Accelerating Finite-Difference Time-Domain (FDTD) Solvers using Voxels-in-Cell Method

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***Abstract*—The Voxels-in-Cell (VIC) method was recently introduced for reducing the computational cost of the finite- difference time-domain (FDTD) method with objects composed with voxels. In this paper, we present some applications of VIC method in electromagnetic problems and show that the accuracy of the method is preserved while large reductions of the computational requirements can be achieved.**

I. Introduction

In recent years, considerable research has been conducted into overcoming the computational burden of the FDTD method. Its computational efficiency is boosted by hardware-based approaches, involving the parallelization of the FDTD algorithm by using multi-core Central Processing Units (CPUs) in distributed or shared memory architectures, or Graphical Processing Units (GPUs). Although this approach enables users to perform the computation of problems at high speed without sacrificing accuracy, it requires additional hardware costs and large clusters or supercomputers to perform the computations. Software-based approaches can alleviate these requirements, by using numerical techniques. Numerous subcell, subgridding and hybrid models have been introduced for a variety of applications [1]. In general, these models tackled very specific problems and materials, mostly good conductive materials. In this paper, we present some applications of VIC method, that was recently introduced for dielectric [2] and dispersive media [3], to speed the FDTD computations while preserving the accuracy.

II. VIC Method

The theoretical details of VIC method are given in [2]. We assume that a non-homogenous cell composed of $M_i M_j M_k$ voxels, each with its own medium and indexes (i_v, j_v, k_v), where $i_v \in (1, \underline{M_i})$, $j_v \in (1, \underline{M_j})$, $k_v \in (1, \underline{M_k})$.

A. Lossless Media

The component E_z can be updated as in [2],

$$E_z^{n+1}(i_g, j_g, k_g) = E_z^n(i_g, j_g, k_g) + \frac{\Delta t}{\varepsilon^{**}(i_g, j_g, k_g)} \left[\frac{H_{y+}^{n+\frac{1}{2}} - H_{y-}^{n+\frac{1}{2}}}{\Delta x} - \frac{H_{x+}^{n+\frac{1}{2}} - H_{x-}^{n+\frac{1}{2}}}{\Delta y} \right] \tag{1}$$

where

$$\varepsilon^{**}(i_g, j_g, k_g) = \frac{M_k}{M_i M_j} \left[\sum_{k_v=1}^{M_k} \frac{1}{\sum_{i_v=1}^{M_i} \sum_{j_v=1}^{M_j} \varepsilon(i_v, j_v, k_v)} \right]^{-1}.$$

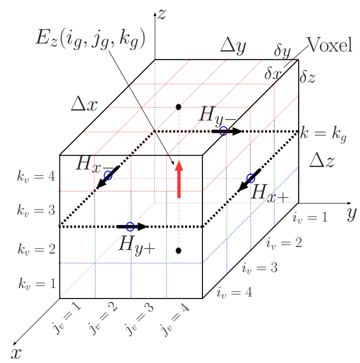

Fig. 1. The FDTD cell composed with voxels of size $\delta x \delta y \delta z$ filled with different media. The component that needs to be updated is E_z, located at the center of a volume with dimensions $\Delta x, \Delta y, \Delta z$. This volume is essentially a cell that has been shifted by $\Delta x/2, \Delta y/2, \Delta z/2$ with respect to the Yee cell typically used in the FDTD simulations. In the plane $k = k_g$ that holds the E_z node, there are nodes H_{y-} and H_{y+} situated $\Delta x/2$ from E_z and nodes H_{x-} and H_{x+} situated $\Delta y/2$ from E_z.

B. Lossy Media to Model Composite Materials

In a homogenous medium, the component E_z can be updated by discretizing the integral form of the Maxwell-Ampere equation with conduction current

$$\iint_{\Delta x \Delta y} \left(\varepsilon \frac{\partial E_z(t)}{\partial t} + \sigma E_z(t) \right) dx dy = C_h(t) \tag{2}$$

where σ is the conductivity, $\Delta x \Delta y$ is the surface perpendicular to E_z at $k = k_g$ and C_h is the integral

$$C_h(t) = \int_L H(t) dl \tag{3}$$

where L is the contour path that surrounds surface $\Delta x \Delta y$ and holds the four H nodes. In the non-homogenous case in Fig.1, we assumed that C_h does not depend on z over the cell, that is C_h is a constant from $z = k_g - \Delta z/2$ to $z = k_g + \Delta z/2$ and E_z is a component tangential to the interfaces between the different media in the slice so that it is continuous through these interfaces and can be considered as uniform in the slice of surface $\Delta x \Delta y$, as in a usual FDTD cell. Therefore, the integral of (2) that yields

$$\sum_{i_v=1}^{M_i} \sum_{j_v=1}^{M_j} \varepsilon(i_v, j_v, k_v) \frac{\partial E_z(k_v, t)}{\partial t} \delta x \delta y + \sum_{i_v=1}^{M_i} \sum_{j_v=1}^{M_j} \sigma(i_v, j_v, k_v) E_z(k_v, t) \delta x \delta y = C_h(t). \tag{4}$$

Discretization of (4) provides us

$$E_z^{n+1}(k_v) = a(k_v)E_z^n(k_v) + \frac{\Delta t}{b(k_v)} \left[\frac{H_{y+}^{n+\frac{1}{2}} - H_{y-}^{n+\frac{1}{2}}}{\Delta x} - \frac{H_{x+}^{n+\frac{1}{2}} - H_{x-}^{n+\frac{1}{2}}}{\Delta y} \right] \quad (5)$$

where

$$a(k_v) = \frac{2 - \Delta t \frac{\sigma^*(k_v)}{\varepsilon^*(k_v)}}{2 + \Delta t \frac{\sigma^*(k_v)}{\varepsilon^*(k_v)}}, \qquad b(k_v) = \frac{\sigma^*(k_v)\frac{\Delta t}{2} + \varepsilon^*(k_v)}{M_i M_j}$$

$$\varepsilon^*(k_v) = \sum_{i_v=1}^{M_i} \sum_{j_v=1}^{M_j} \varepsilon(i_v, j_v, k_v), \sigma^*(k_v) = \sum_{i_v=1}^{M_i} \sum_{j_v=1}^{M_j} \sigma(i_v, j_v, k_v)$$

After updating $E_z(k_v)$ and its x and y counterparts in all the slices of all the FDTD cells, the H components should be updated to complete the FDTD iteration. To update H at the four neighboring nodes represented in Fig. 1, we apply the Maxwell-Faraday law in integral form, which requires integrating of E_z over the segment Δz that contains the $E_z(i_g, j_g, k_g)$ node. In non-homogeneous cells, E_z depends on the slice. However, since the integral is nothing but the average of E_z over Δz multiplied by Δz, and since the permeability is homogeneous in the cells, the standard FDTD algorithm for updating the H components can be left unchanged. We just need to place the average value $< E_z^{n+1} >$ at each original $E_z(i_g, j_g, k_g)$ node of the FDTD cells, that is

$$E_z^{n+1}(i_g, j_g, k_g) = \frac{1}{M_k} \sum_{k_v=1}^{M_k} E_z^{n+1}(k_v). \quad (6)$$

III. Numerical Examples

We present the VIC results for some test cases and compare with fine mesh and AVG results. The AVG method incorporates average parameters, calculated as $x(i_g, j_g, k_g) = \sum_{i_v=1}^{M_i} \sum_{j_v=1}^{M_j} \sum_{k_v=1}^{M_k} x(i_v, j_v, k_v)/(M_i M_j M_k)$ where x is either ε_r or σ.

A. Overlapping Composite Materials

The object in Fig.2 (left side) was placed in vacuum. The FDTD steps were Δx=Δy=Δz=1 mm and Δt=1.83 ps for the fine mesh and Δx=Δy=Δz=2 mm and Δt=3.66 ps for the VIC mesh.

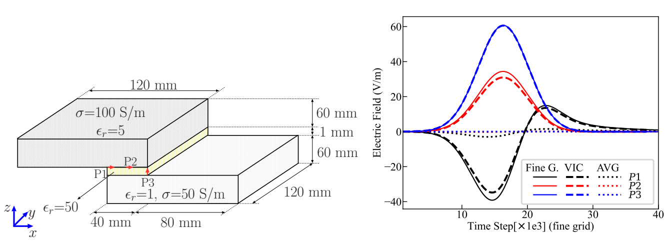

Fig. 2. The results for the overlapping materials. The incident plane wave propagating in y direction and polarized in z direction, with waveform $E_z(t) = 100\exp[-((t - 3\tau)/\tau)^2]$ V/m where τ=10 ns.

B. Lossy Enclosures

We report results for an experiment with the a $60 \times 60 \times 60$-VIC-cell enclosure with walls of thickness 1 mm made with medium (ε_r=50, σ=4.5 S/m).

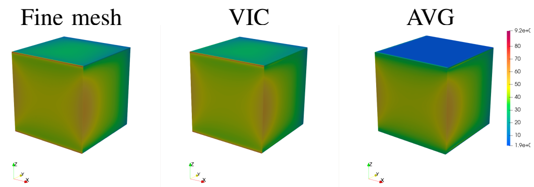

Fig. 3. The observation of peak E_z field. The computational settings are the same as in the previous experiment apart from τ set to 134.5 ps.

C. Bioelectromagnetics

The VIC method [3] was applied to the voxel-based human phantom models [4].

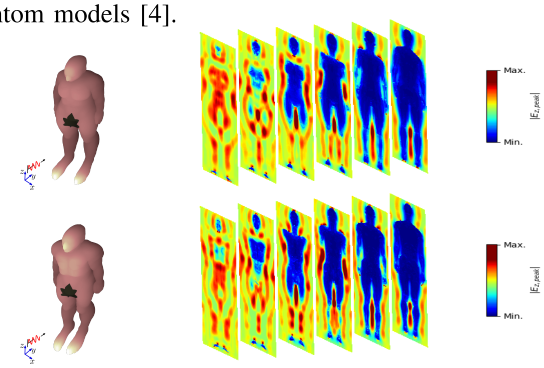

Fig. 4. The observation of peak E_z field in the female phantom (top) and the male phantom (bottom). We introduced the plane wave with sinusoidal time dependence propagating along the y-axis and with the electric field polarized along the z-axis. The frequency of the excitation is 2.4 GHz. The voxel size is 1 mm^3 and VIC cell size is (2 mm)^3. The computational requirements with the serial code are 35 min. CPU time and 23 GB memory for the female phantom, and 46 min. CPU time and 30 GB memory for the male phantom.

IV. Conclusion

With the VIC cell two times larger than the voxels, i.e. with eight voxels in each VIC cell, the overall number of cells is reduced with a factor of eight, and the stability condition is relaxed with a factor of two, resulting in large reduction of memory and CPU time requirements, at least an order of magnitude for the CPU time.

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