Influence of ICHTC correlations on the thermal characterization of façades using the quantitative internal infrared thermography method

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Abstract:

The thermal characterization of façades of the existing building stock is essential to establish optimal energy conservation measures. There are different methods to characterize thermal properties of façades. The quantitative internal infrared thermography method is among those most developed. Given the existing differences in the scientific literature among the proposals of the method, this study analysed the influence of the internal convective heat transfer coefficient (ICHTC). In total, 25 correlations of temperature differences (temperature of the wall and internal air temperature) were analysed, as well as 20 correlations of dimensionless numbers. To do this, an experimental campaign was performed in 3 façades belonging to the most representative building periods of the building stock in Spain. First, a cluster analysis was carried out to determine similarities among the equations analysed, using the Ward method as an agglomerative hierarchical method and the Euclidean distance as an association measurement. In total, 12 and 8 groups were obtained for correlations of temperature difference and of dimensionless numbers, respectively. Afterwards, results associated with each approach were obtained. These results showed that a better adjustment was obtained for correlations of dimensionless numbers by using the approach of convection and radiation, with an average value of representative results higher than 80%.

Keywords:

Quantitative internal infrared thermography method; internal convective heat transfer coefficient (ICHTC); façades; *U*-value.

Nomenclature

Symbols

Gr	The Grashof number [dimensionless]
h_i	Internal convective heat transfer coefficient [W/(m²·K)]
k	Thermal conductivity of the air [W/(m·K)]
L	Height of the wall [m]
Nu	The Nusselt number [dimensionless]
Pr	The Prandtl number [dimensionless]
q	Conductive heat flux [W/m²]
q_c	Convective heat flux [W/m²]
q_r	Radiative heat flux [W/m²]
R^2	Correlation coefficient [dimensionless]
Ra	The Rayleigh number [dimensionless]
$R_{s,i}$	Internal surface thermal resistance [(m²·K)/W]
$R_{s,o}$	External surface thermal resistance [(m²·K)/W]
R_w	Thermal resistance of the wall [(m²·K)/W]
T_i	Internal air temperature [K]
T_o	External air temperature [K]
T_r	Reflected temperature [K]
T_w	Temperature of the wall [K]
U	Thermal transmittance [W/(m²·K)]
U_{6946}	Thermal transmittance obtained by ISO 6946 [W/(m²·K)]
U_{HFM}	Thermal transmittance obtained by ISO 9869 [W/(m²·K)]
U_{IRT}	Thermal transmittance obtained by quantitative internal infrared thermography method [W/(m²·K)]
$U_{Measured}$	Measured thermal transmittance [W/(m²·K)]
$U_{Reference}$	Reference thermal transmittance [W/(m²·K)]

v	Viscosity of the air [$\mathrm{m^2/s}$]
Greek letters	
β	Volumetric temperature expansion coefficient [$\mathrm{K^{-1}}$]
ΔT_{wi}	Difference between T_w and T_i [K]
ε	Emissivity of the wall [dimensionless]
σ	Stefan-Boltzmann constant [$5{,}67\cdot10^{-8}$ W/($\mathrm{m^2\cdot K^4}$)]
Abbreviations	
ICHTC	Internal convective heat transfer coefficient
IRT	Infrared thermography
NBE-CT-79	Basic Building Norm about the Thermal Conditions in Buildings (repealed in 2006)
CTE DB HE	Spanish Technical Building Code

1. Introduction

The building sector is among those with a greater impact on climate change. Approximately, 40% of the energy consumption is attributed to this sector [1]. In quantified data, the building sector was responsible for a world consumption of 23.7 PWh in 2010, with an estimation of 38.4 PWh for 2040 [2]. Furthermore, this sector generates more than 30% of pollutant emissions in the atmosphere [3]. Its deficient behaviour forces therefore to perform in the existing building stock, especially due to predicted future scenarios of climate change [4].

To correct this situation, the European Union has developed different directives with the objective of establishing strategies of energy improvements on the existing buildings in Europe [3], and consequently reducing by 90% pollutant emissions in the atmosphere [5]. With all this, the goal is to achieve a low carbon economy for 2050 [6].

Among the different properties which characterize the thermal performance of the existing buildings, the thermal transmittance (U-value) of their façades is the most influential [7]. The thermal behaviour of these façades is deficient due to their design and to the ageing of materials [8]. Determining correctly the U-value is fundamental to not overestimate the energy consumption [9] as well as to select the energy conservation measures required and to reduce payback periods of measurements to be carried out [10]. Moreover, it is estimated that the improvement of the thermal performance of façades would reduce between 10 and 45% the heat losses of buildings [11].

To determine the thermal transmittance, there are different methods, both theoretical and experimental. The method from ISO 6946 [12] is the theoretical-estimated method with a greater implementation in the field of energy certification [13]. For its application, it is important to know the thickness and thermal properties of the layers of a wall, and it is not necessary to monitor it. Despite its easiness of use, the method presents a high level of uncertainty when the composition of the wall is not determined correctly [14]. Using reliable technical documentation or making endoscopies allow to reduce the uncertainty [14], although the difficulty to have that documentation or damages generated can force to reject the use of the method. Other aspects that advise against using the method are [15,16]: (i) thermal heterogeneities and moisture pathologies, (ii) variation of the thermal conductivity due to the ageing of materials, and (iii) variation of the thermal conductivity value due to environmental conditions.

The method from ISO 9869-1 (also known as heat flow meter method) [17] allows to characterize the *U*-value of a wall by monitoring the variables required for each method. Although valid results can be obtained, there are some limitations. This method needs a high temperature gradient (higher than 10 °C) [18] and a long test duration (more than 3 days) [19]. In addition, other aspects, such as the possible distortion in the thermal behaviour of the wall [18] and the error associated with the heat flux plate [20], can obtain non-representative results.

Given these difficulties, quantitative methods through infrared thermography (IRT) can be an opportunity to characterize the thermal transmittance of façades. IRT was a technique used for the qualitative assessment of thermal heterogeneities of the envelope (e.g. thermal bridges) [21]. However, in recent years several studies have analysed the application of the infrared thermography to characterize the thermal transmittance of a wall, and there are two different approaches: (i) measurement methods from the exterior proposed by Albatici and Tonelli [22] and by Dall'O' et al. [23]; and (ii) measurement methods from the interior proposed by Madding [24], by Fokaides and Kalogirou [25], and by Tejedor et al. [26]. These are passive IRT techniques because they measure temperatures under normal thermal conditions [27]. In addition, they are non-destructive methods which allow to carry out quick measurements, approximately lasting 2 h [28]. Among both kinds of methods, external methods present greater limitations: (i) the main limitation is the great influence that external environmental conditions generate in the representation of results. In this sense, deviations of 50% in the measurement of wind speed as well as of external temperatures have associated errors of 9% and 50%, respectively [28]; and (ii) the external ambient temperature reflected by the wall can present variations up to 20 °C due to the reflection of elements with an unknown thermal state [25].

Concerning internal methods, these limitations do not exist. On the one hand, it is possible a greater control of environmental conditions in which tests are performed [29] and, on the other hand, the reflected ambient temperature can almost be constant [25]. However, despite these advantages of internal methods, the huge variety of the existing proposals in the scientific literature can make the choice of the proposal to be used difficult. This is due to the fact that the main difference in these research studies is the internal convective heat transfer coefficient (ICHTC) chosen, using both correlations of temperature differences [24] and of dimensionless numbers [26]. In the scientific literature, there is only a study analysing the variability that the convective component causes in the heat transfer of the wall. Evangelisti et al. [30] analysed the convective heat flux obtained from 5 ICHTCs correlations as well as its influence on the thermal transmittance. Depending on the ICHTC used, differences between 8.6 and 42.8% were reached. However, the number of existing correlations in the scientific literature is greater than those used in this study, as reflected in reviews done by Khalifa [31], by Peeters et al. [32], and by Obyn and Van Moeseke [33]. Moreover, correlations depending on dimensionless numbers, such as that used in the IRT method of Tejedor et al. [26], were not analysed.

For this reason, the aim of this work is to analyse those ICHTCs correlations that are better adapted for the internal quantitative IRT in walls. This paper is based on the existing theoretical approaches on quantitative methods of IRT, which are analysed by using 25 correlations of temperature differences and 20 correlations of dimensionless numbers. For this purpose, three walls belonging to significant building periods in Spain were monitored, following criteria and recommendations established by several research studies. The type of wall analysed is double leaf brick walls, with air gap in turn with or without thermal insulation according to the building period. Afterwards, a cluster analysis was carried out to group ICHTCs correlations as well as to obtain results of U-value for each test. Results were compared with the reference values obtained from ISO 6946 and ISO 9869-1.

2. Assessment methods of thermal transmittance using IRT from the interior

According to ISO 9869-1, the U-value of a wall is determined by the conductive heat flux (q) and by the difference of internal (T_i) and external air temperatures (T_o) (Eq. (1)). The heat transfer by conduction in steady state between a wall and the ambient can be considered as the sum of the convective (q_c) and radiative (q_r) contributions. By using this assumption (Eq. (2)), a new expression can be obtained to determine the thermal transmittance used by IRT methods (Eq. (3)).

$$U = \frac{q}{T_i - T_o} \tag{1}$$

$$q = q_c + q_r \tag{2}$$

$$U = \frac{q_c + q_r}{T_i - T_o} \tag{3}$$

Based on the formulation of Eq. (3), several methods have arisen in recent years to determine the thermal transmittance from the interior of the building. Given that the heat transfer by convection can be expressed by means of the relationship between ICHTC (h_i) and temperature difference of the wall (T_w) and of the internal air temperature (T_i) (Eq. (4)), the main difference among the three existing approaches is based on the ICHTC used, using both correlations depending on temperature differences and dimensionless numbers.

$$q_c = h_i \Delta T_{wi} \tag{4}$$

The first approach was made by Madding [24] in 2008. In his research, this author proposed to calculate the U-value by using the radiative contribution expressed as a linearization of the Stephan-Boltzmann law and the internal convective contribution (see Eq. (5)). The average temperature of the radiative contribution is the mean between the surface temperature of the wall and the reflected temperature [34].

$$U = \frac{h_i \cdot (\Delta T_{wi}) + 4 \cdot \varepsilon \cdot \sigma \cdot \left(\frac{T_w + T_r}{2}\right)^3 \cdot (T_w - T_r)}{T_i - T_o} \tag{5}$$

Where ε [dimensionless] is the emissivity of the wall, σ is the Stefan-Boltzmann constant [$5{,}67 \cdot 10^{-8}$ W/(m^2·K^4)], and T_r [K] is the reflected temperature. For h_i, Madding recommended the use of convective correlations of Holman [35] and of Earle [36] (see Table 1), which some of them have a characteristic length L [m] (the height of the wall).

Table 1

ICHTCs used in the approach of Madding.

ICHTC		Remark	Reference
$h_i = 1.42 \cdot \left(\frac{\Delta T_{wi}}{L}\right)^{0.25}$	(6)	Laminar flow	Holman [35]
$h_i = 1.31 \cdot (\Delta T_{wi})^{0.33}$	(7)	Turbulent flow	Holman [35]
$h_i = 1.31 \cdot \left(\frac{\Delta T_{wi}}{L}\right)^{0.25}$	(8)	Laminar flow	Earle [36]
$h_i = 1.8 \cdot (\Delta T_{wi})^{0.25}$	(9)	Turbulent flow	Earle [36]

A similar proposal was made by Fokaides and Kalogirou [25] (Eq. (10)). Differences with respect to the method of Madding were both the radiative coefficient used (instead of using the third power of the average of internal surface and reflected temperatures, this third power is only applied to the surface temperature) and the ICHTC selected. For ICHTC, these authors used the tabulated value from ISO 6946 for walls (2.5 W/(m^2·K)).

$$U = \frac{h_i \cdot (\Delta T_{wi}) + 4 \cdot \varepsilon \cdot \sigma \cdot {T_w}^3 \cdot (T_w - T_r)}{T_i - T_o} \tag{10}$$

Recently, a new proposal was suggested for the method from the interior. Tejedor et al. [26,29] used the Nusselt number proposed by Churchill and Chu [37] for vertical surfaces in laminar and turbulent flow regime (the Rayleigh number between 10^{-1} and 10^{12}) (Eqs. (11) and (12)) to suggest a new equation (Eq. (13)).

$$h_i = \frac{Nu \cdot k}{L} \tag{11}$$

$$Nu = \left\{0.825 + \frac{0.387 \cdot Ra^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2 \tag{12}$$

$$U = \frac{\frac{k \cdot \left\{0.825 + \frac{0.387 \cdot Ra^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2}{L} \cdot (\Delta T_{wi}) + \varepsilon \cdot \sigma \cdot (T_r^4 - T_w^4)}{T_i - T_o} \tag{13}$$

Where Nu [dimensionless] is the Nusselt number, Ra [dimensionless] is the Rayleigh number, Pr [dimensionless] is the Prandtl number, and k is the thermal conductivity of the air [W/(m·K)].

In the same study, Tejedor et al. [26] proposed a simplification of Eq. (13) by considering that, in typical test conditions, the internal temperature oscillates between 20 and 25 °C, and therefore the Prandtl number can be established as fixed value (0.73 for dry air under atmospheric pressure at that temperature). This simplification led to the following equation:

$$U = \frac{\frac{k \cdot \left\{0.825 + 0.325 \cdot Ra^{1/6}\right\}^2}{L} \cdot (\Delta T_{wi}) + \varepsilon \cdot \sigma \cdot (T_r^4 - T_w^4)}{T_i - T_o} \tag{14}$$

To apply methods correctly, it is necessary to have specific environmental conditions to guarantee the suitability of results. In this sense, a high temperature differential allows to guarantee a low dispersion in thermal transmittance results [38]. In a recent study, Tejedor et al. [29] determined that the optimal temperature difference between the internal and external air was between 7 and 16 °C [29]. In addition, other environmental aspects should be guaranteed during the performance of the test [24–26,29]: (i) the wind speed should be lower than 1 m/s; (ii) the ideal element under study should be a wall facing north; and (iii) no rainfall before and during the test.

3. ICHTC correlations

Despite the wide variety of methods to determine the thermal transmittance from the interior, there is not a study analysing ICHTC correlations from the interior. In scientific literature, there are various ICHTC correlations for vertical surfaces (walls). Given that IRT methods from the interior use both correlations depending on temperature differences and dimensionless numbers, a detailed compilation of different ICHTCs for the walls used in this study is carried out in subsequent subsections.

3.1. ICHTC correlations depending on temperature differences

In the scientific literature, there are many expressions of correlations depending on temperature differences, some of them compiled by several authors in reviews [31–33,39]. Table 2 includes a list of the different ICHTC correlations depending on temperature differences which were analysed in this paper. The approach used for each correlation varies depending on the research. There are certain correlations based on the similarity that the heat transfer by convection on a vertical surface of a room is the same as the heat transfer of an insulating flat plate. Also, there are correlations which were determined by means of experiments in closed spaces with certain HVAC systems. It must be noted that these ICHTCs do not take into account the effect generated by flank elements to the vertical surface, by gaps or by the room itself [33]. Thus, the use of a fixed ICHTC for all case studies can cause variations in thermal transmittance results.

Table 2

Approximations of ICHTC correlations of temperature differences.

ICHTC		Remarks	Reference
$h_i = 3.05 \cdot (\Delta T_{wi})^{0.12}$	(15)	-	Wilkes and Peterson [40]
$h_i = 2.5 \cdot (\Delta T_{wi})^{0.25}$	(16)	Quoted from the work by Hottinger [41]	Giesecke [42]
$h_i = 1.368 \cdot \left(\frac{\Delta T_{wi}}{L}\right)^{0.25}$	(17)	Laminar flow regime	Min et al. [43]
$h_i = 1.973 \cdot (\Delta T_{wi})^{0.25}$	(18)	Turbulent flow regime	Min et al. [43]
$h_i = 1.664 \cdot (\Delta T_{wi})^{0.27}$	(19)	Quoted from the work by Carroll [44]	Min et al. [43]
$h_i = 1.776 \cdot (\Delta T_{wi})^{0.25}$	(20)	Quoted from the work by McAdams [45]	Min et al. [43]
$h_i = 1.517 \cdot (\Delta T_{wi})^{0.33}$	(21)	Quoted from the work by King [46]	Min et al. [43]
$h_i = \frac{0.0257}{L} \cdot \left(0.825 + 7.01 \cdot (\Delta T_{wi})^{1/6} \cdot L^{3/6}\right)^2$	(22)	-	Churchill and Chu [37]
$h_i = (0.134 \cdot (L)^{-0.5} + 1.11 \cdot (\Delta T_{wi})^{0.17})^2$	(23)	Simplification of the correlation of Churchill and Chu (Eq. (22))	ESDU [47]
$h_i = \left\{\left[1.5 \cdot \left(\frac{\Delta T_{wi}}{L}\right)^{1/4}\right]^6 + \left[1.23 \cdot (\Delta T_{wi})^{1/3}\right]^6\right\}^{1/6}$	(24)	-	Alamdari and Hammond [48]
$h_i = 3.08 \cdot (\Delta T_{wi})^{0.25}$	(25)	-	Li et al. [49]
$h_i = 2.88 \cdot (\Delta T_{wi})^{0.25}$	(26)	-	Li et al. [49]
$h_i = 1.42 \cdot \left(\frac{\Delta T_{wi}}{L}\right)^{1/4}$	(27)	-	Holman [35]
$h_i = 1.98 \cdot (\Delta T_{wi})^{0.32}$	(28)	Wall close to a radiator.	Khalifa and Marshall [50]
$h_i = 2.3 \cdot (\Delta T_{wi})^{0.24}$	(29)	Wall with a radiator under the window.	Khalifa and Marshall [50]
$h_i = 2.92 \cdot (\Delta T_{wi})^{0.25}$	(30)	Wall opposite a blow heater	Khalifa and Marshall [50]
$h_i = 2.03 \cdot (\Delta T_{wi})^{0.14}$	(31)	Wall of great surface with insulation	Khalifa and Marshall [50]
$h_i = 1.57 \cdot (\Delta T_{wi})^{0.31}$	(32)	-	Hatton and Awbi [51]
$h_i = \frac{1.823}{L^{0.121}} \cdot (\Delta T_{wi})^{0.293}$	(33)	-	Awbi and Hatton [52]
$h_i = 1.332 \cdot \left(\frac{\Delta T_{wi}}{L}\right)^{1/4}$	(34)	Laminar flow regime	Fohanno and Polidori [53]

3.2. ICHTC correlations depending on dimensionless numbers

With respect to ICHTC correlations using dimensionless numbers, there is a wide variety of correlations apart from that proposed by Churchill and Chu (Eq. (12)). The Rayleigh number, the Prandtl number or the Grashof number (Gr) are used in these other correlations. Table 3 includes correlations selected for this study due to their suitability for the type of element analysed (walls).

Table 3

Approximations of ICHTC correlations of dimensionless numbers.

ICHTC		Remarks	Reference
$h_i = k \cdot \frac{0.555 \cdot (Ra)^{0.25}}{L}$	(35)	$10^3 < Ra < 10^8$	Jakob [54]
$h_i = k \cdot \frac{0.129 \cdot (Ra)^{0.33}}{L}$	(36)	$10^8 < Ra < 10^{12}$	Jakob [54]
$h_i = k \cdot \frac{0.56 \cdot (Ra)^{0.25}}{L}$	(37)	Laminar flow regime	Fishenden and Saunders [55]
$h_i = k \cdot \frac{0.12 \cdot (Ra)^{0.33}}{L}$	(38)	Turbulent flow regime	Fishenden and Saunders [55]
$h_i = k \cdot \frac{0.548 \cdot (Ra)^{0.25}}{L}$	(39)	Laminar flow regime	McAdams [45]
$h_i = k \cdot \frac{0.52 \cdot (Ra)^{0.25}}{L}$	(40)	$Ra < 3 \cdot 10^8$	McAdams [45]
$h_i = k \cdot \frac{0.59 \cdot (Ra)^{0.25}}{L}$	(41)	$10^4 < Ra < 10^9$	McAdams [45]
$h_i = k \cdot \frac{0.13 \cdot (Ra)^{0.33}}{L}$	(42)	$2 \cdot 10^9 < Ra < 10^{12}$	McAdams [45]
$h_i = k \cdot \frac{0.48 \cdot (Gr)^{0.25}}{L}$	(43)	Laminar flow regime	CIBSE [56]
$h_i = k \cdot \frac{0.119 \cdot (Gr)^{0.33}}{L}$	(44)	Turbulent flow regime	CIBSE [56]
$h_i = k \cdot \frac{0.516 \cdot (Ra)^{0.25}}{L}$	(45)	Laminar flow regime	Wong [57]
$h_i = k \cdot \frac{0.021 \cdot (Ra)^{0.25}}{L}$	(46)	$10^{10} < Ra < 10^{12}$	Wong [57]
$h_i = k \cdot \frac{0.555 \cdot (Ra)^{0.25}}{L}$	(47)	Laminar flow regime	Welty [58]
$h_i = k \cdot \frac{0.021 \cdot (Ra)^{0.40}}{L}$	(48)	Turbulent flow regime	Welty [58]
$h_i = k \cdot \frac{0.508 \cdot \frac{Pr^{0.5} \cdot Gr^{0.25}}{(0.952 + Pr)^{0.25}}}{L}$	(49)	Laminar flow regime	Welty [58]
$h_i = k \cdot \frac{0.678 \cdot \frac{Pr^{0.5} \cdot Gr^{0.25}}{(0.952 + Pr)^{0.25}}}{L}$	(50)	Average laminar flow regime	Welty [58]
$h_i = k \cdot \frac{0.10 \cdot (Ra)^{0.33}}{L}$	(51)	-	Holman [35]
$h_i = k \cdot \frac{0.54 \cdot (Ra)^{0.25}}{L}$	(52)	$1.15 \cdot 10^5 < Ra < 2 \cdot 10^9$	Al-Arabi and Sakr [59]

4. Methodology and experimental campaign

The methodology used in this research consisted of monitoring walls belonging to representative building periods of the building stock in Spain. By means of this monitoring, similarities of ICHTCs were determined by the cluster analysis, and then both thermal transmittance results were obtained and the most adequate approach was determined. For this purpose, the criterion indicated in Section 7.3 from ISO 9869-1 was used [17], considering as valid results those with a deviation of less than 20% between the measured value and the reference value. Fig 1 represents the flow-chart of this study.

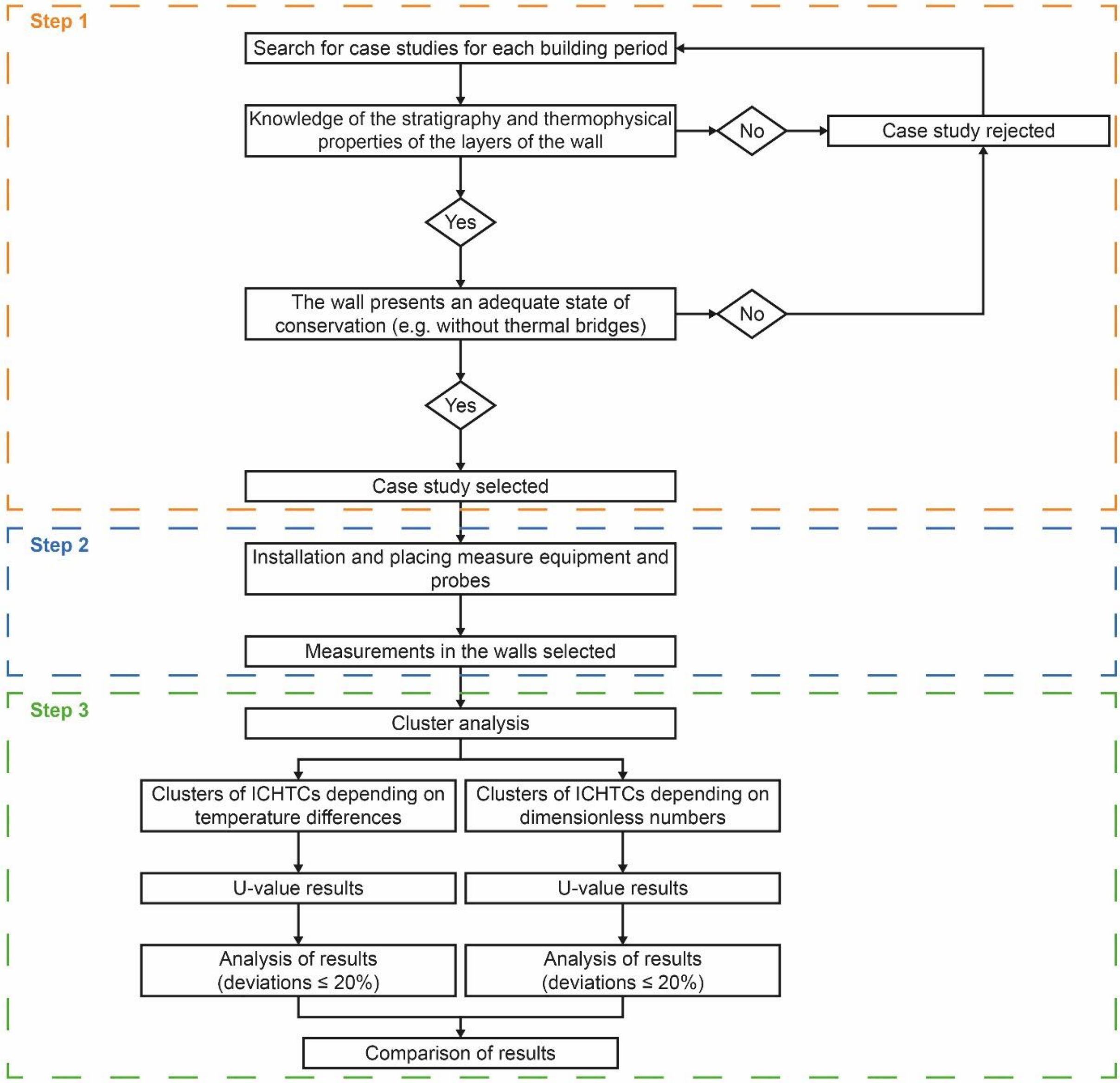


Fig. 1. Flow-chart of this study.

4.1. IRT tests

An experimental campaign was performed in three façades facing north and located in the city of Seville (Csa climate zone according to Köppen-Geiger classification [60]). For selecting these façades, it was guaranteed the consultation of technical documentation which allowed to determine accurately their constructive composition. In addition, the areas analysed were not near the edges of changes in the façade. Likewise, their state of conservation was assessed, as well as the possible presence of moistures by means of qualitative analysis of internal and external surfaces using IRT. Walls belonged to three typical building periods in Spain: (i) Wall A, which belonged to the period before the standard NBE-CT-79 [61], characterized by the construction of double leaf brick walls with air gap and without thermal insulation [62]; (ii)

Wall B, which belonged to the period between the NBE-CT-79 [61] and the CTE-DB-HE [63], characterized by the incorporation of insulating material in building solutions (thicknesses of insulation lower than 3 cm [64]); and (iii) Wall C, after to CTE-DB-HE [63], characterized by very high thermal resistance values (the wall has an insulating material with a lower thermal conductivity and a greater thickness than that in Wall B). Materials, dimensions and thermophysical properties of each wall are represented in Fig. 2. Thermal conductivity values were obtained from the Constructive elements catalogue of the CTE [65]. It is important to highlight that the insulating material was polyurethane (PUR) foam, which had different thermal conductivity values depending on its application in both Walls B and C: PUR foam blown with CO_2 (Wall B), and PUR foam blown with hydrofluorocarbon (Wall C). In addition, it is a typology of wall widely used in other countries, such as Italy [66] or United Kingdom [67], so the importance of analysing these walls is extendable to other regions.

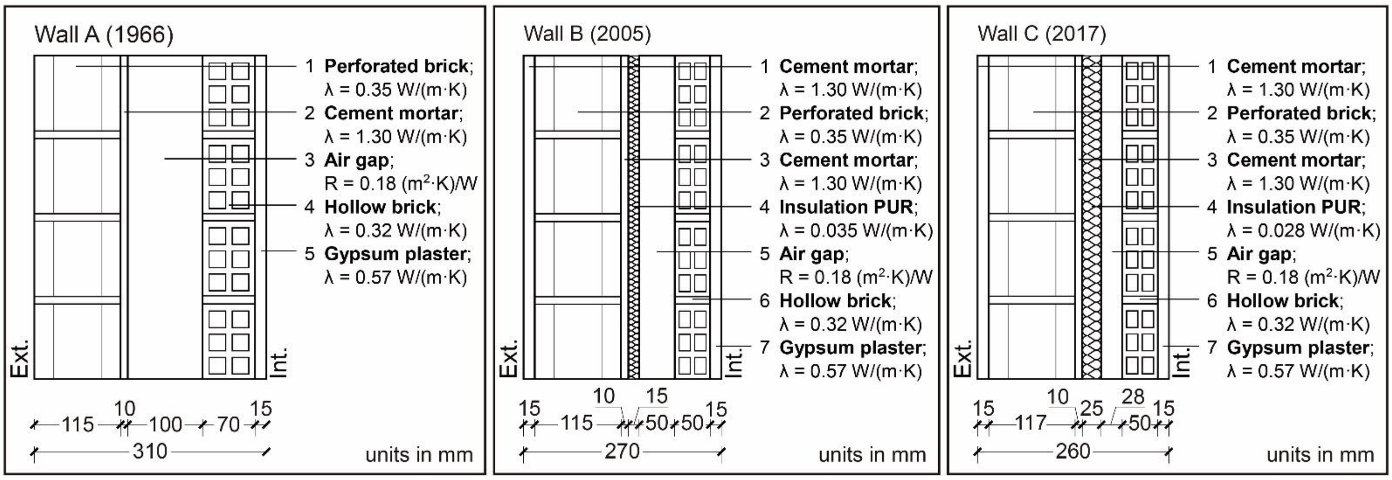


Fig. 2. Configuration of the walls analysed, and properties of materials.

To perform the tests, two different equipment were used. Firstly, the internal surface temperature and the reflected temperature were measured with an infrared camera FLIR E60bx, with a range of measurement from -10 to 105 °C, FOV of 25°x19°, and IFOV of 1.36 mrad. The spectral range of the camera is from 7.5 to 13 μm, and the thermal sensitivity is lower than 0.05 °C at 30 °C. The camera was placed on a tripod, at a perpendicular distance of 1.5 m from the wall and with an inclination angle of 20° to avoid reflections in the wall [26]. To measure the reflected temperature, a reflector foil was placed in the area of measurement, at a height of 1.5 m from the floor. Secondly, internal and external ambient temperatures were measured by using thermocouples T 190-2, with a wire diameter of 0.5 mm. The measurement range of thermocouples is from -10 to 105 °C, with a resolution of 0.1 K and an accuracy of ±0.05 K ±0.05 %. The internal probe was placed at a height of 1.5 m to avoid being affected by the existing thermal gradient between the ceiling and the floor, whereas the external probe was placed at the same height. Moreover, probes were located horizontally aligned, at a distance of 20 cm from the wall to avoid convective effects. Fig. 3 graphically represents those criteria of placing which were used during the experimental campaign.

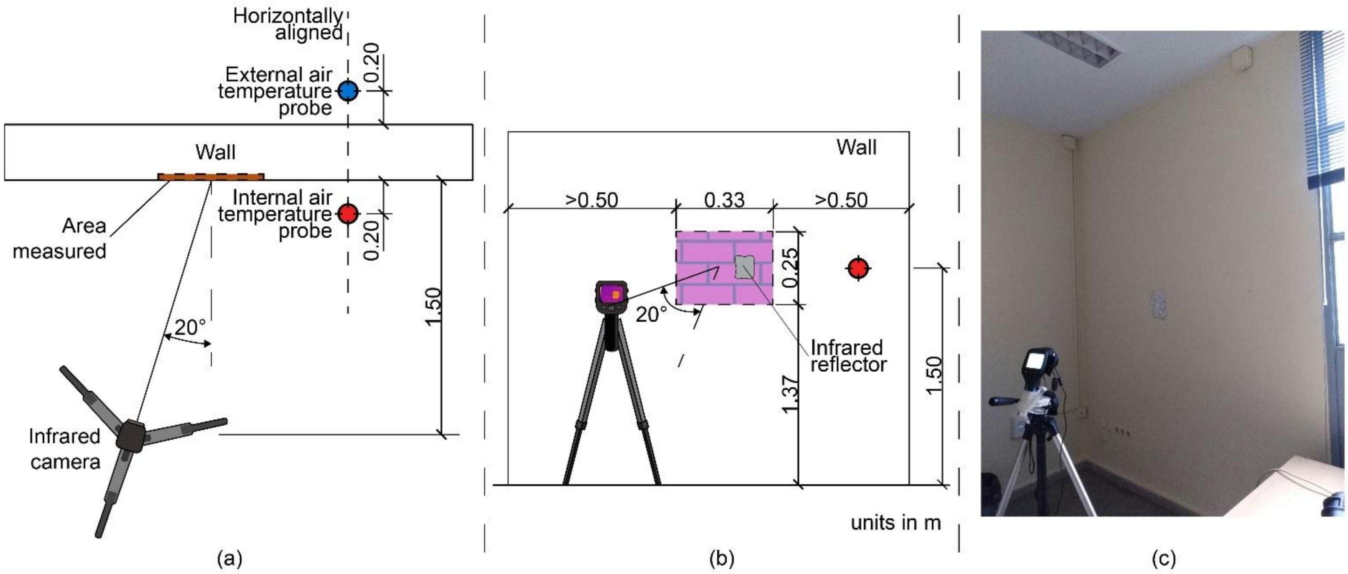


Fig. 3. Criteria of placing the infrared camera and thermocouples: (a) plan view, (b) front view, and (c) photography of the placing of the infrared camera and the reflector foil in Wall A.

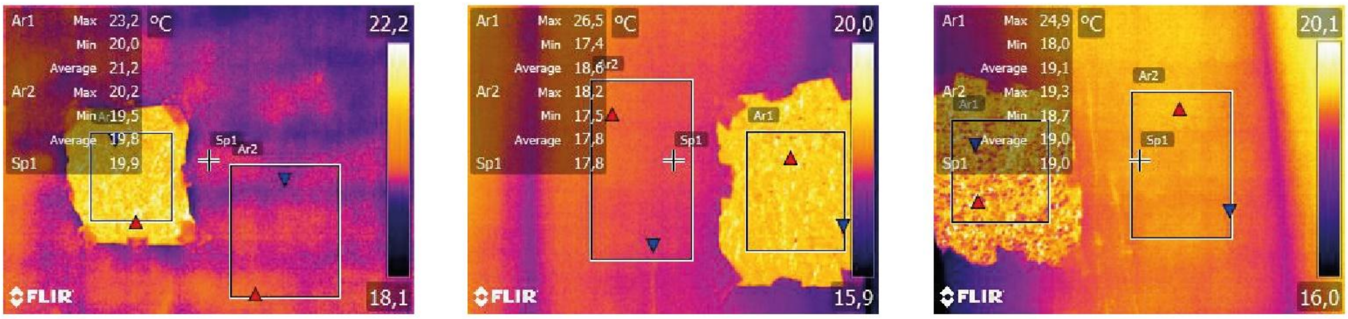
Moreover, it is important to highlight that the emissivity of walls was determined by using a black body tape before beginning their monitorings. Given that ranges of internal temperature did not present variations, it is considered that the emissivity did not vary [68], it was only determined at the beginning of tests. The emissivities obtained were 0.93, 0.94, and 0.93 for Wall A, B, and C, respectively.

Concerning test conditions and duration, it should be noted that tests fulfilled the requirements established in the standard EN 13187 [69] as well as in several research studies [25,26,29]:

- Tests were performed at instants without rainfalls, both at the moment and 48 h before conducting them.
- The wind speed during tests was always lower than 1 m/s.
- Tests were performed in winter, between the previous and posterior hours of the sunrise. All the tests guaranteed a temperature difference between the internal and external air between 10 and 15 °C.
- In total, 4 tests were performed for each wall, and each one lasted 2.5 h in consecutive days. The interval of data acquisition was 15 s.

4.2. Post-processing and data analysis

Two programs were used for the post-processing: (i) the FLIR Tools + software was used for the analysis of thermograms (see Fig. 4); and (ii) the creation, the treatment and the analysis of datasets were carried out using RStudio software. The analysis areas in the thermograms were determined based on those areas in which the temperature was more homogeneous [26].



(a) (b) (c)

Fig. 4. Thermograms analysed for each case study: (a) Wall A; (b) Wall B; and (c) Wall C.

Two different approaches were used to obtain the results: (i) on the one hand, the approach suggested by Dall'O' et al. [23] was used, which is based in the equivalence between the total heat flux of the wall and the heat flux by convection exchanged with the ambient (in this paper, it is called the approach of convection). Regarding inspecting from the interior, similar approaches have been used by Andújar Márquez et al. [70], Kim et al. [71], and Kisilewicz et al. [72]; and (ii) the approach from research studies included in Section 2 was used, which is based on the equivalence of the total heat flux with the convective and radiative heat fluxes (in this paper, it is called the approach of convection and radiation). Given that Madding [24], Fokaides and Kalogirou [25], and Tejedor et al. [26] used different equations for the heat flux by radiation, the existing differences among the 3 equations were firstly analysed. Fig. 5 represents the existing correlations among the values of the radiative heat flux obtained by each approach. The regression analysis was carried out using the function lm of RStudio [73] for the adjustment by minimum blocks. The results obtained by the 3 approaches are almost the same. In this sense, the correlation coefficient (R^2) obtained from the 3 approaches was 1. Only in some observations of the dataset existed deviations of less than 1% between the values of radiative heat flux of the approach of Fokaides and Kalogirou and the values of radiative heat flux of the other two approaches when there is a high difference between T_w and T_r. Thus, there are no differences in the use of these 3 equations. Therefore, the expression for the radiative heat flux of Madding was used for this study.

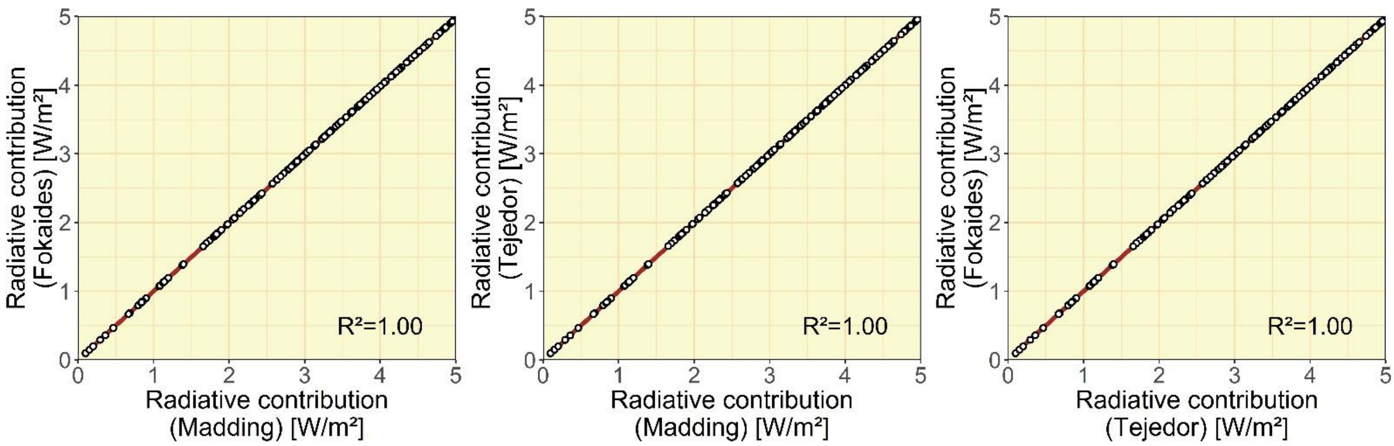


Fig. 5. Correlations among the values of the radiative heat flux obtained according to the equation used by Madding [24], by Fokaides and Kalogirou [25], and by Tejedor et al. [26].

Moreover, regarding the analysis of results, it is important to highlight several aspects:

- For correlations depending on dimensionless numbers, it was necessary to determine variables, such as the Prandtl number, and the viscosity and thermal conductivity of the air. Those values were determined according to the internal temperature by using a technical manual [74]. The Grashof number and the Rayleigh number were determined by means of Eqs. (53) and (54), respectively.
- Due to the huge amount of ICHTCs analysed in this study, possible groups among the different equations were also analysed before obtaining the results. For this purpose, the cluster analysis was used for correlations of temperature differences and of dimensionless numbers. In Section 4.3, the method used is described.
- The uncertainty associated with results was estimated by means of the combined standard uncertainty (Eq. (55)).

$$Gr = \frac{9.8 \cdot \beta \cdot \Delta T_{wi} \cdot L^3}{v^2} \quad (53)$$

$$Ra = Gr \cdot Pr \quad (54)$$

$$u_c(y) = \sqrt{\sum_{i=1}^{n} \left(\frac{\delta f}{\delta x_i}\right)^2 \cdot u^2(x_i)} \quad (55)$$

Where β is the volumetric temperature expansion coefficient [K^{-1}], v is the viscosity of air [m^2/s], $\left(\frac{\delta f}{\delta x_i}\right)$ are the sensitivity coefficients, and $u(x_i)$ are the uncertainties associated with the variables measured.

To assess the representation of the thermal transmittance results obtained for the different ICHTCs, those results obtained by means of ISO 6946 (Eq. (56)) and of ISO 9869-1 (Eq. (57)) were used as representative U-values of each wall. As previously mentioned, the criterion of assessing results from Section 7.3 of ISO 9869-1 was used. Those values presenting a difference lower than 20% between the measured value ($U_{Measured}$) and the reference value ($U_{Reference}$) (Eq. (58)) were valid.

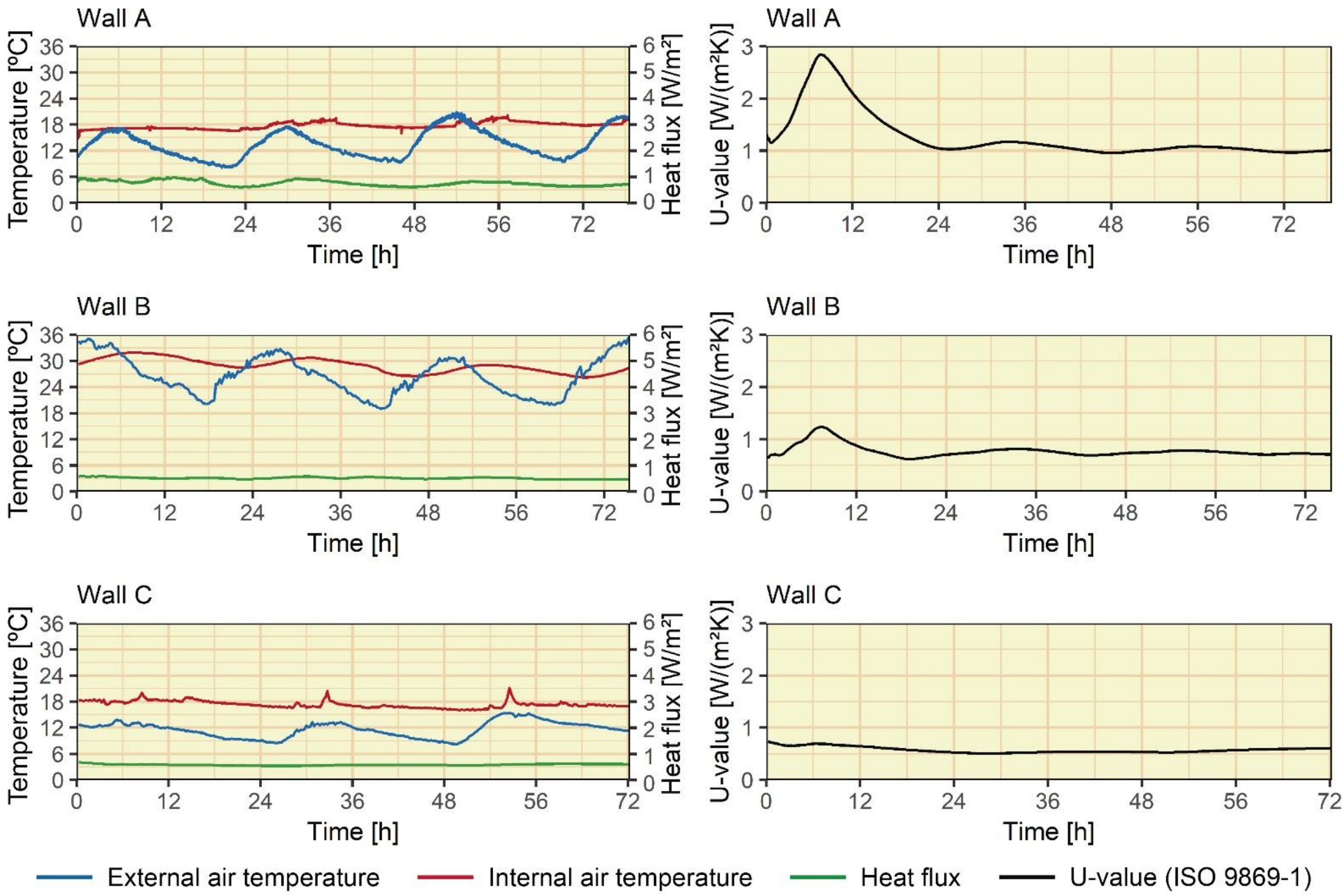
$$U_{6946} = \frac{1}{R_{s,i} + R_w + R_{s,o}} \tag{56}$$

$$U_{HFM} = \frac{\sum_{j=1}^{n} q_j}{\sum_{j=1}^{n}\left(T_{i,j} - T_{o,j}\right)} \tag{57}$$

$$Deviation = \frac{U_{Measured} - U_{Reference}}{U_{Reference}} \tag{58}$$

Where U_{6946} [W/(m²·K)] is the thermal transmittance obtained by ISO 6946, U_{HFM} [W/(m²·K)] is the thermal transmittance obtained by ISO 9869, R_w is the thermal resistance of the wall [(m²·K)/W] obtained by means of the sum of the thermal resistances of each of its layer, and $R_{s,o}$ and $R_{s,i}$ [(m²·K)/W] are the external and internal surface thermal resistances, respectively.

The value of ISO 6946 of each wall was determined by using the technical documentation of the wall and by correcting thermal conductivity values by using the correction factor of 1.0385 published by Pérez-Bella et al. [16] for Seville. For results of ISO 9869-1, the results obtained from a previous experimental campaign were used (see Fig. 6). Tests were performed according to ISO 9869-1. Tests lasted 72 h, and the interval of data acquisition was of 10 min. The temperature probes used were the same as those indicated in Section 4.1. The heat flux was measured by using a heat flux plate FQA018C of 120x120x1.5 mm and the substrate was of epoxy resin. The working temperature ranged from -40 to 80 °C and had an accuracy of 5% in 23 °C. The temperature probes followed the same criteria from Section 4.1. The heat flux plate was placed at a height of 1.5 m above the floor and in the same area analysed during the IRT test. Moreover, the heat flux plate was placed putting silicon grease to ensure good thermal contact between the wall and the probe [75]. As can be seen, the existing difference between U_{6946} and U_{HFM} was lower than 20%, validating the results of U_{HFM} obtained. These deviations are in accordance with some research studies in which the low percentage difference existing in the thermal transmittance results obtained by both standardised methods for heavy walls (walls with a thermal capacity higher than 150 kJ/(m^{2}K) [28]) is highlighted [76].



Wall	Thermal transmittance [W/(m²·K)]		Percentage deviation [%]
	U_{6946}	U_{HFM}	$U_{HFM} - U_{6946}$
Wall A	1.10	1.02	-7.27
Wall B	0.79	0.71	-10.12
Wall C	0.58	0.60	3.45

Fig. 6. Measurement values and *U*-value results obtained by means of ISO 6946 and of ISO 9869-1.

4.3. Cluster analysis

As mentioned above, due to the high number of correlations studied, a cluster analysis (CA) was carried out. CA is a multivariant statistical technique which allows to classify a set of objects in a way that, on the one hand, similar objects are in the same conglomeration and, on the other hand, different objects are in different groups, resulting in different homogeneous groups among them [77]. CA begins with a sampling of individuals from which the value of p variables is known. After making the CA, an X-partition of the sampling of individuals, made by k-groups, is obtained.

To achieve these groups, it is necessary to have association measurements which indicate whether two objects are similar in order to classify them within the same group. These association measurements can be: (i) similarity (two objects are in the same group when they are similar); and (ii) distance (two objects are in the same group when they are near to each other). Among metric distances, there is a wide variety such as the Euclidean distance, the Canberra distance, or the Minkowski distance [78]. For this study, the Euclidean distance was used.

The Euclidean distance between two individuals x_i and x_j, from which the value of p variables is known, is defined as follows:

$$d(x_i, x_j) = \left[\sum_{r=1}^{p} (x_{ir} - x_{jr})^2\right]^{1/2} \tag{59}$$

A third key aspect of CA is the methodology used to classify elements in groups, existing mainly two methods: (i) non-hierarchical methods, whose starting point is the number k of groups intended to be obtained. These groups are initially made at random, and then they are improved in a iterative way until reaching the optimal number k of groups; and (ii) hierarchical methods, which can be classified in agglomerative (starting with a partition of n groups with an individual in each of them, and then they are grouped until obtaining an only group of n individuals) and divisive (starting with a partition of a group of n individuals, and then it is divided until obtaining a partition of n groups of an individual).

In this study, the Ward method [79] was used as methodology. This method belongs to agglomerative hierarchical methods. The Ward method calculates the distance between two clusters as the sum of squares among groups in the ANOVA by adding up over all the variables. Afterwards, it reduces the sum of square within clusters over all the possible partitions obtained by merging two clusters from the previous step. The main advantage of this method is that it creates small clusters, so it was suitable for the objective of this paper. The CA was carried out according to the convective heat flux obtained by each correlation of ICHTC in the different tests.

5. Discussion of the results

5.1. CA of ICHTC correlations

As mentioned above, a CA of the different ICHTC correlations was firstly carried out. CA was made separately for correlations depending on temperature differences and for correlations depending on dimensionless numbers. For this purpose, convective heat fluxes of the different tests performed during the experimental campaign were obtained.

Fig. 7 represents the results obtained for the CA of ICHTCs of temperature differences. In total, 12 different groups were obtained, 7 of them made by 2 or more equations. Moreover, CA allowed to reflect some aspects of similarity among equations. The equations of U-value from other research studies (Eqs. (10), (13) and (14)) and included in the dendrograms refer to the ICHTC used for these equations. Regarding Eq. (10), the tabulated value from ISO 6946 for walls was used to see the similarity of this value with other correlations.

Firstly, most correlations using a characteristic length in their equation (Eqs. (6), (8), (17), (27), and (34)) were grouped in the same cluster (cluster b). The correlations using a characteristic length and not grouped in this cluster were as follows: (i) the equations of Churchill and Chu (Eq. (22)); (ii) the simplification of the correlation of Churchill and Chu (Eq. (23)) carried out by ESDU [47]; and (iii) those of Alamdari and Hammond (Eq. (24)) and of Awbi and Hatton (Eq. (33)). In this sense, the Eq. (22) was grouped with the Eq. (23) in cluster d.

Correlations of the works quoted by Min et al. [43] were grouped with other equations: (i) the correlation of Earle for turbulent flow (Eq. (9)) was grouped with the correlation of McAdams, quoted by Min et al. (Eq. (20)) in cluster e; and (ii) correlations of Awbi and Hatton (Eq. (33)) and of Hatton and Awbi (Eq. (32)) were grouped in cluster f with the correlation of Carroll (Eq. (19)) and with the correlation of King (Eq. (21)), respectively, both quoted by Min et al. [43].

Other groups of correlation are important to be highlighted: (i) in cluster h, equations of Wilkes and Peterson (Eq. (15)) and of Li et al. (Eq. (26)) with the one of Khalifa and Marshall for walls in front of heating equipment (Eq. (30)); and (ii) in cluster i, the correlation of Min et al. for turbulent flow (Eq. (18)) was grouped with correlations of Khalifa and Marshall for walls close to a radiator (Eq. (28)) and for vertical surfaces with insulation (Eq. (31)).

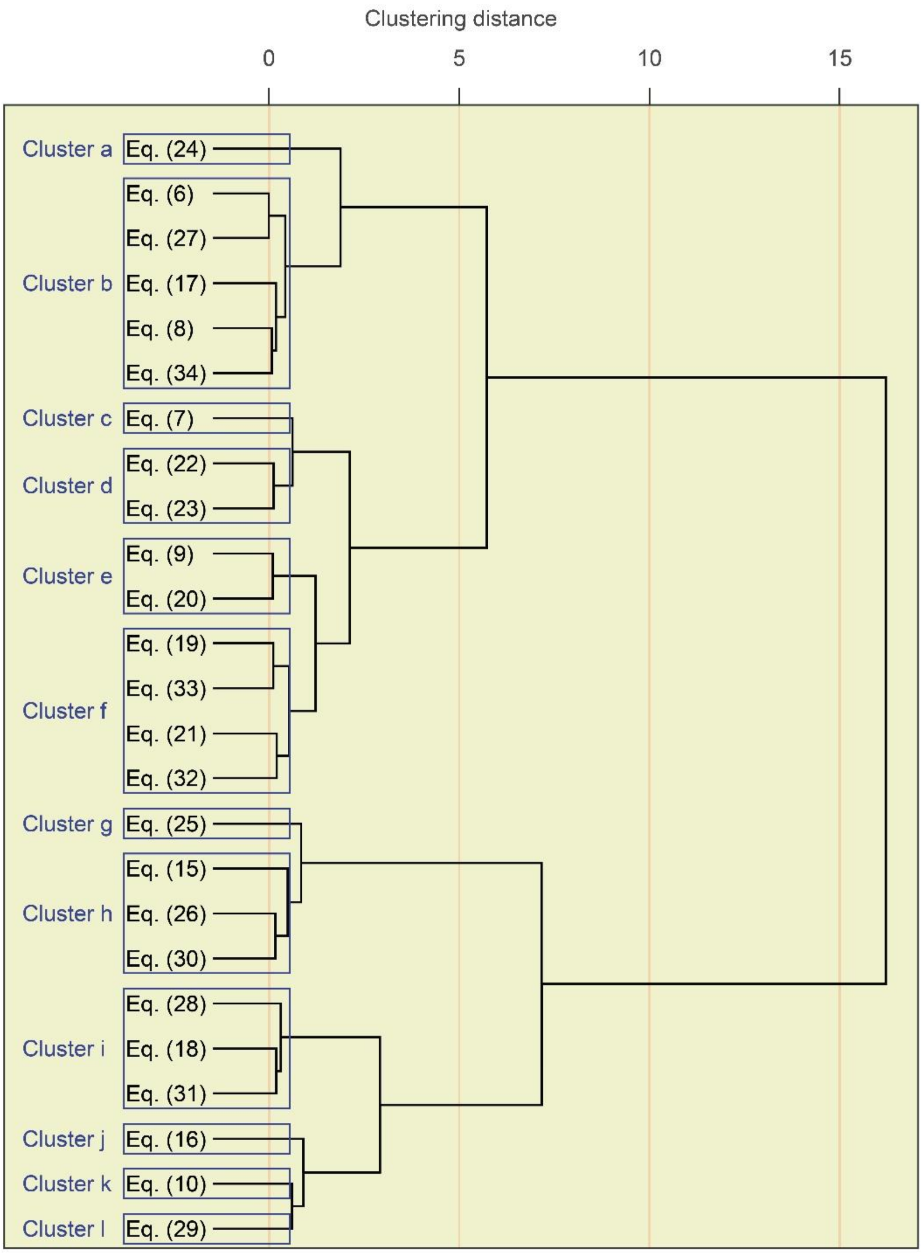


Fig. 7. Clustering dendrogram of convective heat fluxes obtained by ICHTC correlations of temperature differences.

For ICHTC correlations depending on dimensionless numbers, 8 different clusters were obtained (see Fig. 8). As can be seen, 5 clusters had only one equation, and the remaining clusters had 2 or more equations. Regarding these groups, it is important to highlight some aspects: (i) both equations proposed by Tejedor et al. (without simplification (Eq. (13)) and simplified (Eq. (14))) were grouped in cluster b with correlations for laminar flow of CIBSE (Eq. (43)), the correlation of Jakob for a Rayleigh number between 10^8 and 10^{12} (Eq. (36)), and the correlation of McAdams for a Rayleigh number between $2 \cdot 10^9$ and 10^{12} (Eq. (42)). The ranges of Rayleigh numbers associated with correlations of this cluster included both transition fluxes from the laminar flow regime to the turbulent flow regime ($10^8 < Ra < 10^{10}$) and completely developed turbulent flows ($10^9 < Ra < 10^{12}$), which are adapted to the laminar-turbulent regime of the equation of

Churchill and Chu, used by Tejedor et al. ; and (ii) equations of both cluster f and h included correlations for laminar flow ($Ra < 10^8$).

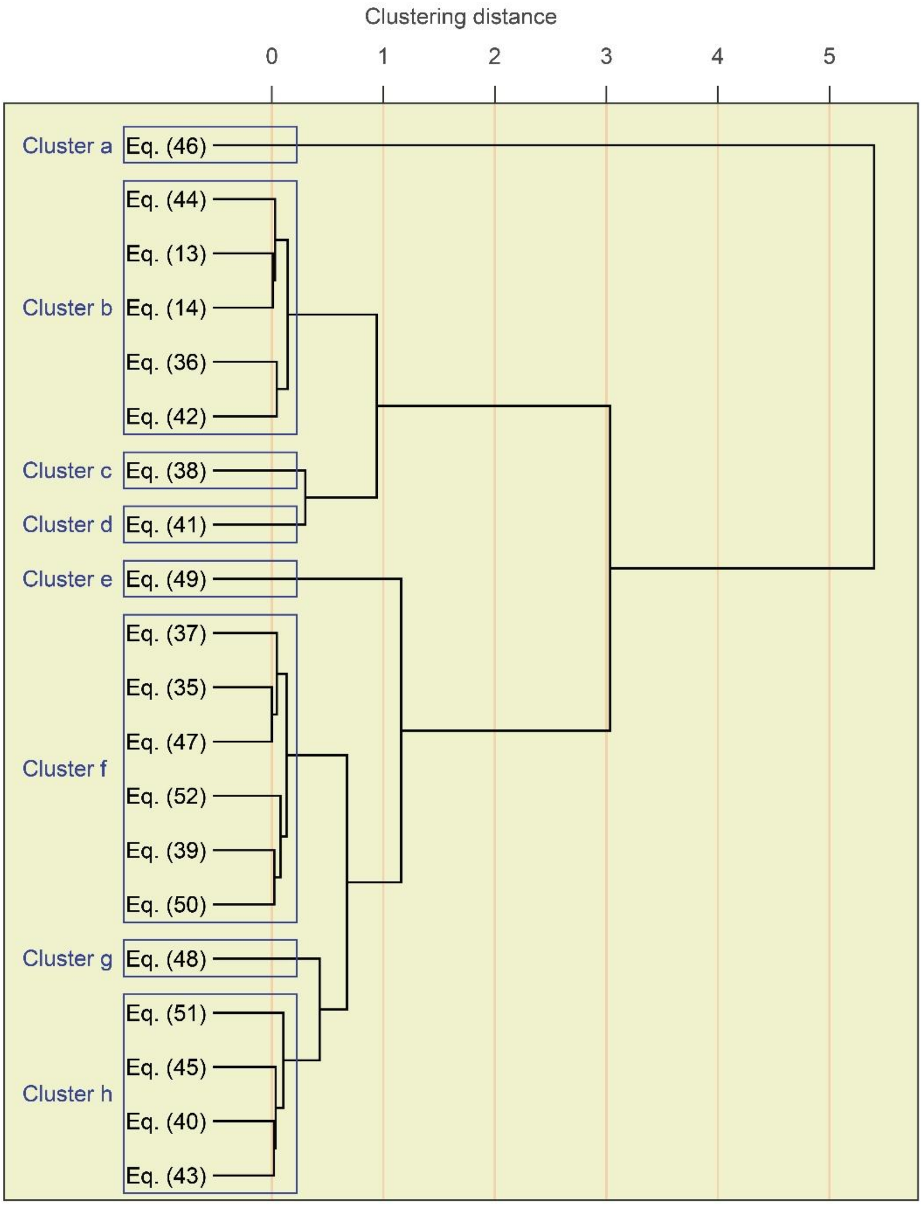


Fig. 8. Clustering dendrogram of convective heat fluxes obtained by ICHTC correlations of dimensionless numbers.

5.2. Results for correlations of temperature differences

After the classification of ICHTC correlations, the results obtained by each cluster were analysed. Firstly, correlations depending on temperature differences were analysed. The total, convective and radiative heat fluxes obtained by each cluster are represented in Fig. 9. As can be seen, values of convective components varied from one cluster to another. Despite percentages of convective terms presented different values among the different case studies, there was a similar tendency in the values obtained for each cluster. In this sense, clusters g, h and j were those which had the greatest convective heat flux, whereas clusters a, b and c had the lowest convective heat flux in all case studies. These clusters with the lowest percentage of contribution of the convective component belong to groups of correlations which include the characteristic length as an independent variable in the equation of ICHTC. Concerning the total heat flux, the value obtained by each cluster was reduced as the building period of the wall under analysis varied (i.e. as the thickness of insulation increased). In this sense, percentage variations between Wall A and Wall B were between -14.82% and -29.76%, whereas percentage variations between Wall A and Wall C were between -51.34% and -57.06%.

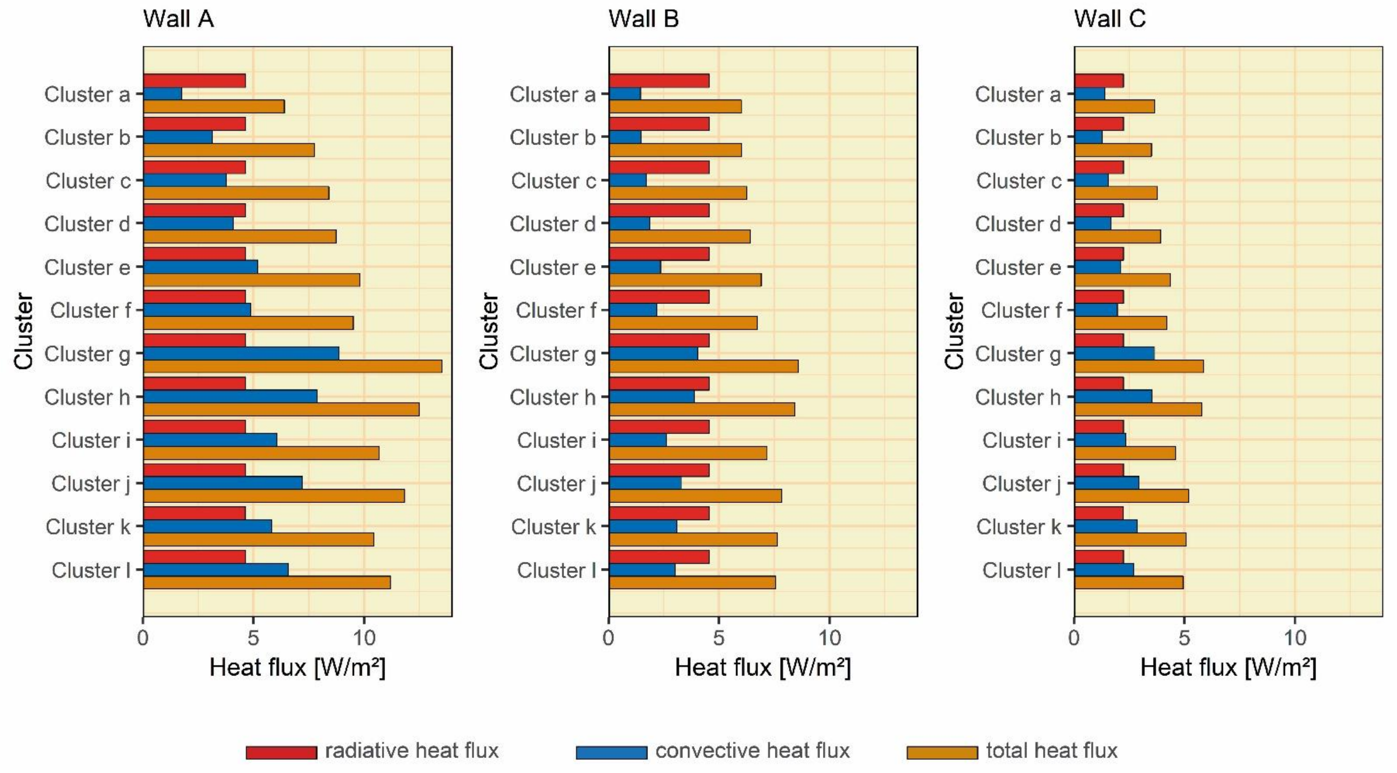


Fig. 9. Convective, radiative and total heat fluxes for clusters of ICHTCs depending on temperature differences.

After analysing the influence of convective and radiative components, thermal transmittance results of each cluster were obtained. As indicated in Section 4, the results obtained by using only the convective component (approach of Dall'O' et al. [23]) were analysed, as well as those obtained by using the convective and radiative components (approach of Madding [24], Fokaides and Kalogirou [25], and Tejedor et al. [26]). Table 4 indicates the thermal transmittance values obtained for the approach of convection, as well as the percentage deviations with respect to reference values. As can be seen, representative results were obtained in some clusters for Wall A and Wall C, whereas valid results were not obtained for Wall B. However, deviations near to ±20% were obtained for Wall B depending on the reference value used. Likewise, the used reference value varied the number of representative results. In this sense, clusters h, j and l obtained deviations of less than ±20% with respect to U_{6946}, whereas clusters j and l obtained deviations of less than ±20% with respect to U_{HFM}. Thus, these two last clusters, which belongs to correlations of Giesecke (Eq. (16)) and of Khalifa and Marshall (Eq. (29)), presented a more adequate behaviour for the approach of convection, reaching deviations of less than ±10%. Finally, clusters g, i and k obtained representative results in one of the case studies analysed. Thus, as can be appreciated, the approach presented a good behaviour in those correlations having a high convective heat flux (see Fig. 9).

Table 4

Thermal transmittance results through IRT method (approach of convection) for correlations depending on temperature differences, and percentage deviations with respect to the values obtained by ISO 6946 and by ISO 9869-1.

Cluster	U_{IRT} [W/(m²·K)]			Percentage deviation $U_{IRT} - U_{6946}$ [%]			Percentage deviation $U_{IRT} - U_{HFM}$ [%]		
	Wall A	Wall B	Wall C	Wall A	Wall B	Wall C	Wall A	Wall B	Wall C
Cluster a	0.28±0.04	0.19±0.04	0.27±0.05	-74.55	-75.95	-53.45	-72.55	-73.24	-55.00
Cluster b	0.50±0.05	0.20±0.04	0.24±0.05	-54.55	-74.68	-58.62	-50.98	-71.83	-60.00
Cluster c	0.61±0.07	0.23±0.04	0.30±0.07	-44.55	-70.89	-48.28	-40.20	-67.61	-50.00
Cluster d	0.66±0.08	0.25±0.05	0.32±0.08	-40.00	-68.35	-44.83	-35.29	-64.79	-46.67
Cluster e	0.65±0.09	0.32±0.06	0.30±0.09	-40.91	-59.49	-48.28	-36.27	-54.93	-50.00
Cluster f	0.78±0.09	0.29±0.05	0.38±0.09	-29.09	-63.29	-34.48	-23.53	-59.15	-36.67
Cluster g	1.42±0.16	0.54±0.10	0.70±0.16	29.09	-31.65	20.69	39.22	-23.94	**16.67**
Cluster h	1.26±0.13	0.52±0.09	0.68±0.14	**14.55**	-34.18	**17.24**	23.53	-26.76	**13.33**
Cluster i	0.97±0.11	0.35±0.07	0.45±0.12	**-11.82**	-55.70	-22.41	**-4.90**	-50.70	-25.00
Cluster j	1.16±0.13	0.44±0.09	0.56±0.13	**5.45**	-44.30	**-3.45**	**13.73**	-38.03	**-6.67**
Cluster k	0.83±0.09	0.42±0.06	0.41±0.09	-24.55	-46.84	-29.31	**-18.63**	-40.85	-31.67
Cluster l	1.05±0.16	0.40±0.10	0.52±0.16	**-4.55**	-49.37	**-10.34**	**2.94**	-43.66	**-13.33**

U_{IRT}: thermal transmittance obtained by quantitative internal infrared thermography method; U_{6946}: thermal transmittance obtained by ISO 6946; U_{HFM}: thermal transmittance obtained by ISO 9869.

The incorporation of the radiative component modified those clusters with representative results (see Table 5). As predicted, the incorporation of that component caused an increase in thermal transmittance values with respect to the approach of convection, with an increase in the U-value higher than 52%. Therefore, those clusters with a low value of the convective heat flux obtained valid results, whereas clusters which obtained valid results with the approach of convection (i.e. clusters with a high value of the convective heat flux) obtained deviations higher than 60% with respect to reference values. This aspect highlighted the importance of the radiative component in this approach. Regarding the case studies, unlike the approach of convection, valid results were obtained in the three walls analysed. Representative results were obtained in one or two case studies with clusters a, b, c, d, e and f. It is important to highlight that two of these clusters (c and e) belongs to correlations of convection recommended by Madding [24]. However, clusters with a larger number of representative results in the 3 case studies were clusters a (belonging to the correlation of Alamdari and Hammond (Eq. (24)) and b (belonging to correlations of Holman (Eqs. (6) and (27)), of Earle (Eq. (8)), of Min et al. (Eq. (17)), and of Fohanno and Polidori (Eq. (34))). The adjustment degree presented by these clusters was quite high, with deviations of less than 10% for walls A and B.

Table 5

Thermal transmittance results through IRT method (approach of convection and radiation) for correlations depending on temperature differences, and percentage deviations with respect to the values obtained by ISO 6946 and by ISO 9869-1.

Cluster	U_{IRT} [W/(m²·K)]			Percentage deviation $U_{IRT} - U_{6946}$ [%]			Percentage deviation $U_{IRT} - U_{HFM}$ [%]		
	Wall A	Wall B	Wall C	Wall A	Wall B	Wall C	Wall A	Wall B	Wall C
Cluster a	1.03±0.21	0.81±0.12	0.70±0.21	**-6.36**	**2.53**	20.69	**0.98**	**14.08**	**16.67**
Cluster b	1.24±0.25	0.81±0.20	0.67±0.21	**12.73**	**2.53**	**15.52**	21.57	**14.08**	**11.67**
Cluster c	1.35±0.25	0.84±0.19	0.72±0.20	22.73	**6.33**	24.14	32.35	**18.31**	**20.00**
Cluster d	1.40±0.24	0.86±0.19	0.75±0.20	27.27	**8.86**	29.31	37.25	21.13	25.00
Cluster e	1.58±0.24	0.93±0.19	0.83±0.19	43.64	**17.72**	43.10	54.90	30.99	38.33
Cluster f	1.53±0.24	0.91±0.19	0.80±0.19	39.09	**15.19**	37.93	50.00	28.17	33.33
Cluster g	2.17±0.23	1.15±0.19	1.12±0.18	97.27	45.57	93.10	112.75	61.97	86.67
Cluster h	2.01±0.24	1.13±0.19	1.11±0.18	82.73	43.04	91.38	97.06	59.15	85.00
Cluster i	1.72±0.24	0.97±0.19	0.88±0.19	56.36	22.78	51.72	68.63	36.62	46.67
Cluster j	1.90±0.23	1.05±0.18	0.99±0.18	72.73	32.91	70.69	86.27	47.89	65.00
Cluster k	1.68±0.24	1.03±0.18	0.97±0.17	52.73	30.38	67.24	64.71	45.07	61.67
Cluster l	1.80±0.23	1.02±0.18	0.95±0.17	63.64	29.11	63.79	76.47	43.66	58.33

Therefore, the thermal transmittance results obtained for ICHTCs depending on temperature differences allowed to establish the suitability of using ICHTCs: (i) for the approach of convection, correlations of Giesecke (Eq. (16)) and of Khalifa and Marshall (Eq. (29)) obtained valid results in two of the case studies, without presenting valid results in Wall B; and (ii) for the approach of convection and radiation, correlations of Holman (Eqs. (6) and (27)), of Earle (Eq. (8)), of Alamdari and Hammond (Eq. (24)), of Min et al. (Eq. (17)), and of Fohanno and Polidori (Eq. (34)) obtained valid results in the three kinds of wall analysed, with an adjustment degree lower than 10% in some of the results.

5.3. Results for correlations of dimensionless numbers

After studying ICHTC correlations depending on temperature differences, the results obtained for correlations depending dimensionless numbers were analysed. Following the same structure of discussion of the results for the other correlations, the heat flux obtained by each cluster were first analysed (see Fig. 10). As can be seen, the level of contribution of the convective component in each cluster presented a similar behaviour in the 3 case studies. In this sense, the order of clusters depending on the level of influence of the convective component was as follow: b, c, d, f, h, g, e and a. It is important to highlight that the value of the convective heat flux was very similar among clusters b, c, d, f, h, g and e, with a standard deviation of 0.49 W/m² in Wall A, of 0.23 W/m² in Wall B, and of 0.18 W/m² in Wall C. Likewise, the percentage of contribution of the convective component oscillated between 16 and 45%, varying those percentages in the same order indicated above. It must be noted the existing difference between cluster a (belonging to Eq.) (46)) and the other clusters.

In this sense, reductions with respect to the following cluster, with a lower convective heat flux (cluster e), were of 1.87 W/m^2 in Wall A, of 0.94 W/m^2 in Wall B, and of 0.77 W/m^2 in Wall C.

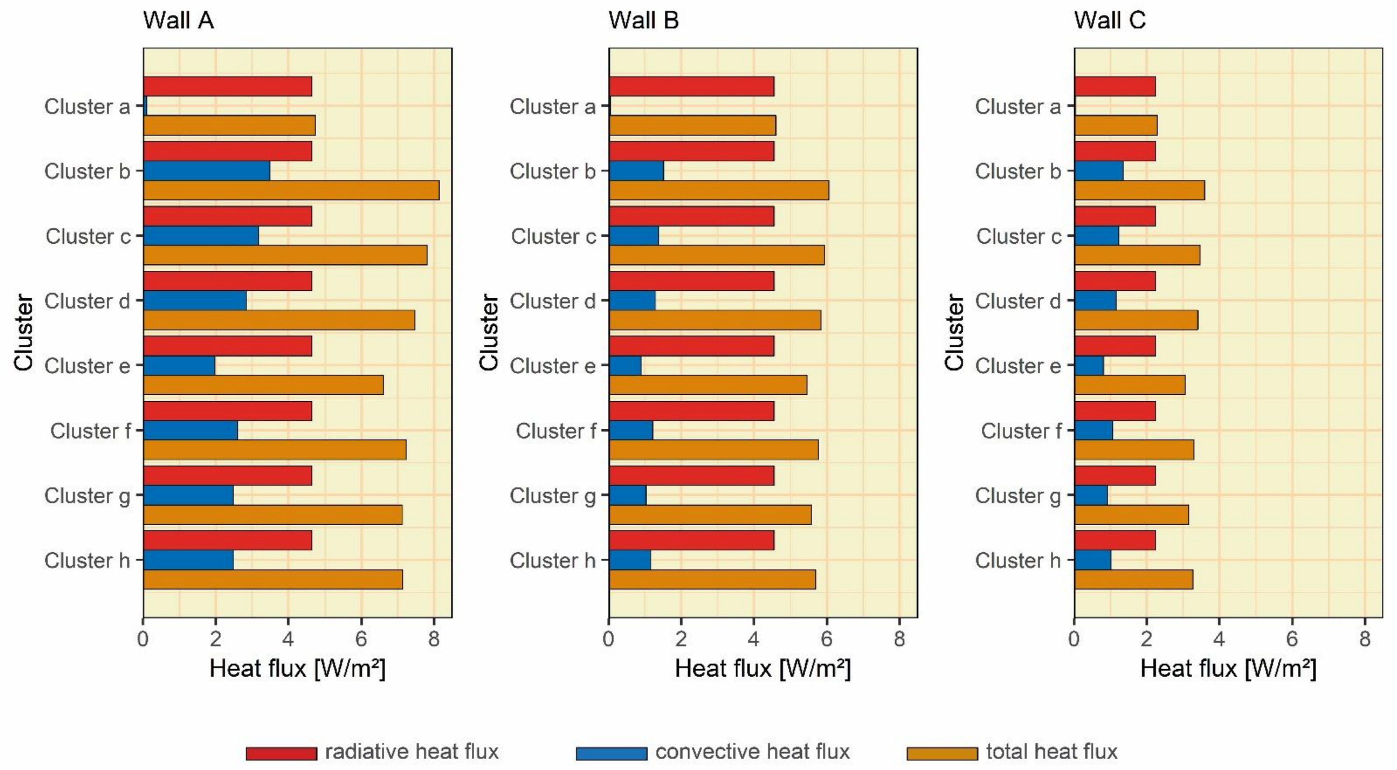


Fig. 10. Convective, radiative and total heat fluxes for clusters of ICHTCs depending on dimensionless numbers.

Afterwards, results for the approaches of convection were obtained (see Table 6) as well as of convection and radiation (see Table 7). As can be appreciated, the approach of convection did not have representative results, with deviations higher than 45% with respect to the reference values. This was due to the low value of the convective heat flux, since, as seen for ICHTC correlations depending on temperature differences, this approach requires a high value of the convective heat flux (see Section 5.2.). Regarding the approach of convection and radiation, representative results were obtained in clusters b-h for the 3 case studies analysed. In this regard, it must be highlighted the high adjustment degree achieved by correlations depending on dimensionless numbers, since they reached 28 results with deviations of less than ±10%, from which 15 had deviations of less than ±5%. Likewise, the used correlations have different value ranges of Ra, so any of correlations could be used in IRT tests. Only cluster a obtained invalid results in the 3 walls analysed, since the total heat flux obtained was lower in comparison with the other clusters. Thus, correlations of dimensionless numbers (except Eq. (46)) presented a very high adjustment degree when these equations were used with the approach of convection and radiation.

Table 6

Thermal transmittance results through IRT method (approach of convection) for correlations depending on dimensionless numbers, and percentage deviations with respect to the values obtained by ISO 6946 and by ISO 9869-1.

Cluster	U_{IRT} [W/(m²·K)]			Percentage deviation $U_{IRT} - U_{6946}$ [%]			Percentage deviation $U_{IRT} - U_{HFM}$ [%]		
	Wall A	Wall B	Wall C	Wall A	Wall B	Wall C	Wall A	Wall B	Wall C
Cluster a	0.02±0.00	0.01±0.00	0.01±0.00	-98.18	-98.73	-98.28	-98.04	-98.59	-98.33
Cluster b	0.56±0.05	0.20±0.03	0.26±0.04	-49.09	-74.68	-55.17	-45.10	-71.83	-56.67
Cluster c	0.51±0.04	0.18±0.02	0.24±0.04	-53.64	-77.22	-58.62	-50.00	-74.65	-60.00
Cluster d	0.45±0.04	0.17±0.02	0.22±0.03	-59.09	-78.48	-62.07	-55.88	-76.06	-63.33
Cluster e	0.32±0.03	0.12±0.01	0.16±0.02	-70.91	-84.81	-72.41	-68.63	-83.10	-73.33
Cluster f	0.42±0.04	0.16±0.02	0.20±0.03	-61.82	-79.75	-65.52	-58.82	-77.46	-66.67
Cluster g	0.40±0.03	0.14±0.02	0.18±0.03	-63.64	-82.28	-68.97	-60.78	-80.28	-70.00
Cluster h	0.40±0.03	0.15±0.02	0.20±0.03	-63.64	-81.01	-65.52	-60.78	-78.87	-66.67

Table 7

Thermal transmittance results through IRT method (approach of convection and radiation) for correlations depending on dimensionless numbers, and percentage deviations with respect to the values obtained by ISO 6946 and by ISO 9869-1.

Cluster	U_{IRT} [W/(m²·K)]			Percentage deviation $U_{IRT} - U_{6946}$ [%]			Percentage deviation $U_{IRT} - U_{HFM}$ [%]		
	Wall A	Wall B	Wall C	Wall A	Wall B	Wall C	Wall A	Wall B	Wall C
Cluster a	0.76±0.18	0.62±0.14	0.44±0.20	-30.91	-21.52	-24.14	-25.49	**-12.68**	-26.67
Cluster b	1.30±0.14	0.81±0.11	0.69±0.16	**18.18**	**2.53**	**18.97**	27.45	**14.08**	**15.00**
Cluster c	1.25±0.15	0.80±0.12	0.66±0.17	**13.64**	**1.27**	**13.79**	22.55	**12.68**	**10.00**
Cluster d	1.20±0.15	0.78±0.12	0.65±0.17	**9.09**	**-1.27**	**12.07**	**17.65**	**9.86**	**8.33**
Cluster e	1.06±0.16	0.73±0.12	0.58±0.18	**-3.64**	**-7.59**	**0.00**	**3.92**	**2.82**	**-3.33**
Cluster f	1.16±0.15	0.78±0.12	0.63±0.17	**5.45**	**-1.27**	**8.62**	**13.73**	**9.86**	**5.00**
Cluster g	1.14±0.15	0.75±0.12	0.60±0.18	**3.64**	**-5.06**	**3.45**	**11.76**	**5.63**	**0.00**
Cluster h	1.17±0.15	0.77±0.12	0.62±0.17	**6.36**	**-2.53**	**6.90**	**14.71**	**8.45**	**3.33**

5.4. Temperature differences vs dimensionless numbers

As seen above, the results obtained by ICHTC correlations among the different approaches analysed presented a different behaviour and adjustment degree. Table 8 represents percentages of the representative results obtained by each approach. As can be seen, very similar percentages of representative results were obtained for ICHTC correlations of temperature differences, both in the approach of convection and the approach of convection and radiation (between 19.44 and 25%). However, these percentages of representative results were lower in comparison with percentages of correlations of dimensionless numbers for the approach of convection and radiation (higher than 80%). Moreover, as seen in Section 5.3, only one of the clusters obtained invalid results, and consequently the percentage of valid results decreased. Therefore, the use of correlations of dimensionless numbers with the approach of convection and radiation was the most efficient approach for the quantitative internal IRT method.

Table 8

Percentage of representative results for both assumptions.

Approach	Case study	Percentage of representative results (reference value U_{6946}) [%]		Percentage of representative results (reference value U_{HFM}) [%]	
		ICHTCs of temperature differences	ICHTCs of dimensionless numbers	ICHTCs of temperature differences	ICHTCs of dimensionless numbers
Approach of convection	Wall A	33.33%	0.00%	33.33%	0.00%
	Wall B	0.00%	0.00%	0.00%	0.00%
	Wall C	25.00%	0.00%	33.33%	0.00%
	All	19.44%	0.00%	22.22%	0.00%
Approach of convection and radiation	Wall A	16.67%	87.50%	8.33%	62.50%
	Wall B	50.00%	87.50%	25.00%	100.00%
	Wall C	8.33%	87.50%	25.00%	87.50%
	All	25.00%	87.50%	19.44%	83.33%

6. Conclusions

In this paper, 45 equations of internal convective heat transfer coefficients (ICHTCs) were analysed for the quantitative internal infrared thermography method by using two different theoretical approaches: (i) the approach of convection; and (ii) the approach of convection and radiation. In total, 25 equations corresponded to correlations of temperature differences, and 20 to correlations of dimensionless numbers. For this purpose, monitorings were carried out in 3 typical façades from the building periods in Spain. For the first time, a cluster analysis of ICHTC correlations was made in order to know the similarity among equations. Afterwards, these clusters were used to obtain thermal transmittance results. The following conclusions can be drawn from the results obtained:

- The cluster analysis of ICHTCs of temperature differences allowed to reflect some aspects of similarity among equations: (i) almost all equations having a characteristic length as independent variable were grouped in the same cluster; and (ii) there were groups of correlations of Min et al. with other correlations published by different authors. In this sense, there were groups of those equations published by Khalifa and Marshall with other equations of ICHTC.
- Regarding the cluster analysis of ICHTCs of dimensionless numbers, only three clusters were generated with two or more equations. Two of the clusters generated are made by correlations for laminar flow ($Ra < 10^8$). However, in the other cluster are included both equations of Churchill and Chu used by Tejedor et al. (with and without simplification), with equations for transition fluxes from the laminar flow regime to the turbulent flow regime and for completely developed turbulent flows.

- The thermal transmittance results obtained for ICHTCs depending on temperature differences allowed to establish the suitability of using equations according to the approach used: (i) for the approach of convection, equations with a high convective heat flux (the equations of Giesecke (Eq. (16)) and of Khalifa and Marshall (Eq. (29))) were those which achieved a larger number of representative results; and (ii) for the approach of convection and radiation, correlations of Alamdari and Hammond (Eq. (24)), of Min et al. (Eq. (17)), of Holman (Eq. (27)), and of Fohanno and Polidori (Eq. (34)) obtained valid results in the three kinds of wall analysed, with an adjustment degree lower than 10% in some of the results.
- Concerning correlations of dimensionless numbers, only the approach of convection and radiation obtained a high number of representative results in the walls analysed, with most deviations of less than 5%. From the 20 equations analysed, only the correlation of Wong (Eq. (46)) did not have representative results.
- The analysis of representative results of the two kinds of ICHTCs allowed to determine that correlations of dimensionless numbers for the approach of convection and radiation presented a percentage of representative results higher than 80%, whereas correlations of temperature differences reached percentages between 19.44 and 25%. Thus, the use of ICHTCs depending on dimensionless numbers was the most efficient approach for the quantitative internal infrared thermography method. Regarding the influence of using ICHTCs depending on dimensionless numbers, walls with insulation obtained a larger number of representative results than the wall without insulation (Wall A).

To conclude, the results obtained in this study could be useful for energy auditors to characterize the thermal transmittance of façades in existing buildings. In this regard, results are useful to determine the adequate approach for the quantitative internal infrared thermography method, and therefore to propose efficient energy conservation measures and to reduce both the energy consumption and the environmental impact. Given that the typology of wall analysed (brick walls with or without insulation) is common in many countries (e.g. Italy and United Kingdom), the results obtained are extrapolated for other regions. Furthermore, the short time required to perform tests of the quantitative internal infrared thermography method guarantees a high assessment tax of the existing building stock in a short period of time. On the other hand, the high adjustment degree obtained with correlations of dimensionless numbers endorses the last research studies on the quantitative internal infrared thermography method [26,29]. Further steps of this study will be focused on the analysis of ICHTC correlations in light walls.

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