Fluid Antenna-Assisted Dirty Multiple Access Channels Over Composite Fading

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Abstract— This letter investigates the application of the emerging fluid antenna (FA) technology in multiuser communication systems when side information (SI) is known at the transmitters. We consider a K-user dirty multiple access channel (DMAC) with non-causally known SI at the transmitters, where K users send independent messages to a common receiver with a FA capable of changing its location depending on the channel condition. By connecting Jakes' model to copula theory through Spearman's ρ rank correlation coefficient, we accurately describe the spatial correlation between the FA channels, and derive a closed-form expression for the outage probability (OP) under Fisher-Snedecor $\mathcal{F}$ fading. Results confirm that a FA-aided receiver can achieve better performance in multiuser communication: (*i*) increasing the number of FA ports as well as FA size can significantly lower the OP; (*ii*) FA can support a large number of users using only one FA at the common receiver in a few wavelengths of space.

Index Terms— Fluid antenna, multiple access channel, Fisher-snedecor $\mathcal{F}$ fading, copula theory, outage probability.

I. INTRODUCTION

MULTIPLE-INPUT multiple-output (MIMO) systems, including multiuser MIMO and massive MIMO, have become one of the greatest advances in mobile communications over recent years. In this regard, deploying multiple antennas far apart at fixed locations at the transmitter and/or receiver sides enables MIMO to multiplex users entirely in the spatial domain and enhances the network capacity, e.g., [1].

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In the fifth-generation (5G) of wireless communications, massive MIMO can efficiently simplify the signal processing of multiuser signals by adopting a massive number of antennas (e.g., over 64) at the base stations (BSs) [2]. The sixth-generation (6G) will also see extra-large MIMO succeed by even utilizing continuous-aperture MIMO (CP-MIMO) [3]. However, despite the outstanding advantages of different types of MIMO systems, there are practical issues such as the complexity of precoding optimization and channel state information (CSI) acquisition. In addition, ensuring the minimum distance between antenna spacing to be half of the wavelength can limit the integration of MIMO systems inside mobile devices due to physical space limitations [4].

In order to address the aforesaid challenges in future mobile communication technology, fluid antenna (FA) systems have recently been introduced in [5] and [6] to obtain additional diversity and multiplexing benefits by exploiting novel dynamic radiating structures. In contrast to conventional antenna systems, a FA is a software-tuneable fluidic (or radio frequency (RF) pixel-based), conductive, or dielectric structure [7] that has the ability to switch its location (i.e., ports) in a given small space. Therefore, these unique features can provide a stronger channel gain, lower interference, and other desirable performance for future wireless communications. In this regard, great efforts have recently been carried out to evaluate the FA system. Initial performance analysis for a single-user FA system was studied in [5] and [8] over correlated Rayleigh fading channels. Additionally, an analytical expression of the outage probability (OP) for a point-to-point (P2P) single FA system under correlated Nakagami-m fading channels was obtained in [9]. Moreover, the closed-form expression of the spatial correlation parameters for a P2P FA system was obtained in [10] and a general eigenvalue-based model to accurately approximate the spatial correlation between FA ports given by Jakes' model was proposed in [11]. Only recently, a general copula-based formulation was proposed in [12] for the joint multivariate distribution of arbitrary correlated fading channels in the FA system which can describe the spatial correlation between FA ports beyond linear dependence structures. Besides, apart from the above-mentioned single P2P FA systems, the authors in [13] evaluated the performance of FA multiple access (FAMA) in terms of the OP and average capacity.

Motivated by the great potential of FA and the importance of analyzing the FA system in realistic propagation environments, this letter aims to investigate the performance of FA-aided multiuser communication systems under generalized fading conditions. Specifically, we consider a K-user dirty multiple access channel (DMAC) in which each user takes advantage of non-causally available side information (SI) and sends an

independent message to a common BS receiver equipped with an FA system through correlated Fisher-Snedecor $\mathcal{F}$ fading channels. In this model, SI is an information-theoretic concept that mainly refers to either CSI or interference awareness, and has the ability to meet the reliability constraint in future wireless networks by reducing the destructive effects of the interference and providing reliable communication at higher rates [14], [15], [16]. In particular, by connecting Jakes' model to copula theory with the help of rank correlation coefficients such as Spearman's ρ, we first provide an analytical expression for the joint cumulative distribution function (CDF) in the considered multiuser FA system under correlated Fisher-Snedecor $\mathcal{F}$ fading channels, and then derive the OP in a closed-form expression. By considering such a mixture of the copula[1] technique and Jakes' model over the FA system, the complexity of mathematical analysis can be remarkably reduced compared with the case when only Jakes' model is applied, thereby achieving more accurate insights into the spatial correlation effects between the FA ports. Our numerical results illustrate how considering the FA in multiuser communication systems can significantly enhance the system performance in terms of the OP by supporting a large number of users.

II. SYSTEM MODEL

We consider a K-user multiple access communication system with known interference S_k, $k \in \{1, \ldots, K\}$, where K single fixed-antenna transmitters t_k aim to send independent messages X_k to a common receiver r equipped with a single FA, respectively. We assume that the interference signals S_k with zero mean and variances Q_k (i.e., $S_k \sim \mathcal{N}(0, Q_k)$) are known non-causally to the transmitters t_k, and the information signals X_k sent by transmitters t_k are subjected to the average power constraint $\mathbb{E}\left[|X_k|^2\right] \leq P$, respectively. Moreover, we suppose that the FA includes only one RF chain and N preset positions (i.e., ports), which are evenly distributed on a linear space of length $W\lambda$ where λ denotes the wavelength of the radiation. Thus, relative to the first port, the distance between the first port and the n-th port is given by

$$d_n = \left(\frac{n-1}{N-1}\right) W\lambda, \quad n = 1, 2, \ldots, N. \quad (1)$$

Furthermore, the received signal at the n-th port of the common receiver r can be written as

$$Y_n = \sum_{k=1}^{K} h_{n,k} X_k + S_k + Z_n \quad (2)$$

in which $h_{n,k}$ denotes the fading channel coefficient of the k-th port from the n-th transmitter and Z_n is the independent identically distributed (i.i.d.) additive white Gaussian noise (AWGN) with zero mean and variance σ^2 at each port. Under this model, we assume for the sake of generality that the square of the channel amplitude $|h_{n,k}|^2$ follows Fisher-Snedecor $\mathcal{F}$ distribution with the following CDF [19]

$$F_{|h_{n,k}|^2}(v) = I_{\frac{m_1 v}{m_1 v + m_2}}\left(\frac{m_1}{2}, \frac{m_2}{2}\right), \quad (3)$$

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[1]Copula theory is a flexible statistical approach that can accurately generate the multivariate distributions of two or more arbitrarily correlated random variables and describe the corresponding dependence structure beyond linear correlation. This has recently become quite popular in the performance analysis of wireless communication systems [17], [18], [19].

where $I_x(a,b)$ is the regularized incomplete beta function and (m_1, m_2) are the corresponding degrees of freedom. In addition, we assume that the FA can always select the best port with the strongest signal for communication, i.e.,

$$h_{\text{FAS},k} = \max\{|h_{1,k}|, |h_{2,k}|, \ldots, |h_{N,k}|\}, \quad (4)$$

where the channel coefficients $h_{n,k}$ for $n \in \{1, \ldots, N\}$ at the selected transmitter t_k are spatially correlated since they can be arbitrarily close to each other. By assuming 2-D isotropic scattering and isotropic receiver ports on the FA, such spatial correlation can be characterized by Jakes' model as [4]

$$\eta_n^{(k)} = \delta^2 J_0\left(\frac{2\pi(n-1)}{N-1}W\right), \quad (5)$$

where $\eta_n^{(k)}$ denotes the linear correlation coefficient that can describe the correlation between the corresponding channel coefficients for each transmitter k, δ^2 accounts for the large-scale fading effect, and $J_0(\cdot)$ denotes the zero-order Bessel function of the first kind. Therefore, under such assumptions, the received signal-to-noise ratio (SNR) at the common FA receiver from the transmitter t_k can be defined as

$$\gamma_k = \frac{P h_{\text{FAS},k}^2}{\sigma^2} = \bar{\gamma} h_{\text{FAS},k}^2, \quad (6)$$

where $\bar{\gamma} = \frac{P}{\sigma^2}$ represents the average SNR.

Further, in a K-user block fading multiple access communications with non-causally known SI at the transmitters t_k and the coherent FA receiver r (i.e., known as DMAC), the instantaneous capacity region is given by [14]

$$\sum_{k=1}^{K} R_k \leq \frac{1}{2} \log_2\left(1 + \min_{k=1,\ldots,K} \frac{P h_{\text{FAS},k}^2}{\sigma^2}\right). \quad (7)$$

III. PERFORMANCE ANALYSIS

Here, we first describe the spatial correlation between the FA ports by combining Jakes' model to copula theory. Then, we derive a closed-form expression of the OP for the considered system under Fisher-Snedecor $\mathcal{F}$ fading channels.

A. Statistical Characterization

Definition 1 (d-dimensional copula): Let $\vec{S} = (S_1, \ldots, S_d)$ be a vector of d random variables (RVs) with marginal and joint CDFs $F_{S_i}(s_i)$ and $F_{S_1,\ldots,S_d}(s_1, \ldots, s_d)$ for $i \in \{1, \ldots, d\}$, respectively. Then the copula function $C(u_1, \ldots, u_d)$ of the random vector $\vec{S}$ defined on the unit hypercube $[0,1]^d$ with uniformly distributed RVs $U_i := F_{S_i}(s_i)$ over $[0,1]$ is given by [20]

$$C(u_1, \ldots, u_d) = \Pr(U_1 \leq u_1, \ldots, U_d \leq u_d), \quad (8)$$

where $u_i = F_{S_i}(s_i)$.

Under such an assumption, Sklar's theorem postulates that there exists one Copula function C such that for all s_i in the extended real line domain $\bar{R}$ [20]

$$F_{S_1,\ldots,S_d}(s_1, \ldots, s_d) = C\left(F_{S_1}(s_1), \ldots, F_{S_d}(s_d)\right). \quad (9)$$

The CDF in (9) is valid for any choice of arbitrary correlated RVs from different families of marginal distributions, where

by applying an appropriate copula function C to it, the corresponding multivariate joint distribution is derived. However, selecting the optimum copula function that perfectly matches to the considered problem is a very challenging task.

In general, there are three main types of copulas that can be used to describe linear/non-linear correlation between RVs, namely: empirical, elliptical, and Archimedean copulas. The empirical class of copula would require a set of real data for analyzing the structure of dependency [21]. Elliptical copulas are defined as the dependence structure of some related elliptical distribution and can be obtained from the respective multivariate distribution function by standardizing the univariate marginal laws, which have a complicated form [22]. Archimedean copulas are often used to correlate a potentially large number of similar RVs by a generator function [23], which have closed-form expressions. Archimedean copulas have also simple structures with nice properties, so that a great variety of families of copulas belong to this class [20]. In this regard, the empirical data analysis shows that the Clayton copula is one of the most popular Archimedean copulas which can perfectly describe any specific non-zero level of low tail dependency between individual RVs. Hence, since the outage of transmission often occurs in deep fading conditions (i.e., tail dependence) over wireless communication systems [24], we here exploit the Clayton copula to describe the structure of dependency between correlated fading channel coefficients and generate the corresponding joint CDF. Our analytical and numerical results will reveal that this choice is justified since it can offer good mathematical tractability and accurately describe the impact of spatial correlation between the FA ports in the considered multiuser communication setup.

Definition 2 (Clayton copula): The d-dimension Clayton copula with a generator function $\phi(t) = \frac{t^{-\beta}-1}{\beta}$ is defined as

$$C_{\mathrm{CL}}(u_1, u_2, \ldots, u_d) = \left[\sum_{j=1}^{d} \left(u_j^{-\beta} - 1 \right) + 1 \right]^{-\frac{1}{\beta}}, \quad (10)$$

where $\beta \in [0, \infty)$ is a dependence structure parameter of Clayton copula. The independent case is achieved if $\beta = 0$.

Here, by utilizing the Clayton copula definition, we derive the CDF of $h_{\mathrm{FAS},k}^2$ in the following lemma.

Lemma 1: The CDF of $h_{\mathrm{FAS},k}^2$ under Fisher-Snedecor $\mathcal{F}$ fading channels is given by

$$F_{h_{\mathrm{FAS},k}^2}(r) = \left[\sum_{n=1}^{N} \left(I_{\frac{m_1 r}{m_1 r + m_2}}^{-\beta} \left(\frac{m_1}{2}, \frac{m_2}{2} \right) - 1 \right) + 1 \right]^{-\frac{1}{\beta}}. \quad (11)$$

Proof: By exploiting the definition of the CDF, $F_{h_{\mathrm{FAS},k}^2}(r)$ can be mathematically defined as

$$
\begin{aligned}
F_{h_{\mathrm{FAS},k}^2}(r) &= \Pr\left(h_{\mathrm{FAS},k}^2 \leq r \right) \\
&= \Pr\left(\max\left\{ |h_{1,k}|^2, \ldots, |h_{N,k}|^2 \right\} \leq r \right) \\
&= F_{|h_{1,k}|^2, \ldots, |h_{N,k}|^2}(r, \ldots, r) \\
&\overset{(a)}{=} C\left(F_{|h_{1,k}|^2}(r), \ldots, F_{|h_{N,k}|^2}(r) \right), \quad (12)
\end{aligned}
$$

where (a) is obtained from (9). Now, by inserting (3) into (12) and considering (10), the proof is completed. $\square$

Note from (11) that the Clayton dependence parameter β can generally capture the positive correlation between the FA ports as well as the independent case. However, it does not necessarily describe the linear correlation because when non-linear transformations are applied to those RVs, the linear correlation cannot be maintained anymore, i.e., copula definition. Additionally, it can be seen that the CDF of $h_{\mathrm{FAS},k}^2$ is only expressed in terms of the number of FA ports N but it is widely accepted that the FA size W has more effects on the spatial correlation between the fading coefficients [10]. In this regard, we need to measure such dependency by connecting the Clayton copula parameter β to the Jakes' model. To this end, we exploit popular rank correlation coefficients such as Spearman's ρ, denoted by ρ_{s}, that can measure the statistical dependence between the ranking of two RVs as

$$\rho_{\mathrm{s}} = 12 \iint_{[0,1]^2} u_1 u_2 dC(u_1, u_2) - 3. \quad (13)$$

The analytical derivation of ρ_{s} is challenging; instead, we compute an alternative rank correlation coefficient [20, Cor. 5.1.4] such as Kendall's $\tau \triangleq 4\mathbb{E}[C(u_1, u_2)] - 1$. Then, we use the approximation that connects $3\tau \approx 2\rho_{\mathrm{s}}$ as described in [20, Cor. 5.1.3], so the integral in (13) is approximated as

$$\rho_{\mathrm{s}} \approx \frac{3\beta}{2(\beta + 2)}. \quad (14)$$

Further, regarding the fact that the Spearman's ρ, i.e., ρ_{s}, for a pair of continuous RVs is identical to Pearson's product moment correlation coefficient for the grades of the corresponding RVs (i.e., $\rho_{\mathrm{s}} = \eta_n^{(k)}$) [20], we can express the Clayton copula parameter β in terms of Jakes' model as

$$\beta = \frac{4\eta_n^{(k)}}{3 - 2\eta_n^{(k)}}. \quad (15)$$

Thus, the CDF of $h_{\mathrm{FAS},k}^2$ can be given in terms of the size W and the number of FA ports N in the following corollary.

Corollary 1: By plugging β from (15) into (11), the CDF of $h_{\mathrm{FAS},k}^2$ under Fisher-Snedecor $\mathcal{F}$ fading channels with the correlation coefficient $\eta_n^{(k)}$ is given by
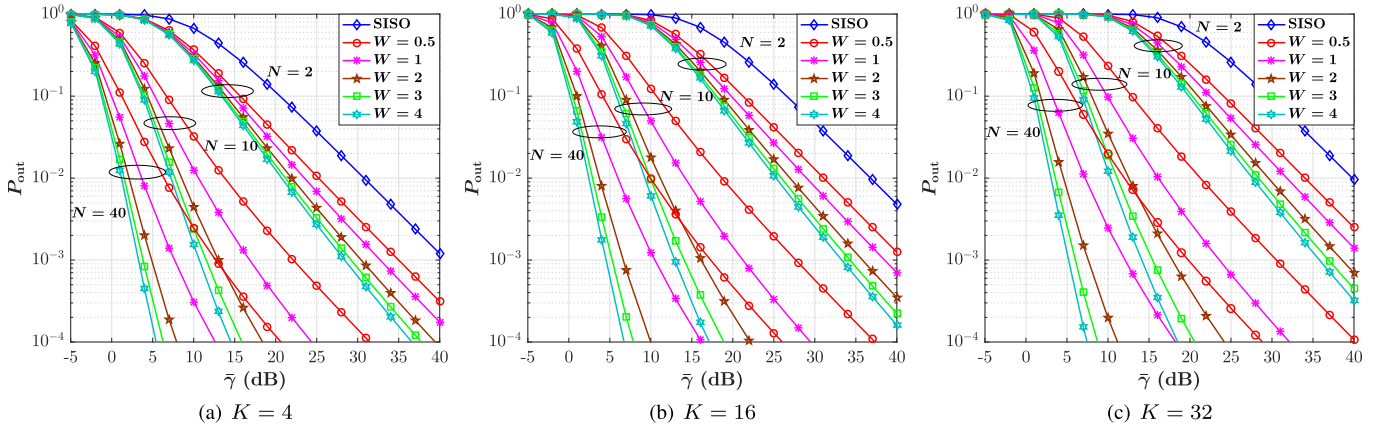
$$F_{h_{\mathrm{FAS},k}^2}(r) = \left[\sum_{n=1}^{N} \left(I_{\frac{m_1 r}{m_1 r + m_2}}^{\frac{4\eta_n^{(k)}}{2\eta_n^{(k)} - 3}} \left(\frac{m_1}{2}, \frac{m_2}{2} \right) - 1 \right) + 1 \right]^{\frac{2\eta_n^{(k)} - 3}{4\eta_n^{(k)}}}, \quad (16)$$

where $\eta_n^{(k)}$ is obtained from (5).

B. OP

Proposition 1: The OP for the considered dirty FA multiuser communication system under Fisher-Snedecor $\mathcal{F}$ fading channels is given by

$$P_{\mathrm{out}}$$

$$= 1 - \left[1 - \left[\sum_{n=1}^{N} \left(I_{\frac{m_1 \gamma_{\mathrm{th}}}{m_1 \gamma_{\mathrm{th}} + m_2}}^{\frac{4\eta_n^{(k)}}{2\eta_n^{(k)} - 3}} \left(\frac{m_1}{2}, \frac{m_2}{2} \right) - 1 \right) + 1 \right]^{\frac{2\eta_n^{(k)} - 3}{4\eta_n^{(k)}}} \right]^{K}. \quad (17)$$

Fig. 1. OP versus average SNR for selected values of the FA size W, number for ports N, and number of users K.
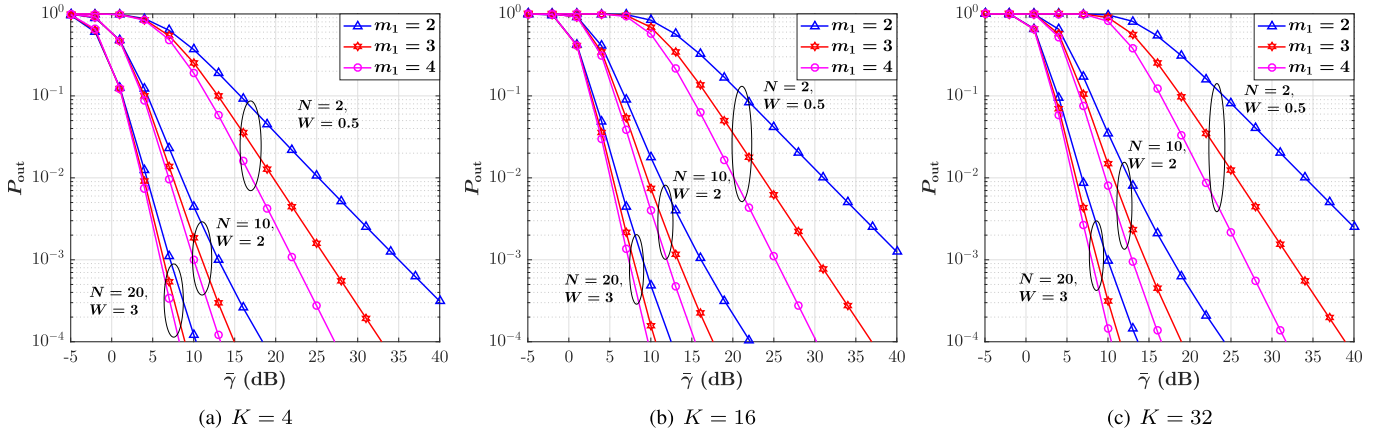
Fig. 2. OP versus average SNR for selected values of the degree of freedom m_1, FA size W, number for ports N, and number of users K.

Proof: By inserting the instantaneous capacity region of the considered dirty FA system from (7) into the OP definition, we have

$$P_{\text{out}} = \Pr\left(\frac{1}{2}\log_2\left(1 + \min_{k=1,\dots,K}\frac{P h_{\text{FAS},k}^2}{\sigma^2}\right) \le R_{\text{th}}\right) \quad (18)$$

$$= \Pr\left(\min_{k=1,\dots,K} h_{\text{FAS},k}^2 \le \gamma_{\text{th}}\right)$$

$$\overset{(b)}{=} 1 - \left[1 - F_{h_{\text{FAS},k}^2}(\gamma_{\text{th}})\right]^K, \quad (19)$$

where $R_{\text{th}} > 0$ is the transmission rate, $\gamma_{\text{th}} = \frac{2^{2R_{\text{th}}}-1}{\bar{\gamma}}$, and (b) is derived by assuming the independence of the fading channels $h_{n,k}$ for $k \in \{1,\dots,K\}$.[2] Now, inserting (16) into (19), (17) is achieved and the proof is completed. $\square$

IV. NUMERICAL RESULTS

Here, we provide numerical results to evaluate the proposed analytical expressions, in a number of multiuser FA set-ups. Parameter values are designed using [5], [16] as a reference. We consider a number of users $K = 4, 16, 32$, and the rest of simulation parameters as $P = 30$dBm, $\sigma^2 = -80$dBm, $R_{\text{th}} = 1$bps/Hz, $\delta^2 = 1$, and $(m_1, m_2) = (2, 4)$. Additionally, to describe the dependence structure between the FA ports and

[2]We assume that the transmitters are far away from each other, and therefore the fading channels between the transmitters and the FA ports at the common receiver are independent.

generate the corresponding correlated Fisher-Snedecor $\mathcal{F}$ RVs (i.e., $h_{n,k}$), we simulate the Clayton copula by exploiting the Marshall and Olkin's approach [25], as shown in Algorithm 1, which is based on the Laplace transform.

Fig. 1 illustrates the performance of the OP in terms of average SNR $\bar{\gamma}$ for given values of the FA size W, the number of FA ports N, and the number of users K under correlated Fisher-Snedecor $\mathcal{F}$ fading. For $K = 4, 16, 32$, it can be seen that by increasing $\bar{\gamma}$, the OP decreases since the channel conditions between the transmitters and the common receiver improve. We can also see that considering the FA in multiuser communication systems can significantly lower the OP compared with the single-input single-output (SISO) system regardless the number of users. Particularly, we observe that for a given number of FA ports (e.g., $N = 2, 10, 40$), the OP reduces as W increases. The reason is that by increasing the spatial separation between the FA ports (i.e., increasing the FA size) for a fixed N, the spatial correlation between the corresponding channels becomes weaker; hence, the system performance in terms of the OP is ameliorated. In addition, we observe that for a fixed FA size W, the OP performance improves as the number of FA ports raises. However, such an improvement is much more noticeable for the large FA size (e.g., $W = 4$) compared to the small values (e.g., $W = 0.5$). It can also be seen that by simultaneously increasing W and N, the best performance for the OP is achieved. Moreover, by comparing the results in Figs. 1(a),

1(b), and 1(c) from the number of users perspective, we can observe that as K grows, the OP performance deteriorates since the network interference increases. Moreover, it can be found that one FA at the common receiver can significantly enhance the system performance in the presence of a large number of users.

Given the importance of accurate fading channel modeling on the performance in real propagation environments, we study the impact of fading severity on the considered multiuser FA system in Fig. 2 by changing the degree of freedom m_1 which mainly refers to as the fading parameter in Fisher-Snedecor $\mathcal{F}$ distribution. For all selected values of K, N, and W, it can be seen that as the fading severity decreases (i.e., m_1 increases), the OP performance is improved, meaning that a lower value of the OP is achieved under a mild fading condition (e.g., $m_1 = 4$) than when a stronger one (e.g., $m_1 = 2$) is considered. In addition, we can observe that the impact of fading parameter m_1 is more tangible on the OP performance when the FA size and the number of FA ports are large. Moreover, it can be found from the results in Figs. 2(a)–2(c) that as the number of users K grows, the effect of fading parameter m_1 becomes minor for a fixed average SNR $\bar{\gamma}$.

Algorithm 1 Clayton Copula Sampling

Step 1. *Generate $J \sim \mathcal{L}^{-1}[\phi(t)]$, such that $J \sim \Gamma\left(\frac{1}{\beta}, \beta\right)$*

Step 2. *Generate $\vec{S} = (S_1, \ldots, S_d)$, such that $S_i \sim \mathcal{U}(0,1)$ for $i = 1, \ldots, d$*

Step 3. *Return $U_i = \phi\left(-\frac{\log(S_i)}{J}\right)$, such that $\phi(t) = \frac{t^{-\beta}-1}{\beta}$*

V. Conclusion

We studied the performance of the FA-aided DMAC with non-causally known SI at the transmitters, where all fading channels follow the Fisher-Snedecor $\mathcal{F}$ distribution. By integrating the copula theory with Jakes' model to describe the inherent spatial correlation among the FA ports, we derived an analytical expression for the joint CDF of the FA multiuser system. Then, we evaluated the considered system performance by obtaining the closed-form expression of the OP. Eventually, our results indicated that considering FA at the receiver can remarkably enhance the system performance in terms of the OP over multiuser communication systems, meaning that the lower values of OP can be achieved as the number of FA ports and FA size increase.

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