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Fuzzy Modelling of Local Linearity in Contours

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***Abstract*—Shape analysis, and particulary the contour study, is a fundamental task for object recognition in images. In this paper, a fuzzy approach for representing the linearity property of a contour segment is proposed. Linearity is a key property related to which degree a contour segment is a curve or a straight line; in addition, it is the basis for modelling other properties like curvature, salience or concavity/convexity. In this framework, firstly, the idea of linearity vs non-linearity, and the meaning of its fulfilment, will be analyzed. Secondly, the definition of a membership function according to that meaning will be proposed on the basis of the coefficient of determination. Finally, we will show the goodness of our proposal by analyzing linearity in a set of shapes with different characteristics.**

I. Introduction

Shape, together with color and texture, is one of the most used properties for image pattern recognition [1]. Its analysis requires a previous process of segmentation for localizing regions of interest (RoI), understood as related groups of pixels, being the characterization of RoI shapes a very important task in the field of computer vision [2][3][4].

Most approaches in the literature for shape analysis and description are mainly divided into two groups: the region-based ones [5], which consider global properties such as area, elongation, etc., and the contour-based ones, which analyze the boundary of the region [6]. The latter is focused on the study of properties such as linearity, curvature, salience, concavity/convexity, etc. and is the one that provides richer information in the shape analysis and description.

Many of the properties associated with a contour are imprecise by nature [7] [8] [9]; for example, if we think about the linearity of a segment, it is clear which is the ideal case of fulfilment (straight line), but linearity is also perceived in segments that, without being straight lines, fulfill the property to some degree. This fact is perceived in the contours showed in the second row of Fig.1, where the first example is a square whose sides are clearly linear segments, while the second shape, without being a square, has a certain degree of linearity and, consequently, a global perception of a squared shape. This is also perceived in other shapes of Fig.1 that could be classified as "square". This imprecision in the properties also generates imprecision in the overall description of the form; for example, if we consider concepts such as "quadrangular" or "triangular", it easy to find examples in Fig.1 that clearly fulfill them, together with intermediate cases and others that do not.

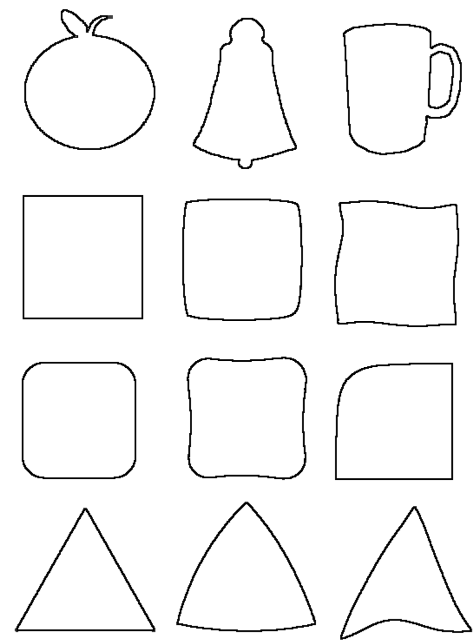


Fig. 1. Examples of contours. The first row corresponds to natural shapes (fruit, bell and cup), while the others represent shapes that could be defined as "quadrangular" or "triangular".

For modelling the imprecision in contour analysis, some works in literature propose the use of fuzzy approaches. Most of these fuzzy techniques [10], [6], [11], [12], [13], [14] are based on active contour models for image segmentation. Within this group, fuzzy tools are often used for characterizing some contour points, but not to model properties; for example, in [12] a fuzzified corner metric based on image intensity is used to identify the feature markers enclosed by the contour for medical image segmentation. Some fuzzy approaches can be also found in the analysis of 3D shapes, where fuzzy logic techniques are employed to describe or localize different types of RoIs [15], [16], [4]. Other interesting fuzzy approaches have been proposed for road line detection [17], shape labeling for automatic image annotation [18], on-line Chinese character recognition [19], pedestrian detection from IR image [20], and modeling of the relation "along" between objects of any shape [21]. However, none of these approaches proposes to model the imprecision associated to the contour properties. To the best of our knowledge, there are no approaches to face this problem in the literature.

In order to contribute to filling this gap, in this paper we will focus on modeling the imprecision associated with the linearity property of a contour segment. This property, besides being useful in the overall description of the shape, is the basis for analyzing other properties, such as curvature or verticity. For this modelling, several issues will be addressed: firstly, the idea of linearity vs non-linearity will analyzed, and the meaning of its fulfilment will be defined (and parameterized). On the other hand, fuzzy models will be proposed according to the previous definition; specifically, line estimation techniques, together with the coefficient of determination as fitting error measure, will be analyzed in order to propose a suitable model for local linearity in contours

The rest of the paper is organized as follow: Section II introduces some notations and preliminary concepts related to contours. After that, the proposal for linearity modelling is described in section III, and some illustrative examples are shown in IV. Finally, section V summarizes the main conclusions and future works.

II. Contours: preliminaries and notations

Let Ω be a connected and bounded image region. Let $C = \partial\Omega$ be the discrete contour of Ω, defined as a cyclically ordered set of points:

$$C = \{\mathbf{p}_i = (x_i, y_i)\}_{1 \le i \le n} \quad (1)$$

where successive boundary pixels in Ω are arranged in a cyclic (clockwise) order. We shall extend the notation to any natural number i by assuming $p_i = p_{i \bmod n}$. Let $\mathcal{C}$ be the set of all discrete contours.

Let $S_{ij}^C = \{\mathbf{p}_k \in C\}_{i \le k \le j}$ be the subset of C representing the segment between $\mathbf{p}_i$ and $\mathbf{p}_j$, and let $\Theta^C = \{S_{ij}^C\}_{i \ne j}$ be the set of all the segments of C. The above definition does not set constraints about the size of the segment. In literature we often find two kind of approaches: those that fix a size, analyzing the contour on the basis of equal size segments, and those that previously segment the contour using some criterion (in that case, segments could be different in size). In the fixed-size approaches, the interpretation depends on the length of the segments relative to the overall length of the contour (i.e, it does not take into account the scale). This problem is faced in the segmentation-based approaches which, in addition, allow to analyze de contour on the basis of simpler primitive segments (for example, convex and concave ones).

III. Fuzzy modelling of linearity

Linearity describes to which degree a contour segment is a line. It is the most simple property of a segment but, in addition, one of the most useful for modelling other properties (like, for example, the curvature). For modelling linearity, two questions need to be analyzed: firstly, the idea of linearity vs non-linearity, and the meaning of its fulfilment (subsection III-A); secondly, the definition of the membership function according to the idea and meaning we have just mentioned (subsection III-B).

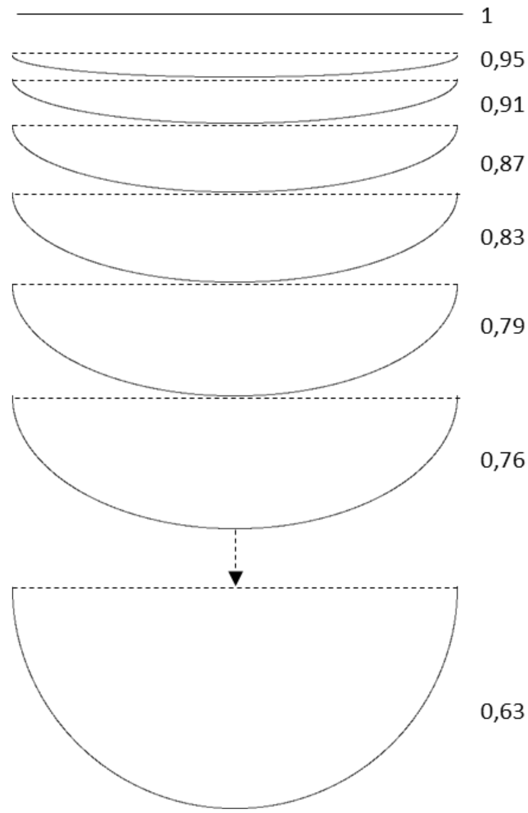


Fig. 2. Sequence of segments representing the transition from line to semi-circumference. This sequence has been generated by applying Eq.(2) with $\alpha = \pi$ and $p \in [0, 1]$. Besides each segment, its coefficient of determination is shown (Eq.(4), with $A(x) = x$)

A. Linearity vs non-linearity

Regarding the idea of linearity, it is easy to think about an example representing the fulfillment of the property (for example, a straight line segment), but, what about something "non-linear"? Taking into account that we are working with contour segments (i.e., with spatial-connected points), it seems natural to postulate that as a straight line segment is transformed into a curve, the degree of linearity should decrease; but, when is the non-fulfilment case reached?

In this paper we propose to associate the non-fulfillment of the property with an arch of a certain angle α (as a particular case, let's think about the semi-circumferential segment, with $\alpha = \pi$). Fig.2 shows an example of a sequence of contour segments ranging from a straight line (top) to a semi-circumference (bottom). If we consider the semi-circumference as the representative case of the non-linear segment, the rest of the segments in Fig.2 would be the intermediate cases (with a membership degree to linearity decreasing from the straight line segment to the semi-circumferential one). Notice that, depending on the particular user perception or the application, the non-fulfillment can be associated to another (usually smaller) angle α.

Contour segments in Fig. 2, understood as sets of points (x_i, y_i) according to the definition 1, have been generated on the basis of the following equation:

$$y^{\alpha,p} = (1-p) + p\sqrt{1-x^2} \tag{2}$$

for all $x \in [-a, a]$, with $a = \sin(\alpha/2)$, and with the parameter $p \in [0, 1]$ representing the transition degree between the straight line ($p = 0$) and the arch ($p = 1$). The value a delimits the range interval of points x that are necessary for generating, using Eq. 2, a certain arch angle α such as its mediatrix is the line $y = 0$; as a particular case, $a = 1$ when $\alpha = \pi$ and we are in the semi-circumference case (see Fig. 2).

B. Membership function

Once the interpretation of linearity and non-linearity is set, we need to define the membership function according to it. Since linearity describes the degree to which a segment is a line, it seems natural to think about some procedure related with a line estimation from the set of segment points and, more specifically, with the fitting error associated to it. Nevertheless, membership degrees in $[0, 1]$ representing the fulfilment of the property are needed, so the fitting error by itself is not enough in this case since i) it is not bounded, and ii) it does not correspond to the interpretation of linearity we have introduced previously.

On the basis of the above idea, in this paper we model the linearity of a segment $S \in \Theta^C$ by means of a fuzzy set $\tilde{L}$ defined as follows:

$$\tilde{L} : \Theta^C \to [0, 1] \tag{3}$$

where the membership degree is defined on the basis of the coefficient of determination as:

$$\tilde{L}(S^C_{ij}) = A\left(1 - \frac{\sum_{k=i}^{j} \|\mathbf{p}_k - \hat{\mathbf{p}}_k\|}{\sum_{k=i}^{j} \|\mathbf{p}_k - \bar{\mathbf{p}}\|}\right) \tag{4}$$

where $\|\cdot\|$ is the Euclidean norm, $\bar{\mathbf{p}}$ is the mean of the points in S^C_{ij}, $\hat{\mathbf{p}}_k$ is the projection of $\mathbf{p}_k$ on the line estimated by means a linear regression process from the set of points S^C_{ij}, and $A()$ is a range adjustment function. The numerator of the ratio represents the sum of squared errors of the fitted model, while the denominator can be thought of as the sum of errors from the null model, that is, a model where each $\mathbf{p}_k$ value is predicted to be the mean of the $\mathbf{p}_k$ values (without having any additional information, the mean would be the best guess if our aim is to minimize the squared differences). Therefore, the ratio is indicative of the degree to which the model parameters improve upon the prediction of the null model. The smaller this ratio, the higher the degree $\tilde{L}(S^C_{ij})$.

C. Range adjustment function

Although the ratio in Eq. (4) is ranging, by definition, in $[0, 1]$, with 0 meaning a perfect fitting, in practice this ratio is in a smaller range when we are working with segment points: remember that they are connected points of a contour, so the extreme case that reflects the denominator (prediction based on the null model) is not reached in practice. Fig. 2 shows, besides each segment, its coefficient of determination (which is equivalent to apply Eq.(4) with $A(x) = x$). It is observed

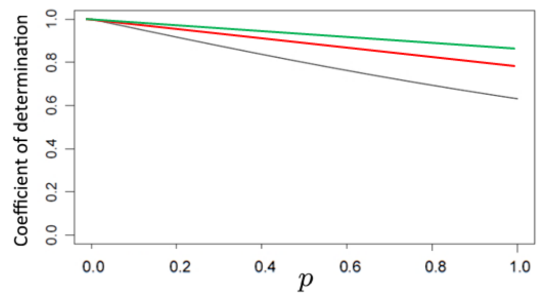


Fig. 3. Coefficient of determination as a function of the parameter p in Eq.(2) for $\alpha = \pi$ (black plot), $\alpha = \pi/2$ (red plot), and $\alpha = \pi/4$ (green plot).

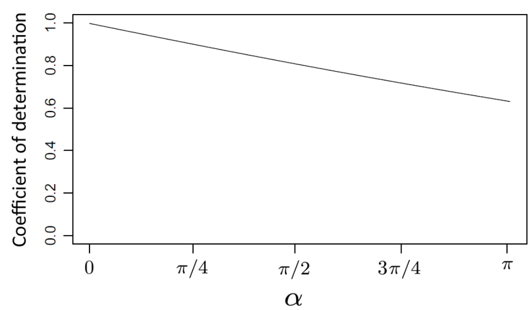


Fig. 4. Coefficient of determination as a function of the parameter α in Eq.(2) for $p = 1$.

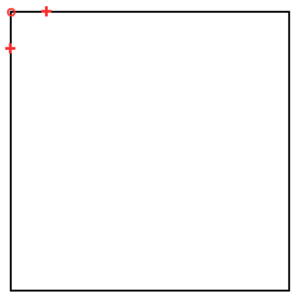
that the value obtained for the semi-circumference is 0.63; in the case of an angle arch $\alpha = \pi/2$ (resp. $\alpha = \pi/4$), the value of the coefficient of determination is 0.81 (resp. 0.91). These examples show the need of a range adjustment function to calculate the membership degree.

Fig. 3 shows, as a function of the parameter p in Eq. (2) and for $\alpha \in \{\pi, \pi/2, \pi/4\}$, the coefficient of determination calculated for the sequence of segments generated following Eq.(2). It is observed that the plotted data has a linear behavior, varying the slope as a function of α (the lower α, the lower the slope). This fact simplifies the selection of the adjustment function, since a linear adjustment will be enough to expand the output range. Thus, in this paper we propose to define the adjustment function $A()$ in Eq.(4) as:

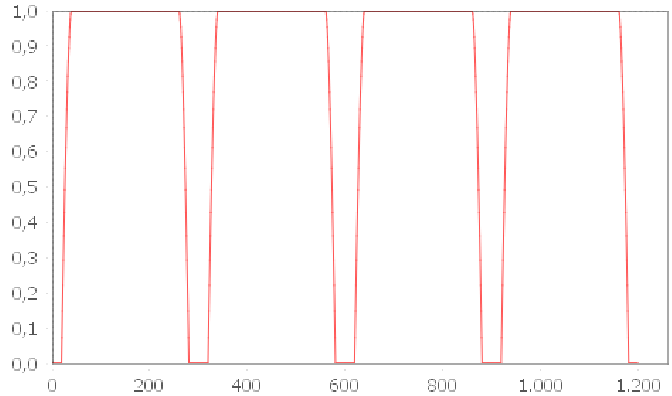
$$A(x; q) = \max\left(\frac{x-q}{1-q}, 0\right) \tag{5}$$

with q being the value at which the function must be zero. In our case, the parameter q is related to the coefficient of determination associated to the non-fulfilment arch (for example, $q = 0.63$ if the non-fulfilment arch is the one associated to $\alpha = \pi$).

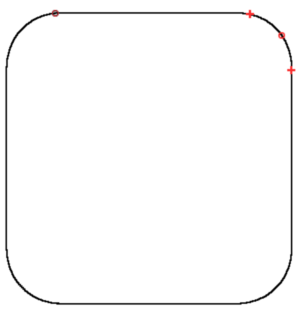
Fig. 4 shows, as a function of the parameter α in Eq.(2) and for $p = 1$, the coefficient of determination calculated for the sequence of segments generated following Eq.(2). Notice that this sequence represents the set of arches that would be selected as non-fulfillment representative of the property; for example, for $\alpha = \pi$, the semi-circumference is generated and the associated coefficient of determination is calculated (0.63



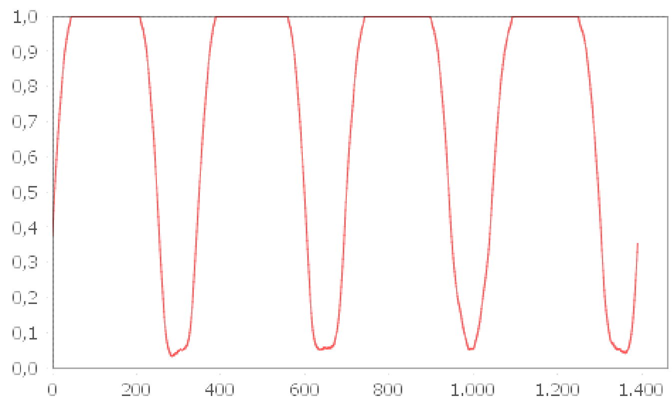
(a)



(b)



(c)



(d)

Fig. 5. Example with two quadrangular shapes: (a) angled corners (b) rounded corners. Beside each contour, a graph ((b),(d)) plotting the linearity membership degree is showed: for each point in the contour, starting in the upper left one (marked with a red point), the segment centered on it is evaluated).

in this case). Analyzing the plotted data, a linear behavior is observed as a function of α. Since the coefficient of determination of the non-fulfillment arches is related to the parameter q in Eq.(5), this behavior leads us to propose an automatic estimation of q as a function of α; specifically, on the basis of a linear fitting, the following equation is obtained:

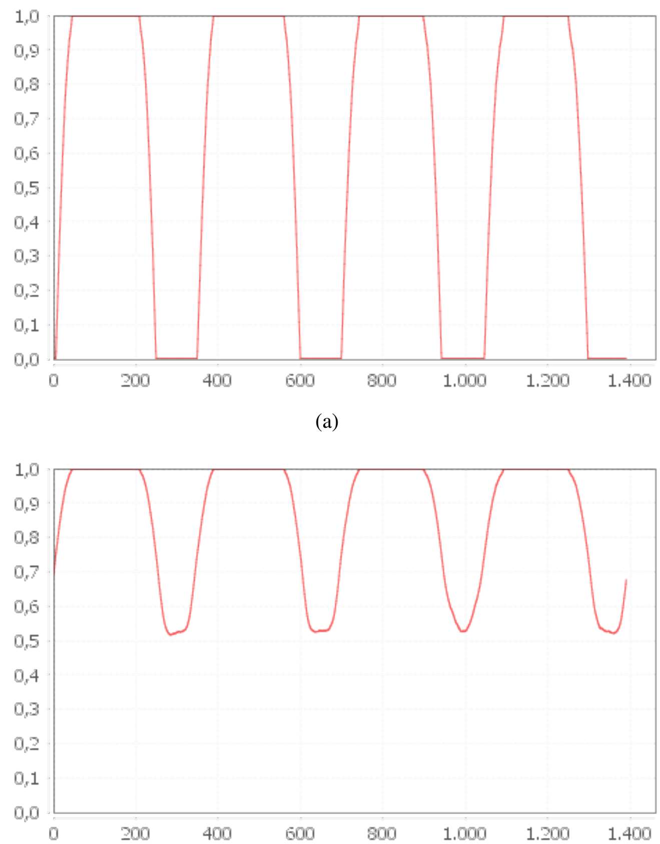
$$q(\alpha) = 1 - \frac{0.37}{\pi}\alpha \tag{6}$$

Then, for applying Eq.(5), we can provide the angle of the arch representing the non-fulfillment of the property (instead of the value q, which is less intuitive).

IV. Results

Figure 5 shows an example with two quadrangular shapes: a perfect square (with angled corners) and a "rounded" square. In this first experiment, we have set $\alpha = \pi/2$, that is, the non-fulfillment is associated with an arch of $\pi/2$ (notice that rounded corners in Fig.5(c) are arches of approximately $\pi/2$). For each point in the contour, starting in the upper left one (red mark in Fig.5(a)(c)), the segment centered on it has been analyzed. In this paper, the segment size is set to $0.1 * n$, with n being the length of the contour (as example, the end points of a segment are marked with two crosses in Fig.5(a)(c)). Graphs in Fig.5(b) and Fig.5(d) show the membership degree to linearity for each of these segments. For the square of Fig.5(a), a maximum membership degree is obtained in the segments located on the sides of the square, whereas this is zero in the segments centered on the corners; for the segments located in intermediate positions, a gradual transition is perceived. This behavior is also observed in Fig.5(d) for the rounded square, although the linearity is lower around the corners; notice that the non-fulfillment is associated to an arch of $\pi/2$, so the rounded corners in Fig.5(c) are expected to have a linearity close to zero.

In the example above, $\alpha = \pi/2$ has been used as parameter value in Eq. (6), but, how does this parameter influence in the final result? As indicated, the value of α corresponds to the arch angle associated to the non-fulfilment of the property; therefore, if it is set to $\alpha = \pi/2$, as in the previous example, it implies that segments with curves of $\pi/2$ or more will be considered non-linear, as we have seen before in the case of Fig.5(c). Fig.6 shows how the previous membership degrees change if we change the value of α. In Fig.6(a), a value of $\alpha = \pi/4$ has been used, which implies that we are more



(a)

(b)

Fig. 6. Influence of the α parameter in the linearity computation. For the shape of Fig.5(c), the linearity is shown for (a) $\alpha = \pi/4$ y (b) $\alpha = \pi$.

restrictive in our perception of linearity, that is, segments with curves of $\pi/4$ or more will already be considered non-linear; for this reason, more segments with a zero degree arise on the graph. On the other hand, in Fig.6(b) a value $\alpha = \pi$ is used, which implies that we are more permissive in our perception of linearity, that is, segments with curves less than π are already considered with some linearity; in the case of rounded corners (arches around $\pi/2$), they are "halfway", so its membership degree is around 0.5.

Finally, Figure 7 shows two examples from the MPEG-7 shape database corresponding to natural contours: a bell and a hat. In both cases, the value $\alpha = \pi/2$ has been used and the segment size has been fixed to $0.1 * n$, with n being the length of the contour. As before, for each point in the contour (starting in the upper left one), the segment centered on it has been analyzed. On the right of the contours, linearity of each point is shown with a black grey level (resp. white) corresponding to a membership degree 1.0 (resp. 0.0). It is observed that the proposed model gives greater degrees to zones where linear segments are located, being zero or practically zero in curved segments. This fact is also noticeable in the graphs shown below each contour: in Fig.7(c) we can see four "hills" corresponding to the main linear areas of the contour (the variation within areas are also represented); a similar behavior ca be observed in Fig.7(f) for the hat shape. Regarding the hat example, one might think that the base of the hat should have a lower degree of linearity, due to its concavity; this would be true for a segment associated to the whole base, but we are working with fixed size segments that, in this case, are smaller pieces of the base. To face this scale issue, it would be necessary to carry out an initial segmentation process that would give us a set of concave/convex segments of non-fixed sizes. This type of preprocessing is proposed for future works.

V. Conclusions

In this paper, a fuzzy approach to model the linearity of a contour segment has been presented. For this, the interpretation and fulfilment of such property has been discussed, concluding that the gradualness from a straight line to an arch of a given angle (and, as a particular case, to the semi-circumference) represents a good starting point for representing the different degrees of fulfillment. Based on this criterion, it has been proposed to define the membership degree on the basis of the coefficient of determination of the segment under study (which gives us a fitting error associated to a line estimation). It has been shown that this coefficient, although it belongs to the $[0, 1]$ interval, does not represents the semantic of the property, being necessary an adjustment for providing a membership function adapted to that semantic. The proposed approach has been tested with different examples, showing its goodness for modelling the imprecision associated to the linearity of a contour segment. To the best of our knowledge, no other approach to model linearity in fuzzy terms is available in the literature.

As future work, the proposed model will be used for analyzing other fuzzy properties. Effectively, from linearity, new contour properties can be derived. For example, the most immediate one is the curvacity, defined as the opposed concept of linearity (i.e, it describes to which degree a contour segment is curved); this property, easily calculable by means a fuzzy negation operator, is useful for selecting saliency features. Another example is the verticity, defined as the degree in which a contour point is a vertex: taking into account that a vertex is a point where two or more lines meet, the linearity is expected to be high both in their left and right sides (and low in any segment centered around it). These are two examples that, although they are not the scope of this paper, shows the usefulness of the modeled property. In addition, on the basis of these primitive properties, it is intended to model more complex ones such as the geometry of the shape. Finally, the model will be applied over a segmentation of the contour, which will allow to analyze segments of non-fixed sizes.

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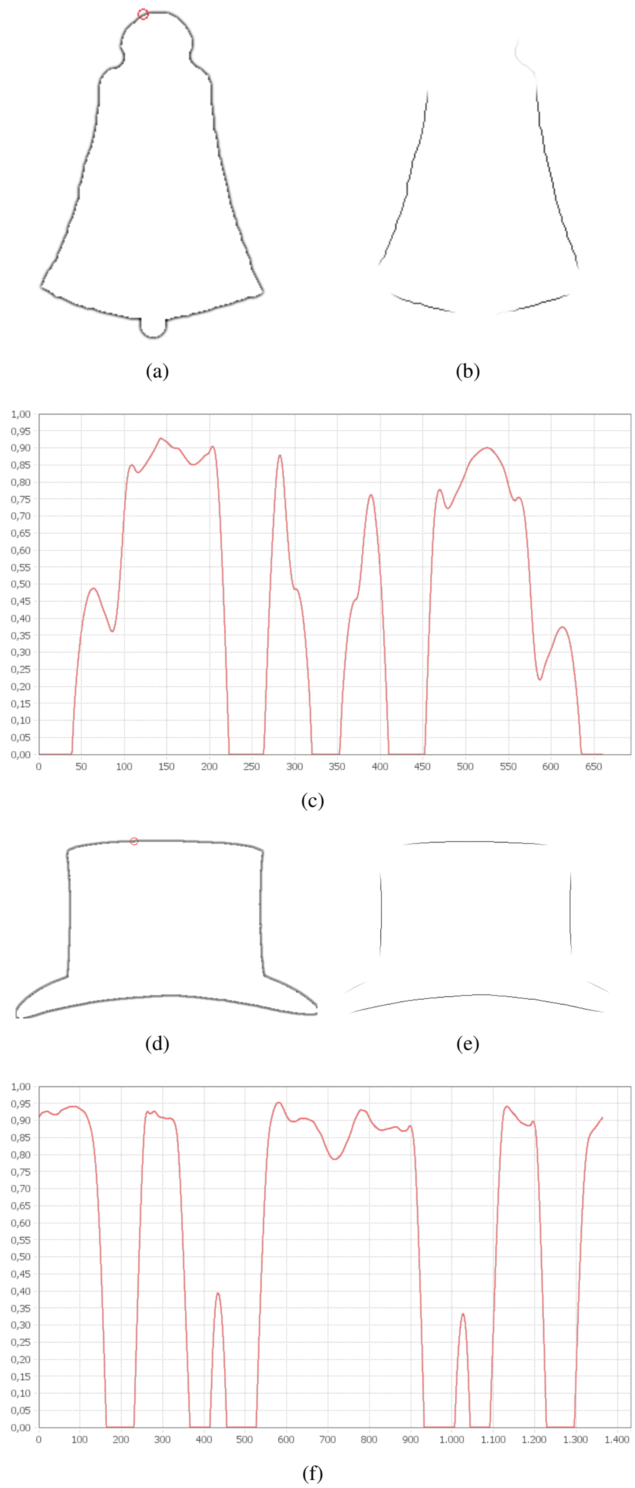

Fig. 7. Examples with two natural shapes: (a) a bell and (d) a hat. On the right of each contour ((b),(e)), linearity of each point is shown with a black grey level (resp. white) corresponding to a membership degree 1.0 (resp. 0.0). Below each contour, a graph ((c),(f)) plotting the linearity membership degree is shown (starting point is marked in red).