consensus index CI_i^a needs be measured after each modification, if $CI_i^a < \delta$, the next iteration will be conducted; otherwise, the collective opinion can be obtained for each subgroup SG_i^a. This collective probabilistic linguistic opinion needs to satisfy the following conditions so as to be precise, concentrated and logical, details can be seen in models[42, 43]:

$$\begin{cases} \sum_{\alpha=0}^{g-1} |x_{\alpha+1}^c - x_\alpha^c| \leq 2 \\ \\ x_0 + x_g \leq 1 \\ \\ \sum_{\alpha=0}^{g} x_\alpha^c \leq \beta \\ \\ x_\alpha^c = \begin{cases} 0, & p_\alpha^c = 0 \\ 1, & p_\alpha^c \neq 0 \end{cases} \end{cases} \tag{18}$$

where $\beta = 2$, x_α^c is a binary variable and the collective preference can be obtained based on p_α^c. This collective preference should be as close as possible to all individual preferences in the subgroup SG_i^a. In addition, the importance of each individual denoted as TC_i^k is derived by Eq.(1), which means the individual who is more important in the network should be considered more. We adopt the absolute distance to measure the above principle reflected by the objective function. This collective probabilistic linguistic preference can be computed according to the following programming model:

$$Min \ \sum_{dm^k \in SG_i^a} TC_i^k |EV(r_i^k) - \sum_{\alpha=0}^{g} \alpha p_\alpha^c|$$

$$s.t. \begin{cases} CD_i^{kc} = \frac{|AV_i^k - AV_i^c|}{2} \leq \phi^{SG} & (19-1) \\ \sum_{\alpha=0}^{g-1} |x_{\alpha+1}^c - x_\alpha^c| \leq 2 & (19-2) \\ x_0^c + x_g^c \leq 1 & (19-3) \\ \sum_{\alpha=0}^{g} x_\alpha^c \leq \beta & (19-4) \\ x_\alpha^c = \begin{cases} 0, & p_\alpha^c = 0 \\ 1, & p_\alpha^c \neq 0 \end{cases} & (19-5) \\ \sum_{\alpha=0}^{g} p_\alpha^c = 1 & (19-6) \\ \text{if} \quad x_\alpha^c = 1 \quad \text{and} \quad x_{\alpha+1}^c = 1, \quad AV_i^c = \begin{cases} -1, & \alpha < g/2 - 1 \\ -p_\alpha^c, & \alpha = g/2 - 1 \\ p_{\alpha+1}^c, & \alpha = g/2 \\ 1, & \alpha > g/2 \end{cases} & (19-7) \\ \text{if} \quad x_\alpha^c = 1 \quad \text{and} \quad x_{\alpha+1}^c = 0, x_{\alpha-1}^c = 0, \quad AV_i^c = \begin{cases} -1, & \alpha < g/2 \\ 0, & \alpha = g/2 \\ 1, & \alpha > g/2 \end{cases} & (19-8) \end{cases} \quad (19)$$

where CD_i^{kc} means the conflict degree between the preference provided by d^k and the collective opinion over alternative x_i, ϕ^{SG} means the conflict threshold set for subgroup and CD_i^{kc} should no more than ϕ^{SG}. Hence $(19-1)$ holds. Constraint $(19-7)$ and $(19-8)$ describe the attitude vector when the number of different linguistic terms is 2 and 1, respectively. Constraints $(19-2)$ - $(19-5)$ are the same as models in previous works[42, 43, 55]. Constraint $(19-6)$ means the collective probabilistic linguistic preference has complete information of probabilistic distribution.

Enumeration method can help to list all the $(2g+1)$ solution situations, when the scale(s) of optimal linguistic collective preference is determined, AV_i^c can be obtained easily. In fact, according to conflict degree constraint, some possibilities of solution can be eliminated, hence calculation complexity is less than $O(2g+1)$. Model(19) is equivalent to:

$$Min \ \sum_{dm^k \in SG_i^a} TC_i^k e(r_i^k)$$

$$s.t. \begin{cases} \frac{\mu_i^k}{2} \leq \phi^{SG} \\ EV(r_i^k) - \sum_{\alpha=0}^{g} \alpha p_\alpha^c \leq e(r_i^k) \\ \sum_{\alpha=0}^{g} \alpha p_\alpha^c - EV(r_i^k) \leq e(r_i^k) \\ AV_i^k - AV_i^c \leq \mu_i^k \\ AV_i^c - AV_i^k \leq \mu_i^k \end{cases} \tag{20}$$

being $e(r_i^k) = |EV(r_i^k) - \sum_{\alpha=0}^{g} \alpha p_\alpha|$ and $\mu_i^k = |AV_i^k - AV_i^c|$. $e(r_i^k), \mu_i^k$ and p_α^c are decision variables. The final unique solution can be achieved after comparison of all the optimizations. Then, the collective preference related to the subgroup SG_i^a is computed as $EV(r^{SG_i^a}) = \sum_{\alpha=0}^{g} \alpha p_\alpha^c$.

5. The consensus reaching process based on global adjustments

Once the consensus has been reached for each subgroup, the next step consists in improving the global consensus index among the subgroups. This part includes four steps: (1) acquiring the trust degrees between subgroups, which can be realized through an optimization model; (2) conducting the global CRP, which includes consensus index based on group min-max programming model and global feedback mechanism; (3) update of trust degrees after sub-group preference modification and (4) acquisition of global collective preference. In the next four subsections we describe these steps in detail.

5.1. Trust degrees between subgroups

Similar to the trust relationships established between DMs, we should obtain the trust relationships between subgroups. Let SG_i be the set containing all the subgroups SG_i^a related to x_i, and $RSG_i^a = \{SG_i^b \mid SG_i^b \in SG_i \wedge SG_i^b \neq SG_i^a\}$. Each DM $dm^k \in SG_i^a$ has a trust degree to each D-M $dm^l \in RSG_i^a$. To obtain the trust degrees between SG_i^a and RSG_i^a, we should consider that the distance of the individual trust $\overline{TD}^{kl}$ and the unified trust $TD^{SG_i^a l}$ should be as close as possible. In addition, the trust shown by the more important DM should be more considered. Therefore, it is worth noting that DMs should be assigned different importance weights TC^k. The following model can help to obtain the trust degrees between subgroups:

$$\begin{aligned} & Min \sum_{dm^k \in SG_i^a} TC^k \sum_{dm^l \in RSG_i^a} |TD^{SG_i^a l} - \overline{TD}^{kl}| \\ & s.t. \left\{ \; 0 \leq TD^{SG_i^a l} \leq 1 \right. \end{aligned} \tag{21}$$

where $TD^{SG_i^a l}$ is the decision variable, denoting trust degree from the subgroup SG_i^a to DM $dm^l \in RSG_i^a$, Let $\nu_i^{kl} = |TD^{SG_i^a l} - \overline{TD}^{kl}|$, then Model(21) is equivalent to:

$$\begin{aligned} & Min \sum_{dm^k \in SG_i^a} TC^k \left(\sum_{dm^l \in RSG_i^a} \nu_i^{kl}\right) \\ & s.t. \begin{cases} TD^{SG_i^a l} - \overline{TD}^{kl} \leq \nu_i^{kl} \\ \overline{TD}^{kl} - TD^{SG_i^a l} \leq \nu_i^{kl} \\ 0 \leq TD^{SG_i^a l} \leq 1 \end{cases} \end{aligned} \tag{22}$$

Once the $TD_i^{SG_i^a l}$ has been computed, the trust degree between the subgroup SG_i^a and the remaining ones SG_i^b can be obtained as:

$$TD^{SG_i^a SG_i^b} = \frac{\sum_{dm^l \in SG_i^b} TD^{SG_i^a l}}{\#(dm^l)} \tag{23}$$

where $\#(dm^l)$ means the number of DMs in SG_i^b.

Then, after obtaining the trust degrees between subgroups, the importance degree of SG_i^a can be calculated as follows:

$$TC^{SG_i^a} = \frac{TD^{SG_i^a}}{\sum_{SG_i^a \in SG_i} TD^{SG_i^a}} \tag{24}$$

where $TD^{SG_i^a}$ is the in-degree centrality index of SG_i^a.

5.2. Global consensus reaching process

After the acquisition of the trust between subgroups, we can conduct the global CRP. Subgroups contributing less to consensus need to modify preference in order to improve consensus. The consensus index of this decision problem is analyzed novelly through a min-max programming model.

5.2.1. Consensus index based on group min-max programming model

Similarly, the collective preference aggregating all the subgroups' preferences should satisfy the above constraint conditions (18). We aim to minimize the maximum distance between the subgroup's preferences and the collective one to measure if the collective preference is acceptable. The optimization model can be defined as follows:

$$
\begin{aligned}
&\min\max\ TC^{SG_i^a}|EV(r^{SG_i^a})-\textstyle\sum_{\alpha=0}^{g}\alpha p_\alpha^G| \\
&s.t.\begin{cases}
CD^{SG_i^aG}=\frac{|AV^{SG_i^a}-AV_i^G|}{2}\le\phi^G \\
\sum_{\alpha=0}^{g-1}|x_{\alpha+1}^G-x_\alpha^G|\le 2 \\
x_0^G+x_g^G\le 1 \\
\sum_{\alpha=0}^{g}x_\alpha^G\le\beta \\
\sum_{\alpha=0}^{g}p_\alpha^G=1 \\
x_\alpha^G=\begin{cases}0, & p_\alpha^G=0\\ 1, & p_\alpha^G\neq 0\end{cases} \\
\text{if}\quad x_\alpha^G=1\quad\text{and}\quad x_{\alpha+1}^G=1,\quad AV_i^G=\begin{cases}-1, & \alpha<g/2-1\\ -p_\alpha, & \alpha=g/2-1\\ p_{\alpha+1}, & \alpha=g/2\\ 1, & \alpha>g/2\end{cases} \\
\text{if}\quad x_\alpha^G=1\quad\text{and}\quad x_{\alpha+1}^G=0,x_{\alpha-1}^G=0,\quad AV_i^G=\begin{cases}-1, & \alpha<g/2\\ 0, & \alpha=g/2\\ 1, & \alpha>g/2\end{cases}
\end{cases}
\end{aligned}
\tag{25}
$$

where $CD^{SG_i^aG}$ means the conflict degree between the preference provided by subgroup SG_i^a and the collective group opinion, ϕ^G means the group conflict threshold and $CD^{SG_i^aG}$ should no more than ϕ^G. The function of x_α^G is the same as x_α^c in Model(19). After the enumeration of possible solutions, Model(25) can be equivalently transformed into the following linear programming model:

$$
\min\ \eta_i
$$

$$
s.t.\begin{cases}
e^{SG_i^a} \leq \eta_i \\
\frac{\mu_i^{SG_i^a}}{2} \leq \phi^G \\
EV(r^{SG_i^a}) - \sum_{\alpha=0}^{g} \alpha p_\alpha^G \leq e^{SG_i^a} \\
\sum_{\alpha=0}^{g} \alpha p_\alpha^G - EV(r^{SG_i^a}) \leq e^{SG_i^a} \\
AV^{SG_i^a} - AV_i^G \leq \mu^{SG_i^a} \\
AV_i^G - AV^{SG_i^a} \leq \mu^{SG_i^a}
\end{cases} \tag{26}
$$

being $e^{SG_i^a} = TC^{SG_i^a}|EV(r^{SG_i^a}) - \sum_{\alpha=0}^{g} \alpha\rho_\alpha^G|$, $\mu^{SG_i^a} = |AV^{SG_i^a} - AV_i^G|$. $\eta_i, e^{SG_i^a}, \mu^{SG_i^a}$ and p_α^G are decision variables. If η_i is no more than the global consensus threshold value η^G, the collective preference can be computed by $EV(r_i^G) = \sum_{\alpha=0}^{g} \alpha p_\alpha^G$, otherwise the global feedback mechanism is necessary.

5.2.2. Global feedback mechanism

To adjust the preferences in order to reach consensus, a method composed of an identification rule, a direction rule and a modification rule from the global point of view is carried out. This method is applied in an iterative way until $\eta_i \leq \eta^G$.

Group identification rule. This rule helps identifying the subgroups who should modify their preferences. SG_i^a with maximum $\eta_i^a > \eta^G$ needs to make modification firstly.

Group direction rule. Evidently, as the existence of importance priority, it is likely that the subgroup preference $EV(r^{SG_i^a})$ needing adjustment may not be the maximum or minimum expected value among all the cluster opinions. Therefore, the referenced subgroups $\overline{RSG_i^a} = \{SG_i^h | SG_i^h \in RSG_i^a\}$ can be obtained: the distance between SG_i^h 's excepted value and the collective preferences is less than $EV(r^{SG_i^a})$'s.

Group modification rule. This rule aims to improve the consensus as it guides the identified subgroups to make adjustments in order to get them closer to the group preference.

The identified subgroup SG_i^a that needs modification has associated a weight $wg^{SG_i^a SG_i^h} \in [0,1]$ related to SG_i^h, which can be calculated as by $w^{SG_i^a SG_i^h} = \frac{TD^{SG_i^a SG_i^h}}{\sum_{SG_i^h \in \overline{RSG}_i^a} TD^{SG_i^a SG_i^h}}$. Then, the collective referenced preference of SG_i^a can be obtained by $EV(\hat{r}^{SG_i^a}) = \sum_{SG_i^h} w^{SG_i^a SG_i^h} \cdot EV(SG_i^h)$.

The following operation is similar as the local modification rule, the final adjusted preference for the group should linearly aggregate the original preference and the referenced one:

$$EV(\overline{r}^{SG_i^a}) = (1 - \rho^{SG_i^a}) \cdot EV(r^{SG_i^a}) + \rho^{SG_i^a} \cdot EV(\hat{r}^{SG_i^a}) \tag{27}$$

where the global feedback parameter $\rho^{SG_i^a}$ depends on two criteria as the determination of local feedback parameter: one is the conflict degree $CD^{SG_i^a}$ of SG_i^a, which is computed by the subgroup preferences aggregated in the local CRP; the other is the willingness reserving ratio $\lambda^{SG_i^a}$, which generates through the cluster whose preference nearest to the identified cluster. Finally, the parameter $\rho^{SG_i^a}$ is obtained as $\rho^{SG_i^a} = \min\{CD^{SG_i^a}, \lambda^{SG_i^a}\}$.

5.3. Update of trust degrees after subgroup preference modification

Similar as Section 4.3, different attitudes after preference modification cause updated trust. The renewed information of a subgroup means all DMs in this subgroup accept this preference, hence we can regard the subgroup's modification as each individual preference. The updated trust relationships between individuals can be computed through two stages, then trust between subgroups can be obtained again by Model(21).

5.4. Acquisition of global collective preference

After the renewed preferences and trust relationships, η_i should be calculated again by Model(25), checking whether it satisfy the predefined η_G. If $\eta_i > \eta_G$, the group CRP needs to be conducted again; otherwise, the collective preference related to η_i is the final global collective information.

6. Example of application

In this section, we illustrate our proposed consensus model for LSGDM through a numerical example. In addition, we discuss its advantages in comparison with other similar approaches.

6.1. Numerical example

Let $DM = \{dm^1, \ldots, dm^{30}\}$ be a group of 30 DMs. According to the preferences communicated by DMs, the objective is to rank the alternatives of the set $X = \{x_1, x_2, x_3\}$ from best to worst to solve a problem. In the following, we describe the steps of our proposed consensus model for this particular example.

6.1.1. Communication of preferences

DMs use the linguistic term set $S = \{S_0 = \text{none}, S_1 = \text{very low}, S_2 = \text{low}, S_3 = \text{medium}, S_4 = \text{high}, S_5 = \text{very high}, S_6 = \text{total}\}$ to evaluate the alternatives by means of PLTSs. Tab.3 and Fig. 3 show the DMs' preferences on the alternatives. We can find that the expected values for x_1 are in the interval (1,4.5), for x_2 are in the interval (3,6) and for x_3 are in the interval (1,5.5). It means there is the smallest difference between preference attitudes of 30 DMs for x_2, and preferences for x_3 show the maximum span. Therefore, we can conclude that it would be easier for x_2 to reach consensus than x_1 and x_3. Moreover, there is no totally accurate preference with preference accuracy as "1" for all alternatives.

6.1.2. Update of trust degrees

Tab.4 and Tab.5 show the initial trust relationships between $dm^1, \ldots, dm^{15}$, and between $dm^{16}, \ldots, dm^{30}$, respectively. As assumed above, there are direct relationships between all DMs. Trust degree is expressed by interval [0,1], when $TD^{kl} = 0$, it means there is no trust between dm^k and dm^l; and when $TD^{kl} = 1$, it means that dm^k totally trusts dm^l. We suppose these initial trust relationships have been established based on DMs' previous knowledge. Fig.4 depicts the average initial trust in-degree of each DM by means of a heatmap. The darker the colour, the higher the trust centrality associated with the DM. It is clear that dm^{29} is the core of this social network, while dm^4, dm^9,dm^{26} and dm^{27}, are those who have achieved a lower trust. Moreover, we can observe the maximum trust in-degree of DMs is about 0.65 and the minimum trust in-degree of DMs is about 0.4. It means there is no evident huge difference between DMs' centralities, and no DM is absolutely trusted or distrusted.

Once the DMs have provided their preferences, the trust between them must be updated. This is done by applying the trust modification mechanism based on conflict effect and assimilation effect developed in Section 3. Here, we assume $\sigma^k = 0.5$ $(k = 1, \ldots, 30)$. Fig.5 depicts the average final updated

Table 3: Preferences provided by the DMs on the alternatives.

	x_1	x_2	x_3
dm^1	$\{S_2(0.4), S_3(0.6)\}$	$\{S_5(0.6), S_6(0.4)\}$	$\{S_1(0.8), S_2(0.2)\}$
dm^2	$\{S_1(0.35), S_2(0.65)\}$	$\{S_5(0.8), S_6(0.2)\}$	$\{S_1(0.2), S_2(0.8)\}$
dm^3	$\{S_3(0.15), S_4(0.85)\}$	$\{S_4(0.4), S_5(0.6)\}$	$\{S_4(0.5), S_5(0.5)\}$
dm^4	$\{S_1(0.15), S_2(0.85)\}$	$\{S_4(0.8), S_5(0.2)\}$	$\{S_1(0.65), S_2(0.35)\}$
dm^5	$\{S_3(0.45), S_4(0.55)\}$	$\{S_3(0.4), S_4(0.6)\}$	$\{S_4(0.85), S_5(0.15)\}$
dm^6	$\{S_4(0.8), S_5(0.2)\}$	$\{S_4(0.55), S_5(0.45)\}$	$\{S_5(0.6), S_6(0.4)\}$
dm^7	$\{S_1(0.65), S_2(0.35)\}$	$\{S_3(0.75), S_4(0.25)\}$	$\{S_1(0.9), S_2(0.1)\}$
dm^8	$\{S_2(0.65), S_3(0.35)\}$	$\{S_5(0.5), S_6(0.5)\}$	$\{S_1(0.85), S_2(0.15)\}$
dm^9	$\{S_1(0.1), S_2(0.9)\}$	$\{S_5(0.85), S_6(0.15)\}$	$\{S_1(0.1), S_2(0.9)\}$
dm^{10}	$\{S_2(0.1), S_3(0.9)\}$	$\{S_5(0.55), S_6(0.45)\}$	$\{S_1(0.9), S_2(0.1)\}$
dm^{11}	$\{S_2(0.55), S_3(0.45)\}$	$\{S_5(0.45), S_6(0.55)\}$	$\{S_2(0.2), S_3(0.8)\}$
dm^{12}	$\{S_3(0.9), S_4(0.1)\}$	$\{S_5(0.1), S_6(0.9)\}$	$\{S_2(0.6), S_3(0.4)\}$
dm^{13}	$\{S_2(0.5), S_3(0.5)\}$	$\{S_5(0.2), S_6(0.8)\}$	$\{S_2(0.15), S_3(0.85)\}$
dm^{14}	$\{S_4(0.85), S_5(0.15)\}$	$\{S_4(0.6), S_5(0.4)\}$	$\{S_4(0.4), S_5(0.6)\}$
dm^{15}	$\{S_2(0.2), S_3(0.8)\}$	$\{S_5(0.15), S_6(0.85)\}$	$\{S_2(0.8), S_3(0.2)\}$
dm^{16}	$\{S_3(0.5), S_4(0.5)\}$	$\{S_4(0.8), S_5(0.2)\}$	$\{S_4(0.9), S_5(0.1)\}$
dm^{17}	$\{S_2(0.1), S_3(0.9)\}$	$\{S_5(0.75), S_6(0.25)\}$	$\{S_2(0.5), S_3(0.5)\}$
dm^{18}	$\{S_2(0.45), S_3(0.55)\}$	$\{S_5(0.4), S_6(0.6)\}$	$\{S_2(0.1), S_3(0.9)\}$
dm^{19}	$\{S_3(0.55), S_4(0.45)\}$	$\{S_3(0.45), S_4(0.55)\}$	$\{S_2(0.55), S_3(0.45)\}$
dm^{20}	$\{S_3(0.6), S_4(0.4)\}$	$\{S_3(0.5), S_4(0.5)\}$	$\{S_2(0.45), S_3(0.55)\}$
dm^{21}	$\{S_3(0.85), S_4(0.15)\}$	$\{S_3(0.55), S_4(0.45)\}$	$\{S_4(0.9), S_5(0.1)\}$
dm^{22}	$\{S_3(0.4), S_4(0.6)\}$	$\{S_3(0.6), S_4(0.4)\}$	$\{S_4(0.8), S_5(0.2)\}$
dm^{23}	$\{S_1(0.6), S_2(0.4)\}$	$\{S_3(0.8), S_4(0.2)\}$	$\{S_1(0.7), S_2(0.3)\}$
dm^{24}	$\{S_2(0.15), S_3(0.85)\}$	$\{S_5(0.7), S_6(0.3)\}$	$\{S_2(0.85), S_3(0.15)\}$
dm^{25}	$\{S_3(0.2), S_4(0.8)\}$	$\{S_4(0.85), S_5(0.15)\}$	$\{S_4(0.5), S_5(0.5)\}$
dm^{26}	$\{S_4(0.9), S_5(0.1)\}$	$\{S_4(0.45), S_5(0.55)\}$	$\{S_4(0.6), S_5(0.4)\}$
dm^{27}	$\{S_3(0.1), S_4(0.9)\}$	$\{S_4(0.5), S_5(0.5)\}$	$\{S_4(0.45), S_5(0.55)\}$
dm^{28}	$\{S_2(0.6), S_3(0.4)\}$	$\{S_5(0.1), S_6(0.9)\}$	$\{S_2(0.9), S_3(0.1)\}$
dm^{29}	$\{S_1(0.4), S_2(0.6)\}$	$\{S_3(0.2), S_4(0.8)\}$	$\{S_1(0.15), S_2(0.85)\}$
dm^{30}	$\{S_1(0.2), S_2(0.8)\}$	$\{S_3(0.7), S_4(0.3)\}$	$\{S_1(0.6), S_2(0.4)\}$

trust in-degree of each DM by means of a heatmap (because of space limitations, we have omitted the computations to obtain the final updated trust degree). It is clear that dm^{29} is the core of this social network, while dm^4, dm^9 and dm^{27}, are those who have achieved a lower trust. Moreover, we can observe this social network does not show a high level of trust between DMs, as the maximum trust degree is about 0.55. Compared Fig.4 and Fig.5, we can find that the trust from Fig.5 is less than it from Fig.4 in general, this is because the conflicts derived from preferences cause the decrease of trusts. Moreover, the chromatism of different DMs in Fig.5 is less than it in Fig.4,

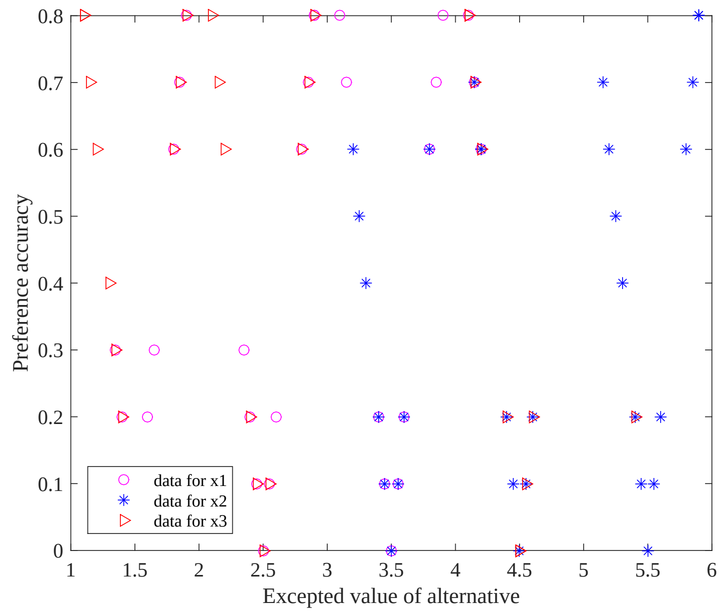

Figure 3: Preferences for alternatives of 30 DMs.

this is because the trust will be concentrated after "assimilation effect".

6.1.3. Clustering

The hierarchical clustering algorithm based on the expected value and the preference accuracy is applied to classify the DMs into different subgroups. This is done for each alternative x_i. According to the silhouette coefficient, the number of subgroups for the alternatives x_1, x_2 and x_3 are 6, 7 and 8, respectively:

- For the alternative x_1 the subgroups obtained are:

$$\begin{aligned}
SG_1^1 &= \{dm^4, dm^9, dm^{30}\} \\
SG_1^2 &= \{dm^{10}, dm^{12}, dm^{15}, dm^{17}, dm^{21}, dm^{24}\} \\
SG_1^3 &= \{dm^3, dm^6, dm^{14}, dm^{25}, dm^{26}, dm^{27}\} \\
SG_1^4 &= \{dm^2, dm^7, dm^{23}, dm^{29}\} \\
SG_1^5 &= \{dm^1, dm^8, dm^{11}, dm^{13}, dm^{18}, dm^{28}\} \\
SG_1^6 &= \{dm^5, dm^{16}, dm^{19}, dm^{20}, dm^{22}\}
\end{aligned}$$

Table 4: Initial trust relationships between $dm^1, \ldots, dm^{30}$ and $dm^1, \ldots, dm^{15}$.

	dm^1	dm^2	dm^3	dm^4	dm^5	dm^6	dm^7	dm^8	dm^9	dm^{10}	dm^{11}	dm^{12}	dm^{13}	dm^{14}	dm^{15}
dm^1		0.7	0.3	0.5	0.8	0.1	0.9	0.7	0.3	0.6	0.4	0.4	0.3	0.9	0.3
dm^2	0.6		0.1	1	0.4	0.6	0.3	0.1	1	1	1	0.7	0.5	0.1	0.5
dm^3	0.4	0.5		0.2	0.3	0.5	0.4	0.8	0.1	0.1	0.2	0.5	0.6	0.9	0.2
dm^4	0.6	0.1	0.5		0.2	0	0.3	0.8	0.5	0	0.1	0.2	1	0	0
dm^5	0.1	0.1	1	0.4		0.1	0.7	1	0.3	1	0.1	0.8	0.7	0.1	0.7
dm^6	0.3	0.7	0.1	0.1	0.1		0.3	0.1	0.5	0.2	0.1	0.4	0.3	0.4	0.5
dm^7	0.9	0.5	0.3	0.1	0.5	0		0.6	0.5	0.5	0.8	0.9	0.6	0.1	0.3
dm^8	0.4	0.3	0.4	0.4	0.6	0.2	1		0.4	0.4	0.2	0.8	0.4	0.2	0.7
dm^9	0.3	0.9	0.5	0.3	0.4	1	0.2	0.8		0.8	0.7	0.7	0.3	0.2	0.8
dm^{10}	0.5	0.9	0.3	0.4	0.8	0.5	0.5	0.8	0.2		0.2	0.5	0.7	0.9	0.3
dm^{11}	0.3	0.3	0.8	0.1	0.5	0.4	0.3	0.4	0.3	0.2		0.2	0.3	0.5	0.3
dm^{12}	0.9	0.7	0.3	0.2	0.9	0.4	0.5	0.8	0.5	0.1	0.2		0.7	0.4	0.2
dm^{13}	0.1	0.8	0.6	0.2	0.3	0.4	0.2	0.9	0.8	0.9	0.9	0.8		0.5	1
dm^{14}	0.9	0.5	0.6	1	0	0.6	0.9	0.1	0	0.8	0.3	0.6	0.7		0.6
dm^{15}	0.9	0.6	0	0	0.8	0.5	0.7	0.1	0.5	0	0.5	0.1	0.4	1	
dm^{16}	0.9	0.1	0.9	0.8	0.6	0.7	0.3	0.7	0.8	1	1	0.1	0.8	1	0.1
dm^{17}	0.1	0	0.4	1	0.9	0.8	0.7	0.9	0.3	0.8	0.8	0.2	0.8	0.7	0.2
dm^{18}	0.9	0	0.4	0.2	1	0.9	0.2	0.5	0.2	0.3	0.1	0.4	0.8	0.1	0.3
dm^{19}	0.4	0.5	0.9	0.1	0.2	0.1	0.1	0.4	0.3	0.3	0.1	0.7	0.7	0.3	1
dm^{20}	0.5	0.7	0.1	0.2	0.7	0.4	0.6	0.5	0	0.8	0.9	0.3	0.1	0.1	0.3
dm^{21}	0.8	0.4	0.2	0.4	0.9	0.4	0.6	0.8	0.3	0.8	0.2	0.9	0.1	0.8	0.6
dm^{22}	0.1	0.4	0.2	0.2	0.8	0.6	0.3	0.3	0	0.7	0.7	0.1	0.6	0.5	0.6
dm^{23}	0.7	0.8	1	0.6	0.7	0.3	0.6	0.4	0.1	0.1	0.8	0.8	0.6	0.9	0.6
dm^{24}	0.2	0.9	0.6	0.8	0.3	0.7	0.3	0	0.3	0.3	1	0	0.4	0.7	0.8
dm^{25}	0.7	0.9	0.3	0.5	0.5	0	0.2	0.4	0.7	0.4	1	0.8	0.5	0.4	0.8
dm^{26}	0.1	0.6	0.2	0.4	0.7	0.7	0.6	0.8	1	0.7	0.9	0.2	0.9	0.1	0.7
dm^{27}	0.9	0.6	0.8	0.6	0.1	0	0.7	0.5	0	0.3	0.2	0.6	0.8	0.2	0.1
dm^{28}	0	1	0.3	0.1	0.9	0.6	0.4	0.1	0.2	0.4	0.7	1	0.2	0.8	0.9
dm^{29}	0.4	0.4	1	0.3	0.3	0.4	0	1	0.7	0.8	0.5	1	0.7	0.2	0.8
dm^{30}	1	0.4	0.7	0.4	0.2	0.6	0.8	0.2	0.4	0.5	0.8	0.9	0.9	0.6	1

- For the alternative x_2 the subgroups obtained are:

$$
\begin{aligned}
SG_2^1 &= \{dm^7, dm^{23}, dm^{30}\} \\
SG_2^2 &= \{dm^4, dm^{16}, dm^{25}, dm^{29}\} \\
SG_2^3 &= \{dm^2, dm^9, dm^{17}, dm^{24}\} \\
SG_2^4 &= \{dm^{12}, dm^{13}, dm^{15}, dm^{28}\} \\
SG_2^5 &= \{dm^5, dm^{19}, dm^{20}, dm^{21}, dm^{22}\} \\
SG_2^6 &= \{dm^3, dm^6, dm^{14}, dm^{26}, dm^{27}\} \\
SG_2^7 &= \{dm^1, dm^8, dm^{10}, dm^{11}, dm^{18}\}
\end{aligned}
$$

Table 5: Initial trust relationships between $dm^1, \ldots, dm^{30}$ and $dm^{16}, \ldots, dm^{30}$.

	dm^{16}	dm^{17}	dm^{18}	dm^{19}	dm^{20}	dm^{21}	dm^{22}	dm^{23}	dm^{24}	dm^{25}	dm^{26}	dm^{27}	dm^{28}	dm^{29}	dm^{30}
dm^1	0.3	1	0.6	0.8	0.7	0.2	0.1	0.8	1	0.8	0.9	0.2	0.5	0.8	0.4
dm^2	0.3	0.2	0.6	0.3	0.8	0.9	0.1	0.6	0.3	0.3	0.8	0.3	0.8	0.7	0.1
dm^3	0	0.5	0.9	0.4	0.2	0.2	0.6	0.7	0.1	0.3	0.2	0.6	0.8	0.9	
dm^4	0.9	0.4	0.4	1	0.3	0	0.4	0.9	0.6	0.7	0.2	0.1	0.8	0.6	0
dm^5	0.6	0.5	0.9	0.2	1	0.4	0.6	0	0.1	0.8	0.3	0.1	0.6	0.6	0.3
dm^6	0.3	0	0.1	0.5	0.7	0.6	0	0.8	0.9	0.5	0.8	0.9	0	0.8	0.1
dm^7	0.5	0.4	0.3	0.8	0.1	0.7	0.5	0.6	1	0.8	0.2	0.8	0.2	0.7	0.7
dm^8	0.3	0.7	0.6	0.2	0.5	0.9	0.9	0.6	0.1	0.7	0.2	0.2	0.3	0.6	0.4
dm^9	0.4	0.1	0	0.7	0.4	0.5	0.6	0.8	1	0.4	0.4	0	0	0.5	0.7
dm^{10}	0.8	0.1	0.4	0.5	0.5	0.9	0.8	0.5	0.7	0.6	0.5	0.9	0.6	0.5	0.8
dm^{11}	0.6	0.9	0	0.7	0.2	0.1	0.7	0.3	0.8	0.8	0.2	0.1	0.5	0.6	0.9
dm^{12}	0.1	0.2	0.7	0.9	0.4	1	0	0.1	0.8	0.7	0.2	0.2	0.8	0.4	0.9
dm^{13}	0.3	0.8	0.5	0.4	0.7	0.3	0.9	0.3	0.6	0.3	0	0.1	0.4	0.7	0.9
dm^{14}	0.4	0.9	1	0.3	0.5	0.6	0.3	0.2	0.7	0	0.4	0.6	0.9	0.8	0.3
dm^{15}	0.4	0.6	0.7	0.2	0.4	0.3	0.7	0.5	0	0.1	0.4	0	0	0.8	0.6
dm^{16}		0.8	0.9	0.2	0.5	0.3	1	0.2	0.1	0.8	0.6	0.4	0.3	0.3	1
dm^{17}	0.9		0.9	0.4	0	0.9	0.1	0.4	0.4	0.3	0.7	0.1	0	0.9	0
dm^{18}	0.5	0.4		0.2	0.9	0.9	0.5	0.9	0.5	0.5	0.1	0.5	0.6	0.9	0.6
dm^{19}	0.4	1	0.6		0.2	0.8	0.5	0.7	0.5	0.4	0.6	0.7	0.8	0.3	0.5
dm^{20}	0.7	0.5	0.2	0.8		0.1	0.1	0.5	0.2	1	0.3	0.1	0.9	0.1	0.5
dm^{21}	0.2	0.5	0.7	0.5	0.4		0.8	1	0.2	0.9	0.8	0.3	0.7	0.1	0.3
dm^{22}	0.7	0.6	1	0.3	0.2	0.3		0.2	0.9	0.3	0.6	0.6	0.2	1	0.3
dm^{23}	0.1	0.1	0.2	0.2	0.4	0.8	0.6		0.1	0.1	0.3	0.7	0.5	0.7	0.6
dm^{24}	0.8	0.7	1	0.5	0.2	0.4	0.1	0.6		0.4	0.1	0.5	1	1	0.3
dm^{25}	1	0.5	0.8	0.8	0.3	0.5	0.7	1	0.4		0.4	0.3	0.2	0.6	0.7
dm^{26}	0.3	0.2	0.3	0.2	1	0.9	0.9	0.8	0.6	0.1		0.9	0.8	0.7	0.4
dm^{27}	0.4	0.6	0.2	0.9	0.7	0.7	0	0.4	1	0.2	0.5		0.7	0.8	0.2
dm^{28}	0.5	0.9	0.8	0.9	0.6	0.6	0.6	0.8	0.4	1	0.9	0.4		0.9	0.7
dm^{29}	0.4	0.9	0	0	0.7	0.8	0.7	0.7	1	0.6	0.4	0.3	0.9		0.3
dm^{30}	0.3	0.9	0.5	0.1	0.8	0.1	1	0.8	0.2	0.3	0.1	0.8	0.8	0.9	

- For the alternative x_3 the subgroups obtained are:

$$
\begin{aligned}
SG_3^1 &= \{dm^1, dm^7, dm^8, dm^{10}\} \\
SG_3^2 &= \{dm^2, dm^9, dm^{15}, dm^{24}, dm^{28}, dm^{29}\} \\
SG_3^3 &= \{dm^{11}, dm^{13}, dm^{18}\} \\
SG_3^4 &= \{dm^5, dm^{16}, dm^{21}, dm^{22}\} \\
SG_3^5 &= \{dm^4, dm^{23}, dm^{30}\} \\
SG_3^6 &= \{dm^{12}, dm^{17}, dm^{19}, dm^{20}\} \\
SG_3^7 &= \{dm^3, dm^{14}, dm^{25}, dm^{26}, dm^{27}\} \\
SG_3^8 &= \{dm^6\}
\end{aligned}
$$

6.1.4. Local consensus reaching process

Suppose $\delta = 0.8$, $\zeta = 0.3$ and $\phi^{SG} = 0.05$ in this example. Using Eq.(13), the consensus index of each subgroup is:

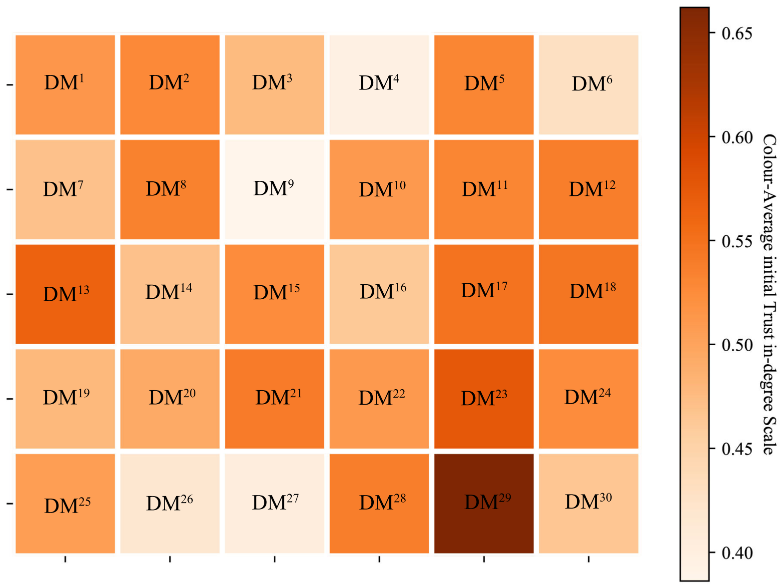

Figure 4: Heatmap related to average initial trust in-degree of DMs.

$$CI_1^1 = 1 \quad CI_1^2 = \frac{14}{15} \quad CI_1^3 = \frac{4}{5} \quad CI_1^4 = 1 \quad CI_1^5 = 1 \quad CI_1^6 = 1$$
$$CI_2^1 = 1 \quad CI_2^2 = \frac{1}{2} \quad CI_2^3 = 1 \quad CI_2^4 = 1 \quad CI_2^5 = 1 \quad CI_2^6 = 1 \quad CI_2^7 = 1$$
$$CI_3^1 = 1 \quad CI_3^2 = \frac{4}{5} \quad CI_3^3 = 1 \quad CI_3^4 = 1 \quad CI_3^5 = 1 \quad CI_3^6 = 1 \quad CI_3^7 = 1 \quad CI_3^8 = 1$$

Because $CI_2^2 < \delta$, the preferences of the subgroup SG_2^2 should be modified. To do so, the local feedback mechanism described in Section 4.2.2 is applied.

According to the local identification rule, we have to identify the pairs of DMs (dm^k, dm^l) whose $d^{kl} > \zeta$. In this example, they are: (dm^{25}, dm^{29}), (dm^{16}, dm^{29}) and (dm^4, dm^{29}). There exists a DM located in more than one pair: dm^{29}. As the other DMs are located in only one pair, we need to compute their average preference distances. Using Eq.(14), we obtain:

$$APD_2^{2(4)} = 0.15 \qquad APD_2^{2(16)} = 0.15 \qquad APD_2^{2(25)} = 0.08$$

Figure 5: Heatmap related to average final updated trust in-degree of DMs.

Therefore, the order in which the DMs must modify their preferences is: $dm^{29}, dm^{16}, dm^4, dm^{25}$.

The set of referenced DMs for dm^{29} can be listed as $\overline{RDM}_2^{2(29)} = \{dm^4, dm^{16}, dm^{25}\}$. As $\overline{TD}_2^{29,4} = 0.3$, $\overline{TD}_2^{29,16} = 0.375$ and $\overline{TD}_2^{29,25} = 0.47$, $w_2^{29,4} = 0.26$, $w_2^{29,16} = 0.33$ and $w_2^{29,25} = 0.41$ can be obtained. Then, $EV(\hat{r}_2^{29}) = 4.18$.

To calculate the final adjusted preference, we need to obtain the feedback parameter ρ_2^{29}. As $CD_2^{29} = 0.1267$ and $\lambda_2^{29} = 0.9210$, $\rho_2^{29} = CD_2^{29}$ and therefore, $EV(\bar{r}_2^{29}) = 3.85$ by Eq.(15).

After this adjustment, $CI_2^2 = \frac{2}{3}$, which is lower than δ. Therefore, the second DM (dm^{16}) in the above ordering must adjust her or his preference. By the same procedure, we get $EV(\bar{r}_2^{16}) = 4.18$. Now, $CI_2^2 = \frac{5}{6}$, which is higher than δ. Therefore, the consensus achieved in this subgroup is enough and no more adjustments are necessary.

Once the consensus is enough within each subgroup, a collective preference for each one of them is obtained. Using the optimization model described in Section 4.4, the following collective preferences are obtained:

$$EV(r^{SG_1^1}) = 1.85 \quad EV(r^{SG_1^2}) = 2.9 \quad EV(r^{SG_1^3}) = 4.1 \quad EV(r^{SG_1^4}) = 1.6$$
$$EV(r^{SG_1^5}) = 2.55 \quad EV(r^{SG_1^6}) = 3.5$$

$$EV(r^{SG_2^1}) = 3.25 \quad EV(r^{SG_2^2}) = 4.15 \quad EV(r^{SG_2^3}) = 5.25 \quad EV(r^{SG_2^4}) = 5.9$$
$$EV(r^{SG_2^5}) = 3.5 \quad EV(r^{SG_2^6}) = 4.5 \quad EV(r^{SG_2^7}) = 5.5$$

$$EV(r^{SG_3^1}) = 1.1 \quad EV(r^{SG_3^2}) = 3 \quad EV(r^{SG_3^3}) = 2.85 \quad EV(r^{SG_3^4}) = 4.15$$
$$EV(r^{SG_3^5}) = 1.35 \quad EV(r^{SG_3^6}) = 2.5 \quad EV(r^{SG_3^7}) = 4.5 \quad EV(r^{SG_3^8}) = 5.4$$

6.1.5. Global consensus reaching process

Suppose $\eta^G = 0.25$ and $\phi^G = 0.6$. By Model(25), the group preference for x_1 is $\{(S_2, 0.15), (S_3, 0.85)\}$ with $\eta_1 = 0.21$; for x_2 is $\{(S_4, 0.425), (S_5, 0.575)\}$ with $\eta_2 = 0.18$; for x_3 is $\{(S_3, 0.8), (S_4, 0.2)\}$ with $\eta_3 = 0.264$. Evidently, the collective preference on x_3 needs adjustment as $\eta_3 > \eta^G$. SG_3^8's preference will be recommended to make modification as the maximum distance. The referenced clusters is $\overline{RSG_i^a} = \{SG_3^2, SG_3^3, SG_3^4, SG_3^5, SG_3^6, SG_3^7\}$ according to Section 5.2.2. Adopting Model(21), the trust degree between clusters on x_3 can be computed as Tab.6. The subgroups' average trust in-degrees for x_3 can be computed from Tab.6: 0.446, 0.470, 0.513, 0.471, 0.390, 0.485, 0.422 and 0.433, respectively. It can be easily found that there is no evident differences between them, which means these subgroups are nearly of equal importance in this trust network. And the referenced clusters' weights of SG_3^8 are:

$$w^{SG_3^8 SG_3^2} = 0.19 \qquad w^{SG_3^8 SG_3^3} = 0.14 \qquad w^{SG_3^8 SG_3^4} = 0.16$$
$$w^{SG_3^8 SG_3^5} = 0.13 \qquad w^{SG_3^8 SG_3^6} = 0.18 \qquad w^{SG_3^8 SG_3^7} = 0.2$$

Then , we can get $EV(\hat{r}^{SG_3^8}) = 3.16$. By calculating $CD^{SG_3^8} = 0.55$, we can obtain $CD^{SG_3^8} > \lambda^{SG_3^8} = 0.40$, so $EV(\overline{r}^{SG_3^8}) = 4.5$ by Eq.(27). As only dm^6 in SG_3^8, and the attitude of dm^6 does not change, trust relationship will not change. After the modification, $\eta^{SG_3^8} = 0.23$, group consensus on x_3 is reached with the collective preference $\{(S_3, 1)\}$. Using the expected value calculation, the ranking of three alternatives is $x_2(4.58) \succ x_3(3) \succ x_1(2.85)$.

Table 6: Trust degrees between subgroups on x_3

	SG_3^1	SG_3^2	SG_3^3	SG_3^4	SG_3^5	SG_3^6	SG_3^7	SG_3^8
SG_3^1		0.503	0.453	0.563	0.453	0.523	0.426	0.375
SG_3^2	0.438		0.534	0.438	0.423	0.57	0.346	0.438
SG_3^3	0.404	0.5		0.478	0.398	0.507	0.385	0.375
SG_3^4	0.495	0.393	0.622		0.354	0.439	0.499	0.592
SG_3^5	0.498	0.491	0.525	0.401		0.441	0.408	0.375
SG_3^6	0.462	0.438	0.542	0.5	0.346		0.387	0.375
SG_3^7	0.45	0.471	0.542	0.488	0.423	0.46		0.5
SG_3^8	0.375	0.493	0.375	0.427	0.332	0.457	0.505	

6.2. Result analysis

In our example with the modified information for x_3, the collective preference is $\{(S_3, 1)\}$. The maximum conflict degree is 0.5, hence the conflict threshold $\phi^G \geq 0.5$ does not work ($\phi^G = 0.6$ in this paper). And there is no solution when $\phi^G < 0.5$. Namely, $\{(S_3, 1)\}$ is the optimized solution no matter what value ϕ^G takes. Therefore, the sensitivity analysis of ϕ^G is infeasible with the modified information for x_3, we will use the initial information for x_3 to analyze the sensitivity of ϕ^G in Model(25). The data is: $EV(r^{SG_3^1}) = 1.1$ with $r^{SG_3^1} = \{(S_1, 0.9), (S_2, 0.1)\}$, $EV(r^{SG_3^2}) = 3$ with $r^{SG_3^2} = \{(S_3, 1)\}$, $EV(r^{SG_3^3}) = 2.85$ with $r^{SG_3^3} = \{(S_2, 0.15), (S_3, 0.85)\}$, $EV(r^{SG_3^4}) = 4.15$ with $r^{SG_3^4} = \{(S_4, 0.85), (S_5, 0.15)\}$, $EV(r^{SG_3^5}) = 1.35$ with $r^{SG_3^5} = \{(S_1, 0.65), (S_2, 0.35)\}$, $EV(r^{SG_3^6}) = 2.5$ with $r^{SG_3^6} = \{(S_2, 0.5), (S_3, 0.5)\}$, $EV(r^{SG_3^7}) = 4.5$ with $r^{SG_3^7} = \{(S_4, 0.5), (S_5, 0.5)\}$, $EV(r^{SG_3^8}) = 5.4$ with $r^{SG_3^8} = \{(S_5, 0.6), (S_6, 0.4)\}$ and $TC^{SG_3^1} = 0.123$, $TC^{SG_3^2} = 0.13$, $TC^{SG_3^3} = 0.141$, $TC^{SG_3^4} = 0.13$, $TC^{SG_3^5} = 0.107$, $TC^{SG_3^6} = 0.133$, $TC^{SG_3^7} = 0.116$, $TC^{SG_3^8} = 0.12$. Tab.7 shows the values of collective preference and mini-maximum centrality-based distance varying with different ϕ^G. As we can see, when $0.61 \leq \phi^G \leq 1$, the collective is $\{(S_3, 0.78), (S_4, 0.22)\}$ with the distance 0.2612; when $0 \leq \phi^G < 0.5$, there is no feasible solution. Hence, we focus on the situation with $0.5 \leq \phi^G \leq 0.61$ (step 0.01): as ϕ^G decreases, the excepted value of collective preference decreases while mini-maximum centrality-based distance increases. The mini-maximum centrality-based distance belongs to SG_3^8 all the time. Obviously, there exists the situation that the subgroup's mini-maximum centrality-based distance has a decrease trend (shown in Tab.8). In addition, although the constraint of conflict will cause the increase of some individuals' conflicts, it controls the maximum individual conflict, achieving a balanced situation. It is worth noting that there may be situations where the distance exceeds the global consensus threshold η^G, the corresponding collective preference is inadvisable. Therefore, the setting of the conflict threshold can balance the collective preference and individual preferences, also it should fully consider the feasibility of model.

Table 7: The collective preference and mini-maximum centrality-based distance varying with different ϕ^G based on x_3's initial data

ϕ^G	collective preference	excepted value	mini-maximum centrality-based distance
0.61-1	$\{(S_3, 0.78), (S_4, 0.22)\}$	3.22	0.2612
0.60	$\{(S_3, 0.80), (S_4, 0.20)\}$	3.20	0.2640
0.59	$\{(S_3, 0.82), (S_4, 0.18)\}$	3.18	0.2664
0.58	$\{(S_3, 0.84), (S_4, 0.16)\}$	3.16	0.2688
0.57	$\{(S_3, 0.86), (S_4, 0.14)\}$	3.14	0.2712
0.56	$\{(S_3, 0.88), (S_4, 0.12)\}$	3.12	0.2736
0.55	$\{(S_3, 0.90), (S_4, 0.10)\}$	3.10	0.2760
0.54	$\{(S_3, 0.92), (S_4, 0.08)\}$	3.08	0.2784
0.53	$\{(S_3, 0.94), (S_4, 0.06)\}$	3.06	0.2808
0.52	$\{(S_3, 0.96), (S_4, 0.04)\}$	3.04	0.2832
0.51	$\{(S_3, 0.98), (S_4, 0.02)\}$	3.02	0.2856
0.50	$\{(S_3, 1)\}$	3.00	0.2880
0-0.49	no solution		

Table 8: The subgroup's centrality-based distance varying with different ϕ^G based on x_3's initial data

ϕ^G	0.61-1	0.60	0.59	0.58	0.57	0.56	0.55	0.54	0.53	0.52	0.51	0.50
SG_3^1	0.2608	0.2583	0.2558	0.2534	0.2509	0.2485	0.2460	0.2435	0.2411	0.2386	0.2362	0.2337
SG_3^2	0.0286	0.0260	0.0234	0.0208	0.0182	0.0156	0.0130	0.0104	0.0078	0.0052	0.0026	0
SG_3^3	0.0522	0.0494	0.0465	0.0437	0.0409	0.0381	0.0353	0.0324	0.0296	0.0268	0.0240	0.0212
SG_3^4	0.1209	0.1235	0.1261	0.1287	0.1313	0.1339	0.1365	0.1391	0.1417	0.1443	0.1469	0.1495
SG_3^5	0.2001	0.1980	0.1958	0.1937	0.1915	0.1894	0.1873	0.1851	0.1830	0.1808	0.1787	0.1766
SG_3^6	0.0958	0.0931	0.0904	0.0878	0.0851	0.0825	0.0798	0.0771	0.0745	0.0718	0.0692	0.0665
SG_3^7	0.1485	0.1508	0.1531	0.1554	0.1578	0.1601	0.1624	0.1647	0.1670	0.1694	0.1717	0.1740
SG_3^8	0.2616	0.2640	0.2664	0.2688	0.2712	0.2736	0.2760	0.2784	0.2808	0.2832	0.2856	0.2880

As stated above about analyzing the original preferences, the possibility of consensus reaching is $x_2 \succ x_1 \succ x_3$. Although in the local CRP, only x_2 needs modification, but it just reflects that the subgroups for x_2 is not as compact as those for x_1 and x_3. In global CRP, we can find that $\eta_1 = 0.21$, $\eta_2 = 0.18$ and $\eta_3 = 0.264$, the order of modification in general is x_3, x_1, x_2. This is consistent with previous analyses. Therefore, a simple analysis of the initial preference information can help to measure the likelihood of consensus reaching. If the range of opinions is too wide and too many preferences show low accuracy, maybe it is needless to conduct CRP, avoiding the unnecessary cost. Moreover, the consensus for x_3 is $\{(S_3, 1)\}$, which is with the complete accuracy "1" while there is no complete accurate original preference. It means the CRP not only obtain a unified opinion, but also improve the accuracy level of the collective preference. However, the collective preference for x_1 is $\{(S_2, 0.15), (S_3, 0.85)\}$ and for x_2 is $\{(S_4, 0.425), (S_5, 0.575)\}$, the accuracy of preferences have not been improved. Therefore, the constraint of accuracy such as " $CL_i \leq \epsilon_i$"(ϵ_i is the preset accuracy threshold) can be introduced into Model(19) and Model(25) to obtain the collective information with high accuracy level.

6.3. Comparative analysis

In this section, we firstly make a descriptive analysis of the proposed trust model and other trust models. Then, we compare our proposal with the existing consensus reaching approaches for LSGDM according to four aspects: (i) the update of the trust relationships; (ii) the way in which the collective preference is obtained by considering trust; (iii) the way in which the collective preference is obtained by considering conflict and (iv) the way in which the collective preference is obtained by goal programming model.

Firstly, we compare the previous trust models with our proposed dynamic trust model, including Bayesian-based trust models, entropy-based trust models, game theory-based trust models, clustering trust models and fuzzy trust models. It is worth noting that Bayesian theory can help to derive trust value, such as Meng et al.[28] used Bayesian inference approach to calculate devices' trust values and identify malicious devices in a healthcare environment; Jin et al.[56] applied Bayesian network into trust evaluation. This method only can work well for the presence of the distribution or probability of observing nodes. Moreover, entropy can also be applied into the acquisition of trust, such as Zhao et al.[57] measured the uncertainty of direct trust values by entropy theory, and indirect trust is introduced to strengthen interaction information when the uncertainty of direct trust is enough high; Ahmed et al.[29] combined similarity-based entropy and previous trust to generate trust recommendations. This approach is appropriate when the missing trust relationships need to be repaired. When it comes to dynamic trust, a multi-strategy game for nodes' actions is proposed to generate updated trust when the nodes' trust behaviors and utility functions are known[58]. There are many works about trust models in the GDM: Wu et al.[23] modeled trust relationships with distributed linguistic terms and provided relevant concepts to construct the minimum adjustment cost feedback mechanism; Capuano et al.[59] used trust statements to estimate missing preferences in fuzzy decision environment; Dong et al.[55] designed a network partition algorithm based on the notion of leadership; Xu et al.[60] applied the Louvain algorithm to classify DMs according to the relationships between nodes in the CRP; and so on. The summarized comparisons is shown as Tab.9. It is obviously found that the most trust models require relevant essential conditions while the proposed model does not need it, which means our model can solve the general situation as well as reflect the dynamic characteristics.

Table 9: Comparative analysis about various trust models

type	function	dynamic trust	essential condition
Bayesian-based[28, 56]	trust evaluation	-	the distribution of nodes
entropy-based[29, 57]	trust evaluation	-	incomplete trust relations
game theory-based[58]	trust updating	$\surd$	trust behaviors and utility
clustering trust[55, 60]	prompt consensus	-	the relations between nodes
fuzzy trust[59]	preference estimation	-	incomplete information
fuzzy trust[23]	prompt consensus	-	-
this paper	prompt consensus	$\surd$	-

Second, different from the traditional trust SNA in the CRP[21, 23, 26], here the trust is modified by the conflict arisen between the DMs and the assimilation effect. Fig.4 and Fig.5 reflect how the updated trust has an influence on the social network: the change of the in-degree centrality leads to a change of the social network. In fact, this is a kind of evolution of social networks[55, 61]. In addition, we emphasize that this trust modification mechanism leads to a dynamic trust across the whole CRP, which is not considered in the existing approaches.

Third, similar to the existing consensus models for social networks, both the local CRP and the global CRP are based on the similarities between the DMs' preferences. However, the proposed approach also takes the DMs' importance into account. That is, the existing approaches considered equally DMs' preferences when making an aggregation[23, 62], but more importance should be given to the preference provided by the DM whose in-degree centrality is higher. As an example, let us consider the global CRP to show the influence of considering DMs' importance. If the in-degree centrality is not consider, the objective function in Model(25) is $\min\max|EV(r^{SG_i^a}) - \sum_{\alpha=0}^{g}\alpha\rho_\alpha^c|$. Assuming the example illustrated in Section 6.1, in this case, both SG_3^1 and SG_3^8 need adjustment as they present the same maximum distance, while in the proposed approach only SG_3^8 need modification. The reason is that SG_3^8 has received more trust than SG_3^1, and therefore SG_3^8's preference has a higher proportion in the aggregation process.

Fourth, if we follows the previous works[42, 43, 55] without considering the conflict between collective and individual preferences, Model(28) can be constructed. We use the initial data from x_3 to make a comparison. The collective preference obtained from Model(28) is $\{(S_3, 0.78), (S_4, 0.22)\}$, the maximum conflict degree between individual and collective preference is $CD^{SG_3^1 G} = CD^{SG_3^5 G} = 0.61$. If the conflict degree is required no more than a threshold to guarantee the individual utility (It is generally assumed that the lager deviation between the attitudes of collective and individual opinions, the lower the individual utility), we can get the collective preference $\{(S_3, 1)\}$ with $\phi^G = 0.5$. The collective preference has been moved from "weakly positive attitude" to "neutral attitude", improving the utilities of individuals locating in the "negative attitude". As stated in Section 6.2, some individuals' utilities will be weaken. However, the balanced solution can be obtained by controlling the maximum conflict. Therefore, our proposed method possesses the evident advantages by considering DM's preference attitude as well as the in-degree centrality-based distance.

$$\min\max\ TC^{SG_i^a}|EV(r^{SG_i^a}) - \textstyle\sum_{\alpha=0}^{g} \alpha p_\alpha^G|$$
$$s.t. \begin{cases} \sum_{\alpha=0}^{g-1} |x_{\alpha+1}^G - x_\alpha^G| \leq 2 \\ x_0^G + x_g^G \leq 1 \\ \sum_{\alpha=0}^{g} x_\alpha^G \leq \beta \\ \sum_{\alpha=0}^{g} p_\alpha^G = 1 \\ x_\alpha^G = \begin{cases} 0, & p_\alpha^G = 0 \\ 1, & p_\alpha^G \neq 0 \end{cases} \end{cases} \tag{28}$$

Lastly, we compare the methods of aggregating individuals' preferences, the data from x_3 is still used with modified $EV(r^{SG_3^8}) = 4.5$. $EV(r^3) = 3.11$

can be obtained from the weighted average method like previous work[25], while $EV(r^3) = 3.22$ from our proposed method (here we do not consider the conflict as the traditional research). Each subgroup's opinion should be as close to the collective opinion as possible. The in-degree centrality-based distances between the subgroups' and collective preferences with $EV(r^3) = 3.11$ are 0.25, 0.01, 0.04, 0.13, 0.19, 0.08 , 0.16 and 0.27 respectively, while with $EV(r^3) = 3.22$ are 0.26, 0.03, 0.05, 0.12, 0.20, 0.10, 0.15 and 0.26 respectively. It is obvious that the maximum distance of traditional method is lager than it of proposed model. As stated above, the closer the collective opinion is to the individual opinion, the higher the individual utility is. Therefore, the proposed method can help to achieve a more reasonable collective opinion.

7. Conclusions and future studies

In this study, we have developed a new consensus model based on SNA for LSGDM with probabilistic linguistic information. We introduce the concepts of conflict effects based on preference attitude, and the 'assimilation effect' based on trust attitude to simulate the dynamic trusts. Also, local and group feedback mechanisms are designed respectively.

- The conflict degrees between DMs composed of three kinds (conflict degree between two experts about one certain alternative; conflict degree between two experts about all the alternatives; conflict degree of one identified expert with all the other DMs about certain alternative) are provided based on the defined preference attitude.

- The dynamic trust mechanism is designed based on conflict degree between two experts about all the alternatives and 'assimilation effect' reflected through trust attitude. Also, each preference adjustment might contribute to the change of conflict and subsequently effect the trust relationship, forming the cyclic dynamic trust-based CRP.

- The consensus measurements of subgroups and group are based on similarity network and min-max goal programming model respectively. The aggregation model considers the individual (subgroup's) in-degree trust centrality as well as the conflict between individuals (subgroups) and collective opinion about certain alternative.

- Two methods of determination about feedback parameter are investigated based on the conflict degree of the identified DM (subgroup) with all the other DMs (subgroups) about certain alternative and comparative reference within DMs (subgroups), and a comparison between them is necessary to guarantee the DM's (subgroup's) utility.

This proposed trust model is universal, as to utilize the proposed dynamic trust model, there is no need of any certain environment. Moreover, comparisons find that the introduction of conflict between preferences can guarantee DM's utility. And the in-degree centrality-based distance between DMs considers individual importance degree in the CRP, unlike treating DMs equally in the traditional models, which is necessary in SNA. Hence our model not only constitutes an extension of conventional methods, but also it shows evident advantages.

There is a increasing need for making easy distributed LSGDM and CRP by software, as the structure of the decision group is decentralized in real-world situations. This research could be continued by means of blockchain technologies, which provide a tool enforcing cost-effectiveness, integrity and security in these distributed decision processes[63]. Blockchain makes the decision process more transparent as it allows us to track the entire decision process.

Conflict of interest

The authors declare that they have no conflict of interest.

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Credit author statement

Xiao Tan was responsible for the model development and first draft.

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