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ORIGINAL PAPER

Examining the role played by resources, goals and orientations in primary teachers' decision- making for problem-solving lesson plans

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Abstract
The decisions that teachers make in transforming the curriculum into specific lesson plans determine the real enactment or otherwise of curricular ideals. These decisions are shaped by the resources available and by each teacher's goals and orientations. This exploratory study employs Schoenfeld's decision-making model to examine how resources, goals and orientations influence lesson planning for mathematics problem solving, for different profiles of primary teachers in Chile. To this purpose, a survey was conducted among 40 teachers of varying degrees of ability and experience: some were beginning teachers, others were experienced but had no further training in teaching problem solving and a third group was composed of experienced teachers with specific training in this question. Interviews with two teachers from each profile revealed important differences between the three groups. Beginning teachers relied more heavily on official resources such as the official curriculum and standard textbooks, aligning themselves with school requirements. Experienced teachers with problem solving training demonstrated a strong inclination towards teaching through a problem solving approach. While beginning teachers acknowledged the importance of promoting problem solving strategies, they did not usually adapt problems to the mathematical content or to the age/competence of their students. Interestingly, all three groups under-utilised sections of curricular resources that emphasise the present curricular focus on problem solving. Finally, the study found that experience alone is not enough to develop a problem solving approach and that focused professional development programmes are needed to equip teachers with the necessary skills. In addition, a diagnostic teaching approach should be incorporated into initial teacher training.

Keywords Decision making · Problem solving · Instructional design · Lesson planning

1 Introduction

Problem solving is a continuously evolving field of study that remains highly topical, in which new subfields continually arise, contributing to existing knowledge. However, despite extensive research in this field, translating theoretical advances into effective classroom practice remains a significant challenge (Liljedahl & Cai, 2021). While historically research has focused on students as problem solvers (Schoenfeld, 2010), in the last decade there has been increasing emphasis on understanding the role of teachers in the problem solving process within the context of school mathematics. Thus, studies have investigated how teachers interact with curriculum materials (Ahl et al., 2015), how problem solving may be used as a pedagogical approach for teaching mathematics (Cai & Hwang, 2019), what kinds of challenges teachers encounter when integrating problem solving into their instruction (Cheeseman, 2018) and shifts in teachers' conceptions of problem posing after participating in professional development activities (Cai et al., 2020; Saadati & Felmer, 2021).

However, few studies have been undertaken to explore other, equally important aspects, such as how teachers with different characteristics approach problem solving. Schoenfeld (2010, 2011, 2012) developed a model that considers teachers' goals, resources and orientations and how these factors influence their decision making for teaching problem solving in mathematics lessons. Moreover, depending on the teacher's level of proficiency, these decisions may be more oriented toward classroom management, creating engaging

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activities for students or diagnostic teaching (Schoenfeld, 2011).

Schoenfeld's model has been applied in research focusing on mathematics classroom instruction. However, it is crucial to note that teachers make decisions not only during teaching but also during other stages of curriculum enactment. Regardless of their profile, primary school teachers must resolve many daily decisions encompassing lesson preparation, implementation and assessment. However, the planning and design of instruction has received relatively little research attention (Bieda et al., 2020). Özyildirim-Gümüs (2022) observed that pre-service teachers often gravitate toward teacher-centred approaches, face challenges in establishing connections among different content areas, struggle to incorporate specific problems and frequently neglect the use of concrete materials in their lesson planning. These findings highlight the specific needs of beginning teachers in instructional planning. Moreover, experienced teachers, even those with extensive problem solving experience, have their own specific requirements.

Given the myriad of factors impacting class planning, there is a significant need for empirical research to understand how teachers develop their planning competence and address the associated challenges (Cevikbas et al., 2023). Furthermore, an integrative model is needed to understand and visualise how resources, goals and orientations interact in preparing problem-solving instruction. In this respect, Schoenfeld's (2011) model of teaching proficiency levels may prove valuable in characterising decision making across various teacher profiles, especially when applied to decisions made before instruction. In the present study, therefore, we address the question: What resources, orientations and goals are enacted by Chilean primary teachers in their lesson planning for problem solving, according to their profile of competence and experience?

2 Literature review

Decision making is generally regarded as a central component of teaching competence. It is complex in nature and incorporates a wide range of cognitive, emotional and contextual factors (Griffith et al., 2013; Lloyd, 2019; Shavelson, 1973). Among various theoretical competence models that have been proposed, decision making usually occupies a significant place. Considering competence as a continuum, decision making, together with perception and interpretation, mediates between disposition (cognition and affect-motivation) and observable behaviour or performance (Blömeke et al., 2015). With respect to teaching, moreover, the competence facets of perception, interpretation and decision making may be crucial in connecting knowledge and attitudes to the classroom situation (Blömeke & Kaiser, 2017).

Several models of decision making by teachers have been proposed. In Schön's (1983) reflective practitioner model, decision making develops in three stages: action, reflection and learning. This cycle is constantly repeated as the teacher encounters new situations and challenges in the classroom and uses that knowledge to improve future decisions. Clark (1984) developed a model incorporating the factors that influence classroom actions, including beliefs, constraints and actions. This model recognises that decision making is a complex process and that teachers make active decisions more or less constantly. The Sullivan and Mousley (2001) model places significant emphasis on adaptability and continuous reflection, similar to Schön's approach but with a specific application in mathematics teaching. This structure helps teachers make informed decisions that promote more effective and personalised learning. Schoenfeld (2011) stresses the importance of teachers' knowledge, resources, goals and beliefs in shaping their decision making. The model is complemented by three levels of proficiency: classroom management, implementing engaging activities and diagnostic teaching, according to their primary focus when decisions must be made during lessons.

Drawing on some of the models mentioned above, various studies have explored how teachers with different professional profiles make decisions in their teaching. Some studies have shown that novice teachers tend to be less creative in problem design, less likely to associate problems with real-life contexts and less efficient in planning, often encountering challenges when attempting student-centred instruction (Borko & Livingston, 1989; Lewis, 2014; Murtafiah et al., 2020). In contrast, experienced teachers appear to be more deliberate in their planning, make active use of assessment information and are more likely to associate problems with real-life contexts (Murtafiah et al., 2020; Sullivan et al., 2012). Additionally, experienced mathematics teachers better interpret their thinking, focus more on higher-order skills and design problems that are more closely aligned with students' experiences, while novice teachers have more difficulties in these areas (Bastian et al., 2022). These differences are attributed to the less elaborate cognitive schemata and less well-developed pedagogical reasoning skills of novices (Borko & Livingston, 1989).

Since decision making is one of the facets of teaching competence, it is necessary to consider both its cognitive and its situated components (Kaiser et al., 2017). Stahnke et al. (2016) highlight the importance of teachers' situation-specific skills, emphasising the positive impact of expertise and experience on noticing and decision-making. Professional development programmes are crucial for enhancing these skills. Santagata and Yeh (2016) focused on the development of elementary beginning teachers' competence, emphasising the significance of perception and interpretation in decision making processes. Bieda et al. (2020) explored how social and institutional contexts influence mathematics lesson planning for early career teachers, and stress the role of contextual factors in shaping planning practices and teacher preparation. Lee and Vongkulluksn (2023) analysed the impact of the professional development activity of mathematics teachers on their teaching practice, highlighting its positive value on self-efficacy, beliefs and pedagogical knowledge of mathematics. Krawec and Montague (2014) observed a marked improvement in the treatment of problem solving in the mathematics classroom following a professional development course focusing on this area.

Given that teachers of different profiles make different decisions, it then becomes necessary to identify specific needs. For example, research has found that teachers' experiences and perceptions of the nature of mathematics, as well as their understanding of curriculum and pedagogy, play a crucial role in shaping their teaching practices (Luitel, 2020). Lloyd (2019) reported that, despite sometimes extensive experience, teachers tend not to use formal reflective frameworks or sources of evidence to inform their decisions, although this pattern of behaviour has not been tested in specific domains, such as mathematical problem solving. Krawec and Montague (2014) noted that the four problems expressed by the teachers who participated in their study are endorsed in the literature, namely: difficulty finding time to incorporate problem solving into the curriculum; the need for sustained support throughout the intervention; greater confidence in the intervention when it is explicitly aligned with known standards and expectations; and dissatisfaction with the intervention's core teaching methodology (p. 132). According to the authors, these issues need to be addressed in professional development programmes.

Finally, Cevikbas et al. (2023) investigated the difficulties that teachers face in making decisions when planning their teaching. Such difficulties may arise in the preparation of learning plans and in determining problems to be solved at the beginning of learning (Nurlaily et al., 2019). In this respect, Carrillo et al. (2019) referred to difficulties involving the purpose of setting a particular problem, the students' level of interest, the applicability of the problem to students' learning, the different ways to approach the problem, potential difficulties students may encounter and the type of assistance needed. Davidson (2016) noted that mathematics lesson planning is complex and highlighted the teacher's mathematical knowledge as a critical factor in the planning process. Hammer and Ufer (2023) affirm that the teacher's professional knowledge is directly related to the quality of the lesson planning process and that this quality can be assessed by noticing and reasoning during the analysis of a learning task.

3 Theoretical framework

3.1 Decision making model

Schoenfeld (2011) describes a model that explains how and why individuals act within well-practised domains, using the constructs of resources, goals, orientations and decision-making. Orientations include beliefs, which play a pivotal role in shaping teachers' behaviour in the classroom. However, rather than an isolated belief, it is a cluster of beliefs that exerts influence. Specific situations act as triggers, activating particular beliefs and subsequently initiating related sets of beliefs. While these clusters contribute to shaping teachers' behaviour, they alone do not provide a complete explanation. In addition, teachers' actions align with their beliefs, taking into account the available resources and prioritising what they deem most important (their goals). In other words, "goals recruit resources" (Schoenfeld, 2011, p. 459). Ultimately, decision making arises through the interplay of beliefs, resources and goals. This involves the selection of goals that align not only with a teacher's available resources but also with their beliefs, or more broadly, their orientations.

3.1.1 Resources

The resources available to a teacher can take various forms, including intellectual, social or material resources. These encompass the reservoir of knowledge that teachers can draw upon, whether this comprises their experience or information that can be provided to them or accessed by them and brought into the classroom. Resources can also encompass material, technological tools, interpersonal connections and personal skills (Schoenfeld, 2011).

Regarding the intellectual resources required for teaching problem solving, the literature emphasises several aspects that must be considered to enable teachers to help students become proficient problem solvers (Olivares et al., 2021a). These aspects include understanding how to develop reasoning through problem solving to enhance the comprehension of mathematical concepts, knowledge of integrating problem solving into assessment and the ability to observe and listen to students effectively while they are solving problems. In addition to these considerations, certain key conditions must be present for this knowledge to be transformed into the effective teaching of problem solving. According to Olivares et al. (2021a), these conditions include sufficient autonomy to make decisions regarding the teaching of problem solving; opportunities for professional development as a problem solver and as a teacher capable of instructing others in this regard; adequate curricular flexibility to take the time necessary for implementing problem-based teaching; and access to resources that facilitate the use of problem solving for learning in diverse areas of school mathematics.

Teacher guides constitute a resource that influences teachers' practices and could be powerful tools in their professional development (Matić & Gracin, 2020). Remillard (2012) emphasised the significance of examining the specific curriculum materials teachers consult and the sections of curricular resources they focus on during engagement. For example, when referring to a teacher guide, a beginning teacher may seek different types of guidance than would a more experienced teacher. Remillard describes this as a form of interaction with curricular resources and notes that to date few research studies have explored these questions.

3.1.2 Orientations

Orientations encompass beliefs, values and preferences (Schoenfeld, 2012). With respect to beliefs, four key aspects have been identified (Skott, 2015): they apply to mental constructs that are subjectively true for the individual; they are imbued with value and commitment; they are relatively stable; and they influence the practices of individuals.

Teachers' beliefs regarding problem solving are particularly significant. For example, beliefs oriented toward a traditional conception of mathematics teaching influence how problem solving is approached, despite curriculum-driven reforms (Boesen et al., 2014). Schroeder and Lester (1989) identified three orientations that the curriculum and teachers can adopt, each of which has implications for mathematics instruction:

- Teaching about problem solving: focused on teaching steps and solving heuristics.
- Teaching for problem solving: emphasises that students should be able to transfer what they have learned from one context to another, using problems as exercises for the learned content.
- Teaching via (through) problem solving: problems are valued not only as the purpose of learning mathematics but also as the primary means of doing so.

3.1.3 Goals

According to Schoenfeld (2011), goals identify what individuals aim to achieve, consciously or unconsciously, using the resources available to them. These goals and the corresponding resources can operate on multiple levels. One approach to working with goals is by setting learning objectives for the lesson. For a learning objective to effectively support knowledge construction, it must, first and foremost, be sufficiently well-specified to suggest a path that helps students achieve it. Secondly, it should be shared and well-understood by all the participants in the teaching and learning process (Jansen et al., 2009). Regarding lesson goals, in a curriculum dissemination system with a centre-periphery structure (Kelly, 2004), such as the Chilean system, objectives are often shaped by proposals found in curriculum materials like the official curriculum and textbooks (Olivares et al., 2021b). However, while the Chilean curriculum stipulates a focus on problem solving, this concept is viewed differently elsewhere (Törner et al., 2007) and even within the same country, it may change over time (Olivares et al., 2021b). Stanic and Kilpatrick (1988) identify three approaches to teaching problem solving, which influence our understanding of the purpose of school mathematics and of problem solving itself:

- Problem solving as a context: problem solving only emerges after teaching the necessary content to tackle problems, often with the aim of transferring them to real-life situations.
- Problem solving as a skill: in this case, the emphasis is on teaching strategies and steps for problem solving.
- Problem solving as an art: viewed as a mathematician would, as a perplexing or challenging matter that involves making an effort to arrive at a solution. As students successfully tackle these challenges, they develop their reasoning capabilities and achieve an understanding of mathematical concepts.

3.2 Schoenfeld's levels of proficiency

Teachers' decision making is influenced by their resources, goals and orientations, and factors such as the level of experience play a crucial role in shaping their professional vision and decision making (Stahnke et al., 2016). Schoenfeld (2011) acknowledges differences among different types of teachers and presents a model of 'levels of proficiency' that identifies three teaching styles.

In the first and most basic style, known as *classroom management*, teachers invest a significant portion of their time in activities to maintain control over the classroom. This approach is typical of beginning teachers. Another substantial portion of time for beginning teachers is spent on *implementing engaging activities*. As teachers become more proficient, they allocate more time to implementing engaging activities than to classroom management. However, a third teaching style characteristic of proficient teachers is *diagnostic teaching*. In this style, teachers invest substantial effort in formative assessment of learning to identify students' level of understanding of mathematical content and make necessary adjustments in teaching. Proficient teachers allocate more of their time to this type of teaching and less to classroom management. Formative assessment involves gathering information about students' understanding before and during instruction to modify lessons and thus enhance engagement (Schoenfeld et al., 2023).

Although this model has been used to analyse classroom activities, we think it is also useful to apply it to with respect to teachers' decisions in the previous stage, i.e. lesson planning for problem solving. Some teachers prioritise operational aspects like classroom management during lesson implementation, while others pay closer attention to these aspects during the planning of instruction. These preparations include having instructional materials ready, ensuring the mathematical content is handled effectively and managing the lessons such that all curriculum requirements are met within the allotted time. While the latter case is not exactly classroom management, it can be termed *curriculum management*. There may also be teachers who, during the planning phase, focus on seeking and selecting interesting problems, and then analysing and preparing them appropriately for classroom instruction. This approach could be considered as *planning engaging activities*. Finally, comparably with the diagnostic teaching discussed by Schoenfeld, we could also consider *diagnostic planning* regarding teachers who seek to incorporate diagnostic strategies into their classes and select problems tailored to the specific needs of their students.

4 Research questions

In this research we address the following general question: What resources, orientations and goals are enacted by Chilean primary teachers in their lesson planning for problem solving? This overarching research question is then divided into the following specific questions:

- SQ 1: What resources, orientations and goals are enacted by teachers in each of the three profiles in their planning for problem-solving classes?
- SQ 2: What kind of decisions are made during this instructional design process?
- SQ 3: How do resources, orientations and goals interact in terms of Schoenfeld's proficiency levels model?

5 Method

With respect to curriculum implementation, this qualitative interpretive study was conducted as an initial approach to examining the decision making processes of teachers in their lesson planning for problem-solving classes.

The study was conducted in three phases. In the first, 40 Chilean elementary school teachers responded to a questionnaire which had two main aims. Firstly, to gain an initial understanding of the resources, goals and orientations of the three profiles of primary education teachers in Chile that are considered in this study. Secondly, the concluding item in the questionnaire identified teachers willing to participate in the second phase of the study. The following three profiles were considered:

- *Beginning Teachers (BT)*: Teachers who had not taken any professional development course on Mathematics Education in general or on problem solving in particular. All had less than ten years' experience. There were eight teachers in this profile.
- *Experienced Teachers (ET)*: Teachers who had participated in one or two professional development courses in Mathematics Education and no more than two specific courses on problem solving. These teachers had more than ten years' experience. There were 21 teachers in this profile.
- *Problem-Solving Trained Teachers (PSTT)*: Teachers who had completed one or two general professional development courses in Mathematics Education and three or more courses specifically focused on problem solving. There were 11 teachers in this profile.

Among the teachers who expressed their willingness to participate in the second phase, two from each group were selected. Semi-structured interviews were conducted with these six teachers to learn more about their resources, goals and orientations, and the decisions they made in their lesson planning for problem solving.

In the third phase, we analysed the information obtained in the first and second phases, seeking to answer the research questions. Figure 1 summarises and illustrates the three phases of the study.

5.1 Data collection instruments

In Phase 1, a closed-ended questionnaire was used to consider the following dimensions regarding the participants:

a) Characteristics.
b) Resources.
c) Goals.
d) Orientations.

Following Schoenfeld's decision-making model, the questionnaire was structured as shown in Table 1.

Each category was transformed into a question. The format was presented to teachers with a series of sentences tailored to different dimensions (some examples are presented in the Results section). An expert was then asked to assess the questionnaire in terms of the coherence, relevance and clarity of each question, and to provide feedback for

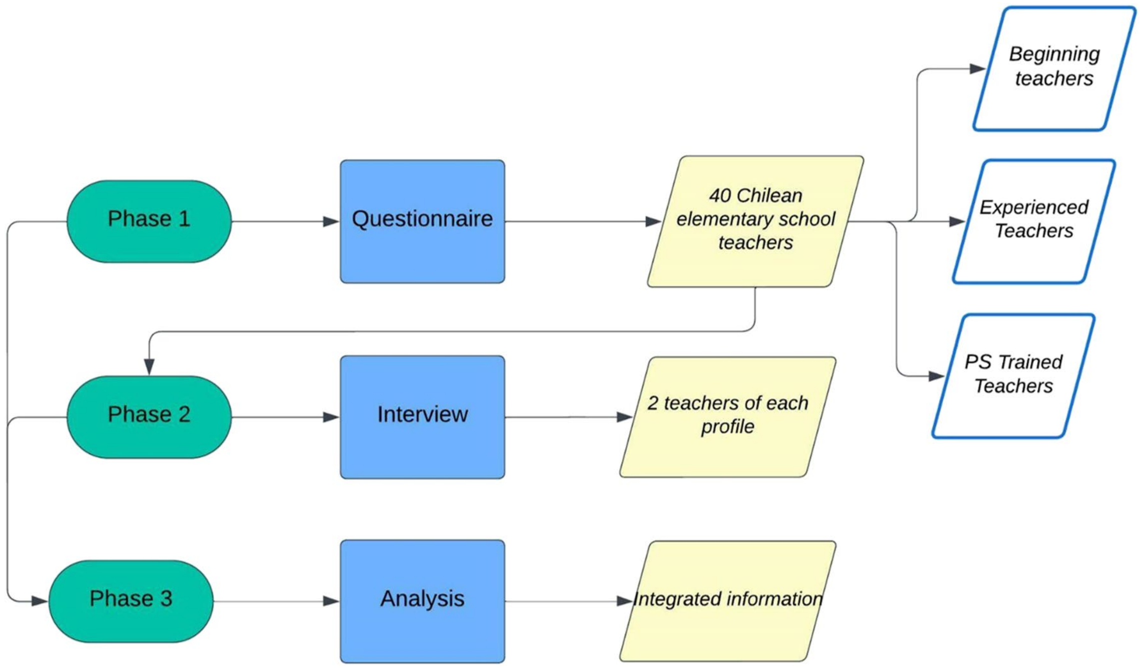

Fig. 1 General structure of the study

improvement. After incorporating the suggested adjustments, a pilot version of the questionnaire was administered to 20 in-service teachers, leading to the final version.

The semi-structured interview guide for the second phase was based on the same questionnaire used in the first phase. Starting with the same questions, the teachers were asked to explain the reasons behind their responses. During these explanations, the interviews were guided to delve deeper into the decisions reflected in each dimension. The interviews were recorded and transcribed, with the consent of the participants. Each teacher was assigned a code according to their profile (for example, BT_1 refers to beginning teacher 1).

5.2 Analysis

The questionnaire responses obtained in Phase 1 were analysed using descriptive statistics, in the view that this approach provides an acceptable approximation of the state of the surveyed group.

The data obtained through the semi-structured interviews in Phase 2 were analysed using the content analysis method and the constant comparison technique (Strauss & Corbin, 2002). The unit of analysis employed for this was the free-flow unit, considering segments of variable size and determining the end of each segment until a complete meaning was established (Hernández-Sampieri & Mendoza, 2018).

For example, concerning the resources dimension, BT_1 reported that the section of curriculum materials she most commonly consulted in planning the teaching of problem solving was the explanation of mathematical content. During the interview, she said:

> *I need to verify certain content. Occasionally, I come across what I believe are errors in the texts and perhaps I lack sufficient knowledge to explain them. In such cases, I rely on the Curriculum to provide a thorough explanation. Additionally, during class, children's questions often arise and if I lack the knowledge to explain, I review the material afterwards.*

In this response (but necessarily in every case), we identified the presence of a decision coded as *Accessing mathematical content*. The same type of decision was also present in the responses of ET_1. On the other hand, when answering the same question, PTT_1 responded:

> *I primarily focus on reviewing the challenges (PS tasks) presented in the text. Initially, it is complex and one of the major challenges is the necessity to prepare for problem solving classes. As some problems are quite challenging, you have to look carefully at what you want to achieve with these types of tasks and how your students can approach them. Different children may choose varying methods, such as solving with paper and pencil, using drawings, or employing cubes. So everything will depend on the skills that the children have.*

Table 1 Dimensions and categories of the questionnaire

Component of the decision-making model	Questionnaire dimension	Categories
Resources (personal)	Participant characteristics	Years of experience Courses taken on mathematics teaching Courses taken on PS
Resources (curricular)	Curricular resources consulted	Official curriculum Public textbooks Commercial textbooks Others
	Curricular sections consulted	Explanations of mathematical content General guidelines Goals Tasks Skills/attitudes Methodological guidelines Examples of evaluation tasks Guidance for evaluation
	Resources provided by the context	Existence of conditions to: Develop reasoning through PS Make autonomous decisions regarding the teaching of PS Access professional development opportunities on PS Incorporate PS as part of the evaluation Work the curriculum flexibly Learn to teach using PS in classes Develop observation and listening skills when students solve problems Access resources to work on PS in all areas of mathematics
Goals	Purpose of learning mathematics according to the curriculum	Learn concepts to transfer to real life problems Learn PS strategies and steps Address challenges to understand relevant mathematical ideas
	Purpose of implementing PS according to the curriculum	Teach concepts and then be able to solve problems Transmit PS strategies Guide the development of reasoning
Orientations	Approaches to teaching PS	Teach how to solve problems Teach about PS Teach through PS
	Interpretations of PS according to curricular resources	A situation that seeks a correct answer A situation in which one must seek, through specific steps, an appropriate response A situation that requires a solution and apparently does not have an immediate path leading to it

PS problem solving

This response reflects a different type of decision made when using curriculum materials, which we term *Selecting the problem solving task according to the characteristics of the students*.

After analysing all the interviews in this manner, we reviewed the categories obtained and ultimately derived seven categories of decisions made by teachers when planning their instruction for problem solving. We concluded the analysis by assigning each category to a planning style according to Schoenfeld's levels of proficiency adapted to the planning stage.

6 Results

6.1 Resources

In this analysis, personal resources were defined by the characteristics of each teacher profile. The BT group had less experience and training in mathematics teaching and problem solving. The PSTT, on the other hand, acknowledged the benefits of problem solving training in their teaching. PSTT_2, for example, recognised the need for in-depth preparation and the thorough study of a new methodology focused on problem solving that was implemented in her school:

> *Well, when the school started implementing the methodology, we received training from the Ministry. But they only gave us a general overview of the topic. We faced many difficulties when we encountered the textbooks. […] The biggest challenge was that most teachers were not familiar with the subject. So, you can't just pick up the textbook and teach a class. You have to review it in advance, anticipate how the children might respond.* (PSTT_2)

The differences among the three teacher profiles extend beyond experience and training. They are also apparent in the specific resources employed during teaching planning. Table 2 outlines the most frequently utilised curricular resources for each teacher profile.

The teachers in the BT group most commonly consulted the textbooks and teaching guides provided by the Ministry of Education. Thus, one observed:

> *Almost always, as a new teacher, you are going to do what the paper says, whether it's following the text... because it's also the first instruction given by the school's Pedagogical Technical Unit: please follow the textbook and go through the units.* (BT_2)

The ET group also consulted the textbooks and teaching guides provided by the State, but to a lesser extent than the BT group. In contrast, the PSTT primarily relied on the official curriculum, subsequently incorporating textbooks, whether public or private editions. In this respect, one said:

> *The official curriculum is like the teacher's Bible. It gets to be torn and marked because it's used week after week when planning classes. The Schedule is like... my constant work, I open it weekly or at least once a week without fail. And also the students' textbook, which I use as a supplement for preparing my classes.* (PSTT_1)

Through the interviews, we observed that both of the PSTT considered textbooks as a tool helping them achieve the learning objectives outlined in the official curriculum. Here, we found an intersection with the goals set by the PSTT group, as the objective pursued in selecting among resources is to comply with the official curriculum:

> *Usually, I use other supporting texts that align with the textbooks used by the Ministry. These materials are from publishers who have implemented programmes with good results, so I try to incorporate them. In the realm of problem solving, I perceive it as pervasive across the entire curriculum. Problem solving is embedded in each unit because the view here in Chile is that we are teaching mathematics for real-life applications. Consequently, focusing on problem solving is essential.* (PSTT_1)

Moreover, within a single information source, there may be various sections of curricular resources that teachers consult when planning their instruction. One of the most often consulted types of section, by all three profiles, is that of the learning objectives of the official curriculum (Table 3).

Learning tasks and methodological guidelines are frequently consulted by teachers in the BT group. Conversely, there are several curricular sections that the ET group make less use of. The BT group tends to seek explanations of the content and introductory sections or general guidelines on approaching curricular materials, and less frequently consult the section on general guidelines. Lastly, we examined whether teachers access additional resources, facilitated in the school environment, to support and improve the planning processes. Such resources should ideally be provided either by the school or by the Ministry of Education (Table 4).

Table 2 Types of curricular resources consulted by teachers

Curricular resources	BT	ET	PSTT
Textbooks and teaching guides provided by the State	3.38	3.05	2.82
Official curriculum and other materials created by the Ministry of Education	3.00	2.95	3.13
Commercial textbooks and teaching guides	1.87	2.12	2.36
Other materials	1.25	1.52	1.27

The scores represent the average of each group on a scale of 1 to 4, where 1 = I hardly ever consult it, 2 = I consult it sometimes, 3 = I consult it frequently and 4 = This is the main material I use

Table 3 Type of curricular section consulted in curricular resources

Type of section	BT	ET	PSTT
Learning objectives	3.88	3.33	3.55
Learning tasks	3.88	3.05	3.18
Skills/attitudes	3.13	3.19	3.18
Methodological guidelines	3.38	2.76	3.09
Assessment tasks	3.00	2.71	3.18
Explanation of mathematical content	3.13	2.81	2.91
Assessment guidelines	2.88	2.43	3.09
Introduction/general guidelines	2.38	2.48	2.36

The scores show the average for each group on a scale from 1 to 4, where 1 = I hardly ever consult it, 2 = I consult them sometimes, 3 = I consult them frequently and 4 = it's the main section I consult

Table 4 Teachers' opinion regarding access to other resources when planning their instruction for problem solving

Resource	BT	ET	PSTT
a. Guidelines for providing reasoning-based instruction through PS	2.63	2.05	1.82
b. Conditions for making autonomous decisions regarding the teaching of PS	2.63	1.86	1.91
c. Professional development opportunities related to teaching PS	2.00	2.05	1.82
d. Curricular flexibility (time, quantity and content of learning objectives, etc.)	2.38	1.86	1.82
e. Learning about ways to teach PS in the classroom	2.25	2.00	2.00
f. Support for developing observation and listening skills towards students while they solve problems	2.38	2.00	1.91
g. Resources for teaching various subjects through PS	2.25	2.05	2.00
h. Resources for incorporating PS as part of assessment	2.50	2.14	2.00

The scores represent the average for each group on a scale from 1 to 4

PS problem solving

Table 5 Teachers' views on the role of problem solving in the teaching of mathematics

Role of problem solving	BT	ET	PSTT
Teaching for problem solving	3.13	2.92	2.59
Teaching about problem solving	3.50	3.06	3.14
Teaching through problem solving	3.25	3.20	3.18

The scores represent the average for each group on a scale from 1 to 4

6.2 Orientations

In this section of the questionnaire, teachers were presented with statements about problem solving, in order to elucidate their orientations regarding this aspect of mathematics education. Table 5 summarises the results obtained.

The BT group exhibited high levels of agreement with all three possible roles of problem solving according to Schroeder and Lester (1989), but awarded the highest score to the statement "Teaching about problem solving". Of the three alternatives offered, the ET group was least strongly oriented towards "Teaching for problem solving". The PSTT group agreed least (among the three alternatives and in comparison with the other two groups) with the statement "Teaching for problem solving". In this respect, PSTT_2 stated:

> *I agree that all three roles are important. Children are indeed fond of mathematical challenges and being able to successfully tackle a challenge brings them a great sense of achievement. However, it is crucial to acknowledge that, to engage with and overcome these challenges, students need to acquire mathematical skills and strategies at some point. These skills and strategies serve as the foundation for their ability to effectively approach and solve mathematical problems.* (PSTT_2)

Teachers commonly use textbooks as curriculum resources, as they provide information on mathematical concepts through tasks and guides. Understanding the concept of a problem is crucial for selecting, designing, implementing, or evaluating problem solving tasks. We asked teachers about their interpretation of the concept of a problem in textbooks (Table 6).

The most notable finding from the results is that none of the PSTT teachers chose the option that defines problems as situations requiring a correct answer. Conversely, the ET group were least likely to select the option indicating that problems do not have an immediate solution.

6.3 Goals

Regarding goals, when we talk about planning and designing teaching, the first thing to consider is the objectives

Table 6 Interpretation of the concept problem proposed in textbooks

Response	BT	ET	PSTT
A situation that seeks an explanation or correct response	25%	23.8%	0%
A situation in which one must seek, through specific steps, an appropriate response	25%	61.9%	45.5%
A situation that requires a solution and apparently does not have an immediate path leading to it	12.5%	9.5%	27.3%
I don't know/It isn't indicated	37.5%	4.8%	27.3%
Total	100%	100%	100%

proposed by the official curriculum for the subject of Mathematics. The role that problem solving will play depends on this purpose. For instance, if the goal of teaching mathematics is the mechanical learning of procedures and algorithms, problem solving will have a secondary role. If, on the contrary, the purpose is related to achieving higher levels of comprehension through mathematical thinking, problems would play a fundamental role. However, even when the curriculum expresses a specific purpose, the interpretation made by each teacher is influenced by their beliefs, resources and personal goals. We presented three statements to the teachers, asking them to indicate which aligned most closely with the purpose of the Mathematics subject according to the official curriculum (Table 7).

The statement concerning the application of concepts and procedures was the most frequently chosen option across all three groups. The emphasis on real-world relevance is evident. Based on these findings, it can be inferred that the participating teachers prioritise teaching students how to apply their learning, particularly within everyday contexts. Regarding this, ET_1 commented:

> *"Let me see... Well, I think... from what I perceive, that Mathematics is not meant to be worked on in isolation. The idea is for it to complement other subjects so that students can use it in a way that enables them to resolve situations, perhaps not only in Mathematics but also in other subjects, in general... in life."* (ET_1)

Other noteworthy findings in this regard include the following: none of the BT group opted for the choice related to achieving understanding through challenges. Among the ET group, this option was selected by a small minority, while rather more of the PSTT group chose this option. Interestingly, some of the BT group appeared to be unfamiliar with the stated purpose of the subject, as outlined in the official curriculum ('to enhance understanding of reality, facilitate the selection of problem solving strategies and foster the development of critical and independent thinking in all students,' Ministry of Education, 2012, p. 86).

In this respect, we also asked the teachers specifically about their interpretation of the official curriculum regarding the purpose they should seek when teaching problem solving (Table 8).

In this instance, preferences differed among the three groups. The ET group primarily opted for teaching concepts for future application. The transmission of problem solving strategies was the least favoured option among the PSTT group. Interestingly, the latter group was more likely to select the option related to guiding student development, compared to the other two groups. And as noted above, some members of the BT group indicated a lack of awareness.

6.4 Decision making in lesson planning for problem solving

Qualitative analysis by constant comparison enabled us to identify seven categories corresponding to the decisions revealed in the interviews (Table 9). Additionally, in accordance with Schoenfeld's (2011) adapted levels of proficiency, we categorised the decisions focused on curriculum management with a lighter colour, those aimed at planning engaging activities with an intermediate colour and those identified for diagnostic planning with a darker colour.

Table 7 Purpose of the Mathematics subject according to the official curriculum

Response	BT	ET	PSTT
To enable students to learn concepts and procedures that can be transferred and applied to the world of work and everyday life	50%	52.4%	45.4%
To enable students to learn to solve problems by incorporating strategies and steps of resolution	37.5%	28.6%	27.3%
To face students with challenges that produce an understanding of mathematical ideas	0%	19%	27.3%
I don't know/It isn't indicated	12.5%	0%	0%
Total	100%	100%	100%

Table 8 Purpose when teaching problem solving according to the official curriculum

Response	BT	ET	PSTT
To teach concepts and procedures effectively so that students can subsequently solve proposed problems	12.5%	52.4%	45.5%
To transmit a set of problem solving strategies to their students	25%	23.8%	9%
To skilfully guide the development of reasoning in their students	37.5%	23.8%	45.5%
I don't know	25%	0%	0%
Total	100%	100%	100%

Table 9 Decision making regarding lesson planning for problem solving

Teacher	Accessing mathematical content	Prioritising problem solving vs. traditional teaching of content	Use of resources	Modify resources	Intentional selection of the PS tasks	Analysis and preparation of problems	Select the PS task according to the characteristics of the students
BT_1	X	X	X	X	-	-	-
BT_2	-	-	X	X	-	-	-
ET_1	X	X	X	X	-		
ET_2	-	-	-	X	-	X	X
PSTT_1	-	-	X	X	X	X	X
PSTT_2	-	-	X	X	X	X	X

- Accessing mathematical content: in the context of planning a problem solving class, this activity involves consulting documents or materials to clarify aspects of mathematical content, including definitions, procedures and explanation.
- Use of resources: determining the type of resource required for support, when to use it and how to integrate it into the instruction. This category was cited in all interviews except by ET_2, who indicated a reliance on professional knowledge to improvise tasks rather than consulting curriculum materials.
- Choosing between implementing problem solving activities or teaching the curriculum content in a traditional way: one teacher from each profile expressed the dilemma of deciding between implementing problem solving tasks or teaching the curriculum content in a traditional way. The primary reason cited for this decision was time pressure. All three teachers agreed that problem solving is often perceived as a separate component of the curriculum.
- Modifying tasks or materials from curricular or instructional resources: in other words, selecting problem solving tasks or particular elements (e.g., images) and modifying or adapting them, typically to adjust their presentation format rather than altering the task itself (e.g., its level of difficulty).
- Selecting a problem solving task intentionally in relation to a teaching goal: purposefully searching for and selecting problem solving tasks to achieve specific teaching objectives within a coherent sequence of tasks and a structured progression of learning. This decision making process was identified in the interviews conducted with the PSTT group.
- Studying and preparing the problem solving task before class: not only selecting tasks with clear objectives but also dedicating time to thoroughly study them in preparation for teaching.
- Selecting the problem solving task based on student characteristics: taking into consideration students' capabilities and prior knowledge when choosing or adapting tasks. This requires the teacher to anticipate possible student responses and to prepare alternative pathways accordingly.

6.5 Interaction between resources, goals and orientations in decision making

Figure 2 summarises the resources, goals, orientations and types of decisions made by the representatives of the three teaching profiles in their lesson planning for problem solving.

This framework should not be seen as a generalisable model; rather, it synthesises the results specific to our study sample. Nonetheless, it can provide useful insights that can be applied in analysing other cases, as a reference or starting point. But it should not be viewed as rigid or absolute. We present the orientations first, followed by goals and then resources to aid our narrative flow. However, we acknowledge that these elements and processes are dynamic and not necessarily linear.

Firstly, represented in different colours, are the three teaching profiles. The planning decisions adopted by the members of each teaching profile are influenced by their particular orientations, goals and resources, in different ways.

For example, the questionnaire results show that the teachers from all three groups expressed orientations toward teaching for, about or through problem solving, albeit to varying degrees. However, not all articulated a goal of challenging their students with tasks fostering mathematical understanding. Indeed, none of the BT expressed such a goal. Conversely, teaching mathematics for everyday life was a goal expressed by teachers from all groups, both in the questionnaire and in the interviews.

According to the views expressed in the interviews, the decisions made by each teacher depend on the resources available. All three types of teachers spoke of the need to use and adapt curriculum resources. Access to official curriculum documents and textbooks allows them to make

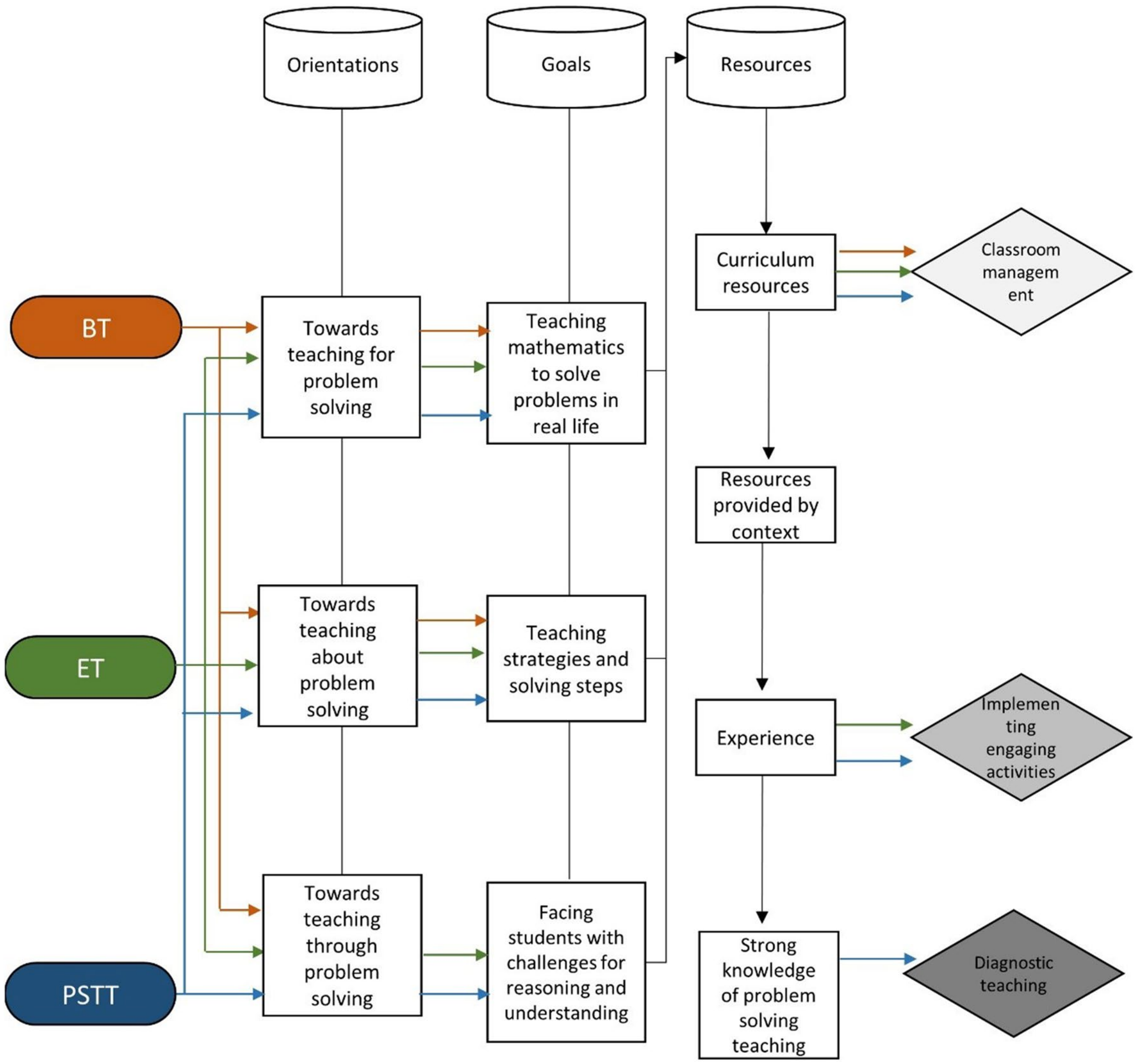

Fig. 2 Resources, goals, orientations and types of decisions made by the three teaching profiles

fundamental decisions in designing their teaching. However, there are other resources that not all teachers have. For example, contextual resources, such as time to prepare lessons or opportunities for collaboration with peers, are uncommon within the Chilean education system. Consequently, even if a teacher recognises the importance of problem solving in mathematics classes, constraints like insufficient preparation time or the need to address students' needs (as in the case of ET_1) may lead them to prioritise traditional content-based teaching methods.

Experience was cited as a crucial factor in designing motivating activities, as it provides teachers with a repertoire of tasks they have tried and validated over the years. Furthermore, a strong background in problem solving is essential for effective diagnostic teaching. Only in the interviews with the two PSTT did we identify decisions specifically regarding the teaching of problem solving. To make such decisions, the teachers needed to familiarise themselves with problem solving methodologies, understand what types of problems would challenge their students, learn how to analyse them and deploy all available resources, in order to maximise the effectiveness of each learning task.

7 Discussion and conclusions

In this study, we examine the resources, goals and orientations that guide different types of teachers in incorporating problem solving into their teaching routine.

We observed variations among the three groups of teachers in terms of resources. Thus, beginning teachers (BT) tend to rely more than the others on official resources like public textbooks and the official curriculum, in order to meet school requirements. This finding is in line with previous work by Santagata and Yeh (2016) and Bieda et al. (2020).

We also found differences in the curricular sections consulted by each profile. Remillard (2012) notes that this is a relevant form of interaction with teaching resources. In Chile, the latest curriculum reform focuses on skills like problem solving, an aspect that is often highlighted in the introductions of curricular materials and general guidelines. However, it is precisely these types of sections that are least often consulted by the teachers, in all three groups.

The problem-solving trained teachers (PSTT) are most likely to adopt the "through problem solving" approach and the least likely to prefer "teaching for problem solving". These teachers are more likely than BT to present challenges as a class objective. BT aim to develop problem-solving strategies and reasoning but do not select problems according to the mathematics being taught or students' learning levels.

The experienced teachers (ET) differed from their PSTT colleagues, which suggests that experience alone is insufficient. As highlighted by Stahnke et al. (2016), Cevikbas et al. (2023) and Saadati and Felmer (2021), professional development programmes are crucial for teachers to acquire greater competence in this respect. Becoming a PSTT requires access to development programmes and time to prepare diagnostic-focused classes. In our opinion, changes, however worthy, in national curriculum design must be supported by sufficient conditions, and professional development cannot be left to teacher discretion.

Our findings diverge from Remillard (2012), for whom experienced teachers either adhered strictly to curriculum materials or adapted them as they saw fit, while novice teachers used guides to impart major mathematical ideas. In contrast, we found that beginning teachers strictly followed the text, while experienced teachers with problem solving training adapted resources while also focusing on curriculum ideas. This indicates that modes of engagement with materials depend on multiple factors: years of experience, initial training, ongoing professional development, prior beliefs and the demands of the cultural context (e.g., educational system requirements or school context). Further research is needed to determine the extent to which each of these factors contributes to modes of engagement with the curriculum.

The present study identifies seven types of decisions made by different types of teachers, facilitating appropriate anticipatory actions. In Chile, public textbooks are standardised throughout the country, which poses a challenge to attempts at their adaptation. BT need access to mathematical content, suggesting they should either be given stronger initial training or be provided with separate materials. Curriculum designers need to take this into account to avoid forcing teachers into a dilemma (the proposed "Prioritised Curriculum" is expected to adopt this approach).

For diagnostic planning, a specific decision type was identified, namely selecting problems based on student characteristics. Future research should explore this question with larger samples to identify other decision types and make appropriate recommendations for teacher training, from the initial stage onwards.

This study is subject to certain limitations, such as the difficulty of isolating teachers' beliefs from the way in which they interpret curricular regulations. The context of Chilean teachers and their initial training is also important, as there may be significant differences among those who work in different regions. It should also be acknowledged that a closed questionnaire, while facilitating data collection and analysis, limits the range of possible responses that can be obtained. Finally, the small size of the group of interviewees is a significant problem. Future work in this field requires more extensive investigation of certain questionnaire topics, with larger and more diverse population samples.

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Declarations

Competing interests José Luis Lupiáñez, Daniela Olivares and Isidoro Segovia all declare they have no relevant financial or non-financial interests to disclose.

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