Modelling Contexts as Fuzzy Propositions in Optimisation Problems

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***Abstract*—Decisions made in areas such as economics, engineering, industry and medical sciences are usually based on finding and interpreting solutions to optimisation problems. When modelling an optimisation problem, it should be clear that people do not make decisions in a vacuum or in isolation from the reality. So, there is always a decision-making context that, in addition to the *natural* constraints of the problem, acts as a filter on the candidate solutions available. If this fact is omitted, optimal but useless solutions to the problem can be obtained. In this paper, we propose a systematic way of modelling contexts based on fuzzy propositions and two approaches (*a priori* and *a posteriori*) for solving optimisation problems under their influence. In the proposed *a priori* approach, the context is explicitly included in the mathematical model of the problem. As this approach may have a limited application due to the increasing number of constraints and their nature, an *a posteriori* approach is proposed, in which a set of solutions, obtained by any means (like exact algorithms, simulation or metaheuristics), are checked for their suitability to the context by using a multi-criteria decision-making methodology. A simple fish harvesting problem in a sustainability context and a tourist trip design problem in a pandemic context were solved for illustration purposes. Our results provide researchers and practitioners with a methodology for more effective optimisation and decision-making.**

***Index Terms*—Optimisation, fuzzy logic, decision-making context, fuzzy proposition**

I. INTRODUCTION

MATHEMATICAL optimisation has pervaded numerous human activities, ranging from finding the fastest route to work or looking for the best house given certain budget constraints [1] to solving complicated problems in economics, engineering, industry, medical sciences, and so forth.

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To solve an optimisation problem, firstly a model that includes (and leaves out) certain features of the real-world should be defined. This model is composed of three basic elements: decision variables, a set that restricts the values taken by the decision variables and an objective function that measures how good a solution to the problem is. The goal is to select an optimal solution (in the sense that it maximises or minimises the objective function) from the set of all feasible ones. However, what is usually omitted in the model definition is the fact that people do not make decisions in a vacuum or in isolation from the reality that surrounds them and the specific problems they are trying to solve—namely, that there is always a decision-making context (hereafter context for short) that dictates which decisions should be made and how to make them. Therefore, ignoring the context in the modelling or solving stages of an optimisation problem almost surely would lead to an optimal but useless solution and, consequently, an unacceptable decision.

Interestingly, as noted in [2], when faced with the same problem, each person could make a different decision (choose a different solution). This is because each person has their own context determined by their experiences and the specific situation in which decisions must be made, and, consequently, what is called an optimal (best) solution may lose this quality when it is analysed from the perspectives of different contexts—clearly, the idea of 'best' is context-dependent.

Nowadays, special attention is put on the increasing development and deployment of the so-called Automated Decision-Making (ADM) systems both in the public and private sectors. According to [3], an ADM system is a system, software, or process that uses computation to aid or replace organisation decisions, judgments, and/or policy implementation that impacts opportunities, access, liberties, rights, and/or safety. A wide variety of technologies are used within these systems to carry out human-like decision-making; these technologies may include optimisation models and algorithms, machine learning, natural language processing, and soft computing [4]. ADM systems are used for 'processing requests for social benefits, for detecting risks of welfare fraud, for profiling unemployed people, for predictive policing purposes by law enforcement authorities or for assessing recidivism risks of parolees…the use of ADM systems offers great potential for public administrations. At the same time, it comes with substantial risks—especially if such systems are not introduced and deployed in a careful manner.'[1] Given the intended penetration of ADM systems in the social tissue, the role of the context in these systems should be carefully analysed, since they may raise ethical concerns [5].

But the question 'What is context?' has no single answer, and researchers from areas such as psychology, human-computer interaction, mobile computing and 'context-aware' computing provide definitions that best fit their needs [6], [7]. Those definitions, however, do not fit the scope of mathematical optimisation. Instead, we follow one of the views in [8] and adopt

> the idea that context consists of a set of features of the environment surrounding generic activities, and that these features can be encoded and made available to a software system alongside an encoding of the activity itself.

For our purposes, we assume the account of context as a representational problem (although Dourish [8] later viewed it as an 'interactional problem') having in mind the following four features [8]: 1) context is a form of information (it can be known, encoded and represented in our systems); 2) it is delineable (we can define what counts as the context of activities that the application supports); 3) it is stable (although the precise elements of a context representation might vary from application to application, they do not vary from instance to instance of an activity or an event) and 4) the context and the problem being solved are separable (the problem is solved *within* a context).

Lamata, Pelta and Verdegay [2], [9] defined the concept of a 'decision-making context' as a set of rules that determine the qualitative characteristics that a solution to a problem must have. They also identified the following ten contexts commonly found in practical situations: Competition, Ethical, Concurrence, Adversarial, Crisis, Stress, Sustainability, Dynamic, Corporate Social Responsibility and Induced. This is not an exhaustive list at all, since other contexts could be identified in more specific situations, including, e.g., a composite of some of them. We stress that the concept of context to which we refer here has nothing to do with the concept of behavior, from the psychological field, nor with that of 'environment', which alludes to the states of nature (uncontrolled future events that affect the outcomes of the alternatives) in a decision problem, but with, e.g., the social, economic, administrative or legal 'scenario' in which the problem we are considering is given.

In [2], the authors illustrated how the optimal solution to a problem may change when we switch from one context to another. However, they did not propose a way of modelling contexts or general operational ways in which contexts could be used to solve optimisation problems. This paper builds upon the theoretical views discussed in [2] and advances this topic by providing answers to the following questions.

- How can the context be formally modelled?
- How to obtain solutions to optimisation problems that conform to a given context?

[1] https://algorithmwatch.org/en/adm-publicsector-recommendation/ (accessed on 10 June 2022)

Furthermore, since contexts are determined, among other elements, by people's experiences and current circumstances (often not fully understood), the information available to describe them is generally incomplete and mostly subjective. Hence, fuzzy logic [10]–[13] methodologies could be useful to handle the inherent imprecision of such contexts, as well as to operate with the rules that define them.

To the best of our knowledge, the antecedents of this work are the papers by Yager [14], where the inference process in bivalent logic was cast as a mathematical programming problem, Castro, Herrera and Verdegay [15], where Yager's [14] approach was extended to the fuzzy case, and [16], where a specific (non-general) approach was illustrated.

Our aim here is to present a systematic way of modelling contexts based on fuzzy propositions and solving optimisation problems under their influence. Consequently, Section II presents basic definitions from fuzzy logic that constitute the theoretical basis of our results. Section III presents a general mathematical model of contexts and two (*a priori* and *a posteriori*) approaches to solve optimisation problems within given contexts. The results of solving a simple fish harvesting problem in a sustainability context and a tourist trip design problem in a pandemic context are provided in Section IV for illustration purposes. Lastly, concluding remarks and ideas for future work are presented in Section V.

II. Preliminaries

This section presents some basic definitions taken from [10]–[12], [17]–[19].

Definition 1: A fuzzy set A in a universe of discourse X, with elements denoted generically by x, is characterised by a membership function $\mu_A : X \to [0,1]$, with the value of $\mu_A(x)$ at x representing the grade of membership of x in A.

Definition 2: A linguistic variable is characterised by a quintuple $(\mathcal{V}, T(\mathcal{V}), X, G, M)$ in which $\mathcal{V}$ is the name of the variable; $T(\mathcal{V})$ is the term-set of $\mathcal{V}$, that is, the collection of its linguistic values; X is a universe of discourse; G is a syntactic rule which generates the terms in $T(\mathcal{V})$; and M is a semantic rule which associates with each linguistic value v its meaning $M(v)$, where $M(v)$ denotes a fuzzy subset of X. The meaning of a linguistic value v is characterised by a compatibility (membership) function $\mu_v : X \to [0,1]$, which associates with each value $x \in X$ its compatibility with v.
An example is the linguistic variable *Height*, whose values could be: *Short*, *Average* and *Tall*; each defined as a fuzzy subset of the universe of discourse X consisting of the height of people.

Definition 3: A fuzzy proposition is a statement p whose truth or falsity is a matter of degree. If truth and falsity are given by values 1 and 0, respectively, then the degree of truth, $\mathrm{truth}(p)$, of a fuzzy proposition is expressed by a number in the interval $[0,1]$.

Definition 4: An aggregation operator in $[0,1]^n$ is a function $A : [0,1]^n \to [0,1]$ that is non-decreasing in each variable and fulfils the boundary conditions $A(0,\ldots,0) = 0$ and $A(1,\ldots,1) = 1$.

Definition 5: An aggregation operator $\triangle : [0,1]^2 \to [0,1]$ is a t-norm if it fulfils the conditions:

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Table I
COMMONLY USED DEFINITIONS OF THE LOGICAL CONNECTIVES [19]

Connective	Łukasiewicz	Zadeh	Gödel	Product
$\triangle(x, y)$	$\max(0, x + y - 1)$	$\min(x, y)$	$\min(x, y)$	$x \cdot y$
$\nabla(x, y)$	$\min(1, x + y)$	$\max(x, y)$	$\max(x, y)$	$x + y - x \cdot y$
$I(x, y)$	$\min(1, 1 - x + y)$	$\max(1 - x, \min(x, y))$	$\begin{cases} 1, & x \leq y \\ y, & \text{otherwise} \end{cases}$	$\begin{cases} 1, & x \leq y \\ y/x, & \text{otherwise} \end{cases}$
$N(x)$	$1 - x$	$1 - x$	$\begin{cases} 1, & \text{otherwise} \\ 0, & x > 0 \end{cases}$	$\begin{cases} 1, & \text{otherwise} \\ 0, & x > 0 \end{cases}$

- **Associativity:** $\triangle(x, \triangle(y, z)) = \triangle(\triangle(x, y), z)$,
- **Symmetry:** $\triangle(x, y) = \triangle(y, x)$,
- **Neutral element:** $\triangle(1, x) = x$.

Definition 6: An aggregation operator $\nabla : [0, 1]^2 \to [0, 1]$ is a t-conorm if it fulfils the conditions:

- **Associativity:** $\nabla(x, \nabla(y, z)) = \nabla(\nabla(x, y), z)$,
- **Symmetry:** $\nabla(x, y) = \nabla(y, x)$,
- **Neutral element:** $\nabla(0, x) = x$.

T-norms are the generalisation of the bivalent logic connective '**and**' and t-conorms generalise the bivalent logic connective '**or**'. Both aggregation operators have found numerous applications in decision-making [17], [20]–[26]. In particular, t-norms (resp. t-conorms) can be used as pessimistic (resp. optimistic) decision rules for solving multi-attribute decision-making problems due to their conjunctive (resp. disjunctive) properties [27].

Remark 1: Associativity of t-norms and t-conorms allows them to be extended to operations with an arbitrary number of arguments (see [17]).

Definition 7: A function $N : [0, 1] \to [0, 1]$ is a negation if it is decreasing and fulfils the boundary conditions $N(0) = 1$ and $N(1) = 0$. N is a strict negation if it is continuous and strictly decreasing. N is a strong negation if it is an involution, i.e., $N(N(x)) = x$.

Definition 8: A function $I : [0, 1]^2 \to [0, 1]$ is called a fuzzy implication if it fulfils, for all x, x_1, x_2, y, y_1, $y_2 \in [0, 1]$, the following conditions:

- if $x_1 \leq x_2$, then $I(x_1, y) \geq I(x_2, y)$, i.e., $I(\cdot, y)$ is decreasing,
- if $y_1 \leq y_2$, then $I(x, y_1) \leq I(x, y_2)$, i.e., $I(x, \cdot)$ is increasing,
- $I(0, 0) = 1$, $I(1, 1) = 1$ and $I(1, 0) = 0$.

Some commonly used definitions of the logical connectives are shown in Table I. In the case of implications, not all of them satisfy the conditions imposed in Definition 8. Nevertheless, they are used as such in the literature (see [18]).

Fuzzy propositions can be classified into the following types [12]:

Unconditional and Unqualified Propositions

The canonical form of these propositions is

$$p : V \text{ is } F,$$

where V is variable that takes values x in a universe of discourse X and F is a fuzzy set on X that represents a fuzzy predicate (linguistic term). The degree of truth of p is the same as the degree of membership of x in F, i.e., $\text{truth}(p) = \mu_F(x)$.

Unconditional and Qualified Propositions

These propositions are characterised by either the canonical form

$$p : (V \text{ is } F) \text{ is } S,$$

for truth-qualified propositions, or the canonical form

$$p : \text{Prob}\,(V \text{ is } F) \text{ is } S,$$

for probability-qualified propositions, where S is a fuzzy set representing either a fuzzy truth qualifier or a fuzzy probability qualifier. The degree of truth of a truth-qualified proposition p is given by $\text{truth}(p) = \mu_S\left(\mu_F(x)\right)$. For a probability distribution f on V, we have that the degree of truth of a probability-qualified proposition p is given by

$$\text{truth}(p) = \mu_S\left(\sum_{x \in V} f(x) \cdot \mu_F(x)\right).$$

Conditional and Unqualified Propositions

The canonical form of these propositions is

$$p : \text{if } V \text{ is } F, \text{ then } W \text{ is } G$$

where V and W are variables that take values in sets X and Y, respectively; F and G are fuzzy sets defined on X and Y, respectively. The degree of truth of p is then given by $\text{truth}(p) = I\left(\mu_F(x), \mu_G(y)\right)$, where $I : [0, 1]^2 \to [0, 1]$ is a fuzzy implication, e.g., the Łukasiewicz implication $I(a, b) = \min(1, 1 - a + b)$.

Conditional and Qualified Propositions

These propositions are characterised by either the canonical form

$$p : (\text{if } V \text{ is } F, \text{ then } W \text{ is } G) \text{ is } S,$$

or the canonical form

$$p : \text{Prob}\,(V \text{ is } F | W \text{ is } G) \text{ is } S,$$

where $\text{Prob}\,(V \text{ is } F | W \text{ is } G)$ is a conditional probability. In addition, we have that V, W, F, G and S are defined as presented previously. The degree of truth of p is then given by

$$\text{truth}(p) = \mu_S\left(I\left(\mu_F(x), \mu_G(y)\right)\right)$$

in the first case and by

$$\text{truth}(p) = \mu_S\left(\sum_{x \in V, y \in G} f(x|y) \cdot I\left(\mu_F(x), \mu_G(y)\right)\right)$$

in the second case.

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Fuzzy propositions can be combined by using the logical connectives ‘**and**’, ‘**or**’, ‘**not**’ and ‘**implies**’ to generate compound propositions. Thus, we may have propositions like

$$p_1 : V \text{ is } F \textbf{ and } (W \text{ is } G \textbf{ or } Y \text{ is } \textbf{not } H)\,,$$

which we could write alternatively as

$$p_1 : \textbf{ and } \Big(V \text{ is } F,\ \textbf{or}\,(W \text{ is } G, Y \text{ is } \textbf{not } H)\Big),$$

and

$$p_2 : V \text{ is } F \textbf{ implies } W \text{ is } \textbf{not } G,$$

written alternatively as

$$p_2 : \textbf{implies}\,(V \text{ is } F,\ W \text{ is } \textbf{not } G)\,.$$

III. Context Modelling

The concept of ‘context’ is generic and depends on each specific research field. Here we focus on the idea introduced in [9], and later defined in [2] as ‘a set of rules …that establish the qualitative characteristics that the available decisions must have.’ In this section, we give a similar definition that is more in the spirit of what we have discussed so far. Then, two approaches are proposed to solve optimisation problems posed within contexts. The first is an *a priori* approach, in which the context is explicitly included in the mathematical model of the problem. The second is an *a posteriori* approach, in which the suitability to the context of some previously calculated solutions to the problem is checked by using a multi-criteria decision-making methodology.

Definition 9: A context, regardless of the nature of the information available, is defined as a non-empty set of propositions, often presented in the form of logical predicates, that establish the qualitative characteristics that the available decisions *must have*.

Definition 10: As predicates may not have clearly defined boundaries, a context $\mathcal{F}$ in fuzzy environment can be expressed as the following set of fuzzy propositions.

$$\mathcal{F} := \left\{ \begin{array}{rl} p_1: & \textbf{and}\Big(\textbf{or}\,(V_{11} \text{ is } B_{11}, V_{12} \text{ is } B_{12}, \ldots)\,, \ldots, \textbf{or}\,(\ldots, V_{1s} \text{ is } B_{1s})\Big), \\ p_2: & \textbf{and}\Big(\textbf{or}\,(V_{21} \text{ is } B_{21}, V_{22} \text{ is } B_{22}, \ldots)\,, \ldots, \textbf{or}\,(\ldots, V_{2s} \text{ is } B_{2s})\Big), \\ \vdots & \vdots \\ p_m: & \textbf{and}\Big(\textbf{or}\,(V_{m1} \text{ is } B_{m1}, V_{m2} \text{ is } B_{m2}, \ldots)\,, \ldots, \textbf{or}\,(\ldots, V_{m,s} \text{ is } B_{m,s})\Big), \\ p_{m+1}: & \textbf{implies}\big(r_{m+1}, q_{m+1}\big), \\ p_{m+2}: & \textbf{implies}\big(r_{m+2}, q_{m+2}\big), \\ \vdots & \vdots \\ p_{m+o}: & \textbf{implies}\big(r_{m+o}, q_{m+o}\big) \end{array} \right\}$$

where V_{jk} is B_{jk} $(j = 1, 2, \ldots, m;\ k = 1, 2, \ldots, s)$ are unconditional fuzzy propositions, and each r_i and q_i has the same structure of p_j, for $j = 1, 2, \ldots, m$.

The choice of logical connectives depends on the specific situation and should provide the best description of the decision-maker’s reasoning with uncertainty. There are experimental and theoretical (choosing according to some reasonable properties) methods to do it, but the problem of choice remains [28].

The general mathematical programming problem, considering a fuzzy context, can be formulated as follows.

$$\begin{aligned} \max(\min)\ & f(\mathbf{x}) \\ \text{s.t. } & \mathbf{G}(\mathbf{x}) \le \mathbf{0}, \\ & \mathbf{H}(\mathbf{x}) = \mathbf{0}, \\ & \mathbf{x} \in \mathcal{F}, \end{aligned} \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$ is a real-valued vector in the n-dimensional Euclidean space. Furthermore, f is a real-valued function in $\mathbb{R}^n$, and $\mathbf{G} = (g_1, g_2, \ldots, g_p)$ and $\mathbf{H} = (h_1, h_2, \ldots, h_r)$ are, respectively, p-dimensional and r-dimensional vectors of real-valued functions in $\mathbb{R}^n$. Lastly, $\mathbf{x} \in \mathcal{F}$ means that $\mathbf{x}$ makes the propositions in $\mathcal{F}$ true at least to a certain degree; clearly, all propositions in $\mathcal{F}$ must be related to $\mathbf{x}$ in some way.

A. A priori approach

According to the definition of context, a solution to problem (1) must have the qualitative characteristics established by $\mathcal{F}$. This means that all propositions in $\mathcal{F}$ must be satisfied. We see this requirement as the conjunction of all propositions in $\mathcal{F}$. Consequently, by using a suitable t-norm aggregation operator, problem (1) is transformed into the following crisp mathematical programming problem.

$$\begin{aligned} \max(\min)\ & f(\mathbf{x}) + (-)\alpha M \\ \text{s.t. } & \mathbf{G}(\mathbf{x}) \le \mathbf{0}, \\ & \mathbf{H}(\mathbf{x}) = \mathbf{0}, \\ & \triangle\big(\operatorname{truth}(p_1), \operatorname{truth}(p_2), \ldots, \\ & \quad \operatorname{truth}(p_{m+o})\big) \ge \alpha, \end{aligned} \tag{2}$$

where α is an auxiliary variable and M is a very large constant. The product αM penalises the solutions that do not satisfy the context.

B. A posteriori approach

Depending on the number of propositions in $\mathcal{F}$, but mainly on the choice of operators that model the logical connectives and the membership functions used, problem (2) can be difficult to solve due to possible discontinuities, non-linearities and non-convexities that can be introduced in the problem model. An alternative approach is to obtain a set of ‘good’ feasible solutions to the problem:

$$\begin{aligned} \max(\min)\ & f(\mathbf{x}) \\ \text{s.t. } & \mathbf{G}(\mathbf{x}) \le \mathbf{0}, \\ & \mathbf{H}(\mathbf{x}) = \mathbf{0}, \end{aligned} \tag{3}$$

by using exact algorithms, simulation or metaheuristics [29] and analyse their suitability according to $\mathcal{F}$. In fact, it may be the case that problem (3) has a special mathematical structure for which efficient solution algorithms exist, and this structure may be lost if additional (context-related) constraints are included. This approach results in the following multi-criteria decision-making problem.

Let $X = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_h\}$ be a non-empty set of feasible solutions to problem (3). For each $\mathbf{x}_i \in X$, the degree of truth of proposition p_j is calculated and denoted by t_{ij}

($j = 1, 2, \ldots, m + o$). Consequently, the decision matrix $T = \left[t_{ij}\right]_{h\times(m+o)}$ is constructed. Next, the solutions/alternatives in X must be ranked and the one that best fits the context $\mathcal{F}$ selected.

Although there are a significant number of multi-criteria decision-making methods for making such ranking and selection [30], here, a method is proposed that is consistent with the formulation of the *a priori* approach. By using a suitable t-norm aggregation operator $\triangle$, a solution $\mathbf{x}^*$ (within the context $\mathcal{F}$) is then drawn from the set $\{\mathbf{x}_k \in X : \triangle\big(t_{i1}, t_{i2}, \ldots, t_{i,m+o}\big) \leq \triangle\big(t_{k1}, t_{k2}, \ldots, t_{k,m+o}\big)$ for all $i = 1, 2, \ldots, h\}$—namely, a solution $\mathbf{x}^* \in X$ is chosen such that it maximises the overall degree of truth of the propositions in $\mathcal{F}$ according to the selected t-norm.

IV. Illustrative Examples

In this section, we present two examples to illustrate the proposed approaches. For simplicity, mostly piecewise linear membership functions are used to characterise the meaning of linguistic terms. Modelling and calculations were performed by using YALMIP toolbox [31] version 20180413, Octave 5.2.0 and Python 3.10.3 on a computer with an Intel® Core™ i3-4005U @ 1.70GHz × 4 and 4GB RAM running Ubuntu 20.04.3 LTS.

A. Fish harvesting problem

Let us consider a simple two-period harvesting model (HM) of a hypothetical fish species with reproduction rate $r_1 = 0.2$ (20%) in the first period and $r_2 = 0.15$ (15%) in the second one. A fraction of the stock will be caught in the first period (denoted by c_1) and another fraction c_2 in the second period. The biomass (in kg) at the beginning of the first and second periods is denoted by I_1 and I_2 , respectively, and by I_3 the biomass at the end of the second period. We have the following equations that relate I_1, I_2 and I_3.

$$\begin{aligned} I_2 &= I_1 + r_1 I_1 - c_1 I_1, \\ I_3 &= I_2 + r_2 I_2 - c_2 I_2. \end{aligned}$$

Suppose that the maximum economic benefit is sought. Therefore, fishermen will try to catch as much as possible and thus to maximise the quantity $c_1 I_1 + c_2 I_2$. In this case, the HM takes the form

$$\begin{aligned} \max\ & c_1 I_1 + c_2 I_2 \\ \text{s.t. } & I_2 = I_1 + r_1 I_1 - c_1 I_1, \\ & I_3 = I_2 + r_2 I_2 - c_2 I_2, \\ & 0 \leq c_1 \leq 1,\ 0 \leq c_2 \leq 1, \\ & I_1 = 1000, \end{aligned} \tag{4}$$

and it is assumed that 1000 units of biomass are present at the beginning of the first period.

By solving HM (4), we get $c_1 = 1$, $c_2 = 1$, $I_2 = 200$ and $I_3 = 30$ with benefit value 1200. Although this solution yields the maximum economic benefit, it leaves the biomass of the species at the end of the second period in a very low level. Consequently, implementing such a solution goes against the recommendations for the conservation of target species and the sustainable exploitation of fish stocks, put forward by the Food and Agriculture Organisation of the United Nations (FAO) [32].

Actually, in FAO's Code of Conduct for Responsible Fisheries [32, p. 1], it reads: 'Fisheries, including aquaculture, provide a vital source of food, employment, recreation, trade and economic well-being for people throughout the world, both for present and future generations and should therefore be conducted in a responsible manner.' Going further into the management objectives, the code establishes that measures should be taken to allow the recovery of depleted stocks or to actively restore them. This and other recommendations aim at a sustainable use of fishery resources; thus guaranteeing not only the conservation of target species, but also sufficient quantities so that the exploitation of the stocks remains economically viable.

Now, let us approach the harvesting problem again, but this time establishing a *sustainability context* with the set of propositions[2]

$$\mathcal{F}_s := \left\{ \begin{array}{ll} p_1: & \text{if } I_1 \text{ is } \textit{High} \textbf{ and } r_1 \text{ is } \textit{Low}, \text{ then } c_1 \text{ is } \textit{Mean}, \\ p_2: & \text{if } I_2 \text{ is } \textit{High} \textbf{ and } r_2 \text{ is } \textit{Low}, \text{ then } c_2 \text{ is } \textit{Mean}, \\ p_3: & I_3 \text{ is } \textit{High} \end{array} \right\}$$

in which Zadeh's conjunction and implication are used (see Table I), and the linguistic terms *Low*, *Mean* and *High* have respectively the membership functions $\mu_{\text{low}}(r_k) = \max\big(0, 1 - 2r_k\big)$, $\mu_{\text{mean}}(c_k) = \min\big(2c_k, 2 - 2c_k\big)$, for $k = 1, 2$, and $\mu_{\text{high}}(I_k) = \max\big(0, \min\big(1, (I_k - 500)/500\big)\big)$ for $k = 1, 2, 3$ (see Figure 1). Thus, the HM in a sustainability context (F-HM) takes the form

$$\begin{aligned} \max\ & c_1 I_1 + c_2 I_2 \\ \text{s.t. } & I_2 = I_1 + r_1 I_1 - c_1 I_1, \\ & I_3 = I_2 + r_2 I_2 - c_2 I_2, \\ & 0 \leq c_1 \leq 1,\ 0 \leq c_2 \leq 1, \\ & I_1 = 1000, \\ & (c_1, c_2, I_1, I_2, I_3) \in \mathcal{F}_s. \end{aligned} \tag{5}$$

Next, both the *a priori* and *a posteriori* approaches are used to solve F-HM (5).

1) Solution via a priori approach: By using Zadeh's connectives from Table I, F-HM (5) is transformed into problem

$$\begin{aligned} \max\ & c_1 I_1 + c_2 I_2 + 10^6 \alpha \\ \text{s.t. } & I_2 = I_1 + r_1 I_1 - c_1 I_1, \\ & I_3 = I_2 + r_2 I_2 - c_2 I_2, \\ & 0 \leq c_1 \leq 1,\ 0 \leq c_2 \leq 1, \\ & I_1 = 1000, \\ & \max\Big[1 - \min\Big(\mu_{\text{high}}(I_1), \mu_{\text{low}}(r_1)\Big), \\ & \quad \min\Big(\min\left(\mu_{\text{high}}(I_1), \mu_{\text{low}}(r_1)\right), \mu_{\text{mean}}(c_1)\Big)\Big] \geq \alpha, \\ & \max\Big[1 - \min\Big(\mu_{\text{high}}(I_2), \mu_{\text{low}}(r_2)\Big), \\ & \quad \min\Big(\min\left(\mu_{\text{high}}(I_2), \mu_{\text{low}}(r_2)\right), \mu_{\text{mean}}(c_2)\Big)\Big] \geq \alpha, \\ & \mu_{\text{high}}(I_3) \geq \alpha,\ \alpha \geq 0, \end{aligned}$$

[2]This set of propositions is not exhaustive and other propositions could be included. However, we keep it this simple for illustration purposes.

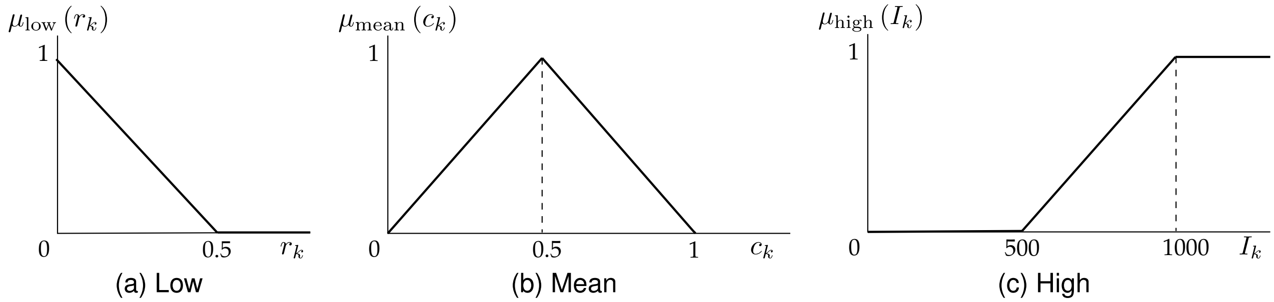

Figure 1. Graphs of the membership functions corresponding to the linguistic values *Low*, *Mean* and *High*.

Table II
BEST SOLUTIONS TO THE HARVESTING PROBLEM IN THE BASIC SETTING AND WITH CONTEXT INCLUDED *a priori* AND *a posteriori*

Approach	Connectives	c_1	c_2	I_1	I_2	I_3	Benefit Value	Overall Truth
Basic model	none	1	1	1000	200	30	1200	0
A priori	Zadeh	0.5	0.007	1000	700	800	505	0.6
A posteriori		0.449	0.021	1000	750.508	846.957	465.619	0.498
A priori	Łukasiewicz	0.294	0.255	1000	905.197	809.972	525.806	0.609
A posteriori		0.449	0.021	1000	750.508	846.957	465.619	0.535

and solved by using Octave code A.1 shown in the Appendix section. Thus, we get $c_1 = 0.5$, $c_2 = 0.0071429$, $I_2 = 700$ and $I_3 = 800$ with benefit value 505; the overall degree of truth corresponding to this solution is 0.6. It is noticeable the fact that this solution provides less economic benefit. However, it does leave enough biomass at the end of the second period, so that the species can reproduce without difficulties and the exploitation of the stock remains economically viable. Alternatively, we may choose Łukasiewicz's connectives to handle the propositions in $\mathcal{F}_s$ and solve F-HM (5), in which case the results shown in Table II are obtained. We see that the biomass level and the fraction to be caught in each period are different from those obtained by using Zadeh's connectives, but the economic benefit and overall degree of truth are almost the same.

2) Solution via a posteriori approach: Let us consider again the initial harvesting model (4). By using simulation (see Octave code A.2 in the Appendix section), we obtained a set of 30 feasible solutions from which 10 are shown in Table III. We followed the approach presented in section III-B with Zadeh's and Łukasiewicz's connectives and obtained the decision matrices shown also in Table III.

According to Zadeh's t-norm aggregation operator, we found that solution No. 6 has an overall degree of truth of 0.498, which is the highest among all simulated solutions and therefore the one that best fits the sustainability context. Interestingly, using Łukasiewicz's connectives with the *a posteriori* approach also leads to choosing solution No. 6 with an overall degree of truth of 0.535. Solution No. 6 has a corresponding benefit value of 465.619. However, it should be noted that the solution given by the *a priori* approach is better than solution No. 6, both in terms of fitting the sustainability context as well as in maximising the economic benefit (see summarised results in Table II).

B. Tourist trip design problem

In 2020, due to the COVID-19 pandemic, the tourism sector declined 49% with a loss of approximately US$4.5 trillion and 62 million jobs.[3] Present Secretary-General of the United Nations, António Guterres, has said that 'It is imperative that we rebuild the tourism sector.' One in every ten people in the world works in this sector, and hundreds of millions more owe their livelihoods to it [33].

Guterres [33] identified five priority areas to aid recovery of the tourism sector. In particular, the third calls us to maximise the use of technology. Using mathematical models to design tourist trips is one of the many ways in which technology could be used to aid recovery. However, at the time of writing these lines, the COVID-19 pandemic is not yet over,[4] and tourist trips may cause mass gatherings, with the subsequent risk of amplifying the transmission of SARS-CoV-2. In this *pandemic context*, solutions provided by mathematical models for the design of tourist trips may not be in accordance with indications given in World Health Organisation's (WHO's) guidelines for holding gatherings during the COVID-19 pandemic [34].

To further illustrate our approach, we present a mathematical model of an NP-hard route planning problem [35], known as tourist trip design problem (TTDP), for tourists interested in visiting multiple points of interest (POIs) in a city. The *a posteriori* approach will be used on a simplified version of the TTDP to obtain routes with characteristics not originally included in its mathematical model. The TTDP model is given

[3]https://research.wttc.org/trending-in-travel (accessed on 10 June 2022)
[4]https://news.un.org/en/story/2022/05/1118752 (accessed on 10 June 2022)

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Table III
SOLUTIONS TO THE HARVESTING PROBLEM OBTAINED BY SIMULATION

Solutions							Zadeh's connectives				Łukasiewicz's connectives			
No.	c_1	c_2	I_1	I_2	I_3	Benefit Value	truth(p_1)	truth(p_2)	truth(p_3)	Overall Truth	truth(p_1)	truth(p_2)	truth(p_3)	Overall Truth
1	0.134	0.233	1000	1065.635	977.097	382.747	0.400	0.466	0.954	0.400	0.668	0.766	0.954	0.389
2	0.847	0.230	1000	352.566	324.055	928.829	0.400	1.000	0.000	0.000	0.705	1.000	0.000	0.000
3	0.763	0.218	1000	436.225	406.221	859.212	0.472	1.000	0.000	0.000	0.872	1.000	0.000	0.000
4	0.255	0.459	1000	944.930	652.377	689.362	0.510	0.700	0.304	0.304	0.910	1.000	0.304	0.214
5	0.495	0.289	1000	704.564	606.079	699.605	0.600	0.590	0.212	0.212	1.000	1.000	0.212	0.212
6	**0.449**	**0.021**	**1000**	**750.508**	**846.957**	**465.619**	**0.600**	**0.498**	**0.693**	**0.498**	**1.000**	**0.841**	**0.693**	**0.535**
7	0.651	0.837	1000	548.407	171.334	1110.926	0.600	0.903	0.000	0.000	1.000	1.000	0.000	0.000
8	0.788	0.556	1000	411.276	244.111	1017.580	0.422	1.000	0.000	0.000	0.822	1.000	0.000	0.000
9	0.093	0.642	1000	1106.140	561.593	804.327	0.400	0.700	0.123	0.123	0.587	1.000	0.123	0.000
10	0.028	0.185	1000	1171.652	1129.582	246.165	0.400	0.371	1.000	0.371	0.456	0.671	1.000	0.128

by Equation (6).

$$\begin{aligned} &\max \sum_{i=1}^{k} S_{\pi(i)} \\ &\text{s.t. } t_{0,\pi(1)} + \left(\sum_{i=1}^{k-1} t_{\pi(i),\pi(i+1)}\right) + t_{\pi(k),0} \\ &\quad + \sum_{i=1}^{k} v_{\pi(i)} \leq T_{\max}, \\ &\quad k \in N = \{1, 2, \ldots, p\}, \end{aligned} \tag{6}$$

where p is the number of POIs, excluding the starting and ending point of a route; S_i and v_i denote the interest in visiting POI i and the time required to visit it, respectively; t_{ij} is the time required to travel from POI i to POI j; and $T_{\max}$ denotes the maximum available time to complete a route. The decision variable is π (route), a permutation of any subset of k elements of N, where $\pi(i)$ is the POI visited at position i of the route and POI $i = 0$ is the starting and ending point of the route. The objective is to find routes with maximum overall interest.

To identify routes suitable for a pandemic context, solutions to TTDP (6) may be analysed according to three factors (duration, location, and compliance with precautionary measures) present in WHO's guidelines [34]. 'Duration' refers to the average time spent visiting the POIs on a route. 'Location' refers to the type of each POI (outdoor or indoor). Lastly, 'compliance with precautionary measures' refers to each POI's adherence to current precautionary measures dictated by health authorities, such as physical distancing, hand sanitiser at the entrance, use of sanitary masks, and ventilation.

Taking into account the previously mentioned factors, we may define a pandemic context with the following fuzzy propositions.

- p_1 : The average time spent visiting the POIs on the route is *Low* or *Mean*,
- p_2 : (The compliance with precautionary measures of the route is *High*) is *Very True*,
- p_3 : If route's occupancy is *Mean* **or** route's occupancy is *High*, then route's ventilation is *High*.

The average time spent visiting the POIs on the route is denoted by V_π. The compliance with precautionary measures of POI i, denoted by m_i, is calculated as the number of precautionary measures present in POI i divided by the total number of such measures (4 in this case); for a route, it is taken as the average over all POIs in the route and is denoted by M_π. Occupancy of POI i is calculated as $o_i = n/C_i$, where n is the number of tourists taking the route (20 in this case) and C_i denotes the capacity (number of people) of POI i; for a route, it is then taken as the average over all POIs in the route and is denoted by O_π. Ventilation of POI i is calculated according to Standard 62.1-2019 of the American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) [36] and then normalised by using Equation (B.1) in the Appendix section. A route's ventilation is calculated as the average normalised ventilation over all POIs in the route and is denoted by $Vent_\pi$. It is assumed that outdoor POIs have occupancy and ventilation values of 0 and 1, respectively.

The linguistic terms *Low*, *Mean*, *High*, and *Very True* have membership functions $\mu_{\text{low}}(x) = \max(0, 1 - 2x)$, $\mu_{\text{mean}}(x) = \min(2x, 2 - 2x)$, $\mu_{\text{high}}(x) = \max(0, 2x - 1)$, and $\mu_{\text{very true}}(x) = x^2$, respectively. By using the previous notation and that of Definition 10, the pandemic context can be written as

$$\left\{ \begin{array}{l} p_1 : \textbf{or}\,(V_\pi \text{ is } \textit{Low}, V_\pi \text{ is } \textit{Mean}), \\ p_2 : (M_\pi \text{ is } High) \text{ is } \textit{Very True}, \\ p_3 : \textbf{implies}\,(\textbf{or}\,(O_\pi \text{ is } \textit{Mean}, O_\pi \text{ is } \textit{High}), Vent_\pi \text{ is } \textit{High}) \end{array} \right\}.$$

Data for solving TTDP (6) are available from https://github.com/cporrasn/TTDP_data and consist of 33 POIs from Granada city (Spain), including museums, parks and religious sites, obtained by using function geometries_from_place('Granada, Spain') from the Python package OSMnx [37]. It should be noted that this simplified model is easier to solve with context included *a posteriori* because no additional constraints are added to the model and the objective function is not modified; thus avoiding the non-linearities present in the propositions.

Table IV shows 10 feasible solutions to TTDP (6) obtained by using the crossover-less evolutionary algorithm described in [29] with population size 30, number of parents 30 and number of generations 100. Table V shows the decision matrices obtained by using Gödel's, Zadeh's and Łukasiewicz's connectives. It can be noticed that results obtained by using Gödel's and Zadeh's connectives lead to choosing route No. 3 with overall degree of truth of 0.660. On the other hand, using Łukasiewicz's connectives leads to choosing route No. 10 with overall degree of truth of 0.734. Interestingly, route No. 10 is the second best route according to Gödel's connectives with overall degree of truth of 0.607 (close to that of route No. 3), and it is ranked fourth according to Zadeh's connectives, but with overall degree of truth of only 0.523. According to Łukasiewicz's connectives, route No. 3 is ranked third, but its overall degree of truth is far from that of route No. 10. It may seem that route No. 3 is the most consistent one, but

Table IV
SOLUTIONS TO TTDP (6) OBTAINED BY USING THE EVOLUTIONARY ALGORITHM DESCRIBED IN [29]

No.	Route	Overall Interest	Visiting Time*	Ventilation*	Compliance*	Occupancy*
1	(6, 28, 17, 13, 29, 11, 16)	15	0.196	0.961	0.857	0.261
2	(6, 9, 14, 17, 11, 13, 33, 16)	14	0.166	0.968	0.843	0.166
3	(16, 33, 17, 28, 11, 26, 13, 30)	14	0.161	0.969	0.906	0.166
4	(28, 9, 17, 16, 10, 13, 11, 6)	14	0.161	0.969	0.843	0.166
5	(19, 17, 16, 28, 11, 33)	14	0.215	0.955	0.875	0.305
6	(29, 16, 19, 11, 17, 13)	14	0.215	0.955	0.875	0.305
7	(16, 11, 29, 13, 17, 28, 33)	15	0.196	0.961	0.857	0.261
8	(7, 17, 28, 16, 11, 13)	14	0.222	0.953	0.916	0.305
9	(16, 9, 28, 17, 26, 10, 11, 13)	14	0.161	0.969	0.875	0.166
10	(16, 29, 30, 11, 13, 26, 28)	14	0.196	0.973	0.928	0.238

* Average values

Table V
DECISION MATRICES OF TTDP (6) CALCULATED BY USING GÖDEL'S, ZADEH'S AND ŁUKASIEWICZ'S CONNECTIVES

No.	Gödel				Zadeh				Łukasiewicz			
	truth(p_1)	truth(p_2)	truth(p_3)	Overall Truth	truth(p_1)	truth(p_2)	truth(p_3)	Overall Truth	truth(p_1)	truth(p_2)	truth(p_3)	Overall Truth
1	0.607	0.510	1.000	0.510	0.607	0.510	0.523	0.510	1.000	0.510	1.000	0.510
2	0.666	0.472	1.000	0.472	0.666	0.472	0.666	0.472	1.000	0.472	1.000	0.472
3	**0.677**	**0.660**	**1.000**	**0.660**	**0.677**	**0.660**	**0.666**	**0.660**	1.000	0.660	1.000	0.660
4	0.677	0.472	1.000	0.472	0.677	0.472	0.666	0.472	1.000	0.472	1.000	0.472
5	0.569	0.562	1.000	0.562	0.569	0.562	0.611	0.562	1.000	0.562	1.000	0.562
6	0.569	0.562	1.000	0.562	0.569	0.562	0.611	0.562	1.000	0.562	1.000	0.562
7	0.607	0.510	1.000	0.510	0.607	0.510	0.523	0.510	1.000	0.510	1.000	0.510
8	0.555	0.694	1.000	0.555	0.555	0.694	0.611	0.555	1.000	0.694	1.000	0.694
9	0.677	0.562	1.000	0.562	0.677	0.562	0.666	0.562	1.000	0.562	1.000	0.562
10	0.607	0.734	1.000	0.607	0.607	0.734	0.523	0.523	**1.000**	**0.734**	**1.000**	**0.734**

as mentioned before choosing the appropriate set of logical connectives is an application-dependent issue still unresolved. Lastly, as expected, routes less suitable for the pandemic context may have higher overall interest (objective function value) than more suitable ones (see routes No. 1 and 7).

V. CONCLUDING REMARKS

Optimal solutions may be useless in practice when they come from models built with no consideration of the contexts in which the problems arise. Hence, modelling such contexts and using the resulting models to effectively assist decision-making should not be overlooked. In this paper, we used fuzzy propositions to model contexts and proposed two approaches to solve optimisation problems posed within such contexts. An *a priori* approach was developed, in which the context is included in the constraint set of the optimisation problem. Optimal solutions obtained in this way always conform (to the highest possible degree) to the context in which the problem has been posed. However, the solving process is complicated by the additional set of constraints included in the problem model. On the other hand, an *a posteriori* approach was developed to alleviate the computational burden. The *a posteriori* approach leaves the problem model intact and uses techniques such as simulation or metaheuristic algorithms to obtain a set of solutions that are checked for their suitability to the context by means of a multi-criteria decision-making methodology. However, this approach cannot guarantee optimal solutions. A fish harvesting problem in a sustainability context and a TTDP in a COVID-19 pandemic context were solved as application examples. The results stemmed from these examples show that 'context-aware' solutions are more useful in practice and contribute to an effective decision-making. Future work will be devoted to applying our results to other problems and analysing solutions from the perspective of different contexts. Extending the theoretical results to multi-objective optimisation is also an interesting research line to explore. Future work will also be devoted to establishing guidelines for choosing the appropriate set of logical connectives to model decision-making contexts, and incorporating these contexts into ADM systems.

APPENDIX A
COMPUTER CODES

```
%% Variables and parameters
% Fraction of the biomass to be removed in each period.
c = sdpvar(1,2);
% Biomass at the beginning of each period.
% Biomass at the beginning of period 1 is known; therefore,
% we only use variables I(2) and I(3).
I = sdpvar(1,3);
% Auxilliary variable for context modelling.
alpha = sdpvar();
% Initial biomass
I1 = 1000;
% Reproduction rate
r = [0.2,0.15];
%% Problem constraints
C = [I1−c(1)∗I1+r(1)∗I1==I(2),
     I(2)−c(2)∗I(2)+r(2)∗I(2)==I(3),
     c(1)>=0, c(1)<=1, c(2)>=0, c(2)<=1];
%% Context modelling
% Membership functions (High)
mI1 = max(0, min(1,(I1−500)/500));
mI2 = max(0, min(1,(I(2)−500)/500));
mI3 = max(0, min(1,(I(3)−500)/500));
% Membership functions (Low)
mr1 = max(0, 1−2∗r(1));
mr2 = max(0, 1−2∗r(2));
% Membership functions (Mean)
mc1 = min(2∗c(1), 2−2∗c(1));
mc2 = min(2∗c(2), 2−2∗c(2));
% Define the context using Zadeh's connectives.
Context = [max(1−min(mI1, mr1),...
           min(min(mI1, mr1),mc1))>=alpha,
```

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```
                    max(1−min(mI2, mr2),...
                    min(min(mI2, mr2),mc2))>=alpha,
                    mI3>=alpha, alpha>=0];
C = [C, Context];
%% Objective function
benefit = c(1)*I1+c(2)*I(2);
objective = benefit+10^6*alpha;
%% Solve the problem using bmibnb with glpk
ops = sdpsettings('solver','bmibnb','bmibnb.lowersolver','glpk',...
              'bmibnb.lpsolver','glpk');
optimize(C, −objective, ops)
%% Results
% Biomass at the beginning of each period.
biomass = [I1, value(I(2)), value(I(3))]
% Fraction of the biomass removed in each period.
fraction = [value(c(1)), value(c(2))]
% Biomass removed in each period.
removed = [value(c(1)*I1), value(c(2)*I(2))]
% Overall degree of truth using Zadeh's conjunction (min).
truth = value(min([max(1−min(mI1, mr1),...
                         min(min(mI1, mr1),mc1)),
                     max(1−min(mI2, mr2),...
                         min(min(mI2, mr2),mc2)),
                     mI3]))
```

Code A.1. Harvesting model in Octave.

```
%% For reproducibility
rand('state',1)
%% Reproduction rate
r = [0.2, 0.15];
%% Membership function (High)
mI = @(x) max(0, min(1,(x−500)/500));
%% Membership function (Low)
mr = @(x) max(0, 1−2*x);
%% Membership function (Mean)
mc = @(x) min(2*x, 2−2*x);
%% Generate N=30 solutions
N = 30;
% Generate random values for c1 and c2
C = rand(N, 2);
% Set I1 = 1000
I1 = 1000*ones(N,1);
% Calculate I2 and I3
I2 = I1 +r(1)*I1−C(:,1).*I1;
I3 = I2 +r(2)*I2−C(:,2).*I2;
% Store all solutions in matrix M
M = [C,I1,I2,I3];
% Calculate the truth value of the propositions
truth_p1 = arrayfun(@(row)max(1−min(mI(M(row,3)),mr(r(1))),...
       min(min(mI(M(row,3)), mr(r(1))),mc(M(row,1)))),(1:N)');
truth_p2 = arrayfun(@(row)max(1−min(mI(M(row,4)),mr(r(2))),...
       min(min(mI(M(row,4)), mr(r(2))),mc(M(row,2)))),(1:N)');
truth_p3 = arrayfun(@(row)mI(M(row,5)),(1:N)');
%% Decision matrix
T = [(1:N)',truth_p1, truth_p2, truth_p3];
%% Use Zadeh's conjunction (min) to aggregate the truth values
% and then sort the solutions in ascending order
sT = sortrows([T(:,1),arrayfun(@(row)min(T(row,2:4)),(1:N)')],2);
%% Select the best solution
best = M(sT(end,1),:)
```

Code A.2. Simulation of the harvesting model in Octave.

Appendix B
Calculation of Normalised Ventilation

The following notation is used to calculate the normalised ventilation of a POI.

- n: Number of tourists taking the route,
- C_i: Capacity (number of people) of POI i,
- A_i: Floor area (m^2) of POI i,
- $(R_p)_i$: Outdoor airflow rate required per person (L/s·person) of POI i,
- $(R_a)_i$: Outdoor airflow rate required per unit area (L/s·m^2) of POI i.

Normalised ventilation is given by

$$vent_i = 1 - \frac{(R_p)_i \times n}{(R_p)_i \times C_i + (R_a)_i \times A_i}, \tag{B.1}$$

where the denominator is the ventilation of a POI i calculated according to Standard 62.1-2019 of ASHRAE [36]. The values of R_p and R_a depend on the categories of indoor POIs (museums, places of religious worship, and so on). Refer to Table 6-1 in [36].

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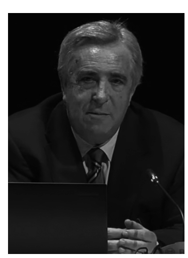

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