Research article

Intimate partner violence: A novel warning system in which the victims' environment alerts to the danger

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ABSTRACT

Intimate Partner Violence (IPV) has become a problem of a great magnitude as it is a leading cause of death for women according to the World Health Organization. This paper presents a novel Early Warning System (EWS) for locating and protecting potential victims of IPV in which the victims' environment plays a central role. Specifically, based on Markov random fields, our EWS measures the danger through the perception that the victims' surroundings have. This new standpoint is of particular relevance since most of the IPV victims do not have a clear perception of the approaching danger.

The EWS has been designed without geographical constrains to be appropriate for use in every country of the world (universal EWS). Importantly, it may be rewritten into computational terms thereby providing a real tool for police investigations. Connections with text mining and sentiment analysis may provide additional devices based on this model.

1. Introduction

Intimate partner violence (IPV)[1] is defined as any behaviour which includes "physical violence, sexual violence, threats of physical or sexual violence, stalking and psychological aggression (including coercive tactics) by a current or former intimate partner", see [2]. Although most people believe that IPV is a substantial public health problem, few agree on its magnitude: Intimate partner violence is a leading cause of death for women, according to the World Health Organization: "violence against women: a global health problem of epidemic proportions", see [15]. Following [16] research on IPV has focus on two lines: on one hand, those studies which identify the primary causes of IP violence (mainly asymmetrical power relationships such as control on financial resources) in the hope that adequate re-educating programs and polices may result in changing the corresponding behaviours. On the other hand, those works which help women identify warning signs (red flags) or abusive attitudes which indicate that a relationship could become a toxic relationship.

Efforts of IPV awareness-raising are essential but other courses of action are needed in order to alert possible victims about increasing (which can evolve to become *imminent*) danger. Actually, a majority of potential victims are not aware of the precise level of danger they are exposed to. To this regard, standard assistance (social services, legal and police support) must be complemented with the implication of community, particularly of those people linked to the victims, who can detect some warning signs that would go unnoticed by the expert' eye. That is, the assessment of IPV risks may be complemented with the standpoint of victims' surroundings. This is the philosophy that underpins the Early Warning System for IPV developed in this paper, an specific tool for locating and protecting IPV victims which rests on the victims'environment. Any kind of support is key in IPV prevention, either in discouraging potential aggressors in continuing-escalating violence or in providing moral support. In this line, the methodology developed in this paper will provide victims with both material and psychological support, i.e., the real protection of an early alert system expressly designed for IPV as well as the phychological support of being sheltered by their environment.

More specifically, this paper presents a novel Early Warning System (EWS) for Intimate Partner Violence in which *the potential victims' neighbourhood plays a central role both in detecting and protecting the victims*. By elaborating a theoretical basis on graphical models (Markov random fields) the victim' environment conforms an informative network such that the EWS shall measure the danger throughout the perception that the victims' surroundings have. On one hand, this new standpoint is

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[1] Also known as Domestic or Gender Violence.

of particular relevance taking into account that most victims have no clear perception of the approaching danger. Actually, in our approach the environment warns of the danger even if the victims are not aware of it. On the other hand, to our best knowledge this is the first time that Markov random fields apply for designing Early Warning Systems aimed at protecting people. As a matter of fact, Markov random fields, [9], are mostly used in Computer Vision and Image Processing, [14], and in Geostatistics or related fields where distribution maps are required [5].

The proposed EWS has been developed through two alarm notifications: the primary one is designed aimed at covering the victims' environment viewed as a spatial network (it is here that Markov random fields play a role). Actually, the entire victims' environment is monitored by an explicit joint probability function which may trigger an alarm whenever a threshold is exceeded. Moreover, such a function has the advantage of depending only on a few significant nodes, the cliques. This data reduction, besides reducing storage costs while preserving the storage capacity for further requirements, shall result in an increase in decision speed. The secondary alarm notification covers the corresponding temporal sequences of the significant nodes thereby providing additional information as to whether the danger persists in time. All model variables may be freely enlarged and/or expanded to meet the needs of each context. Importantly, the model may be rewritten into computational terms thereby providing a *real* instrument for police investigations. Connections with text mining and sentiment analysis (see [1] or [3]) may provide additional devices based on this methodology.

Our objective is to develop an Early Warning System which may successfully protect people (women and their children) *anywhere in the world*. To this end, the current situation of fuzzyness around local demographics may be averted. By this we mean that there are great differences as to how countries define demography according to local specifications. In consequence, standard early warning systems (commonly supported by Geographical Information Systems) which mainly rely on local specifications, must be avoided. Under the objective of working on a *universal* basis, in our EWS local parameters have been changed by global notions, see [6] where the author designed a universal geolocator, "local constrains-free" or [7], where the need for universal tools is detailed.

The truth is that, while the fight against IPV can be conducted on many fronts, the main strands of research address IPV from phychological (model patterns of IPV) or medical standpoints (violence exposure to health-related outcomes, costs of IPV medical care). A more limited number of studies treats the problem from a computational/mathematical point of view. Amongst them, in the paper [10] the author designs a perpetrator's loss of control parameter based on a difference equation. Others are [11] where authors compare text exploration instruments (MDS -Multi-dimensional scaling- and ESOM -Emergent self organizing map) for automating the detection of domestic violence from the unstructured texts comprising the police reports. Or [4] where authors provide a quantitative approach of Intimate Partner Violence based on a system of difference equations. Apart from the present study, the author could not find any references of mathematical/computational methodologies on the IPV prevention and protection aimed at becoming real intervention protocols.

This paper is structured as follows. Section 2 provides the background on Markov random fields. Sections 3 is devoted to fully describing the EWS mathematical fundamentals. In sections 4 and 5, protocols of localisation and protection of IPV victims are developed. Specifically in section 5 the assessment of the information from the victims' neighbourhood provides an Early Warning System capable of alerting when the level of danger exceeds some threshold. In section 6, application and results are given. Section 7 concludes the paper.

2. Background on MRF

In Discrete Mathematics, a graph $G = (V, E)$ consists of a set of vertices (or nodes) V together with a set of edges E, each connecting a pair of vertices. A pair of vertices u and v are *adjacent*, denoted $u \sim v$, if there is one edge connecting them, i.e., $(u, v) \in E$. *Undirected graphs* are those with all the edges bidirectional while a graph with edges pointing in a single direction is called a directed graph. *Neighbourhood of a node* $u \in V$ is formed by all those vertices which are adjacent to u, $N(u) = \{v \in V | u \sim v\}$. It is well known that describing a topology through a system of neighbourhoods is equivalent to describing a graph, i.e., $v \in N(u)$ if and only if there is an edge connecting them, $u \sim v$. Finally, recall that *cliques* in a graph are those subsets of V consisting on either a single node or a set of nodes fully connected amongst them: $C \subseteq V$ is a clique if and only if $C \subseteq \{u, N(u)\}, \forall u \in C$.

Graphical models are graphs where the nodes correspond to random variables and the edges represent the statistical relationships amongst the variables. When viewed as spatial stochastic processes, graphical models are called *random fields*. The most common types of graphical models are Bayesian networks and Markov networks (or Markov random fields). The main difference between them is the underlying graph, directed for Bayesian networks and undirected for MRF. For our purposes of establishing an informative network around potential victims of IPV, we will focus on Markov random fields. Amongst other advantages, MRF (instead of Bayesian networks) will result in a reduction of data which provide both reduction in data storage as well as speed increase. This last feature is particularly relevant when decisions have to be made urgently.

Let $X = \{X_v | v \in V\}$ be a random variable with values in a finite set $V = \{1, 2, \ldots, n\} \times \{1, 2, \ldots, n\}$. $P[X]$ stands for the joint distribution of $X = \{X_v | v \in V\}$ in the sense that

$$P[X] = P[\{X_v = x_v | v \in V\}] = \{P[X_v = x_v] | v \in V\}.$$

Markov random fields (MRF) are a generalisation of the well-known Markov chains as long as they preserve their characteristic memoryless property (i.e., the probability of occurrence of a state depends only on the immediate previous one). Thus, $X = \{X_v | v \in V\}$ is said to be a Markov random field relative to $G = (V, E)$ whenever the joint distribution of X, $P[X]$, depends only on the *nearest nodes* with no inference from the more remote nodes. The specification of *nearest* is commonly done through the neighbourhood of a node v, $N(v)$. Thus $X = \{X_v | v \in V\}$ is a MRF if

$$P[X_v = x_v | X_{V-\{v\}} = x_{V-\{v\}}] = P[X_v = x_v | X_{N(v)} = x_{N(v)}].$$

Also, the concept of neighbourhood of a node results in the notion of *clique* C: $C \subseteq V$ is a clique if and only if $C \subseteq \{u, N(u)\}, \forall u \in C$. In our context of EWS for IPV, cliques in a Markov random field will play a key role in providing information for protecting IPV victims, as we shall see later. Let $X = \{X_v | v \in V\}$ be a MRF. Whenever the joint distribution of X is positive, $P[X]$ may be expressed as a product of functions $\phi_C(X_C)$ called clique potentials since they take only values on the cliques $C \subset V$:

$$P[X] = \frac{1}{Z} \prod_{C \in c} \phi_C(X_C).$$

The clique potentials can take many forms depending on needs. Their usual form is $\phi_C(X_C) = \exp(-f(C))$ where the function $f(C)$ is called *energy function* over values of the clique C and $Z = \sum \prod_{C \in c} \phi_C(X_C)$ is a normalisation constant. Then, $P[X]$ may be expressed as

$$P[X] = \frac{1}{Z} \exp\left[-\sum_{C \in c} f(C)\right], \quad \text{provided } P[X] \geq 0. \tag{1}$$

In literature, such a joint probability distribution is called a Gibbs distribution relative to the graph $G = (V, E)$, in a reference to the so-called Gibbs random fields. The equivalence between MRF and Gibbs random fields is given by the Hammersley-Clifford theorem, which states that both Markov and Gibbs random fields are essentially the same.
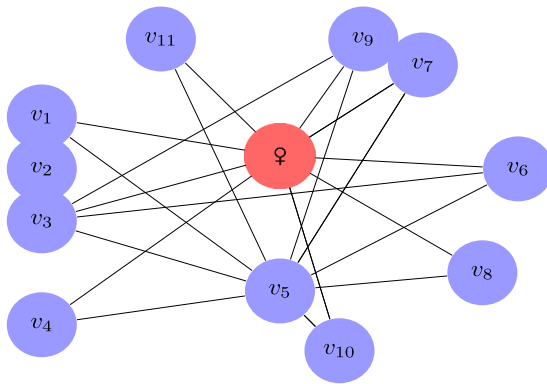
Fig. 1. A potential victim ♀' protecting network.

3. The EWS for intimate partner violence. Fundamentals

This section is aimed at developing the fundamentals of the proposed EWS for protecting potential victims of intimate partner violence (IPV).

Let ♀ be a potential victim of intimate partner violence. For our purposes, we shall consider ♀ as a network, $♀ \equiv (V_♀, E_♀)$, whose nodes consist of people which are connected to ♀ (and eventually, also connected amongst them) by non-tangible links like friendship, neighbourly, labour relationships etc. (see Fig. 1). Specifically,

Definition 3.1. In the network $♀ \equiv (V_♀, E_♀)$, each node $v \in V_♀$ will be identified with a random variable called in abstract X_v, which describes *the v' feeling that ♀ is in danger*.

We shall extensively describe X_v by a collection of features $x_v^k, k = 1, \dots, n$ acting as determinants of the feeling: i.e.,

$$v \simeq X_v \simeq (x_v^1, x_v^2, \dots, x_v^n)^t$$

such that each node is identified with an *n*-tuple *of random variables* $x_v^k, k = 1, \dots, n$. In other words, each node of the network $♀ \sim (V_♀, E_♀)$ (which is a person who somehow knows ♀) is identified with their perception that ♀ is in danger. When necessary, we refer to the random variable X_v as D_v (i.e., the *v*'s perception that ♀ is in danger).

The exhaustive list of either visible or non-visible signs of danger for intimate partner victims is incomprehensible since most of them depend on the particular context. In our model they can be freely selected depending on needs.

The passage from the random variable to a *feature vector*,[2]

$$X_v \simeq (x_v^1, x_v^2, \dots, x_v^n)^t \rightarrow (score_v^1, score_v^2, \dots, score_v^n)^t,$$

may be carried out by any realization of the variable. In general, any process of rating/scoring the feature coordinates $x_v^k, k = 1, \dots, n$ could be considered.

The graphical model $\{X_v, v \in V_♀\}$ would be fully defined once the edges are described. Recall that this task may be performed equivalently through the description of the neighbouring relationship. For this, a definition of distance between (realization of) two nodes should be specified:

Definition 3.2. The distance between two nodes $X_{v_i} = (x_{v_i}^1, \dots, x_{v_i}^n)^t$, $X_{v_j} = (x_{v_j}^1, \dots, x_{v_j}^n)^t$, $i \neq j$, written d_{ij}, is the Euclidean distance between their corresponding feature vectors:

$$d_{ij} = d(X_{v_i}, X_{v_j}) = +\sqrt{\sum_{k=1}^{n}(score_{v_i}^k - score_{v_j}^k)^2}. \qquad (2)$$

The definition of distance is of free choice in such a way that it may be selected as the one which best suits the particularities of each case. Actually, the degrees of freedom of the system as those items that may be freely selected depending on needs are set out below:

Remark 3.3 *(Levels of freedom).* Following items in the model are of free choice:

- **Vector coordinates and number of them**: for the random variable X_v which describes *the v' feeling that ♀ is in danger*, both vector coordinates themselves (i.e., the features $x_v^k, k = 1, \dots, n$) and number of them can be freely selected. Moreover, the random variables $x_v^k, k = 1, \dots, n$ may be considered as *weighted variables*, $w^k \cdot x_v^k, k = 1, \dots, n$ in order to reflect those features which are triggers.
- **The realization of the variable** (i.e., the passage from the variable to the numerical scores).
- **The definition of distance**. Recall that any set (particularly the set of vertices V) is a metric space whenever it is endowed with a function $d : V \times V \rightarrow \mathbb{R}$ holding the usual positivity, symmetrical and transitivity conditions. Thus, apart from the Euclidean one, any definition of such a function d on the set of vertices V could be considered as a measure of similarities amongst nodes.

Previous remark is specially necessary when an order on the set of nodes need not exist.[3] This is the case of $V_♀$, the set of all individuals who have a relationship of any kind with the potential victim. Thus, a metric d on V induces a topology on V. Particularly, a neighbourhood system

$$N = \{N(v) \text{ such that } v \in V\}$$

can be defined, where $N(v)$ denotes the neighbourhood of a node $v \in V$. To this regard, any definition of neighbourhood of a node is allowed as long as the neighbouring relationship holds, i.e., the following conditions must be satisfied: i) $v \notin N(v)$ and ii) $v_j \in N(v_i) \Leftrightarrow v_i \in N(v_j)$.

In our context of preventing from intimate partner violence, the neighbourhood on a node $v \in V_♀$ is defined as follows:

Definition 3.4. The neighbourhood of a node $v_i \in V_♀$, $N(v_i)$, is defined as

$$N(v_i) = \{v_j \in V_♀ \text{ such that } d(v_i, v_j) \leq k, \quad k \in \mathbb{R}, k \neq 0\},$$

where the indicator k should be specified for each case. Note that this definition meets the neighbouring relationship due to $k \neq 0$ and the symmetry of the distance. Hence, for $v_j \in N(v_i)$ an edge will join them (v_i, v_j) if $d(v_i, v_j) \leq k \neq 0$.

Let us consider that the feeling "someone is in danger" is quantified in increasing levels. Accordingly, the neighbourhood of a node may be detailed:

Proposition 3.5 *(Neighbourhoods). The neighbourhood of any node $v \in V_♀$ consists of those people linked to v who has the same level k of perception that v is in danger. That is, those individuals who somehow know v and they have a very similar feeling that v is in danger.*

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[2] In pattern recognition and machine learning, a feature vector is a *n*-dimensional vector of *numerical* scores representing an object (acting as a numerical label).

[3] When the ordering of the elements in V is specified, the neighbourhood can be determined more explicitly, as in Fig. 1.

Proof. From Definition 3.4, it is straightforward that both v_i and v_j' feeling that v is in danger are very similar since the distance between q_{v_i}, q_{v_j} is as short as it is k:

$$N(v_i) = \{v_j \in V \text{ such that } (v_i, v_j) \in E_\female\} = $$
$$= \{v_j \in V \text{ such that } d(v_i, v_j) \le k, \quad k \in \mathbb{R}, k \ne 0\}. \quad \square$$

The neighbourhood of a potential victim shall play a key role in protecting her. Let us detail who compose this security shield:

Corollary 3.6. *For a potential victim of intimate partner violence $\female$, her neighbourhood $N(\female)$ is formed by people with any type of relation, collaboration or interdependence on $\female$ with the same level k of perception that $\female$ is in danger.*

The neighbourhood of a node allows to consider the notion of *cliques* as maximally connected subgraphs of the graph. Recall that the $C \subseteq V$ is a clique iff $C \subseteq \{u, N(u)\}, \forall u \in C$. That is, cliques are groups with different number of members connected amongst them: they may consist of either a single-node $C_1 = \{c_i\}$, a pair of neighbouring nodes $C_2 = \{c_i, c_j\}$, a triple of neighbouring nodes $C_3 = \{c_i, c_j, c_k\}$ in such a way that the collection of all cliques is C:

$$C = C_1 \cup C_2 \cup C_3 \cup \ldots \cup C_i \cup \ldots$$

where C_i represents sets of larger cliques. Then, next result clarifies the notion of clique in our model:

Proposition 3.7 *(Cliques). A clique C_i is a group of individuals linked somehow to $\female$ who agree on the level k_i of danger to which $\female$ is exposed. Importantly, such degree of similarity k_i will vary for each C_i, i.e., nodes $c_1^i, c_2^i \in C_i$ have the same degree k_i while $c_1^j, c_2^j \in C_j$ would have a different degree of similarity $k_i \ne k_j$ whenever $i \ne j$.*

Proof. In general, a clique C is a set of nodes such that every two distinct vertices are adjacent. Hence, such nodes are in their corresponding neighbourhoods and, from Definition 3.4 for nodes $c_1, c_2 \in C_i$, it follows that $d(c_1^i, c_2^i) \le k$ for $k = max\{k_1, k_2\}$. $\quad \square$

Although in colloquial language cliques comprise all sort of individuals whom have a closer relationship amongst them, it is important to remark that in our model, neither neighbourhood nor cliques in $(V_\female, E_\female)$ consist of people who better know about $\female$ but those with *the same* feeling about she is in danger, no matter what type of relationship is involved. In few words, at heart this theoretical framework granulates the neighbourhood of $\female$ into subsets of levels of perception of the danger. Moreover,

Corollary 3.8. *Since cliques are connected groups within the network, each choice of the feature coordinates $x_v^k, k = 1, \ldots, n$ (see Remark 3.3) simulates grouping the nodes under a different criterium.*

Corollary 3.8 has significant implications as far as it makes possible to perform *sensitivity analysis* with respect to different criteria of grouping people who know $\female$ in order to improve the EWS efficiency. Furthermore, it is remarkable that the computation regarding these different possibilities may be executed in parallel thereby reinforcing the system' capability of making good decisions.

Once the basic notions of neighbourhood and clique have been clarified, we can prove the main results. On one hand, following lemma is very useful in practice:

Lemma 3.9. *Cliques in a network, considered as networks themselves, are MRF.*

Proof. The result follows from the definition of clique since they are subsets of some neighbourhood: $C \subseteq V$ is a clique if and only if f $C \subseteq \{v, N(v)\}, \forall v \in C$. Hence, inside cliques, the (Markov) local property holds trivially. $\quad \square$

At this point, recall that the joint distribution of the network is $P[X] = P[\{X_v = x_v | v \in V\}] = \{P[X_v = x_v] | v \in V\}$ or, with a more specific notation, $P[D] = P[\{D_v = x_v | v \in V_\female\}] = \{P[D_v = x_v] | v \in V_\female\}$. Independent variables have a joint probability distribution equal to the product of marginal distributions. However, since nodes in the same clique have a very similar perception of danger, the variables $\{D_v | v \in C_i\}$ (danger as perceived by the nodes in the ith-clique) are not independent. Nevertheless, by previous lemma, the joint probability distribution of each clique C_i, $P_{C_i}[D] = P[\{D_v = x_v | v \in C_i\}]$, is itself a clique potential function $\phi_{C_i}(D_{C_i})$, taking a more concrete form (e.g., quadratic functions) depending on each context. Usually, each $\phi_c(D_{C_i})$ is written in terms of energy functions $f(D_{C_i})$ such as

$$\phi_c(D_{C_i}) = e^{\frac{-1}{T} f(D_{C_i})}.$$

Thus, the level of danger for $\female$ as perceived by ith- clique is given by

$$P_{C_i}[D] = \phi_c(D_{C_i}) = e^{\frac{-1}{T} f(D_{C_i})}. \tag{3}$$

Either way, as such functions express the level of danger for $\female$ as perceived by her environment, this is the proof of the main results of the section:

Theorem 3.10. *Let $\female$ a potential victim of intimate partner violence. Then, each clique (i.e., each group of people with the same level of perception of danger) has associated a function which measures the danger to which $\female$ is exposed. When executed in parallel, once a given threshold is exceeded, they alert to the imminent danger.*

Corollary 3.11. *In the proposed EWS, the potential victim' environment warns of the danger even if the potential victim is not aware of it.*

The practical value of Theorem 3.10 and Corollary 3.11 is that they provide a simple way of specifying the joint probability $P[D]$ by choosing appropriate energy functions f. Moreover, the measure of danger by the potential victim' environment is of particular relevance for those potential victims who have no clear perception of approaching danger. Hence, previous results thereby reflect the spirit of the model by emphasizing the role of the potential victim' environment in alerting when danger is coming. It is also remarkable that both Theorem 3.10 and Corollary 3.11, may be rewritten in terms of a *protocol of action*.

On the other hand, let us pay attention to the whole. In contexts when the whole network is a MRF, the joint probability distribution may be computed in terms of clique potentials:

Theorem 3.12. *Let us suppose that $\female \equiv (V_\female, E_\female)$ is a MRF. Thus, the joint distribution $P[D], \quad D = \{D_v, v \in V_\female\}$ may be expressed as a function of clique potentials:*

$$P[D] = \frac{1}{Z} \prod_{C \in C} \phi_C(D_C) = \frac{1}{Z} e^{\frac{-1}{T} \sum_{c \in C}(D_C)} \tag{4}$$

where T is the temperature (often $T = 1$) and C represent cliques.

Proof. The Hammersley-Clifford theorem establishes the equivalence between Gibbs distributions and Markov random fields. Particularly, $D = \{D_v, v \in V_\female\}$ obeys a Gibbs distribution and the result follows. $\quad \square$

In contexts when the whole network is a MRF an extra-information should be provided. For instance, this allows to weight each clique (see

Table 1

Signs of that someone is potentially threatened.

Visible signs
Reductions in social functioning (derived from social self-isolation: from family, friends and community)
Reduction in purchasing habits (caused by financial control)
Increase in medical-care visits (perceived Health and medical itinerary)
Changes in the daily normal routes (to avoid being localised)
Physical changes Makeup to hide injuries Deterioration of the state of health (thinness, anxiety, depression)

Hao, Z. et al., 2018): specifically, a weight w may assigned to each clique C_i by comparing the joint distribution of each clique to the distribution of the whole network:

$$w_{C_i} = \frac{P[\{D, v \in C_i\}]}{P[\{D, v \in V_{\female}\}]}. \tag{5}$$

4. Localisation of potential victims

Section 3 provides a fruitful methodology for both localising and protecting potential victims of intimate violence. Particularly, this section will be focus on setting a procedure for locatising potential victims. To this regard, the original random variable $X = D$ (perception of danger) might be replaced by a more suitable one: a random variable whose corresponding list of features allows people to discover *significant changes in habits* related to the emergence of a threat. This new variable shall be called CHH.

The localisation procedure comes from a direct application of the theoretical framework developed in section 3. To this end, let CHH be the selected variable, aimed at gathering the most significant <u>ch</u>anges in <u>h</u>abits related to the emergence of a threat (changes in behaviour, physical changes etc.). Hence, CHH stands for "changes in habits" and CHH_v represents *changes in habits as perceived by v*, where v represents an individual.

For our purposes, we will make use of (some of) visible signs of being potentially threatened provided by Social and Psychological literature on IPV victimisation (see for instance [8] and [13]): warning signs of changes in habits which may be detected by outsiders are in Table 1.

On one hand, these visible signs may be considered as significant features for the variable CHH, amongst many others, thereby providing a complete list of features:

$$CHH = (chh_1, chh_n, \dots, chh_n)^t.$$

On the other hand, it should be noticed that the highest scores for "changes in habits" CHH shall correspond to situations where the potential victim is being suffering the highest degrees of control.

To complete the process of localising potential IPV victims, the steps to be taken are the following.

- According to Proposition 3.7, cliques must be extracted from the population under study.
  The process of identification of cliques corresponds to a clustering process on all available data. This gives rise a data granulation into groups (i.e., the cliques) composed by people for whom the variable $X_v = CHH_v$ *changes in habits as perceived by an individual v* takes similar values. Let it be noticed that, depending on the benchmark k (see Definition 3.4), the initial population is divided into groups of people who present different levels k of agreement.
- An assessment on the outcomes in order to sort out the highest ones should be made.

For this, Theorem 3.10 has to be applied. Accordingly, each clique (i.e., each group of people with similar values for changes in habits) has associated a function which measures the likelihood of having recently experimented relevant changes in behaviour (in responding to a potential threat, according to the Social and Psychological literature). This shall reveal the highest degrees of control which shall point out the potential victims in consequence.

Importantly, due to the specific weight of the selected random variable into the overall structural functioning of the model, slight changes either in it or in its features, may result in new information. Since more than one choice of the random variable could be appropriate, that suggests that many variations of the stated model may be executed in parallel in order to cross-reference the outcomes. The result is that different visible signs as features of CHH may be run in parallel in order to get extra information.

5. Protection of potential victims

The theoretical setting of section 3 will enable a protocol to be developed for taking action in situations that IPV victims require protection. Once an IPV potential victim $\female$ has been identified, the following protecting scheme is launched. At heart, the protection of IPV victims relies on two alarm notifications, an spatial alarm and a temporal one supported by a visual system. These are fully described next.

5.1. The spatial monitoring system: primary alarm notification

The primary alarm notification comes from the spatial model and it consists of the following steps.

i) The system starts by firstly identifying people who formed $\female$' environment, i.e., people with any type of relation (friendship, neighbourly, labour relationships), collaboration or interdependence on $\female$, either or not connected amongst them (as in Fig. 1). That gives raise the graphical model $(V_{\female}, E_{\female})$.

ii) Next, the corresponding information about the proximity of danger has to be collected from the environment, i.e., all data about the range of values of the random variable "v's perception that $\female$ is in danger" for every node v in $V_{\female}$. To this regard, recall that variations either in the random variable itself or in its features would produce extra-information. Hence, it is advisable to conduct more than one process. After rating/scoring the information, the neighbourhood of $\female$ becomes granulated into the different cliques in the network.

iii) The system proceeds by computing the corresponding distribution function for each clique: the more valuable information shall correspond to the higher values. Since variations this information may be processed in parallel, these indicators can be cross-referenced. The highest outcomes shall point out the riskiest stages.

To clarify further the point iii), some remarks have to be made. In Decision Making, an assessment process in a group of n people aimed at reaching an agreement is usually divided into two main strands: first, an individual assessment for each member of the group and second, an aggregation process of all individual decisions (for instance, an ordered weighted averaging aggregation operator, OWA). Such aggregation operator may be generally described as a function $F : \mathbb{R}^n \to \mathbb{R}$ which maps lists of real numbers (the n individual decisions) into a single outcome. As a result, a single score is obtained for the whole group, as shown in Fig. 2.

Note that the point iii) is equivalent to an assessment process over a clique: first, the information about the perception of danger is previously collected for individual nodes in the clique (point ii)) and second, the clique' distribution function works as an aggregation method labeling the clique with a single score. These marks (one for each clique) point the best way ahead as illustrated the following example:
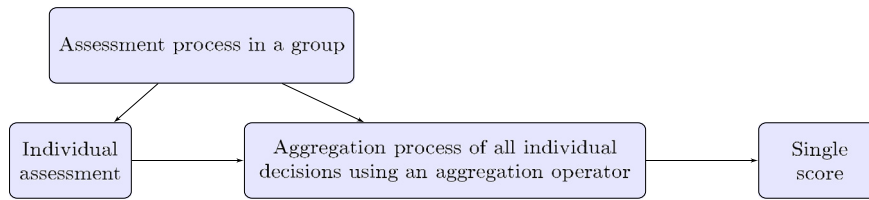
Fig. 2. Assessment process scheme in a group.

Example 5.1. Let us suppose that our universe is composed by 100 individuals, divided into 3 groups: G_l with 20 people, G_m with 30 people and G_h with 50 people. Suppose as well that people in G_1 are sure that there is a low level of danger, people in G_m agree that there is a medium level while people in G_h believe that the level of danger is high. Thus, these data would mean that

G_l with 20 people
G_m with 30 people
G_h with 50 people

$$\Rightarrow \begin{cases} 20\% \text{ people believe that the level of danger is low} \\ 30\% \text{ people believe that the level of danger is medium} \\ 50\% \text{ people feel that there exists a high the level of danger,} \end{cases}$$

what should point the way forward. The same happens when cliques C_1, C_2, C_3 are considered with distribution functions

$$P_{C_1}[D] = e^{\frac{-1}{T}f(D_{C_1})}, \; P_{C_2}[D] = e^{\frac{-1}{T}f(D_{C_2})}, \; P_{C_1}[D] = e^{\frac{-1}{T}f(D_{C_1})},$$

given by equation (3): the highest probability should point the action programme.

It is remarkable that the whole process occurs *at some instant of time t*. That is, *it is a dynamic process* that may suffer from fluctuations. We will return to this point in subsection 5.3. Thus, in order to determine its behaviour in time, a sufficiently large number of trials should be performed over time.

5.2. Visual support: the colour-coding system

Usually EWS are equipped with a colour-coding system thereby allowing to visualize the current state of alarm by triggering a different visual signal corresponding to a probability of occurrence. To this regard, many procedures may be used in order to link certain probability with the desired colour. For simplicity, we use LATEX, a wordprocessor mainly employed for technical documents. This editor shall assign a different colour to each range of probabilities.

Since any probability $P[-]$ should hold $0 \le P[-] \le 1$, a partition $\mathcal{P}$ of $[0,1]$ should be taken. That is, an ordered n-tuple of real numbers $\mathcal{P} = (\alpha_0, \alpha_1, \ldots, \alpha_n)$ such that $0 = \alpha_0 < \alpha_1 < \ldots < \alpha_n = 1$,

$$[0,1] = [0 = \alpha_0, \alpha_1] \cup [\alpha_1, \alpha_2] \cup \ldots \cup [\alpha_{n-2}, \alpha_{n-1}] \cup [\alpha_{n-1}, \alpha_n = 1],$$

such that the editor maps values to colours by assigning warm colours[4] in the scale to those probability values which are approaching or exceeding some threshold. The output is a colour area which correspond to the partition of the interval $[0,1]$:

$$[0,1] = \underbrace{[0, \tfrac{1}{4}]}_{green} \cup \underbrace{[\tfrac{1}{4}, \tfrac{1}{2}]}_{yellow} \cup \underbrace{[\tfrac{1}{2}, \tfrac{3}{4}]}_{orange} \cup \underbrace{[\tfrac{3}{4}, 1]}_{red}$$

---

[4] While the colour meanings vary drastically across languages, certain terms are prevalent like red, which is perceived as a potential danger.

5.3. The temporal monitoring system: secondary alarm notification

In real-life, situations which are potentially at risk require ongoing and careful monitoring. For this reason the proposed EWS has to be complemented with a temporal follow-up. To do so, those high-risky events detected by the spatial monitoring system has to be recorded.

Specifically, while the spatial monitoring system deduces information about the perception that the ♀' environment has on the proximity of the danger (as described in section 5.1), the colour-coding system (section 5.2) helps in visualising those events with highest levels of danger. Such events happen at a certain time instant t but the dynamics of this process may fluctuate over periods of time. In order to assess whether it is just an off-chance event or whether there is real danger coming, a *temporal* monitoring system should provide evidence of this based on a sufficiently large number of trials should be performed over time.

On this assumption, we proceed as follows: for those nodes with warm colours events in their track records, register the temporal sequence formed by their probability values over time, starting from an initial time instant t_0. Such lists-in-time of probabilities (each for each clique) are computed through equation (3) once some *thresholds* th_i have been fixed, as follows:

$$\begin{aligned} P_{C_i}[D \le th_1] &= \sum_{i \le th_1} P[D = i] \\ P_{C_i}[th_1 < D \le th_2] &= \sum_{i \le th_2} P[D = i] - \sum_{i \le th_1} P[D = i] \\ P_{C_i}[D > th_2] &= 1 - P_{C_i}[D \le th_2] = 1 - \sum_{i \le th_2} P[D = i] \end{aligned} \tag{6}$$

Over time, all of these results in a collection of temporal sequences which display how danger is evolving from the behaviour of those time instants associated to potential dangerous situations. Hereinafter such series are referred to as *danger series*. Let it be noticed that associated to each potential IPV victim ♀, there are as many danger series as cliques in the network. The analysis of such series (for instance, whether the number and frequency of high-risky values increase or decrease) would indicate the evolution of the frequency and duration of exposure to the hazard and whether there is a need for action. In Table 2, the evolution of the danger series is displayed for the case of 4 cliques, where the corresponding distributions functions come from equation (3).

For a potential victim ♀, it is normally the case that there exists a high degree of coincidence amongst her danger series. Otherwise, the weight assigned to each clique when applicable (see equation (5)) will allow to evaluate and decide which danger series should be given high priority compared with others.

Besides the weight assigned by the model through equation (5), other forms of weighting cliques may be provided in order to set priorities and make choices: by considering the number of members in the clique (the higher number, the higher weight), the degree of connection with the victim (the closer connection, the higher weight) amongst others.
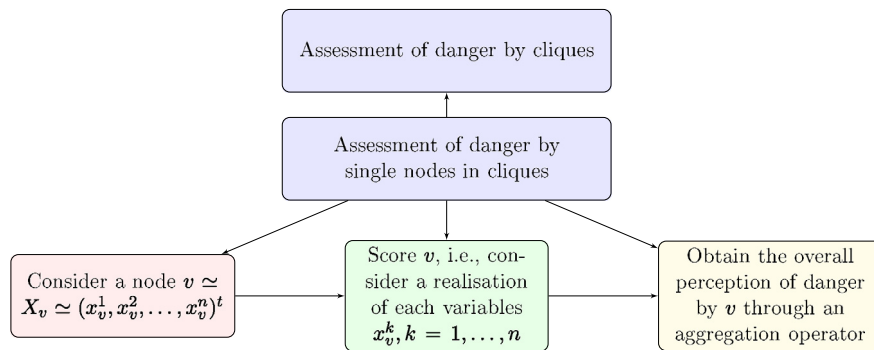
Fig. 3. Assessment of danger by cliques.

Table 2
Evolution of danger in time for 4 cliques.

Cliques' Probability Distribution	Evolution in Time
$\phi_{c_1}(D_{C_1}) = e^{\frac{-1}{T} f(D_{C_1})}$	
$\phi_{c_2}(D_{C_2}) = e^{\frac{-1}{T} f(D_{C_2})}$	
$\phi_{c_3}(D_{C_3}) = e^{\frac{-1}{T} f(D_{C_3})}$	
$\phi_{c_4}(D_{C_4}) = e^{\frac{-1}{T} f(D_{C_4})}$	

6. Application and results

To validate the methodology, an empirical evaluation of the goodness of its performance is needed. To this end, recall that in the network $♀ \equiv (V_♀, E_♀)$, each node $v \in V_♀$ has been identified with the variable *perception that ♀ is in danger*, called in abstract X_v. Recall also that such perception of danger X_v is described by a collection of features $x_v^k, k = 1, \ldots, n$,

$$v \simeq X_v \simeq (x_v^1, x_v^2, \ldots, x_v^n)^t.$$

Now, it may be assumed that the features $x_v^k, k = 1, \ldots, n$ are discrete variables taking values in a set $\{0, 1, \ldots, 10\}$.

Let us evaluate the perception of danger of a clique as a whole. Following the remark made in subsection 5.1 (illustrated by Fig. 2), the corresponding distribution function in a clique works as an aggregation operator. Moreover, an assessment of single nodes in the clique is previously needed. To do so, there are many procedures to assess the overall perception of danger as perceived by a node v when it is disaggregated into its features $v \simeq X_v \simeq (x_v^1, x_v^2, \ldots, x_v^n)^t$. The one which it is assumed here is that the node is scored through a realisation of each $x_v^k, k = 1, \ldots, n$ which converts variables into numerical outcomes. This is shown in Fig. 3.

Let us now proceed with the overall evaluation of danger by cliques. For this, the random variable D (perception of danger) may be assumed to be a discrete variable taking values in a set S,

$$S = \{0, 1, \ldots, d_{pr}, \ldots, d_{im}, \ldots, d_{im} + \epsilon\} \ \epsilon \in \mathbb{Z}^+,$$

where d_{im} represents the level of imminent danger for ♀ -red flag- and d_{pr} stands for the level of probable aggression -yellow/orange flag. To this regard, we can suppose without loss of generality that, despite the probable existence of differences in values d_{im}, d_{pr} amongst cliques, there is a *global scale*: actually, this can be done by taking as d_{im}, d_{pr} the

minimum of the cliques' maximum perception of aggression. Thus, the set $\{d_{im}, \ldots, d_{im} + \epsilon, \ \epsilon \in \mathbb{Z}^+\}$ gathers the levels of maximum likelihood of aggression -when urgent actions must be taken- while $\{d_{pr}, \ldots, d_{im}\}$ includes those levels in which interventions are advisable.

In general the danger between (above, below) two benchmarks is measured by the corresponding cumulative distribution,

$$
\begin{aligned}
P_{C_i}[D \leq d_{pr}] &= \sum_{i \leq d_{pr}} P[D = i]] \\
P_{C_i}[d_{pr} < D \leq d_{im}] &= \sum_{i \leq d_{im}} P[D = i]] - \sum_{i \leq d_{pr}} P[D = i]] \\
P_{C_i}[D > d_{pr}] &= 1 - P_{C_i}[D \leq d_{pr}] = 1 - \sum_{i \leq d_{pr}} P[D = i]],
\end{aligned}
$$

where the level of danger in a specific level $d \in C_i$ is computed as $P_{C_i}[D = d] = e^{\frac{-1}{T} f(d)}$. That is, once thresholds of imminent and probable danger d_{im}, d_{pr} are set by the user, the above expressions foresee approaching danger.

As for the possible actions to be taken in order to protect IPV victims, note that as long as the probabilities $P_{C_i}[D]$ reach higher levels, the likelihood of aggression increases. Hence, from a partition of the unit interval such as $[0, 1] = [0, \alpha_1] \cup \ldots \cup [\alpha_{n-2}, \alpha_{n-1}] \cup [\alpha_{n-1}, \alpha_n = 1]$ the user should set the subintervals which correspond to each kind of action, e.g., $[\alpha_{n-2}, \alpha_{n-1}]$, yellow/orange flag, $[\alpha_{n-1}, \alpha_n = 1]$, red flag.

Back to the assessment of danger by cliques, let it be noticed that in the computation of $P_{C_i}[D = d] = e^{\frac{-1}{T} f(d)}$, the energy functions f may be freely selected as long as they are non-negative functions of its arguments. The choice of such functions f should be made according to the context requirements through sensitivity analysis in such a way that each choice of the energy functions would give raise to an specific configuration of the measurement of danger for ♀. Some instances of energy functions are $f(D_{C_i}) = \frac{(D_{C_i} - \mu_i)^2}{\sigma_i^2}$ (Gaussian with μ_i, σ_i^2 the mean and the variance in the ith-clique) or $f(D_{C_i}) = (D_{C_i})^t A D_{C_i}$ (quadratic form), both non-negative in their arguments.

We focus here in the Gaussian distribution mainly because it covers several contexts: not only those considered as standard but also those where the binomial and the Poisson distributions (for large values of the mean) play a role. A second reason is that it is the most important distribution in the statistical inference processes. Thus, the corresponding measurement of danger (with temperature value 1) is

Energy function	Measurement of danger
$f(D_{C_i}) = \frac{(D_{C_i} - \mu_i)^2}{\sigma_i^2}$	$P_{C_i}[D = d] = e^{\frac{-(D_{C_i} - \mu_i)^2}{\sigma_i^2}}$

In order to cover the majority of situations, we categorize the IPV contexts into two main groups: those where danger exposure is constant in time (probably with small variations in the risk intensity) and those which randomly combine extreme risk exposures with periods of
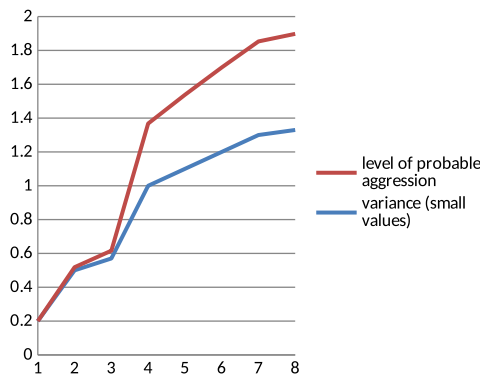
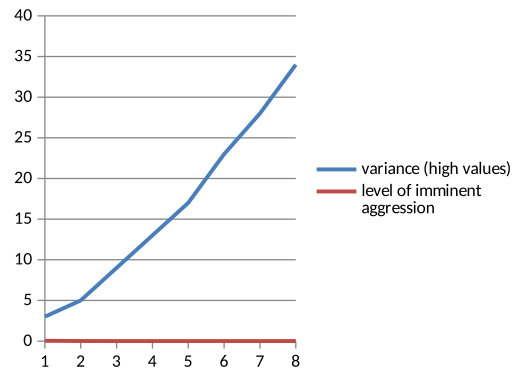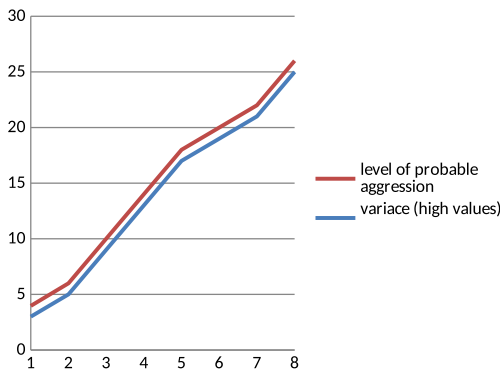**Fig. 4.** Evolution of probable aggressions under σ_i^2 small.

Fig. 5. Evolution of probable aggressions under σ_i^2 high.

Fig. 6. Evolution of imminent aggressions under σ_i^2 small.

Fig. 7. Evolution of imminent aggressions under σ_i^2 high.

7. Conclusions

This paper presents an early warning system for IPV which turns into numerical scores the perception of nearing danger that the victims' surroundings have, thereby enabling for preventive actions. This new standpoint of using the information from the victims's neighbourhood is of particular relevance for those potential victims who have no clear perception of approaching danger (actually, most of the IPV victims do not have awareness of an imminent danger) in such a way that the information from the environment warns even if the victims are not fully aware of the coming danger.

To our best knowledge, this is the first attempt in literature of articulating a mathematical/computational-based model for police investigations in preventing IPV. The practical value of the results of the paper (Theorem 3.10 and Corollary 3.11) is that they provide a simple way of specifying the level of danger (through the joint probability distribution of the selected random variable $X \equiv D$ "someone' perception that ♀ is in danger") by just choosing appropriate potential functions. These results call attention to the spirit of the proposed model by emphasizing the role of the potential victims' environment in alerting when danger is coming (Fig. 8).

The mathematical framework presented in this paper is the technical support which underlies an integral IPV Information System (Location and Protection System) for police interventions. As a matter of fact, this project underway -with the support of social, psychological and sentiment analysis expertise (see [1], [12] for related texts in sentiment analysis)- is the ultimate aim of such a new methodology. This new approach, after all, may assess *any* variable (detailed by means of its features) through each clique' assessment and convert these evaluations into a global numerical score which allows to make decisions (see [7]). This is of particular interest when applied to variables as "change in habits" (section 4) or "perception of being in danger" (section 5) as perceived by the neighbourhood of IPV potential victims.

Although the proposed Early Warning System was in its early days designed for the Spanish case, it is appropriate for preventing IPV in any country of the world since it has been designed under a universal premise (see [6]) in the sense that it does not depend on any local geographical constrains. To stay within that line (universality), all environmental factors could be incorporated to the system by means of a(n *additive*) corrective coefficient c within a range of values $[c_{min}, c_{max}]$ and which should be *added* to the final likelihood. In order to comprise all possible variables concerning environmental factors, the variables will take value 1 if applies and 0 otherwise. Thus, several variables[5] may be taken into account but only those which apply in each case are going to be considered. This fine-tuning results in extra-attributes for the system which could be valuable in certain cases.

relative calm. For both, we analyse how the risk of aggression should evolve in time by using the Gaussian measurement of danger.

Since variance measures how far data are spread out, situations of constant exposure to danger are characterised by a variance σ_i^2 taking low values. On the contrary, those scenarios with big fluctuations in danger exposures (risk curves with peaks and troughs) correspond to higher values for the variance σ_i^2 as a result of a higher dispersion. This is illustrated in the graphics below (Figs. 4 and 5).

From Figs. 4 and 5 (probable levels of aggression) the conclusion is that the evolution of risk exposure appears as an increasing function (under similar conditions of proportion) of σ_i^2 for either small or high values. It is noteworthy that, in Fig. 5 (high level of dispersion), both curves are quite the same. As for imminent levels of danger, the trend remained the same for the case of small dispersion (Fig. 6). However, Fig. 7 seems to indicate that, the more dispersion, the lower probability of imminent aggression.

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[5] Both variables themselves and the number of them may be freely selected.
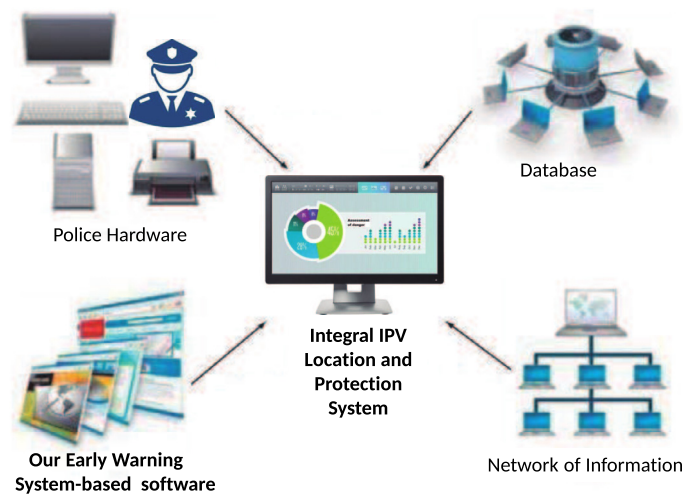
Fig. 8. Integral IPV Location-Protection System.

This methodology can be easily re-written into computational terms thereby providing an easy-to-handle system. It is remarkable that the computation regarding different possibilities (for instance, changes in the random variable) may be executed in parallel thereby reinforcing the system' capability of making decisions. Moreover, the use of cliques instead of the whole data set ensures that the system which could operate under a reduced version of data (only the most significant information). Such data reduction scope will increase the speed of the model, which will be determinant when urgent decisions have to be made.

Declarations

Author contribution statement

The unique author of this paper, Julia García Cabello, did everything (from 1) to 5)):

1) conceived and designed the experiments;
2) performed the experiments;
3) analyzed and interpreted the data;
4) contributed reagents, materials, analysis tools or data;
5) wrote the paper.

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Additional information

No additional information is available for this paper.

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