

Comment on “Computer simulation of Fresnel diffraction from rectangular apertures and obstacles using the Fresnel integrals approach”



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ABSTRACT

The purpose of this comment is to correct the result of the diffracted intensity pattern derived from the application of Babinet's principle to Fresnel diffraction from the rectangular obstacle reported by Abedin et al. (2007), which seems to have been unnoticed. In addition, the correct expression is checked with similar results published.

1. Introduction

Fresnel diffraction from rectangular apertures/obstacles has been largely studied from theoretical, experimental, as well as computational point of view. Regarding this latter, Abedin et al. [1] carry out calculations of the diffraction patterns by using the Fresnel integrals that can be implemented in a computer program such as Matlab™ [2]. As this commercial software (or free alternatives such as GNU Octave [3] or Scilab [4]), as well as others such as Mathematica™ [5], and even computer languages like Python [6] for scientific programming can be accessed today as a research and academic tool, it is very interesting to have on hands implemented codes to get more insights in the phenomenon. Further, this is also very useful in the graduate optics courses in science and engineering when dealing with Fresnel diffraction topic.

However, the authors apply Babinet's principle to obtain an incorrect expression that allows calculating the diffracted intensity pattern from the rectangular obstacle given that from the complementary aperture.

2. Diffracted intensity pattern from the obstacle

When determining the diffracted field at a point P (see Fig. 1 in [1]), in a observation plane at a distance q_0 , from a rectangular aperture illuminated by a monochromatic point source S , located at a distance p_0 , the reported authors' work expression (1) misses the imaginary unit, i , from Fresnel inclination factor (e.g. [7,8]). That is, the diffracted field from the rectangular aperture, E_a , in the Fresnel approximation should be:

$$E_a = -\frac{iE_u}{2} [C(u) + iS(u)]_{x_1}^{x_2} [C(v) + iS(v)]_{y_1}^{y_2} \quad (1)$$

in which we have maintained their paper notation: E_u is the unobstructed field at P , $C(\cdot)$ and $S(\cdot)$ are the Fresnel integrals, $[f(x)]_{x_1}^{x_2} = f(x_2) - f(x_1)$, and

$$u \equiv y \left[\frac{2(q_0 + p_0)}{\lambda q_0 p_0} \right]^{1/2} \quad v \equiv z \left[\frac{2(q_0 + p_0)}{\lambda q_0 p_0} \right]^{1/2}$$

The missing imaginary unit, i , does not indeed affect the calculation of the diffracted intensity from the apertures, and therefore, all the

figures reported showing their diffracted patterns are correct. However, omitting the imaginary unit will have an important role, as we show below, for the pattern obtained from the complementary obstacle.

Babinet's principle [9], as pointed by the authors, allows us calculating the diffracted field from the complementary obstacle, E_o , illuminated in the same conditions as the aperture is. Consequently, if we know the diffracted field from the latter, E_a , since their sum should be equal to that of the unobstructed field, E_u , at the point P , we can determine the diffracted field from the obstacle. This can be written as:

$$E_u = E_a + E_o \quad (2)$$

Referring to Fig. 1 in [1], the unobstructed field from the point source S at the observation point P is given by the spherical wave amplitude in the Fresnel approximation:

$$E_u = \frac{E_0}{p_0 + q_0} \exp[ik(p_0 + q_0)] \exp\left[ik \frac{y^2 + z^2}{2(p_0 + q_0)}\right] \quad (3)$$

Thus, taking into account (2), the diffracted intensity pattern in the vacuum from the obstacle is:

$$\begin{aligned} I_o &= \frac{\varepsilon_0 c}{2} E_o E_o^* = \frac{\varepsilon_0 c}{2} (E_u - E_a)(E_u - E_a)^* \\ &= \frac{\varepsilon_0 c}{2} (E_u E_u^* + E_a E_a^* - 2\Re\{E_u E_a^*\}) \\ &= I_u + I_a - \varepsilon_0 c \Re\{E_u E_a^*\} \end{aligned} \quad (4)$$

in which $*$ denotes complex conjugate, and $\Re\{\cdot\}$ yields the real part.

The authors reported [1] that this resulting diffracted intensity pattern can be determined as:

$$I_o = I_u + I_a - 2\sqrt{I_u I_a} \quad (5)$$

Comparing (4) with (5), notice that the last term differs, and we have to use (4) when the field is complex in general. Therefore, their Fig. 9(b) is not the diffracted intensity pattern from the complementary obstacle of the square aperture. It is puzzling that the authors are not aware of the Fig. 16(b) in [10] (also their Ref. 7) nor Fig. 3(b) in [11], which are not in agreement with their results.

Now, bearing in mind (1) and (3), it follows

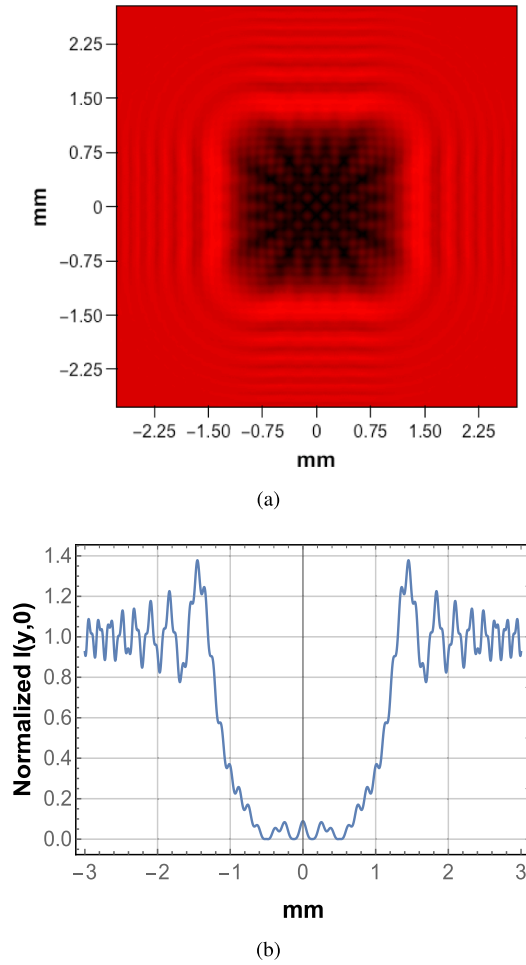


Fig. 1. (a) Simulated Fresnel diffraction intensity pattern from a 2 mm × 2 mm square obstacle at a distance of 400 mm ($\lambda = 632$ nm). (b) Intensity profile along y-axis at $x = 0$. Data obtained by using expression (7) programmed in Mathematica™. The intensity is normalized to I_u .

$$\begin{aligned} \Re\{E_u E_a^*\} &= \frac{|E_u|^2}{2} \Re\{i [C(u) - iS(u)]_{u_1}^{u_2} [C(v) - iS(v)]_{v_1}^{v_2}\} \\ &= \frac{|E_u|^2}{2} \{[S(u)]_{u_1}^{u_2} [C(v)]_{v_1}^{v_2} + [C(u)]_{u_1}^{u_2} [S(v)]_{v_1}^{v_2}\} \end{aligned} \quad (6)$$

and, hence, the correct diffracted intensity pattern from the square obstacle:

$$\begin{aligned} I_o &= I_u + I_a - \varepsilon_o c \Re\{E_u E_a^*\} = I_u + I_a \\ &- I_u \{[S(u)]_{u_1}^{u_2} [C(v)]_{v_1}^{v_2} + [C(u)]_{u_1}^{u_2} [S(v)]_{v_1}^{v_2}\} \end{aligned} \quad (7)$$

where

$$\begin{aligned} I_a &= \frac{I_u}{4} \{([C(u)]_{u_1}^{u_2})^2 + ([S(u)]_{u_1}^{u_2})^2\} \times \\ &\{([C(v)]_{v_1}^{v_2})^2 + ([S(v)]_{v_1}^{v_2})^2\} \end{aligned} \quad (8)$$

and

$$I_u = \frac{\varepsilon_o c E_0^2}{2(p_0 + q_0)^2} \quad (9)$$

Fig. 1 shows the diffraction intensity pattern (panel (a)), normalized to I_u , obtained by using (7), and under the same conditions as those in Fig. 9(b) reported in [1], as well as the intensity profile along the y-axis (panel (b)). We use Mathematica™ [5] software for the calculations, and we can see remarkable differences when comparing both figures.

It is worth to note that now we can observe in Fig. 1 the tendrils described by Dauger 10 and English and George 11 appearing in the corners and edges of the diffracted pattern obtained for the square obstacle. These tendrils are characteristic in the diffraction pattern from these type of obstacles. Nevertheless, they are not present in Fig. 9(b)

reported in [1].

3. Conclusions

We have corrected the calculation of the diffracted intensity pattern from the square obstacle reported previously [1], and provide the correct expression given by (7). Its implementation in Matlab™ is straightforward by taking advantage of the code reported by the authors in their paper Appendix. Thus, we hope this comment helps to complete the conclusions reported in [1] when simulating Fresnel diffraction from rectangular apertures and obstacles by using the Fresnel integrals approach.

Supplementary material

Supplementary data associated with this article can be found, in the online version, at <https://doi.org/10.1016/j.optlastec.2019.105819>.

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