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A Methodology for the Analysis of In-Core Fuel Management Configurations in BWR's

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Abstract

A configuration for the integral problem of in-core fuel management consists of a fuel lattice and a fuel assembly designs, as well as a fuel reload and a control rod pattern design, so that the different criteria for the proper operation of a BWR reactor are met. Such a configuration or solution is not unique. There is a universe of potential candidate solutions that solve the problem in different ways. Departing from a database of candidate configurations, we describe and show the application of a multicriteria analysis methodology based on intervals and possibility function. The nuclear engineer must define his/her preferences in terms of the relevance assigned to the configurations' features and next, using the methodology, those configurations are ranked. Besides this, the methodology can help us to identify the role that play some fuel lattice variables into the in-core fuel problem performance.

Keywords: BWR, Reactor Simulation, Optimization Criteria

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1. Background

The core fuel management in a BWR reactor requires the solution of four problems. These problems are 1) the design of fuel lattice, 2) the design of fuel assemblies, 3) the design of fuel reload, and finally 4) the design of control rod patterns. Solving these problems yields what is called the integral design of an operation cycle.

These problems are modelled as optimization problems, where the aim is to maximize the effective neutron multiplication factor (k_{EOR}) at end of rated while taking into account nuclear reactor's safety limits. The following core parameters are considered in each problem:

- *Fuel lattice design*: local power peaking factor, neutron infinite multiplication factor and average fuel lattice enrichment at the beginning of its lifetime.
- *Axial fuel assembly design*: axial power distribution and average enrichment of the assembly.

- *Fuel reload design*: k_{EOR}, thermal limits at the end of the cycle, cold shutdown margin at the beginning of the cycle and fuel fabrication costs.
- *Control rod pattern design*: thermal limits and effective neutron multiplication factor throughout of the cycle.

Numerous techniques have been employed to solve these problems in the context of the integral design of an operating cycle, typically producing excellent results as in: Ottinger et al (2015), Chien-Hsiang et al (2017), Chung-Yuan et al (2014), Lima-Reinaldo et al (2023)], Ortiz et al (2005), Castillo et al (2007), Francois et al (2013), Castillo et al (2014)].

These works follow an optimization approach: try to obtain the best solution in terms of an objective function. It is important to highlight that the solution for an integral design of an operating cycle is not unique. There is a universe of possible solutions that solve the problem in different ways. For example, solutions that give priority to the safety of reactor operation, at the cost of reducing energy production. Conversely, energy production can be maximized by relaxing or pushing the safety aspects to the limit. Other aspects can also be prioritized, such as those that facilitate reactor operation, for example by reducing control rod movements.

However, the way in which the solution of the integral design of an operation cycle is obtained, in the works cited above, always leads to having one or two solutions to the problem. That solution is the one that solved the problem in the best way according to the objective function used. With this, alternative solutions are discarded, which may be inferior in quality in terms of the objective function but offer other characteristics that could be interesting to analyze.

Departing from a set of candidate solutions (potential reactor' configurations), the aim of this work is to describe and show the application of a methodology that helps the nuclear engineer to select the "best" solutions in terms of his/her preferences. Such preferences are given as a linear order of the criteria/characteristics associated with the solutions.

The paper is structured as follows: we will first explain the way in which we solve each one of the stages of the integral problem of an operation cycle and then, we will explain the methodology that we have already mentioned and that was applied to our set of solutions, followed by the results obtained and finally, the conclusions we reached.

2. Integral design of an operating cycle

In the design of the operation cycle of a BWR, first the fuel lattice is designed to minimize the local power peaking factor (*LPPF*), keeping the neutron infinite multiplication factor (*k*inf) in a predetermined range. The designed fuel lattice is then used in all axial nodes of the fuel assembly, except in one bottom node and two top nodes. Subsequently, the fuel reload and the control rod patterns are optimized with which the fractions to the safety thermal limits are minimized, the cold shutdown margin is maximized and the power generation is maximized by maximizing the k_{EOR}. Since the fuel lattices are axially maintained throughout the assembly, the fuel assembly optimization is not necessary. So, from this point on we will only analyze the three remaining problems.

These optimization problems have three associated objective functions that are employed in order: the fuel lattice optimization function is used first, followed by the fuel reload optimization function, and finally the control rod pattern optimization function. The thermal limits, cold shutdown margin, k_{EOR}, *LPPF*, and *k*inf criteria are used to evaluate the reactor's performance. The distributions of uranium and gadolinium in the fuel lattice, the locations of the fuel assemblies inside the reactor core, and the axial positions of the control rods during the operation cycle make up the independent variables.

The objective functions have been reported in the different papers that our working group has previously published [Castillo et al 2004], [Castillo et al, 2007], [Castillo et al, 2014], [Ortiz et al, 2019].

Objective function for the fuel design problem

It is expected that, if the fuel lattice is good, the reactor core will have a better performance. The function is defined as follows:

$$\min\ F_1 = LPPF \cdot w_1 + K(k\,\mathrm{inf})$$

$$where \qquad (1)$$

$$K(k\,\mathrm{inf}) = \begin{cases} \left|k\,\mathrm{inf}\right| \cdot w_2 & if \quad \left|k\,\mathrm{inf} - k\,\mathrm{inf}_{tar}\right| > 0.005 \\ & 0 \end{cases}$$

where

LPPF : local power peaking factor at the beginning of the fuel lattice life.
*k*inf : neutron infinite multiplication factor at the beginning of the fuel lattice life.
$k\mathrm{inf}_{tar}$: target neutron infinite multiplication factor.

Objective function for the refueling design

$$\max\ F_2 = keff \cdot w_1 + \Delta\lim{}_1 \cdot w_2 + \Delta\lim{}_2 \cdot w_3 + \Delta\lim{}_3 \cdot w_4 + + \Delta\lim{}_4 \cdot w_5 \qquad (2)$$

where

keff : effective multiplication factor at the end of cycle according to a Haling calculation

$\Delta\text{lim}_1 = MFLPD_{lim} - MFLPD_{obtained}$

$\Delta\text{lim}_2 = MAPRAT_{lim} - MAPRAT_{obtained.}$

$\Delta\text{lim}_3 = MFLCPR_{lim} - MFLCPR_{obtained}$

$\Delta\text{lim}_4 = SDM_{obtained} - SDM_{lim}$

and

MFLPD : Fraction to the Linear Heat Generation Rate at the end of cycle according to a Haling calculation

MAPRAT : Fraction to the Average Planar Linear Heat Generation Rate at the end of cycle according to a Haling calculation

MFLCPR : Fraction to the Critical Power Ratio at the end of cycle according to a Haling calculation

SDM : Cold shutdown margin at the beginning of the cycle

Objective function for the control rod pattern design problem

Finally, the objective function for the optimization of the control rod patterns is as follows:

$$\max\ F_3 = k_{EOR}\cdot w_1 - \sum_{i=1}^{n-1}\left|keff^i - k_{crit}\right|\cdot w_2 + \sum_{i=1}^{n}\Delta\lim{}_1^i\cdot w_3 + \sum_{i=1}^{n}\Delta\lim{}_2^i\cdot w_4 + \sum_{i=1}^{n}\Delta\lim{}_3^i\cdot w_5 + \Delta\lim{}_4\cdot w_6 \tag{3}$$

where

k_{EOR} : Obtained Effective Multiplication Factor at the end of rated conditions according to a simulation with control rod patterns

$keff^i$: Obtained Effective Multiplication Factor in each burnup step

k_{crit} : Objective Effective Multiplication Factor

Δlim_1^i : $MFLPD_{i,lim} - MFLPD_{i,obtained}$

Δlim_2^i : $MAPRAT_{i,lim} - MAPRAT_{i,obtained}$

Δlim_3^i : $MFLCPR_{i,lim} - MFLCPR_{i,obtained}$

Δlim_4 : $SDM_{obtained} - SDM_{lim}$

n : number of burnup steps throughout the operation cycle.

MFLPD, *MAPRAT*, *MFLCPR* are obtained for each burnup step and *SDM* is calculated at the beginning of the cycle.

All the above parameters are obtained by applying the commercial 2D and 3D codes respectively CASMO4 [Rhodes and Edenius, 2004] and SIMULATE3 [Dean, 2005].

In the following, we will describe the methodology used and subsequently how it was implemented to our problem.

3. Multicriteria analysis based on intervals and possibility distribution function.

The methodology presented appears in [Torres, 2021] and the starting point is a matrix $A^{n \times m}$ where each row represents a solution to the problem under consideration and each column an evaluation criterion. Thus, each element $a_{ij} \in \Re$ represents the quality of solution i with respect to criterion j (c_j).

Furthermore, we consider that the decision maker can establish an order of importance for the criteria. Without loss of generality, suppose the order is $c_1 \geq c_2 \geq \cdots \geq c_m$, where $c_i \geq c_j$ indicates that criterion i is preferred to criterion j. This translates to a set of weights $w_1 \geq w_2 \geq \cdots \geq w_m$, with $w_j \geq 0 \ \forall j$, and $\sum w_j = 1$.

Then the overall quality or score of a solution i can be calculated as.

$$\sum_{j=0}^{m} w_j \cdot a_{ij} \quad (4)$$

Since there are infinite sets of weights that verify the above constraints, Torres et al [Torres 2021] propose to find the maximum and minimum score values that a solution can reach. For this purpose, two linear programming problems are solved, where w_j are the variables.

In this way, instead of assigning a single score value to a solution i, an interval $[l_i,u_i]$ is assigned where the values represent the minimum/maximum score that the solution can reach. The prioritization of solutions is then based on working with the intervals. For this purpose, a reference solution s^* is identified as the one that has the maximum l_i, $\forall i$. That is, the one that guarantees a minimum score as high as possible. Subsequently, the remaining solutions are compared against s^* using a possibility function which, in simple terms, calculates the overlap of two intervals.

Let's consider two solutions/reactor' configurations X, Y with their corresponding intervals $X = [x_l, x_u], Y = [y_l, y_u], x_l, x_u, y_l, y_u > 0$. The possibility degree of solution X being better than Y is $P(X \geq \ Y)$ is defined considering two cases:

1. If the intervals do not overlap, then

$$P(X \geq \ Y) = \begin{cases} 0, & x_r < y_l \\ 1, & x_l \geq y_r \end{cases}$$

2. If the intervals overlap, then

$$P(X \geq Y) = (x_r - y_l)/(x_r - x_l + y_r - y_l)$$

Thus, if we translate the above methodology to our particular problem, then each criterion (c_j) will be a nuclear parameter involved in the integral problem of an operation cycle, for example, c_1 can be the *LPPF* of the nuclear fuel lattice, so the column $a_{1,j}$ will contain all the *LPPF* values for each of the fuel lattices we have in our database. This explanation applies to all the parameters (criteria) that we consider for our analysis.

In general, the rows of the matrix $A^{n \times m}$ will be all the parameters of a particular solution we have in our database, so *m* will be equal to the number of parameters/characteristics we take into account. It is important to highlight that the methodology allows to involve parameters (criteria) that are not considered in the different objective functions.

Now, the nuclear engineering can define a "profile" for analyzing the alternatives available. For example, let c_1, c_2, c_3 and c_4 be the criteria corresponding to *LPPF*, *k*inf, *U%* and *SDM* respectively. The order

$$c_4 \geq c_2 \geq c_3 \geq c_1$$

indicates that the *SDM* is being assigned a higher priority than the other three parameters, so the methodology gives us a range of values where the *SDM* will be maximized.

4. Case study: results and discussion

We depart from a set of 350 solutions initially evaluated on 8 criteria coming from the integral design problem of an operation cycle. The size of the matrix A is 350x8. Each solution represents a way to operate the reactor in an operation cycle. The 8 criteria are related to the reactor performance according to a computer simulation of the fuel lattice design, the distribution of fuel assemblies inside the core, and the full power control rod patterns that could be used in such a cycle. Also, bad solutions were added to the database to evaluate the performance of the methodology. The solutions were evaluated and qualified with Equations (1) through (3). The criteria used to evaluate the solutions are listed below:

1) Local Power Peaking Factor (*LPPF*) at the beginning of fuel lattice life
2) Neutron Infinite Multiplication Factor (*k*inf) at the beginning of fuel lattice life
3) The average enrichment of the fuel lattice (*U%*)
4) Cold Shutdown Margin (*SDM*) at the beginning of the cycle
5) The greatest fraction of LHGR Thermal Limit (*FLPD*) throughout of the cycle
6) The greatest fraction to APLGHR (*MAPRAT*) throughout of the cycle
7) The greatest fraction to CPR (*FLCPR*) throughout of the cycle
8) The effective neutron multiplication factor at the end of full power operation k_{EOR}

It is important to remember that, once the criteria have been established, a profile is the importance order of the criteria according to expert's judgment, which may not correspond to the criteria's original specification. Now, we will analyze the solutions according with different operations profiles

4.1 Basic profiles

To apply the described methodology, the following basic profiles are defined:

a) **Energy Profile**: where the most relevant criteria is the energy production subject to safety constraints.
b) **Safety Profile:** where the most relevant criteria is the safety margins. This profile should ensure that at least the minimum required amount of energy is produced.

Table 1 shows the order of priority of the criteria for establishing each of these profiles. Note that in the Energy profile, the criterion with the highest priority is k_{EOR}, while in the Safety profile, the *FLPD* criterion has the highest priority.

Table 1. Priority of the variables of the Energy and Safety profiles.

Variable	Energy Profile	Security Profile
LPPF	6	6
*k*inf	7	7
U%	8	8
SDM	3	2
FLPD	2	1
MAPRAT	4	3
MFLCPR	5	4
k_{EOR}	1	5

Figure 1 shows the score interval [l_i, u_i] of each of the 350 solutions in the database corresponding to the Security Profile. Each horizontal line in this figure represents the score interval of each solution. The reference solution s^* is highlighted in green at the top of the graph. It must be remembered that the reference solution is the one with the largest value of l_i, so the dotted vertical line shows the minimum value of s^*. The possibility function indicates the degree of overlap of two intervals. Thus, the solutions in the lower portion of the figure are those that will never be better than the reference solution, because they will never attain higher score values than the reference solution, with any combination of weights for the priority order of criteria established. The solutions near the top of the picture, in comparison, exhibit significant interval overlap with the reference solution.

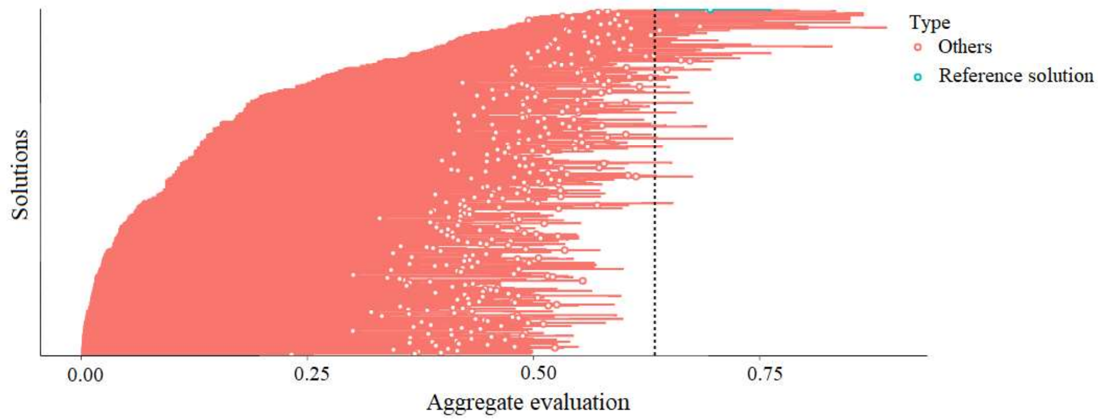

Figure 1. Score intervals of the database solutions according to the Security Profile.

This basic analysis allows to discard most of the available solutions and to concentrate the efforts in the relevant ones. Figure 2 shows the 20 solutions that have the highest interval overlap with the reference one for the Safety Profile. The circular mark in the interval corresponds to the score value of the solution if all weights are equal. The reference solution for this profile is A289, which has a l_i value of 0.634, an u_i value of 0.763, and a score value of 0.697 when all the weights are equal. The u_i value of the A288 solution is greater than that of the reference and l_i value of the A288 solution es lower than the reference one. In other words, there are several weight combinations that make the A288 solution worse than the reference one, but there are several weight combinations that make the A288 solution better than the reference one as well. The nuclear engineer should focus on those solutions having a higher u_i than that of the reference because they have the potential to be better.

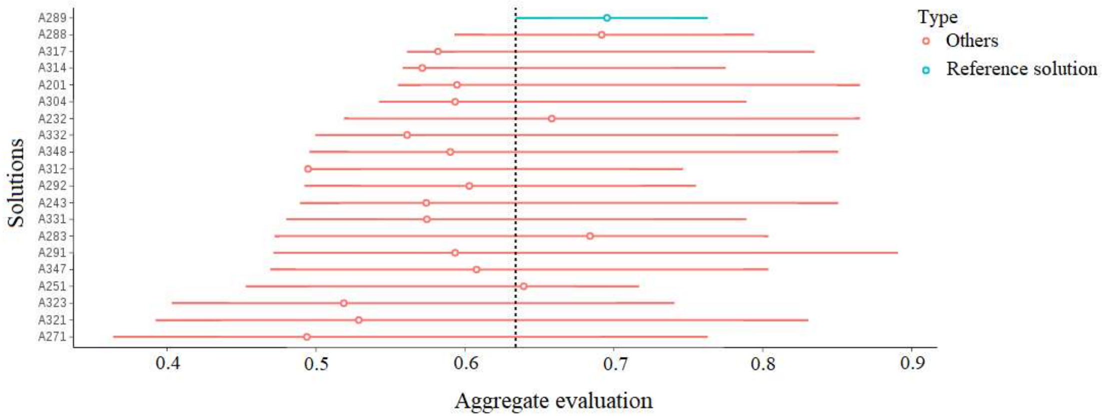

Figure 2. Score intervals of the 20 solutions that have the highest overlap with the reference solution for the Safety Profile.

Next, we analyze those solutions that have high overlap values with the interval of the reference solution. For each profile, the reference solution was identified, as well as the 10 solutions with the highest possibility value. Table 2 shows the results for the Energy Profile and Table 3 shows the results for the Safety Profile. The solutions marked in bold indicate the reference solution. The bottom of both tables displays the five worst answers determined by the methodology.

Table 2. Reference and top 10 solutions for the Energy Profile.

Sol	*LPPF*	*k*inf	*U%*	*SDM*	*FLPD*	*MAPRAT*	*FLCPR*	k_{EOR}
A288	**1.170**	**1.152**	**4.057**	**1.156**	**0.855**	**0.847**	**0.878**	**0.999**
A289	1.170	1.152	4.057	1.215	0.861	0.850	0.911	0.997
A347	1.173	1.161	4.000	1.014	0.853	0.850	0.902	1.000
A250	1.219	1.144	3.997	1.312	0.930	0.902	0.878	0.999
A238	1.222	1.138	3.981	1.116	0.901	0.866	0.879	1.002
A348	1.173	1.161	4.000	1.019	0.842	0.875	0.921	0.998
A296	1.169	1.151	4.054	1.552	0.919	0.926	0.926	0.999
A277	1.201	1.152	4.070	1.126	0.910	0.874	0.884	1.002
A340	1.218	1.160	3.983	1.136	0.884	0.852	0.921	0.999
A350	1.173	1.161	4.000	1.042	0.873	0.887	0.908	0.998
A210	1.248	1.143	3.970	1.358	0.944	0.857	0.908	0.999
A120	1.263	1.161	4.051	1.488	0.991	0.933	0.827	0.989
A088	1.270	1.156	4.052	0.750	1.020	0.947	0.832	0.989
A159	1.250	1.148	4.047	0.849	1.015	1.032	0.829	0.987
A151	1.257	1.162	4.049	0.938	1.024	1.054	0.799	0.986
A222	1.274	1.140	3.982	1.118	0.976	0.903	0.818	0.985

Table 3. Reference solution and top 10 Safety Profile.

Sol	*LPPF*	*k*inf	*U%*	*SDM*	*FLPD*	*MAPRAT*	*FLCPR*	k_{EOR}
A289	**1.170**	**1.152**	**4.057**	**1.215**	**0.861**	**0.850**	**0.911**	**0.997**
A288	1.170	1.152	4.057	1.156	0.855	0.847	0.878	0.999
A317	1.213	1.162	4.027	1.131	0.846	0.802	0.867	0.995
A314	1.213	1.162	4.027	1.297	0.876	0.815	0.866	0.992
A201	1.248	1.143	3.970	1.161	0.838	0.772	0.833	0.989
A304	1.208	1.166	4.077	1.120	0.856	0.814	0.875	0.996
A312	1.213	1.162	4.027	1.176	0.864	0.842	0.856	0.990
A232	1.222	1.138	3.981	1.049	0.838	0.846	0.847	0.993
A318	1.213	1.162	4.027	1.184	0.893	0.897	0.899	0.996
A332	1.218	1.160	3.983	1.100	0.842	0.856	0.884	0.994
A342	1.173	1.161	4.000	1.138	0.879	0.875	0.853	0.993
A178	1.262	1.153	4.097	1.825	1.211	1.050	0.925	1.003
A126	1.232	1.149	4.126	1.960	1.240	1.108	0.962	1.007
A131	1.257	1.168	4.056	1.739	1.266	1.094	0.947	1.001
A189	1.267	1.154	4.049	1.776	1.337	1.136	0.991	1.002
A050	1.226	1.152	4.054	1.112	1.378	1.259	1.113	0.992

We can highlight the following aspects:

- In both profiles, solutions A288 and A289 are chosen within the group of the ten best solutions.
- Table 2 shows that the k_{EOR} values are higher than values in Table 3. This means that, indeed, the methodology was able to identify the solutions that produce more energy.
- Table 2 shows that the fractions to the thermal limits are higher than those reported in Table 3. This indicates that the methodology was able to identify solutions with better safety margins. In the case of the *SDM*, Table 3 has worse values than those in Table 2. This may be the case since we put the thermal limits first in this profile's priority list before the *SDM*.
- Solution A277 has a greater k_{EOR} than the reference solution, as seen in Table 2. Because it generates more energy, solution A277 has a good possibility of outperforming the reference solution. However, because it has smaller safety margins for both thermal limits and cold shutdown margin, it was not chosen as the reference solution. The reactor engineer will now step in and may opt to select solution A277 and disregard the reference one.
- As shown in Table 3, the solution A314 does not fulfill the energy requirement but has better safety margins than the reference. For this profile there is no doubt that the reference solution is the one that should be selected.
- By analyzing both tables, we can see that there is a set of solutions that can be chosen depending on what the expert considers most important.

4.2 Modified Profiles

Tables 2 and 3 show the application of the methodology using the traditional criteria involved into the objective functions. To examine the benefits of the methodology, 7 additional criteria were added to the matrix *A* which now has a size of 350 rows by 15 columns. These new criteria are listed below:

a) The number of different U% enrichments used in the fuel lattice design (NU%).
b) The ¼ symmetry of the fuel lattice (SymFL), which is calculated according to the equation included in Castillo-Méndez et al (2016), section 4.2
c) The number of high enrichment rods next to the water channels (NBUA).
d) The radial power factor in the core (RPF) and its radial position (NodRPF)
e) The radial kinf factor in the core (KIN) and its radial position (NodKIN).

These criteria were chosen because there is an interest to determine if they can help to identify good configurations. In that case, these criteria will be used to improve surrogate models like decision trees.

The values for these conditions were taken from the reactor simulator output files, and for the fuel lattice symmetry scenario, the necessary calculations were done. According to Eqs. (1, 2 and 3) we can verify that these criteria are not part of any of the objective functions that were used to obtain our database. Thus, two modified Energy-M and Safety-M profiles are created with the order of priority of the criteria as shown in Table 4.

Table 4. Priority order of the Energy-M and Safety-M Profiles.

	Energy-M	Segurity-M
LPPF	6	6
*k*inf	7	7
U%	8	8
NU%	10	10
SDM	3	2
FLPD	2	1
MAPRAT	4	3
MFLCPR	5	4
kEOR	1	5
NBUA	11	11
SymFL	9	9
RPF	12	12
NodRPF	13	13
KIN	14	14
NodKIN	15	15

The results for the Energy-M and Safety-M profiles are shown in Tables 5 and 6, respectively Again, the 5 worse solutions are shown at the bottom of each Table.

Table 5. Best alternatives using the Energy-M profile.

Sol	*LPPF*	*k*inf	*U%*	*SDM*	*FLPD*	*MAPRAT*	*FLCPR*	k_{EOR}	NU%	NBUA	Sym FL	RPF	NodRPF	KIN	NodKIN
A288	**1.170**	**1.152**	**4.057**	**1.156**	**0.855**	**0.847**	**0.878**	**0.999**	**13**	**18**	**0.345**	**1.386**	**2**	**1.058**	**5**
A289	1.173	1.161	4.057	1.014	0.853	0.850	0.902	1.000	15	11	0.477	1.429	2	1.060	5
A347	1.173	1.161	4.000	1.019	0.842	0.875	0.921	0.998	15	11	0.477	1.43	2	1.060	5
A250	1.170	1.152	3.997	1.215	0.861	0.85	0.911	0.997	13	18	0.345	1.505	2	1.043	3
A238	1.219	1.144	3.981	1.312	0.930	0.902	0.878	0.999	13	12	0.787	1.444	3	1.050	5
A296	1.169	1.151	4.054	1.552	0.919	0.926	0.926	0.999	12	18	0.309	1.536	1	1.094	1
A348	1.173	1.161	4.000	1.019	0.842	0.875	0.921	0.998	15	11	0.477	1.430	2	1.060	5
A350	1.173	1.161	4.000	1.042	0.873	0.887	0.908	0.998	15	11	0.477	1.450	2	1.060	5
A210	1.222	1.138	3.970	1.116	0.901	0.866	0.879	1.002	14	12	0.884	1.592	2	1.045	5
A349	1.173	1.161	4.000	1.057	0.897	0.892	0.882	0.998	15	11	0.477	1.545	2	1.061	5
A120	1.263	1.161	4.051	1.488	0.991	0.933	0.827	0.989	19	14	1.773	1.461	3	1.061	5
A088	1.270	1.156	4.052	0.750	1.020	0.947	0.832	0.989	18	16	1.809	1.470	2	1.068	5
A159	1.250	1.148	4.047	0.849	1.015	1.032	0.829	0.987	19	11	1.626	1.477	2	1.056	5
A151	1.257	1.162	4.049	0.938	1.024	1.054	0.799	0.986	18	11	1.529	1.364	3	1.064	5
A222	1.274	1.140	3.982	1.118	0.976	0.903	0.818	0.985	13	12	0.858	1.517	1	1.097	1

Table 6. Best alternatives using the M-Security profile.

Sol	*LPPF*	*kinf*	*U%*	*SDM*	*FLPD*	*MAPRAT*	*FLCPR*	k_{EOR}	NU%	NBUA	Sym FL	RPF	NodRPF	KIN	NodKIN
A289	**1.170**	**1.152**	**4.057**	**1.215**	**0.861**	**0.850**	**0.911**	**0.997**	**13**	**18**	**0.345**	**1.505**	**2**	**1.043**	**3**
A288	1.170	1.152	4.057	1.156	0.855	0.847	0.878	0.999	13	18	0.345	1.386	2	1.058	5
A317	1.213	1.162	4.027	1.131	0.846	0.802	0.867	0.995	15	5	0.599	1.385	4	1.061	5
A314	1.213	1.162	4.027	1.297	0.876	0.815	0.866	0.992	15	5	0.599	1.398	4	1.068	5
A201	1.248	1.143	3.970	1.161	0.838	0.772	0.833	0.989	15	11	1.067	1.423	2	1.047	5
A304	1.208	1.166	4.077	1.120	0.856	0.814	0.875	0.996	13	10	0.640	1.482	3	1.054	3
A232	1.222	1.138	3.981	1.049	0.838	0.846	0.847	0.993	14	12	0.884	1.339	4	1.030	6
A318	1.213	1.162	4.027	1.184	0.893	0.897	0.899	0.996	15	5	0.599	1.447	3	1.056	5
A332	1.218	1.160	3.983	1.100	0.842	0.856	0.884	0.994	14	10	0.492	1.456	3	1.068	5
A342	1.173	1.161	4.000	1.138	0.879	0.875	0.853	0.993	15	11	0.477	1.446	1	1.049	5
A178	1.262	1.153	4.097	1.825	1.211	1.050	0.925	1.003	18	16	1.545	1.744	2	1.065	3
A126	1.232	1.149	4.126	1.960	1.240	1.108	0.962	1.007	18	12	1.646	1.692	2	1.064	5
A131	1.257	1.168	4.056	1.739	1.266	1.094	0.947	1.001	19	15	1.560	1.631	2	1.072	3
A189	1.267	1.154	4.049	1.776	1.337	1.136	0.991	1.002	19	11	1.641	1.657	2	1.059	5
A050	1.226	1.152	4.054	1.118	0.976	0.903	0.818	0.985	13	12	0.858	1.517	1	1.097	1

From the above tables, the solutions marked in green are those that also appear in Tables 2 (Energy) and Table 3 (Safety). It can be seen that the order of some solutions changed. In the case of the modified Energy Profile, one of the new solutions (A296) has a better *SDM* value than the rest of the solutions. In the case of the modified Safety Profile, all the solutions are maintained, but with a different order.

Analyzing Tables 5 and 6, an interesting aspect stands out: the fractions of the thermal limits are lower for those cases in which the fuel lattice has a high ¼ symmetry. In a previous work (Castillo-Méndez et al, 2016) it was concluded that, although it cannot be categorically stated that this criterion should be minimized, it can be said that it is desirable to do so. Figures 3 and 4 show thermal limits behavior vs SymFL for Energy and Safety profiles respectively. Each point represents a thermal limit value of a fuel lattice. When the SymFL value decreases the thermal limits also. The best solutions are enclosed in blue rectangle. These figures show that low SymFL values always give good thermal limits, while high SymFL values can give both good or poor thermal limits.

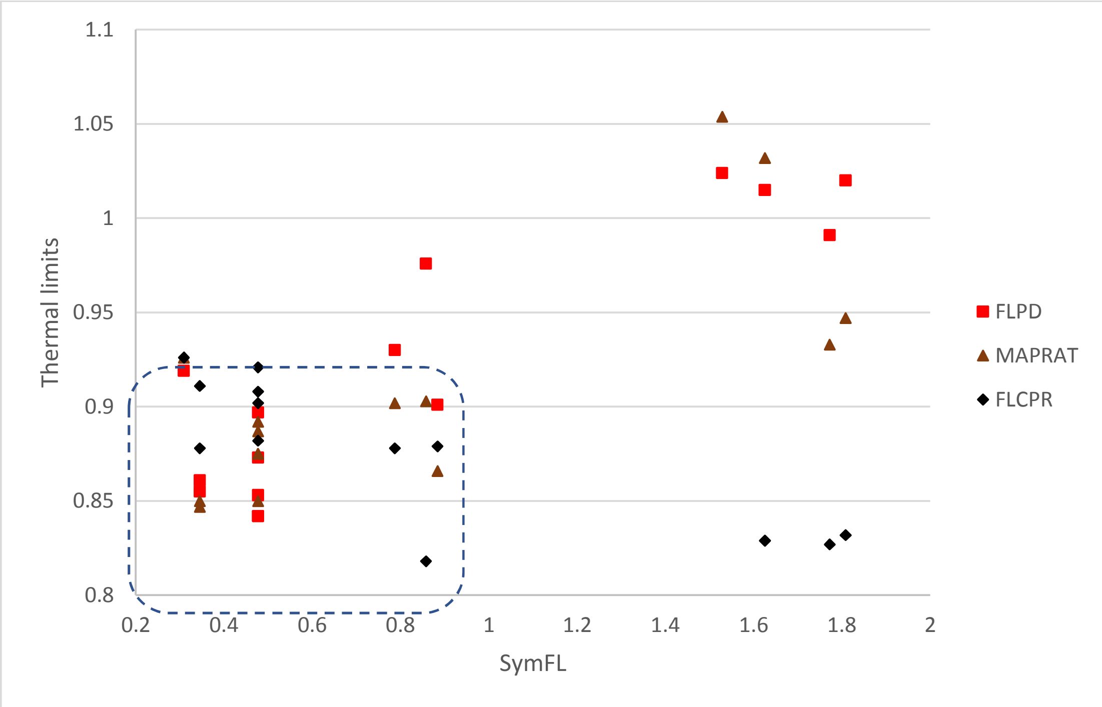

Figure 3. Thermal limits behavior against SymFL for Energy-M profile.

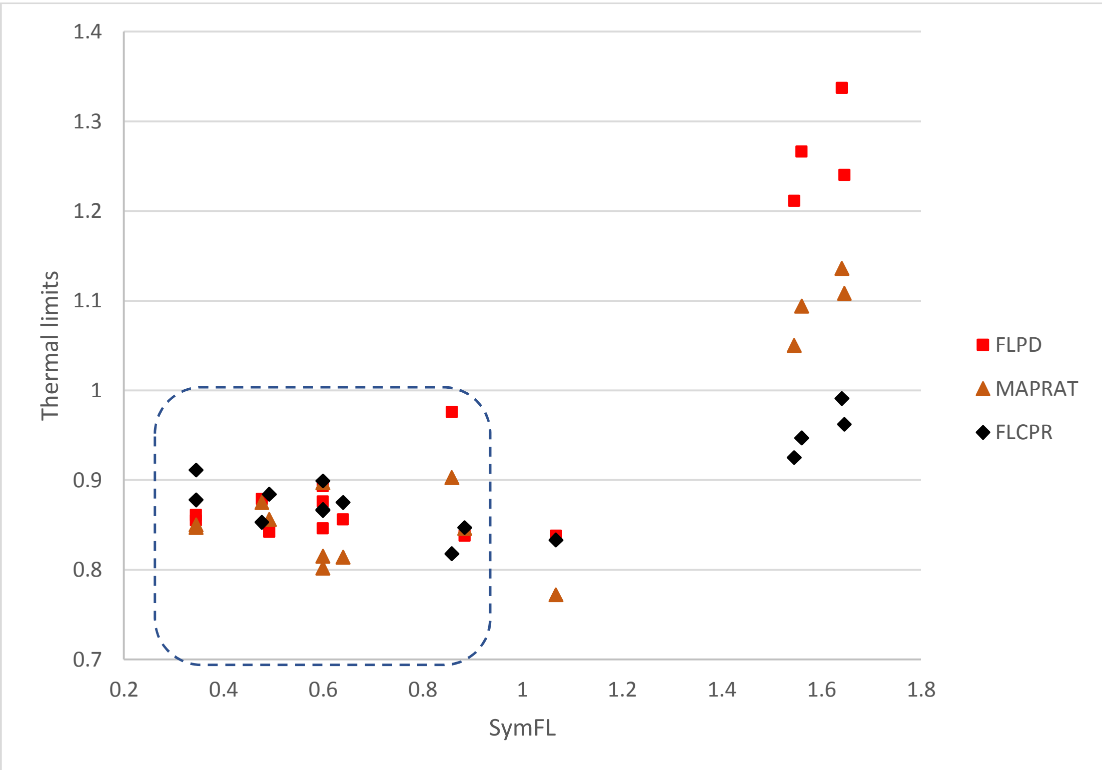

Figure 4. Thermal limits behavior against SymFL for M-Security profile.

It should also be noted that in the profile that promotes reactor safety, giving higher priority to thermal limits, the RPF values are lower than in the worst solutions. In the Energy profile no trend is seen with this variable. Another aspect that can be highlighted is that the best solutions in both profiles have low NU% values, i.e. the number of distinct U% enrichments.

The methodology helps us to identify important criteria which have not included in the objective functions. One may argue that these relevant criteria should be included in the mathematical models of the optimization problems. However, such inclusion may be difficult and moreover, can make the problem more difficult to solve. Using the proposed methodology, those criteria can be added *a posteriori* to the analysis of solutions avoiding the inclusion of additional complexities in the problem resolution.

4.3 Alternative Profiles

Once the way of working of this methodology has been presented, new profiles can be defined in which priority is given to the criteria according to particular interests. For example, giving more priority to the *SDM* criterion over the *FLPD*, let PSeg-SDM be this profile. Table 7 gives the priority order for this profile. The best solutions for this profile in Table 8 are shown. In this profile the worst solutions are no longer included.

Table 7. Priority order of the PSeg-SDM Profile.

	Energy-M
LPPF	6
*k*inf	7
U%	8
NU%	10
SDM	1
FLPD	2
MAPRAT	3
MFLCPR	4
kEOR	5
NBUA	11
SymFL	9
RPF	12
NodRPF	13
KIN	14
NodKIN	15

Table 8. Solutions for the PSeg-SDM profile.

Sol	*FPPL*	*k*inf	*U%*	*SDM*	*FLPD*	*MAPRAT*	*FLCPR*	k_{EOR}	NU%	NBUA	Sym FL	RPF	NodRPF	KIN	NodKIN
A314	**1.213**	**1.162**	**4.027**	**1.297**	**0.876**	**0.815**	**0.866**	**0.992**	**15**	**5**	**0.599**	**1.398**	**4**	**1.068**	**5**
A129	1.219	1.148	4.092	1.510	0.951	0.894	0.875	0.997	19	15	1.377	1.375	3	1.070	5
A289	1.170	1.152	4.057	1.215	0.861	0.850	0.911	0.997	13	18	0.345	1.505	2	1.043	3
A210	1.222	1.138	3.970	1.116	0.901	0.866	0.879	1.002	14	12	0.884	1.592	2	1.045	5
A316	1.213	1.162	4.027	1.348	0.908	0.887	0.856	0.994	15	5	0.599	1.363	4	1.061	5
A250	1.170	1.152	3.997	1.215	0.861	0.850	0.911	0.997	13	18	0.345	1.505	2	1.043	3
A204	1.248	1.143	3.970	1.432	0.953	0.908	0.883	0.997	15	11	1.067	1.535	2	1.064	5
A292	1.169	1.151	4.054	1.332	0.876	0.853	0.954	0.994	12	18	0.309	1.514	1	1.094	1
A296	1.169	1.151	4.054	1.552	0.919	0.926	0.926	0.999	12	18	0.309	1.536	1	1.094	1
A020	1.234	1.146	4.048	1.792	0.975	0.904	0.915	0.999	19	11	1.428	1.384	3	1.068	5

The best configuration, according to Table 6, is A289. However, Table 8 demonstrates that A314 significantly enhances the *SDM* without compromising the three thermal constraints. This solution was already included in Table 4.

Finally, a profile in which *SDM* and k_{EOR} are the two most important parameters can be proposed, which is namely PSegSDM-$\mathrm{k}_{\mathrm{EOR}}$ profile. Table 9 gives the priority order for this profile. The solutions corresponding to this profile are shown in Table 10. Again, in this profile the worst solutions are no longer included, and the reference solution is marked in bold. Table 10 no longer shows solution A314, which has a low value of k_{EOR}.

Table 9. Priority order of the PSeg-SDM-$\mathrm{k}_{\mathrm{EOR}}$ Profile.

	Energy-M
LPPF	6
*k*inf	7
U%	8
NU%	10
SDM	1
FLPD	3
MAPRAT	4
MFLCPR	5
kEOR	2
NBUA	11
SymFL	9
RPF	12
NodRPF	13
KIN	14
NodKIN	15

Table 10. Solutions for the PSegSDM-k_{EOR} Profile.

Sol	*FPPL*	kinf	*U%*	*SDM*	*FLPD*	*MAPRAT*	*FLCPR*	k_{EOR}	NU%	NBUA	Sym FL	RPF	NodRPF	KIN	NodKIN
A250	**1.17**	**1.152**	**3.997**	**1.215**	**0.861**	**0.85**	**0.911**	**0.997**	**13**	**18**	**0.345**	**1.505**	**2**	**1.043**	**3**
A296	1.169	1.151	4.054	1.552	0.919	0.926	0.926	0.999	12	18	0.309	1.536	1	1.094	1
A210	1.222	1.138	3.970	1.116	0.901	0.866	0.879	1.002	14	12	0.884	1.592	2	1.045	5
A129	1.219	1.148	4.092	1.51	0.951	0.894	0.875	0.997	19	15	1.377	1.375	3	1.07	5
A205	1.248	1.143	3.970	1.215	0.935	0.866	0.897	0.999	15	11	1.067	1.513	2	1.069	5
A289	1.17	1.152	4.057	1.215	0.861	0.85	0.911	0.997	13	18	0.345	1.505	2	1.043	3
A021	1.243	1.140	4.063	1.586	0.991	0.949	0.857	0.997	19	14	1.062	1.379	2	1.066	5
A249	1.219	1.144	3.997	1.312	0.939	0.908	0.926	0.998	13	12	0.787	1.444	3	1.050	5
A167	1.263	1.137	4.048	1.557	0.990	0.945	0.852	0.999	17	15	1.489	1.400	3	1.045	5
A295	1.169	1.151	4.054	1.228	0.916	0.912	0.931	0.997	12	18	0.309	1.556	1	1.092	1

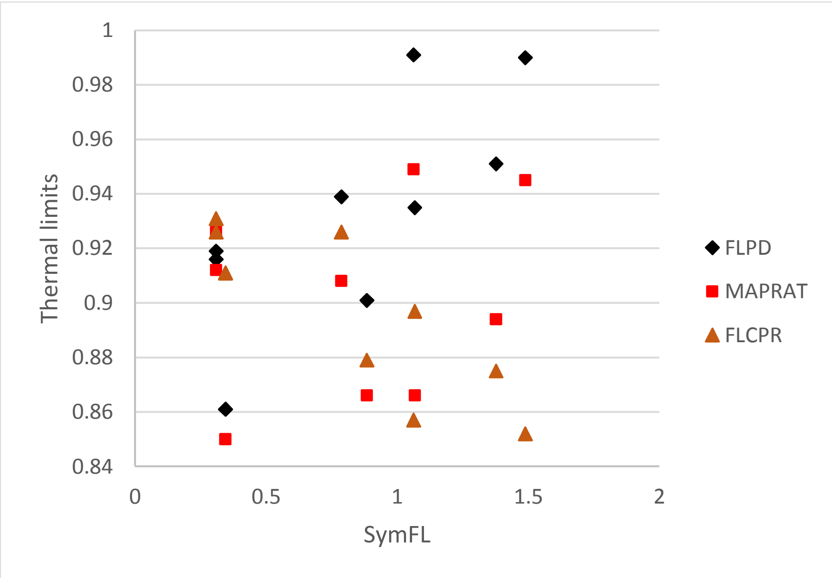

Figure 7. Thermal limits behavior against SymFL for best profile.

In summary, we have several alternatives on how to operate the reactor in a new cycle. These solutions are listed in Table 11. In Figure 5 the thermal limits behavior versus SymFL for the solutions of Table 11 is shown.

Table 11. Best solutions as a function of the profile.

	Energy Profile	Security Profile	Energy-M Profile	Security-M Profile	Security SDM Profile	SecSDM -kEOR Profile
Solution	A288	A289	A288	A289	A314	A250
LPPF	1.170	1.170	1.170	1.170	1.213	1.170
*k*inf	1.152	1.152	1.152	1.152	1.162	1.152
U%	4.057	4.057	4.057	4.057	4.027	3.997
SDM	1.156	1.215	1.156	1.215	1.297	1.215
FLPD	0.855	0.861	0.855	0.861	0.876	0.861
MAPRAT	0.847	0.850	0.847	0.850	0.815	0.850
FLCPR	0.878	0.911	0.878	0.911	0.866	0.911
k_{EOR}	0.999	0.997	0.999	0.997	0.992	0.997
NBUA	18	18	18	18	5	18
NU%	13	13	13	13	15	13
SymFL	0.345	0.345	0.345	0.345	0.599	0.345
RPF	1.386	1.505	1.386	1.505	1.398	1.505
NodRPF	2	2	2	2	4	2
KIN	1.058	1.043	1.058	1.043	1.068	1.043
NodKIN	5	3	5	3	5	3

5. Conclusions

The analysis methodology showed its usefulness in two aspects:

1. From a database of solutions for the integral problem of the operation cycle it discovered several solutions that were different to those found by the optimization system. They are solutions that, in general, allows the operation of a nuclear reactor's towards maximizing the energy production while satisfying the safety constraints. Instead of having to pick from a population of 50 or 100 alternatives, the technique makes it easy for the reactor owner to decide from a smaller set of potential configurations.
2. The identification of some criteria that were not considered in the optimization problems models but that indicate a tendency in those solutions that provide superior performance is another encouraging finding from the use of the technique. For instance:

    a) It is well known that low values of radial peaking factor in the core improve the thermal limits. The solutions found by the methodology fulfill this aspect.
    b) The integral fuel management problem's best solutions have a greater 1/4 fuel lattice symmetry.

c) Lower amount of different enrichment levels of uranium is found in the best integrated fuel management strategies.
d) Additional criteria can be computed or retrieved from the simulator output files to extend the database to create new profiles according to other interests.
e) We cannot identify relationships between variables NodRPF, KIN and NodKIN and the other core parameters.

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