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Effect of Mathematical Content Knowledge on Mathematical Knowledge for Teaching

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ABSTRACT
Several studies and related literature stated that in an effective teaching process, teachers need to have mathematical content knowledge and pedagogical knowledge to be able to teach mathematics deeply and broadly.In this study, researchers worked with pre-service primary teachers through teaching practice activities to see how mathematical content knowledge influenced their mathematical knowledge for teaching.Data collection was carried out through observation, interviews, and field notes.The collected data were analyzed by induction and deduction methods.The results of the study showed that when pre-service primary teachers were lacking in mastering mathematical content knowledge, it hindered pedagogical content knowledge and mathematical knowledge for teaching as well.Their weakness in mathematical content knowledge can be seen when they interpreted and evaluated students' explanations.They also found it difficult to provide feedback to students.For research subjects who had good mathematical content knowledge, but not optimal pedagogical content knowledge, learning was also not effective.They seemed to have difficulty bringing students to contextual situations. As a result, they used the knowledge that students have not understood yet.

Keywords: Content Knowledge, Mathematical Knowledge for Teaching, Pre-Service Teachers

INTRODUCTION

Content knowledge includes knowledge related to a subject and its organizational structure(Shulman, 1986), and refers to facts, concepts, theories, and principles(Bakar et al., 2022; Metz, 2021). Content knowledge is knowledge about the material taught in the form of facts, theories, concepts, procedures, and relationships between concepts, evidence, and content as well(Koehler, M. J., Mishra, 2008). Thus, mastery of content knowledge can be characterized by scientific mastery that can be used to deepen certain materials.

One of the competencies that prospective teachers should possess is broad and in-depth mastery of content knowledge(Kunter et al., 2013; Orazbayeva, 2016). One of the indicators is that prospective teachers master scientific substance to deepen knowledge or material in a particular field of study. Teachers and prospective teachers must master the concept of what will be taught as one of the competencies they must possess (Haylock, 2007; Loewenberg Ball et al., 2008). Teachers' mathematical knowledge significantly influences what and how the teachers teach and what and how students learn (Masingila et al., 2018). Prospective teachers' weak content knowledge mastery can hinder their ability to analyze students' thinking and their ability to involve in an ideal learning process(Norton, 2013).

However, it is not enough to only master content knowledge for effective learning. Teachers and prospective teachers need to have deep and broad pedagogical content knowledge(Guerrero-Roldán & Noguera, 2018). Some previous studies reported that teachers (especially elementary school teachers) do not understand mathematics well enough. On the other hand, we know that understanding the material is important to be able to teach mathematics because it involves managing instructive interactions so that all activities carried out by teachers and students focus on understanding the material(Browning et al., 2014; Delaney, S., & Phelps, 2008; Hoover et al., 2016). Prospective teachers' mathematics knowledge is one of the essential things that must be possessed by teachers to be able to teach effectively(Li & Kulm, 2008; Novikasari, 2020; Setyaningrum et al., 2018). In addition, researchers have shown that mathematical content knowledge correlates with pedagogic abilities(Ekawati et al., 2018; Yang et al., 2021). Previous studies have also shown that pedagogical knowledge of mathematics is directly correlated with knowledge of mathematical content(Hill et al., 2008; Koehler, M. J., Mishra, 2008; Loewenberg Ball et al., 2008).

Mathematical knowledge for teaching (MKT) covers the mathematical knowledge required by teachers to teach mathematics by taking into account knowledge of mathematics from a teaching perspective, including knowledge of the structure of the subject, the rules of how it works, and careful thought about the content and relationship between mathematics and the way of teaching it(Carrillo et al., 2013; Hill et al., 2008; Nolan et al., 2015). The knowledge is grouped into subject matter knowledge (SMK) and pedagogical content knowledge (PCK).SMK includes knowledge that must be possessed and developed when teaching mathematics, whilepedagogical content knowledge (PCK) includes knowledge about how students understand the material, and how to design and managethe learning process(Shulman, 1986). It is undeniable that the problems of learning mathematics, especially in elementary schools, are not only on the students' side but also on teachers and their learning media.The emergence of problems in learning is due to the gap between the theory obtained during lectures and real learning practices in the field (Leikin & Levav-Waynberg, 2007). The current condition indicates that many elementary school teacher candidates still lack mathematics knowledge. Besides, they also lack knowledge of how to teach mathematics properly.

This study describes how the mathematical content knowledge of prospective teachers influences the components of mathematical knowledge for teaching.This describes, for example, how the prospective teachers' content knowledge influences the way they design a lesson.This research investigates the ability of prospective teachers in their teaching practice activities to obtain more comprehensive data when they conduct learning in real classes.Therefore, this study tries to describe how the mathematical content knowledge of prospective teachers influences their mathematical knowledge for teaching.

THEORETICAL FRAMEWORK

A study based on the results of learning experience and Shulman's work adoption by adding subject matter knowledge as one of its aspects yielded a new theory, namely mathematical content knowledge (Hill et al., 2008). These theoretical ideas include how teachers interact in the pedagogic context in classes, design assignments, and check students' work.

Constructing and mastering a mathematical concept requires a long time and process.To overcome the problems of elementary school teachers who teach mathematics, it is necessary to improve teachers' and prospective teachers' mastery of mathematical concepts and their pedagogic abilities.There are three main components of knowledge that teachers must have: subject matter knowledge, pedagogical content knowledge, and general pedagogical knowledge(Attorps, 2006; Hill et al., 2008). These three components are interrelated. It means that teachers and prospective teachers do not only have to master content knowledge but alsomathematical knowledge which is inseparable from mastering content knowledge for teaching individuals.Prospective teachers' content knowledge can be used to see how prospective teachers master a concept and how they organize certain content.Content knowledge is knowledge about the material to be taught which is in the form of facts, theories, concepts, procedures, relationships between concepts, evidence, and content as well(Koehler, M. J., Mishra, 2008). Meanwhile, subject matter knowledge, in this case, mathematical content knowledge, is material that contains other knowledge, namely knowledge of content, knowledge of substantive structure, knowledge of syntactic structure, conceptual knowledge, and procedural knowledge.Knowledge of content can be seen from the amount of mathematical knowledge possessed by teachers and how they organize this knowledge.The amount of knowledge possessed by teachers will greatly determine the development of material.Knowledge of substantive structure is a way to organize facts, concepts, and principles in mathematics.Knowledge is closely related to previous knowledge and requires experience in learning.Knowledge of syntactic structure is an understanding of the rules of proof in mathematics so that teachers can believe that they convey the truth. In short, conceptual knowledge is computational ability in mathematics, while procedural knowledge is the ability to identify components, algorithms, and mathematical definitions (Attorps, 2006).

Mathematical knowledge for teaching (MKT) is the mathematical knowledge required by teachers to teach mathematics by taking into account knowledge of mathematics from a teaching perspective, including knowledge of the structure of the subject, the rules of how it works, and careful thought about the content and relationship between mathematics and the way of teaching it(Carrillo et al., 2013; Hill et al., 2005; Nolan et al., 2015). MKT is the knowledge needed by teachers to understand tasks in teaching mathematics on an ongoing basis.Knowledge of MKT helps teachers to understand the mathematical problems encountered in the learning process.Individuals' mathematical knowledge of teaching varies and depends on teachers' experience and ability to design learning(Herbst & Kosko, 2012). It is important to develop MKT because it can help teachers design concepts and skills. Besides, it allows teachers to link ideas within and across components of mathematics and encourages teachers to communicate mathematics in ways that students can understand and use. In addition, it

allows teachers to use appropriate practices in teaching mathematics.(Masingila et al., 2018). MKT also provides a boundary between each of its components which is divided into two, namely subject matter knowledge and pedagogical content knowledge.In summary, the MKT components are described in Figure 1 below.

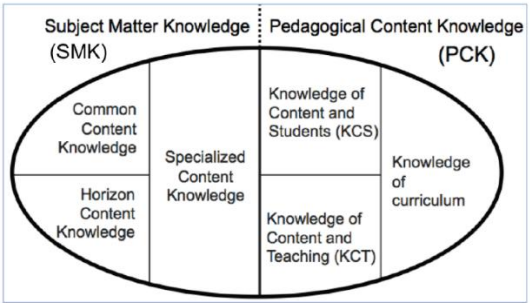


Figure 1: Components of Mathematical Knowledge for Teaching (Ball, 2008)

METHODS

Context of the Study and Participants

This research was designed in the context of a case study in a qualitative research review.Researchers were involved in the process from the point of view of educators and researchers.This study involved 20 elementary school teacher candidates in lectures and learning practices.Mathematics Material Deepening Lectures and Mathematics Learning Development courses were taken by participants at the end of the third year and the beginning of the fourth year of lectures.Furthermore, participants carried out learning practices in a real classroom environment at the end of the lecture year.The prospective teachers knew their participation and the purpose of this research.

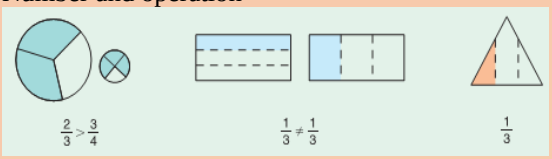
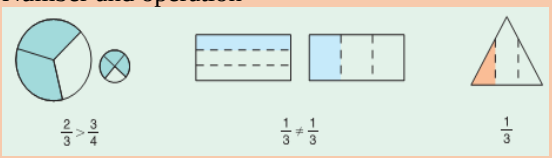
Data Sources

Data collection techniques used in this study were observation, interviews, and field notes.Observations were conducted by observing the learning activities carried out by the prospective teachers during their learning practices.During the observation activities, field notes were taken.Field notes were also used to see the difficulties faced by the prospective teachers during the learning practices.Interview activities were carried out after the end of the learning practice session. The interviews focused on instructions during learning. Besides, interviews were also conducted so that prospective teachers could evaluate the learning practices.

Data Analysis

The analysis used in this study was inductive and deductive analysis methods using triangulation of observations, interviews, and field notes(Bengtsson, 2016). The MKT components were determined based on theoretical studies which were divided into several sub-components of subject matter knowledge and pedagogical content knowledge.The two components were broken down into several subcomponents which are listed in Table 1. Table 1 describes the sub-components that are included in the question items to see the subjects' initial abilities.To get an overview of the relationship between mathematical content knowledge and mathematical knowledge for teaching, tests related to the depth of knowledge were administered. (Webb, 2002). As stated by Webb, item recall (Level 1) assumes direct knowledge of a simple fact, definition, term, or procedure, and skills in performing a simple algorithm or applying a formula.Skills and concepts (Level 2) items involve more than just remembering familiar responses, as they may require the solver to make a decision, recognize a need, organize information, apply procedures or definitions in an unknown setting,or solve multi-step problems.Strategic thinking items (Level 3) require reasoning, connecting ideas, making connections, drawing conclusions, using concepts, or offering explanations of thoughts.Extended thinking items require reasoning, and the ability to think over a long time to see the application of conceptual understanding and higher-order thinking. The results of the analysis of Table 1 bring out the subcomponents which, among others, are shown in Table 2. The picture that emerges is seen from teaching practice, behavior, difficulties, and situations encountered during teaching practice so that a more definite subcomponent can be obtained.

Table 1: MKT Components (Note: sample items of SMK and PCK from mathematical knowledge of teaching instrument)

Components	Subcomponents	Indicators	Items
Subject matter knowledge	Common Content Knowledge	Understanding the contents of the elementary school mathematics curriculum. Analyzing inaccuracies in the definition of learning resources.	Number and operation:  When students have to answer such a question, in your opinion, how will they answer it?And in your opinion, what is the expected resolution?Is the context of the question inaccurate?
	Specialized Content Knowledge	Selecting, using, and creating representations for a particular concept.	Number and operation: In your opinion, is there a fraction that lies between 5/6 and 6/7? How do you create the representation? Number and operation  When a student concludes such a picture, in your opinion, why can it happen?
Pedagogical Content Knowledge	Knowledge of Content and Students	Designing anticipatory activities for students' mathematical thinking processes. Identifying students' general knowledge and misconceptions that students may have.	Number and operation  If this happens in the learning process, what anticipation or design should a teacher do?
	Knowledge of Content and Teaching	Providing problems that help students construct knowledge.	Number and operation:  What can a teacher do to help students understand the concept of pre-algebra in this case? Geometry Elementary school students learn to calculate the area of rectangles, parallelograms, triangles, and trapeziums. To help students understand the formulas and their relationships, the sequence of shapes studied is…

FINDINGS AND DISCUSSIONS

The results of the initial identification carried out yielded subcomponents which, among others, are shown in Table 2. Table 2 illustrates the relationship between mathematical content knowledge and mathematical knowledge for teaching.

Table 2: MKT Subcomponents

Components	Subcomponents	Codes
Subject matter knowledge	Common Content Knowledge	Identifying inaccuracies in learning resources.
	Specialized Content Knowledge	Choosing activities that show a correct representation of a concept.
Pedagogical Content Knowledge	Knowledge of Content and Students	Identifying concepts that allow students to experience misunderstandings.
	Knowledge of Content and Teaching	How to create instructions and explanations to build a concept?

The findings are presented in the number and operations section only.Based on the test results given, the following is the example of the response given to the common content knowledge subcomponent:

Dian: In picture 1, students will choose 5 as the answer because the numbers have been prepared.Whereas, the instructions given in Figure 2 are not correct, what should be calculated?When the objects described are different, there will be two possibilities. Therefore, students will count all the objects or they will group these objects.Pictures 1 and 2 can direct students that they can do additions even though the objects are different. However, students should be given reinforcement that they can do it with the same objects only.

Intania: Students can determine the number of objects with the teachers' guidance. Therefore, the teachers should direct them to find out number of objects.

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Intania: Students can determine the number of objects with the teachers' guidance. Therefore, the teachers should direct them to find out number of objects.thing occurs in other topics. The researcher asked several prospective teachers to represent 2/3 and ¾, some of the drawn results are as follows:

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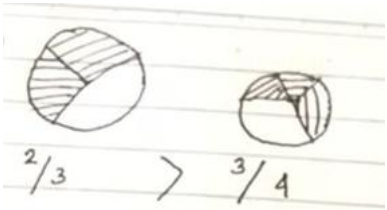

Figure 1

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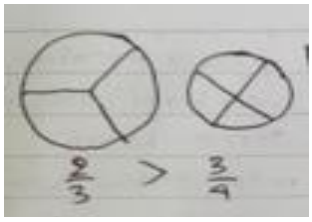

Figure 2

These two examples show that in representing the value of 2/3 and ¾, prospective teachers simply divided an area, regardless of whether they already have the same proportion of areas. The second image does not show the size of the area that represents the value of the number. The initial problems were given to the prospective teachers to find out how deep their understanding was to present the value of a fractional number. One of the abilities that teachers and prospective teachers must have is how to choose a representation for a particular purpose (Hill et al., 2008) to help students understand the meaning of a fractional value. The prospective teachers must have the ability to choose a representation for a particular purpose. The prospective teachers try to re-understand that one of the meanings of a fraction is an equal part of the whole. Furthermore, they were asked to analyze why the following problems can arise:

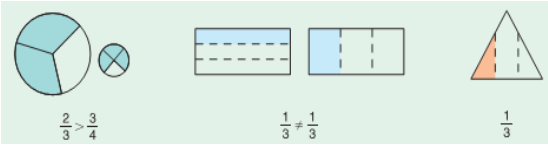

Figure 3

If in the process of introducing fractional values teachers do not present a fractional value correctly, these problems can occur.
Silmi: The first case occurs because the images used have different sizes.
Robi: It's like comparing a watermelon to a tomato, right?
Silmi: Well, we can give the illustration to the students, right?
The first case can be observed by both prospective teachers using illustrations of objects around them. Then, they are asked to find ideas to decide on what they should do to avoid misunderstandings in students' understanding. Robi: At the beginning, we used the same illustration, for example, white bread. Then, we drew it in the same size.
Silmi: We explained to them that to compare objects, they should use the same objects and shapes.
From this discussion, the researcher continued the discussion to see the responses found in the subcomponent specialized content knowledge on the topic of numbers and operation.
Silmi: 5/6 and 6/7 look like two consecutive numbers.
Roby: If the denominators are different, can they be considered as two consecutive fractions?
Teacher: If we describe them, what will the representation of the two numbers look like?
Silmi: As you can see, they are not sequential numbers.However, to find fractions between 5/6 and 6/7, you need to find equivalent fractions of these two numbers.The equivalent fractions of 5/6 and 6/7 are 35/42 and 36/42.
Roby: There are no fractions between the two fractions. It seems we need to find another equivalent fraction.
Silmi: The fraction between 5/6 and 6/7 is 71/84
Silmi: In conclusion, we can tell students that finding the value of a fraction among other fractions can be done by describing it first. Then, they should look for fractions among other fractions until they find the equivalent fraction.
The concept of fractional value is known as a material that is difficult to learn and solve(Newton, 2008). Besides, it is difficult how to represent the value of a fraction. Specialized content knowledge is different from common content knowledge that adults should have. Generally, teaching ability needs specialized content knowledge regardless of contexts and students' knowledge.In this research context, specialized content knowledge does not consider aspects of how prospective teachers can explain meaning and goals mathematically

to students(Hill et al., 2008), but consider how prospective teachers can have the ability to represent a certain concept so that learning is meaningful for students(Ding, 2016).

A knowledge process built by prospective teachers allows the difference between each type of knowledge that has almost no boundaries.When prospective teachers can recognize incorrect answers or concepts in the context of common content knowledge, they can also measure misrepresentation in the context of specialized content knowledge.When the prospective teachers try to use most of their experience to see the potential and the emergence of students' understanding errors, knowledge of content, and students will work.In the end, the prospective teachers can decide what the best step that might be able to correct mistakes, that can lead to potential misunderstandings, and where knowledge of content and teaching will work.

In a learning process, prospective teachers are hoped to be able to sequence content so that they can decide where to start.For example, when a prospective teacher is asked to determine a number between 2 fractional values, he or she tries to understand where to work from.It is closely related to what instructions should be given in the learning process, and what examples can be given at the beginning of learning. From the results of the classroom practice, prospective teachers need to evaluate the instructions and questions given to students so that the prospective teachers can decide when to give new questions to build students' prior knowledge and when to provide clarifications to students.Each of these pieces of knowledge interacts with each other to overcome pedagogical problems. In the end, the interaction can influence students' learning outcomes.

CONCLUSIONS

This study attempts to qualitatively describe how mathematical content knowledge becomes an integral part of mathematical knowledge for teaching. Therefore, this study investigates the connection between mathematical content knowledge and the subcomponents of mathematical knowledge for teaching.

Based on the findings, the limitations of mathematical content knowledge of elementary school teacher candidates have an impact on the learning process in classrooms, especially on the interactions that occur in the learning process.For example, when a prospective teacher tries to analyze inaccuracies in a learning resource, he or she is not aware of the mistakes.This will have an impact on how he or she selects and designs learning resources for a learning activity and provides explanations in classes.The knowledge possessed by the prospective teachers also influences how the stimulus given to students and how they utilize students' prior knowledge as part of the learning process.Mathematical abilities and pedagogic abilities greatly affect the ability to interpret students' feedback(Guerriero, 2013). Based on the findings, feedback given by prospective teachers on learning practices is also influenced by their mathematical knowledge.

Regarding cases in which the prospective teachers' mathematical content knowledge is good, but their pedagogical abilities are not optimal, the learning practices carried out are also less effective.This can be seen when prospective teachers do not understand how they should give feedback and explanations to students.Some research results showed that it is not enough to have mathematical content knowledge only. Prospective teachers must also master good pedagogical content knowledge(Blömeke et al., 2022)because it will also have an impact on students' understanding.

Prospective teachers' mastery of content knowledge that is built through several courses should be something that can be done to overcome problems related to their mathematical content knowledge so that it can assist them in carrying out learning practices.In other words, deficiencies in learning practices reflect the state of the teacher candidates' mastery of certain subjects.Prospective teachers need a lot of time to reflect during lectures and practice learning so that their mathematical knowledge for teaching can be effectively applied to learning practice.

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