Alternative Ranking-Based Clustering and Reliability Index-Based Consensus Reaching Process for Hesitant Fuzzy Large Scale Group Decision Making

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***Abstract*—Large scale group decision making (LSGDM) problems are becoming one of the hotspots in recent research fields. This paper focuses on the hesitant fuzzy LSGDM problems, where decision makers (DMs) use hesitant fuzzy reciprocal preference relations (HFPRs) to express their assessment information. The HFPRs can well represent the fuzziness and hesitancy of DM's assessment information. To improve the efficiency of hesitant fuzzy LSGDM problems, we propose a reliability index-based consensus reaching process (RI-CRP). By assessing the ordinal consistency of DM's assessment information and measuring the deviation with the collective opinion, the DM's opinion reliability index is given. To avoid unreliable information, we propose an unreliable DMs management method to be used in the RI-CRP, based on the computation of the DM's opinion reliability index. Moreover, an alternative ranking-based clustering (ARC) method with HFPRs is proposed to improve the efficiency of the RI-CRP. The similarity index between two DMs' opinions is provided, to ensure the ARC method can be effectively implemented. Compared with those clustering methods which need to preset several correlated parameters, the presented ARC method is more objective with a different approach based on the alternative ranking. Finally, a numerical example proves that the proposed ARC method and the RI-CRP are feasible and effective for hesitant fuzzy LSGDM problems.**

***Index Terms*—Large scale group decision making (LSGDM), reliability index (RI), alternative ranking-based clustering (ARC), consensus reaching process (CRP).**

This work was supported by the National Natural Science Foundation of China (NSFC) (Grant No. 71471056 and No. 71772136), the Key Project of National Natural Science Foundation of China (No.71433003), sponsored by Qing Lan Project of Jiangsu Province, and the China Scholarship Council.

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Draft 02-2018.

I. Introduction

WITH the rapid development and applications of science and technology, such as e-democracy [1], social networks [2],[3], and public participation [4], more and more decision makers (DMs) are involved in decision making problems. It makes the large scale group decision making (LSGDM) problems becoming a hotspot in the related research fields [5]-[7]. As the number of DMs involved in LSGDM problems is huge, thus, it is of great importance to effectively manage the DMs and improve the efficiency of LSGDM problems.

In LSGDM problems, there are a large number of DMs involved. They may have different culture, education backgrounds and personal interest preferences. Meanwhile, there are also fuzziness and hesitancy natures in human judgement. Thus, when expressing their assessment information, DMs may have several possible numerical values and may perform hesitance to give the decisions [8]. We focused our attention on the hesitant fuzzy set (HFS) [9]-[12].

In the decision making process, preference relation is one of the most usual preference structures to be used in expressing DM's assessment information. Hesitant fuzzy preference relation (HFPR) [13] is an effective tool to express DM's hesitancy and fuzziness. Meanwhile, the HFPR is widely used in the decision making events [11],[14]-[16]. In HFPR, DM's assessment information consists of hesitant fuzzy elements (HFEs), which denotes all possible preference values, and can be utilized to well express DM's hesitant and fuzzy information in LSGDM problems.

As we all know, in LSGDM problems, it is really hard to ensure the final decision can be accepted by all the DMs since there are a large number of DMs participated. Thus, the consensus reaching processes (CRPs) [17],[18] were proposed to improve the efficiency of LSGDM problems [5]-[7], [19]-[22]. Additionally, to improve the efficiency of CRPs, clustering methods were proposed and widely used in the CRPs for LSGDM problems. All the existing CRPs models play an important role in improving the efficiency of LSGDM problems. However, there are still some flaws that in the research for LSGDM problems that need to be discussed:

(a) All the existing CRPs models are almost based on the hypothesis that all the DMs' opinions provided for LSGDM problems are reliable. Their assessment information is used directly in the decision making process without checking the reliability of them. Actually, it is very difficult to ensure that each DM's assessment information is reliable in LSGDM problems. The reason is that there are a huge number of DMs participated in LSGDM problems, and some of them may give dishonest or contradictory opinions, which are presented only for their interests. Once the unreliable opinions are utilized in the CRPs, the validity and reliability of the final decision will be great decreased for LSGDM problems.

(b) Most of the existing clustering methods for LSGDM problems are almost the expansions of fuzzy-c means [6],[20],[23],[24], and interval fuzzy c-means clustering [25]. All these methods usually need to preset several subjective clustering coefficients, which may reduce the objectiveness of the clustering results. Additionally, some innovative clustering methods are provided with fuzzy set [26], interval-valued intuitionistic fuzzy set [21], interval type-2 fuzzy [22], rather than HFS. Whether they are applicable to hesitant fuzzy LSGDM problems or not, it needs further verification.

(c) In the CRPs for LSGDM problems, some clusters' opinions may be far from the collective opinion and the DMs in them may do not make any compromise despite the guidance of the moderator. Those DMs prefer to stick with their own opinions, which are good for their own interests. We call the cluster's behavior that contains these DMs as "non-cooperative behavior". To achieve a high level of consensus and improve the efficiency of the CRPs in LSGDM problems, these non-cooperative clusters need to be managed reasonably.

In order to tackle these three gaps in LSGDM problems mentioned above, we propose an alternative ranking-based clustering (ARC) method with HFPRs, and a corresponding reliability index-based CRP (RI-CRP). The proposed ARC method and the RI-CRP for hesitant fuzzy LSGDM problems are mainly based on the following hypotheses:

(1) As DMs always give the assessment information which is conducive to their own interests, the assessment information given by DMs may be unreliable in hesitant fuzzy LSGDM problems. The unreliability can be reflected by the following two aspects. One is the contradictory views in DMs' assessment information, and the other is the excessive deviation between individual and collective opinions. Furthermore, if the unreliable DMs' opinions are used in the decision making process, the validity and efficiency of the CRPs for hesitant fuzzy LSGDM problems will be greatly reduced.

(2) The aim of clustering method is to classify the DMs who provide similar opinions into a group. Generally, the similarity between two DMs' opinions can be reflected by the DMs' HFPRs alternative ranking. According to the majority principle in decision making, those DMs who express the majority of similar opinions in the HFPRs alternative ranking should be classified into the same group. We introduce the similarity index (SI) between two DMs' opinions. The implementation of ARC method can well improve the efficiency of the CRPs for hesitant fuzzy LSGDM problems. This is a greatly different hypothesis in the comparison with the literatures, grouping experts by their preferences instead of alternative ranking.

(3) Although the DMs, which provide the reliable assessment information, can participate in the further hesitant fuzzy LSGDM process, some of them may do not make any compromise to protect their interests in the CRPs. It makes the clusters which contains those non-cooperative DMs contribute less for the consensus. Thus, in order to reach a high level of consensus for hesitant fuzzy LSGDM problems, the clusters that contain these DMs need to be managed reasonably.

The improvements of the ARC method and the RI-CRP for hesitant fuzzy LSGDM problems in this paper can be mainly listed as the following three aspects:

(1) By assessing the ordinal consistency of DMs' opinions and measuring the deviation between individual and collective opinions, the DM's opinion reliability index (ORI) is given. Meanwhile, the algorithm of DM's opinion reliability detection is provided. By checking the DM's ORI, the unreliable DMs reasonable management processes are proposed. This allows us to guarantee all the DMs involved in the CRPs can provide reliable assessment information, which ensures that the final decision is reasonable and reliable.

(2) An ARC method is given with DMs' HFPRs alternative ranking. By comparing the number of alternatives with the same position (NASP) between two DMs, the SI between them is provided, which ensure the ARC method can be effectively implemented. Additionally, the Algorithm of the ARC method for hesitant fuzzy LSGDM problems is proposed. Compared with those clustering methods which need to preset several correlated parameters, the ARC method is more objective with a different approach based on the alternative ranking.

(3) In the RI-CRP, the group consensus index (GCI) is given to measure the consensus level. To achieve a high level of consensus, the management processes for non-cooperative clusters in the RI-CRP are proposed. For the non-cooperative clusters which are unwilling to make any compromise, their weights will be punished. The implementations of the weight punishment make the RI-CRP more efficient for hesitant fuzzy LSGDM problems.

The proposed ARC method and the RI-CRP for hesitant fuzzy LSGDM problems are examined by a numerical example. The example shows that the utilization of ARC method and RI-CRP can effectively improve the efficiency of the hesitant fuzzy LSGDM problems. From the example results, we can show that unreliable DMs and the non-cooperative clusters are effectively managed and the consensus is reached up to the threshold in a limited three rounds of the RI-CRP, which shows the efficiency of the proposed ARC method and the RI-CRP for hesitant fuzzy LSGDM problems.

The rest of this paper is organized as follows. In Section II, some preliminaries related to fuzzy reciprocal preference relation (FPR), HFS, HFPR, the score functions of HFPR, and the assessment method of ordinal consistency for FPR are reviewed. In Section III, DM's opinion reliability detection processes are proposed, and the corresponding reasonable management methods for unreliable DMs are given. In Section IV, the ARC method is given, detailing the steps for clustering processes. In Section V, the RI-CRP for hesitant fuzzy LSGDM problems is proposed. A numerical example and analysis of the proposed ARC method and RI-CRP for hesitant fuzzy LSGDM problems are shown in Section VI. Finally, some conclusions of this paper are summarized in Section VII.

II. Preliminaries

Before giving the ARC method and the RI-CRP, some related preliminaries are presented in this section. In Section II.A, we first provide the preliminary knowledge regarding the FPR, HFS and HFPR. Subsequently, we review the score function of HFEs and HFPR in Section II.B, which provide the basis for ARC method of hesitant fuzzy LSGDM problems. Finally, the assessment method of ordinal consistency for FPR is provided, which is used in the RI-CRP to detect the DM's opinion reliability. For simplicity, we denote $N=\{1,2,\ldots,n\}$ as the number of the alternatives.

A. Basic concepts of FPR, HFS and HFPR

Definition 1 (*see [27]*). An additive FPR R on a finite set of alternatives $X=\{x_1,x_2,\ldots,x_n\}$ is a fuzzy relation on the product set $X\times X$ with membership function $\mu_R: X\times X\to[0,1]$, $\mu_R(x_i,x_j)=r_{ij}$, verifying:

$$r_{ij}+r_{ji}=1,\ r_{ii}=0.5,\ i,j\in N\ .$$

Generally, an FPR is represented by an $n\times n$ matrix $R=(r_{ij})_{n\times n}$, in which r_{ij} denotes the preference degree of x_i over x_j. Where $r_{ij}=0.5$ implies indifference between x_i and x_j $(x_i\sim x_j)$; $r_{ij}=1$ indicates that x_i definitely preferred to x_j $(x_i\succ x_j)$; $0.5<r_{ij}<1$ means that x_i is preferred to x_j $(x_i\succ x_j)$; $0\le r_{ij}<0.5$ indicates that x_j is preferred to x_i, the smaller r_{ij} the stronger the preference of x_j over x_i.

Definition 2 (*see [12]*). Let $X=\{x_1,x_2,\ldots,x_n\}$ be a fixed set of alternatives. An HFS A on X is characterized by a membership function $h_A(x)$ that when applied to X returns a subset of [0,1], which can be represented by a mathematical expression:

$$A=\{<x,h_A(x)>|\,x\in X\},$$

where $h_A(x)$ is a set of some different values in [0,1], denoting the possible hesitant membership degree of the elements $x\in X$ to A. For convenience, $h=h_A(x)$ is called an HFE.

A detailed review on HFS and the further use are provided in [28],[29]. Based on HFS and FPR, the concept of the HFPR is defined by Xu et al. [11] as follows:

Definition 3 (*see [11]*). Let $X=\{x_1,x_2,\ldots,x_n\}$ be a fixed set of alternatives, then an HFPR H on X is represented by a matrix $H=(h_{ij})_{n\times n}\subset X\times X$, where $h_{ij}=\{h_{ij}^{(l)}\,|\,l=1,\ldots,\#h_{ij}\}$ ($\#h_{ij}$ is the number of elements in h_{ij}) is an HFE, which indicates all the possible values of preference degree of the alternative x_i over x_j. For all $i,j\in N$, h_{ij} should satisfy the following conditions:

$$h_{ij}^{(l)}+h_{ji}^{(l)}=1,\ h_{ii}=\{0.5\},\ \#h_{ij}=\#h_{ji},$$

where $h_{ij}^{(l)}$ and $h_{ji}^{(l)}$ are the l th elements in h_{ij}, respectively.

Remark 1. The Definition 3 is different from [30]'s definition of HFPR, it does not have the constraint that the values in h_{ij} are supposed to be arranged in ascending order, i.e., $h_{ij}^{(l)}<h_{ij}^{(l+1)}$, $h_{ji}^{(l+1)}<h_{ji}^{(l)}$. The detailed explanation can be seen in Remark 1 of [11]. Generally, the number of values in different h_{ij} is different. In order to operate correctly, there exists a normalization process in [11],[31], which make the different HFEs with the same number of values. In this paper, we assume the HFPRs offered by the DMs are normalized.

B. Basic concepts of the score function of HFE and HFPR

To compare the HFEs, Xia and Xu [32] defined the following comparison laws:

Definition 4 (*see [32]*). For an HFE h, $s(h)=\frac{1}{\#h}\sum_{\gamma\in h}\gamma$ is called the score function of h, and $\#h$ is the number of the elements in h. For two HFEs h_1 and h_2, if $s(h_1)>s(h_2)$, then h_1 is superior to h_2, denoted by $h_1\succ h_2$; if $s(h_1)=s(h_2)$, then h_1 is indifferent to h_2, denoted by $h_1\sim h_2$.

According to the Definitions 3 and 4, suppose an HFPR $H=(h_{ij})_{n\times n}$, where h_{ij} represents the preference degree between alternative x_i and x_j, $\#h_{ij}$ is the number of the elements in h_{ij}, and $X=\{x_1,x_2,\ldots,x_n\}$ be a fixed set of alternatives, then the score value of each alternative $s(x_i)$, $i\in N$, can be calculated as follows:

$$s(x_i)=\frac{1}{\#h_{ij}}\sum_{j=1}^{n}\sum_{\gamma\in h_{ij}}\gamma\ . \tag{1}$$

Then, the alternative ranking with HFPRs can be obtained based on the overall score values, as well as the best alternative(s) can be selected.

C. The ordinal consistency with FPR

In [33], the definition of ordinal consistency for FPR is introduced as follows:

Definition 5 (*see[33]*). Let $R=(r_{ij})_{n\times n}$ be a FPR, for all $i,j,k\in N$, $i\neq j\neq k$,

(1) if $r_{ik}>0.5$, $r_{kj}\ge 0.5$; or $r_{ik}\ge 0.5$, $r_{kj}>0.5$, we have $r_{ij}>0.5$;

(2) if $r_{ik}=0.5$, and $r_{kj}=0.5$, we have $r_{ij}=0.5$.

Then FPR R is said to have ordinal consistency.

Remark 2. Definition 5 is the minimum requirement that a consistent FPR should possess, and it is the usual transitivity condition that a logical and consistent DM should use if he/she does not want to provide contradictory opinions.

Then, Xu et al. [33] discuss the ordinal consistency of FPR from the perspective of graph theory, and present some basic theory of digraph as follow:

Definition 6 (*see [33]*). Let $R=(r_{ij})_{n\times n}$ be a FPR, the adjacency matrix $E=(e_{ij})_{n\times n}$ of R is defined as follows:

$$e_{ij}=\begin{cases}1, & r_{ij}\geq 0.5, (i\neq j);\\ 0, & otherwise.\end{cases} \quad (2)$$

Then, a digraph $G=(V,A)$ of R is constructed, where $V=\{v_1,v_2,\ldots,v_n\}$ denotes the node set and $A=\{(V_i,V_j)\mid i\neq j, r_{ij}\geq 0.5\}$ denotes the arc set. That is, if $i\neq j$, $r_{ij}>0.5$, then there is a directed arc in G from v_i to v_j, denoted by (v_i,v_j) or $v_i\rightarrow v_j$, r_{ij} is called the weight of the arc (v_i,v_j). Therefore, if $r_{ij}=0.5$ $(i\neq j)$, then there exist two arcs between v_i and v_j, one from v_i to v_j, and another form v_j to v_i. A directed path ρ in a digraph G is a sequence of arcs $v_{i_1},v_{i_2},v_{i_3}\ldots$ in G, where the nodes v_{i_k} are different. The length of a directed path is the number of successive arcs in the directed path. A cycle is a directed path that begins and ends at the same node.

According to the Definition 5 of ordinal consistency for a FPR R, if R does not have ordinal consistency, then there exist some unreasonable judgment elements in R, satisfying one of the following:

(a) $r_{ik}\geq 0.5$, $r_{kj}>0.5$, but $r_{ij}\leq 0.5$;

(b) $r_{ik}>0.5$, $r_{kj}\geq 0.5$, but $r_{ij}\leq 0.5$;

(c) $r_{ik}=0.5$, $r_{kj}=0.5$, but $r_{ij}\neq 0.5$.

In each situation, there is a directed cycle of length 3 (simplified 3-cycle) $(v_i\rightarrow v_k\rightarrow v_j\rightarrow v_i)$ in the digraph G of R, That is, the inconsistent judgments could be represented by 3-cycle in G.

Theorem 1 (*see [33]*). Let $R=(r_{ij})_{n\times n}$ be a FPR, there is a directed 3-cycle $(v_i\rightarrow v_k\rightarrow v_j\rightarrow v_i)$ in the digraph G of R, if and only if there exist the elements r_{ik},r_{kj},r_{ji} $(i\neq j\neq k)$, satisfying one of the following:

(a) $r_{ik}>0.5$, $r_{kj}\geq 0.5$, or $r_{ik}\geq 0.5$, $r_{kj}>0.5$, but $r_{ji}\geq 0.5$;

(b) $r_{ik}=0.5$, $r_{kj}=0.5$, but $r_{ji}\neq 0.5$;

(c) $r_{ik}=0.5$, $r_{kj}=0.5$, $r_{ji}=0.5$.

Remark 3. Theorem 1 shows that a directed 3-cycle $(v_i\rightarrow v_k\rightarrow v_j\rightarrow v_i)$ would be determined from the above three cases. When r_{ik}, r_{kj} and r_{ji} satisfy the third case of Theorem 1, there would be two 3-cycles $(v_i\rightarrow v_j\rightarrow v_k\rightarrow v_i, v_i\rightarrow v_k\rightarrow v_j\rightarrow v_i)$ in the digraph G. But these judgment elements would be considered reasonable, because x_i, x_k and x_j are indifferent $(x_i\sim x_k\sim x_j)$. Thus, these two 3-cycles would not result in order inconsistency.

Based on the analysis of 3-cycle, Xu et al. [33] introduce the ordinal consistency index (OCI) of FPR as follows:

Definition 7 (*see [33]*). Let $R=(r_{ij})_{n\times n}$ be a FPR, $E=(e_{ij})_{n\times n}$ is the adjacency matrix of R, and $B=(b_{ij})_{n\times n}=E^2\circ E^T$, we call

$$OCI=\frac{\sum_{i=1}^{n}\sum_{j=1}^{n}b_{ij}}{3}-l \quad (3)$$

is the OCI of R, where l is the number of 3-cycles that satisfies the condition (c) in Theorem 1. Meanwhile, the $B=(b_{ij})_{n\times n}$ is the *Hadamard* product of E^2 and E^T. Suppose that $E^2=(e_{ij}^2)_{n\times n}$ and $E^T=(e_{ij}^T)_{n\times n}$, then, $b_{ij}=(e_{ij}^2)\times(e_{ij}^T)$.

Theorem 2 (*see [33]*). Let R be a FPR, R has ordinal consistency if and only if $OCI=0$.

Proof. The proof process of Theorem 2 can be seen in [33].

III. Opinion reliability detection and the management for unreliable DMs

Almost the remaining methods for LSGDM problems are based on the hypothesis that the assessment information provided by DMs is reliable and can be utilized in the decision making process directly. Actually, some DMs may give unreliable opinions in LSGDM problems, which may reduce the reliability of final decision. Thus, it is of great importance to detect the opinion reliability of DMs and to manage the unreliable DMs in LSGDM problems. In this section, we provide the DM's opinion reliability detection, the specific processes are shown in Section III.A. For unreliable DMs, the corresponding management methods are presented in Section III.B.

A. Opinion reliability detection

Usually, if a DM provides unreliable opinion, it can be reflected by the following two aspects:

- DM's opinion is contradictory, that is, the preference relation given by the DM does not have ordinal consistency.
- The deviation between individual and collective opinions is excessively large. Namely, the DM's contribution to the CRPs may be lower than those DMs which have low deviation level with the collective opinion.

Compared with the second aspect, the contradictory remained in the DMs' opinions can reflect more objectively the unreliability of DMs. Thus, in the DMs' opinions reliability detection processes, we firstly assess the contradictory degree of DMs' HFPRs based on the ordinal consistency.

(1) DM's opinion contradictory detection based on the ordinal consistency

As mentioned, the contradiction remained in the DM's opinion can be detected by assessing the DM's HFPR ordinal consistency. Thus, we expands [33]'s method in this paper.

Let $D=\{d_1,d_2,\ldots,d_m\}$ denotes the DMs set, $M=\{1,2,\ldots,m\}$ represents the number of DMs, and the $H_\varphi=(h_{ij,\varphi})_{n\times n}$ $(\varphi\in M)$ be the HFPRs provided by DM d_φ. Then, based on the Theorem 2, the contradictory degree of

DM's HFPR defined as follows:

Definition 8. Let $H_\varphi = (h_{ij,\varphi})_{n\times n}$ be an HFPR, and $R^{H_\varphi}_{\upsilon\in(\# h_{ij})} = (r_{\upsilon,ij})_{n\times n}$ denotes the FPRs transformed by $H_\varphi = (h_{ij,\varphi})_{n\times n}$, where $r_{\upsilon,ij} \in \{h_{ij,\varphi}\}$ and $\# h_{ij}$ is the number of the elements in $h_{ij,\varphi}$. Then the contradictory degree of DM d_φ 's HFPR is defined as

$$\tau_{(\varphi)} = \# h_{ij} - N(OCI(R^{H_\varphi}_{\upsilon\in(\# h_{ij})}) = 0), \tag{4}$$

Remark 4. In the Definition 8, the $N(OCI(R^{H_\varphi}_{\upsilon\in(\# h_{ij})}) = 0)$ is the number of $OCI(R^{H_\varphi}_{\upsilon\in(\# h_{ij})}) = 0$ in HFPR $H_\varphi = (h_{ij,\varphi})_{n\times n}$, and the $OCI(R^{H_\varphi}_{\upsilon\in(\# h_{ij})})$ can be calculated by Eq. (3).

Obviously, $\tau_{(\varphi)}$ have the following characteristics:

(1) $0 \le \tau_{(\varphi)} \le \# h_{ij}$.

(2) If $\tau_{(\varphi)} = 0$, all the FPRs transformed by HFPR are of ordinal consistency. Then, the opinion provided by d_φ is completely logical opinion without any contradiction.

(3) If $0 < \tau_{(\varphi)} < \# h_{ij}$, some of FPRs transformed by HFPR are of ordinal consistency. Then, the opinion provided by d_φ is considered partially contradictory.

(4) If $\tau_{(\varphi)} = \# h_{ij}$, all the FPRs transformed by HFPR are ordinal inconsistent. Then, the opinion provided by d_φ is completely contradictory opinion, which is regarded as completely unreliable opinion.

In a hesitant fuzzy LSGDM problem, DM's opinion is completely contradictory, or completely logical, belonging to two relatively extreme phenomena. Thus, we consider the acceptable ordinal consistency as a way to assess the contradictory degree of DM's opinion in this paper. We assume that if $0 \le \tau_{(\varphi)} < [(\# h_{ij})\times\alpha]$, (where α is an acceptable ordinal consistency parameter, and $\alpha\in[0,1]$), then d_φ is considered to provide an acceptable ordinal consistency HFPR. For those DMs which $[(\# h_{ij})\times\alpha] \le \tau_{(\varphi)} \le \# h_{ij}$, their opinions are regarded as unreliable.

Based on the majority principle, we supposed that $\alpha = 0.5$ in this paper. That is, if $0 \le \tau_{(\varphi)} \le [(\# h_{ij})\times 0.5]$, then d_φ 's HFPR is considered of acceptable ordinal consistency, and d_φ can participate in the next stage of DM's opinion reliability detection. If d_φ gives partly contradictory opinions, but not within the acceptable level, namely, $(\# h_{ij})\times 0.5 \le \tau_{(\varphi)} < \# h_{ij}$. Then, d_φ will be involved in the management process for unreliable DMs. Moreover, if $\tau_{(\varphi)} = \# h_{ij}$, then d_φ 's opinion will be directly rejected. See the Example 1 for the detail calculation process.

Example 1. Assume that there are four alternatives $X = \{x_1, x_2, x_3, x_4\}$ for a hesitant fuzzy LSGDM problem, and DM d_1 provides his/her HFPR as follows:

$$H_1 = \begin{bmatrix} \{0.5\} & \{0.7,0.1\} & \{0.9,0.6\} & \{0.5,0.7\} \\ \{0.3,0.9\} & \{0.5\} & \{0.6,0.8\} & \{0.7,0.4\} \\ \{0.1,0.4\} & \{0.4,0.2\} & \{0.5\} & \{0.8,0.9\} \\ \{0.5,0.3\} & \{0.3,0.6\} & \{0.2,0.1\} & \{0.5\} \end{bmatrix}.$$

Firstly, this HFPR can be transform into two FPRs as follow:

$$R^{H_1}_1 = \begin{bmatrix} 0.5 & 0.7 & 0.9 & 0.5 \\ 0.3 & 0.5 & 0.6 & 0.7 \\ 0.1 & 0.4 & 0.5 & 0.8 \\ 0.5 & 0.3 & 0.2 & 0.5 \end{bmatrix},\ R^{H_1}_2 = \begin{bmatrix} 0.5 & 0.1 & 0.6 & 0.7 \\ 0.9 & 0.5 & 0.8 & 0.4 \\ 0.4 & 0.2 & 0.5 & 0.9 \\ 0.3 & 0.6 & 0.1 & 0.5 \end{bmatrix}.$$

By Eq. (3), we can calculate $OCI(R^{H_1}_1) = 2$, $OCI(R^{H_1}_2) = 2$. Obviously, we have $N(OCI(R^{H_1}_{v\in(1,2)}) = 0) = 0$. Then, utilizing Eq. (4), we have $\tau_{(1)} = 2 > [(\# h_{ij})\times\alpha] = 1$ ($\alpha = 0.5$), namely, all the FPRs $R^{H_1}_{v\in(1,2)}$ transformed by HFPR H_1 are ordinal inconsistent. Then, we concluded that d_1 's opinion is completely a contradictory opinion.

Remark 5. Actually, for the Example 1, based on the Definition 1, we can also clearly find the contradiction of d_1's opinion. Such as in $R^{H_1}_1$, d_1 provides $r^{H_1}_{12} = 0.7$, means $x_1 \succ x_2$; $r^{H_1}_{23} = 0.6$, indicates $x_2 \succ x_3$; and $r^{H_1}_{34} = 0.8$, representation $x_3 \succ x_4$, then according to the Definition 5, we should have $x_1 \succ x_4$. However, the d_1 gives the $r^{H_1}_{14} = 0.5$, implies $x_1 \sim x_4$. It is obvious that d_1's opinion is illogical and contradictory. In the same way, we can easily find there are contradictions in $R^{H_1}_2$ between x_2 and x_4.

(2) Deviation measure between individual and collective opinions

To achieve a high level of consensus in the CRPs for hesitant fuzzy LSGDM problems, after the contradiction detection processes, we need to further detect the DM's opinion reliability by measuring the deviation level (*DL*) between the individual opinion and the collective opinion.

Let $\lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_m\}^T$ denotes the weight vector of DMs, where $\sum_{\varphi=1}^m \lambda_\varphi = 1$, $\lambda_\varphi \in [0,1]$, $\varphi \in M$. Considering the fairness of the decision making, we suppose the weights of DMs are equal as $\lambda_1 = \lambda_2 = \cdots = \lambda_m = 1/m$. Meanwhile, let $H_\varphi = (h_{ij,\varphi})_{n\times n}$ $(\varphi\in M)$ be the HFPR given by DM d_φ, and suppose all of the HFPRs are of an acceptable ordinal consistency. By using the weighted arithmetic average (WAA) operator, the collective preference relation $H = (h_{ij})_{n\times n}$ can be calculated as follows:

$$h_{ij} = \sum_{i,j=1}^{n}\sum_{\varphi=1}^{m} \lambda_\varphi h_{ij,\varphi}. \tag{5}$$

Then the *DL* between individual and collective opinions defined as follows:

Definition 9. Let $H_\varphi = (h_{ij,\varphi})_{n\times n}$, $H = (h_{ij})_{n\times n}$ be the individual HFPR and the collective opinion, respectively. Then, the $DL_{(\varphi)}$ can be calculated by the following:

$$DL_{(\varphi)} = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} d(h_{ij,\varphi}, h_{ij}), \tag{6}$$

where $d(h_{ij,\varphi}, h_{ij}) = \frac{1}{\# h_{ij}}\sum_{h_{ij,\varphi}^{(l)}\in h_{ij,\varphi};h_{ij}^{(l)}\in h_{ij}} |h_{ij,\varphi}^{(l)} - h_{ij}^{(l)}|$, $\# h_{ij}$ is the number of the elements in h_{ij}. $h_{ij,\varphi}^{(l)}$, $h_{ij}^{(l)}$ are the l-th elements in $h_{ij,\varphi}$ and h_{ij}, respectively. Obviously, the $DL_{(\varphi)} \in [0,1]$.

As with DM's opinion contradiction detection, we should consider an acceptable reliability level. Then, the *ORI* of DM d_φ is defined as follows:

Definition 10. Let $H_\varphi = (h_{ij,\varphi})_{n\times n}$, $H = (h_{ij})_{n\times n}$ be the individual preferences and collective opinion, respectively, and $DL_{(\varphi)}$ as calculated by Eq. (6). Then the DM's *ORI* is given by

$$ORI_{(\varphi)} = \sigma - DL_{(\varphi)}, \tag{7}$$

where σ is the acceptable deviation threshold, and $\sigma \in [0,1]$.

Additionally, we suppose that:

(1) If $ORI_{(\varphi)} \geq 0$, then d_φ provide the acceptable reliability opinion.

(2) If $ORI_{(\varphi)} < 0$, then d_φ is considered to provide unreliable opinion.

Based on the above analysis, we present the detail processes of DM's opinion reliability detection in Algorithm 1.

ALGORITHM 1 DM'S OPINION RELIABILITY DETECTION

Step 1 Transform the HFPR $H_\varphi = (h_{ij,\varphi})_{n\times n}$ into FPRs $R_{\upsilon\in(\# h_{ij})}^{H_\varphi} = (r_{\upsilon,ij})_{n\times n}$, $r_{\upsilon,ij} \in \{h_{ij,\varphi}\}$, $\# h_{ij}$ is the number elements in $h_{ij,\varphi}$.

Step 2 Compute the $OCI(R_{\upsilon\in(\# h_{ij})}^{H_\varphi})$ with Eq. (3).

Step 3 Compute the $\tau_{(\varphi)}$ of d_φ with Eq. (4).

- If $0 \leq \tau_{(\varphi)} < [(\# h_{ij}) \times \alpha]$, $\alpha \in [0,1]$, then d_φ is considered to provide acceptable ordinal consistency preference relation. Turn to Step 4.
- Otherwise, d_φ's opinion is contradictory and d_φ need to be managed reasonable.

Step 4 By Eq. (6) and Eq. (7), we have the $ORI_{(\varphi)}$ of d_φ.

- If $ORI_{(\varphi)} \geq 0$, then d_φ provide acceptable reliability opinion, and allowed to participate in further LSGDM processes.
- Otherwise, d_φ is considered to provide unreliable opinion and needs to be managed reasonable.

Output: The reliable set and the unreliable set.

B. The unreliable DMs management process

To ensure the fairness and democracy in hesitant fuzzy LSGDM problems, a moderator is introduced to persuade the DMs with unreliable opinions to make some modifications. Additionally, in order to guarantee the efficiency of the RI-CRP in hesitant fuzzy LSGDM problems, we need to preset the maximum modification rounds $T_{\max}$. By checking the DM's opinion reliability, we can obtain an unreliable DMs set. The corresponding management methods for those unreliable DMs are provided in this section.

The unreliable DMs are obtained considering two aspects: one is the DMs with unacceptable contradictory in the Step 3 of Algorithm 1; and the other is the DMs which are too biased against collective opinion obtained in Step 4 of Algorithm 1.

Correspondingly, the unreliable DMs management is carried out in the following two parts.

(1) The management for DMs who offer contradictory opinions

By Eq. (4), we can obtain the $\tau_{(\varphi)}$ of d_φ. For the DMs $[(\# h_{ij}) \times \alpha] \leq \tau_{(\varphi)} \leq \# h_{ij}$, $\alpha \in [0,1]$, their opinions are regarded as contradictory and they need the following management to modify their opinions.

- If $\tau_{(\varphi)} = \# h_{ij}$, DM d_φ provides completely contradictory opinion, then his/her opinion will be directly rejected to ensure the final decision reliability.
- If $[(\# h_{ij}) \times \alpha] \leq \tau_{(\varphi)} < \# h_{ij}$, $\alpha \in [0,1]$, DM d_φ gives part contradictory opinions. A moderator will be introduced to persuade d_φ to make some modifications:
  i. If d_φ follows the persuasion, then d_φ's HFPR ordinal consistency degree will be reconsidered after he/she makes a modification within the maximum modification rounds. If d_φ's revised preference relation is of ordinal consistency, then d_φ's opinion reliability will be further redetected by measuring the deviation with collective opinion.
  ii. If d_φ is unwilling to make any adjustment. Or in the maximum permissible modification rounds, d_φ's revised opinion still does not possess ordinal consistency. Then, d_φ's opinion will be rejected directly.

Additionally, in order to retain the original preference information of DMs as much as possible, we allow the DMs to select how many FPR they want to modify in the HFPR, but the minimum cannot be lower than the acceptable ordinal consistency level. For example, an HFPR can be transformed into 4 FPRs, that is, $\# h_{ij} = 4$, and by Eq. (4), we have $\tau_{(\varphi)} = 3$, ($\alpha = 0.5$). Thus, the DM d_φ needs to modify at least one of the FPRs to meet the acceptable ordinal consistency requirements.

(2) The management for DMs who are too biased against collective opinion

According to the deviation measure shown in Step 4 in Algorithm 1, if $ORI_{(\varphi)} < 0$, then d_φ is considered to provide unreliable opinion. The moderator will try to persuade d_φ to make some modifications on his/her opinion.

- For those DMs with unreliable opinions and willing to modify their opinions, we redetect d_φ's opinion reliability after he/she made modification, and allow d_φ participating in the next decision making when $ORI_{(\varphi)} \geq 0$ within the maximum modification rounds limit.

- For the DMs who unwilling to make compromise, or in the case of maximum permissible modification rounds, the DM's revised opinion still does not meet the reliability requirement. Then, their opinions will be rejected directly to ensure the final decision is reliability.

After apply the unreliable DMs management process, only the DMs who provide acceptable reliable opinions can enter the following decision making process. To clarify, the specific processes of DM's opinion reliability detection and the unreliable DMs management are depicted in Fig. 1.

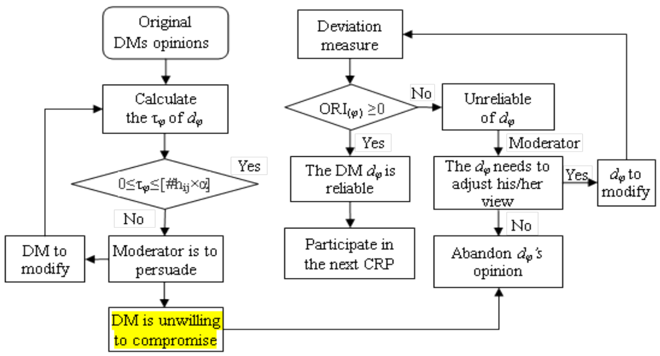

Fig 1. The processes of DM's opinion reliability detection and the unreliable DMs management.

IV. Alternative ranking-based cluster method for hesitant fuzzy LSGDM problems

Clustering methods aim to classify the DMs who provide similar opinions into a group to improve the CRPs efficiency in LSGDM problems. A novel ARC method for hesitant fuzzy LSGDM problems, based on the DM's HFPR alternative ranking, is presented in this section.

Based on the score function of HFPRs in Section II, the DMs' alternative ranking can be obtained, which can be used to get the alternatives position order for each DM, as shown in Example 2.

Example 2. Suppose there are four alternatives $X = \{x_1, x_2, x_3, x_4\}$ can be selected, DM d_1 provides his/her normalized HFPR $H_1 = (h_{ij,1})_{4\times 4}$ as follows:

$$H_1 = \begin{bmatrix} \{0.5\} & \{0.2, 0.3\} & \{0.4, 0.3\} & \{0.1, 0.3\} \\ \{0.8, 0.7\} & \{0.5\} & \{0.6, 0.7\} & \{0.4, 0.2\} \\ \{0.6, 0.7\} & \{0.4, 0.3\} & \{0.5\} & \{0.3, 0.1\} \\ \{0.9, 0.7\} & \{0.6, 0.8\} & \{0.7, 0.9\} & \{0.5\} \end{bmatrix}.$$

By Eq. (1), we have the score value of each alternative $s(x_i)$ ($i = 1, 2, 3, 4$),

$$s(x_1) = 1.3,\ s(x_2) = 2.2,\ s(x_3) = 1.7,\ s(x_4) = 2.8.$$

Thus, the alternative ranking for d_1 is $x_4 \succ x_2 \succ x_3 \succ x_1$, and the alternatives position order of d_1 is $O(d_1) = (4, 2, 3, 1)$.

In addition, if there are equivalent alternatives given by DM, in order to obtain a reasonable clustering result, we propose to consider all the possible alternatives position orders. For instance, the alternative ranking of d_1 is $x_4 \succ x_2 \sim x_3 \succ x_1$, then we have $O^1_{(d_1)} = (4, 2, 3, 1)$ and $O^2_{(d_1)} = (4, 3, 2, 1)$ at the same time.

The alternative position order can be used to find the similarity degree of each pair of DMs, which can be further used in the clustering method based on alternative ranking.

Definition 11. Let $n \in N$ be the number of alternatives, for two DMs $d_\varphi, d_m \in D$, their opinions SI is defined as:

$$SI(d_\varphi, d_m) = NASP(d_\varphi, d_m),\ SI \in N^+. \tag{8}$$

Where $\mu \in [1/n, (n-1)/n]$, and the $NASP(d_\varphi, d_m)$ is the number of alternatives with the same position between d_φ and d_s. If $SI(d_\varphi, d_m) \geq \mu \times n$, then we consider that there exist similar or completely consistent opinion between d_φ and d_s.

Remark 6. For Eq. (8), the $SI(d_\varphi, d_m) = n-1$ already means that the opinions between d_φ and d_m are completely consistent. Thus, we set $\mu \in [1/n, (n-1)/n]$, instead of $\mu \in [1/n, 1]$.

Considering the feasibility of the numerical example in section VI.A, we assume that $\mu = 0.5$ in this paper. It means that if $SI(d_\varphi, d_m) = NASP(d_\varphi, d_m) \geq 0.5 \times n$, then we consider that d_φ and d_m have a majority of the same opinions, and classify them into one group. The detail cluster analysis steps can be seen in Algorithm 2.

ALGORITHM 2 THE ARC METHOD FOR HESITANT FUZZY LSGDM PROBLEMS

Step 1 According to Eq. (1), we can calculate the alternatives score function values of DMs with HFPRs. Then the alternatives position order of DMs can be obtained;

Step 2 Firstly, cluster the DMs with the completely consistent alternative position order, and the remaining DMs are considered as a unique cluster, then we have the initial clustering results C_s, $1 \leq s \leq m$.

Step 3 The clusters C_s are then compared with each other to obtain the SI between them. Suppose that C_s $(s = 1, 2, 3)$:

- If $SI(C_1, C_2) \geq \mu \times n$, $SI(C_2, C_3) \geq \mu \times n$, and $SI(C_1, C_3) \geq \mu \times n$, then C_s $(s = 1, 2, 3)$ are divided into one group.
- If $SI(C_1, C_2) \geq \mu \times n$, $SI(C_2, C_3) \geq \mu \times n$, but $SI(C_1, C_3) < \mu \times n$, then:
  - i. If $SI(C_1, C_2) > SI(C_2, C_3)$, then, C_1 and C_2 are divided into one group.
  - ii. If $SI(C_1, C_2) < SI(C_2, C_3)$, then, C_2 and C_3 are divided into one group.
  - iii. If $SI(C_1, C_2) = SI(C_2, C_3)$, then:
    - a) if $d(f(C_1), f(C_2)) > d(f(C_2), f(C_3))$, divide C_2 and C_3 into one group;
    - b) if $d(f(C_1), f(C_2)) < d(f(C_2), f(C_3))$ classify C_1 and

C_2 into one group.

Step 4 End.

Remark 7. To reduce the complexity of alternative position order comparisons among DMs, in Algorithm 2, we firstly classify the DMs who have the completely consistent opinions. Furthermore, the $f(C_1)$ represents the average of the cluster C_1. The $d(f(C_1), f(C_2))$ denotes the distance between $f(C_1)$ and $f(C_2)$, which can be obtained by Eq. (6). Others symbols which have the same formula with $d(f(C_1), f(C_2))$ and $f(C_1)$ are also have the similar meaning with them.

V. THE RI-CRP FOR HESITANT FUZZY LSGDM PROBLEMS

In this section, we first present the consensus measure for hesitant fuzzy LSGDM problems, and the GCI is proposed based on the consensus measure presented at Section V.A. Subsequently, the management processes for non-cooperative clusters in the RI-CRP are given in Section V.B. Finally, the flowchart of RI-CRP for hesitant fuzzy LSGDM problems is provided in Section V.C.

A. The consensus measure for hesitant fuzzy LSGDM problems

Using the ARC method which is provided in Section IV, the remaining DMs can be divided into S $(1 \le S \le M)$ clusters, denoted as C_s ($s \in S$). Based on two rules: (a) DMs in the same cluster can be assigned the same weight because they have similar opinions, and (b) clusters that have large number DMs should be assigned larger weights based on the majority principle. Then, the weight of DM d_φ in different clusters is calculated as follows:

$$\hat{\lambda}_\varphi = 1/o_s,$$

where $d_\varphi \in C_s$, $\varphi = 1,2,\ldots o_s$, o_s is the number of DMs in cluster C_s. The weight of cluster C_s can be obtained:

$$w_s = o_s / \sum_{s=1}^{S} o_s. \tag{9}$$

It is obvious that $0 < w_s \le 1$ and $\sum_{s=1}^{S} w_s = 1$. Meanwhile, the decision matrix of cluster C_s can be obtained as $P^s = (p_{ij}^s)_{n\times n}$:

$$p_{ij}^s = \hat{\lambda}_\varphi \times \sum_{\varphi=1}^{o_s} h_{ij,\varphi}. \tag{10}$$

Similarly, the group decision matrix can be represented as $G^c = (g_{ij}^c)_{n\times n}$:

$$g_{ij}^c = \sum_{s=1}^{S} w_s p_{ij}^s. \tag{11}$$

In order to obtain the GCI, we give the following definition based on distance measure:

Definition 9. Let $P^s = (p_{ij}^s)_{n\times n}$ be the decision matrix of cluster C_s, and $G^c = (g_{ij}^c)_{n\times n}$ be the group decision matrix obtained by Eq. (10) and Eq. (11), respectively. Then the deviation degree between the individual cluster matrix P^s and the group decision matrix G^c is defined as

$$\vartheta^s = d(P^s, G^c) = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} d(p_{ij}^s, g_{ij}^c), \tag{12}$$

where the $d(p_{ij}^s, g_{ij}^c) = \frac{1}{Y}\sum_{p_{ij,l}^s \in p_{ij}^s, g_{ij,l}^c \in g_{ij}^c} | p_{ij,l}^s - g_{ij,l}^c |$, and Y is the number of the elements in p_{ij}^s and g_{ij}^c. Furthermore, $p_{ij,l}^s$ and $g_{ij,l}^c$ are the l-th elements in p_{ij}^s and g_{ij}^c, respectively.

It is clear that, ϑ^s has the following properties:

(1) $0 \le \vartheta^s \le 1$;

(2) $\vartheta^s = 0$, if and only if $P^s = G^c$, namely, there is no deviation between P^s and G^c.

(3) $\vartheta^s = 1$, if and only if P^s and G^c are completely dissimilar, that is, they are contrary.

Accordingly, the weighted sum of all the ϑ^s, then the GCI can be defined as follows:

Definition 10. Let w_s and ϑ^s be the weight and deviation degree of cluster C_s, respectively. By the WAA operator, the GCI can be calculated as

$$GCI = \sum_{s=1}^{S} w_s \vartheta^s. \tag{13}$$

Obviously, if $GCI = 0$, there is no deviation between clusters' opinions. Generally, we suppose that if $GCI \le \delta$ (where δ is consensus threshold), then the acceptable consensus is reached in the RI-CRP for hesitant fuzzy LSGDM problems.

B. The management processes for non-cooperative clusters in the RI-CRP

In the RI-CRP for hesitant fuzzy LSGDM problems, some clusters' opinions may be far from the collective opinion, and the DMs in them may do not make any compromise despite the guidance of the moderator. We call the clusters' behavior that contains these DMs as "non-cooperative behavior". To achieve a high level of consensus and improve the efficiency of the RI-CRP, these non-cooperative clusters need to be managed reasonably. The detail RI-CRP for hesitant fuzzy LSGDM problems can be seen in Algorithm 3.

ALGORITHM 3 THE RI-CRP FOR HESITANT FUZZY LSGDM PROBLEMS

Input: The alternatives set $X = \{x_1, x_2, \ldots, x_n\}$, the individual HFPRs $H_\varphi = (h_{ij,\varphi})_{n\times n}$ $(\varphi \in M)$, the DMs set $D = \{d_1, d_2, \ldots, d_m\}$, the initial weights vector of DMs $\lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_m\}^T$, the maximum modification rounds $T_{\max} \ge 1$ in the reliability detection process, the maximum number of iterations $t_{\max} \ge 1$ in the RI-CRP, the predefined deviation threshold σ, consensus threshold δ. Meanwhile, the acceptable ordinal consistency parameter α, and the *SI* parameter μ.

Step 1 Use the Algorithm 1 to detect the DM's opinion reliability. Let ψ_1 denotes the DM set that provides completely contradictory opinions. Let ψ_2 denotes the DM set that provides partial contradictory opinions, and ψ means the DM set that provides acceptable ordinal consistency opinions. DMs in ψ_1 are directly rejected.

Step 2 For DMs in ψ_2, a moderator is introduced to persuade them to make some adjustments of their opinions, and then go to Step 1. If the DM is unwilling to make compromise, or in the case of $T_{\max} \geq 1$, the revised preference relations that are still ordinal inconsistent, then their views will be rejected directly.

Step 3 Let Ω denotes the DM set that provides reliable opinions, and ψ_3 denotes the unreliable DMs set. For the DM d_φ in ψ, we use Eq. (7) to calculate d_φ 's $ORI_{(\varphi)}$. If $ORI_{(\varphi)} \geq 0$, then d_φ belongs to Ω; otherwise, d_φ belongs to ψ_3.

Step 4 Similar to Step 2, for the DMs that belongs to ψ_3, the moderator has to persuade them to make modifications of their opinions, if they follow the persuasion and then go to Step 3. Otherwise, reject their opinions directly.

Step 5 Suppose that there are remaining q $(q \in M)$ DMs after *Step 4*. Using the Algorithm 2, the remaining DMs are divided into S $(1 \leq S \leq M)$ small clusters C_s $(s \in S)$. By Eq. (9) and Eq. (10), the weights w_s and the decision matrix $P^{s(t)} = (p_{ij}^{s(t)})_{n\times n}$ of C_s can be obtained, respectively.

Step 6 The group decision matrix $G^{c(t)} = (g_{ij}^{c(t)})_{n\times n}$ can be obtained by Eq. (11). The $GCI^{(t)}$ can be calculated by Eq. (13). If $GCI^{(t)} \leq \delta$, then go to Step 8; otherwise go to the next step.

Step 7 Find the cluster C_s which has the largest deviation from the collective opinion, namely, the one that has the maximal value of $\vartheta_{\max}^s$. Let the moderator to persuade the DMs in this cluster to modify their preferences. For the non-cooperative clusters which are unwilling to make any compromise, their weights will be punished. The principle of punishment is as follows:

$$w_s^{(t+1)} = w_s^{(t)} \times \zeta, \tag{14}$$

where the ζ is the punishment parameter, and $\zeta \in [0,1]$. Let Q be the number of clusters excluding non-cooperative clusters. Then, their weights are accordingly as

$$w_{S-s}^{(t+1)} = w_{S-s}^{(t)} + \frac{(1-\zeta)w_s^{(t)}}{Q}, \tag{15}$$

set $t = t+1$ and go to Step 6.

Step 8 Let $\overline{G}^c = G^{c(t)}$, according to the Eq. (1), the alternatives score values of the collective opinion $\overline{G}^c$ can be calculated, and the best alternative(s) can be selected.

Step 9 End.

Output: The group consensus level $GCI^{(t)}$, the number of iteration t, and the best alternative(s) selection.

C. Flowchart of RI-CRP for hesitant fuzzy LSGDM problems

Once the consensus among DMs is reached, the selection process based on the score function which was introduced in Section II is employed to obtain the group alternative ranking. Then, the final decision can be obtained. The detail RI-CRP for hesitant fuzzy LSGDM problems is depicted in Fig 2.

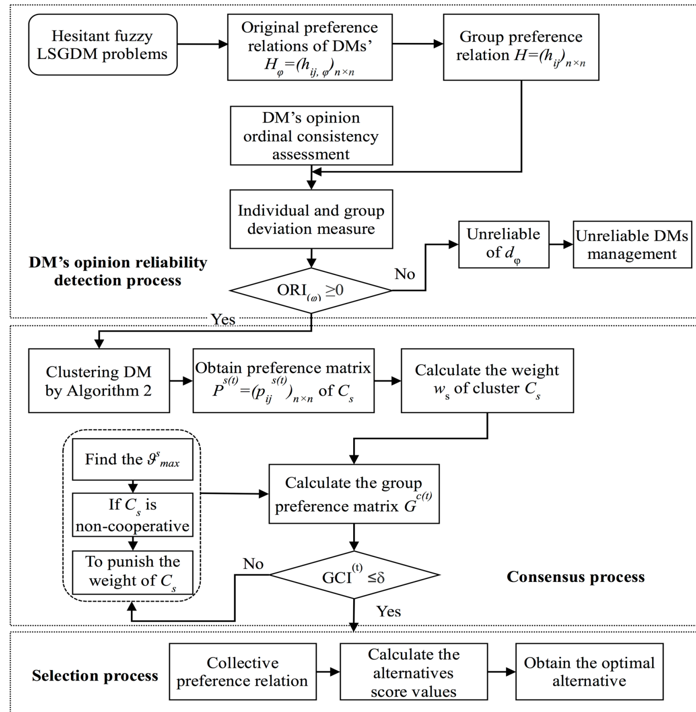

Fig. 2. Flowchart of the RI-CRP for hesitant fuzzy LSGDM problems.

VI. A NUMERICAL EXAMPLE AND ANALYSIS

In this section, a numerical example is provided to examine the proposed ARC method and the RI-CRP for hesitant fuzzy LSGDM problems utility and applicability. Meanwhile, the analysis of the ARC method is given in Section VI.B.

A. Numerical example

Suppose there are five alternatives $X = \{x_1, x_2, x_3, x_4, x_5\}$ for a hesitant fuzzy LSGDM problem, and 30 DMs $D = \{d_1, d_2, \ldots, d_{30}\}$ are involved. All the DMs use the normalized HFPRs to make the comparisons of alternatives and give their assessment information. Then we can obtain 30 original HFPRs $H_\varphi = (h_{ij,\varphi})_{n\times n}$ $(\varphi = 1,2,\ldots,30)$, and the specific preference information can be seen in the Table I . Moreover, some related parameters are set as follows:

(1) The maximum modification round is set to $T_{\max} = 3$, the acceptable ordinal consistency parameter $\alpha = 0.5$, and the acceptable deviation threshold $\sigma = 0.12$ in the DM's opinion reliability detection process. Meanwhile, the *SI* parameter is $\mu = 0.5$.

(2) The minimum level of consensus threshold $\delta = 0.05$, and the maximum number of iterations is $t_{\max} = 3$ in the RI-CRP.

(3) The punishment parameter for updating the weight is $\zeta = 0.5$.

Step 1 Apply Algorithm 1 to detect the DM's opinion reliability. Then, we have $\psi_1=\{d_5,d_{10},d_{12}\}$, $\psi_2=\{d_1,d_3\}$, $\psi=\{d_\varphi\}$, $d_\varphi \in D$, and $d_\varphi \notin (\psi_1 \cup \psi_2)$.

Step 2 DMs in ψ_1 are rejected directly. A moderator is introduced to persuade the DMs in ψ_2 to make some adjustments. Only the DM d_1 is willing to follow the advice of moderator to revise his/her preference relation. At $T=1$, d_1's revised opinion have ordinal consistency. Thus, DM d_1 is classified into ψ. In this step, we use the repairing ordinal inconsistency method provided in [33] to effectively modify the inconsistency. The specific steps of this method can be seen in the Algorithm 2 of [33]. Furthermore, d_3's opinion is rejected directly due to he/she is unwilling to make any modifications. The revised results of d_1 can be seen in Table II.

Table II
THE REVISED PREFERENCE RELTION OF THE DM d_1.

d_1	x_1	x_2	x_3	x_4	x_5
x_1	{0.5}	{0.7,0.1,0.6}	{0.9,0.6,0.7}	{0.6,0.7,0.4}	{0.4,0.2,0.3}
x_2	{0.3,0.9,0.4}	{0.5}	{0.6,0.8,0.7}	{0.7,0.6,0.3}	{0.1,0.2,0.3}
x_3	{0.1,0.4,0.3}	{0.4,0.2,0.3}	{0.5}	{0.8,0.9,0.4}	{0.1,0.4,0.2}
x_4	{0.4,0.3,0.6}	{0.3,0.4,0.7}	{0.2,0.1,0.6}	{0.5}	{0.2,0.4,0.3}
x_5	{0.6,0.8,0.7}	{0.9,0.8,0.7}	{0.9,0.6,0.8}	{0.8,0.6,0.7}	{0.5}

Step 3 The remaining 26 DMs' HFPRs are acceptable ordinal consistent and the weights of DMs are $\lambda_\varphi = 1/26$ $(\varphi \neq 3,5,10,12)$. By Eq. (5), the collective preference relation $H=(h_{ij})_{n\times n}$ can be obtained. Then using Eq. (6) and Eq. (7), we have the $DL_{(\varphi)}$ and $ORI_{(\varphi)}$ of DMs d_φ $(\varphi \neq 3,5,10,12)$. Furthermore, we have the unreliable DMs set $\psi_3=\{d_2,d_4,d_7,d_{11}\}$. The detailed results are seen in Table III.

TABLE III
THE RESULTS OF DL AND ORI OF THE DMS d_φ $(\varphi \in [1,30]; \varphi \neq 3,5,10,12)$.

d_φ	$DL_{(\varphi)}$	$ORI_{(\varphi)}$
d_1	0.1088	0.0112
d_2	0.2477	−0.1277
…	…	…
d_{30}	0.1000	0.0200

Step 4 In ψ_3, the DMs d_φ $(\varphi=2,4)$ are not willing to change despite the guidance of the moderator. Thus, their opinions are rejected directly. The DMs d_φ $(\varphi=7,11)$ are willing to make some modifications by the advice of the moderator, and in the $T_{\max}=3$ limitation, their revised preference relations satisfy the reliability requirement. Then, d_φ $(\varphi=7,11)$ are classified in to the reliable DMs set Ω.. In this step, we take the principle which is provided in [34] to repair the deviation between the individual opinion and the collective opinion. Finding the position i_τ and j_τ of the maximum elements $h_{i_\tau j_\tau,\varphi}$ of d_φ, where $h_{i_\tau j_\tau,\varphi} = \max_{i,j} d(h_{i_\tau j_\tau,\varphi}, h_{ij})$ $(i,j\in N)$, and return H_φ to d_φ to

TABLE I
THE ORIGINAL PREFERENCE INFORMATION MATRIX OF DMS

		x_1	x_2	x_3	x_4	x_5
d_1	x_1	{0.5}	{0.7,0.1,0.6}	{0.9,0.6,0.7}	{0.5,0.7,0.4}	{0.4,0.2,0.3}
	x_2	{0.3,0.9,0.4}	{0.5}	{0.6,0.8,0.7}	{0.7,0.4,0.3}	{0.1,0.2,0.3}
	x_3	{0.1,0.4,0.3}	{0.4,0.2,0.3}	{0.5}	{0.8,0.9,0.4}	{0.1,0.4,0.2}
	x_4	{0.5,0.3,0.6}	{0.3,0.6,0.7}	{0.2,0.1,0.6}	{0.5}	{0.2,0.4,0.3}
	x_5	{0.6,0.8,0.7}	{0.9,0.8,0.7}	{0.9,0.6,0.8}	{0.8,0.6,0.7}	{0.5}
d_2	x_1	{0.5}	{0.6,0.7,0.8}	{0.7,0.9,0.6}	{0.4,0.5,0.6}	{0.6,0.8,0.9}
	x_2	{0.4,0.3,0.2}	{0.5}	{0.5,0.6,0.7}	{0.3,0.4,0.2}	{0.8,0.7,0.6}
	x_3	{0.3,0.1,0.4}	{0.5,0.4,0.3}	{0.5}	{0.2,0.3,0.1}	{0.7,0.8,0.9}
	x_4	{0.6,0.5,0.4}	{0.7,0.6,0.8}	{0.8,0.7,0.9}	{0.5}	{0.7,0.9,0.6}
	x_5	{0.4,0.2,0.1}	{0.2,0.3,0.4}	{0.3,0.2,0.1}	{0.3,0.1,0.4}	{0.5}
				…		
d_{30}	x_1	{0.5}	{0.8,0.9,0.7}	{0.6,0.9,0.7}	{0.8,0.7,0.6}	{0.1,0.4,0.3}
	x_2	{0.2,0.1,0.3}	{0.5}	{0.5,0.7,0.8}	{0.6,0.8,0.7}	{0.1,0.3,0.2}
	x_3	{0.4,0.1,0.3}	{0.5,0.3,0.2}	{0.5}	{0.7,0.9,0.8}	{0.3,0.2,0.4}
	x_4	{0.2,0.3,0.4}	{0.4,0.2,0.3}	{0.3,0.1,0.2}	{0.5}	{0.2,0.4,0.1}
	x_5	{0.9,0.6,0.7}	{0.9,0.7,0.8}	{0.7,0.8,0.6}	{0.8,0.6,0.9}	{0.5}

construct a new HFPR $\bar{H}_\varphi = (\bar{h}_{ij,\varphi})_{n\times n}$ according to d_φ's new judgment, where

$$\bar{h}_{ij,\varphi} = \begin{cases} h_{ij}, & if \quad i=\tau, j=\tau; \\ h_{ij,\varphi}, & oterwise. \end{cases}$$

This repairing method does not only can satisfy the reliability requirements, but also could preserve the initial DM's preference information as much as possible. The detail results are shown in Table IV.

TABLE IV
THE REVISED RESULTS OF THE DMS d_φ $(\varphi=7,11)$.

d_φ	$h_{ij,\varphi}^{(T)}$	$ORI_{(\varphi)}^{(T)}$
d_7	$h_{34,7}^{(1)} \to \{0.6231, 0.7231, 0.5962\}$	$RI_{(7)}^{(1)} = -0.0085$
	$h_{43,7}^{(1)} \to \{0.3769, 0.2769, 0.4038\}$	$RI_{(7)}^{(2)} = 0.0229$
	$h_{24,7}^{(1)} \to \{0.6458, 0.7417, 0.7167\}$	
	$h_{42,7}^{(1)} \to \{0.3542, 0.2583, 0.2833\}$	
d_{11}	$h_{25,11}^{(1)} \to \{0.1731, 0.3615, 0.2962\}$	$RI_{(11)}^{(1)} = -0.0675$
	$h_{52,11}^{(1)} \to \{0.8269, 0.6385, 0.7038\}$	$RI_{(11)}^{(2)} = -0.0330$
	$h_{15,11}^{(2)} \to \{0.3958, 0.2125, 0.3167\}$	$RI_{(11)}^{(3)} = 0.00170$
	$h_{51,11}^{(2)} \to \{0.6042, 0.7875, 0.6833\}$	
	$h_{45,11}^{(3)} \to \{0.2167, 0.2125, 0.3125\}$	
	$h_{54,11}^{(3)} \to \{0.7833, 0.7875, 0.6875\}$	

Step 5 Applying the Algorithm 2, the remaining 24 provides reliable opinions DMs are divided into four clusters, and the cluster results as follows:

$$C_1=\{d_1,d_7,d_9,d_{19},d_{21},d_{26},d_{28},d_{30}\},$$
$$C_2=\{d_6,d_8,d_{13},d_{16},d_{17},d_{22},d_{23},d_{27},d_{29}\},$$
$$C_3=\{d_{11}\},\ C_4=\{d_{14},d_{15},d_{18},d_{20},d_{24},d_{25}\}.$$

According to Eq. (9), we have the weights w_s of cluster C_s:

$$w_1^{(0)}=8/24,\ w_2^{(0)}=9/24,\ w_3^{(0)}=1/24,\ w_4^{(0)}=6/24.$$

Then, the decision matrix $P^{s(0)}=(p_{ij}^{s(0)})_{n\times n}$ of cluster C_s can be obtained by Eq. (10). Detailed results are omitted due to space constrictions.

Step 6 Utilizing Eq. (11), the collective decision matrix $G^{c(0)}=(g_{ij}^{c(0)})_{n\times n}$ can be obtained, and then by Eqs. (12) and (13), we have the $\vartheta^{s(0)}$ and $GCI^{(0)}$ as follow:

$$\vartheta^{1(0)}=0.0505,\ \ \vartheta^{2(0)}=0.0440,\ \vartheta^{3(0)}=0.1183,\ \ \vartheta^{4(0)}=0.0560,$$

$$GCI^{(0)}=0.0523>\delta=0.05\,.$$

Then go to Step 7.

Step 7 Cluster C_3 has the largest deviation from collective, $\vartheta^{3(0)}=0.1183$. DMs in C_3 in this round are unwilling to make any comprise. Thus, the weight of C_3 will be punished, and return to the Step 6, we have $GCI^{(1)}=0.0509>\delta=0.05$, $\vartheta^{3(1)}=0.1209$, C_3 still needs to make adjustments. In the next rounds, the DMs in C_3 are willing to make some modifications. Similar to the modification method in Step 4, we find the position i_τ and j_τ of the maximum elements $o_{i_\tau j_\tau,s}^{(t)}$ in C_s, which has the largest deviation from the group's opinion. That is, the C_s having the maximal value of $\vartheta_{\max}^{s}$, where $o_{i_\tau j_\tau,s}^{(t)}=\max_{i,j} d(p_{ij}^{s(t)},g_{ij}^{c(t)})$ $(i,j\in N)$, return $P^{s(t)}$ to the cluster C_s to construct a new preference relation $P^{s(t+1)}=(p_{ij}^{s(t+1)})_{n\times n}$ according to C_s's new judgment, where

$$p_{ij}^{s(t+1)}=\begin{cases} g_{ij}^{c(t)}, & if \quad i=\tau, j=\tau;\\ p_{ij}^{s(t)}, & oterwise.\end{cases}$$

Return to the Step 6. After 3 modify rounds, we have $GCI^{(3)}=0.0498<\delta=0.05$, then the acceptable consensus is reached. The detail results of the RI-CRP are shown in Table V.

TABLE V
THE DETAIL RESULTS OF THE RI-CRP $(s=1,2,3,4)$.

t	$w_s^{(t)}$	$p^{3(t)}$	$\vartheta^{s(t)}$, $GCI^{(t)}$
0	$w_1^{(0)}=8/24$,	…	$\vartheta^{1(0)}=0.0505$,
	$w_2^{(0)}=9/24$,		$\vartheta^{2(0)}=0.0440$,
	$w_3^{(0)}=1/24$,		$\vartheta^{3(0)}=0.1183$,
	$w_4^{(0)}=6/24$		$\vartheta^{4(0)}=0.0560$,
			$GCI^{(0)}=0.0523$
1	$w_1^{(1)}=0.3403$,	…	$\vartheta^{1(1)}=0.0497$,
	$w_2^{(1)}=0.3819$,		$\vartheta^{2(1)}=0.0457$,
	$w_3^{(1)}=0.0208$,		$\vartheta^{3(1)}=0.1209$,
	$w_4^{(1)}=0.2569$		$\vartheta^{4(1)}=0.0544$,
			$GCI^{(1)}=0.0509$
2	$w_1^{(2)}=0.3403$,	$p_{35}^{3(2)}\to\{0.2913,0.2379,0.3532\}$	$\vartheta^{1(2)}=0.0497$,
	$w_2^{(2)}=0.3819$,	$p_{53}^{3(2)}\to\{0.7087,0.7621,0.6468\}$	$\vartheta^{2(2)}=0.0461$,
	$w_3^{(2)}=0.0208$,		$\vartheta^{3(2)}=0.0891$,
	$w_4^{(2)}=0.2569$		$\vartheta^{4(2)}=0.0545$,
			$GCI^{(2)}=0.0504$
3	$w_1^{(3)}=0.3403$,	$p_{34}^{3(3)}\to\{0.6609,0.7652,0.6403\}$	$\vartheta^{1(3)}=0.0492$,
	$w_2^{(3)}=0.3819$,	$p_{43}^{3(3)}\to\{0.3391,0.2348,0.3597\}$	$\vartheta^{2(3)}=0.0466$,
	$w_3^{(3)}=0.0208$,		$\vartheta^{3(3)}=0.0665$,
	$w_4^{(3)}=0.2569$		$\vartheta^{4(3)}=0.0540$,
			$GCI^{(3)}=0.0498$

Finally, by Eq. (1), we can calculate the alternatives score values of $GCI^{(3)}$ as follow:

$s(x_1)=2.4815$, $s(x_2)=2.5277$, $s(x_3)=2.3158$, $s(x_4)=1.6844$, $s(x_5)=3.4829$.

Thus, we have $x_5\succ x_2\succ x_1\succ x_3\succ x_4$, then the optimal consensus alternative is x_5.

B. Analysis of the ARC method in the numerical example

In the numerical example of Section VI.A, considering the feasibility of the numerical example, we assume $\mu=0.5$. By applying Eq. (8), we have $SI(d_\varphi,d_m)=3$, which denotes that d_φ and d_m have a majority of the same opinions. Thus, we classify them into one group. Actually, all the $SI(d_\varphi,d_m)$ possible values in this numerical example are $\{1,2,3,4\}$. Moreover, we can obtain the different number cluster based on the different value of *SI*. The detailed results can be seen in the Fig. 3.

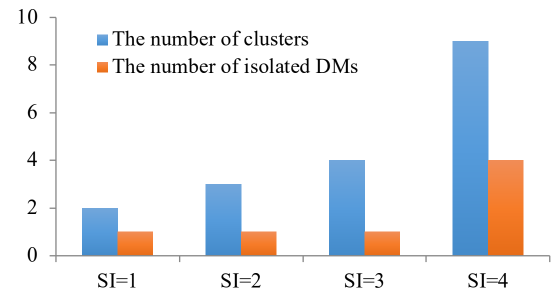

Fig. 3. The ARC results with different SI for numerical example in VI.A.

From Fig. 3, we conclude that the different value of *SI*, the different number of cluster can be obtained. Meanwhile, the higher of *SI*, the more number of clusters, that is, the possible number of isolated DMs is greater. Obviously, the value of *SI* is directly affected by the selection of μ. Thus, in the real hesitant fuzzy LSGDM problems, the DMs can select the value of $\mu\in[1/n,(n-1)/n]$ based on the actual needs, to get a reasonable clustering result and improve the efficiency of the RI-CRP.

VII. CONCLUSION

This paper focuses on the hesitant fuzzy LSGDM problems, and presents the ARC method and RI-CRP to improve the efficiency of the hesitant fuzzy LSGDM problems. The major contributions of this paper are concluded as follow:

(1) By assessing the DMs' HFPRs ordinal consistency and measuring the deviation with the collective opinion, the DM's ORI is proposed, so that in the reliability detection process, it is easy to detect the unreliable DMs by calculating the DM's ORI. For the unreliable DMs, a moderator is introduced to work on them, and a relatively reasonable limited modification round is given to save the costs. By detecting the DM's opinion reliability and managing the unreliable DMs, we can avoid the unreliable DMs involved in the further CRPs, thus ensuring that the final decision is reasonable and reliable.

(2) To improve the efficiency of the RI-CRP for hesitant fuzzy LSGDM problems, an ARC method is proposed with

HFPRs in this paper. Meanwhile, the *SI* between two DMs' opinions is provided, to ensure the ARC method can be effectively implemented. Compared with those clustering methods which need to preset some correlated parameters, the presented ARC method is more objective with a different approach based on the alternative ranking.

(3) In the RI-CRP of hesitant LSGDM problems, a weight penalizing mechanism is implemented for the non-cooperative clusters. The implementation of the weight punishment makes the RI-CRP more efficient for hesitant LSGDM problems.

In some real LSGDM problems, due to time pressure, lack of knowledge, and the DM's limited experience related with the problem domain, DMs may provide the incomplete assessment information [17],[35]-[37]. Thus, in further work, we will try to extend the proposed ARC method and RI-CRP to the LSGDM with incomplete assessment information, to further verify the validity of them.

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