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Linear Concrete Tension Stiffening Model for Reinforced Concrete Elements

Modelo lineal de la rigidez a tracción del hormigón para elementos de hormigón estructural

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ABSTRACT

Nowadays, linear expressions for the tension stiffening of concrete are widely used in structural analysis software packages. Nevertheless, more complex expressions of tension stiffening available in the literature have been used successfully in the past. The use of linear approximations is justified since the influence of the tension stiffening effect on the deformation of a structure is small but not negligible. Therefore, linear approximations are simpler and can be accurate enough. However, the linear approximation, in its present form, does not provide good results in terms of deformations of reinforced concrete beam-column elements. This paper proposes a linear expression of tension stiffening supported by the experimental formulation used to predict the flexural behavior of concrete beams. A detailed example is presented.

KEYWORDS: Tension stiffening of concrete; deflections of RC elements; concrete model.

RESUMEN

Expresiones lineales de la rigidez a tracción del hormigón están siendo ampliamente utilizadas en paquetes de software de análisis estructural. Sin embargo, en el pasado se han utilizado con éxito expresiones más elaboradas de la rigidez a tracción del hormigón. El uso de aproximaciones lineales está justificado ya que la influencia del efecto de rigidez a tracción del hormigón en la deformación de las estructuras de hormigón es pequeña pero no despreciable. En este sentido, las aproximaciones lineales son más simples y pueden ser lo suficientemente precisas. Sin embargo, esta aproximación lineal, en su forma actual, no proporciona buenos resultados en términos de deformaciones a nivel de elementos estructurales. Esta publicación propone una expresión lineal de la rigidez a tracción apoyada por la formulación experimental utilizada para predecir el comportamiento a flexión de vigas de hormigón. Se presenta un ejemplo detallado.

PALABRAS CLAVE: : Rigidez a tracción del hormigón, deflexión de elementos de hormigón armado, modelos de hormigón

1. INTRODUCTION

The phenomenon of concrete tensile contribution, for tensile strains greater than the cracking strain, is called tension stiffening of concrete. This contribution is caused by the bond effect between the concrete and the reinforcing bars. Comprehensive studies of this phenomenon can be found in [1], [2], [3], and [4]. Tension stiffening models have been widely used in the study of the shear response of both reinforced and prestressed concrete elements by using the two main theories: the MCFT (Modified Compression Field Theory [2]) and the RA-STM (Rotating Angle Softened Truss Model [3]). There are several discrepancies between these two main theories, which have been the subject of an exhaustive justification by other authors [5], [6].

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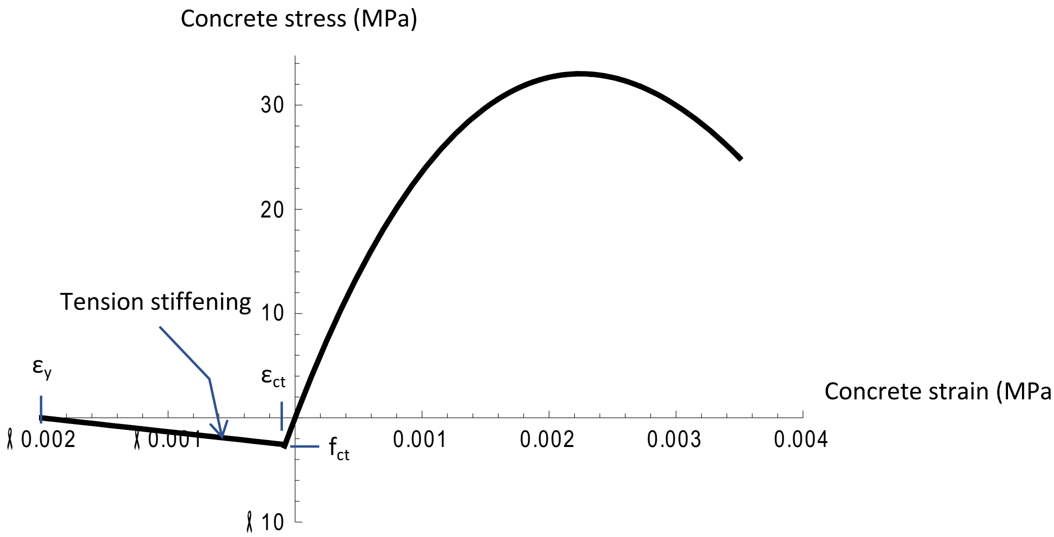

Figure 1. Strain-Stress curve of concrete, including tension stiffening.

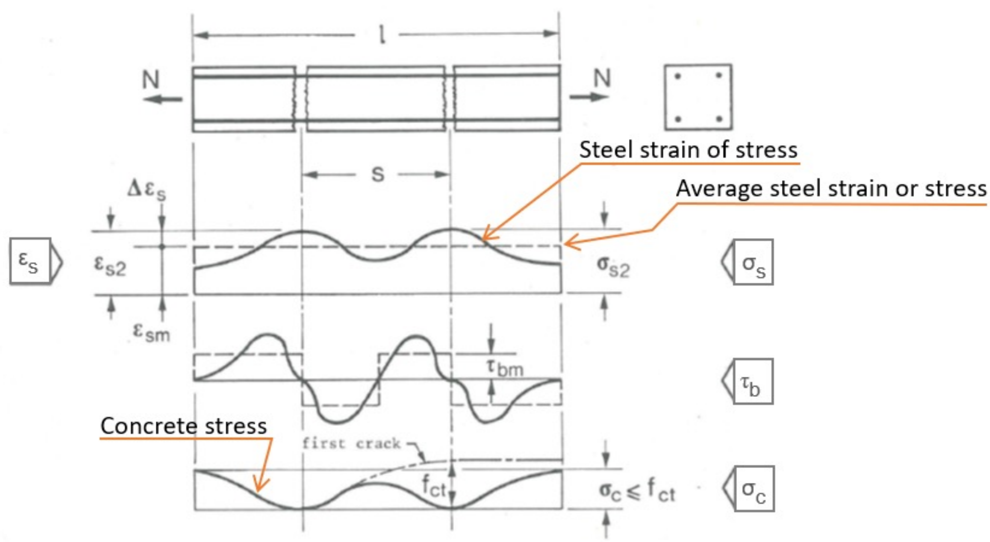

Figure 2. Stress and strain distributions along the reinforcing bars and the effective concrete area, adapted from [11].

International software for structural analysis, such as OpenSees [7] and Sap2000 [8], use approximated linear expressions for the tension stiffening model, e.g. model "Concrete02" of OpenSees. When using Concrete02, the post-cracking behavior in tension is described by a line that goes from the tensile strength to 0 in a constant slope which is usually deduced by assuming that the tension stiffening capacity is null at a value of the strain equal to the yield strain of steel (ε_y). See in the line between the points Figure1: ($-\varepsilon_{ct}$, $-f_{ct}$) and ($-\varepsilon_y$, 0).

However, as shown below, it has been found that the deflections calculated when using the model described in Figure 1 are lower than those predicted by the formulations proposed by current regulations (EN 1992[9], ACI-318[10]), which are based on large experimental campaigns.

Considering that the tension stiffening effect is primarily caused by the bond mechanism between concrete and reinforcement [11], the premise that it is zero at $\varepsilon=\varepsilon_y$ [1] is well established. Alternatively, some authors consider that small amount of tensile strength of concrete between cracks beyond yielding [12] remains, with a consequent increase in the rigidity of the RC element beyond the yielding of the reinforcement.

So, if tension stiffening is assumed to be zero at $\varepsilon=\varepsilon_y$, then the only variable that remains for modeling the tension stiffening effect is the tension strength of concrete, f_{ct}. In this paper, a ψ factor is introduced to adjust f_{ct} so that the theoretical deflection matches the result proposed by the current standards.

It is known that for tension strains larger (in absolute value) than the one that corresponds to cracking (note that cracking happens at ($-\varepsilon_{ct}$, $-f_{ct}$), see Figure 1), the stress and strain distributions along the length of the reinforcing bars are no longer constant for either steel or concrete, see Figure 2, adapted from [11].

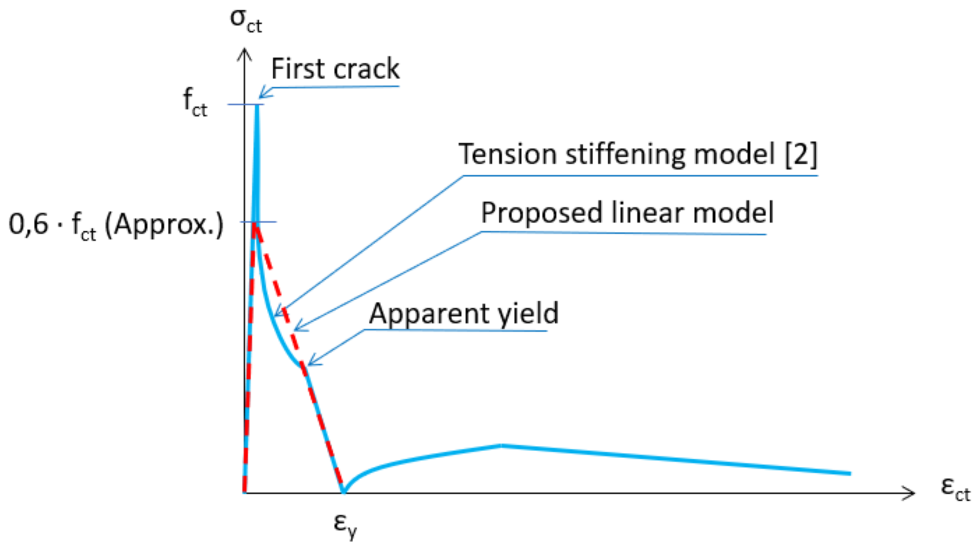

Figure 3. Variation of average tensile stress in concrete up to failure

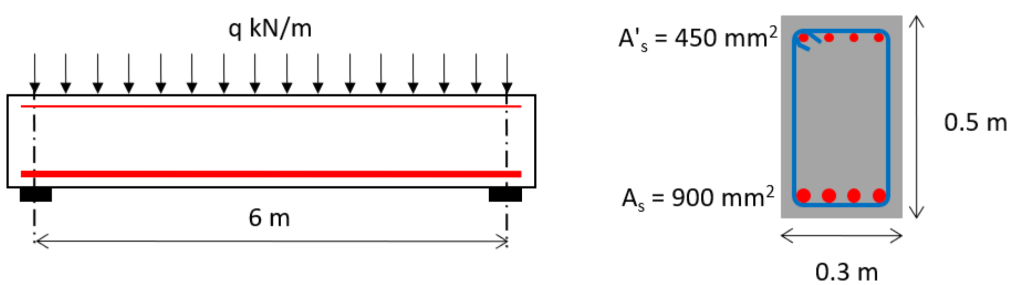

Figure 4. Example studied of reinforced concrete beam.

Due to this variability, the strain of the bar could be ε_y at the crack location while in sections between cracks (where concrete in tension is undamaged), the bar stress is lower than f_y. This implies that the average stress-strain response of the reinforcement exhibits yielding at an average stress that is below f_y. The average steel stress and strain associated with the yielding at a crack location is called "apparent yield" [3],[5], see Figure 3. Thus, before the average strain (i.e., calculated along a length intersecting several cracks) reaches ε_y, the steel reinforcement will have already yielded at the cracks. Beyond the apparent yield point, the tension stiffening phenomenon presents a large drop in its slope [1], [11] which cannot be observed when using the linear approach, see Figure 3 adapted from [2]. This drawback can be overcome by appropriately correcting the value of the tensile strength of concrete, as proposed in this work.

2.
DEFLECTION FORMULATION FOR BEAM ELEMENTS

The linear tension stiffness model proposed is going to be evaluated using results from the deflections of beams. The deflections used in this study are obtained from formulations endorsed by ACI-318 and EN 1992, which represent the experience and approval of the engineering community.

ACI-318 uses an interpolation function to calculate effective inertia, [13], while EN 1992 uses an interpolation function that is applicable to the deflections calculated with the gross inertia and with the cracked inertia, [11]. Recently, slenderness limits have been proposed to avoid the explicit calculation of cracking and deformation. [14].

EN 1992 presents an explicit formulation of the concrete stress-strain curve, and therefore this study has been developed using the European code (e.g. Figure 1 shows the stress-strain formulation given by EN 1992 for a concrete with a characteristic strength of 25 MPa).

Example

The example studied is a simply supported beam with a 6-meter span, which is of the beams used in the construction of buildings. This example, shown in Figure 4, has been adapted from [11].

The values of the effective depth, the modulus of elasticity, the characteristic compressive strength of concrete, and the steel yield stress are d=450 mm, E_{cm}=30 500 MPa, f_{ck}= 25 MPa and f_y=400 MPa, respectively.

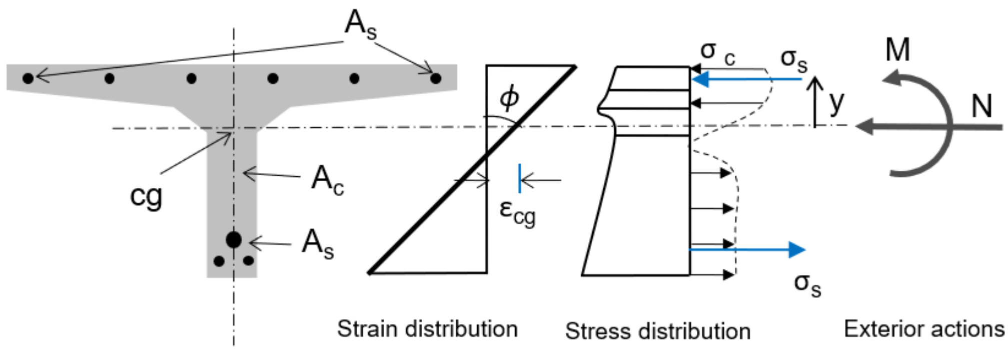

Figure 5. Plane section hypothesis. Stress distribution. External actions.

In the original example, the beam was subjected to a uniform load (q) that is equal to 16.9 kN/m. The maximum immediate deflection caused by this load, calculated in accordance withACI-318 [10], is 10.7 mm while the value of the maximum immediate deflection calculated using the simplified method in EN 1992 [9] is 9.9 mm. The implicit consideration of shrinkage in the ACI-318 approach justifies this discrepancy. A detailed resolution of these results can be seen on YouTube (in Spanish): (https://www.youtube.com/watch?v=_YInm-fDRXs&t=1037s and https://www.youtube.com/watch?v=BkXO6s2Ktwk) for ACI-318 [10] and EN 1992 [9], respectively.

3.
ANALYTICAL PROCEDURE

As an alternative to the formulations proposed by EN 1992 [9] and ACI-318 [10], an analytical procedure for obtaining the immediate deflections of a reinforced or prestressed concrete element has been applied as follow:

1) The equilibrium equations, of both axial forces and bending moments at cross-sectional level, are

$$N=\int_{A_c} \sigma_c(\varepsilon_{cg}+\phi y)\,dA_c+\sum_j \sigma_{s,j}(\varepsilon_{cg}+\phi y_j)\,A_{\phi,j} \qquad (1a)$$

$$N=\int_{A_c} \sigma_c(\varepsilon_{cg}+\phi y)\,y\,dA_c+\sum_j y_j\,\sigma_{s,j}(\varepsilon_{cg}+\phi y_j)\,A_{\phi,j} \qquad (1b)$$

with N and M as the external axial force and bending moment, respectively. A_c is the area of concrete, σ_c is the concrete stress. The plane section hypothesis is assumed (Figure 5), so the strain can be expressed as $\varepsilon=\varepsilon_{cg}+\Phi y$, where ε_{cg} is the strain at the center of gravity (cg), Φ is the curvature, and y is the distance to the center of gravity. $\sigma_{s,j}$ is the stress of the j bar, y_j is the distance of the j bar to the center of gravity, and $A_{\phi,j}$ is the area of the *j* bar.

In the equilibrium equations, the model of concrete used is the short-term model proposed by EN 1992 [9] plus a linear tension stiffening model (see Figure 1) which corresponds to a concrete with a characteristic strength (f_{ck}) of 25 MPa. A bilinear steel model with no strain hardening has been adopted.

For a fixed value of axial force *N* and a given value of curvature Φ, ε_{cg} is calculated from the axial force equilibrium (Eq. 1a). *M* is calculated by introducing the calculated value of ε_{cg} in Eq.1b. In doing so, a pair (M,Φ) is obtained. By incrementing the value of the curvature for the same value of the axial force and solving Eqs. 1a and 1b, new pairs (M,Φ) are obtained, and the moment-curvature diagram of the cross-section can be formed.

2) The values of the bending moment along the length of the concrete member are transformed into curvature values by using the moment-curvature diagram, and
3) By assuming small deformations, the curvature values are integrated twice to obtain the deflection of the element. This is done using finite differences. The formulation for the length of element *L* when divided into *n*+1 segments of equal length, is shown in Eq. 2. The supports are located at *i*=0 and *i*=*n*+1, and the deflection at these points is zero.

$$\mathbf{y''}=\mathbf{f}\mathbf{y}$$

$$\mathbf{f}=\left(\frac{n+1}{L}\right)^2\begin{bmatrix} -2 & 1 & & & & & & & & \\ 1 & -2 & 1 & & & & & & & \\ & 1 & -2 & 1 & & & & & & \\ & & 1 & -2 & 1 & & & & & \\ & & & 1 & -2 & 1 & & & & \\ & & & & 1 & -2 & 1 & & & \\ & & & & & 1 & -2 & 1 & & \\ & & & & & & 1 & -2 & 1 & \\ & & & & & & & 1 & -2 & 1 \\ & & & & & & & & 1 & -2 \end{bmatrix};\ \mathbf{y''}=\begin{bmatrix} y''_1 \\ y''_2 \\ y''_3 \\ . \\ . \\ . \\ \\ y''_n \end{bmatrix};\mathbf{y}=\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ . \\ . \\ . \\ \\ y_n \end{bmatrix} \qquad (2)$$

The deflection is calculated by inverting the **f** matrix as: $\mathbf{y}=\mathbf{f}^{-1}\mathbf{y''}$

Continue previous example

For the linear tension stiffening model, a reduction factor (ψ) for the tensile strength is considered:

$$f_{ct}=\psi\,f_{ctm}$$

$$where\ f_{ctm}=0.3\,f_{ck}^{2/3} \qquad (3)$$

With f_{ctm} as the mean axial tensile strength of concrete formulated in EN 1992 [9]. The linear models of tensile concrete for values of ψ =1.0, 0.5, 0.3 and 0.25 are shown in Figure 6.

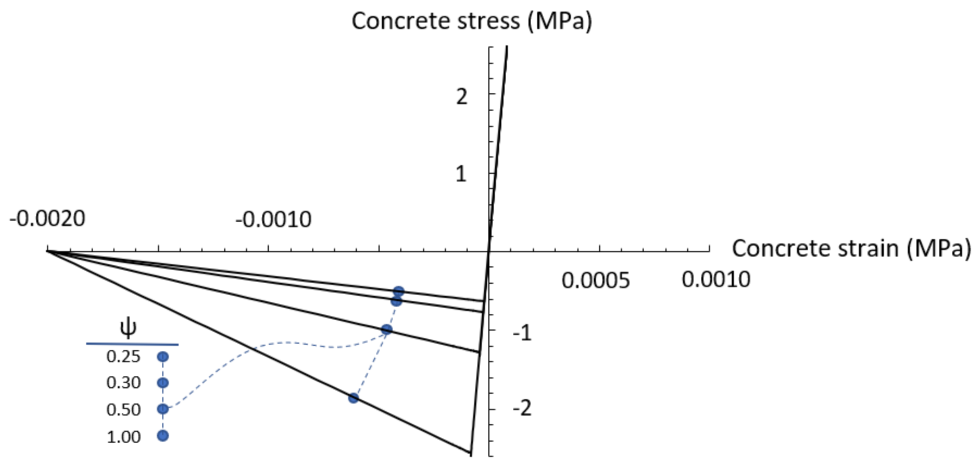

Figure 6. Linear approximations of tensile strength of concrete (tension stiffening).

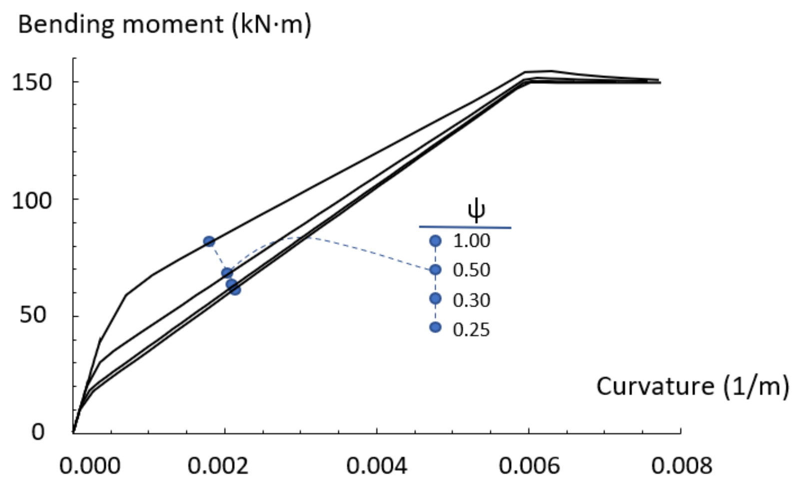

Figure 7. Moment curvature diagrams for the tensile model of concrete in Figure 6.

The moment-curvature diagrams of the cross-section for the different values of ψ are calculated introducing each one of the linear approximations of the tension stiffening in Figure 6 in the equilibrium equations, see Figure 7.

The theoretical deflection of the beam in Figure 4 is solved by the integration of curvatures using finite-differences. The beam was divided into 20 segments. Table 1 summarized the values obtained for both the moments and curvatures at cracking (M_{cr} and Φ_{cr}, respectively) and the maximum deflection (δ_{max}). The computer code is available on request.

TABLE 1.
Maximum deflections

ψ	M_{cr} (kN·m)	ϕ_{cr} (1/m)	δ_{max} (mm)
1.0	38.64	0.00035	4.68
0.5	19.82	0.00018	8.41
0.3	12.08	0.00011	9.70
0.25	9.91	0.00009	10.00

As can be seen in Table 1, the linear approximation (i.e. ψ=1) leads to a maximum deflection of less than half of the deflection predicted by the regulation. By ensuring that the theoretical deflection equals the deflection given by the regulation (EN 1992 [9] in this case) (δ_{max}=9.9 mm) a linear interpolation between the two bottom rows results in ψ=0.27.

The new version of Eurocode 2 (prEN 1992) has reduced the effective area of concrete influenced by tensile stress in concrete, from the traditional value of a square area centered on the reinforcing bar of 15Φ side to 10Φ side. This is a variation of the effective concrete area of 44%. Additionally, in order to compensate for the sudden drop in tensile stress for strains greater than those that correspond to the first crack (Figure 3), given by most of the existing models, a continuous linear function is proposed with a reduction in the tensile capacity of 0.6. If these two values (0.44 and 0.6) are fitted into a single parameter ψ (i.e. 0.44·0.6=0.26), the result obtained of 0.27 is similar to this value.

4. CONCLUSIONS

Due to the relative importance of the tension stiffening phenomenon, for the sake of simplicity, complex expressions of tension stiffening are being substituted with simpler linear approximations. Although linear approximations of tension stiffening lead to more rigid solutions in term of deflections, the simplicity of these approximations means that they are being widely used in computer software.

This work presents a new linear approximation that matches the results given by current standards in terms of deflections and this overcomes the problem caused by underestimating the deflections inherent in bilinear approximations.

Conflict of interest

On behalf of all the authors, the corresponding author states that there is no conflict of interest.

Data Availability Statement

A computer code has been developed for the development of this paper; the code is available on request from any of the authors .

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