Analysis of self-confidence indices-based additive consistency for fuzzy preference relations with self-confidence and its application in group decision making

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Abstract

Preference relations have been widely used in group decision making (GDM) problems. Recently, a new kind of preference relations called fuzzy preference relations with self-confidence (FPRs-SC) has been introduced, which allow experts to express multiple self-confidence levels when providing their preferences. This paper focuses on the analysis of additive consistency for FPRs-SC and its application in GDM problems. To do that, some operational laws for FPRs-SC are proposed. Subsequently, an additive consistency index which considers both the fuzzy values and self-confidence is presented to measure the consistency level of an FPR-SC. Moreover, an iterative algorithm which adjusts both the fuzzy values and self-confidence levels is proposed to repair the inconsistency of FPRs-SC. When an acceptable additive consistency level for FPRs-SC is achieved, the collective FPR-SC can be computed. We aggregate the individual FPRs-SC using a self-confidence indices-based induced ordered weighted averaging (SCI-IOWA) operator. The inherent rule for aggregation is to give more importance to the most self-confident experts. Additionally, a self-confidence score function for FPRs-SC is designed to obtain the best alternative in GDM with FPRs-SC. Finally, the feasibility and validity of the research is demonstrated with an illustrative example and some comparison analyses.

Keywords: Fuzzy preference relations with self-confidence (FPRs-SC); additive consistency; self-confidence levels; induced ordered weighted averaging (IOWA) operator;

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1. Introduction

In group decision making (GDM) problems, a preference relation is the most common representation of information, because it is a useful tool in modeling decision processes. The main advantage of preference relations is that individuals can focus exclusively on two alternatives at a time, which facilitates the expression of their opinions [1, 2] and then makes them more accurate than non-pairwise methods [3]. To date, many different types of preference relations have been proposed and widely used in decision making [4-9].

Applicability of GDM in real contexts implies fields of knowledge such as mathematics, psychology and behavior, relating the computational processes with the behavior of experts [10, 11]. Self-confidence is one of the psychological behaviors of people and has important influence on decision making [12-14]. For example, Zarnoth and Sniezek [15] introduced self-confidence as a person's belief that a statement represents the best possible response. Guha and Chakraborty [16] proposed that in any real GDM situations, when experts give their responses to a particular alternative, their self-confidence levels regarding the opinions are very important. Thus, it would be of great importance to discuss the influence of experts' self-confidence levels on decision making. To do this, Liu, et al. [17] introduced a new kind of preference relations called fuzzy preference relations with self-confidence (FPRs-SC), which allow experts to express multiple self-confidence levels when providing their preferences. In an FPR-SC, elements are composed by two components: the preference degree between a pair of alternatives and the self-confidence level associated with the given preference.

An important issue regarding preference relations is the consistency of the information provided by experts. Consistency is recognized as experts are being neither random nor illogical in their expression of pairwise comparisons. Moreover, consistency has direct influence on the ranking results of final decision [18]. Lack of consistency in preference information can lead to unreliable results and misleading ranking of alternatives [19, 20]. Therefore, in decision making process, it would be of great importance to analyze the consistency of preference relations. Generally, there are two main reasons that make consistency difficult to achieve:

(1) The decision problem is complex, and experts do not possess a precise or sufficient level of knowledge of part of the problem.

(2) The preference relations are based on pairwise comparisons of alternatives,

instead of being handled as a whole set of alternatives holistically.

In order to obtain a rational result, a valid approach is to modify the original preference relations till they are consistent. In addition, we propose that the final improved preference relations should not only satisfy the consistency requirements but also preserve the initial preference information as much as possible.

Up to now, many approaches have been proposed to discuss the consistency of different kinds of preference relations [2, 21-26]. Dong, et al. [21] proposed a consistency for linguistic preference relations. Herrera-Viedma, et al. [19] presented a study of the consistency for fuzzy preference relations (FPRs). An interval consistency index is introduced by Li, et al. [25] for hesitant fuzzy linguistic preference relations. Xu and Wei [27] proposed a consistency improving method in the analytic hierarchy process, and Xu, et al. [2] presented the inconsistency repair methods for FPRs. All the existing consistency studies have made considerable progress. However, they do not consider experts' self-confidence levels and thus cannot analyze directly the additive consistency of FPRs-SC.

To fill the gap mentioned above, we focus on the analysis of additive consistency for FPRs-SC and its application in GDM problems. The main novelty and contributions of this paper is listed as follows:

(1) A new kind of preference relations, i.e., FPRs-SC, which take into account experts' self-confidence levels, is extended to decision making problems. Meanwhile, some new operation laws for FPRs-SC are presented to analyze the additive consistency of FPRs-SC.

(2) An additive consistency index (ACI) which considers both the fuzzy values and self-confidence levels is proposed to measure the consistency level of FPRs-SC. In case of unacceptable consistency, we give an iterative algorithm to improve the consistency of FPR-SC, which is able to adjust both components, the fuzzy values and the self-confidence levels.

(3) After an acceptable additive consistency is reached for FPRs-SC, a self-confidence indices-based induced ordered weighted averaging (SCI-IOWA) operator is proposed to aggregate the individuals' FPRs-SC into a collective one. The rule of aggregation is that more importance is given to the most self-confident ones.

(4) A self-confidence score (SCS) function for FPRs-SC is designed to obtain the best alternatives in GDM with FRPs-SC. We rank alternatives by

computing the *SCS* of the collective FPR-SC. The best alternative as the one with the highest *SCS*.

The feasibility and validity of this research is demonstrated by an illustrative example and some comparison analyses. From the results, we learn that adjusting both the fuzzy values and self-confidence levels will accelerate the inconsistency repair for FPRs-SC, while experts' original preference information also can be retained as much as possible. In addition, we also find that experts' self-confidence levels have influence on the results of alternative ranking of GDM with FPRs-SC.

The rest of this paper is organized as follows. In Section 2, some preliminaries related to FPRs, 2-tuple linguistic ordinal scale model, and FPRs-SC are reviewed. In Section 3, additive consistency analysis of FPRs-SC is given, including consistency measurement and inconsistent improvement. In Section 4, the SCI-IOWA operator is introduced, and a detailed description of the selection process for GDM with FPRs-SC is presented. In Section 5, the illustrative example and comparison analyses are given to show the feasibility and validity of this study. The concluding remarks are pointed out in Section 6.

2. Preliminaries

Before introducing the additive consistency of FPRs-SC and its application in GDM problems, some related preliminaries regarding to the FPRs, the 2-tuple linguistic ordinal scale model, and the FPRs-SC are presented in this section.

2.1. Fuzzy preference relations and 2-tuple linguistic ordinal scale model

For simplicity, let $M=\{1,2,\ldots,m\}$, $N=\{1,2,\ldots,n\}$. Let $X=\{x_1,x_2,\ldots,x_n\}$ be a finite set of alternatives, where x_i denotes the i^{th} alternative. The definition of FPR is given below.

Definition 1 [4]. A matrix $P=(p_{ij})_{n\times n}$ is called an FPR, if $p_{ij}+p_{ji}=1$, $p_{ii}=0.5$, for $\forall i,j$, where p_{ij} denotes the preference degree of alternative x_i over x_j. It is assumed that $p_{ij}+p_{ji}=1$, $p_{ii}=0.5$, for all $i,j\in N$.

For an FPR, Tanino [28] introduced the additive consistent FPR as follows:

Definition 2 [28]. Let $P=(p_{ij})_{n\times n}$ be an FPR, then P has additive consistency if $p_{ij}=p_{ik}-p_{jk}+0.5$, for $\forall i,j,k\in N$.

To carry out ordinal computing with words when dealing with the linguistic self-confidence levels of experts in this study, the 2-tuple linguistic ordinal scale model is reviewed as follows.

Let $S=\{s_i \mid i=0,1,\ldots,g\}$ be a linguistic term set. The term s_i denotes a possible value of a linguistic variable. The ordinal ordering on set *S* is assumed that $s_i>s_j$ if and only if $i>j$. Herrera and Martínez [29] introduced the concept of 2-tuple fuzzy linguistic model as follows:

Definition 3 [29]. Let $\beta\in[0,g]$ be a number in the granularity interval of the linguistic term set *S*. Let $i=round(\beta)$ and $\alpha=\beta-i$ be two values such that $i\in[0,g]$, and $\alpha\in[-0.5,0.5)$, then α is called a *symbolic translation*, and the *round* is the usual round operation.

Afterwards, Herrera and Martínez [29] developed a linguistic representation model which represents the linguistic information by means of 2-tuples (s_i,α), $s_i\in S$ and $\alpha\in[-0.5,0.5)$. It is obvious that 2-tuple linguistic model defines a function to convert between linguistic 2-tuples and numerical values.

Definition 4 [29, 30]. Let *S* be a linguistic term set with the granularity interval $[0,g]$. Then the 2-tuple that expresses the equivalent information to $\beta\in[0,g]$ is obtained with the following function: $\Delta:[0,g]\to S\times[-0.5,0.5)$, where

$$\Delta(\beta)=(s_i,\alpha),\text{ with }\begin{cases} s_i, & i=round(\beta) \\ \alpha=\beta-i, & \alpha\in[-0.5,0.5). \end{cases}$$

Obviously, Δ is one to one mapping function. Afterwards, Herrera and Martínez [29] proposed that for a linguistic term set *S*, and a 2-tuple (s_i,α), there is awalys an inverse function Δ^{-} such that from a 2-tuple it returns its equivalent numerical value $\beta\in[0,g]$:

$$\Delta^{-}:S\times[-0.5,0.5)\to[0,g]$$

$$\Delta^{-}(s_i, \alpha) = i + \alpha = \beta .$$

Clearly, the conversion of a linguistic term into a linguistic 2-tuple consist of adding a value zero as symbolic translation $s_i \in S \Rightarrow (s_i, 0)$, i.e., $\Delta^{-}(s_i, 0) = \Delta^{-}(s_i)$. In addition, some computations and operators were presented to deal with 2-tuple linguistic information in [29-31] as follows:

(1) 2-tuples comparison operator: Let (s_k, α) and (s_l, γ) be two 2-tuples, then:

- if $k < l$, then (s_k, α) is smaller than (s_l, γ);
- if $k = l$, then
  a) if $\alpha = \gamma$, then (s_k, α), (s_l, γ) represents the same information;
  b) if $\alpha < \gamma$, then (s_k, α) is smaller than (s_l, γ);

(2) Negation operator over 2-tuples as

$$Neg(s_i, \alpha) = \Delta\left(g - \Delta^{-}(s_i, \alpha)\right) \tag{1}$$

2.2. Fuzzy preference relations with self-confidence

Suppose a linguistic term $S^{SL} = \{s_i \mid i = 0,1,\ldots,g\}$ is used to characterize experts' self-confidence levels over preference values. Without loss of generality, this paper assumes that experts use a set of nine linguistic terms $S^{SL} = \{s_0, s_1, \ldots, s_8\}$ to express their self-confidence levels. Detailed information of S^{SL} is shown in Fig. 1.

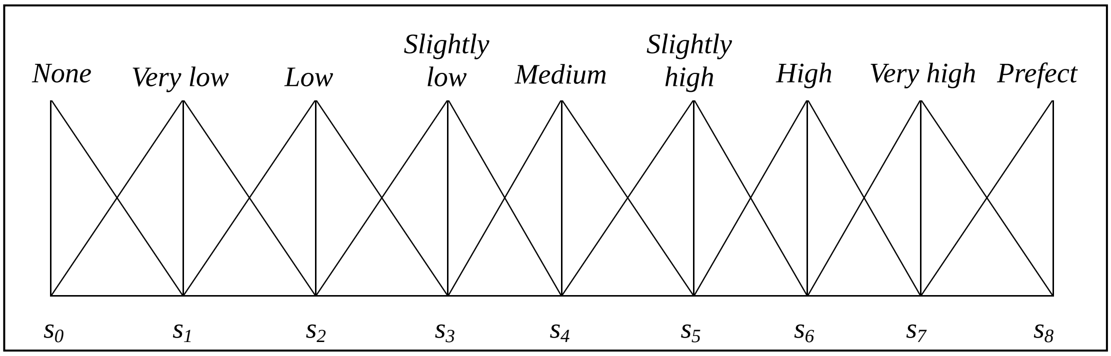

Fig. 1. The expert's self-confidence level language labels.

The definition of FPR-SC proposed by [17] is given below:

Definition 5 [17]. A matrix $\tilde{P} = (p_{ij}, l_{ij})_{n \times n}$ is called an FPR-SC if its elements have two components: the first component, $p_{ij} \in [0,1]$ represents the preference degree of the alternative x_i over x_j, and the second element $l_{ij} \in S^{SL}$ denotes the self-confidence level associated to the first component p_{ij}. The following conditions are assumed: $p_{ij} + p_{ji} = 1$, $p_{ii} = 0.5$, $l_{ij} = l_{ji}$ and $l_{ii} = s_g$ for $\forall i, j \in N$.

Remark 1. Considering Definition 5, any fuzzy value p_{ij} can be transformed into the following form:

$$p_{ij} \Rightarrow (p_{ij}, s_g),$$

i.e., the traditional FPRs are a special case with prefect self-confidence level. The expert is absolutely self-confident of his/her comparisons. In addition, Liu, et al. [17] pointed that FPRs-SC can be considered, in some sense, a Z-number given by Zadeh [32] that both representations relate with reliability of information.

Example 1. Let $X = \{x_1, x_2, x_3, x_4\}$ be a set of alternatives and $S^{SL} = \{s_0, \ldots, s_8\}$ be a linguistic term set used to characterize experts' self-confidence levels over the preference values. Then, an expert can provide her/his FPR-SC $\tilde{P} = (p_{ij}, l_{ij})_{4\times 4}$, $l_{ij} \in S^{SL}$ over the four alternatives in *X*. Assume the FPR-SC $\tilde{P} = (p_{ij}, l_{ij})_{4\times 4}$ given by the expert is as follows:

$$\tilde{P} = \begin{pmatrix} (0.5, s_8) & (0.1, s_5) & (0.6, s_7) & (0.7, s_8) \\ (0.9, s_5) & (0.5, s_8) & (0.8, s_6) & (0.6, s_4) \\ (0.4, s_7) & (0.2, s_6) & (0.5, s_8) & (0.6, s_5) \\ (0.3, s_8) & (0.4, s_4) & (0.4, s_5) & (0.5, s_8) \end{pmatrix}.$$

In the FPR-SC $\tilde{P}$, $p_{12} = 0.1$ means the preference degree of the alternative x_1 over x_2 is 0.1, and the $l_{12} = s_5$ represents the expert's self-confidence level associated with p_{12} is s_5, i.e., the expert has slightly high confident for her/his judgment. Consequently, the rest of elements in $\tilde{P}$ can be explained in this way.

Based on Definition 2, for an FPR-SC, Liu, et al. [17] introduced the definition of additive consistent FPR-SC as follows.

Definition 6 [17]. Let $\tilde{P}=(p_{ij},l_{ij})_{n\times n}$ be an FPR-SC. If $p_{ij}=p_{ik}-p_{jk}+0.5$ for $\forall i,j,k$ and $l_{ij}\geq s$ for $\forall i,j$, then $\tilde{P}$ has additive consistency at the self-confidence level $s\in S^{SL}$.

Remark 2. Specifically, for any FPR-SC $\tilde{P}=(p_{ij},l_{ij})_{n\times n}$, if $p_{ij}=p_{ik}-p_{jk}+0.5$ for $\forall i,j,k$, then it must have additive consistency at the self-confidence level $s_0\in S^{SL}$.

3. The additive consistency analysis for FPRs-SC

Consistency analysis relates to experts to express their preference information without contradictions. In this section, we propose an additive consistency analysis for FPRs-SC. The additive consistency measure for FPRs-SC is introduced in Section 3.1. Afterwards, the *ACI* which considers self-confidence levels for FPRs-SC is given in Section 3.2. In Section 3.3, a method for repairing the inconsistency of FPRs-SC is presented.

3.1. Additive consistency measure of FPRs-SC

Before showing the additive consistency measure for FPRs-SC, some necessary new operational laws of 2-tuples in FPRs-SC are presented as follows.

Definition 7. Assume (p_k,l_k) (p_i,l_i) be two 2-tuples, p_i, p_k are the fuzzy values, and l_i, l_k are corresponding self-confidence values, where $l_i,l_k\in S^{SL}$, $\lambda\in[0,1]$. Then, we have the following operations:

a) $(p_k,l_k)+(p_i,l_i)=(p_k+p_i,\min\{l_k,l_i\})$;

b) $(p_k,l_k)-(p_i,l_i)=(p_k-p_i,\min\{l_k,l_i\})$;

c) $(p_k,l_k)-\lambda=(p_k-\lambda,l_k)$;

d) $(p_k,l_k)^{\lambda}=((p_k)^{\lambda},l_k)$;

e) $\lambda(p_k,l_k)=(\lambda p_k,l_k)$.

Then, based on Definition 6 and Definition 7, we redefine the additive consistent FPR-SC as follows:

Definition 8. Let $\tilde{P}=(p_{ij},l_{ij})_{n\times n}$ be an FPR-SC, and if the elements in $\tilde{P}$ satisfy:

$$p_{ij}=p_{ik}+p_{kj}-0.5, \tag{2}$$

for $\forall i,j,k\in N$. Then, we call $\tilde{P}$ has additive consistency at self-confidence level s, where $s=\min_{i,j}\{l_{ij}\}$, and $s,l_{ij}\in S^{SL}$.

Remark 3. For the additive consistency property in [17], it only needs $l_{ij}\geq s$, and the value s is not specified. As we mentioned in Remark 2, if an FPR-SC satisfy Eq. (2), it must be additive consistency at the self-confidence level s_0. The difference between Definition 6 and Definition 8 is that if an FPR-SC satisfies Eq. (2), then the highest self-confidence level is $s=\min_{i,j}\{l_{ij}\}$ in Definition 8, while the self-confidence level s in Definition 6 could be between s_0 and $\min_{i,j}\{l_{ij}\}$.

As mentioned in Section 2, traditional FPRs can be seen as the special case of FPRs-SC in which experts are completely self-confidence for their comparisons. Let $P=(p_{ij})_{n\times n}$ and $Z=(z_{ij})_{n\times n}$ be two FPRs, where $z_{ij}=\frac{1}{n}\sum_{k=1}^{n}(p_{ik}-p_{jk})+0.5$, Xu, et al. [2] introduced the consistency index (CI) of P as follows:

$$CI(P)=d(P,Z)=\sqrt{\frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(p_{ij}-z_{ij})^2} \tag{3}$$

Obviously, the *CI* of FPRs proposed in [2] is computed by using the distance measure between FPRs and the corresponding additive consistent FPRs. However, Eq. (3) does not consider the self-confidence levels of experts. Therefore, we propose the new additive consistency measure of FPRs-SC which considers experts' self-confidence levels.

By Definition 8 and Eq. (3), we define the deviation level (*DL*) between FPRs-SC and the corresponding consistent FPRs-SC.

Definition 9. Let $\tilde{P}=(p_{ij},l_{ij})_{n\times n}$, and $\tilde{Z}=(\tilde{z}_{ij},\tilde{l}_{ij})_{n\times n}$ be two FPRs-SC, where $(\tilde{z}_{ij},\tilde{l}_{ij})=(\frac{1}{n}\sum_{k=1}^{n}(p_{ik}+p_{kj})-0.5,\min\{l_{ik},l_{kj}\mid k=1,\ldots,n\})$ ($i\neq j$), $l_{ij},\tilde{l}_{ij}\in S^{SL}$. Then, the $DL(\tilde{P})$ is defined as follows:

$$DL(\tilde{P}) = d(\tilde{P}, \tilde{Z}) = \sqrt{\frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left((p_{ij}, l_{ij}) - (\tilde{z}_{ij}, \tilde{l}_{ij}) \right)^2}, \tag{4}$$

where $\left((p_{ij}, l_{ij}) - (\tilde{z}_{ij}, \tilde{l}_{ij}) \right)^2 = \left((p_{ij} - z_{ij})^2, \min\{l_{ij}, \tilde{l}_{ij}\} \right)$.

Remark 4. Clearly, the value of $DL(\tilde{P})$ is also a 2-tuple since it is composed of a fuzzy value and the corresponding self-confidence level. Let us note $DL(\tilde{P}) = (q, v)$, then we have the following considerations:

a) $q \in [0,1]$, and $v \in S^{SL}$;

b) if $q = 0$, it means that the fuzzy values are fully consistent between $\tilde{P}$ and $\tilde{Z}$ at the self-confidence level v; Otherwise, the greater q is, the more inconsistent of the fuzzy values between $\tilde{P}$ and $\tilde{Z}$ will be.

For simplicity, we call q the consistency level of fuzzy values in $\tilde{P}$, and v represents the corresponding self-confidence level of experts.

3.2. The additive consistency with self-confidence index of FPRs-SC

According to the analysis in Section 3.1, we define the new *ACI* of an FPR-SC that considers the self-confidence levels of expert as follows:

Definition 10. Let $\tilde{P} = (p_{ij}, l_{ij})_{n \times n}$ and $\tilde{Z} = (\tilde{z}_{ij}, \tilde{l}_{ij})_{n \times n}$ be two FPRs-SC, where $(\tilde{z}_{ij}, \tilde{l}_{ij}) = \left(\frac{1}{n} \sum_{k=1}^{n} (p_{ik} + p_{kj}) - 0.5, \min\{l_{ik}, l_{kj} \mid k = 1, \ldots, n\} \right)$ ($i \neq j$), $l_{ij}, \tilde{l}_{ij} \in S^{SL}$. Let $DL(\tilde{P}) = (q, v)$, which is obtained by Eq. (4). Then, the $ACI(\tilde{P})$ is defined as:

$$ACI(\tilde{P}) = \gamma(1-q) + (1-\gamma) \frac{\Delta^{-}(v)}{g}, \tag{5}$$

where $\gamma \in [0,1]$ is a parameter to control the weight of both the consistency level of fuzzy values and the self-confidence level of expert.

Remark 5. In Definition 10, we have $ACI(\tilde{P}) \in [0,1]$. Moreover, the higher the value of $ACI(\tilde{P})$ is, the more consistent $\tilde{P}$ will be. In addition, in this paper, we assume that the consistency level of fuzzy values and the self-confidence levels of experts are equally important in GDM problems. Thus, we have $\gamma = 0.5$.

3.3. A method for repairing the inconsistency of FPRs-SC

The lack of consistency in preference information can lead to unreliable results and misleading rankings of alternatives [20]. Thus, to repair the inconsistency of FPRs-SC is an important and necessary work. Generally, in real GDM problems, due to the time pressure, lack of knowledge, and the limited experience, it is really difficult for experts to provide FPRs-SC with prefect additive consistency. Thus, in this paper, we only consider an acceptable *ACI* for FPRs-SC. We assume that if the *ACI* of FPRs-SC is equal to or greater than a predefined additive consistency threshold δ ($\delta \in [0,1]$), i.e., $ACI \geq \delta$. Then, an acceptable *ACI* of FPR-SC is reached. Otherwise, the FPR-SC is inconsistent, and needs to be improved. The consistency improving processes for FPRs-SC are depicted in Fig. 2.

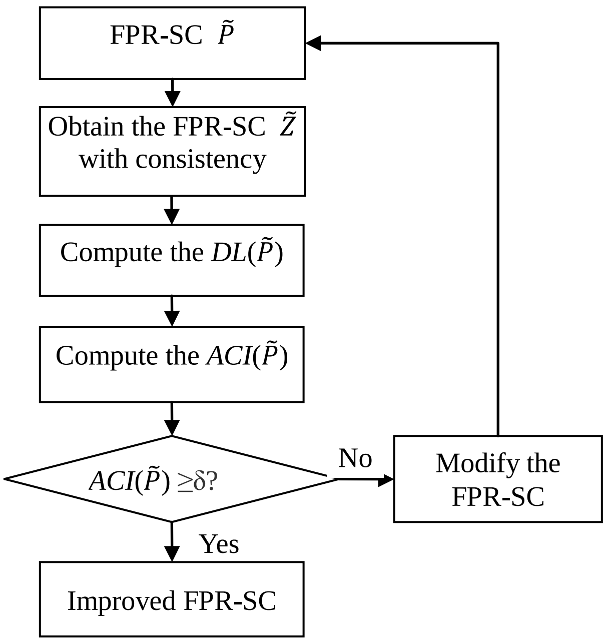

Fig. 2. Flowchart of the consistency improving process for FPRs-SC.

Usually, in real GDM problems, self-confidence levels represent experts' recognition of their knowledge, abilities and experiences. If an expert express low level of self-confidence for her/his assessment information, it means that she/he may not have enough knowledge or evidence to justify her/his judgment. In other words, the expert's assessment information may lack of reliability. Based on this hypothesis, we propose to repair the inconsistency of FPRs-SC based on experts' self-confidence levels. That is, to modify the fuzzy values in which the expert has the lowest self-confidence level. The detailed iterative algorithm which is used to repair the inconsistency of FPRs-SC is presented in Algorithm 1.

Algorithm 1. The detailed repair of inconsistency for FRPs-SC.

Input: The linguistic term set $S^{SL} = \{s_0, \ldots, s_g\}$ in which self-confidence levels are expressed, the FPR-SC $\tilde{P} = (p_{ij}, l_{ij})_{n\times n}$, $l_{ij} \in S^{SL}$, the number of iterations t, the parameter γ and the acceptable additive consistency threshold δ.

Step 1. Let $\tilde{Z} = (\tilde{z}_{ij}, \tilde{l}_{ij})_{n\times n}$ be an FPR-SC, where

$(\tilde{z}_{ij}, \tilde{l}_{ij}) = (\frac{1}{n}\sum_{k=1}^{n}(p_{ik} + p_{kj}) - 0.5, \min\{l_{ik}, l_{kj} \mid k = 1, \ldots, n\})$ $(i \neq j)$, and t=0.

Step 2. Calculate the $DL(\tilde{P})$ by using Eq. (4). Afterwards, utilize Eq. (5) to compute the $ACI(\tilde{P}^{(t)})$.

Step 3. If $ACI(\tilde{P}^{(t)}) \geq \delta$, then go to Step 5; Otherwise, go to the next step.

Step 4. Find the position of self-confidence level $l^{(t)}_{i_\tau j_\tau}$, where $l^{(t)}_{i_\tau j_\tau} = \min_{i,j}\{l_{ij}\}$, i.e., the smallest expert's self-confidence level on her/his preference. Then, return $\tilde{P}^{(t)}$ to the expert to construct a new FPR-SC $\tilde{P}^{(t+1)} = (p^{(t+1)}_{ij}, l^{(t+1)}_{ij})_{n\times n}$. If there exist two self-confidence elements that are equal, i.e., $l^{(t)}_{i_\tau j_\tau} = l^{(t)}_{i'_\tau j'_\tau}$, then find the fuzzy values $p^{(t)}_{i_\tau j_\tau}$ and $p^{(t)}_{i'_\tau j'_\tau}$ which are corresponding to these two self-confidence levels, respectively. We have the following rules:

a) if $d^{(t)}_{i_\tau j_\tau} < d^{(t)}_{i'_\tau j'_\tau}$, then expert needs to improve $(p^{(t)}_{i'_\tau j'_\tau}, l^{(t)}_{i'_\tau j'_\tau})$;

b) if $d^{(t)}_{i_\tau j_\tau} > d^{(t)}_{i'_\tau j'_\tau}$, then expert needs to modify $(p^{(t)}_{i_\tau j_\tau}, l^{(t)}_{i_\tau j_\tau})$;

c) if $d^{(t)}_{i_\tau j_\tau} = d^{(t)}_{i'_\tau j'_\tau}$, then expert can randomly choose any $(p^{(t)}_{i_\tau j_\tau}, l^{(t)}_{i_\tau j_\tau})$ and $(p^{(t)}_{i'_\tau j'_\tau}, l^{(t)}_{i'_\tau j'_\tau})$ to repair;

where $d^{(t)}_{i_\tau j_\tau} = (p^{(t)}_{i_\tau j_\tau} - \tilde{z}^{(t)}_{i_\tau j_\tau})^2$, and $d^{(t)}_{i'_\tau j'_\tau} = (p^{(t)}_{i'_\tau j'_\tau} - \tilde{z}^{(t)}_{i'_\tau j'_\tau})^2$.

In order to repair the inconsistency of FPRs-SC while retain the original experts' information as much as possible, we present the following rules to construct the new FPR-SC $\tilde{P}^{(t+1)} = (p_{ij}^{(t+1)}, l_{ij}^{(t+1)})_{n\times n}$:

$$p_{ij}^{(t+1)} = \begin{cases} \tilde{z}_{ij}^{(t)}, & if \quad i, j = \tau; \\ p_{ij}^{(t)}, & otherwise. \end{cases} \tag{6}$$

$$l_{ij}^{(t+1)} = \begin{cases} \Delta((\Delta^{-}(s_g) - \Delta^{-}(\tilde{l}_{ij}^{(t)})), & if \quad \tilde{l}_{ij}^{(t)} \le s_{\frac{g}{2}}, \quad i, j = \tau; \\ \tilde{l}_{ij}^{(t)}, & otherwise. \end{cases} \tag{7}$$

Meanwhile, $p_{ij}^{(t+1)} + p_{ji}^{(t+1)} = 1$, $l_{ij}^{(t+1)} = l_{ji}^{(t+1)}$, and $\tilde{P}^{(t+1)} \ne \tilde{P}^{(t)}$. Let $t = t+1$ and return to Step 1.

Step 5. Output the adjusted FPR-SC $P^{*} = (p_{ij}^{*}, l_{ij}^{*})$ which has acceptable additive consistency, and the values $ACI(P^{*})$ of P^{*}.

Remark 6. In Algorithm 1, when replacing a fuzzy values $p_{ij}^{(t+1)}$ with $\tilde{z}_{ij}^{(t)}$, the expert is advised to increase her/his corresponding self-confidence level, which will consist of a more acceptable additive consistency. Meanwhile, to ensure that expert's minimum self-confidence (M-SC) level can been improved, i.e., $l_{ij}^{(t+1)} \ge l_{ij}^{(t)}$ while the original expert's information are retained as much as possible, we propose to modify the expert's M-SC by comparing with the intermediate level $s_{g/2}$. That is, if the M-SC of an expert is equal to or greater than $s_{g/2}$, then, we assume that the M-SC of this expert is moderate. Note that, we only repair the fuzzy values while retain the self-confidence unchanged. Otherwise, the M-SC of expert needs to be modified in order to ensure that an acceptable additive consistency can be reached.

4. Selection process for GDM with FPRs-SC

In GDM problems, the selection process is usually composed of the aggregation process and the exploitation process [6]. The aim of the aggregation process is to obtain a collective preference relation, which denotes the group preference between every ordered pair of alternatives. The goal of the exploitation process is to choose the best alternative for the GDM problem by transforming the collective preference information about the alternatives into a collective ranking of them.

In this section, we propose the selection process for GDM with FRPs-SC. The aggregation process for FPRs-SC is provided in Section 4.1, and the exploitation process for GDM with FPRs-SC is given in Section 4.2. Additionally, Fig. 3 shows the flowchart of the selection process for GDM with FPRs-SC.

4.1. Aggregation process

The aggregate operator is an effective tool in aggregation process for decision making [8, 33-36]. The IOWA operator introduced by Yager [37], which is guided by fuzzy linguistic quantifiers, is one of the most effective operators to be used in the aggregation process of GDM [6, 7, 38]. The definition of the IOWA operator is given below:

Definition 11 [37, 39]. An IOWA operator of dimension m is a mapping $IOWA: R^m \rightarrow R$ that has an associated weighting vector $W = (w_1, \ldots, w_m)^T$ of dimension m, such that $\sum_{i=1}^{m} w_i = 1$ and $w_i \in [0,1]$, it is expressed as follows:

$$\Phi_W(< u_1, a_1 >, < u_2, a_2 >, \ldots, < u_m, a_m >) = \sum_{i=1}^{m} w_i a_{\sigma(i)} , \tag{8}$$

where σ is a permutation of $\{1, 2, \ldots, m\}$ such that $u_{\sigma(i)} \geq u_{\sigma(i+1)}$ for $\forall i = 1, \ldots, m-1$. That is, $< u_{\sigma(i)}, a_{\sigma(i)} >$ is a 2-tuple with $u_{\sigma(i)}$ is the i^{th} largest value in the set $\{u_1, \ldots, u_m\}$.

The order inducing value is a parameter of the IOWA operator for a better control over the aggregation stage, as it introduces some semantics in the aggregation to guide the ordering process. As mentioned in Section 3, the self-confidence levels represent experts' recognition of their knowledge, abilities and experiences. The higher the self-confidence levels of expert, the more the reliable of the expert's assessment information. Thus, based on this hypothesis, we propose to make the aggregation process of GDM with FPRs-SC by measuring the overall self-confidence index (SCI) of the FPRs-SC provided by experts. Meanwhile, the *SCI* can be obtained by the deviation measure between the expert's self-confidence matrix and the maximum self-confidence matrix.

Generally, the FPR-SC $\tilde{P} = (p_{ij}, l_{ij})_{n \times n}$ given by an expert can be seen as the combination of an FPR $P = (p_{ij})_{n \times n}$ and a self-confidence matrix $L = (l_{ij})_{n \times n}$. Then, we define the self-confidence deviation level (SCDL) between the self-confidence matrix of FPR-SC and the maximal self-confidence matrix as follows:

Definition 12. Let $\tilde{P}=(p_{ij},l_{ij})_{n\times n}$ be an FPR-SC, and let $L=(l_{ij})_{n\times n}$ be the corresponding self-confidence matrix of $\tilde{P}$. We assume that $\tilde{L}=(s_g)_{n\times n}$ is the maximal self-confidence matrix, where $l_{ij},s_g\in S^{SL}$. Then, the *SCDL* of $\tilde{P}$ can be computed as follows:

$$SCDL(\tilde{P})=\frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}d(L,\tilde{L})=\frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{|\Delta^{-}(l_{ij})-\Delta^{-}(s_g)|}{g}. \tag{9}$$

Clearly, the value of the $SCDL(\tilde{P})$ has the following characteristic:

a) $SCDL(\tilde{P})\in[0,1]$;

b) if $SCDL(\tilde{P})=0$, it means that expert is fully self-confident in all of her/his preferences. In addition, the higher the value of $SCDL(\tilde{P})$, the smaller the level of self-confidence of expert on her/his valuations.

Then, the *SCI* of the $\tilde{P}=(p_{ij},l_{ij})_{n\times n}$ provided by the expert is computed by:

$$SCI(\tilde{P})=1-SCDL(\tilde{P}) \tag{10}$$

Similarly, we have $SCI(\tilde{P})\in[0,1]$. The larger the value of the $SCI(\tilde{P})$, the higher the self-confidence level of expert for her/his opinions. Moreover, according to the rule that the higher the value of *SCI* of the FPR-SC provided by expert, the greater the weight of the expert should be assigned. The definition of the SCI-IOWA operator is given below:

Definition 13. Let $E=\{e_1,e_2,\ldots,e_m\}$ be a set of experts, $X=\{x_1,x_2,\ldots,x_n\}$ be a set of alternatives. The experts give their comparison information using FPRs-SC $\tilde{P}_1,\tilde{P}_2,\ldots,\tilde{P}_m$. The SCI-IOWA operator of dimension *m*, Φ_W^{SCI}, is an IOWA operator whose set of order inducing values is the *SCI* of the experts' FPRs-SC, denoted as $\{SCI_1,SCI_2,\ldots,SCI_m\}$, and represents the overall self-confidence of experts. Thus, the collective FPR-SC $P^{SCI}=(p_{ij}^{SCI},l_{ij}^{SCI})$ can be computed as:

$$\begin{aligned}\Phi_W^{SCI}&=(<SCI_1,(p_{ij,1},l_{ij,1})>,<SCI_2,(p_{ij,2},l_{ij,2})>,\ldots,<SCI_m,(p_{ij,m},l_{ij,m})>)\\&=\sum_{h=1}^{m}w_h(p_{ij,\sigma(h)},l_{ij,\sigma(h)}),\end{aligned}\tag{11}$$

with $SCI_{\sigma(h-1)}\geq SCI_{\sigma(h)}$ and $\sum_{h=1}^{m}w_h=1$.

Using the SCI-IOWA operator to aggregate the individual information, the important problem is to set its associated weighting vector. Yager [40] introduced a fuzzy linguistic quantifier Q to compute the weights of the IOWA operator as follows:

$$w_i=Q\left(\frac{\sum_{k'=1}^{i}u_{\sigma(k')}}{T}\right)-Q\left(\frac{\sum_{k'=1}^{i-1}u_{\sigma(k')}}{T}\right),\tag{12}$$

where $T=\sum_{k'=1}^{m}u_{\sigma(k')}$ denotes the overall sum of importance.

Similarly, by Eq. (12), the weights of the SCI-IOWA operator are computed by:

$$w_h=Q\left(\frac{\sum_{k'=1}^{h}SCI_{\sigma(k')}}{T}\right)-Q\left(\frac{\sum_{k'=1}^{h-1}SCI_{\sigma(k')}}{T}\right)\tag{13}$$

with $T=\sum_{k'=1}^{m}SCI_{\sigma(k')}$ and $SCI_{\sigma(k')}$ is the $(k')^{th}$ largest value of the set $\{SCI_1,\ldots,SCI_m\}$.

4.2. Exploitation process

When the aggregation process is completed, the exploitation process follows to choose the best alternative for GDM with FPRs-SC. To do this, we define the SCS function of FPR-SC as follows:

Definition 14. Let $X=\{x_1,\ldots,x_n\}$ be an alternatives set, and $\tilde{P}=(p_{ij},l_{ij})_{n\times n}$ be the associated FPR-SC, then the *SCS* function for any alternatives is defined as:

$$SCS(x_i)=\frac{1}{n}\sum_{j=1}^{n}(p_{ij}\times\Delta^{-}(l_{ij})),\quad i=1,2,...,n\tag{14}$$

Remark 7. In Definition 14, the higher the value of $SCS(x_i)$, the more expert's self-confidence on the alternative x_i. That is, if we have $SCS(x_i) > SCS(x_j)$, then $x_i \succ x_j$, $x_i, x_j \in X$.

In addition, the selection process for GDM with FPRs-SC is depicted in Fig. 3 as follows:

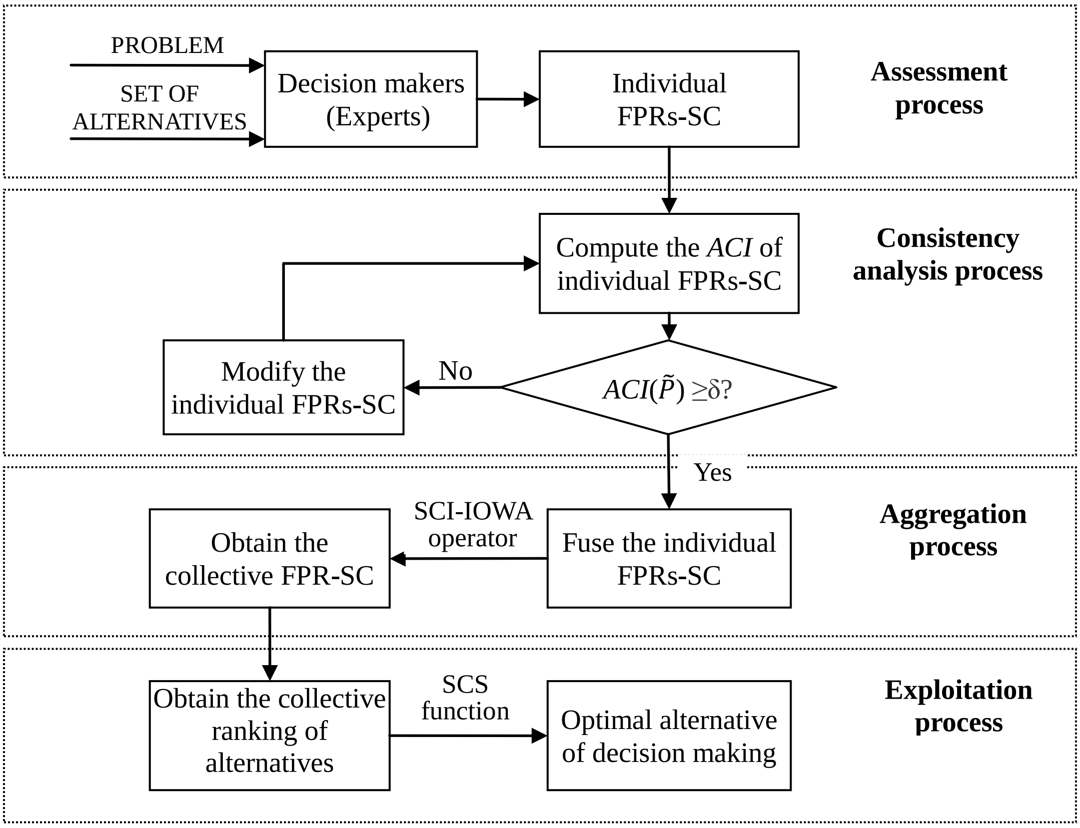

Fig. 3. The flowchart of selection process for GDM with FPRs-SC.

5. Illustrative example and comparative analysis

An illustrative example is provided in Section 5.1 to show how to select the best alternative for GDM with FPRs-SC. A comparative analysis is presented in Section 5.2 to show that self-confidence levels have influence on decision making, and the proposed inconsistency improvements are effective for FPRs-SC.

5.1. An illustrative example

Let $S^{SL} = \{s_0, s_2, \ldots, s_8\}$ be a linguistic term set which is used to express experts' self-confidence levels for their judgments (the detailed information of S^{SL} can be found in Fig. 1) and $X = \{x_1, x_2, x_3, x_4\}$ be a possible solution for a GDM with

FPRs-SC. There are four experts $E=\{e_1,e_2,e_3,e_4\}$ invited to make a pairwise comparison for the alternatives in X using FPRs-SC. Suppose the FPRs-SC $\tilde{P}_m=(p_{ij,m},l_{ij,m})$ $(m=1,2,3,4)$ are as follows:

$$\tilde{P}_1=\begin{pmatrix}(0.5,s_8) & (0.1,s_5) & (0.6,s_7) & (0.7,s_8)\\(0.9,s_5) & (0.5,s_8) & (0.8,s_6) & (0.6,s_4)\\(0.4,s_7) & (0.2,s_6) & (0.5,s_8) & (0.6,s_5)\\(0.3,s_8) & (0.4,s_4) & (0.4,s_5) & (0.5,s_8)\end{pmatrix},$$

$$\tilde{P}_2=\begin{pmatrix}(0.5,s_8) & (0.6,s_3) & (0.6,s_5) & (0.2,s_2)\\(0.4,s_3) & (0.5,s_8) & (0.6,s_4) & (0.7,s_6)\\(0.4,s_5) & (0.4,s_4) & (0.5,s_8) & (0.4,s_3)\\(0.8,s_2) & (0.3,s_6) & (0.6,s_3) & (0.5,s_8)\end{pmatrix},$$

$$\tilde{P}_3=\begin{pmatrix}(0.5,s_8) & (0.3,s_5) & (0.4,s_7) & (0.7,s_4)\\(0.7,s_5) & (0.5,s_8) & (0.2,s_6) & (0.3,s_3)\\(0.6,s_7) & (0.8,s_6) & (0.5,s_8) & (0.6,s_2)\\(0.3,s_4) & (0.7,s_3) & (0.4,s_2) & (0.5,s_8)\end{pmatrix},$$

$$\tilde{P}_4=\begin{pmatrix}(0.5,s_8) & (0.4,s_5) & (0.2,s_4) & (0.1,s_5)\\(0.6,s_5) & (0.5,s_8) & (0.6,s_6) & (0.2,s_3)\\(0.8,s_4) & (0.4,s_6) & (0.5,s_8) & (0.1,s_7)\\(0.9,s_5) & (0.8,s_3) & (0.9,s_7) & (0.5,s_8)\end{pmatrix}.$$

To ensure that the original expert's information can be maintained as much as possible while an acceptable additive consistency is achieved, we assume an acceptable consistency threshold $\delta=0.7$ and a maximum number of iterations $t^*=6$. Then, the detailed processes of the proposed selection process for GDM with FPRs-SC are as follows:

Step 1. Consistency measurement process.

Compute $\tilde{Z}_m^{(0)}$ $(m=1,2,3,4)$ by using Definition 9:

$$\tilde{Z}_1^{(0)}=\begin{pmatrix}(0.5,s_8) & (0.275,s_4) & (0.550,s_5) & (0.575,s_4)\\(0.725,s_4) & (0.5,s_8) & (0.775,s_4) & (0.800,s_4)\\(0.450,s_5) & (0.225,s_4) & (0.5,s_8) & (0.525,s_4)\\(0.425,s_4) & (0.200,s_4) & (0.475,s_4) & (0.5,s_8)\end{pmatrix},$$

$$\tilde{Z}_2^{(0)}=\begin{pmatrix}(0.5,s_8) & (0.425,s_2) & (0.550,s_2) & (0.425,s_2)\\(0.575,s_2) & (0.5,s_8) & (0.625,s_3) & (0.500,s_2)\\(0.450,s_2) & (0.375,s_3) & (0.5,s_8) & (0.375,s_2)\\(0.575,s_2) & (0.500,s_2) & (0.625,s_2) & (0.5,s_8)\end{pmatrix},$$

$$\tilde{Z}_3^{(0)}=\begin{pmatrix}(0.5,s_8) & (0.550,s_3) & (0.350,s_2) & (0.500,s_2)\\(0.450,s_3) & (0.5,s_8) & (0.300,s_2) & (0.450,s_2)\\(0.650,s_2) & (0.700,s_2) & (0.5,s_8) & (0.650,s_2)\\(0.500,s_2) & (0.550,s_2) & (0.350,s_2) & (0.5,s_8)\end{pmatrix},$$

$$\tilde{Z}_4^{(0)}=\begin{pmatrix}(0.5,s_8) & (0.325,s_3) & (0.350,s_4) & (0.025,s_3)\\(0.675,s_3) & (0.5,s_8) & (0.525,s_3) & (0.200,s_3)\\(0.650,s_4) & (0.475,s_3) & (0.5,s_8) & (0.175,s_3)\\(0.975,s_3) & (0.800,s_3) & (0.825,s_3) & (0.5,s_8)\end{pmatrix}.$$

Then, compute the ACI of $\tilde{P}_m^{(0)}$ $(m=1,2,3,4)$ by using Eq. (5):

$$ACI(\tilde{P}_1^{(0)})=0.6865,\quad ACI(\tilde{P}_2^{(0)})=0.5525,$$
$$ACI(\tilde{P}_3^{(0)})=0.5480,\quad ACI(\tilde{P}_4^{(0)})=0.6430.$$

Experts e_m $(m=1,2,3,4)$ are advised to make adjustments for their FPRs-SC since the $ACI(\tilde{P}_m^{(0)})<\delta$ $(m=1,2,3,4)$.

Step 2. Consistency improvement process.

According to the proposed Algorithm 1, the adjustments of $\tilde{P}_1^{(0)}$ and $\tilde{P}_2^{(0)}$ are given below:

(1) Inconsistency repair of $\tilde{P}_1^{(0)}$.

Find the position of self-confidence level $l_{i_\tau j_\tau,1}^{(0)}$, where $l_{i_\tau j_\tau,1}^{(0)}=\min_{i,j}\{l_{ij}\}$. Since $l_{24,1}^{(0)}=l_{42,1}^{(0)}=\min_{24,1}\{l_{ij}\}=s_4$, expert e_1 has the lowest self-confidence on fuzzy values $p_{24,1}^{(0)}$ and $p_{42,1}^{(0)}$. Then, replacing these two fuzzy values with the corresponding elements in $\tilde{Z}_1^{(0)}$. Meanwhile, the corresponding self-confidence levels are also changed to $l_{24,1}^{(0)}=l_{42,1}^{(0)}\to s_4$ by using Eq. (7). Then, we have $\tilde{P}_1^{(1)}$:

$$\tilde{P}_1^{(1)}=\begin{pmatrix}(0.5,s_8) & (0.1,s_5) & (0.6,s_7) & (0.7,s_8)\\ (0.9,s_5) & (0.5,s_8) & (0.8,s_6) & (0.8,s_4)\\ (0.4,s_7) & (0.2,s_6) & (0.5,s_8) & (0.6,s_5)\\ (0.3,s_8) & (0.2,s_4) & (0.4,s_5) & (0.5,s_8)\end{pmatrix}.$$

Moreover, the $\tilde{Z}_1^{(1)}$ is computed by Definition 9 as follows:

$$\tilde{Z}_2^{(1)}=\begin{pmatrix}(0.5,s_8) & (0.225,s_4) & (0.550,s_5) & (0.625,s_4)\\ (0.775,s_4) & (0.5,s_8) & (0.825,s_4) & (0.900,s_4)\\ (0.450,s_5) & (0.175,s_4) & (0.5,s_8) & (0.575,s_4)\\ (0.375,s_4) & (0.100,s_4) & (0.425,s_4) & (0.5,s_8)\end{pmatrix}.$$

By using Eq. (5) we have $ACI(\tilde{P}_1^{(1)})=0.712>\delta$. Thus, the iteration of $\tilde{P}_1^{(1)}$ is ended.

(2) Inconsistency repair of $\tilde{P}_2^{(0)}$

Since $l_{14,2}^{(0)}=l_{41,2}^{(0)}=\min_{14,2}\{l_{ij}\}=s_2$, we replace the fuzzy values $p_{14,2}^{(0)}$ and $p_{41,2}^{(0)}$ with the corresponding elements in $\tilde{Z}_2^{(0)}$. The corresponding self-confidence levels are changed to $l_{14,2}^{(0)}=l_{41,2}^{(0)}\to s_6$ by using Eq. (7). Then, we have $\tilde{P}_2^{(1)}$:

$$\tilde{P}_2^{(1)}=\begin{pmatrix}(0.5,s_8) & (0.6,s_3) & (0.6,s_5) & (0.425,s_6)\\ (0.4,s_3) & (0.5,s_8) & (0.6,s_4) & (0.7,s_6)\\ (0.4,s_5) & (0.4,s_4) & (0.5,s_8) & (0.4,s_3)\\ (0.575,s_6) & (0.3,s_6) & (0.6,s_3) & (0.5,s_8)\end{pmatrix}.$$

Meanwhile, the $\tilde{Z}_2^{(1)}$ can be calculated by Definition 9:

$$\tilde{Z}_2^{(1)}=\begin{pmatrix}(0.5,s_8) & (0.481,s_3) & (0.606,s_3) & (0.538,s_3)\\ (0.519,s_3) & (0.5,s_8) & (0.625,s_3) & (0.556,s_3)\\ (0.394,s_3) & (0.375,s_3) & (0.5,s_8) & (0.431,s_3)\\ (0.463,s_3) & (0.444,s_3) & (0.569,s_3) & (0.5,s_8)\end{pmatrix}.$$

The $ACI(\tilde{P}_2^{(1)})=0.6425<\delta$ by using Eq. (5). Thus, the iteration of $\tilde{P}_2^{(0)}$ is continued.

In the second round of iterations, we find that the lowest self-confidence level of expert e_2 is $l_{12,2}^{(1)} = l_{21,2}^{(1)} = l_{34,2}^{(1)} = l_{43,2}^{(1)} = s_3$. Meanwhile, we have $d_{12,2}^{(1)} > d_{34,2}^{(1)}$. Thus, the elements $(p_{12,2}^{(1)}, l_{12,2}^{(1)})$ and $(p_{21,2}^{(1)}, l_{21,2}^{(1)})$ in $\tilde{P}_2^{(1)}$ should be adjusted. Replacing the fuzzy values with the corresponding elements in $\tilde{Z}_2^{(1)}$. And the $l_{12,2}^{(1)} = l_{21,2}^{(1)} \to s_5$ by using Eq. (7). After three iterations, we compute the $ACI(\tilde{P}_2^{(3)}) = 0.7260 > \delta$ by utilizing Eq. (5). It denotes that $\tilde{P}_2^{(3)}$ is of acceptable consistency, thus the iteration ends. The detailed iterative processes for $\tilde{P}_2$ are depicted in Table 1.

Table 1. The detailed consistency improving processes for $\tilde{P}_2$.

t	$\tilde{P}_2^{(t)}$	$ACI(\tilde{P}_2^{(t)})$	$(p_{ij,2}^{(t)}, l_{ij,2}^{(t)})$
0	$\tilde{P}_2^{(0)} = \begin{pmatrix} (0.5, s_8) & (0.6, s_3) & (0.6, s_5) & (0.2, s_2) \\ (0.4, s_3) & (0.5, s_8) & (0.6, s_4) & (0.7, s_6) \\ (0.4, s_5) & (0.4, s_4) & (0.5, s_8) & (0.4, s_3) \\ (0.8, s_2) & (0.3, s_6) & (0.6, s_3) & (0.5, s_8) \end{pmatrix}$	$ACI(\tilde{P}_2^{(0)}) = 0.5525$	$p_{14,2}^{(0)} \to 0.425$ $p_{41,2}^{(0)} \to 0.575$ $l_{14,2}^{(0)} = l_{41,2}^{(0)} \to s_6$
1	$\tilde{P}_2^{(1)} = \begin{pmatrix} (0.5, s_8) & (0.6, s_3) & (0.6, s_5) & (0.425, s_6) \\ (0.4, s_3) & (0.5, s_8) & (0.6, s_4) & (0.7, s_6) \\ (0.4, s_5) & (0.4, s_4) & (0.5, s_8) & (0.4, s_3) \\ (0.575, s_6) & (0.3, s_6) & (0.6, s_3) & (0.5, s_8) \end{pmatrix}$	$ACI(\tilde{P}_2^{(1)}) = 0.6425$	$p_{12,2}^{(1)} \to 0.481$ $p_{21,2}^{(1)} \to 0.519$ $l_{12,2}^{(1)} = l_{21,2}^{(1)} \to s_5$
2	$\tilde{P}_2^{(2)} = \begin{pmatrix} (0.5, s_8) & (0.481, s_3) & (0.6, s_5) & (0.425, s_6) \\ (0.519, s_3) & (0.5, s_8) & (0.6, s_4) & (0.7, s_6) \\ (0.4, s_5) & (0.4, s_4) & (0.5, s_8) & (0.4, s_3) \\ (0.575, s_6) & (0.3, s_6) & (0.6, s_3) & (0.5, s_8) \end{pmatrix}$	$ACI(\tilde{P}_2^{(2)}) = 0.6535$	$p_{34,2}^{(2)} \to 0.431$ $p_{43,2}^{(2)} \to 0.569$ $l_{34,2}^{(2)} = l_{43,2}^{(2)} \to s_5$
3	$\tilde{P}_2^{(3)} = \begin{pmatrix} (0.5, s_8) & (0.481, s_3) & (0.6, s_5) & (0.425, s_6) \\ (0.519, s_3) & (0.5, s_8) & (0.6, s_4) & (0.7, s_6) \\ (0.4, s_5) & (0.4, s_4) & (0.5, s_8) & (0.431, s_3) \\ (0.575, s_6) & (0.3, s_6) & (0.569, s_3) & (0.5, s_8) \end{pmatrix}$	$ACI(\tilde{P}_2^{(3)}) = 0.7260$	

Similarly, $\tilde{P}_3$ and $\tilde{P}_4$ are of acceptable additive consistency after four and one iterations, respectively. Details are given in Tables 2-3, respectively.

Table 2. The detailed consistency improving processes for $\tilde{P}_3$.

t	$\tilde{P}_3^{(t)}$	$ACI(\tilde{P}_3^{(t)})$	$(p_{ij,3}^{(t)}, l_{ij,3}^{(t)})$
0	$\tilde{P}_3^{(0)} = \begin{pmatrix} (0.5, s_8) & (0.3, s_5) & (0.4, s_7) & (0.7, s_4) \\ (0.7, s_5) & (0.5, s_8) & (0.2, s_6) & (0.3, s_3) \\ (0.6, s_7) & (0.8, s_6) & (0.5, s_8) & (0.6, s_2) \\ (0.3, s_4) & (0.7, s_3) & (0.4, s_2) & (0.5, s_8) \end{pmatrix}$	$ACI(\tilde{P}_3^{(0)}) = 0.5480$	$p_{34,3}^{(0)} \to 0.650$ $p_{43,3}^{(0)} \to 0.350$ $l_{14,3}^{(0)} = l_{41,3}^{(0)} \to s_6$

1	$\tilde{P}_3^{(1)} = \begin{pmatrix} (0.5,s_8) & (0.3,s_5) & (0.4,s_7) & (0.7,s_4) \\ (0.7,s_5) & (0.5,s_8) & (0.2,s_6) & (0.3,s_3) \\ (0.6,s_7) & (0.8,s_6) & (0.5,s_8) & (0.65,s_2) \\ (0.3,s_4) & (0.7,s_3) & (0.35,s_2) & (0.5,s_8) \end{pmatrix}$	$ACI(\tilde{P}_3^{(1)}) = 0.6120$	$p_{24,3}^{(1)} \to 0.463$ $p_{42,3}^{(1)} \to 0.537$ $l_{24,3}^{(1)} = l_{42,3}^{(1)} \to s_5$
2	$\tilde{P}_3^{(2)} = \begin{pmatrix} (0.5,s_8) & (0.3,s_5) & (0.4,s_7) & (0.7,s_4) \\ (0.7,s_5) & (0.5,s_8) & (0.2,s_6) & (0.463,s_3) \\ (0.6,s_7) & (0.8,s_6) & (0.5,s_8) & (0.65,s_2) \\ (0.3,s_4) & (0.537,s_3) & (0.35,s_2) & (0.5,s_8) \end{pmatrix}$	$ACI(\tilde{P}_3^{(2)}) = 0.6865$	$p_{14,3}^{(2)} \to 0.553$ $p_{41,3}^{(2)} \to 0.447$ $l_{14,3}^{(2)} = l_{41,3}^{(2)} \to s_4$
3	$\tilde{P}_3^{(3)} = \begin{pmatrix} (0.5,s_8) & (0.3,s_5) & (0.4,s_7) & (0.553,s_4) \\ (0.7,s_5) & (0.5,s_8) & (0.2,s_6) & (0.463,s_3) \\ (0.6,s_7) & (0.8,s_6) & (0.5,s_8) & (0.65,s_2) \\ (0.447,s_4) & (0.537,s_3) & (0.35,s_2) & (0.5,s_8) \end{pmatrix}$	$ACI(\tilde{P}_3^{(3)}) = 0.7015$	$p_{14,3}^{(3)} \to 0.480$ $p_{41,3}^{(3)} \to 0.520$ $l_{14,3}^{(3)} = l_{41,3}^{(3)} \to s_4$
4	$\tilde{P}_3^{(4)} = \begin{pmatrix} (0.5,s_8) & (0.3,s_5) & (0.4,s_7) & (0.480,s_4) \\ (0.7,s_5) & (0.5,s_8) & (0.2,s_6) & (0.463,s_3) \\ (0.6,s_7) & (0.8,s_6) & (0.5,s_8) & (0.65,s_2) \\ (0.520,s_4) & (0.537,s_3) & (0.35,s_2) & (0.5,s_8) \end{pmatrix}$	$ACI(\tilde{P}_3^{(4)}) = 0.7010$	

Table 3. The detailed consistency improving processes for $\tilde{P}_4$.

t	$\tilde{P}_4^{(t)}$	$ACI(\tilde{P}_4^{(t)})$	$(p_{ij,4}^{(t)}, l_{ij,4}^{(t)})$
0	$\tilde{P}_4^{(0)} = \begin{pmatrix} (0.5,s_8) & (0.4,s_5) & (0.2,s_4) & (0.1,s_5) \\ (0.6,s_5) & (0.5,s_8) & (0.6,s_6) & (0.2,s_3) \\ (0.8,s_4) & (0.4,s_6) & (0.5,s_8) & (0.1,s_7) \\ (0.9,s_5) & (0.8,s_3) & (0.9,s_7) & (0.5,s_8) \end{pmatrix}$	$ACI(\tilde{P}_4^{(0)}) = 0.6430$	$p_{24,4}^{(0)} \to 0.2$ $p_{42,4}^{(0)} \to 0.8$ $l_{24,4}^{(0)} = l_{42,4}^{(0)} \to s_5$
1	$\tilde{P}_4^{(1)} = \begin{pmatrix} (0.5,s_8) & (0.4,s_5) & (0.2,s_4) & (0.1,s_5) \\ (0.6,s_5) & (0.5,s_8) & (0.6,s_6) & (0.2,s_3) \\ (0.8,s_4) & (0.4,s_6) & (0.5,s_8) & (0.1,s_7) \\ (0.9,s_5) & (0.8,s_3) & (0.9,s_7) & (0.5,s_8) \end{pmatrix}$	$ACI(\tilde{P}_4^{(1)}) = 0.7065$	

The adjusted FPRs-SC P_m^* $(m=1,2,3,4)$ are offered in the following:

$$P_1^* = \begin{pmatrix} (0.5,s_8) & (0.1,s_5) & (0.6,s_7) & (0.7,s_8) \\ (0.9,s_5) & (0.5,s_8) & (0.8,s_6) & (0.8,s_4) \\ (0.4,s_7) & (0.2,s_6) & (0.5,s_8) & (0.6,s_5) \\ (0.3,s_8) & (0.2,s_4) & (0.4,s_5) & (0.5,s_8) \end{pmatrix},$$

$$P_2^* = \begin{pmatrix} (0.5,s_8) & (0.481,s_3) & (0.6,s_5) & (0.425,s_6) \\ (0.519,s_3) & (0.5,s_8) & (0.6,s_4) & (0.7,s_6) \\ (0.4,s_5) & (0.4,s_4) & (0.5,s_8) & (0.431,s_3) \\ (0.575,s_6) & (0.3,s_6) & (0.569,s_3) & (0.5,s_8) \end{pmatrix},$$

$$P_3^* = \begin{pmatrix} (0.5, s_8) & (0.3, s_5) & (0.4, s_7) & (0.480, s_4) \\ (0.7, s_5) & (0.5, s_8) & (0.2, s_6) & (0.463, s_3) \\ (0.6, s_7) & (0.8, s_6) & (0.5, s_8) & (0.65, s_2) \\ (0.520, s_4) & (0.537, s_3) & (0.35, s_2) & (0.5, s_8) \end{pmatrix},$$

$$P_4^* = \begin{pmatrix} (0.5, s_8) & (0.4, s_5) & (0.2, s_4) & (0.1, s_5) \\ (0.6, s_5) & (0.5, s_8) & (0.6, s_6) & (0.2, s_3) \\ (0.8, s_4) & (0.4, s_6) & (0.5, s_8) & (0.1, s_7) \\ (0.9, s_5) & (0.8, s_3) & (0.9, s_7) & (0.5, s_8) \end{pmatrix}.$$

Step 3. Aggregation process.

For each P_m^* $(m = 1, 2, 3, 4)$, we have the self-confidence matrix $L_m = (l_{ij,m})$ of each expert e_m $(m = 1, 2, 3, 4)$ as follows:

$$L_1 = \begin{pmatrix} s_8 & s_5 & s_7 & s_8 \\ s_5 & s_8 & s_6 & s_4 \\ s_7 & s_6 & s_8 & s_5 \\ s_8 & s_4 & s_5 & s_8 \end{pmatrix}, \quad L_2 = \begin{pmatrix} s_8 & s_5 & s_5 & s_6 \\ s_5 & s_8 & s_4 & s_6 \\ s_5 & s_4 & s_8 & s_5 \\ s_6 & s_6 & s_5 & s_8 \end{pmatrix},$$

$$L_3 = \begin{pmatrix} s_8 & s_5 & s_7 & s_4 \\ s_5 & s_8 & s_6 & s_5 \\ s_7 & s_6 & s_8 & s_6 \\ s_4 & s_5 & s_6 & s_8 \end{pmatrix}, \quad L_4 = \begin{pmatrix} s_8 & s_5 & s_4 & s_5 \\ s_5 & s_8 & s_6 & s_5 \\ s_4 & s_6 & s_8 & s_7 \\ s_5 & s_5 & s_7 & s_8 \end{pmatrix}.$$

Then, compute the values of *SCDL* and *SCI* of e_m $(m = 1, 2, 3, 4)$ by using Eqs. (9) and (10), respectively. Results are given in Table 4.

Table 4. The results of *SCDL* and *SCI* of the FPRs-SC given by e_m $(m = 1, 2, 3, 4)$.

	e_1	e_2	e_3	e_4
SCDL	0.2709	0.3542	0.3126	0.3334
SCI	0.7291	0.6458	0.6874	0.6666

By using Eq. (13), we have $T = \sum_{k=1}^{4} SCI_{\sigma(k)}(P_k^*) = 2.7289$, $\sigma(1) = 1$, $\sigma(2) = 3$, $\sigma(3) = 4$, and $\sigma(4) = 2$.

In addition, the linguistic quantifier "most of" provided in [41] is utilized to generate the weighting vector, i.e., $Q(\varphi) = \varphi^{\frac{1}{2}}$. Thus, the weights of experts e_m

$(m = 1, 2, 3, 4)$ are as follows:

$$w_1 = 0.5169,\ \ w_2 = 0.2036,\ \ w_3 = 0.1532,\ \ w_4 = 0.1263.$$

Then, by using Eq. (11), the collective FPR-SC $P^{SCL} = (p_{ij}^{SCL}, l_{ij}^{SCL})$ is computed as follows:

$$P^{SCL} = \begin{pmatrix} (0.5, s_8) & (0.2438, s_5) & (0.4980, s_4) & (0.5285, s_4) \\ (0.7652, s_5) & (0.5, s_8) & (0.6219, s_4) & (0.6268, s_4) \\ (0.5020, s_4) & (0.3781, s_4) & (0.5, s_8) & (0.5122, s_5) \\ (0.4715, s_4) & (0.3732, s_4) & (0.4878, s_5) & (0.5, s_8) \end{pmatrix}.$$

Step 4. Exploitation process.

To calculate the $SCS(x_i)$ $(i = 1, 2, 3, 4)$ by using Eq. (14). Then, we have:

$$SCS(x_1) = 2.320,\ \ SCS(x_2) = 3.205,$$
$$SCS(x_3) = 2.520,\ \ SCS(x_4) = 2.454.$$

Thus, the alternative ranking of collective is $x_2 \succ x_3 \succ x_4 \succ x_1$, and the best alternative is x_2.

5.2 A comparative analysis

To further show the advantages and contributions of this paper, a comparative analysis is given below.

(1) Comparison project

As far as we know, the FPRs denote that experts are fully self-confident of their judgments. The self-confidence levels related to all fuzzy values are the same, that is, $l_{ij} = s_g$ for $\forall i, j \in N$. Generally, the self-confidence levels are omitted for notation simplification in FPRs [17]. Thus, the FPRs can be seen a special case of FPRs-SC. Many kinds of additive consistency improvements for FPRs have been proposed [2, 18]. Here, we compare the inconsistency repairs in [18] with the Algorithm 1 proposed in this paper.

(2) Comparison criteria and results

In this paper, we mainly focus on the study of FPRs-SC. Firstly, it is our aim to demonstrate that the self-confidence levels of experts have influence on decision making. In other words, the new kind of preference relation FPR-SC is useful in GDM problems. Secondly, for the repair of inconsistencies of FPRs-SC, we hope that the original experts' information retained as much as possible while the acceptable consistency is achieved. Therefore, we propose the following three comparison criteria to evaluate the influence of self-confidence levels on decision results in GDM problems, as well as the advantage and utility of the proposed Algorithm 1.

a) Experts' self-confidence levels influence the results of alternative ranking

Consider the special case of the illustrative example proposed in Section 5.1, in which, the self-confidence levels of experts are the same, i.e., $l_{ij,m} = s_8$ for all $i, j, m = \{1, 2, 3, 4\}$. Thus, it means that the weights of experts e_m are equal with $w_m = 1/4$. The detailed decision results are shown in Table 5.

Table 5. The decision results that considers the special case of proposed illustrative example.

m	w_m	ACI_m	Collective preference relation	Alternative ranking of collective
1	0.25	0.9365	$\begin{pmatrix} 0.5 & 0.35 & 0.45 & 0.43 \\ 0.65 & 0.5 & 0.55 & 0.45 \\ 0.55 & 0.45 & 0.5 & 0.43 \\ 0.57 & 0.55 & 0.57 & 0.5 \end{pmatrix}$	$x_4 \succ x_2 \succ x_3 \succ x_1$
2	0.25	0.9275		
3	0.25	0.9230		
4	0.25	0.9555		

From Table 5, we observe that if experts are fully confident of their preferences, the alternative ranking of collective is $x_4 \succ x_2 \succ x_3 \succ x_1$, and the best alternative is not x_2 but x_4. It validates that the self-confidence levels of experts have influence on the final decision in GDM problems.

b) The adjustment degree (AD) and adjustment ratio (AR) between original and modified FPRs-SC

The *AD* represents the degree of difference between the original FPR-SC and the modified FPR-SC. The larger *AD* is, the less the original FPR-SC is retained. In addition, the *AR* represents the ratio of the elements which are modified in FPR-SC. The larger *AR* is, the more elements in FPR-SC are modified. Here, the *AD* and *AR* are computed by the following way, respectively:

Let $\tilde{P}_m^{(0)} = (p_{ij,m}^{(0)}, l_{ij,m}^{(0)})$ and $P_m^* = (p_{ij,m}^*, l_{ij,m}^*)$ defined as before ($i, j \in N$, $m \in M$). The *AD* for the $\tilde{P}_m$ given by expert e_m is:

$$AD_m = \tau \frac{\sum_{i,j=1}^{n} | p_{ij,m}^{(0)} - p_{ij,m}^* |}{\sum_{i,j=1}^{n} p_{ij,m}^{(0)}} + (1-\tau) \frac{\sum_{i,j=1}^{n} | \Delta^-(l_{ij,m}^{(0)}) - \Delta^-(l_{ij,m}^*) |}{\sum_{i,j=1}^{n} \Delta^-(l_{ij,m}^{(0)})}. \quad (15)$$

The *AR* of the $\tilde{P}_m$ given by expert e_m is computed by

$$AR_m = \tau \frac{\sum_{i,j=1}^{n} f_{ij,m}}{n^2} + (1-\tau) \frac{\sum_{i,j=1}^{n} \tilde{f}_{ij,m}}{n^2} \quad (16)$$

where $f_{ij,m} = \begin{cases} 0, & p_{ij,m}^{(0)} = p_{ij,m}^* \\ 1, & otherwise \end{cases}$, $\tilde{f}_{ij,m} = \begin{cases} 0, & l_{ij,m}^{(0)} = l_{ij,m}^* \\ 1, & otherwise \end{cases}$, and $\tau \in [0,1]$ is a parameter to control the *AD* and *AR* of both the fuzzy values and the self-confidence levels of experts. As we mentioned in Section 3, this paper assumes that the fuzzy values and the self-confidence levels are equally important in GDM problems, thus, we have $\tau = 0.5$.

If we use Eq. (10) which is utilized by [18] as the FPRs-SC modifying strategy, that is, only adjust the fuzzy values while retain the self-confidence levels unchanged, we can obtain the consistency iteration process of expert e_1. The detailed result is depicted in Table 6 (setting the related parameter $\eta = 0.5$).

Table 6. The consistency iteration of $\tilde{P}_1$ by using Eq. (10) in [18].

t	$\tilde{P}_1^{(t)}$	$ACI(\tilde{P}_1^{(t)})$
0	$\tilde{P}_1^{(0)} = \begin{pmatrix} (0.5, s_8) & (0.1, s_5) & (0.6, s_7) & (0.7, s_8) \\ (0.9, s_5) & (0.5, s_8) & (0.8, s_6) & (0.6, s_4) \\ (0.4, s_7) & (0.2, s_6) & (0.5, s_8) & (0.6, s_5) \\ (0.3, s_8) & (0.4, s_4) & (0.4, s_5) & (0.5, s_8) \end{pmatrix}$	$ACI(\tilde{P}_1^{(0)}) = 0.6865$
1	$\tilde{P}_1^{(1)} = \begin{pmatrix} (0.5, s_8) & (0.188, s_5) & (0.575, s_7) & (0.638, s_8) \\ (0.812, s_5) & (0.5, s_8) & (0.788, s_6) & (0.7, s_4) \\ (0.425, s_7) & (0.213, s_6) & (0.5, s_8) & (0.563, s_5) \\ (0.363, s_8) & (0.3, s_4) & (0.438, s_5) & (0.5, s_8) \end{pmatrix}$	$ACI(\tilde{P}_1^{(1)}) = 0.7185$

Then, the values of the *AD* and *AR* of expert e_1 by utilizing the proposed

Algorithm 1 and the Eq. (10) in [18], respectively, are shown in Table 7.

Table 7. The values of the AD and AR of expert e_1.

Utilize the proposed Algorithm 1		Utilize Eq. (10) in [18]	
AD_1	AR_1	AD_1	AR_1
0.025	0.063	0.041	0.375

Clearly, from Table 7, we find that the AD_1 by using Algorithm 1 is smaller than using the Eq. (10) in [18]. Thus, it demonstrates that our proposed Algorithm 1 keeps more original information than the method given in [18]. Meanwhile, the value of AR_1 by using the Algorithm 1 is 0.063, which means that 93.7% elements in $\tilde{P}_1$ are retained. However, the value of AR_1 by using the Eq. (10) in [18] is 0.375, it means that only 43.8% elements in $\tilde{P}_1$ are unchanged. Therefore, it also shows that our proposed Algorithm 1 retains the experts' original information as much as possible.

To save the space, the detailed consistency iteration processes for the other three FPRs-SC $\tilde{P}_m$ $(m=2,3,4)$ are omitted here. We only show the values of the *ACI* of $\tilde{P}_m$ $(m=2,3,4)$ in Table 8.

Table 8. The values of *ACI* of $\tilde{P}_m$ $(m=2,3,4)$ by using Eq. (10) in [18].

t	$ACI(\tilde{P}_2^{(t)})$	$ACI(\tilde{P}_3^{(t)})$	$ACI(\tilde{P}_4^{(t)})$
0	0.5525	0.5480	0.6430
1	0.5890	0.5870	0.6660
2	0.6065	0.6055	0.6765
3	0.6160	0.6160	0.6810
4	0.6185	0.6185	0.6875
5	0.6250	0.6250	0.6875
6	0.6250	0.6250	0.6875

From Table 8, we see that after six iterations, the $ACI(\tilde{P}_m^{(6)}) < \delta = 0.7$, $(m=2,3,4)$, which means that at the maximum number of iterations $t^* = 6$, the acceptable additive consistency of $\tilde{P}_m$ $(m=2,3,4)$ is not achieved by using Eq. (10) in [18]. This represents that the inconsistency repair that only adjust the fuzzy values of FPRs-SC would not ensure that an acceptable consistency can be achieved.

In addition, the trends of additive consistency iterations for experts e_m

($m=2,3,4$) by using the Algorithm 1 are shown in Figs. 4-6, respectively.

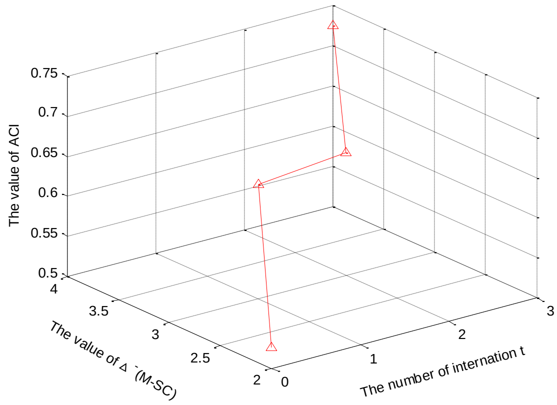

Fig. 4. The trend of consistency iteration for e_2.

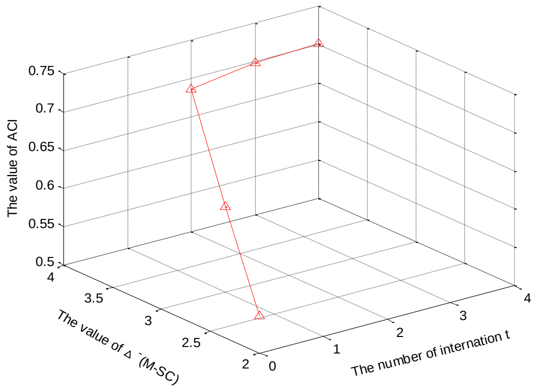

Fig. 5. The trend of consistency iteration for e_3

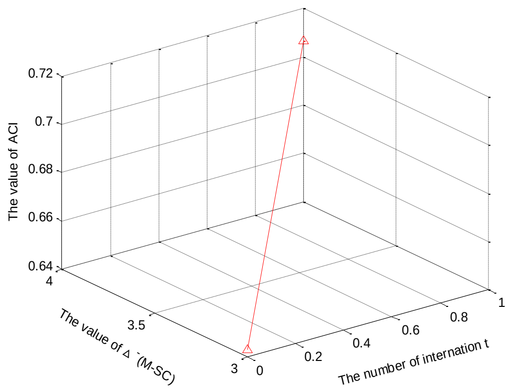

Fig. 6. The trend of consistency iteration for e_4.

Obviously, from Figs. 4-6, we observe that with the increases of the number of iterations, the values of the *ACI* and M-SC of experts e_m $(m=2,3,4)$ are generally increased. It means that the Algorithm 1 is feasible and effective for the inconsistency repair of FPRs-SC. Moreover, Fig. 7 shows the change trend between the values of

ACI and M-SC of experts e_m $(m = 2, 3, 4)$ in inconsistency repair process of the proposed illustrative example. From Fig. 7, we see that the values of ACI increase along with the increases of the expert's M-SC. It shows that adjustments in expert's M-SC will effectively improve the additive consistency of FPR-SC provided by experts. Meanwhile, it is also consistent with our previous hypothesis, i.e., the higher the levels of experts' self-confidence, the greater the likelihood that they may grasp of the GDM problem, and then the assessment information given by the experts improves in terms of consistency.

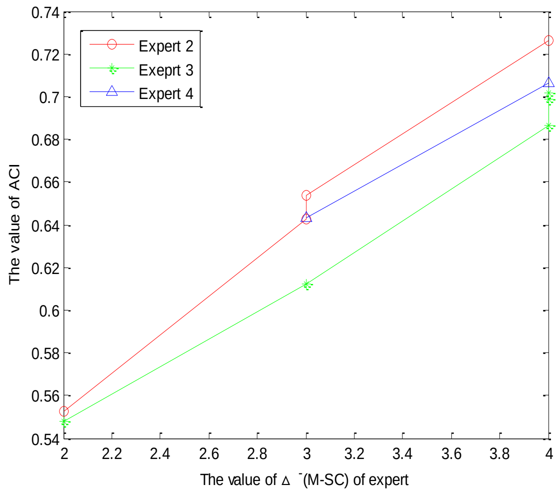

Fig. 7. The change trend between the values of M-SC and ACI of e_m $(m = 2, 3, 4)$

6. Concluding remarks

This paper focuses on a new kind of preference relations, i.e., FPRs-SC, and proposes an analysis of additive consistency for FPRs-SC. And then studies its application in GDM problems. The major contributions of this paper are concluded as follows:

(1) Some new operational laws of FPRs-SC have been presented to analyze the additive consistency and its application in GDM problems. Meanwhile, a new ACI which considers experts' self-confidence levels has been proposed to measure the additive consistency level of FPRs-SC.

(2) An iterative algorithm which adjusts both the fuzzy values and self-confidence levels of experts is provided to repair the inconsistency of FPRs-SC. From the comparison results, we find that the proposed

inconsistency improvement can ensure experts' original information be retained as much as possible while an acceptable additive consistency is achieved.

(3) An *SCI* is proposed to measure the self-confidence degree of expert for her/his preferences. And then, an SCI-IOWA operator, which makes the more importance give those experts which have more self-confident, is presented to aggregate individual FPRs-SC into a collective one.

(4) An *SCS* function of FPR-SC is designed to obtain the alternative ranking of collective in GDM with FPRs-SC. Subsequently, we can select the best alternative as the one with the highest *SCS* value.

With the rapid developments and applications of science and technology [42], more and more experts are involved in GDM problems. It makes large-scale group decision making (LSGDM) problems becoming a hotspot [43-48]. In addition, due to time pressure, lack of knowledge, and limited experience, experts may overestimate their judgments, i.e., experts may show overconfidence behaviors in decision making processes [49]. Thus, in future work, we will address to extend FPRs-SC to LSGDM problems and discuss the influence of over-confidence behaviors on decision making.

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