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Multiobjective Optimization-Based Collective Opinion Generation With Fairness Concern

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***Abstract*—The generation of collective opinion based on probability distribution function (PDF) aggregation is gradually becoming a critical approach for tackling immense and delicate assessment and evaluation tasks in decision analysis. However, the existing collective opinion generation approaches fail to model the behavioral characteristics associated with individuals, and thus, cannot reflect the fairness concerns among them when they consciously or unconsciously incorporate their judgments on the fairness level of distribution into the formulations of individual opinions. In this study, we propose a multiobjective optimization-driven collective opinion generation approach that generalizes the bi-objective optimization-based PDF aggregation paradigm. In doing so, we adapt the notion of fairness concern utility function to characterize the influence of fairness inclusion and take its maximization as an additional objective, together with the criteria of consensus and confidence levels, to achieve in generating collective opinion. The formulation of fairness concern is then transformed into the congregation of individual fairness concern utilities in the use of aggregation functions. We regard the generalized extended Bonferroni mean (BM) as an elaborated framework for aggregating individual fairness concern utilities. In such way, we establish the concept of BM-type collective fairness concern utility to empower multiobjective optimization-driven collective opinion generation approach with the capacity of modeling different structures associated with the expert group with fairness concern. The application of the proposed fairness-aware framework in the maturity assessment of building information modeling demonstrates the effectiveness and efficiency of multiobjective optimization-driven approach for generating collective opinion when accomplishing complicated assessment and evaluation tasks with data scarcity.**

***Index Terms*—Collective opinion generation, fairness concern, multiobjective optimization, probability distribution function (PDF).**

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I. Introduction

THE WHOLE world has and will still be witnessing how the digital technology dramatically revolutionizes nearly every area of human existence, including but not limited to, shopping, communications, the workplace, entertainment, etc. [1], [2] Digital technologies have progressed rapidly to level the playing field in the means of improving connection, financial inclusion, trade access, and access to public services. Understanding the potential and impact of the introduction and implementation of digital technologies and the reimagining of business in the digital age is of great significance to accelerating the fostering and development of emerging industries [3]. Decision analysis in such contexts becomes extremely tricky as few objective data are available for analyzing digital technologies introduction and implementation. Experience and knowledge from professionals and stakeholders occupy crucial roles in rendering robust and reliable decision support for decision analysis faced with unforeseeable or partially unknown future in this particular scenario.

Expert judgments are the main manifestation of the experience and knowledge from professionals and stakeholders and provide useful information for forecasting, risk assessment, and decision making [4], [5], [6]. The adoption of judgments from subject matter experts (SMEs) is common, and often inevitable, when there are no empirical data or information available on the variables of interest [7]. For the past decades, judgments of various kinds, such as forecasts, estimates, and probability assessments, dominate the field of knowledge representation under uncertainty [8], [9], [10]. The elicitation and aggregation processes of expert judgments become central tasks for accomplishing probabilistic risk analysis and probabilistic-forecast-based prediction [11], [12]. Expert opinion elicitation involves the discussion and investigation of formal protocols, comprehensive procedures, and guidelines, which are rather mature in their developments [13], [14], [15]. The aggregation, or combination, of the expert judgments, however, faces more challenges now than ever as generating collective opinion to model diversifying decision-making scenarios requires the accurate determination of decision variables during the aggregation process with several criteria constraints.

The focus of this article will be given to the topic of collective opinion generation in which establishing rationale and efficient aggregation paradigms for subjectively assessed probability distributions has been the central commitment.[1] In general, a well-recognized classification of such aggregation paradigms is to group them into behavioral and mathematical approaches. Behavioral approaches usually require diverse forms of negotiation such that the SMEs from a group are able to reach a consensus or they obtain a satisfactory manifestation of their collective opinion with the help of a facilitator or a group leader. The behavioral methods encourage the SMEs to interact with one another in some fashion and share their opinions in the forms of face-to-face group meetings, interaction by computer, or sharing of information in other ways. Detailed review and discussions of behavioral methods can be accessed in Wright and Ayton [19], [20], Clemen and Winkler [21], and Armstrong [22]. In addition, when SMEs' opinions take the forms of fuzzy entities, text reviews, or some other counterparts, collective opinion generation may follow context-specific aggregation paradigms, such as aggregation functions, multidimensional utility, and so on. We will not restrict our discussions to the detailed development and possible advancements for this topic and refer the interested readers to relevant monographs and books [23], [24].

Mathematical approaches, however, advocate the expression of SMEs' individual assessments on an uncertain quantity with subjective probabilities and aggregate them in the use of various mathematical constructions. The mostly commonly used and investigated mathematical approaches include, but not limited to, linear opinion pool [25], logarithmic opinion pool (LOP) [26], Bayesian framework [27], and Median quantile aggregation [28]. The linear opinion pool satisfies the properties of unanimity and marginalization, and the LOP satisfies the principle of external Bayesianity. Both of these two approaches belong to the class of axiom-based aggregation formulas, however, the failure of them has been verified by Lindley [29] with interpretative examples demonstrating that marginalization ignores important information and external Bayesianity requires the consistency of the pooling function. In addition, introducing more assumptions to generate desirable and derivative rules that are consistent with the axioms and properties leads to the impossibility theorem [30], [31]. The Bayesian framework is another well-established approach for collective opinion generation. Many of the methods based on the Bayesian framework require the estimation of parameters of the likelihood function with the assumption that the past data will be used for such estimation. Unfortunately, the past data are often unavailable for decision analysis, especially in the cases of probabilistic risk analysis, probabilistic-forecast-based prediction and uncertain assessment tasks with limited information at hand. The Median quantile aggregation paradigm shares an agreement preservation property with the aggregation of quantiles and does not require normalization. Known as a special case of trimmed aggregation, it also demonstrates good calibration and sharpness properties. The Median quantile aggregation can provide unacceptable results when applied to discrete distribution functions in the case that the number of aggregates is even.

Both axiom-based and statistical learning-driven approaches have their pros and cons in the accomplishment of collective opinion generation tasks [4], [21]. The general obstacle for implementing these approaches in practical and complicated collective decision-making contexts is, however, that they fail to model behavioral constructs of SMEs that may exert a great influence on shaping the collective opinion. Empowering them with the flexibility to incorporate behavioral aspects involved in an overall aggregation process suffers from several inevitable restrictions of either satisfying multiple strict properties or the dependence on massive historical data. Recent researches have shifted their priorities from advancing the axiom-based and statistical learning-driven approaches to exploring the optimization-based aggregation paradigm for generating collective opinion as the latter was endowed with the capacity to model behavioral characteristics associated with SMEs. This observation encourages us to accelerate the development of existing optimization-based aggregation paradigms to generate reliable and representative collective opinions, which forms the main motivation of this work.

The first attempt on establishing the optimization-based aggregation paradigm for collective opinion generation was made by Liu et al. [32], in which the built optimization model meets the aggregation criterion that the aggregated subjectively assessed probability distributions should maximize the total similarity of each individual probability distribution and the aggregated probability distribution. The idea in [32] creates the notion of "overlapping area" to model the nonlinear relationship between each individual's opinion and the collective opinion, based on which an iteration scheme [33] was further developed in a bid to approximate the optimization model with dynamic expert weight assignments. However, as pointed out by Ji et al. [34], the aforementioned optimization models suffers from several weaknesses, such as the failure to assign different weights when the total overlapping areas remain unchanged in the case that only two SMEs are involved, the dominance of experts with dispersed opinions to the generated collective opinion, and the difficulty to reach

[1] The terminologies of "collective opinion generation" and "collective opinion formation" are not interchangeable in this work. In particular, collective opinion formation has been more frequently used in the contexts of opinion dynamics in which it emphasizes that agents modify their opinions in accordance with the interaction with other agents [16], [17]. Collective opinion generation, however, is concentrated on the representative and unbiased collective opinions given a set of judgments from SMEs generally in the form of subjectively assessed probability distributions [4], [18]. This terminology is self-explanatory and context-dependent in many ways. In this article, "collective opinion generation" refers to the establishment of aggregation strategies, models, and methods for individual opinions collected from a group of SMEs or other collective decision-making body, which are usually in the form of subjectively assessed probability distributions.

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final decision given that the aggregated probability distribution might be multimodal. To address these concerns, the bi-objective optimization model in [34] succeeds in considering simultaneously the improvement of the representativeness and concentration of the generated collective opinion. The additional criterion added to the single objective optimization model is consistent with the aggregation rule named "sharpness" for nonoptimization-oriented collective opinion generation approaches. Unfortunately, the existing optimization-based aggregation paradigm for generating collective opinion fails to model the behavioral characteristics associated with individuals. In collective decision-making contexts, behavioral characteristics associated with individuals emerge from the coordination of individuals and will affect the group productivity and performance. Behavioral collective opinion generation examine how individual's strategic decision-making behavior could be shaped by their social preferences, social utility, and other psychological factors. The most common behavioral factors in such contexts include, but not limited to, social loafing, fairness, cooperative/noncooperative mechanisms, neglect of altruism, framing effect, etc. Among them, fairness has been the major concern among SMEs and is reflected on their reactions to the decision outcomes and the information provided in regards to the outcomes or decision-making procedures. Factoring into fairness concern among SMEs is essential for the overall acceptance of a decision, and the failure to consider fairness requirements fails to reflect the most critical behavioral characteristic of SMEs when they consciously or unconsciously incorporate their judgments on the fairness level of distribution into the formation of individual opinions. In this sense, the collective opinion may not be considered fair to all invited SMEs as they might evaluate how much of their provided advice has been taken into account when making the final decision on the basis of the generated collective opinion. The objective of this article is, therefore, to advance the bi-objective optimization model for collective opinion generation to exhibit the capacity of modeling the fairness concern among SMEs.

The main contribution of this work is twofold.

1) Redefining the individual and collective fairness concern utility functions (CFCUFs) to enrich fairness preference theory in the context of collective opinion generation based on probability distribution function (PDF) aggregation.
2) Proposing a novel multiobjective optimization-based model for collective opinion generation with the capacity to model fairness concerns among SMEs that increase the overall acceptance of a decision to them.

The remainder of this article is organized as follows. Section II reviews the bi-objective optimization model for PDF aggregation and the aggregation functions that will be used to establish the fairness concern utilities. Section III proposes the notion of fairness concern utility function and formulate the problem of collective opinion generation with fairness concern with a multiobjective optimization model. Section IV provides a numerical example to demonstrate the effectiveness and efficiency of the proposed model and compares the proposed collective opinion generation approach with the bi-objective optimization model-based model to explore the advantages of the multiobjective approach taking into account fairness concern. Section V concludes this article.

II. Preliminaries

In this section, we will review the original bi-objective optimization model for collective opinion generation and introduce the notion of aggregation functions that is crucial to modeling CFCUF.

A. Bi-Objective Optimization Model for Collective Opinion Generation

Before elaborating on the explicit construction of our proposed multiobjective optimization model, we herein recall the mathematical formulation of the PDF aggregation-based collective opinion generation problem, as analogously discussed in [65]. Eliciting opinions from $\mathcal{N}$ experts requires the task-intensive self-construction of their assessments on a particular event (e.g., risk evaluation) in the form of probability distributions. The PDF and cumulative distribution function (CDF) for the ith SME is formulated as $f_i(x)$ and $F_i(x)$ for $i \in [\mathcal{N}]$. The core task of PDF aggregation is to combine probabilistic opinions to generate the collective opinion, i.e., the aggregated PDF $f(x)$ and CDF $F(x)$, that includes a diversity of individual opinions for certain analysis purpose. The aggregated PDF $f(x)$ based on the PDFs elicited from individual SMEs is given as follows:

$$f(x) = \Phi(f_i(x) | i \in [\mathcal{N}])$$

where Φ is a mapping $\mathbb{R}^{\mathcal{N}} \rightarrow \mathbb{R}^{+}$, and $\int_{-\infty}^{+\infty} f(x)dx = 1$.

The most frequently used approach for obtaining the aggregated PDF is to search for the optimal f that minimize its distance to each SME's PDF f_i that is expressed by $\sum_i^{\mathcal{N}} \|f_i - f\|$, where $\| \cdot \|$ is the generalized norm.[2] Recently, Ji et al. [34] brought about the idea of bi-objective optimization based on two criteria, i.e., consensus and confidence levels, to enhance the representativeness and concentration of the aggregated PDF. In what follows, we will first introduce the idea of the bi-objective optimization-based model for PDF aggregation. The first criterion, i.e., the consensus level, is characterized by the total overlapping area $\mathscr{A} = \sum_i \mathscr{A}_i$, where $\mathscr{A}_i$ is the overlapping area between the PDF of ith expert and the aggregated PDF, and its explicit form is given by $\mathscr{A}_i = \int_X \mathrm{Min}\{f_i(x), f(x)\}dx = \int_X ([f_i(x) + f(x) - |f_i(x) - f(x)|]/2)dx$. It should be noted that there exist a significant number of literature on measurement of consensus level, such as [35], [36], and [37]. However, none of these consensus measures are suitable for subjectively assessed probability distributions, and thus we stick to the choice as made in [32] and [34]. The second criterion, i.e., the confidence level, uses the variance of the aggregated PDF as an indicator of the reliability of the aggregated opinion. In order to achieve the best performance of the PDF aggregation model, the maximum of consensus and confidence levels were regarded as two objectives, and the weight vector of

[2] In some cases, the standard and Frobenius norms are commonly applied.

invited experts, $\mathbf{w} \in [0,1]^{\mathcal{N}}$, was taken as the decision variable. The generalized bi-objective optimization model for PDF aggregation is constructed as follows:

Model 1:

$$\text{Max} \quad (\mathscr{A}(\mathbf{w}), -\mathscr{V}(\mathbf{w}))$$
$$\text{s.t.} \begin{cases} \mathscr{A}_i(\mathbf{w}) = \int_X \text{Min}\{f_i(x), f(x)\}dx \\ \mathscr{A}(\mathbf{w}) = \sum_i \int_X \text{Min}\{f_i(x), f(x)\}dx \\ \mathscr{V}(\mathbf{w}) = \int_X \left(x - \int_X xf(x)dx\right)^2 f(x)dx \\ f(x) = \frac{d(F(x))}{dx} \\ \sum_i w_i = 1, w_i \geq 0 \text{ for } i \in [\mathcal{N}]. \end{cases}$$

where $-\mathscr{V}(\mathbf{w})$ is the established objective for modeling confidence level.

The choice of appropriate approaches to aggregating individual PDFs requires the deliberate comparisons among them. Three most frequently used ones in the literature are linear combination (LC), LOP, and quantile averaging (QA) aggregation [38], [39], [40]. However, the implementation of LC and LOP suffers from several weaknesses. The LC aggregation fails to retain the unimodality of the aggregation outcome, which indicates the possibility of obtaining more than one recommendations for the collective decision, and subsequently, fails the decision-making process that requires a resounding optimal decision suggestion. The weakness of the LOP aggregation formula stems from the absorbing element zero, which nullifies the aggregated PDF as it will also be zero when one expert assigns the absorbing element as the probability to an event x. Experimental analysis also supports the benefits of the QA aggregation paradigm and proved that it does not suffer from the above-mentioned weaknesses. In this sense, the explicit form of the CDF $F_{\text{QA}}(x) = F_{\text{QA}}(\sum_i w_i(F_i^{-1}(p)))$, where $F_i^{-1}(p)$ denotes the inverse function (quantile) of the ith expert's CDF, $p = F_i(x), p \in [0,1]$ indicates that the aggregated quantile function $F_i^{-1}(p)$ is the weighted average of each individual's quantile. Thus, Model 1 can be reformulated as follows.

Model 2:

$$\text{Max} \quad (\mathscr{A}(\mathbf{w}), -\mathscr{V}(\mathbf{w}))$$
$$\text{s.t.} \begin{cases} \mathscr{A}_i(\mathbf{w}) = \int_X \text{Min}\left\{f_i(x), f_{\text{QA}}(x)\right\}dx \\ \mathscr{A}(\mathbf{w}) = \sum_i \int_X \text{Min}\left\{f_i(x), f_{\text{QA}}(x)\right\}dx \\ \mathscr{V}(\mathbf{w}) = \int_X \left(x - \int_X xf_{\text{QA}}(x)dx\right)^2 f_{\text{QA}}(x)dx \\ f_{\text{QA}}(x) = \frac{d\left(F_{\text{QA}}\left(\sum_i w_i\left(F_i^{-1}(p)\right)\right)\right)}{dx} \\ \sum_i w_i = 1, w_i \geq 0 \quad \text{for} \quad i \in [\mathcal{N}]. \end{cases}$$

Note that the reformulated Model 2 is slightly different from its original version in [34]. The constraint $f_{\text{QA}}(x) = ([d((\sum_i w_i(F_i^{-1}(p))))]/dx)$ in the original model is replaced by $f_{\text{QA}}(x) = ([d(F_{\text{QA}}(\sum_i w_i(F_i^{-1}(p))))]/dx)$ in this article as the missing F_{QA} will make the expression $([d((\sum_i w_i(F_i^{-1}(p))))]/dx)$ meaningless. The bi-objective optimization model for PDF aggregation inherently obtains its solution by combining all the objectives into a single objective. In this sense, the bounds for the two objectives are crucial to guarantee the validation of the objective combination. In terms of the first objective, the bounds for the consensus level, $\mathscr{A}(\omega)$, can be obtained by the following constrained nonlinear optimization models.

Model 3:

$$\text{Max} \quad \mathscr{A}(\mathbf{w})$$
$$\begin{cases} \mathscr{A}_i(\mathbf{w}) = \int_X \text{Min}\left\{f_i(x), f_{\text{QA}}(x)\right\}dx \\ \mathscr{A}(\mathbf{w}) = \sum_i \int_X \text{Min}\left\{f_i(x), f_{\text{QA}}(x)\right\}dx \\ f_{\text{QA}}(x) = \frac{d\left(F_{\text{QA}}\left(\sum_i w_i\left(F_i^{-1}(p)\right)\right)\right)}{dx} \\ \sum_i w_i = 1, w_i \geq 0 \text{ for } i \in [\mathcal{N}]. \end{cases}$$

Model 4:

$$\text{Min} \quad \mathscr{A}(\mathbf{w})$$
$$\begin{cases} \mathscr{A}_i(\mathbf{w}) = \int_X \text{Min}\left\{f_i(x), f_{\text{QA}}(x)\right\}dx \\ \mathscr{A}(\mathbf{w}) = \sum_i \int_X \text{Min}\left\{f_i(x), f_{\text{QA}}(x)\right\}dx \\ f_{\text{QA}}(x) = \frac{d\left(F_{\text{QA}}\left(\sum_i w_i\left(F_i^{-1}(p)\right)\right)\right)}{dx} \\ \sum_i w_i = 1, w_i \geq 0 \, for \, i \in [\mathcal{N}]. \end{cases}$$

The bounds for the second objective, $\mathscr{V}(\mathbf{w})$, can be obtained from the following Theorem 1.

Theorem 1 [34]: Let $F_i(x) (i \in [\mathcal{N}], x \in X)$ be the CDFs of individual experts, and $\mu_i, \mathscr{V}_i$ be the corresponding expectation value and variance of the CDF $F_i(x)$. If the aggregated PDF is obtained by the QA formula, then the expectation value μ_{QA} and the variance $\mathscr{V}_{\text{QA}}$ of the aggregated CDF $F_{\text{QA}}(x)$ satisfy $\mu_{\text{QA}} = \sum_i w_i \mu_i$ and $\mathscr{V}_{\text{QA}} \leq \text{Max}\{\mathscr{V}_i | i \in [\mathcal{N}]\}$.

The proof of Theorem 1 has been provided in Ji et al. [34]. With Models 3 and 4 and Theorem 1, the Min–Max normalization of the two objectives can be clarified. In the case that the two objectives could be assigned with different weights, we further transform Model 2 into a single objective optimization model.

Model 5:

$$\text{Max} \quad g(\mathbf{w}) = \alpha \bar{\mathscr{A}}(\mathbf{w}) - \beta \bar{\mathscr{V}}(\mathbf{w})$$
$$\text{s.t.} \begin{cases} \mathscr{A}_i(\mathbf{w}) = \int_X \text{Min}\left\{f_i(x), f_{\text{QA}}(x)\right\}dx \\ \mathscr{A}(\mathbf{w}) = \sum_i \int_X \text{Min}\left\{f_i(x), f_{\text{QA}}(x)\right\}dx \\ \mathscr{V}(\mathbf{w}) = \int_X \left(x - \int_X xf_{\text{QA}}(x)dx\right)^2 f_{\text{QA}}(x)dx \\ \bar{\mathscr{A}}(\mathbf{w}) = \frac{\mathscr{A}(\mathbf{w}) - \text{Min}\,\mathscr{A}(\mathbf{w})}{\text{Max}\mathscr{A}(\mathbf{w}) - \text{Min}\mathscr{A}(\mathbf{w})} \\ \bar{\mathscr{V}}(\mathbf{w}) = \frac{\mathscr{V}(\mathbf{w}) - \text{Min}\,\mathscr{V}(\mathbf{w})}{\text{Max}\mathscr{V}(\mathbf{w}) - \text{Min}\mathscr{V}(\mathbf{w})} \\ f_{\text{QA}}(x) = \frac{d\left(F_{\text{QA}}\left(\sum_i w_i\left(F_i^{-1}(p)\right)\right)\right)}{dx} \\ \alpha + \beta = 1 \\ \sum_i w_i = 1, w_i \geq 0 \text{ for } i \in [\mathcal{N}]. \end{cases}$$

The single objective can be further expressed as $\text{Max}\{\bar{\mathscr{A}}(\mathbf{w}) - \bar{\mathscr{V}}(\mathbf{w})\}$ or $\text{Min}\{-\bar{\mathscr{A}}(\mathbf{w}) + \bar{\mathscr{V}}(\mathbf{w})\}$ given that both consensus and confidence levels are considered to be equally important to the aggregation process. Implementing Model 5 obtains the optimal weight assignments for expert forecasts and, subsequently, generates the collection opinion as per the QA aggregation paradigm.

B. Aggregation Functions

Aggregation functions are deemed to be at the heart of a number of information fusion processes and indicate the summarization of a certain number of input values into a single representative value [24]. These kinds of functions are generally of special properties and, in the context of decision

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making under uncertainty, are supposed to deal with inputs in the forms of degrees of membership, degrees of preference, strength of evidence, and support of a hypothesis, etc. In what follows, we review briefly the notion of aggregation functions and present one of the most recent powerful generalizations that will be used in the subsequent development of the proposed model.

Definition 1 [24]: An aggregation function in $[0,1]^n$ is a function denoted by $\mathsf{A}^{(n)}:[0,1]^n \to [0,1]$ that: 1) does not decrease (in each variable) and 2) fulfils the following boundary conditions: $\mathsf{A}^{(n)}(n\cdot 0)=0$ and $\mathsf{A}^{(n)}(n\cdot 1)=1$.

The integer n represents the arity of the aggregation function; that is, the number of its variables. When no confusion can arise, the aggregation functions can simply be written as A instead of $\mathsf{A}^{(n)}$. The diagonal section of any aggregation function $\mathsf{A}^{(n)}:[0,1]^n\to[0,1]$ is the unary function $\delta_{\mathsf{A}}:[0,1]^n\to[0,1]$ defined as $\delta_{\mathsf{A}}(x):=\mathsf{A}(n\cdot x)$ for all $x\in[0,1]$.

Considering the functions Min and Max as the dominating functions for generating three main classes of aggregation functions: 1) conjunctive; 2) disjunctive; and 3) averaging. Aggregation functions A that do not belong to these three classes are called mixed aggregation functions. Aggregation of inputs with heterogeneities may require its capacity to deal with a family of functions of $n=2,3,\ldots,$ arguments with the same underlying property. This is common in both theory and practice as we may need to empower the aggregation with the ability to model heterogenous interactions or mandatory requirements for certain outputs. The notion of extended aggregation function is proposed in such context to define and work with such families of functions of any number of arguments [24].

Definition 2 [24]: An extended aggregation function A in $\bigcup_{n\in\mathbb{N}}[0,1]^n$ is a mapping $A:\bigcup_{n\in\mathbb{N}}[0,1]^n\to[0,1]$, whose restriction $\mathsf{A}^{(n)}:=\mathsf{A}|_{[0,1]^n}$ to $[0,1]^n$ is an aggregation function in $[0,1]^n$ for $n\in\mathbb{N}$, with the convention $\mathsf{A}(x)=x$ for $n=1$.

Definition 3 [41]: A strong negation N is a mapping denoted by $\mathsf{N}:[0,1]\to[0,1]$ that satisfies the following conditions: 1) *Boundary:* $\mathsf{N}(0)=1$ and $\mathsf{N}(1)=0$; 2) *Monotonicity:* for all $x,y\in[0,1]$, if $x\le y$, then $\mathsf{N}(x)\ge\mathsf{N}(y)$; 3) *Continuity*; and 4) *Involution:* $\mathsf{N}(\mathsf{N}(x))=x$ for all $x\in[0,1]$.

Definition 4 [24]: Let $\mathsf{N}:[0,1]\to[0,1]$ be a strong negation and $A:[0,1]^n\to[0,1]$ be an aggregation function. Then, the dual of an aggregation function with respect to N is given by

$$\mathsf{A}_{\mathrm{d}}(x_1,\ldots,x_n)=\mathsf{N}(\mathsf{A}(\mathsf{N}(x_1),\ldots,\mathsf{N}(x_n))).$$

The explicit form of A_{d} is determined by the use of negation. In addition, the duality of a conjunctive aggregation function is clearly disjunctive, and vice versa, irrespective of the adopted strong negation [24]. Conjunctive and disjunctive functions are well known to be important for modeling logical connectives and other types of aggregations [42]. Archetypical examples of conjunctive and disjunctive functions are triangular norms (t-norms) and their dual triangular conforms (t-conorms). Another concept of great importance is the additive generator of t-norms. An additive generator is a strictly decreasing function expressed as $g:[0,1]\to[0,\infty]$, such that $g(1)=0$.

Proposition 1 [41]: Let T be a t-norm, S be its dual t-conorm, and $g:[0,1]\to[0,\infty]$ be an additive generator of T. The continuous strictly increasing function $h:[0,1]\to[0,\infty]$ defined by $h(t)=g(1-t)$ is an additive generator of S.

There are plenty of monographs and more than hundreds of publications in terms of additive generators. In this case, we restrict ourselves to their basic definitions and related properties. For more details in this aspect, we recommend interested readers to refer to [24] and [41]. An important branch of the aggregation function is the so-called generalized extended Bonferroni mean (GEBM), which is a generalization of Bonferroni mean (BM) [43], geometric BM (GBM) [44], the extended BM (EBM), and many of their variants.

Definition 5 [45]: We let $\mathbb{M}$ denote a quintuple aggregation function $\langle\mathsf{M}_1,\mathsf{M}_2,\mathsf{M}_3,\mathsf{M}_4,\mathsf{C}\rangle$, with $\mathsf{M}_1:[0,1]^{[\![\neg I']\!]}\to[0,1]$, $\mathsf{M}_2:[0,1]^{[\![I_i]\!]}\to[0,1]$, $\mathsf{M}_3:[0,1]^{[\![I']\!]}\to[0,1]$, $\mathsf{M}_4:[0,1]^2\to[0,1]$, and $\mathsf{C}:[0,1]^2\to[0,1]$, with the diagonal of C denoted by $\delta_{\mathsf{C}}(t)=\mathsf{C}(t,t)$ and inverse diagonal δ_{C}^{-1}. The GEBM is given by

$$\begin{aligned}&\mathsf{B}_{\mathbb{M}}^{\mathrm{G\&E}}(\mathbf{x})=\\&\mathsf{M}_4\Big(\delta_{\mathsf{C}}^{-1}\Big(\mathsf{M}_1\Big(\mathbf{C}^{[\![\neg I']\!]}\big(x_{i\in\neg I'},\mathsf{M}_2\big(\mathbf{x}|_{I_i}\big)\big)\Big)\Big),\mathsf{M}_3(\mathbf{x}|_{I'})\Big)\end{aligned}$$

where $\mathbf{C}^{[\![\neg I']\!]}(x_{i\in\neg I'},\mathsf{M}_2(\mathbf{x}|_{I_i}))$ is the $[\![\neg I']\!]$-tuple $(\mathsf{C}(x_i,\mathsf{M}_2(\mathbf{x}|_{I_i}))|i\in\neg I')$.

The functions M_1, M_2, M_3, M_4, and C generalize the arithmetic mean, the power mean with power q, the power mean with power p, the weighted power mean with power p, and the product of the EBM, respectively. Utilizing the GEBM will assist in recognizing the inherent structure and potential extensions of EBM. In terms of the weighted forms of GEBM, the weighting triangle [46] is commonly used and applied to obtain weighting vectors of different dimensions for different components of the GEBM. Several important features of the GEBM have been introduced and summarized in [45]. The orness of the GEBM increases if C becomes more conjunctive. The GEBM $\mathsf{B}_{\mathbb{M}}^{\mathrm{G\&E}}(\mathbf{x})$ can be reduced to its components based on some reasonable and different assumptions. In this sense, the GEBM has been empowered with enough flexibility of modeling complex aggregation mechanism as per the actual needs.

III. Collective Opinion Generation With Fairness Concern

The bi-objective optimization model for PDF aggregation succeeds in generating the collective opinion that takes both the consensus and confidence levels of the experts into account. These two criteria are critical to ensure that the combined PDF agrees with all individuals' PDFs and achieves the maximal concentration or sharpness in its shape. The bi-objective optimization-driven collection opinion generation approach tradeoffs between the objectivity and reliability of the combined PDF to a great extent. However, the problem faced by a central decision maker is that the objective is not only to maximize the representativeness of the combined PDF, but also to increase marginally the sense of identity among the invited experts to reach an agreement that will be considered as fair to each individual. In this section, we will elaborate on how to extend the bi-objective optimization-driven collection opinion generation approach for the inclusion of the additional objective—fairness concern among experts.

A. Fairness Utility Function

The importance of fairness concern in decision analysis has been recognized and well studied in a variety of settings [47], [48], [49], [50]. These range from social sciences, welfare economics, to engineering. The existing literature provides a plethora of fairness criteria and multiple (subjective) interpretations of the concepts of fairness. Defining a fairness scheme for decision analysis is challenging as it requires the consideration of different characteristics associated with complicated decision-making problems. In general, there are three main types of theories for characterizing fairness concern in fairness preference theory: 1) the F-S and ERC models [51], [52]; 2) reciprocal fairness preference [53]; and 3) the fairness preference integrating income distribution and reciprocity psychology [54]. Category 1) indicates that people care about whether the distribution is fair, not only their own income, but also the income of others. Category 2) suggests that people concern about the motives behind behaviors and will sacrifice part of their income to repay goodwill or revenge for hostile behavior. Each of these types of fairness concern models has their own advantages and disadvantages, there is no single mathematical construction that is universally accepted.

Taking into account the fairness concern among SMEs has been one of the dominant endeavors in advancing the group decision-making theory in recent years. Jing and Chao [55] introduced the game theory perspective into the consensus-reaching process and viewed it as a strategic decision-making behavior generated from the interplay between the SMEs and the moderator. Proving the existence of the Nash Equilibrium between the SMEs and the moderator was another major achievement in their work, accompanied by a well-established consensus-reaching model with minimum costs for compensation. The equilibrium solution was also obtained in the use of a refined simulated annealing algorithm. Du et al. [50] brought about the notion of consensus fairness and established the generalized form of limited cost consensus model with fairness concern for group negotiation. The fairness-aware limited cost consensus model characterizes the situation in which the SMEs constantly compare their consensus compensation with others also in the discussed group to judge whether it is fair in terms of their own gains and losses as well as others. Gong et al. [56] redefined the fairness degree of SMEs from the perspectives of social comparison theory and Gini coefficient and proposed a maximum fairness-based and cost-constrained consensus model to derive balanced consensual solution. It is worth mentioning that researchers have deliberated over the fairness issues in varying decision-making scenarios from decades ago [57], [58], but mathematically constructs for fairness in group decision making are relatively new and we can only access a few of them. However, the rapid growth in fairness-aware group decision making is foreseeable in the near future, supported by the full-fledged theoretical foundations.

The fairness concern theory is also introduced into extending the bi-objective optimization model herein to obtain more reasonable collective opinion for diversifying decision-making purposes. In this research, we adopt the F-S model, and the "absolute income" concerned by the SMEs is the overlapping area $\mathcal{A}_i$. $\mathcal{A}_i$ indicates the consistency between the ith SME's own opinion and the collective opinion about the object being evaluated. The larger the value is, the closer the experts' opinion will be to the generated collective opinion and the more fair they will feel. Otherwise, the SMEs may feel unfair and disregard the consensus that has been reached. Therefore, we choose $\mathcal{A}_i$ as the independent variable to observe the fairness concern behavior in the process of collective opinion aggregation and continuously optimize the aggregation results by adjusting the distribution of SMEs' weights within the group.

Definition 6: Let $f_i(x)$ and $F_i(x)$ be the PDF and CDF for $i \in [\mathcal{N}]$, $\mathscr{A}_i$ be the overlapping area between the PDF of ith SME and the aggregated PDF $f_{\mathrm{QA}}(x)$. Then, the individual fairness concern utility function (IFCUF) for the ith SME is defined by

$$\mathscr{F}_i^{(\rho_i,\vartheta_i)}(\mathbf{w}) = \mathscr{A}_i(\mathbf{w}) - \frac{\rho_i}{\mathcal{N}-1}\sum_{j\neq i}\mathsf{Max}\big(\big(\mathscr{A}_j(\mathbf{w}) - \mathscr{A}_i(\mathbf{w})\big), 0\big) - \frac{\vartheta_i}{\mathcal{N}-1}\sum_{j\neq i}\mathsf{Max}\big(\big(\mathscr{A}_i(\mathbf{w}) - \mathscr{A}_j(\mathbf{w})\big), 0\big)$$

where $\mathscr{A}_i(\mathbf{w}) = \int_X \mathsf{Min}\{f_i(x), f_{\mathrm{QA}}(x)\}dx$, $f_{\mathrm{QA}}(x) = ([d(F_{\mathrm{QA}}(\sum_i w_i(F_i^{-1}(p))))]/dx)$, $0 \le \rho_i \le 1$, and $-1 \le \vartheta_i \le 1$. ρ_i is jealousy preference coefficient, ϑ_i is pride preference coefficient $(-1 \le \vartheta_i \le 0)$ or sympathy preference coefficient $(0 < \vartheta_i \le 1)$.

The IFCUF indicates the absolute fairness utility, which is not bounded and cannot be utilized directly for fairness measurement purpose. In the case that $\mathscr{A}_i(\mathbf{w}) > 0$ and $\mathscr{A}_i(\mathbf{w}) < \mathscr{F}_i^{(\rho_i,\vartheta_i)}(\mathbf{w})$, the excessive fairness concern will not increase the marginal fairness utility of each individual, and thus, the IFCUF reaches its maximum. In addition, the SMEs are generally sensitive to the changes of their fairness perceptions, and their fairness utilities sharply decreases in a nonlinear manner with the increasing deviations between the SME and other SMEs in the considered group. The parabola utility functions succeed in modeling these characteristics as to determine the boundary of the IFCUF and its piecewise nonlinear functional changes. The normalized IFCUF is defined as follows:

$$\bar{\mathscr{F}}_i^{(\rho_i,\vartheta_i)}(\mathbf{w}) = \begin{cases} 0, & \mathscr{F}_i^{(\rho_i,\vartheta_i)}(\mathbf{w}) < 0 \\ \left[\dfrac{\mathscr{F}_i^{(\rho_i,\vartheta_i)}(\mathbf{w})}{\mathscr{A}_i(\mathbf{w})}\right]^{\kappa}, & 0 \le \mathscr{F}_i^{(\rho_i,\vartheta_i)}(\mathbf{w}) \le \mathscr{A}_i(\mathbf{w}) \\ 1, & \mathscr{A}_i(\mathbf{w}) < \mathscr{F}_i^{(\rho_i,\vartheta_i)}(\mathbf{w}) \end{cases}$$

where the parameter κ for parabola utility function is usually set as 2 based on randomized experiment [59].

Several critical properties of the IFCUF and its normalized version are established as follows.

Theorem 2: Let $\mathscr{A}_i$ be the overlapping area between the PDF of ith SME and the aggregated PDF. If $\mathscr{F}_i^{(\rho_i,\vartheta_i)}(\mathbf{w}) > \mathscr{A}_i$, then $-1 \le \vartheta_i < 0$.

Theorem 2 indicates that when the individual fairness concern utility of the ith SME is larger than his/her absolute fairness utility, the SME will exhibit a fairness concern behavior of pride preference for accomplishing the assessment and evaluation task. Nevertheless, if the ith SME manifests a pride preference, the individual fairness concern utility for the SME

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is not necessarily larger than his/her absolute fairness utility. To conclude, $\mathscr{F}_i^{(\rho_i,\vartheta_i)}(\mathbf{w}) > \mathscr{A}_i$ is sufficient but unnecessary for $-1 \le \vartheta_i < 0$.

Theorem 3: Let $\mathscr{A}_i$ be the overlapping area between the PDF of ith SME and the aggregated PDF. For $0 \le \mathscr{F}_i^{(\rho_i,\vartheta_i)}(\mathbf{w}) \le \mathscr{A}_i(\mathbf{w})$, $0 \le \rho_i \le 1$, and $-1 \le \vartheta_i < 1$, if $\mathscr{A}_1(\mathbf{w}) \le \cdots \le \mathscr{A}_i(\mathbf{w}) \le \cdots \le \mathscr{A}_{\mathcal{N}}(\mathbf{w})$, then we have:

1) given that $-1 \le \vartheta_i < 0$, then $\bar{\mathscr{F}}_i^{(\rho_i,\vartheta_i)}(\mathbf{w})$ is increasing with respect to $\mathscr{A}_i(\mathbf{w})$;
2) given that $0 \le \vartheta_i \le 1$ and $(\rho_i/\vartheta_i) > ([\sum_{j=1}^{i-1}\mathscr{A}_j(\mathbf{w})]/[\sum_{j=i+1}^{\mathcal{N}}\mathscr{A}_j(\mathbf{w})])$, then $\bar{\mathscr{F}}_i^{(\rho_i,\vartheta_i)}(\mathbf{w})$ is increasing with respect to $\mathscr{A}_i(\mathbf{w})$;
3) given that $0 \le \vartheta_i \le 1$ and $(\rho_i/\vartheta_i) < ([\sum_{j=1}^{i-1}\mathscr{A}_j(\mathbf{w})]/[\sum_{j=i+1}^{\mathcal{N}}\mathscr{A}_j(\mathbf{w})])$, then $\bar{\mathscr{F}}_i^{(\rho_i,\vartheta_i)}(\mathbf{w})$ is decreasing with respect to $\mathscr{A}_i(\mathbf{w})$.

The proof of Theorem 3 has been provided in Appendix A of the supplementary material. This theorem demonstrates that SMEs' fairness preference has a direct impact on their individual fairness concern utilities. The fairness perception diverges even when the SMEs exhibit the consistent fairness preference behavior. In this sense, it becomes feasible and meaningful for future endeavors to be devoted to dealing with fairness concern via the balance of the consensus and confidence levels with weight manipulation techniques [60] under diversifying fairness preference behavior.

B. Multiobjective Optimization Model Formulation

Designing a fairness-aware method for collective opinion generation is the central task in this work. Based on the proposed fairness concern utility function in Section III-A, the bi-objective optimization model will be extended to a multiobjective optimization model for PDF aggregation with fairness concern. In what follows, we build the multiobjective optimization model for collective opinion generation, which is as follows.

Model 6:

$$\mathsf{Max}\ \left(\bar{\mathscr{A}}(\mathbf{w}), -\bar{\mathscr{V}}(\mathbf{w}), \bar{\mathscr{F}}(\mathbf{w})\right)$$

$$\text{s.t.}\begin{cases} \mathscr{A}_i(\mathbf{w}) = \int_X \mathsf{Min}\{f_i(x), f_{\mathrm{QA}}(x)\}dx \\ \mathscr{A}(\mathbf{w}) = \sum_i \int_X \mathsf{Min}\{f_i(x), f_{\mathrm{QA}}(x)\}dx \\ \mathscr{V}(\mathbf{w}) = \int_X \left(x - \int_X x f_{\mathrm{QA}}(x)dx\right)^2 f_{\mathrm{QA}}(x)dx, \\ \bar{\mathscr{A}}(\mathbf{w}) = \frac{\mathscr{A}(\mathbf{w}) - \mathsf{Min}\,\mathscr{A}(\mathbf{w})}{\mathsf{Max}\mathscr{A}(\mathbf{w}) - \mathsf{Min}\mathscr{A}(\mathbf{w})} \\ \bar{\mathscr{V}}(\mathbf{w}) = \frac{\mathscr{V}(\mathbf{w}) - \mathsf{Min}\,\mathscr{V}(\mathbf{w})}{\mathsf{Max}\mathscr{V}(\mathbf{w}) - \mathsf{Min}\mathscr{V}(\mathbf{w})} \\ \bar{\mathscr{F}}(\mathbf{w}) = \mathbb{AGG}\left(\bar{\mathscr{F}}_i^{(\rho_i,\vartheta_i)}(\mathbf{w})\middle| i \in [\mathcal{N}]\right) \\ f_{\mathrm{QA}}(x) = \frac{d\left(F_{\mathrm{QA}}\left(\sum_i w_i\left(F_i^{-1}(p)\right)\right)\right)}{dx} \\ \mathscr{F}_i^{(\rho_i,\vartheta_i)}(\mathbf{w}) = \mathscr{A}_i(\mathbf{w}) - \\ \frac{\rho_i}{\mathcal{N}-1}\sum\limits_{j\neq i}\mathsf{Max}\left(\left(\mathscr{A}_j(\mathbf{w}) - \mathscr{A}_i(\mathbf{w})\right), 0\right) \\ -\frac{\vartheta_i}{\mathcal{N}-1}\sum\limits_{j\neq i}\mathsf{Max}\left(\left(\mathscr{A}_i(\mathbf{w}) - \mathscr{A}_j(\mathbf{w})\right), 0\right) \\ \bar{\mathscr{F}}_i^{(\rho_i,\vartheta_i)}(\mathbf{w}) \\ = \begin{cases} 0, & \mathscr{F}_i^{(\rho_i,\vartheta_i)}(\mathbf{w}) < 0 \\ \left[\frac{\mathscr{F}_i^{(\rho_i,\vartheta_i)}(\mathbf{w})}{\mathscr{A}_i(\mathbf{w})}\right]^{\kappa}, & 0 \le \mathscr{F}_i^{(\rho_i,\vartheta_i)}(\mathbf{w}) \le \mathscr{A}_i(\mathbf{w}) \\ 1, & \mathscr{A}_i(\mathbf{w}) < \mathscr{F}_i^{(\rho_i,\vartheta_i)}(\mathbf{w}) \end{cases} \\ \sum_i w_i = 1, w_i \ge 0 \quad \text{for} \quad i \in [\mathcal{N}]. \end{cases}$$

The construction of third objective, maximizing the overall fairness concern utility of all SMEs, involves the definition of collective fairness concern utility. The notion of aggregation function plays a significant role toward this effort as it guarantees the flexible and manageable reflection of collective fairness perceptions of the SMEs. In addition, a plethora of generalizable extensions has been proposed under the umbrella of the term of aggregation function and will benefit the accurate and context-aware construction of collective fairness concern utility. The multiobjective optimization model can be equivalently transformed into the single objective optimization model with the refined objective function $\mathsf{Max}\, g(\mathbf{w}) = \alpha\bar{\mathscr{A}}(\mathbf{w}) - \beta\bar{\mathscr{V}}(\mathbf{w}) + \gamma\bar{\mathscr{F}}(\mathbf{w})$, where $\alpha+\beta+\gamma = 1$. Unless more information are provided, the three objectives will be treated equally as all of them are decisive for producing the reasonable and rational outcomes. In particular, the function $\mathbb{AGG}$ in control of fairness concern among different experts occupies a significant position in generating collective opinions. In what follows, we discuss the choice of $\mathbb{AGG}$ in different contexts and how such diversity in this choice will impact the final outcome of the multiobjective optimization model. We choose the GEBM as the general way to model the congregation of SMEs' fairness concern utility.

Definition 7: Given that $\bar{\mathscr{F}}_i^{(\rho_i,\vartheta_i)}(\mathbf{w})$ indicates the IFCUF of ith expert, we call $\mathbf{F} = (\bar{\mathscr{F}}_i^{(\rho_i,\vartheta_i)}(\mathbf{w})|i \in [\mathcal{N}])$ the fairness concern utility vector. Let $\mathbb{M}$ denote a quintuple aggregation function $\langle \mathsf{M}_1, \mathsf{M}_2, \mathsf{M}_3, \mathsf{M}_4, \mathsf{C}\rangle$, with $\mathsf{M}_1 : [0,1]^{[\![\neg I']\!]} \to [0,1]$, $\mathsf{M}_2 : [0,1]^{[\![I_i]\!]} \to [0,1]$, $\mathsf{M}_3 : [0,1]^{[\![I']\!]} \to [0,1]$, $\mathsf{M}_4 : [0,1]^2 \to [0,1]$, and $\mathsf{C} : [0,1]^2 \to [0,1]$, with the diagonal of C denoted by $\delta_{\mathsf{C}}(t) = \mathsf{C}(t,t)$ and inverse diagonal δ_{C}^{-1}. The BM-type CFCUF defined on $\mathbf{F}$ is given in

$$\begin{aligned} &\mathcal{CF}_{\mathbb{M}}(\mathbf{F}) \\ &= \mathsf{M}_4\left(\delta_{\mathsf{C}}^{-1}\left(\mathsf{M}_1\left(\mathbf{C}^{[\![\neg I']\!]}\left(\bar{\mathscr{F}}_{i\in\neg I'}^{(\rho_i,\vartheta_i)}(\mathbf{w}), \mathsf{M}_2\left(\mathbf{F}|_{I_i}\right)\right)\right)\right), \mathsf{M}_3\left(\mathbf{F}|_{I'}\right)\right) \end{aligned} \quad (1)$$

where $\mathbf{C}^{[\![\neg I']\!]}(\bar{\mathscr{F}}_{i\in\neg I'}^{(\rho_i,\vartheta_i)}(\mathbf{w}), \mathsf{M}_2(\mathbf{F}|_{I_i}))$ is the $[\![\neg I']\!]$-tuple $(\mathsf{C}(\bar{\mathscr{F}}_{i\in\neg I'}^{(\rho_i,\vartheta_i)}(\mathbf{w}), \mathsf{M}_2(\mathbf{F}|_{I_i}))|i \in \neg I')$.

The CFCUF retains the desired properties of the GEBM. In what follows, we elaborate on some of them and offer the explicit implications of these theorems and propositions.

Theorem 4: For any $\mathbb{M} = \langle \mathsf{M}_1, \mathsf{M}_2, \mathsf{M}_3, \mathsf{M}_4, \mathsf{C}\rangle$, with M_1, M_2, M_3, and M_4 being averaging aggregation functions, $\mathscr{C}_{\mathbb{M}}(\mathbf{F})$ is an averaging aggregation function, independent of C.

Proposition 2: Let M_1, M_3, M_4, and C be aggregation functions with the same absorbing element $\mathcal{U}$, then $\mathcal{U}$ is an absorbing element of $\mathscr{C}_{\mathbb{M}}(\mathbf{F})$, independent of M_2.

Proposition 3: Let C be an aggregation function with an absorbing element $\mathcal{E}$. If M_2 and M_3 are aggregation functions with a neutral element $\mathcal{E}$, then $\mathcal{E}$ is a neutral element of $\mathscr{C}_{\mathbb{M}}(\mathbf{F})$, independent of M_1 and M_4.

The proofs of Theorem 4 and Propositions 2 and 3 have been provided in Appendixes B–D of the supplementary material. These elaborate theorems and propositions will benefit the mathematical construction of CFCUF considering special social network structures. The SMEs usually concern only a relatively small proportion of people who are socially related

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TABLE I
INITIAL PROBABILISTIC ASSESSMENTS GIVEN BY THE 15 SMES AND FITTING PARAMETERS

	Initial SMEs' data inputs							Fitted parameters for WD				Fitted parameters for ND			
SME no.	L_1	L_2	L_3	L_4	L_5	L_6	L_7	a	b	SSE	R^2	μ	$\mathscr{V}$	SSE	R^2
SME_1	0.05	0.07	0.10	0.40	0.20	0.10	0.08	4.07	4.59	0.03	0.69	4.24	1.14	0.02	0.73
SME_2	0.00	0.2	0.40	0.30	0.10	0.00	0.00	3.74	3.57	0.00	0.99	3.24	0.98	0.00	0.99
SME_3	0.00	0.00	0.00	0.30	0.65	0.05	0.00	8.82	4.96	0.00	1.00	4.73	0.54	0.00	1.00
SME_4	0.00	0.00	0.20	0.50	0.30	0.00	0.00	6.16	4.38	0.00	1.00	4.13	0.79	0.00	0.99
SME_5	0.00	0.11	0.16	0.36	0.19	0.11	0.07	3.51	4.55	0.01	0.84	4.11	1.31	0.01	0.85
SME_6	0.00	0.1	0.20	0.50	0.10	0.10	0.00	5.73	4.08	0.02	0.88	3.87	0.77	0.02	0.86
SME_7	0.00	0.3	0.50	0.20	0.00	0.00	0.00	4.13	3.14	0.00	0.99	2.87	0.79	0.00	0.99
SME_8	0.00	0.00	0.00	0.30	0.50	0.20	0.00	6.90	5.14	0.01	0.98	5.42	0.30	0.09	0.62
SME_9	0.00	0.00	0.20	0.52	0.14	0.14	0.00	6.20	4.17	0.02	0.88	3.95	0.74	0.02	0.90
SME_{10}	0.00	0.00	0.17	0.55	0.18	0.10	0.00	6.73	4.25	0.01	0.95	4.02	0.71	0.01	0.96
SME_{11}	0.00	0.00	0.20	0.50	0.20	0.10	0.00	5.93	4.25	0.01	0.94	4.03	0.80	0.01	0.96
SME_{12}	0.00	0.32	0.45	0.23	0.00	0.00	0.00	3.79	3.17	0.00	0.98	2.87	0.86	0.01	0.97
SME_{13}	0.00	0.00	0.20	0.60	0.20	0.00	0.00	6.97	4.25	0.00	1.00	4.00	0.67	0.00	1.00
SME_{14}	0.00	0.10	0.60	0.30	0.00	0.00	0.00	6.45	3.43	0.00	1.00	3.22	0.63	0.00	1.00
SME_{15}	0.00	0.00	0.30	0.50	0.20	0.00	0.00	5.52	4.14	0.00	0.98	3.87	0.79	0.00	0.99

to them and will impact their judgments. In this sense, exerting the power of aggregation functions with certain properties avoid the overestimation or poor fitting of the fairness concerns of SMEs in real-life scenarios. For instance, the neutral element guarantees that the CFCUF will reflect the fairness concern utility of SMEs in the case that they omit partially the comparisons with a subset of the SME group without influencing the generated collective opinion. The flexility of GEBM could be used to parameterize and to validate the expressive power of the established CFCUF in diversified applications. In addition to using aggregation functions to build up CFCUF, we may also consider the application of soft computing logic aggregators to define CFCUF. The advantages of such graded logic-based CFCUF construct include [42]: 1) enabling the establishment of justifiable mathematical models that satisfy observable properties of human evaluation reasoning and 2) offering the systematic tool to characterize heterogenous fairness levels of distribution based on dispersed group structures and internal relationships from a decomposable perspective. The discussion of the CFCUF under social network and its generalizations is not the focus of this article and will be investigated in the continued work of this article.

IV. ILLUSTRATIVE EXAMPLES

In this section, we will demonstrate the effectiveness and efficiency of the proposed multiobjective optimization model with an illustrative example with specific application background. Digital construction management, mainly through building information modeling (BIM) and a connected data environment, enables both broad and detailed views of all phases of a project, helping mitigate or avoid problems that can stall a project [61]. BIM is creating the path for digital transformation in construction industry and is also shaping the way that all types of infrastructure is being designed for safety improvement, schedule optimization, cost reduction during construction, and better asset performance throughout the entire life cycle [62]. However, the imperfect development among various aspects of BIM adoption in each project may result in different BIM maturity levels or even failure. Creating BIM capability and maturity model to facilitate the measurement and further improvement of BIM utilization performance will help in understanding the development of BIM and its ability to be both predictive and reactive in project management [63]. We apply the proposed multiobjective optimization model to measuring the maturity of BIM application in a real-life project.

The illustrative example concerns the Jiangxia Sewage Treatment Plant, which is located in Jiangxia District, Wuhan City, Hubei Province. To stay focused and save the space, the detailed overview of the project can be found in official website of this project.[3] BIM has been applied in the whole building cycle of the project, but was subjected to many weaknesses or encountered problems, such as the limited interoperability between systems, stakeholders lacking professional knowledge and training, data quality issues (inaccurate, incomplete, or unnecessary), and unclear division of responsibilities and rights among stakeholders, etc. The project manager is eager to draw a general picture of the current state of the BIM application to improve the BIM adoption in the follow-up construction management tasks during the project development. Fifteen SMEs were invited to evaluate the maturity of the BIM adoption in this project. The level of development includes seven grades that are L_1 [Not present, $[0, 1)$], L_2 [Initial, $[1, 2)$], L_3 [Adjusted, $[2, 3)$], L_4 [Standard, $[3, 4)$], L_5 [Mature, $[4, 5)$], L_6 [Optimized, $[5, 6)$], and L_7 [Perfect, $[6, 7)$]. The data source is provided in Table I.

Using probability distribution that can represent the SMEs' opinions requires the exploration of appropriate structures for them. Empirical analysis in this regard suggests that the assumption of SME's opinion obeys a specified family of parametric distributions avoids the introduction of too many parameters or unnecessary bias due to nonparametric approaches and eases the elicitation of fewer uncertain quintile assessments for fitting the SMEs' data. The mostly used PDFs for fitting the SMEs' data are of the types of Normal distribution, Weibull distribution, Beta distribution, etc. [32], [33], [34], among which the Normal and Weibull distributions are the most commonly implemented ones because of their advantages as

[3]For more information, readers are suggested to access the following link: https://www.engineering.citic.

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TABLE II
INTERMEDIATE RESULTS OBTAINED FROM THE TEN COLLECTIVE OPINION GENERATION MODELS

	Initial SMEs' data inputs														
Types of models	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}	w_{11}	w_{12}	w_{13}	w_{14}	w_{15}
SOO-N	0.0458	0.0398	0.0138	0.0209	0.1278	0.0216	0.0275	0.0103	0.0202	0.0190	0.0218	0.0323	0.0179	0.0193	0.0220
SOO-W	0.0491	0.0359	0.0156	0.0201	0.1207	0.0208	0.0253	0.0194	0.0194	0.0183	0.0206	0.0290	0.0176	0.0172	0.0220
BOO-N	0.0181	0.0219	0.0466	0.0285	0.0154	0.0292	0.0289	0.1280	0.0308	0.0327	0.0280	0.0258	0.0354	0.0392	0.0286
BOO-W	0.0154	0.0199	0.0758	0.0361	0.0126	0.0355	0.0297	0.0338	0.0407	0.0484	0.0353	0.0249	0.0536	0.0774	0.0317
BM-MOO-N	0.0182	0.0218	0.0469	0.0289	0.0155	0.0296	0.0287	0.1256	0.0313	0.0332	0.0284	0.0256	0.0360	0.0393	0.0289
BM-MOO-W	0.0164	0.0211	0.0688	0.0364	0.0135	0.0360	0.0309	0.0339	0.0406	0.0476	0.0357	0.0261	0.0517	0.0730	0.0324
GBM-MOO-N	0.0183	0.0222	0.0466	0.0288	0.0156	0.0295	0.0292	0.1256	0.0310	0.0329	0.0283	0.0261	0.0355	0.0394	0.0289
GBM-MOO-W	0.0165	0.0211	0.0675	0.0362	0.0138	0.0358	0.0308	0.0343	0.0406	0.0471	0.0356	0.0266	0.0516	0.0720	0.0327

mentioned in [4]. In this article, we choose the Normal distribution and Weibull distribution for fitting SME's opinions. The reason for using these two distinct PDFs is that we need to catch the difference between them and to explore their potential impact on the derived collective opinion. The Normal distribution is $\varphi(x; \mathscr{V}, \mu) = e^{(-([(x-\mu)^2]/2\mathscr{V}))}/\sqrt{2\pi\mathscr{V}}$, and the generalized two-parameter Weibull distribution is $f(x, a, b) = (a/b)(x/b)^{a-1}e^{-(x/b)^a}, x \geq 0$. The fitted parameters for the Normal distribution and Weibull distribution are also provided in Table I. The relevant fitting results are sufficiently good in terms of the evaluation metrics as shown in the table.

To facilitate model analysis and comparisons with existing optimization models established in Liu et al. [32] and Ji et al. [34], we exploit Models 3, 5, and 6 to explore the generation of collective opinion under the two different PDFs. Model 3 is actually a single objective optimization model that measures only the consensus level $\bar{\mathscr{A}}$, and we denote it by SOO-N and SOO-W when Normal and Weibull distributions are, respectively, applied. Moreover, the objective functions of Models 5 and 6 are assumed to be of equal importance. This leads to the use of Model 5 with Normal and Weibull distributions, which we will call them BOO-N and BOO-W. In addition, we choose the BM and the GBM, two special cases of the GEBM, in terms of the choice of $\mathbb{AGG}$ in Model 6. In particular, the explicit forms of the BM and GBM are given in Appendix E of the supplementary material, where $\rho_i = 0.2$ and $\vartheta_i = -0.2$ for $i \in [\mathcal{N}]$, and $p = q = 1$ without loss of generality. We name Model 6 with BM and the Normal distribution and Weibull distribution as BM-MOO-N and BM-MOO-W, respectively. Similarly, we call, respectively, Model 6 with GBM and the Normal distribution and Weibull distribution as GBM-MOO-N and GBM-MOO-W.

We implement the eight models with the collected data from the 15 SMEs and obtain the optimal solutions for them using the MATLAB R2022a software. The BIM maturity level for the project under evaluation obtained from SOO-N, BOO-N, BM-MOO-N, and GBM-MOO-N is 3.9, which is *Standard* level and implies that "BIM can be well applied. The project team has formulated more detailed rules and regulations, and has a professional BIM management team and professional hardware and software equipment. The leadership attaches great importance to the value development of BIM, and has established an incentive mechanism to encourage the use of BIM. The acquisition of information not only focuses on real time and accuracy, but also pays more attention to the effectiveness of information acquisition to obtain useful data rather than blindly obtain data. The scope of BIM activities has been further expanded." In addition, the BIM maturity level obtained from SOO-W, BOO-W, BM-MOO-W, and GBM-MOO-W is 4.2, which is *Mature* level and can be interpreted as "BIM can integrate information effectively and efficiently to predict or control management problems. The establishment of the BIM system, process, and usage standards is complete. It can effectively solve the interoperability of the BIM platform with other systems. A clear division of ownership of data and information, rights to access and use information, and liability for incorrect information exists. BIM can be maturely applied in various management activities." The slight differences between the results using different models lie in the fact that different PDFs have been applied to fit the data.

The intermediate results have been included in Table II. The ideal objective spaces of BOO-N, BOO-W, BM-MOO-N, BM-MOO-W, GBM-MOO-N, and GBM-MOO-W are shown in Fig. 1. The decision variables have converged within a relatively small range, the embedded algorithms proved to be effective in obtaining optimal solutions in a reasonably short period of time. The convergence index depicted in Fig. 1 mainly reflects the closeness (or distance) of the solution set to the real Pareto frontier (i.e., the highlighted dots). Fig. 1(b) and (e) have demonstrated that, in the objective three-dimensional spaces, the solution sets of BM-MOO-N and GBM-MOO-N exhibit high diversity, although the convergence index is at a low level. The diversity of solution set for in Fig. 1(c) and (f) is low, and the index is high, indicating that the BM-MOO-W and GBM-MOO-W models have the better convergence of the solution set. The optimization-based approaches for generating collective opinion have generally succeeded in achieving satisfactory balance among different objectives. It is also observed in Fig. 3 (See Appendix F in the supplementary material) that the consensus and confidence levels remain in high values even though the third objective was taken into account, and the collective fairness concern utility is high in all multiobjective optimization-driven collective opinion generation models. The highlighted numbers in Table III suggest that the four models considering fairness concern among SMEs achieve the better performance in almost all objectives than the single objective and bi-objective optimization-driven collective opinion generation approaches. In the cases of SOO-N and SOO-W, if we calculate the confidence and collective fairness utility level based on the overlapping areas, the resulting optimal $\bar{\mathscr{V}}$ are 0.5797 and 0.6020, respectively, and the resulting optimal $\bar{\mathscr{F}}$ are 0.9350 and 0.9511, respectively. In the cases of BOO-N and BOO-W,

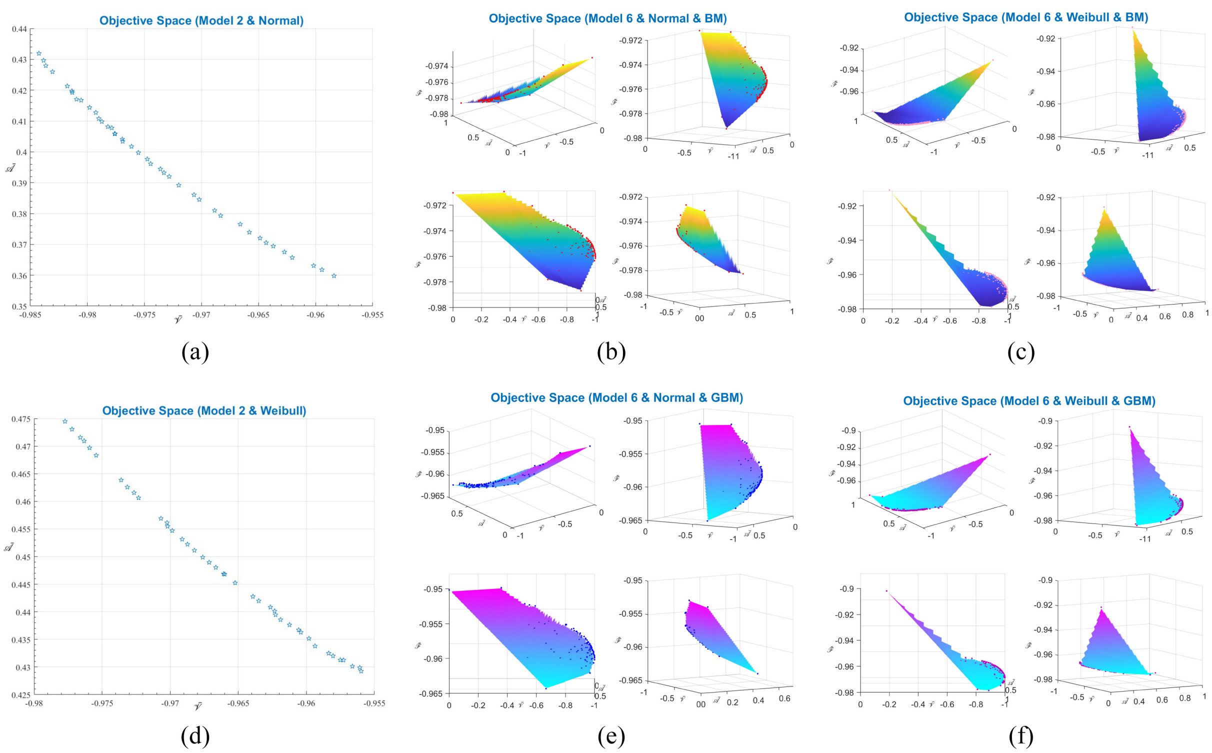

Fig. 1. Objective spaces for the six counterpart collective opinion generation models. (a) BOO-N. (b) BM-MOO-N. (c) BM-MOO-W. (d) BOO-W. (e) GBM-MOO-N. (f) GBM-MOO-W.

TABLE III
COMPARISON ANALYSIS OF THE EIGHT COLLECTIVE OPINION GENERATION MODELS

	Optimal of the objectives				
Types of models	Opt$\mathscr{A}$	Opt$\bar{\mathscr{A}}$	Opt$\mathscr{V}$	Opt$\bar{\mathscr{V}}$	Opt$\bar{\mathscr{F}}$
SOO-N	**21.0400**	**1.0000**	–	–	–
SOO-W	**21.6447**	**1.0000**	–	–	–
BOO-N	19.7098	0.9062	0.4873	0.2845	–
BOO-W	21.0556	0.9110	0.6201	0.3705	–
BM-MOO-N	19.7308	0.9077	0.4899	0.2860	**0.9606**
BM-MOO-W	21.0891	0.9161	0.6289	0.3758	**0.9679**
GBM-MOO-N	**19.7546**	**0.9093**	0.4928	0.2877	**0.9578**
GBM-MOO-W	**21.0947**	**0.9169**	0.6303	0.3767	**0.9675**

the resulting optimal $\bar{\mathscr{F}}$ are 0.9311 and 0.9378. These outcomes suggest that the single objective optimization-driven collective opinion generation approaches suffers from low confidence level and mediocre collective fairness utility in spite of achieving the highest consensus levels. The results also indicate that the bi-objective optimization-driven collective opinion generation approaches suffers from low consensus level and ediocre collective fairness utility in spite of achieving the highest confidence levels. This implies that the four models considering fairness concern among SMEs outperform the single objective and bi-objective optimization-driven counterparts with necessary balances among all objectives. In particular, the GBM-MOO-W performs the best in terms of the objectives of maximizing consensus level and collective fairness utility. This observation is consistent with the existing finding that consensus-driven aggregation framework for decision analysis strengths the fairness increment. The integration of fairness concern to optimization-based collective opinion generation paradigm also impedes introducing or exacerbating bias disadvantaging particular SMEs.

The awareness of roles that the parameters p and q play in the proposed multiobjective optimization model for the collective opinion generation framework require deeper knowledge on how various sources of uncertainty from these two parameters contribute to the model's overall uncertainty. The Sobol indices, as a frequently used form of global sensitivity analysis [64], serves for this purpose to help decompose the variance of the decision output of the proposed model into fractions which can be attributed to the respective or joint impacts from the parameters p and q. The justification behind this choice of Sobol indices lies in the fact that variance-based measures of sensitivity reflect sensitivity across the whole input space and can deal with nonlinear responses. In terms of the GEBM-based CFCUF, the Sobol indices enable measuring the effect of interactions because of the nonadditive characteristics of the collective fairness concern utility. The results of the Sobol indices for the proposed model when taking $p, q \in [0, 1000]$ are presented in Fig. 2. The Sobol-S indices exhibit the first-order sensitivities of p and q, as shown in Fig. 2(a), (c), (e), and (g). The multiobjective optimization-driven collective opinion generation framework is apparently more sensitive to the variance of p, but the convergence rate shapely becomes steady for GBM-MOO-N and GBM-MOO-W for both parameters of p and q. The observations here agree with our previous discussions on the objective spaces of these models. The Sobol-ST indices, as a reflection of the total effect when taking into account the interactions between p and q shown in Fig. 2(b), (d), (f), and (h), confirm further that the sensitive tendency of these

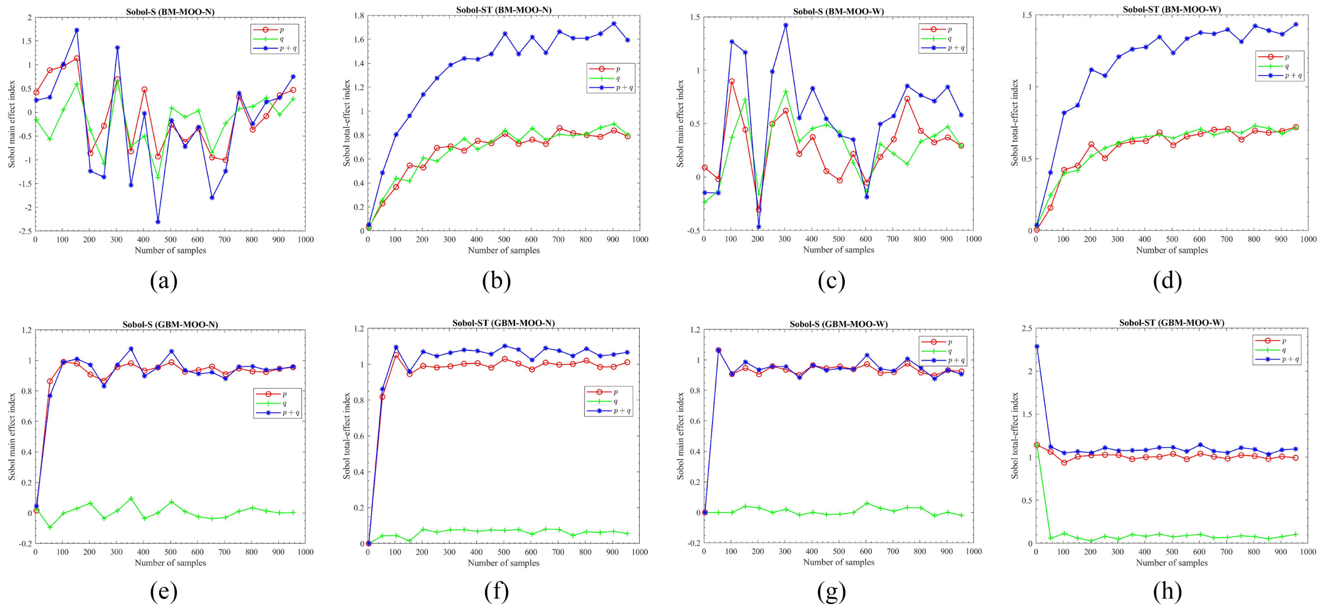

Fig. 2. Global sensitivity analysis of the parameters p and q for collective opinion generation models. (a) BM-MOO-N. (b) BM-MOO-N. (c) BM-MOO-W. (d) BM-MOO-W. (e) GBM-MOO-N. (f) GBM-MOO-N. (g) GBM-MOO-W. (h) GBM-MOO-W.

parameters and a robust performance with the fairness increments driven by the proposed collective opinion generation models.

V. Conclusion

Generating collective opinion based on PDF aggregation is critical in situations where major decisions are to be made in the presence of substantial uncertainty and where genuinely expert judgment is deemed essential to minimize and characterize the uncertainty. In addition to the axiom-based and Bayesian aggregation paradigms, the optimization-based PDF aggregation approach has dominated the recent advances in the field of uncertain judgment-based decision analysis. The recently developed bi-objective optimization model combines both the objective (i.e., consensus level) and reliability (i.e., confidence level) in the process of subjectively assessed probability distributions. Unfortunately, the bi-objective optimization-driven paradigm cannot reflect the fairness concerns among them when they consciously or unconsciously incorporate their judgments on the fairness level of distribution into the formulations of individual opinions. The failure to include behavioral characteristics associated with individuals may diminish the possibility of increasing the performance of the established objectives and may also introduce or exacerbate bias disadvantaging particular SMEs [48].

The current study included the fairness concern among SMEs as an additional criterion for uncertain judgments aggregation, and then, establishes a multiobjective optimization-driven collective opinion generation approach. We proposed the notion of IFCUF and the aggregation function-based CFCUF, which enable the accurate construction of fairness concerns among experts in a socially related expert group. Even more, we proved several properties of these novel quantitative constructs and demonstrate its effectiveness and efficiency of the proposed model via its application in the maturity assessment of building information modeling in a practical project.

Our future efforts will be devoted to the following aspects.

1) The search space will increase considerably with the growing dimension of decision variables, we will explore ways of balancing the convergence and diversity performance given limited computation budget.
2) As there are plenty of popular aggregation functions for choice purpose, it is necessary to investigate algorithmically the performance of the proposed multiobjective optimization-driven collective opinion generation approach and to build mathematical constructs for characterizing fairness concern in different contexts.
3) In the big data era, it is meaningful to develop large-scale multiobjective optimization-based decision support models that are capable of handling huge amounts of data with uncertainty.

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