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Optical characterization of heliostat facets based on Computational Optimization

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Abstract

In solar power tower plants, knowing the optical quality of heliostats makes it possible to predict relevant information on the receiver surface, such as the irradiance concentration factor and spillage. However, there are no standardized routines for optical characterization in commercial facilities because the process is challenging, multidisciplinary, and time-demanding. This article revises the traditional optical characterization methodology followed at the Solar Platform of Almería (PSA). The process starts with the acquisition of the image of the studied optical system. After that, the picture must be fitted to an analytical model, which requires finding the variables that best reproduce the reality. The traditional method for accomplishing this task is iterative, semi-automatic, and contains trial-and-error components. This work studies how to replace this part with heuristic optimizers and considers using the state-of-the-art methods TLBO, UEGO, and Multi-Start Interior-Point (MSIP). Their effectiveness has been compared to the results manually achieved by an expert with three different heliostat facets. According to the results obtained, the parameter sets found by TLBO and UEGO outperform those obtained through the traditional method.

Keywords: Solar Power Tower Plants, Optical Characterization, Computational Optimization, Numerical Methods

1. Introduction

The increase in the world population and its demands require replacing the traditional and polluting energy sources with renewable and clean ones (Gallego and Camacho, 2018; Sah et al., 2020). Since solar energy the most abundant option (Gallego and Camacho, 2018), there is great interest in its exploitation (Kabir et al., 2018; Reddy et al., 2013). Among the existing technologies for

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7 this purpose (Kabir et al., 2018; Reddy et al., 2013), concentrated solar power
8 systems are promising because of their compatibility with hybridization, large-
9 scale production, and storage capabilities (Reddy et al., 2013; Saghafifar et al.,
10 2019). Solar Power Tower Plants (SPTP) stand out from this kind of system
11 because of their higher efficiency and lower Levelized Cost of Electricity (LCOE)
12 (de la Calle et al., 2020).

13 A typical SPTP consists of multiple sun-tracking mirrors, known as he-
14 liostats, which dynamically concentrate solar radiation on a receiver. The latter
15 transfers the energy to a heat-transfer fluid used to generate electricity in a
16 power cycle (Cruz et al., 2020; Reddy et al., 2013). Despite their conceptual
17 simplicity, heliostats are not simple mirrors. They are complex systems con-
18 sisting of multiple facets and have an impact on the performance of facilities
19 (Fernández-Reche, 2006). For this reason, it is convenient to know the optical
20 quality of heliostats, which is known as their Optical Characterization (OC in
21 what follows). This information, coupled with image simulation models (Cruz
22 et al., 2017), makes it possible to predict valuable data on the receiver. Some
23 examples are the irradiance concentration factor, the image orientation and elon-
24 gation, the image radius containing a certain percentage of the incoming power,
25 and spillage (García et al., 2015). It is also valuable to detect malfunctions in
26 heliostats that could reduce the throughput of the plant.

27 The process of OC consists in determining the set of parameters that best
28 describe the behavior of the system, e.g., a complete heliostat or one of its
29 facets. Thus, it is necessary to compare the real image of the characterized
30 system to different synthetic replicas generated with an optical model and the
31 appropriate parameters, e.g., the focal length for concave facets. The parameters
32 that produce the most realistic synthetic image will define the OC. However, this
33 procedure is time-demanding and challenging because it requires knowledge of
34 image acquisition and analysis methods. For these reasons, there are no periodic
35 OC programs in commercial SPTP, and this kind of task is generally carried
36 out in heliostat prototypes at applied research centers, such as the *Plataforma*
37 *Solar de Almería* (Spanish for Solar Platform of Almería) (PSA) in Spain.

38 For a commercial plant, it would be advisable to perform an OC campaign
39 at several critical moments. The first one would be at the commissioning of the
40 plant, i.e., before starting its regular operation, to confirm that it fulfills the
41 specifications. Secondly, the solstices would be interesting because it is when
42 the sun is at extreme positions. Thirdly, at the design point of the plant, which
43 is likely to be one of the equinoxes. Fourthly, if the plant location has extreme
44 temperatures, it would be advisable to test the optical quality with the highest
45 and lowest temperatures to check their impact on the field. These tests would
46 let us know the plant in detail. After that, the aging of the heliostats and their
47 materials would determine further necessities, e.g., every two years, to ensure
48 that the field is capable enough.

49 Despite not being an actively studied subject, there are several techniques
50 in the literature for assessing the optical quality. In the 1980s, the intensity
51 distribution reflected by heliostats was estimated by convolving several distri-
52 butions coming from independent error sources. The fundamental ones were the

53 sun shape, mirror waviness, and astigmatism. In this context, the error analysis
54 was performed in the Fourier space (Kiera, 1985). In the 1990s, Sandia Na-
55 tional Laboratories carried out different optical characterization campaigns of
56 heliostats (Jones et al., 1995; Strachan and Houser, 1993). They used the BCS
57 measurement system (Strachan, 1993) for real image acquisition and the HE-
58 LIOS simulation software (Biggs and Vittitoe, 1979) for estimating the images
59 produced under different parametric configurations, but the parameters to try
60 were manually supervised by a human expert. Likewise, at the PSA, heliostat
61 characterization has been performed since the 1990s to the present (Cordes,
62 1994; Monterreal, 1997) with the ProHERMES measurement system and the
63 Fiat_Lux (Monterreal, 1999) heliostat image simulator. However, all these tech-
64 niques share the handicap of heavily relying on specialized staff and considerable
65 dedication to direct the parametric search and obtain reliable results.

66 In general, optical quality assessment techniques can be divided into two
67 main classes: Indirect and Direct methods. The former studies the image of
68 the sun reflected by the surface of the heliostat with an image analysis target
69 and a measurement system. The most relevant parameters concerning the op-
70 tical properties of the concentrator are then computed from the experimental
71 data with different strategies. For instance, after modeling the heliostat in a
72 proper simulator (King, 1982; Strachan and Houser, 1993), the model can be
73 used along with the data to unfold an error distribution characterizing the non-
74 deterministic slope errors of the heliostat. It is done by iterating through the
75 dispersion (standard deviation) of a circular normal distribution in the simulator
76 until the shape of the measured and predicted beams match (King, 1982; Stra-
77 chan and Houser, 1993). It is also possible to compute the radiation intercepted
78 by the receiver of a real concentrator by the convolution of the angular accep-
79 tance function, the optical error distribution, and the angular brightness of the
80 radiation source. The parametrization of the theoretical distribution function
81 makes it possible to iteratively seek the model parameters that best replicate
82 the real image (Avellaner, 1980; Bendt and Rabl, 1981; Kiera and Schiel, 1989).
83 Regarding Direct methods, they study either the heliostat surface or an image
84 reflected on it, but from an artificial light source instead of the sun. It groups
85 the most recent research lines in optical characterization (Kammel, 2003; Mon-
86 terreal et al., 2017; Pottler et al., 2005).

87 The methodology currently followed at the PSA falls into the group of In-
88 direct methods. The source of light reflected on the target directly comes from
89 the sun. It is taken with a high-resolution digital camera at the characterization
90 instant. The theoretical parametric model of the optical quality of the heliostat
91 combines both variables of statistical nature, such as the slope error, and de-
92 terministic ones, such as the focal distance. King (1982); Strachan and Houser
93 (1993) used the HELIOS software package, while Fiat_Lux is the choice at the
94 PSA (Cruz et al., 2017). For its simulations, HELIOS follows a cone optics
95 approach in which the flux density results from combining the error cone of the
96 reflected rays and the sun shape by convolving independent distributions with
97 the Fourier transform (Garcia et al., 2008). For its part, Fiat_Lux simulates
98 the flux distributions produced by mirrors using normally-distributed random

99 values of slope error and computing the trajectories for the bundle of reflected
100 rays coming from the solar disk according to geometric optics laws. The latter
101 is done for each unitary normal vector to the reflective surface (Garcia et al.,
102 2008; Monterreal, 1999). Fiat Lux also stands out due to its capability of taking
103 as input a real image of the sun experimentally acquired, while other tools rely
104 on synthetic images of the sun.

105 Regardless, the OC methodology is still semi-automatic due to the required
106 parametric search. It relies on an expert to guide the definition of parameter
107 sets to compare to real images (Monterreal, 1997). Although this process has
108 been successfully applied for years, it is significantly time-demanding, and the
109 explored parameter sets are inherently biased by the expert’s vision. This work
110 aims to update this method by approaching OC as an optimization problem
111 (Nocedal and Wright, 2006; Törn and Žilinskas, 1989) focused on finding the set
112 of parameters that results in the most realistic synthetic image. Therefore, this
113 work continues the research line of modernization recently opened by Monterreal
114 et al. (2022) and proposes applying computational optimization methods to
115 replace the manual part of the OC strategy of the PSA.

116 The use of global optimization algorithms is expected to improve the qual-
117 ity of solutions and simplify their obtaining. Since the OC workflow involves
118 applying non-linear models with different accuracy degrees and stochasticity,
119 the optimizers considered are heuristics (Salhi, 2017). This sort of optimiza-
120 tion algorithm relies on intuitive ideas and randomness rather than on rigorous
121 certainty. They are known to provide a trade-off between computational effort
122 and the quality of solutions. Namely, the optimizers studied in this work as a
123 replacement of the manual work of experts are the following: i) TLBO (Rao
124 et al., 2012), ii) UEGO (Jelasity et al., 2001), and iii) Multi-start Interior-Point
125 (MSIP) (Griva et al., 2009; Salhi, 2017). Their results have been compared to
126 those obtained through the traditional method of the PSA with three different
127 heliostat facets.

128 The remainder of the paper has the following structure: Section 2 describes
129 the materials and methods considered in this work, including the experimental
130 setup and the OC strategies. After that, Section 3 shows the results obtained
131 with each end method and the three facets considered. Finally, Section 4 con-
132 tains the conclusions and states the future work.

133 **2. Materials and methods**

134 As introduced, OC is a multidisciplinary process covering several fields, from
135 image acquisition to optical simulation and parameter fitting. These tasks rely
136 on the use of specific hardware and software. This section describes the envi-
137 ronment and equipment for OC at the PSA, both for the traditional approach
138 and the new one based on Computational Optimization.

139 *2.1. Image acquisition and simulation tools*

140 The traditional OC methodology followed at the PSA has been applied with
141 heliostat prototypes and facets since the nineties. It obtains the information

142 about the studied optical system by comparing the sun image that it reflects
143 on a target to the synthetic one generated by the appropriate models (Cruz
144 et al., 2017). The revision of the method proposed in this work also inherits
145 this background. Therefore, image acquisition is highly relevant for OC.

146 Roughly speaking, the images are obtained by making the optical system
147 reflect the sun image onto a target in the tower of the CESA-I heliostat field
148 (Cruz et al., 2018; Gallego and Camacho, 2018). Figure 1 provides a general
149 overview of the referred facility and the elements involved in this work, including
150 the three types of facets to be characterized. More precisely, the whole system
151 is known as ProHERMES 2A (Monterreal and Neumann, 1994), and it consists
152 of the following hardware and software components:

- 153 • FUJINON LENS. Model: H14X10.5A-R11. Zoom range: 10,5-147 mm.
- 154 • CCD camera: Hamamatsu Photonics. Model: ORCA-II C-4742-98 series.
- 155 • Filters: Omega Optical. Type: Neutral Density. Flat response range
156 [400-700] nm.
- 157 • Image acquisition and processing software: ImagePro Plus 4.1 by Media
158 Cybernetics ®.
- 159 • White Lambertian target coating: Amercoat 741. Area: 12 m × 12 m.
- 160 • Meteorological station.

161 The simulation of the optical system under study is carried out with the
162 Fiat_Lux software package (Cruz et al., 2017; Monterreal, 1999), which runs in
163 the MATLAB environment (Higham and Higham, 2016). Once configured to
164 use the parameterized model appropriate to the type of device, e.g., that of
165 spherical facets when so is the studied system, Fiat_Lux allows the creation of
166 synthetic images to compare them with reality.

167 As introduced, the chosen facets belong to heliostats in the CESA-I field of
168 the PSA. The mirrors have a spherical surface and have been manufactured by
169 the Guardian company using a second-surface glass of 4 mm width with low con-
170 tents of iron. Their size is approximately 3000 mm x 1000 mm, as detailed in the
171 next section, and they are stuck to a steel stretcher with moorings provided with
172 silicone. The reason for choosing facets with different focal lengths as their main
173 differentiating factor is to check if the curvature degree could affect the quality
174 of the results. Thus, three ranges of focal length were selected, i.e., medium,
175 long, and very long. Figure 2 shows the three facets mentioned. It includes
176 their physical appearance, their real images acquired, their synthetic equiva-
177 lents generated by Fiat_Lux according to the technical specifications provided
178 by their manufacturer, and their comparison. Notice the difference between the
179 iso-intensity lines of the real and the synthetic images. It supports the fact that
180 the analyzed facets significantly deviate from their specifications.



Figure 1: Image acquisition setup at the Plataforma Solar de Almería. From left to right: Image analysis target (where the real projected shapes are measured), Medium-short and Medium-long distance mirror (where the F62 and F42 facets are mounted, respectively), Image measuring system (image acquisition equipment), and Long distance mirror (where the LDM facet is mounted).

Table 1: Nominal specifications of the facets under characterization.

Facet	Surface	Dimensions (W x H) (mm)	Focal length (m)	X (m)	Y (m)	Z (m)
H0803_F62	Spherical	2995 x 1102	175	-15.144	135.200	0.893
H1100_F42	Spherical	2995 x 1102	220	-0.158	192.200	1.570
PSA_LDM	Spherical	2800 x 1605	420	-19.000	440.000	5.000

181 *2.2. Optical model of the facets*

182 As previously mentioned, the optical simulation tool needs to be configured
 183 to use the geometric and optical error model that correspond to the facets
 184 under characterization. The geometric model is fixed by the manufacturer based
 185 on the following factors: i) type of surface, ii) curvature or focal length (F),
 186 and iii) dimensions (wide (W) and height (H)). Table 1 contains the nominal
 187 specifications of the three facets considered in the present study. It also includes
 188 the East (X), North (Y) and Zenith (Z) coordinates of the heliostat in which
 189 they are mounted, measured from the tower base of the CESA-I field.

190 Regarding the optical error, ε , it is modeled as the standard deviation, σ ,
 191 of the Gaussian probability distribution describing the divergence of the real
 192 normal vector of the reflecting surface from the expected one. Figure 3 depicts
 193 the facet geometry and the slope error model. While the focal length is a
 194 geometric and deterministic variable, the slope error is an angular and statistical
 195 one. It is hence necessary to take into account their different nature throughout
 196 the process of OC.

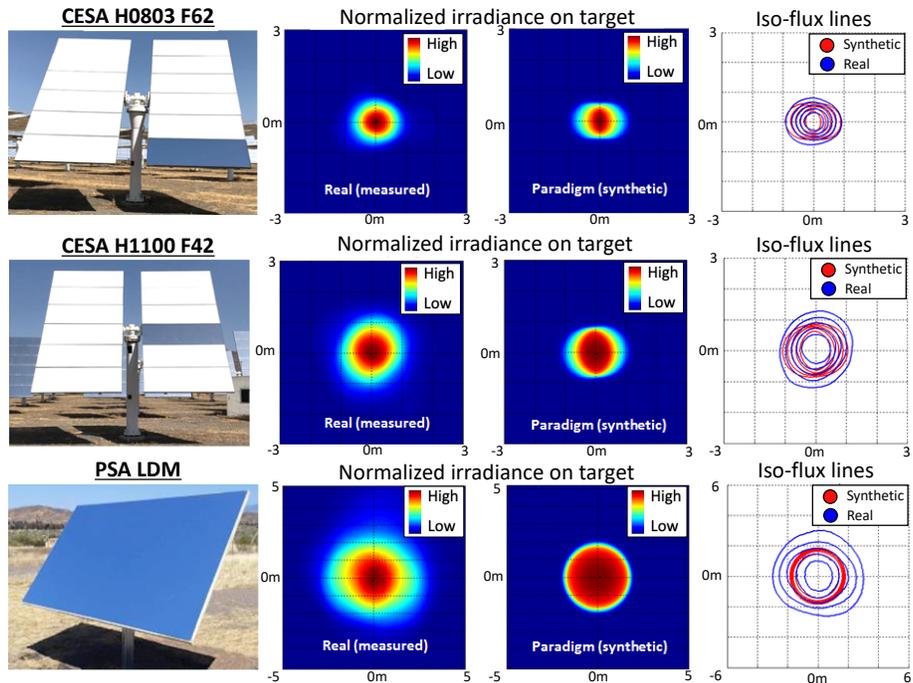


Figure 2: Real facets (left column), acquired images from measurement (central-left column), predicted images according to the manufacturers specifications (central-right column), and comparison of their iso-flux lines (right column).

197 *2.3. Image processing and comparison*

198 Both the traditional OC methodology and the proposed alternatives rely on
 199 comparing the real and synthetic parameter-based images. Thus, the images
 200 must be processed to make meaningful comparisons. It is also necessary to
 201 define metrics that allow assessing the similitude between real samples and
 202 synthetic proposals. All the characterization strategies share the framework
 203 defined for this purpose. It consists of the following steps described below:
 204 image normalization, segmentation and comparison.

205 *2.3.1. Image normalization*

206 The real and synthetic images come from different sources, i.e., ProHermes
 207 2A and Fiat_Lux, respectively. For this reason, their gray levels (gl) have arbi-
 208 trary references and cannot be directly compared. To allow their comparison,
 209 the real image and the synthetic one, which are required to be defined by mat-
 210 trices of the same dimension, must be modified as follows:

$$gl_{i,j}^{Norm} = \frac{gl_{i,j}}{\sum_{i,j} gl_{i,j}} \quad (1)$$

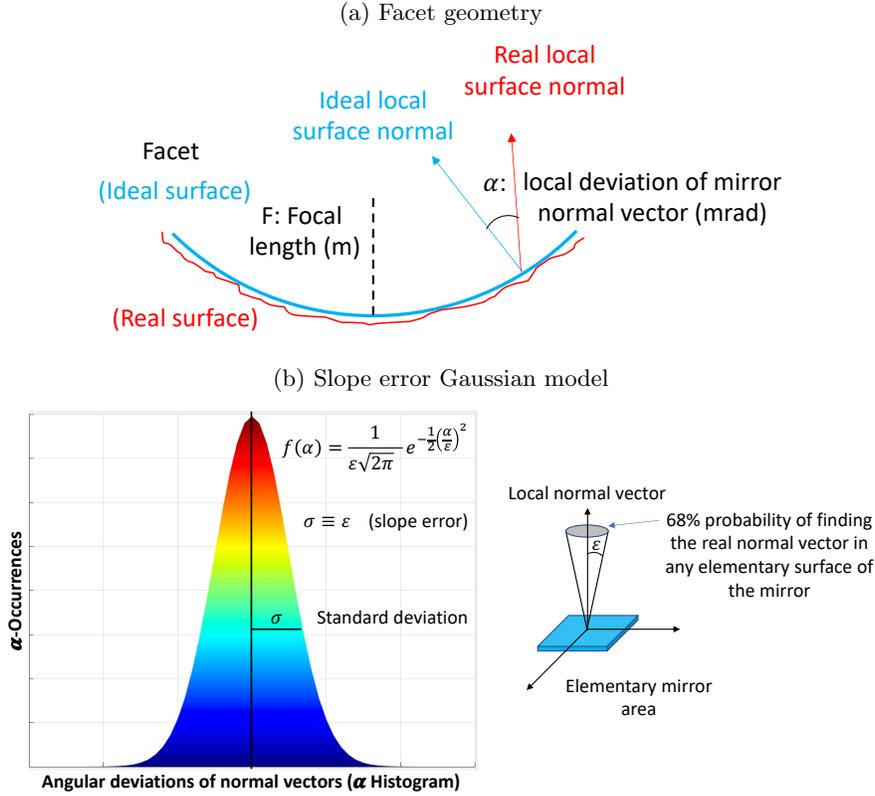


Figure 3: Facet model for characterization: a) Definition of the nominal geometry of the facet and the focal length including mirror imperfections. The latter are represented by the deviation of the normal vector from its surface at any point, also called mirror slope error. b) Population of mirror slope errors statistically described by a Gaussian distribution. Its standard deviation represents the so-called facet optical error.

211 where $gl_{i,j}$ refers to the gray level at pixel i,j of the image being processed,
 212 and $gl_{i,j}^{Norm}$ is the corresponding pixel after normalization. Thus, the resulting
 213 image meets the following condition:

$$\sum_{i,j} gl_{i,j}^{Norm} = 1 \quad (2)$$

214 In what follows, GL will designate the gray levels of the pixels of the real
 215 image (also known as the benchmark). Analogously, gl will represent those
 216 of any synthetic image, i.e., predicted by the model. This notation allows for
 217 distinguishing both types of images.

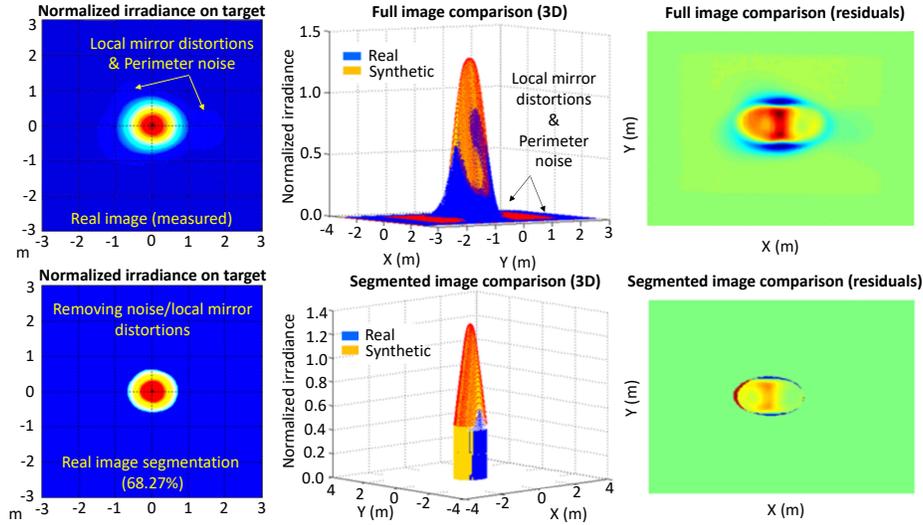


Figure 4: Understanding the comparison of sun-reflected images on target. Top-left: Occasional undesirable effects on the real image due to local distortions of the mirror and residual background noise on target. Top-central: 3D view of both real and synthetic images. Top-right: Deviation of the real image from the synthetic one by residual calculation. Down-left: removing undesirable effects on the real image. Down-central: 3D view of the real and synthetic images, free of undesirable effects. Down-right: Deviation between the real and synthetic images by means of residual calculation after removing undesirable effects.

218 *2.3.2. Image segmentation*

219 Real images can contain undesirable effects. For the scope of this work, the
 220 most relevant ones are the haloes caused by local mirror dislocations (or dis-
 221 tortions) and the perimeter noise on the surface of the image analysis target.
 222 Mirror dislocations do not necessarily occur. They are very localized effects
 223 caused by the mechanical tension at the glass clamping devices that fix it to
 224 the steel frame of the facet. Their spurious nature makes them unsupported by
 225 Fiat_Lux and probably by all current simulators. The involved reflective surface
 226 is small, and so are their effects on the reflected image. These deformations are
 227 of the same order of magnitude as the noise produced by the target. They will
 228 generally appear near the image borders and feature low intensity. Regarding
 229 the perimeter noise, it is caused by the instrumentation (readout noise) and
 230 aspects such as the irregular surface of the Lambertian target. It consumes cal-
 231 culation time on irrelevant pixels and could misguide comparisons. Fortunately,
 232 the image segmentation process eliminates both. It considers only the part of
 233 the image that is relevant in the comparison for the OC process. Figure 4 shows
 234 the mentioned effects and how segmentation affects the image.

235 In terms of implementation, segmentation modifies the pixels of an input

236 image GL^{Norm} indexed by subscripts l, m as follows:

$$\text{Segmented } GL = \begin{cases} GL_{l,m}^{Norm} = GL_{l,m}^{Norm} & \text{if } GL_{l,m}^{Norm} \geq GL_{Isoline} \\ GL_{l,m}^{Norm} = 0 & \text{Otherwise} \end{cases} \quad (3)$$

237 where $GL_{Isoline}$ is the threshold gray level required so that $\sum_{l,m} GL_{l,m}^{Norm} = P$,
 238 i.e., the gray level from what the summation of pixels with values equal or
 239 above is equal to the given P. Accordingly, only the pixels whose value is equal
 240 or greater than $GL_{Isoline}$ remain unaltered, while the others are set to 0. The
 241 isoline is generally chosen so that $P = 0.6827$ (68.27%). Thus, the summation
 242 of the pixels selected (taken or kept) will represent 68.27% of the intensity
 243 (accumulated gray level). The complete summation (no segmentation) would
 244 be 1, i.e., 100%. The value of P has been empirically adjusted in relation
 245 to the probability of pixels being selected according to their gray level. It
 246 results in an adequate segmentation and avoids undesirable effects (local mirror
 247 distortion and background noise) without relevant information loss for the latter
 248 comparison.

249 2.3.3. Image comparison

250 As introduced, provided that the surface type and size are known, the spherical
 251 facets under study are characterized by their slope error, ε , and focal length,
 252 F . Therefore, every set of parameters ε, F allows generating a synthetic image
 253 with the optical model to compare to reality and becomes a possible result
 254 of OC. The characterization quality increases as the difference between reality
 255 (benchmark) and the synthetic prediction decreases. This value is computed
 256 as the Root Mean Square Error (RMSE) between the benchmark and the syn-
 257 thetic image that results from the parameter set considered. It is formulated as
 258 follows:

$$RMSE_{segm} = \sqrt{\frac{\sum_{l,m} (gl_{l,m} - GL_{l,m})^2}{N}} \quad (4)$$

259 where N is the number of pixels selected at segmentation. It is also possible
 260 to normalize the RMSE (nRMSE) by multiplying it by $100/M$, where M is the
 261 average value of the reference or benchmark image, i.e., \overline{GL} . This transforma-
 262 tion allows working with values bigger than those of the plain RMSE, which are
 263 easier to remember and plot (e.g., 45.7 in nRMSE in contrast to 0.0002597 in
 264 RMSE), without altering the meaning of the results.

265 2.4. Optical characterization methods

266 Based on the previous sections, it is possible to define OC as the search for
 267 the optical parameters that minimize the difference between their corresponding
 268 synthetic image and the real or benchmark one. Hence, in practical terms, it
 269 can be addressed as an optimization problem in which the objective function
 270 to minimize is Eq. (4), and the variables are the optical parameters defining

271 different synthetic images to compare. This section explains the alternatives
 272 considered for this process, starting from the traditional method of the PSA to
 273 the optimization algorithms studied to replace it, i.e., TLBO, UEGO, and MSIP.
 274 All of them share the operational context previously described and depicted
 275 in Figure 5. The only change is how they generate the candidate parametric
 276 combinations. The section ends with an overview of the most relevant properties
 277 of each method for their use. Thus, the reader not interested in their internals
 278 can directly go to that part.

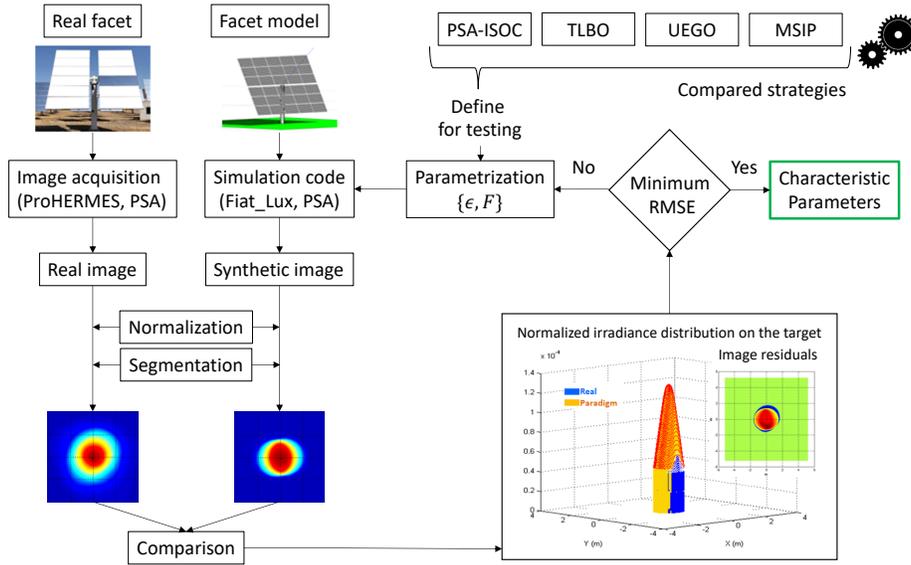


Figure 5: Common context for optical characterization. Top-left: real facet to register its image. It is next to the equivalent model to generate the synthetic image to compare after processing (bottom-left). Top-right: The OC strategies generate and study different parameter sets for the model to produce a synthetic image with the aim of finding the parameter set resulting in the minimum RMSE after comparison (bottom- right).

279 *2.4.1. PSA Iterative Search for Optical Characterization (PSA-ISOC)*

280 The methodology traditionally followed at the PSA, known as PSA Iterative
 281 Search for Optical Characterization (PSA-ISOC), has been improved through-
 282 out the years. Currently, the approach combines the experience of operators
 283 with the toolboxes and scripting capabilities provided by the MATLAB envi-
 284 ronment.

285 The characterization process starts by regularly sampling the dimensions
 286 involved, i.e., variables F and ϵ for the studied facet, to form a grid. The nominal
 287 value of F remains in the center of its range. The limits are generally defined
 288 in terms of percentages below and above the nominal value. For instance, 50%
 289 below and above 220 m results in a search space from 110 to 330 m. The step
 290 size is also a user-defined user parameter. It is defined considering that slight

291 variations might not be measurable. Thus, the initial step size might be 10 or
292 20 m. Regarding ε , if it is known to be near a particular value, the dimension
293 is defined the same. Otherwise, the range covers from 0 (no slope error) to
294 a user-defined upper bound. The step size is another variable to set, and the
295 value has to be of few milliradians in this case. The sampling technique can be
296 generalized to all the variables involved.

297 After the previous definitions, the characterization process fixes the slope
298 error to zero and explores all the sampled values for F. The method keeps a
299 record with the best choice. According to Figure 5, assessing every combina-
300 tion implies generating the corresponding synthetic image, pre-processing, and
301 comparing it to the real one to compute the RMSE. After that, F is fixed, and
302 the exploration is repeated while focusing on the slope error this time. Finding
303 the most appropriate slope error after having set the focal length ends the first
304 iteration.

305 Once the first iteration ends, the user might decide to execute a second one.
306 For this purpose, the ranges are centered around the previous result. The step
307 sizes must be reduced to provide the search with more resolution. It is also
308 possible to reduce the percentages that define the upper and lower bounds to
309 reduce the search space. The user can execute as many iterations as desired
310 according to the evolution of the results. However, it is not usual to perform
311 more than three or four complete iterations. It is also relevant to highlight that
312 the user might decide to fix the focal length variables before those linked to
313 the slope errors, i.e., to invert the order of some iterations. It depends on the
314 consideration of the results achieved, and it is one of the main reasons for this
315 process to be demanding and uncertain.

316 *2.4.2. Teaching-Learning-based Optimization (TLBO)*

317 Teaching-Learning-based Optimization (TLBO) is a numerical optimization
318 algorithm proposed by Rao et al. (2012). It belongs to the group of meta-
319 heuristics (Boussaïd et al., 2013; Lindfield and Penny, 2017; Salhi, 2017), which
320 are problem-independent optimizers. They cannot guarantee optimal solutions
321 yet are known to achieve acceptable ones by relying on general principles. More
322 accurately, TLBO is a population-based meta-heuristic into the Swarm Intelli-
323 gence subgroup because it works with a population of candidate solutions and
324 simulates their social interaction. Namely, it treats each solution as a student
325 that learns from the rest and becomes a better option. This algorithm has be-
326 come very popular due to its simplicity of implementation and configuration
327 (Rao, 2016).

328 The configuration of TLBO consists of two parameters: the population size
329 and the number of iterations. The optimizer starts by randomly generating
330 as many candidate solutions as defined by the population size. Each one is a
331 vector that contains a valid value for every variable under optimization. It is
332 also necessary to compute the RMSE for all of them according to the procedure
333 depicted in Figure 5. This is the value of the cost function in optimization terms.
334 The range of each variable and the evaluation function must be provided by the
335 user as part of the context information, but this occurs with all the options

336 taken into account. In general, the range of variables should be the same as the
 337 widest ones considered with the traditional PSA-ISOC.

338 After the previous initialization stage, TLBO executes as many iterations
 339 as required. Each one consists of two consecutive steps: the teacher and the
 340 learner stages. The former simulates how students learn from their teachers and
 341 improve their skills. It tries to shift all the solutions in the population towards
 342 the best one, i.e., that with the lowest RMSE, which becomes the teacher, T .
 343 Namely, after identifying the best solution, TLBO computes a vector M in
 344 which the i component is the average of variable i in the population. Next, the
 345 optimizer applies Eq. (5) to create a modified version, S' , from every candidate
 346 solution, S . This equation aims at vectors, so i refers to the i component of
 347 candidate solutions. r_i is a random real number in the range $[0, 1]$ for each
 348 component. T_F , known as ‘teaching factor’, is a random integer affecting the
 349 potential amplitude of movement. Rao et al. (2012) empirically defined it to be
 350 either 1 or 2 when designing the method. Notice that every candidate solution
 351 at a particular stage and iteration shares the same random factors computed
 352 at run time. The modified solutions that outperform their original ones will
 353 replace them, while the rest are discarded.

$$S'_i = S_i + r_i (T_i - T_F M_i) \quad (5)$$

354 Regarding the learner step, it models the interaction between students. For
 355 this purpose, TLBO pairs every candidate solution with another one. Then,
 356 it applies Eq. (6) to create a modified version, S' , of every existing candidate
 357 solution, S , considering the effect of its pair, W_S . The equation aims at vectors
 358 like Eq. (5). Thus, r_i refers to a real random number in the range $[0, 1]$
 359 and linked to the i component. The set of random factors is recomputed and
 360 remains the same for all the interactions at the present stage and iteration.
 361 For every pair, the learning stage tries to shift the worst solution towards the
 362 best. This step concludes with the replacement of the current solutions that are
 363 outperformed by their modified versions. The rest do not change.

$$S'_i = \begin{cases} S_i + r_i (S_i - W_i) & \text{if } error(S) < error(W) \\ S_i + r_i (W_i - S_i) & \text{otherwise} \end{cases} \quad (6)$$

364 After the method has run the requested number of cycles, it returns the best
 365 candidate solution in the population as the final result. The interested reader
 366 can find in (Rao et al., 2012) a numerical example of this optimizer in action
 367 for a better understanding.

368 2.4.3. Universal Evolutionary Global Optimizer (UEGO)

369 The Universal Evolutionary Global Optimizer, known by its acronym UEGO,
 370 is a meta-heuristic global optimizer proposed by Jelasity et al. (2001); Ortigosa
 371 et al. (2001). Like TLBO, UEGO is a population-based algorithm, but it belongs
 372 to the branch of Evolutionary Computation (Boussaïd et al., 2013). Accordingly,
 373 the algorithm manages different solutions concurrently and treats them as indi-
 374 viduals subject to Darwinian evolution. The solutions improve their quality as

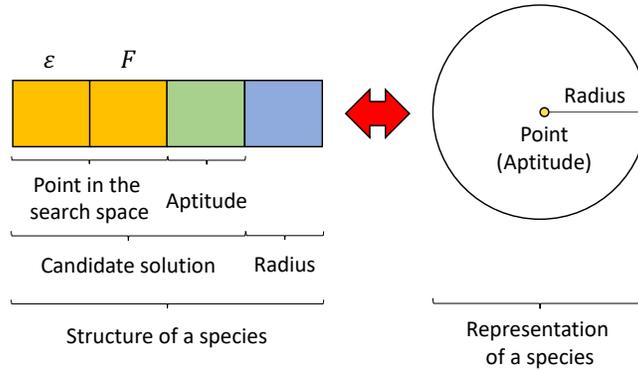


Figure 6: Depiction of species for UEGO and the target problem. In implementation terms (left), each species is an array consisting of the decision variables (ϵ , F), the value or aptitude of that parameter set as a solution (RMSE, i.e., the lower, the better), and the radius of that point as a species (Euclidean distance from the decision variables to consider nearby points equivalent). Conceptually (right), a species is a window in the search space defined by its center (the duple of (ϵ, F) for the species), and the referred distance. There are two decision variables, so the search space is bi-dimensional, and species are represented by circumferences.

375 they evolve. This method can be further classified as memetic (Molina et al.,
 376 2011). Thus, it adds a local search component to the simulated biological context
 377 so that solutions can improve autonomously. At the same time, UEGO is a
 378 multi-modal optimizer, which means that it seeks different optima in the search
 379 space. This algorithm has been successfully applied to many problems, such as
 380 protein folding (García-Martínez et al., 2015) and heliostat field design (Cruz
 381 et al., 2018).

382 The fundamental working unit of this algorithm is the species, which combines
 383 a candidate solution with an attraction radius around it. Provided a distance
 384 metric, such as the Euclidean distance for continuous variables, species are like
 385 ‘windows’ in the search space that focus the optimization process on different
 386 regions. These species can be created, eliminated, moved, and merged
 387 throughout the operation of UEGO. In fact, in practical terms, the optimizer
 388 can be thought of as a method for managing a list, i.e., a species population.
 389 Figure 6 shows the structure and meaning of a species considering the slope
 390 error (ϵ) and the focal distance (F) of a hypothetical facet under OC as the
 391 optimization variables. The slope error and the focal distance defining the central
 392 point of the species also form a solution to the problem. Hence, its value as a
 393 solution, known as aptitude in Evolutionary Computation, is also registered. As
 394 it refers to the RMSE obtained after simulating the corresponding parameter
 395 set, the lower this value is, the better solution this configuration represents.

396 Aside from the problem context, i.e., the variable bounds and the objective
 397 function, UEGO takes the following parameters: i) the maximum number of
 398 species, ii) the maximum number of evaluations of the objective function, iii)
 399 the minimum radius between species, and iv) the number of iterations (search
 400 levels in UEGO). Nevertheless, take into account that the selected local search

401 algorithm might require extra parameters.

402 The algorithm starts by creating an initial species. Its center is a random
403 feasible point, and its radius is set to the diameter of the search space to cover it
404 completely. Then, UEGO launches the selected local optimizer from the center
405 of this species. This point will be updated as the method finds better ones,
406 which is equivalent to moving the species. These steps define the first level of
407 search.

408 After the initialization, UEGO executes a loop with the remaining levels of
409 search. Each one consists of the following steps. Firstly, UEGO computes the
410 radius to be assigned to any new species created at that level. Radii decrease
411 with the search level according to a geometrical progression, and the last one
412 is the user-given input. This strategy corresponds to a cooling component that
413 progressively reduces mobility to promote convergence. UEGO also computes
414 the budget of objective function calls to create and locally optimize species. The
415 former is three times the maximum number of species, and the latter increases
416 with the level, as radii decrease. However, UEGO has mechanisms to save function
417 evaluations, such as removing redundant species, so it might not consume
418 all of them.

419 Secondly, UEGO creates as many random species in the regions of the exist-
420 ing ones as allowed by the budget. Then, the algorithm fuses any species whose
421 centers are nearer than the radius assigned to the current level. When fusing
422 two species, the resulting one keeps the longest radius, which tries to avoid pre-
423 mature convergence, and takes the better candidate solution as its new center.
424 Next, if the population is larger than allowed, the excess is removed, starting
425 with the species having the shortest radius. After that, UEGO launches the
426 local optimization component from each species and considering the function
427 evaluation budget. To conclude every search level, as the local optimizer moves
428 the species centers to better candidate solutions independently, UEGO repeats
429 the fusing process.

430 With respect to the local search component, the Solis and Wet's algorithm
431 has been selected (Molina et al., 2011; Solis and Wets, 1981). This method is a
432 stochastic hill-climber that starts at a given point, i.e., the center of a species
433 when coupled to UEGO, and takes improving steps in random directions. The
434 amplitude of jumps is scaled with the number of consecutive successful (improving)
435 and discarded (non-improving) movements. This local search algorithm has
436 been selected because it has no specific requirements from the objective function,
437 and it has already been successfully coupled with UEGO (Cruz et al., 2018;
438 Jelasity et al., 2001).

439 After having executed as many search levels as requested, the output of
440 UEGO is the final list of species. According to the multi-modal approach of the
441 algorithm and provided that it has converged, the surviving species are expected
442 to be different optimal solutions.

443 *2.4.4. Multi-Start Interior-Point (MSIP)*

444 The optimizer referred to as Multi-Start Interior-Point, abbreviated as MSIP,
445 results from coupling an Interior-Point algorithm (Griva et al., 2009) with a

446 stochastic Multi-Start technique (Redondo et al., 2013; Salhi, 2017).

447 Interior-Point methods form a group of algorithms for addressing linear and
448 non-linear optimization problems (Griva et al., 2009). They are characterized
449 by keeping the exploration in the feasible region of the search space employing
450 different methods. The Interior-Point algorithm considered in this work belongs
451 to the FMinCon (FMC) solver included in the Optimization Toolbox of MAT-
452 LAB (Branch and Grace, 2020; López, 2014). The method addresses the target
453 problem by solving a sequence of approximations that result from adding slack
454 variables and a barrier function (Byrd et al., 2000). The resulting instances are
455 simpler to solve than the original one.

456 FMC takes as input an initial point to start the search, and it might impact
457 the final result depending on the problem type. At each iteration, it can choose
458 one of two alternatives to solve the approximate problem. The first option,
459 which is also the preferred one, is to take a direct or Newton step that applies
460 a linear approximation. The second one is to take a conjugate gradient step
461 with a trust region. This latter option is only selected when it is not possible to
462 apply the previous one, for example, because the approximate problem is not
463 locally convex near the current position.

464 The referred Interior-Point algorithm will only find the optimal solution
465 for convex problems. Otherwise, the result might be a local optimum. The
466 convexity of the target problem has not been certified. Moreover, according to
467 preliminary experimentation, the solutions found by FMC vary with the starting
468 points. Thus, the multi-start component is in charge of randomly generating
469 different initial points and launching FMC independently from each one. It
470 controls the total number of function evaluations and records the best result
471 achieved so far, which ultimately becomes the problem solution. This approach
472 serves to escape from local optima by focusing the seek on different regions of
473 the search space.

474 *2.4.5. Overview of the OC methods*

475 After explaining the four options considered for finding the optical param-
476 eters of the studied facets, Table 2 provides the reader with a summary of the
477 main properties of each. The first column contains the most relevant aspects
478 to take into consideration. After that, there is a column per method with the
479 corresponding details. PSA-ISOC stands out as the only deterministic method,
480 i.e., it always returns the same result (as long as the same expert decisions are
481 taken). However, it is also the most tedious and difficult to apply due to its
482 inherent link to an expert. The others are solvers that only need to be appropri-
483 ately configured and launched. Nevertheless, their stochastic nature can make
484 their output vary among executions, so several runs might be needed. Among
485 them, UEGO seems the hardest to configure after a first glance, but configuring
486 the FMC part of MSIP can be challenging, as it has more than ten parameters
487 according to the official documentation. Hence, TLBO is the best balanced in
488 this concern as it only takes two parameters. Finally, notice that PSA-ISOC
489 cannot be independently applied without an expert and time, which is one of the
490 problems that this work aims to correct, and MSIP requires a software license

Table 2: Main properties of the different methods considered for OC.

	PSA-ISOC	TLBO	UEGO	MSIP
Type	Semi-automatic grid search	Meta-heuristic (Swarm Intelligence)	Meta-heuristic (Evolutionary Computation)	Interior-Point in Multi-Start component
Output stability	Deterministic (Taking the same decisions)	Stochastic	Stochastic	Stochastic
Human interaction	High (Interactive stages)	Minimal (Setup)	Minimal (Setup)	Minimal (Setup)
Setup complexity	High (Under continuous adaption)	Low (2 parameters)	Medium (4 parameters)	Minimal (1 parameter, using the defaults of FMinCon)
Availability	Not applicable (Expert-dependent)	Open-source implementation at Cruz (2021a)	Open-source implementation at Cruz (2021b)	Optimization Toolbox required

491 for FMC. On the contrary, we have open-source implementations of TLBO and
 492 UEGO, simplifying their use (or even modification).

493 3. Experimentation and results

494 3.1. Environment and configuration

495 The present study has considered three spherical heliostat facets to charac-
 496 terize: F62, F42, and LDM. After recording the benchmark image of each facet
 497 as described in Section 1 and Section 2, the different characterization processes
 498 have been carried out in a computational environment. At that point, the tools
 499 used are MATLAB 2018b, the auxiliary routines for image management, the
 500 Fiat_Lux optical model from the PSA, and the optimizers involved.

501 There are four alternative OC strategies, the traditional and semi-automatic
 502 PSA-ISOC method, and the optimizers TLBO, UEGO, and MSIP, which explore
 503 the parameter search space autonomously after being configured. The first
 504 one has been executed, as usual, by one of the experts in charge of OC at
 505 the PSA. The others have been launched independently at the University of
 506 Almería (UAL). The MATLAB implementations of TLBO and UEGO have
 507 been developed at the UAL. The same occurs with the multi-start component
 508 that manages the FMinCon software package by MathWorks used to define
 509 MSIP. The computer used at the PSA features an Intel Core i5-8265U 4-core
 510 processor and 16 GB of RAM. The one used at the UAL has an Intel i7-4790
 511 4-core processor and 32 GB of RAM.

512 The PSA-ISOC procedure was applied at the PSA, as usual, to minimize
 513 the same objective or cost function as the optimizers. It covered the following
 514 aspects:

- 515 1. Facet geometry: Spherical

- 516 (a) Variable linked to the search: Focal distance (m).
- 517 (b) Nature of the variable: Deterministic.
- 518 (c) Starting Value (SV) (manufacturer's data):
 - 519 i. 220.0 m (F42)
 - 520 ii. 175.0 m (F62)
 - 521 iii. 420.0 (LDM)
- 522 (d) Search range:
 - 523 i. $[F_{min} = 90.0, F_{max} = 330.0]$ (F42, F62)
 - 524 ii. $[F_{min} = 150.0, F_{max} = 650.0]$ (LDM)
- 525 2. Mirror surface state.
 - 526 (a) Variable linked to the search: slope error (mrad)
 - 527 (b) Nature of the variable: Statistical.
 - 528 (c) SV: 0.0 mrad (no manufacturer's data).
 - 529 (d) Search range:
 - 530 i. $[\epsilon_{min} = 0.0, \epsilon_{max} = 3.0]$ (F42, F62)
 - 531 ii. $[\epsilon_{min} = 0.0, \epsilon_{max} = 2.5]$ (LDM)

532 Two variants of PSA-ISOC were tested for each facet:

533 **Direct:** It starts at the initial configuration. It assumes that the SV of the
 534 slope error is correct, so the search starts by testing the focal length of
 535 the facet, which is the variable of deterministic nature.

536 **Inverse:** It starts at the initial configuration. It assumes that the focal F of the
 537 facet is correct and its SV is the nominal one given by the manufacturer,
 538 so the search starts by testing the slope error, which is the variable of
 539 statistical nature.

540 In both, the best value found for the variable considered is recorded and
 541 used when switching to the other dimension, and the process is repeated until
 542 finding the lowest error. Each variant generates its own final result, and the one
 543 ultimately selected is that producing the smallest error.

544 The optimizers inherit the initial ranges of each variable and facet described
 545 above. TLBO has been configured to work with a population of 12 candidate
 546 solutions for 30 cycles in all the cases. This configuration approximately results
 547 in 730 objective function evaluations. It implies the same number of simula-
 548 tions with Fiat_Lux generating a synthetic image to compare to the benchmark.
 549 This value is based on the number of evaluations that the simplest PSA-ISOC
 550 would execute, according to the expert. MSIP and UEGO have been config-
 551 ured to approximately make the same number of function evaluations. More
 552 specifically, UEGO has been configured to consume up to 1400 function eval-
 553 uations, maintain up to 12 individuals, run 16 levels of search, and consider a
 554 minimum radius of 4.0 in the search space. Since it includes strategies to save
 555 function evaluations after convergence, the number stays in the desired range.
 556 Finally, MSIP has been directly provided with the reference number of function
 557 evaluations.

558 Regarding the computational effort, the methods considered can be divided
559 into two groups. Let the traditional expert-based PSA-ISOC be on one side and
560 the standard optimization algorithms UEGO, TLBO, and MSIP on the other.
561 This division is equivalent to the degree of human interaction detailed in Table
562 2. It can be reformulated as the methods of high human interaction (PSA-
563 ISOC) compared to those mainly automatic (only requiring to be configured
564 once), i.e., UEGO, TLBO, and MSIP. In general, the time required by the
565 traditional expert-based PSA-ISOC will be significantly higher than the rest due
566 to the necessity of supervising its progress and making decisions. Hence, PSA-
567 ISOC could only compete with the standard optimization algorithms in terms
568 of quality. More specifically, the expert invested 2 days of work applying the
569 PSA-ISOC to every facet, i.e., 16 hours for each case. On the contrary, the time
570 of any optimizer mainly depends on the number of function evaluations, which
571 makes them equivalent in terms of computational effort. The reason is that
572 the objective function is computationally demanding (e.g., its evaluation takes
573 approximately 45 seconds for F42), which makes the cost of the internal steps of
574 any optimizer negligible. As all the standard optimizers have been configured to
575 launch approximately 730 evaluations, their running time is virtually the same.
576 Namely, all the optimizers took approximately 9.5 hours for F42, 8.5 hours for
577 F62, and 4.5 hours for LDM in the non-dedicated UAL computer. Thus, the
578 execution time is not relevant for choosing one of the optimizers.

579 *3.2. Problem considerations*

580 In contrast to a classic optimization problem analytically addressed, e.g.,
581 finding the minimum of the parabola $f(x) = x^2$, we face two levels of stochas-
582 ticity. The first one is linked to the objective function evaluation, which involves
583 the simulation an generation of synthetic images to compare. More specifically,
584 if one thinks of evaluating the referred parabola, the values computed are deter-
585 ministic and do not vary. For instance, $f(5) = 5^2 = 25$, and the result does not
586 change. On the contrary, the results of generating and comparing synthetic im-
587 ages to the real one vary. The reason is the slope error, modeled by a Gaussian
588 distribution. This aspect affects the creation of synthetic images by Fiat_Lux
589 with random yet normally distributed values. In this context, given two similar
590 parameter sets, it is possible that one evaluation using Fiat_Lux returns than
591 the first one is slightly better, while the next one results in the opposite inter-
592 pretation. It is like looking for the minimum point of a function that moves
593 slightly. We face this problem by making the objective function (its internal
594 simulations) deterministic artificially. Namely, we fix the same seed at evalua-
595 tion, i.e., the one used by Fiat_Lux, for a complete execution of any optimizer.
596 Roughly speaking, the seed in Computer Science is the input of random number
597 generation algorithms defining an infinite sequence of pseudo-random numbers
598 (Matsumoto and Nishimura, 1998). For the same seed, they produce the same
599 numbers. Hence, this is like taking a snapshot of the moving surface of the
600 objective function or freezing it.

601 The second level of stochasticity is linked to the optimization methods con-
602 sidered. Even with a fixed surface to explore, they also use random numbers,

603 i.e., they are stochastic. This property means that a certain optimizer might re-
604 turn a different result for the same problem and configuration. In other words,
605 its results are not deterministic either (also in contrast to the analytical ap-
606 proach of the example above). This behavior is coupled with the variation of
607 the seed defining the objective function. Regardless, the stochastic component
608 of the Fiat_Lux simulations is enough to impact the whole OC process. Thus,
609 it is necessary to handle this situation.

610 For this purpose, every optimizer has been independently executed with five
611 different fixed seeds for the objective function (Fiat_Lux simulation). Accord-
612 ingly, for each facet and method, we ultimately have 5 equivalent parameter
613 sets. We cannot say that one is better than the rest because they were obtained
614 from a different snapshot of the objective function. Thus, the final parameter
615 set from the particular optimizer an facet is the average of their independent
616 results. For example, assuming a particular facet and two simulation seeds vary-
617 ing the shape of the objective function, we could have registered from one of
618 the optimizers the following results: (0.52 mrad, 175 m) and (0.58 mrad, 185
619 m). Then, its result for the studied facet would be (0.55 mrad, 180 m), i.e., the
620 average of the variables.

621 However, that parameter set might perform bad in reality, especially if the
622 optimizer mis-converged. They cannot even be certainly ranked: each param-
623 eter set has its own value for its frozen function, but the quality of the average
624 point cannot be computed in that way. Thus, each parameter set is ultimately
625 evaluated an compared by studying it under 100 different seeds for the sim-
626 ulation, i.e., by evaluating with 100 different snapshots of the same moving
627 objective function (which cannot be fixed due to its statistical definition). If
628 the parameter sets adapt well to the different variations of the function, they are
629 robust enough. In other words, the robustness of every result must be assessed
630 by considering multiple evaluations (synthetic image generation and compar-
631 ison) with different random numbers. For this reason, it is always advisable to
632 vary them during the seek, even if the OC methods are deterministic.

633 It is interesting to end this subsection with a more detailed explanation of
634 how Fiat_Lux works in practical terms. It receives as input a picture of the sun,
635 and it is taken with the same digital camera used for the images of the facets
636 over the target. It is an angularly-calibrated image, which means that we know
637 the mrad subtended for any pixel in the picture of the sun from the camera.
638 This can be applied to any element in the concentrator too, given the proximity
639 between both. In this regard, Fiat_Lux differs from standard Monte-Carlo Ray-
640 Tracing (MCRT), which associates the uncertainty of the optical system to the
641 direction of the incident ray. For it, the uncertainty at simulation is caused by
642 the deviation in the orientation of the vector perpendicular to every element of
643 the surface, dS , on the mirror (slope error). This orientation differs from the
644 nominal one and is modeled by a normal distribution statistically describing
645 the deviation of the corresponding perpendicular vector, and ultimately affects
646 the direction of every ray reflected at every dS . Therefore, the unavoidable
647 uncertainty is handled by considering multiple seeds, as detailed above. Each
648 Fiat_Lux simulation has processed approximately 930 000 000 rays.

649 *3.3. Optical characterization results*

650 Figure 7 depicts intermediate results of the PSA-ISOC process for F42. It
 651 allows visualizing the impact on the successive approximations of the horizontal
 652 and vertical normalized intensity profiles of the real and synthetic images.
 653 Figure 8 displays the slope error and focal length through the stages of the Di-
 654 rect and Inverse variants of PSA-ISOC for the same F42 facet. The asymptotic
 655 evolution of the variables shows the proximity of the minimum error. The be-
 656 havior is similar for F62 and LDM, so they are omitted due to space limitations.
 657 Regardless, the PSA-ISOC method has been outperformed by optimization for
 658 every facet.

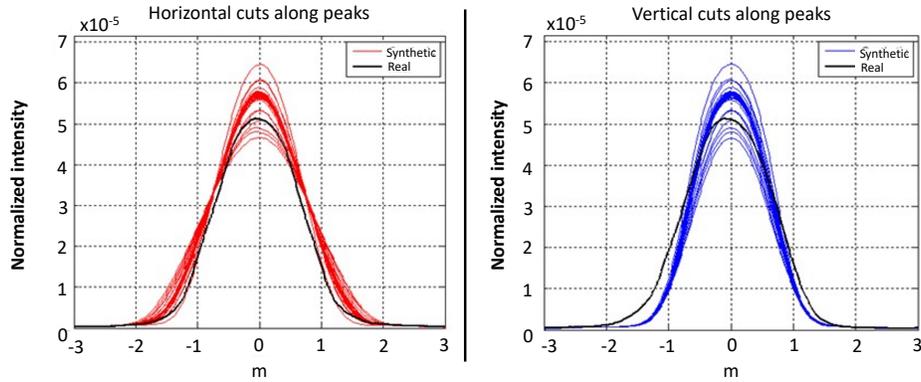


Figure 7: Horizontal (left) and vertical (right) slices across the centroid of the normalized intensity of the real image (black) acquired from the F42 facet. The red and blue slices correspond to synthetic images generated with different model parameters considered by the PSA-ISOC method.

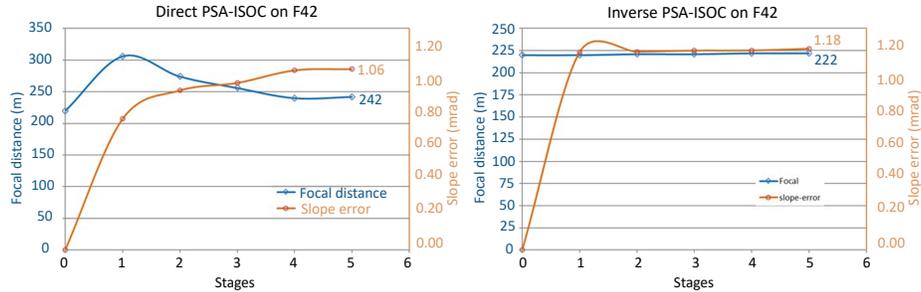


Figure 8: Partial results of PSA-ISOC for F42. Left (Direct PSA-ISOC): The blue line represents the evolution of the searched focal length of the F42 facet from its initial (nominal) value to the best value minimizing the cost function. The brown line shows this evolution for the slope error. Right: It contains the same information for the Inverse PSA-ISOC variant.

659 Regarding the results of advanced optimization, Table 3 contains the nominal
 660 values of the parameters defining each facet, i.e., slope error and focal distance,
 661 alongside the corresponding values achieved through characterization. It also

Facet	Geometry	Nominal optical parameters		Best optical parameters		Method	nRMSE	Confidence Interval (90%)
		Slope error (mrad)	Focal length (m)	Slope error (mrad)	Focal length (m)			
F62	Spherical	Unknown	175	0.76	171	UEGO	89.46	18.1%
F42	Spherical	Unknown	220	1.18	211	TLBO	34.20	10.7%
LDM	Spherical	Unknown	420	0.81	185	TLBO	17.98	6.7%

Table 3: Final results of the optical characterization process.

662 includes the method that found the best solution, represented by its nRMSE and
663 90% confidence interval. Unknown values refer to the fact that manufacturers
664 did not provide information about this parameter for the facets built.

665 Figure 9 shows the robustness analysis of the final result of each method
666 and facet. Each solution is represented by its average nRMSE registered after
667 100 independent evaluations with different seeds. The figure also displays the
668 corresponding 90% confidence interval of each sample. As previously explained,
669 any of the 100 nRMSE instances could be the most realistic one. Thus, they are
670 replaced with two representative statistics, i.e., the average and the standard
671 deviation, as the most probable value and the observed scattering, respectively.

672 Figure 9 confirms the viability of replacing the traditional characterization
673 approach with optimization algorithms, which was a fundamental goal in the
674 present study. More specifically:

- 675 • The nRMSE of PSA-ISOC is similar to those of the best-performing numerical
676 optimizers, i.e., TLBO and UEGO. More specifically, one can
677 compare the nRMSE obtained in the test of 100 independent evaluations
678 for each result found by PSA-ISOC to that of the best-performing opti-
679 mizer, i.e., lowest average nRMSE. For F62, the result of UEGO deviates
680 0.55% up from that of PSA-ISOC. For F42, the result of TLBO deviates
681 up 5.32%. Finally, for LDM, TLBO deviates up 1.87%.
- 682 • The solutions of PSA-ISOC fall into the confidence interval defined by the
683 best-performing numerical optimizers.
- 684 • For all the facets, the lowest average nRMSE comes from a numerical
685 optimizer. Hence, they could replace the traditional PSA-ISOC while
686 providing similar quality and requiring less supervision by the experts.

687 Figure 10 extends Figure 2 by comparing the iso-intensity lines of the real
688 images of the facets to their synthetic equivalents according to the best param-
689 eters found after characterization. As can be seen, the iso-intensity lines of the
690 real images are more similar to those of the synthetic ones generated with the
691 characterized configuration than to those using the nominal values.

692 The similarity between the actual images of the facets and their synthetic
693 equivalent can be further studied with statistical tools. In this context, the
694 pixel-to-pixel deviation ($px2px$) between the real image and the one simulated
695 with the parameters obtained after characterization has been analyzed. Figure
696 11 shows this study (left side) and the corresponding histograms of occurrences
697 (right side) for the three facets considered.

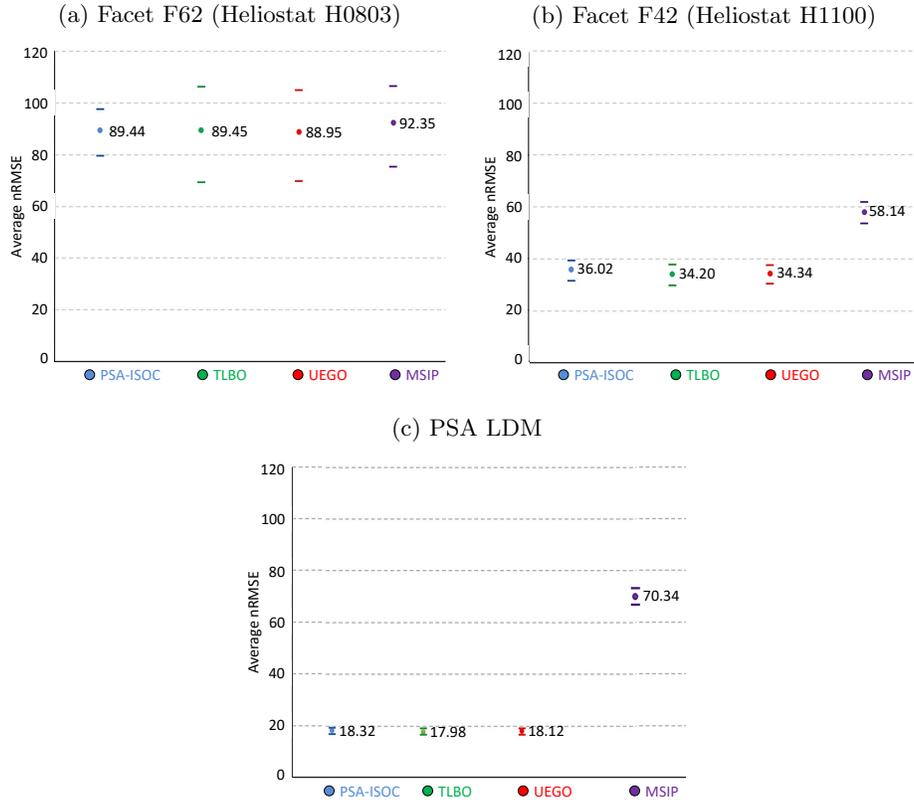


Figure 9: Quality of the results. The optical parameters found by each method for every facet have been used to compare the resulting synthetic image to its reference under 100 different random seeds to confirm their generality. The numbers correspond to the average nRMSE obtained with each parameter set after the 100 independent comparisons. The associated bars indicate the 90% confidence interval of these results.

698 Table 4 shows three different metrics of pixel-to-pixel deviation between
 699 the actual images and their synthetic equivalents found after characterization
 700 and their occurrence percentages. Namely, Type A refers to the most frequent
 701 deviation of the synthetic image from the actual one. Type B is the name of the
 702 highest deviation above the real image. Type C is how the highest deviations
 703 below reality are tagged.

704 The first group shows if the characterized image has either overestimated or
 705 underestimated the intensity in the benchmark, as well as the magnitude of the
 706 most frequent deviation. For instance, the optical parameters found for the facet
 707 F62 cause 96.75% of the pixels in the synthetic image to be 0.33% above the
 708 actual values. Accordingly, the characterization predominantly overestimates
 709 the intensity. However, the opposite situation occurs with F42 and LDM.

710 This analysis would be biased without considering the magnitude and fre-

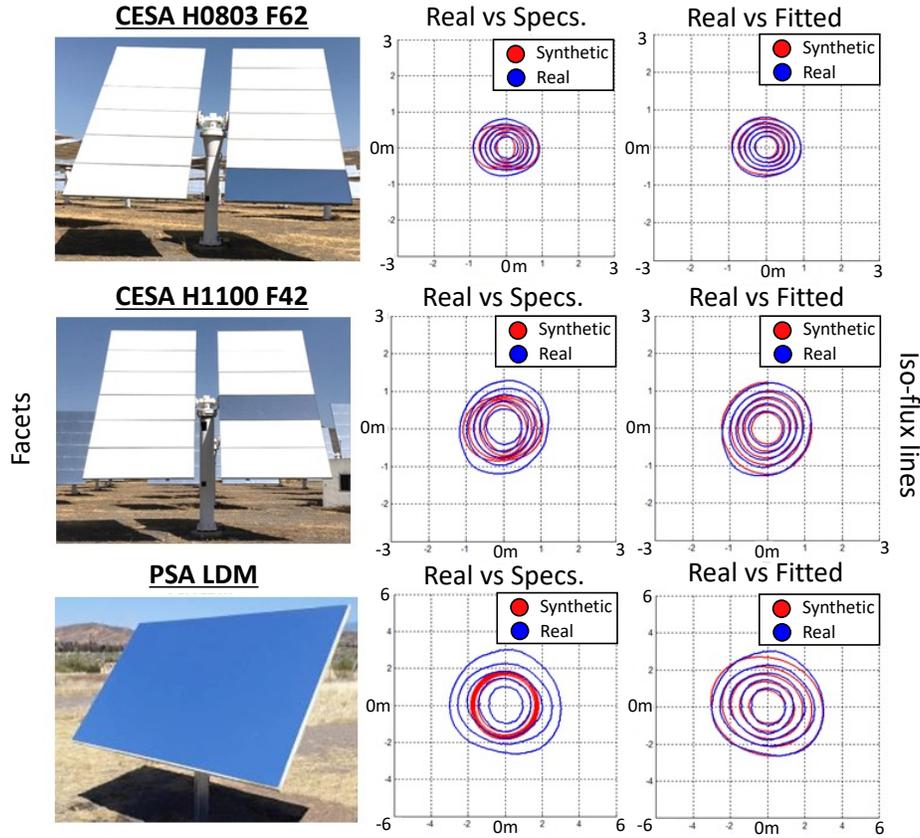


Figure 10: Iso-intensity lines of the real images of each facet (left) compared to the synthetic equivalent using: the nominal parameters (center) and those obtained through characterization (right).

Facet	Type A px2px deviation	Occurrence	Type B px2px deviation	Occurrence	Type C px2px deviation	Occurrence	Others
F62	0.33%	96.75%	13.9%	0.007%	-13.2%	0.006%	3.24%
F42	-0.32%	93.39%	20.0%	0.009%	-20.7%	0.017%	6.58%
LDM	-0.62%	88.16%	15.9%	0.001%	-14.1%	0.001%	11.86%

Table 4: Statistical analysis of the best synthetic images obtained through characterization.

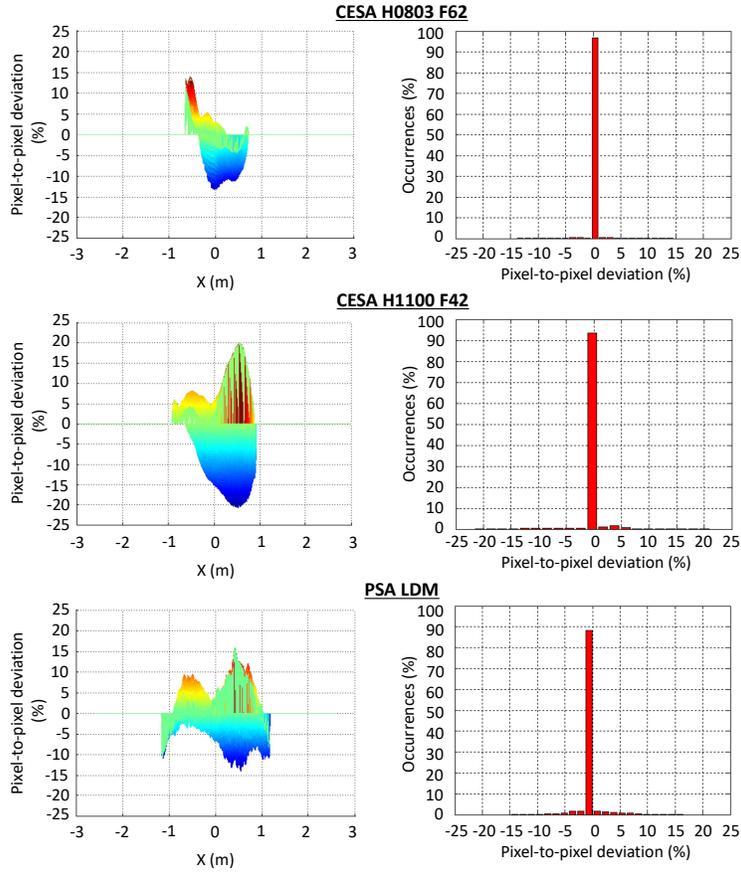


Figure 11: Pixel-to-pixel deviation between the real image and the best characterization result (left) and corresponding histogram of occurrences (right) for each facet. The best parameter set found through optimization for every facet has been used to generate the corresponding synthetic image with Fiat_Lux. These images have been compared pixel by pixel to their corresponding reference. The comparison computes the percentage deviation of each pixel in the synthetic image with respect to its equivalent in the one experimentally obtained (reference) generating a histogram of percentage deviations.

711 quency of the highest deviations between both sorts of images, which can be
712 determinant to assess characterization. For example, the result of F42 shows
713 deviations up to 20.0% above its benchmark, with an occurrence of 0.009%. For
714 LDM, 15.9% are above its benchmark and the occurrence is 0.001%, i.e., 9 times
715 less. Regarding the highest deviations below the benchmark, the percentages
716 are similar (-20.7% and -14.1%, respectively). However, their occurrences are
717 0.017% and 0.001%, respectively, i.e., 17 times less for the facet LDM. There-
718 fore, it can be concluded that LDM is the facet that has been characterized the
719 best. This idea is supported by Figure 11 and the position of its results in the
720 nRMSE ranking, in which its most pronounced deviations from the benchmark
721 are also rare.

722 4. Conclusions and future work

723 This work has two main goals. The first one is to warn about the neces-
724 sity of both an initial and a routine assessment of the optical quality of the
725 heliostats in commercial solar power tower plants throughout their useful life.
726 The second one is to demonstrate that this process can be carried out as long as
727 effective alternatives to traditional indirect optical characterization, such as the
728 method followed at the Plataforma Solar de Almería (PSA), are developed. The
729 viability of this approach depends on minimizing two fundamental aspects of
730 current strategies: i) the requirement of constant participation of highly qual-
731 ified staff, and ii) the time linked to traditional methods, which are mainly
732 iterative searches with notable heuristic or expert-specific components.

733 A common experimentation framework has been defined with the experts
734 of the PSA. In this context, the traditional iterative optical characterization
735 method of the PSA (PSA-ISOC) has been compared to using three existing
736 numerical optimizers, UEGO, TLBO, and MSIP. The study has not been lim-
737 ited to obtaining an independent result to optical characterization instances.
738 It has also covered how to find the optimal solution among the proposals of
739 each method. The experimentation has considered three heliostat facets. Two
740 of them belong to standard heliostats of the CESA-I field of the PSA, while
741 the third one is a facet prototype featuring a long focal distance. The results
742 obtained confirm that for simple optical systems, such as heliostat facets, the
743 numerical optimizers TLBO and UEGO achieve the optimal solution for the
744 optical characterization problem. More precisely, for the three facets consid-
745 ered, the traditional semi-automatic process of the PSA, PSA-ISOC, deviates
746 0.55%, 1.87%, and 5.32% in nRMSE, respectively, from the results obtained by
747 the automated execution of the optimizers. Besides, the PSA-ISOC results fall
748 into the confidence interval linked to those achieved by the optimizers. There-
749 fore, these methods seem valid to approach optical characterization processes
750 without requiring constant human interaction. The decision between them is
751 up to the user, but TLBO is simpler to tune and implement.

752 For future work, the aim is to generalize the optical characterization method
753 to cover complete facet-based heliostats. Upon success, it will be possible to in-
754 clude the technique as part of the routine revision and maintenance tasks of

755 heliostats in solar power tower plants. This update allows monitoring their op-
756 tical performance throughout their useful life and, thus, controlling the economic
757 expectations linked to their real production capacity.

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