Social network group decision making: Managing self-confidence-based consensus model with dynamic importance degree of experts and trust-based feedback mechanism

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Abstract

Social network group decision-making (SNGDM) is increasingly valued since the advancement and development of intelligent decision-making. Generally, real SNGDM cases involve not only the mathematical formulation of the social network analysis but the experts' psychological behaviors. Self-confidence, an expert's psychological implication of self-statement, is a meaningful topic in SNGDM problems, while it is overlooked in the most existing researches. To fill this gap, this study takes experts' self-confidence into account in SNGDM. All experts are allowed to use self-confident fuzzy preference relations (SC-FPRs) to express their opinions. Subsequently, we develop a novel self-confidence-based consensus approach for SNGDM with SC-FPRs, in which a dynamic importance degree determination of experts combining the external trust and internal self-confidence to assign their weights. A consensus index considering self-confidence is defined to assess the consensus level among experts. Meanwhile, a trust-based feedback mechanism is presented to improve the consensus efficiency. The rule of the feedback mechanism is that experts are allowed to dynamically adjust their self-confidence levels while revising the preferences. Using self-confidence score function, an alternative that has the highest self-confidence score can be selected as the best solution. An illustrative example and some comparisons are given to verify the feasibility and effectiveness of the proposed method.

Keywords: Social network group decision making; Self-confident fuzzy preference relations; Consensus; Trust; Self-confidence.

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1. Introduction

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Social network group decision making (SNGDM) can be described as a scenario that some experts (decision makers) participate in decision-making through a social networking platform and then to choose the best solution from given alternatives. Nowadays, SNGDM gained increasing attention because of the advancement and development of intelligent decision-making [1-7]. In SNGDM events, preference relation is useful to voice the assessment information of experts since its advantages of pairwise comparison [8, 9]. To date, many different types of preference relations have been presented and applied to decision-making analyses [10-17].

Generally, real SNGDM scenarios involve not only the mathematical formulation of the social network analysis but the human psychological factors [18, 19]. In reality, experts usually own differentiated decision-making habits, risk preferences or knowledge backgrounds. As a result, they may have different psychological cognition which will influence their expression of preference information. For example, in SNGDM process, some experts may not be able or unwilling to provide complete preference relations when making a pairwise comparison among alternatives. The possible reasons are as follows:

(a) Some experts may own limited knowledge or experience referring the SNGDM problem domain.

(b) Some experts may show a negative attitude in the decision-making process since they are unwilling to take the responsibility for the failure of the final decision or possible negative impact.

Consequently, in some real SNGDM scenarios, experts may provide incomplete preference relations. That is, experts only give the preference information that they are fully sure of, while for the uncertain information, they prefer to keep the elements missing [20]. In some senses, this behavior can be called self-confidence psychological in the decision-making process [21, 22]. The preference information that experts provide indicates that they are fully self-confident while the missing elements mean that experts have no self-confidence.

Self-confidence is a person's psychological implication of self-statement, and can reflect one's knowledge, experience or attitude in SNGDM process [22, 23]. Hence, it would be an interesting and valuable research to explore the impact of expert's self-confidence on SNGDM. Although incomplete preference relations can reflect expert's self-confidence to some extent, it is still not the best preference tool to analyze the expert's self-confidence in SNGDM events. The main reason is that different experts usually own different self-confidence levels. Some experts may have varying self-confidence degrees in their own preference information. In other words, experts may have multiple self-confidence levels except as the absolute confidence or total lack of confidence. In order to deeply explore the effect of experts' multiple self-confidence on decision-making, a novel self-confident fuzzy preference relation (SC-FPR) was introduced in [20]. The main advantage of the SC-FPRs is that experts can simultaneously voice their preferences and multiple self-confidence. And then, enabling them truthfully voices their assessment information as well as guaranteeing that the final decision can be closer to the truth.

Self-confident preference relations provide a more general theoretical background in researching multiple self-confidence in decision sciences and have aroused widely attention. For example, Dong, Liu, Chiclana, Kou and Herrera-Viedma [24] validated that compared with incomplete preference relations, in most situations self-confident preference relations can improve the quality of the final decision(s). Liu, Zhang, Chen and Yu [25] proposed a new consensus approach to group decision making based on self-confident multiplicative preference relations. Liu, Xu, Montes, Dong and Herrera [26] presented a novel additive consistency measurement and improvement methods for SC-FPRs, and then introduced a novel SCI-IOWA operator applied to group decision making. After that, Liu, Xu and Herrera [27] developed a novel consensus model which detecting and managing overconfidence behaviors under large-scale group decision-making scenarios. Additionally, Liu, Xu, Ge, Zhang and Herrera [28] presented a new group decision making method considering self-confidence behaviors, and applied it to environmental pollution emergency management. All the studies on self-confident preference relations mentioned above have strongly demonstrated the necessity of considering the multiple self-confidence of experts in decision-making analysis.

Consensus reaching process (CRP) aims to reduce the objections of the minority while pursuing the agreements of the majority. To date, many consensus models have been developed for decision-making events [29-38]. Especially, the existing CRPs referring the SNGDM problems mainly include the following categories:

(a) Preference relations-based consensus. For instance, Wu and Chiclana [39] introduced a CRP considering trust relationships for SNGDM with interval-valued fuzzy reciprocal preference relations. And then, Wu, Chiclana and Herrera-Viedma [40] explored a trust-based CRP for SNGDM with incomplete linguistic information. In addition, Wu, Chang, Cao and Liang [41] developed a trust propagation and collaborative filtering method for incomplete information in SNGDM with type-2 linguistic trust, and presented a novel CRPs for SNGDM with incomplete information.

(b) Consensus model based on minimum adjustment cost. Generally, in the CRPs of SNGDM events, the minimum adjustment cost is an important rule. To date, Dong, Ding, Martínez and Herrera [42] explored a leadership-based consensus with a tactic adding a minimum interactions time. Additionally, based on SNGDM with distributed linguistic trust, Wu, Dai, Chiclana, Fujita and Herrera-Viedma [6] provided a consensus based on minimum adjustment cost feedback mechanism.

(c) Managing non-cooperative behaviors-based consensus model. Usually, non-cooperative behavior refers to a series of negative behaviors of experts in SNGDM process to protect personal or alliance interests. To reduce the negative impact of non-cooperative behavior on SNGDM, Zhang, Palomares, Dong and Wang [19] presented a consensus tactic to manage the non-cooperative for multi-attributes SNGDM scenarios.

(d) Other consensus researches for SNGDM problems. Wu, Chiclana, Fujita and Herrera-Viedma [1] presented a visual interaction consensus approach to SNGDM problems considering trust propagation. Dong, Zha, Zhang, Kou, Fujita, Chiclana and Herrera-Viedma [2] reviewed the CRPs in SNGDM, and proposed some new challenges for future research in SNGDM. In addition, Ding, Chen, Dong and Herrera [43] investigated the influence of the self-confidence level and the node degree on CRPs and the consensus convergence speed in the social network DeGroot model.

The above four kinds of research on the consensus model have enriched the theory and application of SNGDM. Nevertheless, using self-confidence preference relations to analyze the CRPs in SNGDM have not been concerned. As aforementioned, real SNGDM cases involve not only the mathematical formulation of the social network analysis but the experts’ self-confidence psychological behaviors. Hence, in order to ensure the rationality and authenticity of the final decision(s), it would be of great importance to consider the experts' inner multiple self-confidence in SNGDM scenarios. In addition, if we introduce the multiple self-confidence levels of experts into SNGDM, the following points are concerned in our research:

(1) In the context of social networks, how to reasonably allocate the importance degree of experts based on their trust relationships and self-confidence levels, and then to aggregate their information into a collective one?
(2) How to make the consensus measure in SNGDM considering experts' multiple self-confidence levels?
(3) How to utilize the trust relationships among experts and their self-confidence levels to effectively promote the negotiation in the CRPs, and then to reach a high level of consensus?
(4) Finally, how does the experts' self-confidence influence the final decision in SNGDM scenarios

In summary, this paper aims to explore a new CRP considering the multiple self-confidence levels of experts in SNGDM as well as to resolve the above research concerns. To do so, we propose a novel self-confidence-based consensus model with a dynamic importance degree of experts and trust-based feedback mechanism. The proposed method considers the external trust and internal self-confidence for defining the dynamic importance degrees of experts and the feedback adjustment in the CRPs. The main innovations are:

(1) Experts' multiple self-confidence levels are discussed in SNGDM scenarios. We allow experts to use SC-FPRs to voice their evaluations, and then to analyze the influence of self-confidence on SNGDM.
(2) A novel trust and self-confidence-based consensus model is proposed. In the proposed model, a dynamic importance degree determination of experts which combining the trust and self-confidence is provided to assign their weights. That is, if an expert is highly trusted while she/he also has a high level of self-confidence, then it means that she/he not only has a good reputation in social networks, but may have rich knowledge or experience referring to the SNGDM domain. Thus, she/he plays an important role in SNGDM and should be assigned high weight.
(3) A new group consensus index (GCI) which considers both the external trust and internal self-confidence of experts is given to measure the consensus level

among them.

(4) A trust-based feedback adjustment mechanism is presented to allow experts to modify their opinions according to their most trusts participants (MTPs). To accelerate the consensus, experts are suggested to dynamically adjust their self-confidence levels while revising their preferences in the CRPs. In addition, the importance degree of experts also changed based on the changing of their self-confidence levels.

The remainder of this paper is as follows. Section 2 reviews some preliminaries regarding the 2-tuple linguistic model, fuzzy preference relation (FPR), SC-FPRs, and SNGDM problems. Section 3 presents a self-confidence-based consensus reaching with dynamic importance degree of experts and trust-based feedback mechanism in SNGDM with SC-FPRs. Section 4 gives an illustrative example. Section 5 provides some comparisons and discussions. The concluding remarks are outlined in Section 6.

2. Preliminaries

Some preliminaries are reviewed in this section. In Section 2.1, the 2-tuple linguistic model is provided. In Section 2.2, the concepts of FPR and SC-FPR are given. The SNGDM problems are described in Section 2.3.

2.1. 2-tuple linguistic model

The 2-tuple linguistic model, which is utilized to express the experts' multiple self-confidence levels, is reviewed in this section. Suppose a linguistic term set $S=\{s_i \mid i=0,1,\ldots,g\}$, and s_i represents a probable value of a linguistic variable. For any s_i, s_j in *S*, they are assumed that $s_i > s_j$ iff $i > j$. Herrera and Martínez [44] first proposed the 2-tuple linguistic model:

Definition 1 [44]. Assume $\beta \in [0,g]$ be a value in *S*. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, where $i \in [0,g]$, $\alpha \in [-0.5,0.5)$. Then α is known as a *symbolic translation*, and the *round* is the general round operation.

Herrera and Martínez [44] proposed to use 2-tuple (s_i,α), $s_i \in S$ and $\alpha \in [-0.5,0.5)$ to represent the linguistic information. It is worth pointing out that the 2-tuple linguistic model gives a one-to-one mapping function to ensure the linguistic 2-tuple and quantitative value conversion mutually.

Definition 2 [44, 45]. Suppose a linguistic term set *S*, and its granularity interval is $[0,g]$. Then, the 2-tuple that expresses the equal information to $\beta \in [0,g]$ is obtained by the following function:

$$\Delta : [0,g] \to S \times [-0.5, 0.5),$$

$$\Delta(\beta) = (s_i, \alpha), \text{ with } \begin{cases} s_i, i = round(\beta) \\ \alpha = \beta - i, \alpha \in [-0.5, 0.5) \end{cases}.$$

Moreover, an inverse function Δ^{-1} of the *S* and (s_i, α) is also provided in [44]:

$$\Delta^{-1} : S \times [-0.5, 0.5) \to [0,g],$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta.$$

Especially, $\Delta^{-1}(s_i, 0) = \Delta^{-1}(s_i)$. Let (s_{l_1}, α) and (s_{l_2}, γ) be two 2-tuples, some operators were introduced by [44-46]:

(1) Operations:

- if $l_1 < l_2$, then (s_{l_1}, α) is smaller than (s_{l_2}, γ);
- if $l_1 = l_2$, then

(a) if $\alpha = \gamma$, then (s_{l_1}, α), (s_{l_2}, γ) denotes the equal information;

(b) if $\alpha < \gamma$, then (s_{l_1}, α) is smaller than (s_{l_2}, γ).

(2) Negation operator: $Neg(s_i, \alpha) = \Delta\left(g - \Delta^{-1}(s_i, \alpha)\right)$.

2.2. The concepts of FPR and SC-FPR

Suppose that $X = \{x_1, x_2, \ldots, x_n\}$ is an alternative set, $M = \{1, 2, \ldots m\}$ and $N = \{1, 2, \ldots n\}$.

Definition 3 [47]. Let $R = (r_{ij})_{n \times n}$ be a matrix, where r_{ij} denotes the preference degree of x_i over x_j. If the elements in *R* satisfy $r_{ij} + r_{ji} = 1$, $r_{ii} = 0.5$, for $\forall i, j \in N$, then *R* is called an FPR.

Let $S^{SL} = \{s_i \mid i = 0, 1, \ldots, g\}$ be a linguistic term set, which is used to denote the multiple self-confidence levels of experts. Then, the SC-FPR is defined as below:

Definition 4 [20]. Let $P=(p_{ij},l_{ij})_{n\times n}$ be an FPR with self-confidence on a set of alternatives X. P has two components, the first one indicates the preference degree of x_i over x_j, and the second one indicates the self-confidence level associated to the first component. Meanwhile, the components in P satisfy the following conditions:

$$\begin{cases} p_{ij}+p_{ji}=1, \ p_{ii}=0.5 \\ l_{ij}=l_{ji}, \qquad l_{ii}=s_g \\ p_{ij}\in[0,1], \ l_{ij}\in S^{SL} \end{cases}, \text{ for } \forall i,j\in N,$$

then P can be called an SC-FPR.

This study supposes that experts use $S^{SL}=\{s_0,s_1,\ldots,s_8\}$ to express multiple self-confidence levels. Fig.1 shows the language labels of the S^{SL}. To effectively fuse the SC-FPRs in SNGDM problems, Liu, Xu, Montes, Dong and Herrera [26] introduced some new operations of the 2-tuples in SC-FPRs:

Definition 5 [26]. Assume two 2-tuples (p_i,l_i), (p_k,l_k), p_i, p_k are the fuzzy preference values, and l_i, l_k are the self-confidence levels associated with p_i and p_k, where $l_i,l_k\in S^{SL}$, $\lambda\in[0,1]$. The operations of the 2-tuples are given below:

(1) $(p_k,l_k)+(p_i,l_i)=(p_k+p_i,\min\{l_k,l_i\})$;

(2) $(p_k,l_k)-(p_i,l_i)=(p_k-p_i,\min\{l_k,l_i\})$;

(3) $(p_i,l_i)-\lambda=(p_i-\lambda,l_i)$;

(4) $(p_i,l_i)^{\lambda}=((p_i)^{\lambda},l_i)$;

(5) $\lambda(p_i,l_i)=(\lambda p_i,l_i)$.

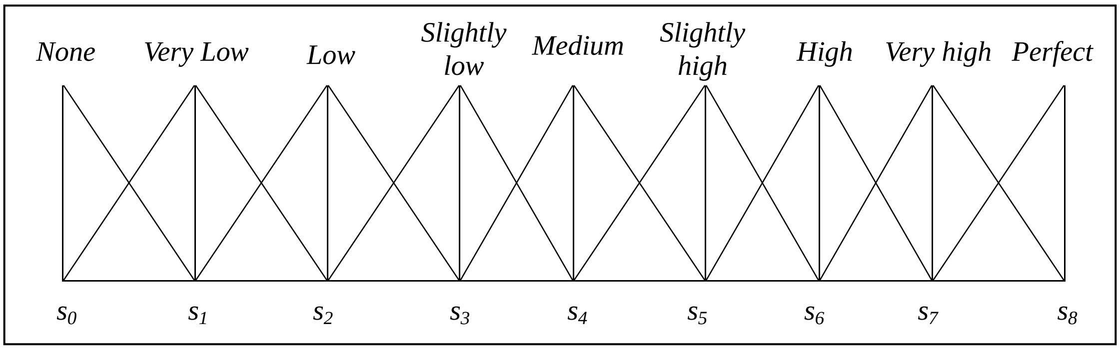
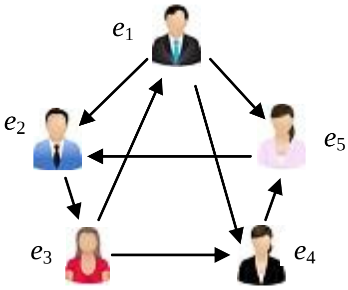


Fig. 1. The language labels of the multiple self-confidence levels of experts.

2.3. SNGDM problems

Social network can be seen as a platform that users can share information and communicate with each other. Meanwhile, the relationships among users can be obtained by social network analysis. Generally, in a social network, there are three main elements: a) the user set; b) the user relationship; and c) the user property. Detailed information is shown in Table 1.

Table 1. Different representation schemes in social networks.

Graph	Sociometric	Algebraic
	$A=\begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$	e_1Re_2 e_3Re_1 e_1Re_4 e_3Re_4 e_1Re_5 e_4Re_5 e_2Re_3 e_5Re_2

Additionally, there are three elements commonly used in social networks are described below:

(a) Sociometric: Socio-matrix $A=(a_{kh})_{m\times m}$ ($a_{kh}\in\{0,1\}$) is used to denote the relationships among experts in social networks. $a_{kh}=1$ denotes that there exists a direct trust relationship from experts e_k to e_h.

(b) Graph theoretical: The directed graph is used to view the social network . In the directed graph, nodes denote the experts, $e_k \to e_h$ denotes expert e_k trusts expert e_h.

(c) Algebraic: this notation allows distinguishing several unique relations and representing the combinations of them.

Actually, a social network can be denoted as a graph $G(E,V)$. $E=\{e_1,e_2,\ldots,e_m\}$ is the set of experts, denotes the nodes of experts. V is the set of edges, denotes the social relations among experts. The detailed definitions associated with SNGDM are given below:

Definition 6 [42, 48]. A social network is a directed graph $G(E,V)$, where $E=\{e_1,e_2,\ldots,e_m\}$ denotes the experts set, V is a set of an ordered pair of elements of E and edge $(e_k,e_h)\in V$ represents that expert e_k directly trusts expert e_h ($k,h\in M$).

Definition 7 [42, 48]. A sociometric $A=(a_{kh})_{m\times m}$ is utilized to represent $G(E,V)$ such that

$$a_{kh}=\begin{cases}1, & (e_k,e_h)\in V\\ 0, & otherwise\end{cases},$$

where $a_{kh}=1$ indicates expert e_k directly trusts e_h ($k,h\in M$).

It is obvious that Definition 7 can only describe two relations among experts in social networks: total trust or no trust at all. Whereas, in some cases, it is hard to crisply describe the trust relationships among experts since there exists indetermination in trust relationship description in social networks [19, 40]. Thus, a fuzzy sociometric was introduced in [2, 19, 42] as follows:

Definition 8 [2, 19, 42]. A fuzzy sociometric $A=(a_{kh})_{m\times m}$ on E is a relation in $E\times E$ with $U_A:E\times E\to[0,1]$, and $u_A(e_k,e_h)=a_{kh}$, where $a_{kh}\in[0,1]$ denotes the trust degree that e_k allocates to e_h ($k,h\in M$).

Without loss of generality, this paper adopts the fuzzy sociometric to describe the trust relationships among experts. Example 1 is illustrated to explain this.

Example 1. For the five experts depicted in Table 1, the $A=(a_{kh})_{5\times 5}$ (*k,h=1,2,3,4,5*; $k\neq h$) denotes the trust relationships among them:

$$A=\begin{pmatrix} - & 0.7 & 0 & 0.8 & 0.75 \\ 0 & - & 0.82 & 0 & 0 \\ 0.9 & 0 & - & 0.88 & 0 \\ 0 & 0 & 0 & - & 0.9 \\ 0 & 0.6 & 0 & 0 & - \end{pmatrix}.$$

3. Self-confidence-based consensus model with dynamic importance degree of experts and trust-based feedback mechanism

This section mainly presents the self-confidence-based CRPs with dynamic importance degree of experts and trust-based feedback mechanism. In Section 3.1, a method to dynamically determine the experts' importance degree is proposed. In Section 3.2, a consensus measure is given. After that, a trust-based feedback adjustment is given in Section 3.3. Additionally, the self-confidence-based consensus model for SNGDM with SC-FPRs is implemented in Algorithm 1. Finally, a selection process for SNGDM with SC-FPRs is provided in Section 3.4.

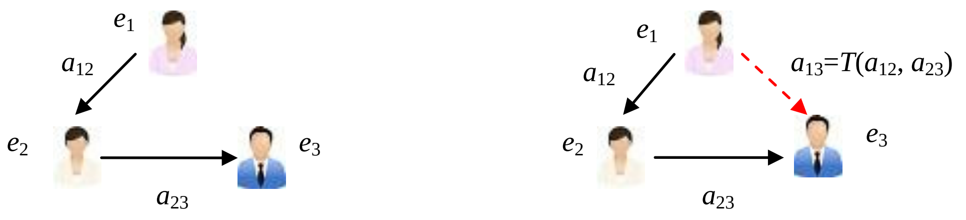
3.1. Determination of the dynamic importance degree of expert

To aggregate individual's assessment information into a group one, and then to explore the CRPs in SNGDM, the first step is to determine each expert's importance degree. In the following, we propose to combine the external trust of experts gained from others and their internal self-confidence to determine their importance degrees. Detailed processes are given below:

(1) Calculate the trust degree of expert

In SNGDM scenario, the external trusts of an expert gained from others can reflect her/his reputation in the decision-making process. Generally, the more trusted an expert is, the more important or influential she/he will be. Thus, the external trusts of an expert gained from others can be regarded as an objective importance degree to determine her/his weight.

To measure the trust degree of an expert, it needs to obtain the corresponding fuzzy sociometric firstly. However, it is generally incomplete because some experts may not give a direct trust value for a specific expert as shown in Fig. 2 (a). Clearly, there is no direct trust relationship between e_1 and e_3 in Fig. 2(a). Nevertheless, the indirect trust between e_1 and e_3 can be inferred based on the transitivity. In order to evaluate the missing trust values in the fuzzy sociometric, a *t*-norms-based trust propagation is introduced in [48, 49]:



(a) No direct trust between e_1 and e_3 (b) Trust propagation between e_1 and e_3 via e_2

Fig. 2. Trust propagation through indirect trust path.

Definition 9 [48, 49]. Let $e_k \xrightarrow{1} e_{\sigma(1)} \xrightarrow{2} e_{\sigma(2)} \xrightarrow{3} \ldots \xrightarrow{q} e_{\sigma(q)} \xrightarrow{q+1} e_h$ be a path from e_k to e_h, and the length is $q+1$. The trust value a_{kh} can be evaluated using *t*-norm:

$$a_{kh} = T(a_{k,\sigma(1)}, a_{\sigma(1),\sigma(2)}, \ldots, a_{\sigma(q),h})$$

$$= \frac{2 \cdot a_{k,\sigma(1)} \cdot a_{\sigma(q),h} \prod_{z=1}^{q-1} a_{\sigma(z),\sigma(z+1)}}{(2-a_{k,\sigma(1)})(2-a_{\sigma(q),h}) \prod_{z=1}^{q-1} (2-a_{\sigma(z),\sigma(z+1)}) + a_{k,\sigma(1)} \cdot a_{\sigma(q),h} \prod_{z=1}^{q-1} a_{\sigma(z),\sigma(z+1)}}. \quad (1)$$

Specifically, if the length is two from e_k to e_h, that is $q=1$. Then, the trust value a_{kh} is computed by:

$$a_{kh} = T(a_{k,\sigma(1)}, a_{\sigma(1),h}) = \frac{a_{k\sigma(1)} a_{\sigma(1)h}}{1+(1-a_{k\sigma(1)})(1-a_{\sigma(1)h})}. \quad (2)$$

Example 2. In Fig. 2(b), suppose that $a_{12} = 0.75$ and $a_{23} = 0.88$. Then, utilizing Eq. (2), the trust value a_{13} between e_1 and e_3 is 0.641.

There is usually more than one trust path between two experts in a social network. Fig. 3 illustrates the trust relationships among five experts in an SNGDM problem. Clearly, there are three paths from expert e_1 to expert e_5:

(a) $e_1 \to e_2 \to e_3 \to e_5$;

(b) $e_1 \to e_3 \to e_5$;

(c) $e_1 \to e_2 \to e_4 \to e_1 \to e_3 \to e_5$.

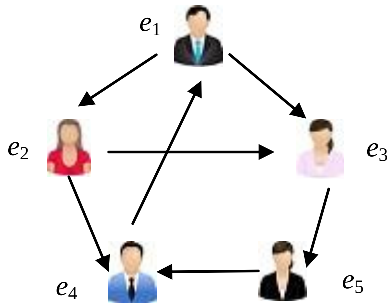


Fig. 3. The trust relationships among five experts in an SNGDM problem

As aforementioned, the trust degrees of experts gained from others represent their possible influence in SNGDM process. To compute a more accurate trust value, we suggest to consider all the trust propagation paths between two experts. Hence, as per the case (c) mentioned in Fig. 3, although the trust path goes back to e_1 in the middle, this trust path from e_1 to e_5 still needs to be considered. The reason is that as the node of trust delivery, e_2 generates two trust delivery paths to e_5, i.e., $e_2 \to e_3$ and $e_2 \to e_4$, respectively. In order to obtain the accurate trust degree of e_1, that is, to measure the possible influence of expert e_1 more authentically, the case (c) is considered in this study.

For the case that there are multiple trust paths among experts, we need to aggregate different trust values into a total trust degree. To do so, a trust aggregation using the ordered weighted averaging (OWA) operator [50] is presented:

Definition 10 [50]. Assume that there are χ trust paths from e_k to e_h, and the $a_{kh}^1, a_{kh}^2, \ldots, a_{kh}^\chi$ denote the trust values. Then, a_{kh} can be computed by:

$$a_{kh} = OWA(a_{kh}^1, a_{kh}^2, \ldots, a_{kh}^\chi) = \sum_{z=1}^{\chi} \varphi_z a_{kh}^{\sigma(z)}, \tag{3}$$

where $a_{kh}^{\sigma(z)}$ is the z^{th} largest value in $\{a_{kh}^1, a_{kh}^2, \ldots, a_{kh}^\chi\}$, and $\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_z)^T$ is the weight vector such that $\varphi_z \geq 0$ and $\sum_{z=1}^{\chi} \varphi_z = 1$.

To compute the trust value a_{kh} between e_k and e_h, an important issue is to get the weight vector $\varphi=(\varphi_1,\varphi_2,\ldots,\varphi_z)^T$. This paper adopts the quantifier-guided method based on the linguistic quantifiers Q proposed by Zadeh [51] to compute the weights φ_z ($z=1,2,...,\chi$):

$$\varphi_z=Q\left(\frac{z}{\chi}\right)-Q\left(\frac{z-1}{\chi}\right),\quad z=1,2,\ldots,\chi, \tag{4}$$

where $Q(b)$ can be denoted as

$$Q(b)=\begin{cases}0, & b<f,\\ \dfrac{b-f}{c-f}, & f\le b\le c,\\ 1, & b>c,\end{cases} \tag{5}$$

with $b,c,f\in[0,1]$. Meanwhile, the parameters (f,c) are (0,1), (0.3,0.8), (0,0.5), and (0.5,1)), denoting the "All", "Most", "At least half", "As many as possible" of the proportional quantifiers respectively.

Then, the complete sociometric denoted as $A=(a_{kh})_{m\times m}$, ($k,h\in M$, $k\neq h$) can be obtained. Subsequently, the definition of the trust degree of an expert in SNGDM with SC-FPRs is given below.

Definition 11. Let $A=(a_{kh})_{m\times m}$ ($k,h\in M$, $k\neq h$) be a complete sociemetric matrix, then the trust degree $TD(e_h)$ of an expert e_h in SNGDM with SC-FPRs can be calculated by:

$$TD(e_h)=\frac{1}{m-1}\sum_{k=1,h\neq k}^{m}a_{kh}. \tag{6}$$

Example 3. Suppose that the five experts e_1,e_2,e_3,e_4,e_5 which are depicted in Fig. 3. The corresponding fuzzy sociometric are assumed as follows:

$$A=\begin{pmatrix} - & 0.7 & 0.6 & - & - \\ - & - & 0.8 & 0.9 & - \\ - & - & - & - & 0.7 \\ 0.6 & - & - & - & - \\ - & - & - & 0.8 & - \end{pmatrix}.$$

From Fig. 3, we observe that at least one path exists between any two experts who are not directly connected. The detailed indirect linkages for the incomplete trust values of Example 3 are shown in Table 2.

Table 2. The detailed indirect linkage for incomplete trust values in Example 3

	Indirect linkage		Indirect linkage
$\tilde{a}_{14}$	$e_1 \to e_2 \to e_4$ $e_1 \to e_3 \to e_5 \to e_4$ $e_1 \to e_2 \to e_3 \to e_5 \to e_4$	$\tilde{a}_{15}$	$e_1 \to e_2 \to e_3 \to e_5$ $e_1 \to e_3 \to e_5$ $e_1 \to e_2 \to e_4 \to e_1 \to e_3 \to e_5$
$\tilde{a}_{21}$	$e_2 \to e_3 \to e_5 \to e_4 \to e_1$ $e_2 \to e_4 \to e_1$	$\tilde{a}_{25}$	$e_2 \to e_3 \to e_5$ $e_2 \to e_4 \to e_1 \to e_3 \to e_5$
$\tilde{a}_{31}$	$e_3 \to e_5 \to e_4 \to e_1$	$\tilde{a}_{32}$	$e_3 \to e_5 \to e_4 \to e_1 \to e_2$
$\tilde{a}_{34}$	$e_3 \to e_5 \to e_4$	$\tilde{a}_{42}$	$e_4 \to e_1 \to e_2$
$\tilde{a}_{43}$	$e_4 \to e_1 \to e_3$ $e_4 \to e_1 \to e_2 \to e_3$	$\tilde{a}_{45}$	$e_4 \to e_1 \to e_2 \to e_3 \to e_5$ $e_4 \to e_1 \to e_3 \to e_5$
$\tilde{a}_{51}$	$e_5 \to e_4 \to e_1$	$\tilde{a}_{52}$	$e_5 \to e_4 \to e_1 \to e_2$
$\tilde{a}_{53}$	$e_5 \to e_4 \to e_1 \to e_2 \to e_3$ $e_5 \to e_4 \to e_1 \to e_3$ $e_5 \to e_4 \to e_1 \to e_2 \to e_4 \to e_1 \to e_3$		

Then, utilizing Definitions 9 and 10, the complete sociometric $A=(a_{kh})_{5\times 5}$ (*k*,*h=1,2,3,4,5*) can be computed. For example, as for the a_{14}, using Eqs. (1) and (2), we have:

$$a_{14}^1=\frac{a_{12}\cdot a_{24}}{1+(1-a_{12})\cdot(1-a_{24})}=0.612,$$

$$a_{14}^2=\frac{2\cdot a_{13}\cdot a_{54}\cdot a_{35}}{(2-a_{13})\cdot(2-a_{54})\cdot(2-a_{35})+a_{13}\cdot a_{54}\cdot a_{35}}=0.267;$$

$$a_{14}^3=\frac{2\cdot a_{12}\cdot a_{54}\cdot(a_{23}\cdot a_{35})}{(2-a_{12})\cdot(2-a_{54})\cdot(2-a_{23})\cdot(2-a_{35})+a_{12}\cdot a_{54}\cdot(a_{23}\cdot a_{35})}=0.228.$$

After that, using Eqs. (4) and (5), the φ_z (z=1,2,3) are (here we use the linguistic quantifier “Most”):

$$\varphi_1 = 0.067,\ \ \varphi_2 = 0.666,\ \ \varphi_3 = 0.267.$$

Then, the $a_{14} = 0.28$ by Eq. (3). All other values in $A = (a_{kh})_{5\times5}$ (*k*,*h*=*1*,*2*,*3*,*4*,*5*) can be determined similarly. The final *A* is derived as follows:

$$A = \begin{pmatrix} - & 0.7 & 0.6 & 0.28 & 0.26 \\ 0.32 & - & 0.8 & 0.9 & 0.3 \\ 0.27 & 0.15 & - & 0.53 & 0.7 \\ 0.6 & 0.38 & 0.28 & - & 0.16 \\ 0.44 & 0.27 & 0.16 & 0.8 & - \end{pmatrix}.$$

Thus, using Eq. (6), the $TD(e_h)$ (*h*=*1*,*2*,*3*,*4*,*5*) in Fig. 3 can be obtained:

$$TD(e_1) = 0.41,\ TD(e_2) = 0.38,\ TD(e_3) = 0.46,\ TD(e_4) = 0.63,\ TD(e_5) = 0.36.$$

(2) Measure the self-confidence degree of an expert

Due to the time pressure, limited knowledge or experience referring the SNGDM domain, experts may give different self-confidence in the decision-making process. Consequently, the self-confidence degree of an expert also needs to be considered. The self-confidence generally means one's self-affirmation on her/his opinion. The higher the degree of self-confidence, the more knowledge or experience the expert is likely to own. Hence, the self-confidence levels of experts can be seen as the subjective importance degree to determine their weights. However, as far as we know, the existing methods only consider the external trust, while ignoring the internal self-confidence of experts. For Example 3, it is clear that the diagonal elements that represent the self-confidence of experts themselves are missing in A. As aforementioned, self-confidence can reflect one's knowledge, experience or attitude in SNGDM process [23]. Thus, the authenticity and objectivity of the measurement of expert's importance degree can be further ensured by combining the multiple self-confidence.

For an SC-FPR $P_k = (p_{ij,k}, l_{ij,k})_{n\times n}$ provided by an expert e_k, the overall self-confidence matrix $L_k = (l_{ij,k})_{n\times n}$ of e_k can be obtained. Then, according to the distance measure, the self-confidence deviation level (SCDL) of e_k is defined as:

Definition 12. Assume that e_k provides an SC-FPR $P_k = (p_{ij,k}, l_{ij,k})_{n\times n}$. Then,

$L_k=(l_{ij,k})_{n\times n}$ of e_k can be obtained, and let $L=(s_g)_{n\times n}$ be the maximal self-confidence matrix such that $l_{ij,k}, s_g \in S^{SL}$. Then, the $SCDL$ of e_k ($k\in M$) is calculated by:

$$SCDL(e_k)=\frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} d(L_k,L)$$
$$=\frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{|\Delta^{-1}(l_{ij,k})-\Delta^{-1}(s_g)|}{g}. \quad (7)$$

And then, the self-confidence degree $SCD(e_k)$ of e_k is:

$$SCD(e_k)=1-SCDL(e_k). \quad (8)$$

Clearly, $SCD(e_k)\in[0,1]$. The larger the $SCD(e_k)$, the higher the self-confidence of e_k will be.

Example 4. In an SNGDM problem, let $E=\{e_1,e_2,e_3,e_4,e_5\}$ be a set of five experts in Fig. 3, and $S^{SL}=\{s_0,s_2,\ldots,s_8\}$ is utilized to express the multiple self-confidence levels of experts. Each expert provides her/his SC-FPRs $P_k=(p_{ij,k},l_{ij,k})_{n\times n}$ (k=*1,2,3,4,5*) over a set of four alternatives $X=\{x_1,x_2,x_3,x_4\}$ as follows:

$$P_1=\begin{pmatrix}(0.5,s_8) & (0.1,s_5) & (0.6,s_7) & (0.7,s_8)\\ (0.9,s_5) & (0.5,s_8) & (0.8,s_6) & (0.6,s_4)\\ (0.4,s_7) & (0.2,s_6) & (0.5,s_8) & (0.6,s_5)\\ (0.3,s_8) & (0.4,s_4) & (0.4,s_5) & (0.5,s_8)\end{pmatrix},$$

$$P_2=\begin{pmatrix}(0.5,s_8) & (0.6,s_3) & (0.8,s_5) & (0.2,s_2)\\ (0.4,s_3) & (0.5,s_8) & (0.6,s_4) & (0.7,s_6)\\ (0.2,s_5) & (0.4,s_4) & (0.5,s_8) & (0.4,s_3)\\ (0.8,s_2) & (0.3,s_6) & (0.6,s_3) & (0.5,s_8)\end{pmatrix},$$

$$P_3=\begin{pmatrix}(0.5,s_8) & (0.3,s_5) & (0.4,s_7) & (0.7,s_4)\\ (0.7,s_5) & (0.5,s_8) & (0.2,s_6) & (0.4,s_3)\\ (0.6,s_7) & (0.8,s_6) & (0.5,s_8) & (0.9,s_2)\\ (0.3,s_4) & (0.6,s_3) & (0.1,s_2) & (0.5,s_8)\end{pmatrix},$$

$$P_4 = \begin{pmatrix} (0.5, s_8) & (0.4, s_5) & (0.2, s_4) & (0.1, s_5) \\ (0.6, s_5) & (0.5, s_8) & (0.6, s_6) & (0.5, s_3) \\ (0.8, s_4) & (0.4, s_6) & (0.5, s_8) & (0.3, s_7) \\ (0.9, s_5) & (0.5, s_3) & (0.7, s_7) & (0.5, s_8) \end{pmatrix},$$

$$P_5 = \begin{pmatrix} (0.5, s_8) & (0.6, s_6) & (0.3, s_4) & (0.4, s_6) \\ (0.4, s_6) & (0.5, s_8) & (0.6, s_5) & (0.7, s_4) \\ (0.7, s_4) & (0.4, s_5) & (0.5, s_8) & (0.3, s_7) \\ (0.6, s_6) & (0.3, s_4) & (0.7, s_7) & (0.5, s_8) \end{pmatrix}.$$

Then, the self-confidence matrices $L_k = (l_{ij,k})_{n \times n}$ of experts e_k (*k=1,2,3,4,5*) are:

$$L_1 = \begin{pmatrix} s_8 & s_5 & s_7 & s_8 \\ s_5 & s_8 & s_6 & s_4 \\ s_7 & s_6 & s_8 & s_5 \\ s_8 & s_4 & s_5 & s_8 \end{pmatrix}, \quad L_2 = \begin{pmatrix} s_8 & s_3 & s_5 & s_2 \\ s_3 & s_8 & s_4 & s_6 \\ s_5 & s_4 & s_8 & s_3 \\ s_2 & s_6 & s_3 & s_8 \end{pmatrix},$$

$$L_3 = \begin{pmatrix} s_8 & s_5 & s_7 & s_4 \\ s_5 & s_8 & s_6 & s_3 \\ s_7 & s_6 & s_8 & s_2 \\ s_4 & s_3 & s_2 & s_8 \end{pmatrix}, \quad L_4 = \begin{pmatrix} s_8 & s_5 & s_4 & s_5 \\ s_5 & s_8 & s_6 & s_3 \\ s_4 & s_6 & s_8 & s_7 \\ s_5 & s_3 & s_7 & s_8 \end{pmatrix}, \quad L_5 = \begin{pmatrix} s_8 & s_3 & s_4 & s_7 \\ s_3 & s_8 & s_6 & s_3 \\ s_4 & s_6 & s_8 & s_4 \\ s_7 & s_3 & s_4 & s_8 \end{pmatrix}.$$

Using Eqs. (7) and (8), $SCD(e_k)$ (*k=1,2,3,4,5*) can be obtained:

$$SCD(e_1) = 0.72, \quad SCD(e_2) = 0.47,$$

$$SCD(e_3) = 0.55, \quad SCD(e_4) = 0.61, \quad SCD(e_5) = 0.66.$$

(3) The dynamic importance degree of an expert

After the trust degree and self-confidence degree of an expert have been obtained, the dynamic importance degree of expert combining the trust degree and the self-confidence degree is given below:

Definition 13. Let $TD(e_k)$ be the trust degree of e_k in an SNGDM with SC-FPRs, and $SCD(e_k)$ be the self-confidence degree of e_k in her/his opinions. Then, the importance degree $\psi(e_k)$ of e_k ($k \in M$) is defined by:

$$\psi(e_k) = \vartheta TD(e_k) + (1 - \vartheta) SCD(e_k), \tag{9}$$

where $\vartheta \in [0,1]$ is a parameter to control the weight of $TD(e_k)$ and $SCD(e_k)$. Specially, we set $\vartheta = 0.5$ since this paper supposes that the trust degree and self-confidence degree of an expert is of equal importance.

Remark 1. As per Definition 13, we have $\psi(e_k) \in [0,1]$. The smaller the value of $\psi(e_k)$, the less the importance degree of expert e_k will be.

Afterwards, in an SNGDM with SC-FPRs, let $\{\psi(e_1), \psi(e_2), \dots, \psi(e_m)\}$ be the set of importance degrees of experts $E = \{e_1, e_2, \dots, e_m\}$. The normalized weight w_k of an expert e_k ($k \in M$) can be obtained by:

$$w_k = \frac{\psi(e_k)}{\sum_{k=1}^{m} \psi(e_k)}. \tag{10}$$

Example 5 (Example 4 continuation). The $\psi(e_k)$ (*k=1,2,3,4,5*) can be obtained by Eq. (9):

$$\psi(e_1) = 0.57,\ \psi(e_2) = 0.42,\ \psi(e_3) = 0.51,\ \psi(e_4) = 0.64,\ \psi(e_5) = 0.18.$$

Then, utilizing Eq. (10), the weights w_k of e_k (*k=1,2,3,4,5*) are:

$$w_1 = 0.24,\ w_2 = 0.18,\ w_3 = 0.22,\ w_4 = 0.28,\ w_5 = 0.08.$$

3.2. Consensus measure

After the initial experts' weights are obtained by Eq. (10), we can compute the temporary collective SC-FPR $P_c = (p_{ij,c}, l_{ij,c})_{n\times n}$ using the WA operator:

Definition 14. Let $P_k = (p_{ij,k}, l_{ij,k})_{n\times n}$ ($k \in M$) be the SC-FPRs given by experts in an SNGDM problem, and $W = (w_1, w_2, \dots, w_m)^T$ be the expert's weighting vector. Then, the collective SC-FPR $P_c = (p_{ij,c}, l_{ij,c})_{n\times n}$ is:

$$P_c = \sum_{k=1}^{m} w_k P_k. \tag{11}$$

Then, the individual consensus index (ICI) between individual and collective SC-FPR can be obtained.

Definition 15. Let $P_k = (p_{ij,k}, l_{ij,k})_{n\times n}$ ($k \in M$) be an SC-FPR of e_k , and $P_c = (p_{ij,c}, l_{ij,c})_{n\times n}$ be the collective SC-FPR. Then, the *ICI* is defined by:

$$ICI(P_k) = 1 - d(P_k, P_c), \tag{12}$$

where $d(P_k, P_c)$ is the distance between P_k and P_c:

$$d(P_k, P_c) = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} |(p_{ij,k}, l_{ij,k}) - (p_{ij,c}, l_{ij,c})|. \tag{13}$$

And then, the GCI can be calculated as:

$$GCI = \sum_{k=1}^{m} w_k \times ICI(P_k). \tag{14}$$

Obviously, $GCI \in [0,1]$. The larger the value of GCI, the higher the consensus among experts. Generally, to achieve an unanimous agree among experts is impossible. Therefore, a soft consensus is adopted in the consensus reaching process. In order to achieve soft consensus in SNGDM, a consensus threshold $\theta \in [0,1]$ is predefined to measure the consensus degree among experts. At the same time, if $GCI \geq \theta$, then a soft consensus for SNGDM with SC-FPRs is achieved. Otherwise, the feedback process is carried out to promote a soft consensus achieved.

3.3. Trust-based feedback mechanism

To achieve a soft consensus, a trust-based feedback mechanism is presented in this section. The experts are suggested to dynamically adjust their self-confidence while revising the preferences according to the MTPs. It mainly contains the following three processes:

(a) Identification of the MTP of each expert;
(b) Identification of the preferences which has the minimal self-confidence for each expert;
(c) Generation of the adjustments.

The detailed processes are described as follows:

(1) Identification of the MTP of each expert. The MTP of an expert can be

determined by the values of a_{kh} in A:

$$MTP(e_k) = \{e_h \mid \max_{h,h\neq k}\{a_{kh}\}\}, \quad k \in M . \tag{15}$$

(2) Identification of the preferences that has minimal self-confidence for each expert: self-confidence usually reveals the experts' knowledge, abilities or experiences referring the SNGDM domain. Generally, if an expert expresses a low self-confidence, it indicates that she/he may lack sufficient knowledge or experience. In other words, the fuzzy preference value provided by the expert may lack reliability. Based on this hypothesis, after identifying the MTP of each expert, we suggest experts to modify their fuzzy preference values which they own minimal self-confidence in each round of feedback. That is, find the position i_τ and j_τ of the minimal elements $l^{(\eta)}_{i_\tau j_\tau,k}$ (η is the η^{th} iteration in the CRPs), for each expert e_k ($k \in M$), where

$$l^{(\eta)}_{i_\tau j_\tau,k} = \min_{i,j}\{l^{(\eta)}_{ij,k}\}, \quad i, j \in N . \tag{16}$$

And then, the $p^{(\eta)}_{i_\tau j_\tau,k}$ corresponding to $l^{(\eta)}_{i_\tau j_\tau,k}$ can be determined.

(3) Generation of the adjustments. The elements in $P_k = (p_{ij,k}, l_{ij,k})_{n\times n}$ of e_k ($k \in M$) are suggested to be:

$$p^{(\eta+1)}_{ij,k} = \begin{cases} p^{(\eta)}_{ij,h}, & \text{if } i = i_\tau,\ j = j_\tau \\ p^{(\eta)}_{ij,k}, & \text{otherwise} \end{cases}, \tag{17}$$

$$l^{(\eta+1)}_{ij,k} = \begin{cases} l^{(\eta)}_{ij,h}, & \text{if } i = i_\tau,\ j = j_\tau \\ l^{(\eta)}_{ij,k}, & \text{otherwise} \end{cases} .$$

(18)

Especially, if there exist two minimal self-confidence levels that are equal, i.e., $l^{(\eta)}_{i_\tau j_\tau,k} = l^{(\eta)}_{i'_\tau j'_\tau,k}$, then find $p^{(\eta)}_{i_\tau j_\tau,k}$ and $p^{(\eta)}_{i'_\tau j'_\tau,k}$, make the modifications based on the following rules:

- if $d^{(\eta)}_{i_\tau j_\tau,k} < d^{(\eta)}_{i'_\tau j'_\tau,k}$, then e_k needs to improve $(p^{(\eta)}_{i'_\tau j'_\tau,k}, l^{(\eta)}_{i'_\tau j'_\tau,k})$;
- if $d^{(\eta)}_{i_\tau j_\tau,k} > d^{(\eta)}_{i'_\tau j'_\tau,k}$, then e_k needs to modify $(p^{(\eta)}_{i_\tau j_\tau,k}, l^{(\eta)}_{i_\tau j_\tau,k})$;

- if $d^{(\eta)}_{i_\tau j_\tau,k} = d^{(\eta)}_{i'_\tau j'_\tau,k}$, then e_k can randomly choose any $(p^{(\eta)}_{i_\tau j_\tau,k}, l^{(\eta)}_{i_\tau j_\tau,k})$ and $(p^{(\eta)}_{i'_\tau j'_\tau,k}, l^{(\eta)}_{i'_\tau j'_\tau,k})$ to repair;

where $d^{(\eta)}_{i_\tau j_\tau,k} = | p^{(\eta)}_{i_\tau j_\tau,k} - p^{(\eta)}_{i_\tau j_\tau,c} |$, and $d^{(\eta)}_{i'_\tau j'_\tau,k} = | p^{(\eta)}_{i'_\tau j'_\tau,k} - p^{(\eta)}_{i'_\tau j'_\tau,c} |$.

Moreover, Algorithm 1 depictes the self-confidence-based consensus model for SNGDM with SC-FPRs.

Algorithm 1. Self-confidence-based consensus model for SNGDM with SC-FPRs

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Input: the individual SC-FPRs $P_k = (p_{ij,k}, l_{ij,k})_{n\times n}$ ($k \in M$), the maximum number of iterations η^*. The consensus threshold θ and the parameter ϑ.

Output: the collective SC-FPR $P_c = (p_{ij,c}, l_{ij,c})_{n\times n}$, terminal iteration step η and $GCI^{(\eta)}$.

Step 1. Compute $TD(e_k)$ of e_k using Eq. (6).

Step 2. Let $\eta = 0$, calculate $SCD(e_k)$ using Eq. (8). And then, using Eqs. (9) and (10) to obtain the weight $w_k^{(0)}$ of e_k.

Step 3. Compute the collective SC-FPR $P_c^{(\eta)} = (p^{(\eta)}_{ij,c}, l^{(\eta)}_{ij,c})_{n\times n}$ using Eq. (11).

Step 4. Using Eq. (14) to compute the $GCI^{(\eta)}$. If $GCI^{(\eta)} \ge \theta$, go to Step 6. Otherwise, go to the next step.

Step 5. Utilize Eq. (15) to identify the MTP of each expert e_k ($k \in M$). Subsequently, use Eq. (16) to identify the preference that has minimal self-confidence for each e_k ($k \in M$). And then, advice e_k to revise her/his SC-FPR by Eqs. (17) and (18).

Then, let $\eta = \eta + 1$ and go to Step 2.

Step 6. End.

Remark 2. As per Algorithm 1, suppose that the MTP of e_k is e_h, it is worth pointing that if the elements $(p^{(\eta)}_{ij,h}, l^{(\eta)}_{ij,h})$ of e_h needs to be modified at the iteration $\eta = \eta + 1$, the adjusted elements of e_k should adopt the following rules:

$$p^{(\eta+1)}_{ij,k} = \begin{cases} p^{(\eta+1)}_{ij,h}, \text{if } i = i_\tau,\ j = j_\tau \\ p^{(\eta)}_{ij,k}, \text{otherwise} \end{cases}, \tag{19}$$

$$l_{ij,k}^{(\eta+1)} = \begin{cases} l_{ij,h}^{(\eta+1)}, \text{if } i = i_\tau,\ j = j_\tau \\ l_{ij,k}^{(\eta)}, \text{otherwise} \end{cases}. \quad (20)$$

3.4. Selection process

After an acceptable consensus is reached, we can compute the final collective SC-FPR $P_c = (p_{ij,c}, l_{ij,c})_{n\times n}$. Then, a selection process is activated for SNGDM with SC-FPRs.

Definition 16. Suppose that an alternative set $X = \{x_1, \ldots, x_n\}$, and $P_c = (p_{ij,c}, l_{ij,c})_{n\times n}$ is the collective SC-FPR, then the alternative *SCS* function is defined by:

$$\tilde{s}(x_i) = \frac{1}{n}\sum_{j=1}^{n}(p_{ij,c} \times \Delta^{-1}(l_{ij,c})),\ \ i = 1,2,...,n. \quad (21)$$

Clearly, the larger the value of $\tilde{s}(x_i)$, the more the self-confidence of an expert on x_i will be. As a result, if $\tilde{s}(x_i) > \tilde{s}(x_j)$, then $x_i \succ x_j$. Additionally, the decision process for SNGDM with SC-FPRs: trust and self-confidence-based consensus reaching and selection processes are depicted in Fig. 4.

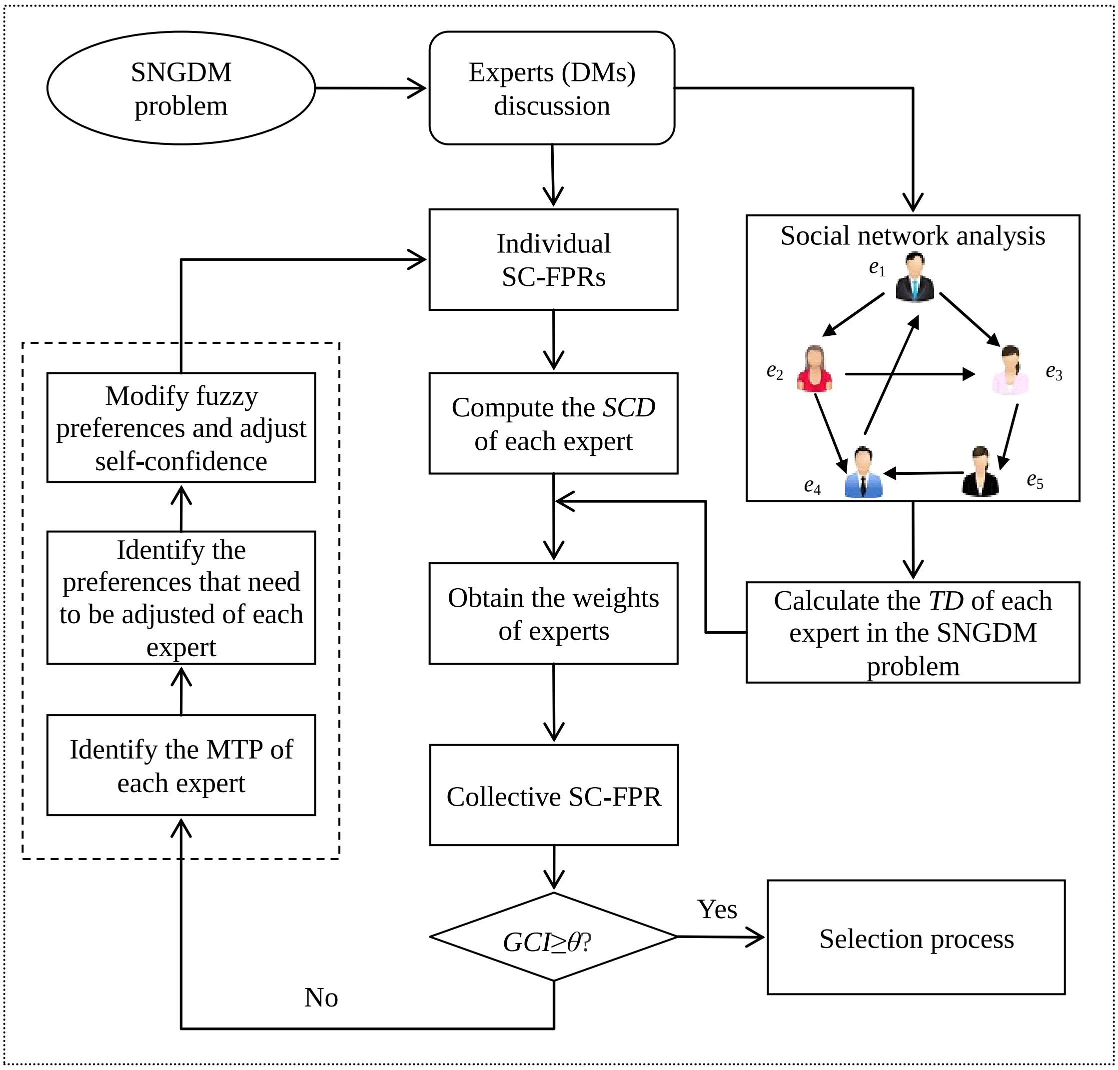

Fig. 4. Decision process for SNGDM with SC-FPRs: Trust and self-confidence-based consensus reaching and selection processes

4. Illustrative example

An example is provided in this section to show how the proposed consensus model works in SNGDM scenario. For simplicity, we continue Example 5, and suppose $\eta^* = 6$, $\theta = 0.7$ and $\vartheta = 0.5$, respectively. Then, detailed trust and self-confidence-based CRPs for SNGDM with SC-FPRs are given below:

Step 1. In Example 5, we have obtained the original weights of e_k (k=1,2,3,4,5)

$$w_1^{(0)} = 0.24,\ w_2^{(0)} = 0.18,\ w_3^{(0)} = 0.22,\ w_4^{(0)} = 0.28,\ w_5^{(0)} = 0.08.$$

Then, using Eq. (11) to compute the temporary collective SC-FPR

$P_c^{(0)} = (p_{ij,c}^{(0)}, l_{ij,c}^{(0)})_{4\times 4}$ is:

$$P_c^{(0)} = \begin{pmatrix} (0.5, s_8) & (0.36, s_3) & (0.46, s_4) & (0.42, s_2) \\ (0.64, s_3) & (0.5, s_8) & (0.56, s_4) & (0.55, s_3) \\ (0.54, s_4) & (0.44, s_4) & (0.5, s_8) & (0.52, s_2) \\ (0.58, s_2) & (0.45, s_3) & (0.48, s_2) & (0.5, s_8) \end{pmatrix}.$$

Step 2. Compute the $ICI(P_k^{(0)})$ (*k=1,2,3,4,5*) utilizing Eq. (12):

$$ICI(P_1^{(0)}) = 0.5375,\ ICI(P_2^{(0)}) = 0.5325,\ ICI(P_3^{(0)}) = 0.5175,$$
$$ICI(P_4^{(0)}) = 0.5475,\ ICI(P_5^{(0)}) = 0.5560.$$

Then, $GCI^{(0)} = 0.5361 < 0.7$, the iteration is continued.

Step 3. Identify the MTP of each expert using Eq. (15). For example, for e_1, the MTP of e_1 is e_2 since $MTP(e_1) = \{e_2 | a_{12} = \max\{a_{1h}\}=0.7\}$ (*h=1,2,3,4,5*). Similarly, the MTPs of e_k (*k=2,3,4,5*) are

$$MTP(e_2) = e_4,\ MTP(e_3) = e_5,\ MTP(e_4) = e_1,\ MTP(e_5) = e_4.$$

Additionally, Table 4 shows the detailed results.

Table 4. The MTP of each expert e_k in the SNGDM (*k=1,2,3,4,5*).

k	*MTP(e_k)*	Trust value
1	e_2	0.7
2	e_4	0.9
3	e_5	0.7
4	e_1	0.6
5	e_4	0.8

Step 4. Using Eq. (16) to find the position i_τ and j_τ of the $l_{i_\tau j_\tau,k}^{(t)}$, for each e_k (*k=1,2,3,4,5*). For $P_1^{(0)}$, since $l_{24,1}^{(0)} = \min_{i,j}\{l_{ij,1}^{(0)}\} = s_4$, replacing the element with the corresponding element in $P_2^{(0)}$: $p_{24,1}^{(0)} = p_{24,2}^{(0)} = 0.7$, $l_{24,1}^{(0)} = l_{24,2}^{(0)} = s_6$ by Eqs. (17) and (18). The same program is applied to update the other experts' assessment information:

$$p_{14,2}^{(0)} \to p_{14,4}^{(0)} = 0.1,\ l_{14,2}^{(0)} \to l_{14,4}^{(0)} = s_5,$$
$$p_{34,3}^{(0)} \to p_{34,5}^{(0)} = 0.7,\ l_{34,3}^{(0)} \to l_{34,5}^{(0)} = s_7,$$

$$p_{24,4}^{(0)} \to p_{24,1}^{(0)} = 0.7, \ l_{24,4}^{(0)} \to l_{24,1}^{(0)} = s_6,$$

$$p_{13,5}^{(0)} \to p_{13,4}^{(0)} = 0.2, \ l_{13,5}^{(0)} \to l_{13,5}^{(0)} = s_4.$$

Let $\eta = \eta + 1$, then go to Step 1. After four iterations, the CRPs terminate. Table 5 shows the detailed results. The final collective SC-FPR $P_c^{(4)} = (p_{ij,c}^{(4)}, l_{ij,c}^{(4)})_{4\times 4}$ is:

$$P_c^{(4)} = \begin{pmatrix} (0.5, s_8) & (0.42, s_5) & (0.6, s_5) & (0.48, s_5) \\ (0.58, s_5) & (0.5, s_8) & (0.37, s_6) & (0.7, s_6) \\ (0.4, s_5) & (0.63, s_6) & (0.5, s_8) & (0.3, s_7) \\ (0.52, s_5) & (0.3, s_6) & (0.7, s_7) & (0.5, s_8) \end{pmatrix}.$$

Meanwhile, the $ICI(P_k^{(4)})$ (*k=1,2,3,4,5*) of the ultimate modified individual SC-FPRs are:

$$ICI(P_1^{(4)}) = 0.7300, \ ICI(P_2^{(4)}) = 0.7135, \ ICI(P_3^{(4)}) = 0.7600,$$

$$ICI(P_4^{(4)}) = 0.7585, \ ICI(P_5^{(4)}) = 0.7665.$$

From Table 5, we find that after four iterations $GCI^{(4)} = 0.7468 > 0.7$. Thus, a soft consensus has been reached. Then, utilize Eq. (21) to compute the scores of alternatives in collective SC-FPR $P_c^{(4)} = (p_{ij,c}^{(4)}, l_{ij,c}^{(4)})_{4\times 4}$ are:

$$\tilde{s}(x_1) = 2.8631, \ \tilde{s}(x_2) = 3.3326, \ \tilde{s}(x_3) = 2.9775, \ \tilde{s}(x_4) = 3.3288.$$

Thus, the alternative ranking is $x_2 \succ x_4 \succ x_3 \succ x_1$, and x_2 is the best alternative.

Table 5. The detailed iterative process of the proposed method.

η	$w_k^{(\eta)}$	$P_c^{(\eta)}$	$GCI^{(\eta)}$	$(p_{ij,k}^{(\eta)}, l_{ij,k}^{(\eta)})$
0	0.24 0.18 0.22 0.28 0.08	$P_c^{(0)} = \begin{pmatrix} (0.5, s_8) & (0.36, s_3) & (0.46, s_4) & (0.42, s_2) \\ (0.64, s_3) & (0.5, s_8) & (0.56, s_4) & (0.55, s_3) \\ (0.54, s_4) & (0.44, s_4) & (0.5, s_8) & (0.52, s_2) \\ (0.58, s_2) & (0.45, s_3) & (0.48, s_2) & (0.5, s_8) \end{pmatrix}$	0.5361	$(p_{24,1}^{(0)} \to 0.7, l_{24,1}^{(0)} \to s_6)$ $(p_{14,2}^{(0)} \to 0.1, l_{14,2}^{(0)} \to s_5)$ $(p_{34,3}^{(0)} \to 0.3, l_{34,3}^{(0)} \to s_7)$ $(p_{24,4}^{(0)} \to 0.7, l_{24,4}^{(0)} \to s_6)$ $(p_{13,5}^{(0)} \to 0.2, l_{13,5}^{(0)} \to s_4)$
1	0.21 0.17 0.20 0.24 0.18	$P_c^{(1)} = \begin{pmatrix} (0.5, s_8) & (0.39, s_3) & (0.43, s_4) & (0.4, s_4) \\ (0.61, s_3) & (0.5, s_8) & (0.57, s_4) & (0.64, s_3) \\ (0.58, s_4) & (0.44, s_4) & (0.5, s_8) & (0.38, s_3) \\ (0.6, s_4) & (0.36, s_3) & (0.62, s_3) & (0.5, s_8) \end{pmatrix}$	0.6133	$(p_{12,1}^{(1)} \to 0.4, l_{12,1}^{(1)} \to s_5)$ $(p_{12,2}^{(1)} \to 0.4, l_{12,2}^{(1)} \to s_5)$ $(p_{24,3}^{(1)} \to 0.7, l_{24,3}^{(1)} \to s_4)$ $(p_{13,4}^{(1)} \to 0.6, l_{13,4}^{(1)} \to s_7)$ $(p_{13,5}^{(1)} \to 0.6, l_{13,5}^{(1)} \to s_7)$

2	0.20 0.17 0.20 0.24 0.19	$P_c^{(2)}=\begin{pmatrix}(0.5,s_8) & (0.42,s_5) & (0.59,s_5) & (0.4,s_4)\\(0.58,s_5) & (0.5,s_8) & (0.56,s_4) & (0.7,s_4)\\(0.41,s_5) & (0.44,s_4) & (0.5,s_8) & (0.37,s_3)\\(0.6,s_4) & (0.3,s_4) & (0.63,s_3) & (0.5,s_8)\end{pmatrix}$	0.6190	$(p_{34,1}^{(2)}\to 0.3, l_{34,1}^{(2)}\to s_7)$ $(p_{34,2}^{(2)}\to 0.3, l_{34,2}^{(2)}\to s_7)$ $(p_{14,3}^{(2)}\to 0.4, l_{14,3}^{(2)}\to s_6)$ $(p_{14,4}^{(2)}\to 0.7, l_{14,4}^{(2)}\to s_8)$ $(p_{24,5}^{(2)}\to 0.7, l_{24,5}^{(2)}\to s_5)$
3	0.20 0.17 0.20 0.24 0.19	$P_c^{(3)}=\begin{pmatrix}(0.5,s_8) & (0.42,s_5) & (0.59,s_5) & (0.48,s_5)\\(0.58,s_5) & (0.5,s_8) & (0.56,s_4) & (0.7,s_4)\\(0.41,s_5) & (0.44,s_4) & (0.5,s_8) & (0.3,s_7)\\(0.52,s_5) & (0.3,s_4) & (0.7,s_7) & (0.5,s_8)\end{pmatrix}$	0.6876	$(p_{23,1}^{(3)}\to 0.2, l_{23,1}^{(3)}\to s_6)$ $(p_{23,2}^{(3)}\to 0.2, l_{23,2}^{(3)}\to s_6)$ $(p_{24,3}^{(3)}\to 0.7, l_{24,3}^{(3)}\to s_6)$ $(p_{12,4}^{(3)}\to 0.4, l_{12,4}^{(3)}\to s_5)$ $(p_{23,5}^{(3)}\to 0.6, l_{23,5}^{(3)}\to s_6)$
4	0.20 0.18 0.20 0.24 0.18	$P_c^{(4)}=\begin{pmatrix}(0.5,s_8) & (0.42,s_5) & (0.6,s_5) & (0.48,s_5)\\(0.58,s_5) & (0.5,s_8) & (0.37,s_6) & (0.7,s_6)\\(0.4,s_5) & (0.63,s_6) & (0.5,s_8) & (0.3,s_7)\\(0.52,s_5) & (0.3,s_6) & (0.7,s_7) & (0.5,s_8)\end{pmatrix}$	0.7468	

5. Comparative analyses and discussions

In this section, the comparative analyses are conducted to show the reasonableness and reliableness of the proposed approach. It includes the advantages of the presented consensus model, the discussion of experts' self-confidence effect on the alternative ranking, and the analysis of the influence of SC-FPRs and FPRs on consensus efficiency.

(1) The advantages of the presented consensus model

In real SNGDM cases, if an expert owns absolute self-confidence in her/his judgment, then a compete preference relation can be obtained. As a result, an FPR is actually a particular case for SC-FPR. That is, experts' self-confidence levels are $l_{ij}=s_g$ for $\forall i,j\in N$ [20]. Here, we compare the consensus improvements provided in Wu and Xu [52] with the proposed consensus model in this paper. To do this, Step 2 and Step 5 in Algorithm 1 are replaced by Step 2A and Step 5A, respectively.

Step 2A. Let $\eta=0$. Weights generation without considering the values of self-confidence degrees of experts are computed by:

$$w_k=\frac{TD(e_k)}{\sum_{k=1}^{m}TD(e_k)}, \tag{22}$$

Step 5A. Construct a new SC-FPR $P_k^{(\eta+1)}=(p_{ij,k}^{(\eta+1)},l_{ij,k}^{(\eta+1)})$ of expert e_k by using the

Eq. (17) in [52], then

$$p_{ij,k}^{(\eta+1)} = \begin{cases} \xi\, p_{ij,k}^{(\eta)} + (1-\xi)\, p_{ij,c}^{(\eta)}, \text{if } i = i_\tau, j = j_\tau \\ p_{ij,k}^{(\eta)}, \text{otherwise} \end{cases}, \tag{23}$$

and $l_{ij,k}^{(\eta+1)} = l_{ij,k}^{(\eta)}$, for $\forall i, j \in N$, and $\forall k \in M$.

Generally, in the CRPs, the experts' original information should be distorted as little as possible while an acceptable consensus is reached. In order to measure the information distortion, the adjustment degree (AD) and adjustment ratio (AR) are adopted. To do that, let $P_k^{(0)} = (p_{ij,k}^{(0)}, l_{ij,k}^{(0)})$ and $P_k^{*} = (p_{ij,k}^{*}, l_{ij,k}^{*})$ ($i, j \in N$) be the original SC-FPR and the final adjusted SC-FPR of e_k ($k \in M$) respectively. Then, the *AD* and *AR* for the P_k are:

$$AD_k = \rho \frac{\sum_{i,j=1}^{n} | p_{ij,k}^{(0)} - p_{ij,k}^{*} |}{\sum_{i,j=1}^{n} p_{ij,k}^{(0)}} + (1-\rho) \frac{\sum_{i,j=1}^{n} | \Delta^{-1}(l_{ij,k}^{(0)}) - \Delta^{-1}(l_{ij,k}^{*}) |}{\sum_{i,j=1}^{n} \Delta^{-1}(l_{ij,k}^{(0)})} \tag{24}$$

$$AR_k = \rho \frac{\sum_{i,j=1}^{n} f_{ij,k}}{n^2} + (1-\rho) \frac{\sum_{i,j=1}^{n} \tilde{f}_{ij,k}}{n^2}, \tag{25}$$

where $f_{ij,k} = \begin{cases} 0, & p_{ij,k}^{(0)} = p_{ij,k}^{*} \\ 1, & \text{otherwise} \end{cases}$, $\tilde{f}_{ij,k} = \begin{cases} 0, & l_{ij,k}^{(0)} = l_{ij,k}^{*} \\ 1, & \text{otherwise} \end{cases}$, and $\rho \in [0,1]$.

Remark 3. The *AD* denotes the difference degree between the initial and the revised SC-FPR. The smaller the value of the *AD*, the more the initial information is reserved. The *AR* means the elements adjustment ratio in the expert's initial information. The smaller the value of the *AR*, the fewer the elements in the initial information of expert are adjusted. In addition, this paper supposes that the preference values and the self-confidence are of equal importance, thus the $\rho = 0.5$.

If using Step 2A to generate the weights, we have

$$w_1 = 0.18,\ w_2 = 0.17,\ w_3 = 0.21,\ w_4 = 0.28,\ w_5 = 0.16.$$

Afterwards, utilize Step 5A to adjust the SC-FPRs, we can obtain the GCIs in Table 6 ($\xi = 0.5$).

Table 6. The *GCIs* of Wu and Xu [52]'s method (ξ=0.5)

η	$GCI^{(\eta)}$	η	$GCI^{(\eta)}$
0	0.5313	4	0.6075
1	0.5723	5	0.6099
2	0.5925	6	0.6116
3	0.6025		

In Table 6, it is clear that the $GCI^{(6)} < \delta = 0.7$. It denotes a soft consensus is not achieved using Eq. (17) provided by Wu and Xu [52]'s method when $\eta^* = 6$.

Suppose the acceptable consensus threshold $\delta = 0.6116$, that is, the acceptable consensus level is achieved by Wu and Xu [52]'s method. The *AD* and *AR* are depicted in Table 7.

Table 7. The *AD* and *AR* for Wu and Xu [52]'s method

	e_1	e_2	e_3	e_4	e_5
AD	0.14	0.13	0.16	0.10	0.09
AR	0.38	0.38	0.38	0.38	0.38

From Table 5, if the proposed method is performed, the consensus is reached in the first iteration ($\eta = 1$) when $\delta = 0.6116$. Moreover, Table 8 shows the corresponding *AD* and *AR* of the proposed method.

Table 8. The *AD* and *AR* for the proposed consensus model

	e_1	e_2	e_3	e_4	e_5
AD	0.03	0.05	0.13	0.06	0.01
AR	0.13	0.13	0.13	0.13	0.06

Comparing the values in Tables 7 and 8, it is obviously that all the values AD_k and AR_k in Table 8 are smaller than that are Table 7, denoting that a smaller information distortion than that by Wu and Xu [52]'s method. It shows that the proposed method has better performance than Wu and Xu [52]'s method in the above two criteria.

Additionally, different from the existing method of importance degrees determination of experts in SNGDM problems, we present a dynamic weight updating based on the changing of experts' self-confidence in the CRPs for SNGDM with SC-FPRs. Fig. 5 shows the trends of the *GCIs* with the changing of experts' weights.

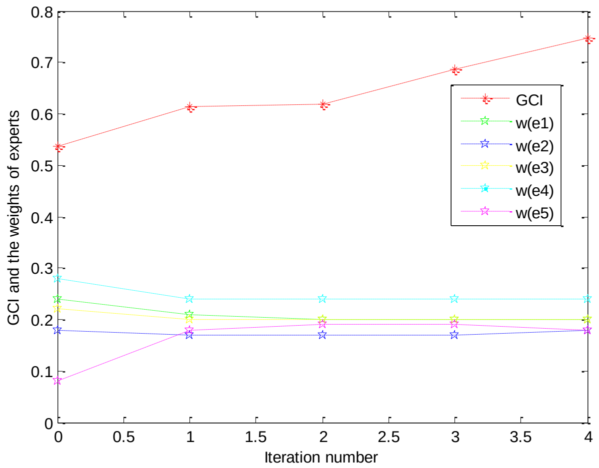

Fig. 5. The GCIs of SNGDM in SC-FPRs with the changing of experts' weights.

Clearly, Fig. 5 shows that the *GCI* is improved by the changing of expert's weights. Therefore, it signifies that the proposed dynamic importance degree generation in the CRPs is effective for SNGDM with SC-FPRs.

(2) The discussion of experts' self-confidence effect on the alternative ranking

Suppose the consensus threshold $\delta = 0.6116$, the final alternative ranking for Wu and Xu [52]'s method is depicted in Table 9.

Table 9. The results of alternative ranking for Wu and Xu [52]'s method (i=1,2,3,4; δ=0.6116)

Collective SC-FPR	$SCS(x_i)$	Alternative ranking
$P_c^{(6)} = \begin{pmatrix} (0.5, s_8) & (0.39, s_3) & (0.43, s_4) & (0.4, s_2) \\ (0.61, s_3) & (0.5, s_8) & (0.55, s_4) & (0.56, s_3) \\ (0.57, s_4) & (0.45, s_4) & (0.5, s_8) & (0.5, s_2) \\ (0.6, s_2) & (0.44, s_3) & (0.51, s_2) & (0.5, s_8) \end{pmatrix}$	1.925 2.435 2.261 1.881	$x_2 \succ x_3 \succ x_1 \succ x_4$

Similarly, Table 10 shows the alternative ranking of the proposed method in our research.

Table 10. Alternative ranking of the proposed method in this research (i=1,2,3,4; δ=0.6116)

Collective SC-FPR	$SCS(x_i)$	Alternative ranking of collective
$P_c^{(1)} = \begin{pmatrix} (0.5, s_8) & (0.39, s_3) & (0.43, s_4) & (0.4, s_4) \\ (0.61, s_3) & (0.5, s_8) & (0.57, s_4) & (0.64, s_3) \\ (0.58, s_4) & (0.44, s_4) & (0.5, s_8) & (0.38, s_3) \\ (0.6, s_4) & (0.36, s_3) & (0.62, s_3) & (0.5, s_8) \end{pmatrix}$	2.132 2.518 2.312 2.348	$x_2 \succ x_4 \succ x_3 \succ x_1$

Obviously, the alternative rankings in Tables 9 and 10 are different. It denotes that the experts' self-confidence levels will influence the alternative ranking in SNGDM, and then will lead to different rankings.

(3) Analysis of the influence of SC-FPRs and FPRs on consensus efficiency

To further verify the effectiveness and necessity of considering multiple self-confidences of experts in SNGDM scenarios, an analysis of the influence of SC-FPRs and FPRs on consensus efficiency is provided. As aforementioned, an FPR is actually a particular case of the SC-FPR. From the above analysis, we have obtained the consensus iterations under the conditions of SC-FPRs and FPRs respectively. Based on the detailed results which are shown in Tables 5 and 6, the gradient changes of the GCIs for SC-FPRs and FPRs in Section 4 are depicted in Fig.6.

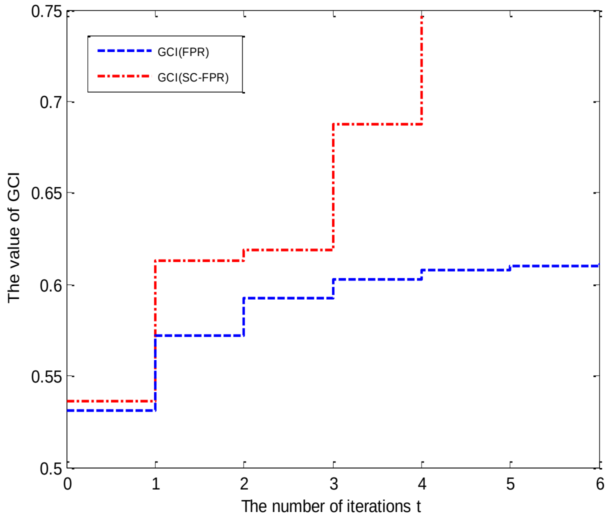

Fig. 6. The gradient changes of the GCI for SC-FPRs and FPRs

From Fig.6, it is obviously that the consensus variation gradient for SC-FPRs is significantly larger than that for FPRs. Moreover, the GCI of SC-FPRs will reach to a higher degree than that of FPRs. It means that in real SNGDM cases, considering the multiple self-confidence of experts is more conducive to the reaching of soft consensus. In other words, compared with the FPRs, allowing experts using SC-FPRs to express their assessment information is closer to real SNGDM situations. As a result, the quality and efficiency of the decision-making is effectively improved. This is also consistent with the conclusions in [24]. Additionally, Fig. 6 also can reflect that in real SNGDM cases, considering both the external trust and internal self-confidence of experts can effectively mobilize their enthusiasm in consensus negotiation, and then improve the consensus efficiency. Similarly, compared with incomplete fuzzy preference relations, we should draw the same conclusions as in [24]. That is, the

SC-FPRs is more conducive to the improvement of the quality and efficiency of SNGDM in most cases.

6. Concluding remarks

In this research, we focus on the self-confidence-based CRPs with dynamic importance degree of experts and trust-based feedback mechanism for SNGDM with SC-FPRs. All experts are allowed to use SC-FPRs to express their preference information. In the CRPs, experts are suggested to dynamically adjust their self-confidence levels while revising preference values. And then all experts reached an acceptable consensus level. With the changing of experts' self-confidence, a dynamic importance degree determination is utilized to assign their weights in the CRPs. Some comparisons with the existing methods are offered to demonstrate the effectiveness of the proposed method.

In some real SNGDM cases, the experts may have complex decision behaviors, such as non-cooperative, overconfidence or self-concern [27, 53, 54]. It is very interesting in any future research to explore the proposed consensus model considering these complex behaviors in SNGDM problems.

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