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Anomalous strangeness transport

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Abstract: Nondissipative transport of strangeness is studied in a chiral hadronic plasma with three flavors. In the phase in which chiral symmetry is preserved, strangeness transport is found to be driven by both an external magnetic field and fluid vorticity. As for the constitutive relations of the baryon and electromagnetic currents, they exhibit vortical terms proportional to the strangeness chemical potential. In the superfluid phase, transverse nondissipative diffusion of the baryon, electromagnetic, and strangeness charges is found, which survives in the limit of vanishing chiral imbalance and mixes in a fashion similar to standard dissipative diffusion in quark-gluon plasma.

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Contents

1 Introduction and general setup

Anomalous transport [1–6] is known to be operative in a variety of physical setups where chiral imbalance is present (see also [7, 8] for reviews). Among them is nuclear matter [9–15] and in particular the dynamics of the quark-gluon plasma experimentally produced in heavy ion collisions [16–23], where chirality imbalance may results from vacuum configurations with nonzero topological number [24, 25]. This points to the possibility of detecting nondissipative charge transport induced either by a magnetic field or a nonzero fluid vorticity, a current undertaking in experimental studies of heavy ion collisions [26–30].

Historically, strangeness production played a key role in the early attempts to find experimental signatures of quark-gluon plasma production [31]. Strangeness difussion, however, is not disentangled from that of the electric and baryon charges, as a gradient in the density of one of them induces dissipative diffusion in the others [32]. In this short note, we explore *nondissipative* transport of strangeness in nuclear matter, modelized as a chiral hadronic fluid with three flavors in equilibrium and coupled to an external axial-vector gauge field affected by a 't Hooft anomaly, thus implementing chiral imbalance in the medium.

The addition of a third flavor, besides making calculations more involved with respect to the $N_f = 2$ case studied in [33], raises an important physical issue. It concerns the justification of the chiral approximation in the case of the strange quark, whose mass is similar to the effective temperature of the plasma, $m_s/T \sim 1/3$. As a result, chirality flip due to the strange quark mass would result in a dissipation of chirality in the sector with nonzero strangeness, effectively decoupling the strange quark from chiral transport which would be fully dominated by the two light flavors. It was argued however in [34, 35] that finite mass effects scale rather like $(m_s/T)^2$, bringing about a stronger suppression of chirality dissipation and making the three-flavor approximation a reliable starting point for the analysis of the system.

Keeping this in mind, we study the constitutive relations of the strangeness current, as well as the effect of the strange flavor on other currents. Our model consists in a fermion fluid coupled to Abelian vector and axial-vector external gauge fields denoted respectively by $\mathcal{V}$ and $\mathcal{A}$. In flavor space, the associated vector field one-forms are expanded in the basis spanned by the baryon number (B), electric charge (Q), and strangeness (S) matrices, given in terms of the Gell-Mann matrices λ_a ($a = 1, \ldots, 8$) and the 3×3 identity matrix $\mathbb{1}$ by

$$B = \frac{1}{3}\mathbb{1}, \qquad Q = \frac{e}{2}\left(\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8\right), \qquad S = \frac{1}{\sqrt{3}}\lambda_8 - \frac{1}{3}\mathbb{1}, \tag{1.1}$$

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with e the elementary charge. The isospin matrix I, on the other hand, is obtained from the previous ones through the Gell-Mann–Nishijima relation $\frac{1}{e}Q = \frac{1}{2}(B+S)+I$.

In addition to this, all form fields will be written using the electric-magnetic decomposition with respect to the fluid four-velocity u. For the vector gauge field, we have

$$\mathcal{V} = \boldsymbol{V}Q + iu(\mu_B B + \mu_Q Q + \mu_S S) \equiv \boldsymbol{\mathcal{V}}_M + u\mathcal{V}_E, \tag{1.2}$$

where $\boldsymbol{V}$ is the magnetic piece of the electromagnetic one-form potential and μ_B, μ_Q, and μ_S are the baryon number, charge, and strangeness chemical potentials respectively. Regarding the axial-vector gauge field, on the other hand, we follow [33, 36] and take it to be purely electric and proportional to the identity matrix

$$\mathcal{A} = iu\mu_5\mathbb{1} \equiv u\mathcal{A}_E, \tag{1.3}$$

with μ_5 the chiral chemical potential controlling chiral imbalance in the system. Moreover, the time component of the axial-vector gauge fields is taken to be constant, which amounts to the condition

$$(d+\mathfrak{a})\mu_5 = 0, \tag{1.4}$$

where $\mathfrak{a} \equiv \imath_u du$ is the fluid acceleration one-form, expressed here in terms of the differential forms interior product [37]. Using all previous relations, the field strengths of the vector and axial-vector gauge fields can be written as

$$\begin{aligned}\mathcal{F}_V &= 2i\mu_B\omega B + \mathbb{B}Q + 2i\mu_S\omega S - iuT\left[d\left(\frac{\mu_B}{T}\right)B + d\left(\frac{\mu_Q}{T}\right)Q + d\left(\frac{\mu_S}{T}\right)S\right] \\ &\equiv \boldsymbol{F}_V + uE_V, \\ \mathcal{F}_A &= 2i\mu_5\omega\mathbb{1} \equiv \boldsymbol{F}_A,\end{aligned} \tag{1.5}$$

where $\omega = \frac{1}{2}(du + u\mathfrak{a})$ is the vorticity two-form, T is the equilibrium local temperature, and $\mathbb{B} = d\boldsymbol{V} + 2i\mu_Q\omega$ denotes the magnetic field two-form.[1]

After these preliminaries, we are ready to start discussing a hadronic fluid in the symmetric phase. The method to be employed here is the same one devised in ref. [36], and applied in [33] to the case of a two-flavor hadronic fluid. Our departing point is the Abelian Bardeen Chern-Simons form

$$\omega_5^0(\mathcal{A},\mathcal{F}_V,\mathcal{F}_A) = -\frac{i}{4\pi^2}\mathrm{Tr}\left[\mathcal{A}\left(\mathcal{F}_V^2 + \frac{1}{3}\mathcal{F}_A^2\right)\right], \tag{1.6}$$

from where the equilibrium partition function W_{eq} is computed by applying the Mañes-Stora-Zumino transgression formula [40] to a one-parameter family of connections interpolating between $\mathcal{V} = \boldsymbol{V}Q$, $\mathcal{A} = 0$ and the configuration given in eqs. (1.2) and (1.3) (see [33, 36] for full details). The resulting partition function has the structure $W_{\text{eq}} = W_{\text{bulk}} + W_{\text{bdy}}$, where the first (nonlocal) piece defined on the five-dimensional bulk $\mathcal{M}_5$ has the form

$$\begin{aligned}W_{\text{bulk}} = &-\frac{i}{4\pi^2}\int_{\mathcal{M}_5} u\mathrm{Tr}\Big\{\mathcal{A}_E\boldsymbol{F}_A^2 + \mathcal{A}_E\boldsymbol{F}_V^2 + 2\mathcal{V}_E\boldsymbol{F}_A\boldsymbol{F}_V \\ &-2\Big[\boldsymbol{F}_A(\mathcal{A}_E^2 + \mathcal{V}_E^2) + 2\boldsymbol{F}_V\mathcal{A}_E\mathcal{V}_E\Big]\omega + \frac{4}{3}\mathcal{A}_E\Big(\mathcal{A}_E^2 + 3\mathcal{V}_E^2\Big)\omega^2\Big\}.\end{aligned} \tag{1.7}$$

[1] Here we follow the conventions of [33, 36], with the only exception that our definition of the magnetic field includes the vorticity-dependent term $2i\mu_Q\omega$ (see also [38, 39]).

The local term W_{bdy}, on the other hand, is defined on the boundary $\partial\mathcal{M}_5$, identified with the physical spacetime, and in our case it can be shown to be zero. This is a consequence of having chosen a purely electric axial-vector gauge field in eq. (1.3). Notice that, since in the following we are going to be concerned only with the constitutive relations of vector currents, there is no problem in setting $\boldsymbol{A} = 0$ in the effective action before taking any variations.

2 The strangeness covariant current

The vector and axial-vector (dual) covariant currents are computed by varying the bulk piece of the equilibrium partition function with respect to the field strengths $\boldsymbol{F}_V$ and $\boldsymbol{F}_A$ and keeping the boundary contributions [33, 36, 38]. For the vector current, the result is

$$\begin{aligned}\langle\star\boldsymbol{J}_V\rangle_{\text{cov}} &= \frac{i}{2\pi^2}u\Big(\mathcal{A}_E\boldsymbol{F}_V + \boldsymbol{F}_A\mathcal{V}_E - 2\mathcal{A}_E\mathcal{V}_E\omega\Big)\\ &= -\frac{1}{2\pi^2}\mu_5 u\Big(2i\mu_B\omega B + \mathbb{B}Q + 2i\mu_S\omega S\Big).\end{aligned} \tag{2.1}$$

To compute the strangeness current from here, we take the trace of the product of the vector current with the strangeness matrix, $\langle\star\boldsymbol{J}_S\rangle_{\text{cov}} = \operatorname{Tr}\left(S\langle\star\boldsymbol{J}_V\rangle_{\text{cov}}\right)$. Since the dual vector current (2.1) is purely electrical, taking a further Hodge dual leads to a four-vector whose covariant time component vanishes, $\langle J_{S,0}\rangle_{\text{cov}} = 0$, whereas the contravariant spatial components are given by

$$\langle J_S^i\rangle_{\text{cov}} = \frac{N_c}{6\pi^2}\mu_5\Big(2\mu_B\omega^i + e\mathbb{B}^i - 6\mu_S\omega^i\Big), \tag{2.2}$$

with N_c the number of colors. In addition, $\mathbb{B}^i$ and ω^i are expressed in terms of the components of the two-forms $\mathbb{B}$ and ω introduced above by

$$\begin{aligned}\mathbb{B}^i &= \frac{1}{2}\epsilon^{ijk}\mathbb{B}_{jk},\\ \omega^i &= \frac{1}{2}\epsilon^{ijk}\omega_{jk}.\end{aligned} \tag{2.3}$$

The result for the strangeness current in the symmetric phase shows the existence of chiral nondissipative transport of strangeness charge driven by both an external magnetic field and fluid vorticity.

To gain some further physical insight about the different terms in (2.2), we have to take into account that the strange quark carries both baryon and electric charge, so strangeness transport is entangled with the transport of the other two charges. From a formal viewpoint, this results from the nonorthogonality of the $\{B, Q, S\}$ flavor basis. Going back to eq. (2.1), we notice that the term containing the magnetic field comes from the contribution proportional to $\operatorname{Tr}(QS\mathbb{1})$, whose origin is a triangle diagram with one axial-vector, one electromagnetic, and one strangeness current insertion. Nondissipative transport of strangeness associated with this term is therefore consequence of the standard chiral magnetic effect affecting the strange quarks. Something similar happens with the vortical baryonic contribution, stemming from a term proportional to $\operatorname{Tr}(SB\mathbb{1})$. Its diagrammatic origin lies in a triangle with one axial-vector, one baryonic, and one strangeness currents. Here again, the chiral vortical effect for the baryonic current (cf. the results of ref. [33]) induces strangeness transport due to the nonzero baryon number of the strange quark. The upshot of all previous considerations is that the only "genuine" anomalous strangeness transport comes from the vortical term in (2.2) weighted by the strangeness chemical potential μ_S, originated in a triangle diagram with one axial-vector and two strange current insertions, whose flavor factor is $\operatorname{Tr}(S^2\mathbb{1})$.

The Bardeen-Zumino (BZ) currents, on the other hand, can be evaluated from the Chern-Simons form (1.6) using the explicit expressions given in ref. [36]. In particular, we find that in our case the BZ vector current is identical to the corresponding covariant current

$$\langle \star \boldsymbol{J}_V \rangle_{\text{BZ}} = \langle \star \boldsymbol{J}_V \rangle_{\text{cov}}, \tag{2.4}$$

in agreement with the fact that the boundary partition function is zero and so are all consistent vector currents.[2]

Let us now turn to the analysis of the system after chiral symmetry breaking. The computation of the covariant currents in this phase can be carried out by the appropriate transformation of the BZ currents in the symmetric phase using the Nambu-Goldstone boson matrix U, as shown in eqs. (6.26) and (6.29) of ref. [36]. This matrix is parametrized in terms of the pion, kaon, and η_8-meson fields by

$$U = \exp\left[\frac{i\sqrt{2}}{f_\pi}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 \end{pmatrix}\right], \tag{2.5}$$

where $f_\pi \approx 92\,\text{MeV}$ is the pion decay constant. Unlike in the unbroken phase studied earlier where the (dual) currents were purely electrical, now the vector and axial-vector currents have both electric and magnetic components. Here we are ultimately interested in the contravariant spatial components, so we only need to evaluate the electric part of the corresponding three-form currents, the magnetic parts giving the covariant time components upon taking the Hodge dual. The calculation is long and involved but follows the same steps detailed in ref. [33]. It leads to the following result for the covariant strangeness current

$$\begin{aligned}
\langle J_S^i \rangle_{\text{cov}} &= \frac{N_c}{6\sqrt{3}\pi^2 f_\pi} T\epsilon^{ijk}\partial_j\eta_8\partial_k\left(\frac{\mu_B}{T}\right) - \frac{eN_c}{6\sqrt{3}\pi^2 f_\pi} T\epsilon^{ijk}\partial_j\eta_8\partial_k\left(\frac{\mu_Q}{T}\right) \\
&\quad - \frac{N_c}{2\sqrt{3}\pi^2 f_\pi} T\epsilon^{ijk}\partial_j\eta_8\partial_k\left(\frac{\mu_S}{T}\right) + \frac{N_c}{6\pi^2 f_\pi^2}\mu_5\epsilon^{ijk}\Big[i\partial_j K^+\partial_k K^- + e\mathbb{V}_j\partial_k(K^+K^-)\Big] \\
&\quad - \frac{eN_c}{3\pi^2 f_\pi^2}\mu_5 K^+K^-(e\mathbb{B}^i + \mu_Q\omega^i) + \frac{iN_c}{6\pi^2 f_\pi^2}\mu_5\epsilon^{ijk}\partial_j K^0\partial_k\overline{K}^0 \\
&\quad + \frac{N_c}{3\pi^2 f_\pi^2}\mu_5\mu_S\Big(K^+K^- + K^0\overline{K}^0\Big)\omega^i + \frac{N_c}{6\pi^2}\mu_5\Big(e\mathbb{B}^i + 2\mu_B\omega^i - 6\mu_S\omega^i\Big),
\end{aligned} \tag{2.6}$$

where $\mathbb{V}_i$ are the components of the electromagnetic potential and we have dropped terms of third order and higher in the meson fields. Notice that, despite the explicit appearance of the electromagnetic potential, the previous expression remains gauge invariant.

[2] The consistent axial-vector current, however, is nonzero, since the magnetic part of the axial-vector gauge field in the boundary partition function cannot be set to zero prior to taking variations with respect to $\mathcal{A}$.

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To check that the anomalous strangeness transport encoded in the constitutive relations (2.2) and (2.6) is indeed a nondissipative phenomenon, we look at whether the different transport coefficients remain invariant under the time reversal operation T. From the transformation of the classical Nambu-Goldstone matrix field (2.5), $\mathsf{T} : U \to U^\dagger$ (see, for example, [41]), we find $\mathsf{T} : (\pi^0, \eta_8) \to (-\pi^0, -\eta_8)$, $\mathsf{T} : (\pi^\pm, K^\pm) \longrightarrow (-\pi^\mp, -K^\mp)$, and $\mathsf{T} : (K^0, \overline{K}^0) \to (-\overline{K}^0, -K^0)$, whereas the vorticity and the gauge and magnetic fields satisfy $\mathsf{T} : (\omega^i, \mathbb{V}_i, \mathbb{B}^i) \to (-\omega^i, -\mathbb{V}_i, -\mathbb{B}^i)$. Combining these transformations with the one for the strangeness current, $\mathsf{T} : J_S^i \to -J_S^i$, we conclude that all transport coefficients in eqs. (2.2) and (2.6) are T-even.[3] Notice that all chemical potentials are invariant under time reversal, since they are proportional to the time component of the corresponding background gauge fields.

The first three terms in (2.6), driven by the gradients of the chemical potentials, can be interpreted as describing nondissipative strangeness diffusion. Unlike the dissipative case studied in [32] in which the current points along the gradient, this is mediated by the gradient of the T-odd η_8-meson and is itself normal to the charge gradient. Interestingly, these are the only contributions surviving in the absence of chiral imbalance. In fact, all terms in (2.6), apart form the ones depending on μ_S have to be interpreted as resulting from the mixing between strangeness S and electric charge Q and baryon number B. Notice however the conspicuous absence of terms depending on the meson fields and proportional to the baryon chemical potential μ_B, indicating that baryon anomalous transport only contributes to the strangeness covariant current through the meson-independent BZ terms.

3 Other currents

A nonvanishing strangeness chemical potential μ_S also has effects on the constitutive relations of other currents. In the unbroken phase, an explicit evaluation of the electromagnetic current $\langle \boldsymbol{J}_{\rm em}\rangle_{\rm cov} = \mathrm{Tr}\,(Q\langle \boldsymbol{J}_V\rangle_{\rm cov})$ shows the existence of a chiral vortical effect mediated by $\mu_5\mu_S$

$$\langle J^i_{\rm em}\rangle_{\rm cov} = \frac{eN_c}{3\pi^2}\mu_5\Big(e\mathbb{B}^i - \mu_S\omega^i\Big). \tag{3.1}$$

While the first, chiral magnetic effect term has its source in the standard triangle diagram with one axial-vector and two electromagnetic currents, the second one has the same diagrammatic origin as the first term in eq. (2.2). A further peculiarity of $N_f = 3$ is that $\mathrm{Tr}\,Q = 0$, which removes from the constitutive relations of the electromagnetic covariant current a vortical term proportional to the baryon number chemical potential μ_B, that is however present for $N_f = 2$ (notice that this contribution was not explicitly computed in ref. [33], where the baryon chemical potential was set to zero from the start).

[3]These same transformations, together with the T-odd character of the electromagnetic and baryonic currents, imply as well the nondissipative character of the anomalous transport phenomena to be analyzed in the next section, both in the symmetric and the broken phases (cf. the discussion of the two-flavor case in ref. [33]).

In the broken phase, the electromagnetic and baryon currents are obtained along the same lines as the strangeness current shown in eq. (2.2). For the first one, we find

$$\begin{aligned}\langle J^i_{\rm em}\rangle_{\rm cov} &= \frac{eN_c}{12\pi^2 f_\pi}T\epsilon^{ijk}\left(\partial_j\pi^0+\frac{1}{\sqrt{3}}\partial_j\eta_8\right)\partial_k\left(\frac{\mu_B}{T}\right)\\ &+\frac{e^2N_c}{12\pi^2 f_\pi}T\epsilon^{ijk}\left(\partial_j\pi^0+\frac{1}{\sqrt{3}}\partial_j\eta_8\right)\partial_k\left(\frac{\mu_Q}{T}\right)\\ &-\frac{eN_c}{6\sqrt{3}\pi^2 f_\pi}T\epsilon^{ijk}\partial_j\eta_8\partial_k\left(\frac{\mu_S}{T}\right)-\frac{eN_c}{3\pi^2 f_\pi^2}\mu_5\Big(\pi^+\pi^-+K^+K^-\Big)(e\mathbb{B}^i+\mu_Q\omega^i)\\ &+\frac{eN_c}{6\pi^2 f_\pi^2}\mu_5\epsilon^{ijk}\Big[i\Big(\partial_j\pi^+\partial_k\pi^-+\partial_jK^+\partial_kK^-\Big)+e\mathbb{V}_j\partial_k\Big(\pi^+\pi^-+K^+K^-\Big)\Big]\\ &+\frac{eN_c}{3\pi^2 f_\pi^2}\mu_5\mu_SK^+K^-\omega^i+\frac{eN_c}{3\pi^2}\mu_5\Big(e\mathbb{B}^i-\mu_S\omega^i\Big).\end{aligned}\tag{3.2}$$

Again, the first three terms proportional to the chemical potentials gradients give rise to a nondissipative transverse diffusion of electric charge, similar to the corresponding effect spotted in the constitutive relations for the strangeness covariant current (2.6). As for the remaining contributions, only the terms proportional to $\mu_5\mu_S$ represent electric charge transport induced by the anomalous transport of other conserved charges, in this case strangeness (here, as a consequence of $\operatorname{Tr} Q=0$, there are no contributions resulting from the mixing between electric and baryonic charges). Finally, the meson-independent term is the BZ electromagnetic current that, as we explained above, coincides in our model with the covariant electomagnetic current in the symmetric phase given in eq. (3.1).

We complete our analysis with the calculation of the constitutive relations for the baryonic covariant current. In the symmetric phase, the result is

$$\langle J^i_{\rm bar}\rangle_{\rm cov}=-\frac{N_c}{3\pi^2}\mu_5(\mu_B-\mu_S)\omega^i.\tag{3.3}$$

Here we find a vortical effect similar to the one encountered in the electromagnetic current (3.1) proportional to $\mu_5\mu_S$, this time in combination with the one driven by a nonvanishing baryonic chemical potential. Thus, a nonzero strangeness chemical potential gives rise to a vortical term proportional to $\operatorname{Tr}(B\mathbb{1}S)$, originating in a triangle with an axial-vector, a baryonic, and a strangeness current. Once more, the identity $\operatorname{Tr} Q=0$ eliminates any electromagnetic contribution to the anomalous transport of the baryonic charge.

Chiral symmetry breaking adds transverse nondissipative diffusion terms to the constitutive relation for the baryonic current, driven by the gradients of the electric charge and strangeness chemical potentials

$$\begin{aligned}\langle J^i_{\rm bar}\rangle_{\rm cov} &= \frac{eN_c}{12\pi^2 f_\pi}T\epsilon^{ijk}\left(\partial_j\pi^0+\frac{1}{\sqrt{3}}\partial_j\eta_8\right)\partial_k\left(\frac{\mu_Q}{T}\right)+\frac{N_c}{6\sqrt{3}\pi^2 f_\pi}T\epsilon^{ijk}\partial_j\eta_8\partial_k\left(\frac{\mu_S}{T}\right)\\ &-\frac{N_c}{3\pi^2}\mu_5(\mu_B-\mu_S)\omega^i.\end{aligned}\tag{3.4}$$

Notice the absence in this case of meson-dependent contributions proportional to the chiral chemical potential μ_5.

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4 Closing remarks

We studied the anomalous transport of strangeness in a chiral hadronic fluid with three flavors at equilibrium, analyzing the cases where chiral symmetry is preserved and spontaneously broken. In the symmetric case, our main conclusion is that there are nondissipative mechanisms of strangeness transport driven by vorticity and an external magnetic field.

After chiral spontaneous symmetry breaking, we found a number of contributions to the constitutive relations mediated by the meson fields and/or their gradients. More remarkably, we showed how all three currents (2.6), (3.2), and (3.4) contain terms that survive the limit of vanishing chiral imbalance ($\mu_5 \to 0$), depending on the gradients of the three chemical potential. This can be interpreted as describing transverse nondissipative diffusion in the chiral hadronic fluid (i.e., normal to the direction set by the charge gradient). As a matter of fact, the contributions mentioned can be written in a way very much reminiscent of the structure found in [32] for dissipative diffusion

$$\left.\begin{pmatrix} \langle \vec{J}_{\rm bar}\rangle_{\rm cov} \\ \langle \vec{J}_{\rm em}\rangle_{\rm cov} \\ \langle \vec{J}_{S}\rangle_{\rm cov} \end{pmatrix}\right|_{\mu_5=0} = \begin{pmatrix} \vec{\kappa}_{BB} & \vec{\kappa}_{BQ} & \vec{\kappa}_{BS} \\ \vec{\kappa}_{QB} & \vec{\kappa}_{QQ} & \vec{\kappa}_{QS} \\ \vec{\kappa}_{SB} & \vec{\kappa}_{SQ} & \vec{\kappa}_{SS} \end{pmatrix} \times \begin{pmatrix} \vec{\nabla}\left(\frac{\mu_B}{T}\right) \\ \vec{\nabla}\left(\frac{\mu_Q}{T}\right) \\ \vec{\nabla}\left(\frac{\mu_S}{T}\right) \end{pmatrix}, \tag{4.1}$$

where the notation indicates the matrix product should be carried out using the three-dimensional vector cross-product between the corresponding entries. The calculations presented above show that most entries in the (vectorial) diffusion coefficient matrix are nonzero and determined by the gradients of the π^0 and η_8 meson fields, namely

$$\begin{aligned} &\vec{\kappa}_{BB} = \vec{0}, \\ &\vec{\kappa}_{QQ} = e\vec{\kappa}_{QB} = e\vec{\kappa}_{BQ} = \frac{e^2 N_c}{12\pi^2 f_\pi} T\vec{\nabla}\left(\pi^0 + \frac{1}{\sqrt{3}}\eta_8\right), \\ &e\vec{\kappa}_{SS} = 3\vec{\kappa}_{QS} = 3\vec{\kappa}_{SQ} = -3e\vec{\kappa}_{BS} = -3e\vec{\kappa}_{SB} = -\frac{eN_c}{2\sqrt{3}\pi^2 f_\pi} T\vec{\nabla}\eta_8. \end{aligned} \tag{4.2}$$

We see that, as in the dissipative case, anomaly-mediated diffusion mixes the different gradients among themselves. This is again a consequence of the use of a nonorthogonal basis of conserved charges, which implies microscopic degrees of freedom carry all three quantum numbers. Notice moreover that all terms in (4.1) have their origin in the axial anomaly responsible for the electromagnetic decays of neutral Nambu-Goldstone mesons $\pi^0 \to 2\gamma$ and $\eta \to 2\gamma$.

Since this mixed nondissipative transport of conserved charges is at work even in the absence of chirality imbalance, it would be interesting to explore this phenomenon in more precise modelizations of the quark-gluon plasma in order to decide whether they can be detected in current heavy-ion collision facilities. To the extend that the model used here provides a reliable description of the physics of quark-gluon plasma produced in heavy ion collisions, our results might point to a novel way of searching for nondissipative phenomena in these physical systems by focusing on strangeness transport.

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References

[1] K. Fukushima, D.E. Kharzeev and H.J. Warringa, *The Chiral Magnetic Effect*, *Phys. Rev. D* **78** (2008) 074033 [arXiv:0808.3382] [INSPIRE].

[2] D.T. Son and P. Surówka, *Hydrodynamics with Triangle Anomalies*, *Phys. Rev. Lett.* **103** (2009) 191601 [arXiv:0906.5044] [INSPIRE].

[3] A.V. Sadofyev and M.V. Isachenkov, *The Chiral magnetic effect in hydrodynamical approach*, *Phys. Lett. B* **697** (2011) 404 [arXiv:1010.1550] [INSPIRE].

[4] Y. Neiman and Y. Oz, *Relativistic Hydrodynamics with General Anomalous Charges*, *JHEP* **03** (2011) 023 [arXiv:1011.5107] [INSPIRE].

[5] K. Landsteiner, E. Megías and F. Pena-Benitez, *Gravitational Anomaly and Transport*, *Phys. Rev. Lett.* **107** (2011) 021601 [arXiv:1103.5006] [INSPIRE].

[6] V.P. Kirilin, A.V. Sadofyev and V.I. Zakharov, *Chiral Vortical Effect in Superfluid*, *Phys. Rev. D* **86** (2012) 025021 [arXiv:1203.6312] [INSPIRE].

[7] K. Fukushima, *Views of the Chiral Magnetic Effect*, *Lect. Notes Phys.* **871** (2013) 241 [arXiv:1209.5064] [INSPIRE].

[8] K. Landsteiner, *Notes on Anomaly Induced Transport*, *Acta Phys. Polon. B* **47** (2016) 2617 [arXiv:1610.04413] [INSPIRE].

[9] D.T. Son and A.R. Zhitnitsky, *Quantum anomalies in dense matter*, *Phys. Rev. D* **70** (2004) 074018 [hep-ph/0405216] [INSPIRE].

[10] D.T. Son and M.A. Stephanov, *Axial anomaly and magnetism of nuclear and quark matter*, *Phys. Rev. D* **77** (2008) 014021 [arXiv:0710.1084] [INSPIRE].

[11] R. Gatto and M. Ruggieri, *Hot Quark Matter with an Axial Chemical Potential*, *Phys. Rev. D* **85** (2012) 054013 [arXiv:1110.4904] [INSPIRE].

[12] D. Capasso, V.P. Nair and J. Tekel, *The Isospin Asymmetry in Anomalous Fluid Dynamics*, *Phys. Rev. D* **88** (2013) 085025 [arXiv:1307.7610] [INSPIRE].

[13] K. Fukushima and Y. Hidaka, *Resummation for the Field-theoretical Derivation of the Negative Magnetoresistance*, *JHEP* **04** (2020) 162 [arXiv:1906.02683] [INSPIRE].

[14] K. Fukushima and A. Okutsu, *Electric conductivity with the magnetic field and the chiral anomaly in a holographic QCD model*, *Phys. Rev. D* **105** (2022) 054016 [arXiv:2106.07968] [INSPIRE].

[15] G.W. Evans and A. Schmitt, *Chiral anomaly induces superconducting baryon crystal*, *JHEP* **09** (2022) 192 [arXiv:2206.01227] [INSPIRE].

[16] D.E. Kharzeev and D.T. Son, *Testing the chiral magnetic and chiral vortical effects in heavy ion collisions*, *Phys. Rev. Lett.* **106** (2011) 062301 [arXiv:1010.0038] [INSPIRE].

[17] M. Hongo, Y. Hirono and T. Hirano, *Anomalous-hydrodynamic analysis of charge-dependent elliptic flow in heavy-ion collisions*, *Phys. Lett. B* **775** (2017) 266 [arXiv:1309.2823] [INSPIRE].

[18] J. Liao, *Anomalous transport effects and possible environmental symmetry "violation" in heavy-ion collisions*, *Pramana* **84** (2015) 901 [arXiv:1401.2500] [INSPIRE].

[19] D.E. Kharzeev, J. Liao, S.A. Voloshin and G. Wang, *Chiral magnetic and vortical effects in high-energy nuclear collisions — A status report*, *Prog. Part. Nucl. Phys.* **88** (2016) 1 [arXiv:1511.04050] [INSPIRE].

[20] X.-G. Huang, *Electromagnetic fields and anomalous transports in heavy-ion collisions — A pedagogical review*, *Rept. Prog. Phys.* **79** (2016) 076302 [arXiv:1509.04073] [INSPIRE].

[21] W. Li and G. Wang, *Chiral Magnetic Effects in Nuclear Collisions*, *Ann. Rev. Nucl. Part. Sci.* **70** (2020) 293 [arXiv:2002.10397] [INSPIRE].

[22] D.E. Kharzeev, J. Liao and S. Shi, *Implications of the isobar-run results for the chiral magnetic effect in heavy-ion collisions*, *Phys. Rev. C* **106** (2022) L051903 [arXiv:2205.00120] [INSPIRE].

[23] S. Grieninger and S. Morales-Tejera, *Real-time dynamics of axial charge and chiral magnetic current in a non-Abelian expanding plasma*, *Phys. Rev. D* **108** (2023) 126010 [arXiv:2308.14829] [INSPIRE].

[24] D. Kharzeev, A. Krasnitz and R. Venugopalan, *Anomalous chirality fluctuations in the initial stage of heavy ion collisions and parity odd bubbles*, *Phys. Lett. B* **545** (2002) 298 [hep-ph/0109253] [INSPIRE].

[25] D.E. Kharzeev, L.D. McLerran and H.J. Warringa, *The Effects of topological charge change in heavy ion collisions: 'Event by event $\mathscr{P}$ and $\mathscr{CP}$ violation'*, *Nucl. Phys. A* **803** (2008) 227 [arXiv:0711.0950] [INSPIRE].

[26] CMS collaboration, *Search for the chiral magnetic effect at the LHC with the CMS experiment*, *Nucl. Phys. A* **982** (2019) 527 [INSPIRE].

[27] ALICE collaboration, *Constraining the Chiral Magnetic Effect with charge-dependent azimuthal correlations in Pb-Pb collisions at $\sqrt{s_{\rm NN}} = 2.76$ and 5.02 TeV*, *JHEP* **09** (2020) 160 [arXiv:2005.14640] [INSPIRE].

[28] STAR collaboration, *Search for the chiral magnetic effect with isobar collisions at $\sqrt{s_{NN}} = 200$ GeV by the STAR Collaboration at the BNL Relativistic Heavy Ion Collider*, *Phys. Rev. C* **105** (2022) 014901 [arXiv:2109.00131] [INSPIRE].

[29] ALICE collaboration, *Search for the Chiral Magnetic Effect with charge-dependent azimuthal correlations in Xe-Xe collisions at $\sqrt{s_{\rm NN}} = 5.44$ TeV*, arXiv:2210.15383 [INSPIRE].

[30] STAR collaboration, *Upper Limit on the Chiral Magnetic Effect in Isobar Collisions at the Relativistic Heavy-Ion Collider*, arXiv:2308.16846 [INSPIRE].

[31] J. Rafelski, *Discovery of Quark-Gluon-Plasma: Strangeness Diaries*, *Eur. Phys. J. ST* **229** (2020) 1 [arXiv:1911.00831] [INSPIRE].

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[32] M. Greif, J.A. Fotakis, G.S. Denicol and C. Greiner, *Diffusion of conserved charges in relativistic heavy ion collisions*, Phys. Rev. Lett. **120** (2018) 242301 [arXiv:1711.08680] [INSPIRE].

[33] J.L. Mañes, E. Megías, M. Valle and M.Á. Vázquez-Mozo, *Anomalous Currents and Constitutive Relations of a Chiral Hadronic Superfluid*, JHEP **12** (2019) 018 [arXiv:1910.04013] [INSPIRE].

[34] E.-D. Guo and S. Lin, *Quark mass effect on axial charge dynamics*, Phys. Rev. D **93** (2016) 105001 [arXiv:1602.03952] [INSPIRE].

[35] S. Shi, Y. Jiang, E. Lilleskov and J. Liao, *Anomalous Chiral Transport in Heavy Ion Collisions from Anomalous-Viscous Fluid Dynamics*, Annals Phys. **394** (2018) 50 [arXiv:1711.02496] [INSPIRE].

[36] J.L. Mañes, E. Megías, M. Valle and M.Á. Vázquez-Mozo, *Non-Abelian Anomalous (Super)Fluids in Thermal Equilibrium from Differential Geometry*, JHEP **11** (2018) 076 [arXiv:1806.07647] [INSPIRE].

[37] M. Nakahara, *Geometry, Topology and Physics*, second edition, Taylor & Francis (2003).

[38] K. Jensen, R. Loganayagam and A. Yarom, *Anomaly inflow and thermal equilibrium*, JHEP **05** (2014) 134 [arXiv:1310.7024] [INSPIRE].

[39] J.L. Mañes, M. Valle and M.Á. Vázquez-Mozo, *Chiral torsional effects in anomalous fluids in thermal equilibrium*, JHEP **05** (2021) 209 [arXiv:2012.08449] [INSPIRE].

[40] J. Mañes, R. Stora and B. Zumino, *Algebraic Study of Chiral Anomalies*, Commun. Math. Phys. **102** (1985) 157 [INSPIRE].

[41] L. Álvarez-Gaumé and M.Á. Vázquez-Mozo, *An invitation to quantum field theory*, Springer (2012) [DOI:10.1007/978-3-642-23728-7] [INSPIRE].