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Supervised Multi-person Multi-criteria Decision-Making Methodologies under Risk and Uncertainty: Engineering Applications

Tesis Doctoral

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MEMORIA PRESENTADA POR

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Summary

In a realistic decision-making process, there are often multiple subjects in a problem. Each subject can include various criteria. In traditional decision-making, multiple experts usually tackle the problems; however, they are supposed to have all-inclusive competence in each subject. Moreover, human decisions are subjected to risk of cognitive imperfections. In other problems, rationality of decision-makers is vague in real life. Besides, the data of decision-making problems are associated with uncertainty. Therefore, for multidisciplinary decision-making problems, the most important challenges are:

- Segmentation is needed for experts as they may not have competence in all subjects of problem.
- Risk of imperfect decisions should be considered as rationality of experts is under question.
- Uncertainty of decision-making problems should be tackled with robust approaches to avoid loss of uncertain information.

We attempt to efficiently tackle the above difficulties, as follows:

- Multi-person structures supervised by a director are considered for our methodologies. Subject-oriented expert segments structures are employed when multiple subjects exist in decision-making problem.
- We use the concept of fuzzy rationality as a cognitive aid to represent the degree of rationality of expert decisions in risky situations.
- New uncertain measures are introduced to reach complete uncertain computation and preserve all data.

Therefore, the main objectives of this doctoral thesis are studying: (1) supervised multi-person structures with subject-oriented expert segments, (2) risk in uncertain environment, and (3) uncertainty to preserve all of data in decision-making problems with multiple criteria and alternatives.

In this regard, three methodologies are proposed, as follows:

- An interval supervised multi-person multi-criteria decision-making methodology is presented. This methodology is without subject-oriented expert segments and has a non-risky decision-making process. The computation is based on interval distances of interval numbers and interval preference matrix.
- A risky fuzzy supervised multi-person multi-criteria decision-making methodology is developed supported on fuzzy-rationality-based fuzzy prospect theory. This methodology is without subject-oriented expert segments and formulated based on proposed fuzzy-rationality-based fuzzy prospect theory. The computation is based on fuzzy distances of trapezoidal fuzzy numbers and fuzzy distance matrix for extremum and ranking.

- A dynamic interval supervised multi-person multi-criteria decision-making methodology with subject-oriented expert segments is introduced. This methodology includes the proposed Subject-oriented Expert Segments and has a non-risky decision-making process. The parametric representation is employed for interval preference in interval optimization model.

The aforementioned methodologies are derived by introducing the following decision-making models:

- BWM-based models for weighting process.
- MULTIMOORA-based models for ranking process.

We use the aforementioned methodologies and models to tackle engineering problems in the area of industrial, biomedical, and energy sectors.

Resumen

En un proceso de toma de decisiones realista, a menudo hay múltiples temas en un problema. Cada tema puede involucrar diferentes criterios. En la toma de decisiones tradicional, se dispone de múltiples expertos que suelen abordar los problemas; sin embargo, se parte del supuesto de que tienen competencias omnicomprensivas en cada tema. Además, las decisiones humanas están sujetas al riesgo de imperfecciones cognitivas. En otras palabras, la racionalidad de los decisores es imprecisa en la vida real. Asimismo, los datos de los problemas de toma de decisiones están asociados a la incertidumbre. Por lo tanto, para los problemas de toma de decisiones multidisciplinares, los retos importantes más destacados son:

- Se requiere de una segmentación de los expertos, ya que pueden no tener competencia en todos los temas del problema.
- Hay que tener en cuenta el riesgo de decisiones imperfectas, ya que se cuestiona la racionalidad de los expertos.
- La incertidumbre de los problemas de toma de decisiones debe abordarse con enfoques sólidos para evitar la pérdida de información incierta.

Intentamos abordar eficazmente las dificultades anteriores de la siguiente manera:

- Para nuestras metodologías se consideran estructuras multipersonales supervisadas por un director. Se emplean estructuras de segmentos expertos orientadas a sujetos cuando existen múltiples sujetos en el problema de toma de decisiones.

- Utilizamos el concepto de racionalidad difusa como ayuda cognitiva para representar el grado de racionalidad de las decisiones de los expertos en situaciones de riesgo.
- Se introducen nuevas medidas de incertidumbre para alcanzar un cálculo incierto completo y preservar todos los datos.

Por lo tanto, los principales objetivos de esta tesis doctoral son estudiar: (1) las estructuras supervisadas multipersona con segmentos de expertos orientados por temas, (2) el riesgo en un entorno incierto, y (3) la incertidumbre para preservar todos los datos en problemas de toma de decisiones con múltiples criterios y alternativas.

A este respecto, se proponen las tres metodologías siguientes:

- Se presenta una metodología de toma de decisiones multicriterio multipersona supervisada por intervalos. Esta metodología carece de segmentos de expertos orientados por temas y dispone de un proceso de toma de decisiones no arriesgado. El cálculo se basa en distancias de intervalo de números de intervalo y matriz de preferencias de intervalo.
- Se desarrolla una metodología de toma de decisiones multicriterio multipersona supervisada de riesgo difuso basada en la Teoría de la Perspectiva Difusa basada en la Racionalidad Difusa. Esta metodología carece de segmentos de expertos orientados por temas y se formula a partir de la Teoría difusa de las perspectivas basada en la racionalidad difusa propuesta. El cálculo se basa en distancias difusas de números trapezoidales difusos y matrices de distancias difusas para el extremo y la clasificación.
- Se introduce una metodología dinámica de toma de decisiones multicriterio supervisada por intervalos con segmentos expertos orientados a temas. Esta metodología incluye los segmentos expertos orientados a temas propuestos y tiene un proceso de toma de decisiones no arriesgado. La representación paramétrica se emplea para la preferencia de intervalo en el modelo de optimización de intervalo.

Las metodologías mencionadas se derivan introduciendo los siguientes modelos de toma de decisiones:

- Modelos basados en BWM para el proceso de ponderación.
- Modelos basados en MULTIMOORA para el proceso de clasificación.

Utilizamos las metodologías y modelos mencionados para abordar problemas de ingeniería en el ámbito de los sectores industrial, biomédico y energético.

Introduction

In this section, we present the context of Supervised Multi-person Multi-criteria Decision-Making (S-MpMcDM) considering risk and various concepts of uncertainty, motivation and hypothesis, objectives, methodologies and models, and structure of the thesis.

Context

The context of Supervised Multi-person Multi-criteria Decision-Making (S-MpMcDM) under risk and uncertainty deals with the significance of supervision of multiple experts and the impact of various theories of vagueness in decision-making area. In such problems, multiple experts supervised by a director are responsible for evaluating alternatives according to multiple criteria in a vague situation subjected to risk and uncetainty. The ideas of “supervised multi-person structures”, “risk”, and “uncertainty” in S-MpMcDM and its engineering applications are clarified as follows.

In real-life, many socio-economic and industrial companies deal with multidisciplinary S-MpMcDM problems. In this regard, multiple groups of experts with special competence in each subject are often required [1]. Such decision-making process can be reinforced by the supervision of a director. The director can have multiple roles to evaluate competence of experts and also participates in assessment of criteria and alternatives of the problem [2].

In the models of S-MpMcDM, the term of risk can be utilized and interpreted differently. In some cases, risk may be the indicator of the degree of optimism about the information available in the problem. Additionally, risk can be interpreted as the intensity and probability of unfavorable effects in other cases [3]. The uncertain conditions can incur risk to the problem. Risk can also be interpreted as vagueness of rationality of decision-makers [4].

In many practical S-MpMcDM problems, the available data are associated with some degrees of vagueness [5], [6]. In this regard, different kinds of uncertain sets are applied to tackle the vagueness of data, including interval numbers, fuzzy sets, and their extensions [7].

In industrial, biomedical, and energy applications of S-MpMcDM, supervised multi-person structure can result in robust decision-making process. Moreover, in many practical engineering problems, the input data are reported as uncertain values which may also entail some risks [8].

Contexto

El contexto de la Toma de Decisiones Multicriterio Supervisada por Múltiples Personas (S-MpMcDM) bajo riesgo e incertidumbre aborda la importancia de la supervisión de múltiples expertos y el impacto de diversas teorías de vaguedad en el ámbito de la toma de decisiones. En este tipo de problemas, múltiples expertos supervisados por un director se encargan de evaluar alternativas según múltiples criterios en una situación de vaguedad sometida a riesgo e incertidumbre. Las ideas de "estructuras multipersonales supervisadas", "riesgo" e "incertidumbre" en el proceso S-MpMcDM se aclaran como sigue.

En la vida real, muchas empresas socioeconómicas e industriales se enfrentan a problemas multidisciplinares de S-MpMcDM. En este sentido, a menudo se requieren múltiples grupos de expertos con competencias especiales en cada materia [1]. Este proceso de toma de decisiones puede verse reforzado por la supervisión de un director. El director puede desempeñar múltiples funciones para evaluar la competencia de los expertos y también participa en la evaluación de criterios y alternativas del problema [2].

En los modelos para abordar el S-MpMcDM, el término de riesgo puede utilizarse e interpretarse de forma diferente. En algunos casos, el riesgo puede ser el indicador del grado de optimismo sobre la información disponible en el problema. Además, en otros casos, el riesgo puede interpretarse como la intensidad y probabilidad de efectos desfavorables [3]. Las condiciones de incertidumbre pueden suponer un riesgo para el problema. El riesgo también puede interpretarse como la vaguedad de la racionalidad de los decisores [4].

En muchos problemas prácticos de S-MpMcDM, los datos disponibles están asociados con algunos grados de vaguedad [5], [6]. En este sentido, se aplican diferentes tipos de conjuntos inciertos para abordar la vaguedad de los datos, incluidos los números de intervalo, los conjuntos difusos y sus extensiones [7].

En las aplicaciones industriales, biomédicas y energéticas de S-MpMcDM, la estructura multipersonal supervisada puede dar lugar a un proceso de toma de decisiones robusto. Además, en muchos problemas prácticos de ingeniería, los datos de entrada se presentan como valores inciertos que también pueden conllevar algunos riesgos [8].

Motivation and hypothesis

Although many researches have been conducted on S-MpMcDM under risk and uncertainty, serious research gaps still exist in this regard, as follows:

- **Supervision analysis in S-MpMcDM:** In traditional S-MpMcDM, decision-making with multi-person structure is analyzed in very limited studies [2]. Director can also participate in downstream decisions of the problem. If there exist multiple subjects in decision-making problem, experts can be segmented based on their competence in the subjects. Director may additionally evaluate the relative significance of the problem subjects.
- **Risk analysis in S-MpMcDM:** The concept of risk factors are usually presented as independent criteria in the context of MpMcDM methods [9]; however, there are a few studies that include risk factors directly into the formulation [10]. As rationality of human decisions is imperfect in real-life, considering the risk of the degrees of rationality is an interesting topic in the S-MpMcDM context. In some real cases, decisions are subject to the risks posed from the lack of or limited knowledge of problem conditions.
- **Uncertainty analysis in S-MpMcDM:** There are many studies in the field of uncertain decision-making problems with multiple person structure; however, some research gaps still exist in this regard. For considering uncertainty, the thesis focuses on interval numbers and fuzzy sets for suggesting new developments. Interval S-MpMcDM approaches have got rather lower attention comparing the fuzzy methods despite their privileges especially in practical industrial application [11]. Research gaps exist regarding complete uncertain computation based on interval and fuzzy data.

This study attempts to plug the aforementioned research gaps, as follows:

- **Supervision analysis in S-MpMcDM:** We employ two supervised multi-person structures for S-MpMcDM process. The first structure consists of one director and multiple experts. For the second structure, multiple experts are segmented based on problem subjects. That is, the second structure comprises one director and multiple expert segments. For the second structure, we coin the expression "Subject-oriented Expert Segments (SoESs)" and introduce it into the S-MpMcDM context. The SoESs structure is a multi-person decision-making framework in which experts are segmented based on their competence in problem subjects.

- **Risk analysis in S-MpMcDM:** We introduce the "Fuzzy-Rationality-based Fuzzy Prospect (FRFP)" theory to evaluate the risk of experts decisions. We use the concept of fuzzy rationality as an cognitive aid to represent the degree of rationality of expert decisions in risky S-MpMcDM. That is, fuzzy rationality is used to indicate risk of expert decision in realistic problems.
- **Uncertainty analysis in S-MpMcDM:** We tackle uncertainty for S-MpMcDM based on theories of interval and trapezoidal fuzzy numbers. In the theory of interval numbers, we propose interval distance and interval preference matrix to make pairwise comparison and obtain extremum and ranking, respectively. In the theory of trapezoidal fuzzy numbers, we develop fuzzy distance to make pairwise comparison. To compute extremum and ranking of trapezoidal fuzzy numbers, we first calculate α-cuts to reach the corresponding interval numbers and then employ the proposed interval preference matrix.

Objectives

The main objectives of this thesis is to develop Supervised Multi-person Multi-criteria Decision-Making (S-MpMcDM) methodologies under risk and uncertainty. Besides, the related studies in the literature are reviewed comprehensively and the proposed methodologies are utilized in practical engineering problems.

We present the objectives of this thesis, as follows:

- ***Objective 1:*** Overview of the MpMcDM methodologies with specific interest in uncertainty, risk, and their applications.
- ***Objective 2:*** Study of "supervised multi-person structures" in S-MpMcDM process with subject-oriented expert segments.
- ***Objective 3:*** Study of "risk" in S-MpMcDM process based on the prospect theory in uncertain environment.
- ***Objective 4:*** Study of "uncertainty" in S-MpMcDM process based on the theories of interval and fuzzy numbers.
- ***Objective 5:*** Analysis of engineering applications of S-MpMcDM problems in industrial, biomedical, and energy sectors.

Methodologies and models

As it is aforementioned in Objectives Section, the focus of theory of this thesis is on the study of "supervised multi-person structures", "risk", and "uncertainty" in S-MpMcDM process. In this regard, three methodologies are proposed, as follows:

- ***Methodology 1*:** We present the *"Interval Supervised Multi-person Multi-criteria Decision-Making (I_S-MpMcDM)"* methodology. This methodology is without subject-oriented expert segments and has a non-risky decision-making process. The computation is based on interval distances of interval numbers and interval preference matrix.
- ***Methodology 2*:** We develop the *"Risky Fuzzy Supervised Multi-person Multi-criteria Decision-Making methodology supported on Fuzzy-Rationality-based Fuzzy Prospect Theory (RF_S-MpMcDM_FRFP)"*. This methodology is without subject-oriented expert segments and formulated based on "Proposed Fuzzy-Rationality-

based Fuzzy Prospect Theory (FRFP)". The computation is based on fuzzy distances of trapezoidal fuzzy numbers and and fuzzy distance matrix for extremum and ranking.

- ***Methodology 3*: We introduce the *"Dynamic Interval Supervised Multi-person Multi-criteria Decision-Making methodology with Subject-oriented Expert Segments (DI_S-MpMcDM_SoESs)"*. This methodology includes the proposed "Subject-oriented Expert Segments (SoESs)" and has a non-risky decision-making process. The parametric representation is employed for interval preference in interval optimization model.

The abovementioned methodologies will be explained and clarified with details in Methodologies Section. The proposed methodologies are derived by introducing the following decision-making models:

1) ***BWM-based models for weighting process:*** We derive models based on Best-Worst Method (BWM) to compute weights needed for the proposed methodologies. For the I_S-MpMcDM methodology, two interval BWM-based model is generated to weight experts and criteria. For the RF_S-MpMcDM_FRFP methodology, two risky fuzzy BWM-based model supported on the FRFP theory is generated to weight experts and criteria. For the DI_S-MpMcDM_SoESs methodology, three interval BWM-based models supported on the SoESs structure are generated to calculate dynamic weights of subjects, experts, and alternatives.
2) ***MULTIMOORA-based models for ranking process:*** We develop models based on Multi-Objective Optimization by Ratio Analysis plus the full MULTIplicative form (MULTIMOORA) to calculate ranking of alternatives in the eventual step of the proposed methodologies. The obtained weights in the BWM-models are used in derivation of the MULTIMOORA-based models. For the I_S-MpMcDM methodology, the final rankings of alternatives are obtained utilizing the interval MULTIMOORA-Borda model. For the RF_S-MpMcDM_FRFP methodology, subordinate rankings are generated based on the fuzzy MULTIMOORA model and the final ranking is obtained employing the fuzzy distance matrix. For the DI_S-MpMcDM_SoESs methodology, the dynamic assessment values of alternatives are aggregated based on the interval MULTIMOORA-Borda model.

Structure of the thesis

The remainder of this thesis is arranged into six chapters as follows:

- Chapter I gives preliminaries on MpMcDM, uncertainty, and risk.
- Chapter II is allocated to the overview on MULTIMOORA for risky uncertain MpMcDM and its applications.
- Chapter III presents the I_S-MpMcDM methodology and its industrial application.
- Chapter IV discusses the RF_S-MpMcDM_FRFP methodology and its biomedical application.
- Chapter V presents the DI_S-MpMcDM_SoESs methodology and its energy application.
- Chapter VI gives the concluding remarks, research papers published regarding the overview and methodologies, as well as the directions for future works.

Chapter I

Preliminaries on MpMcDM, uncertainty, and risk

In this chapter, we present the preliminaries required for derivation of methodologies. Sections 1, 3, and 2 give fundamentals on MpMcDM, uncertainty, and risk, respectively.

1. MpMcDM methods

We discuss two important decision-making methods in the context of MpMcDM methods, MULTIMOORA and BWM in Sections 1.1 and 1.2, respectively.

1.1. MULTIMOORA method for MpMcDM

MULTIMOORA is a robust decision-making approach employing three subordinate methods and a rank aggregation function. The subordinate techniques are Ratio System, Reference Point Approach, and Full Multiplicative Form. Ratio System and Full Multiplicative Form effectively solve problems with independent and dependent criteria, respectively. Reference Point Approach leads to a conservative ranking of alternatives [12].

Earlier studies have exploited the approach of MULTIMOORA in many MpMcDM problems. Wang et al. [13] put forward a trust model for preference of experts based on the multi-person MULTIMOORA. Zolfaghari and Mousavi [14] presented a risky multi-person MULTIMOORA extension employing failure mode and effect analysis. Deng et al. [15] introduced an optimization-based consensus model supported on the multi-person MULTIMOORA.

A review of the researches on MULTIMOORA, including its developments and applications, has been conducted by Hafezalkotob et al. [16].

1.2. BWM for MpMcDM

BWM is an important decision-making technique for obtaining relative preferences of decision items as: (i) it needs less number of comparisons in contrast to Analytic Hierarchy Process (AHP); (ii) BWM has higher consistency than AHP; (iii) consistency of BWM also represents the confidence level; and (iv) BWM is straightforward for decision-makers from the viewpoint of mathematical computation [17].

Many researchers developed MpMcDM approaches based on the BWM. Hafezalkotob *et al.* [18] suggested a multi-person BWM model in which a director supervises an expert panel. Yazdi *et al.* [19] developed a risky multi-person BWM approach with a democratic-autocratic decision-making style. Omrani *et al.* [20] employed the concept of data envelopment analysis for a multi-person BWM approach.

A review of the studies on BWM comprising its extensions and applications has been presented by Mi *et al.* [17].

2. Uncertainty theories

We dicuss the important theories and our novelties related to interval numbers nand trapezoidal fuzzy numbers in Sections 2.1 and 2.2, respectively.

2.1. Interval numbers

Mathematical theories of interval numbers are given in this section including basic definitions, interval distance, and preference matrix used for extremum and rankings.

- ***Basic definitions of interval numbers***

The midpoint of $\overline{A}$ is defined as [21]:

$$pm\left(\overline{A}\right) = A^M = \frac{A^L + A^U}{2} \tag{I.1}$$

For two non-negative real interval numbers $\overline{A} = \left[A^L, A^U\right]$ and $\overline{B} = \left[B^L, B^U\right]$ and a non-negative real number r, the following arithmetic operations were defined [22]:

$$\begin{aligned}
&\overline{A} + \overline{B} = \left[A^L + B^L,\ A^U + B^U\right], \overline{A} - \overline{B} = \left[A^L - B^U,\ A^U - B^L\right] \\
&\overline{A}\overline{B} = \left[A^L B^L,\ A^U B^U\right] \\
&\overline{A}/\overline{B} = \left[A^L/B^U,\ A^U/B^L\right],\ \text{with } B^L \text{ and } B^U \neq 0 \\
&r\overline{A} = \left[rA^L,\ rA^U\right],\ \overline{A}^r = \left[\left(A^L\right)^r, \left(A^U\right)^r\right]
\end{aligned} \tag{I.2}$$

- ***Interval distance between two interval numbers***

The interval distance between interval numbers $\overline{A}$ and $\overline{B}$ was defined by Trindade et al. [21]:

$$\overline{d}(\overline{A},\overline{B}) = \begin{cases} \Big[\min\left\{\left|A^L - B^U\right|, \left|A^U - B^L\right|\right\} \\ \quad \max\left\{\left|A^L - B^U\right|, \left|A^U - B^L\right|\right\}\Big],\ \text{if } \overline{A} \cap \overline{B} = \varnothing \\ \Big[0,\ \max\left\{\left|A^L - B^U\right|, \left|A^U - B^L\right|\right\}\Big], \text{if } \overline{A} \cap \overline{B} \neq \varnothing \end{cases} \tag{I.3}$$

Eq. (I.3) has some shortcomings and thus Hafezalkotob and Hafezalkotob [23] improved it to the following form:

$$\overline{d}^*(\overline{A},\overline{B}) = \begin{cases} \Big[\min\left\{\left|A^L - B^U\right|, \left|A^U - B^L\right|\right\}, \left|A^M - B^M\right|\Big], \text{if } \overline{A} \cap \overline{B} = \varnothing \\ \Big[0,\ \left|A^M - B^M\right|\Big], \qquad \text{if } \overline{A} \cap \overline{B} \neq \varnothing \end{cases} \tag{I.4}$$

To show the differences between $\overline{d}$ and $\overline{d}^*$, we present some numerical examples as listed in Table I.1. The general point which can be conceived from Table I.1 is that $\overline{d}^*$ leads to smaller interval distance comparing with $\overline{d}$. Considering the second to fifth rows of Table I.1, it is obvious that $\overline{d}$ may not be sensitive to the degrees of intersection and inclusion of two interval numbers. However, $\overline{d}^*$ changes with the degrees of intersection and inclusion. The last row of Table I.1

shows that the interval distance $\overline{d}$ between two equal interval numbers is a given interval number; however, it may not be rational. In contrast to $\overline{d}$, the interval distance $\overline{d}^*$ between two equal interval numbers is exactly zero.

Table I.1. The differences between the interval distance $\overline{d}$ and $\overline{d}^*$.

$\overline{A}$	$\overline{B}$	$\overline{A}$ vs. $\overline{B}$	Relation	$\overline{d}(\overline{A},\overline{B})$	$\overline{d}^*(\overline{A},\overline{B})$
[1, 2]	[3, 7]	$\overline{A}<\overline{B}\vert_{\overline{A}\cap\overline{B}=\varnothing}$	Non-intersection	[1, 6]	[1, 3.5]
[1, 4]	[3, 7]	$\overline{A}<\overline{B}\vert_{\overline{A}\cap\overline{B}\neq\varnothing}$	Intersection	[0, 6]	[0, 2.5]
[1, 6]	[3, 7]	$\overline{A}<\overline{B}\vert_{\overline{A}\cap\overline{B}\neq\varnothing}$	Intersection	[0, 6]	[0, 1.5]
[3.5, 5.5]	[3, 7]	$\overline{A}\subset\overline{B}$	Inclusion	[0, 3.5]	[0, 0.5]
[3.5, 6]	[3, 7]	$\overline{A}\subset\overline{B}$	Inclusion	[0, 3.5]	[0, 0.25]
[3, 7]	[3, 7]	$\overline{A}=\overline{B}$	Equality	[0, 4]	0

- ***Preference matrix for finding the extremum and rankings of interval numbers***

The preference degree of $\overline{A}=\left[A^L,A^U\right]$ over $\overline{B}=\left[B^L,B^U\right]$ is defined as [24]:

$$P(\overline{A}>\overline{B})=\frac{\max\left\{0,A^U-B^L\right\}-\max\left\{0,A^L-B^U\right\}}{A^U-A^L+B^U-B^L} \tag{I.5}$$

The following properties hold [25]:

– $P(\overline{B}>\overline{A})=1-P(\overline{A}>\overline{B})$. If $\overline{A}=\overline{B}$ then $P(\overline{A}>\overline{B})=P(\overline{B}>\overline{A})=0.5$.

– If $P(\overline{A}>\overline{B})>P(\overline{B}>\overline{A})$, then $\overline{A}$ is said to be superior to $\overline{B}$ to the degree of $P(\overline{A}>\overline{B})$, represented as $\overline{A}\overset{P(\overline{A}>\overline{B})}{\succ}\overline{B}$; If $P(\overline{A}>\overline{B})=P(\overline{B}>\overline{A})=0.5$, then $\overline{A}$ is said to be indifferent to $\overline{B}$, indicated as $\overline{A}\sim\overline{B}$; If $P(\overline{B}>\overline{A})>P(\overline{A}>\overline{B})$, then $\overline{A}$ is said to be inferior to $\overline{B}$ to the degree of $P(\overline{B}>\overline{A})$, displayed as $\overline{A}\overset{P(\overline{B}>\overline{A})}{\prec}\overline{B}$.

– If $\overline{A}\leq\overline{B}$ and $\overline{A}\cap\overline{B}=\varnothing$, then $P(\overline{A}\geq\overline{B})=0$. If $\overline{A}\leq\overline{B}$ and $\overline{A}\cap\overline{B}\neq\varnothing$, then $0\leq P(\overline{A}\geq\overline{B})\leq 0.5$.

– If $\overline{A}\geq\overline{B}$ and $\overline{A}\cap\overline{B}=\varnothing$, then $P(\overline{A}\geq\overline{B})=1$. If $\overline{A}\geq\overline{B}$ and $AP(\varphi)$, then $0.5\leq P(\overline{A}\geq\overline{B})\leq 1$.

To rank a set of interval numbers $\left\{\overline{A},\overline{B},\ \ldots,\overline{N}\right\}$, a preference degree matrix is defined as Table I.2.

Table I.2. Preference degree matrix.

	$\overline{A}$	$\overline{B}$	$\cdots$	$\overline{N}$
$\overline{A}$	$P(\overline{A}>\overline{A})$	$P(\overline{A}>\overline{B})$	$\cdots$	$P(\overline{A}>\overline{N})$
$\overline{B}$	$P(\overline{B}>\overline{A})$	$P(\overline{B}>\overline{B})$	$\cdots$	$P(\overline{B}>\overline{N})$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\overline{N}$	$P(\overline{N}>\overline{A})$	$P(\overline{N}>\overline{B})$	$\cdots$	$P(\overline{N}>\overline{N})$

In the next step, the relative preference $P_{\varphi\theta}$ is generated as:

$$P_{\varphi\theta}=\begin{cases}1, & \text{if } P(\varphi>\theta)>0.5,\\ 0, & \text{if } P(\varphi>\theta)\leq 0.5,\end{cases}\qquad \varphi,\theta=\overline{A},\overline{B},\ \ldots,\overline{N} \tag{I.6}$$

Table I.3 shows the preference matrix for calculating extremum and ranking of interval numbers.

Table I.3. Preference matrix.

Relative preferences					Aggregated preferences
	$\bar{A}$	$\bar{B}$	$\cdots$	$\bar{N}$	
$\bar{A}$	$P_{\bar{A}\bar{A}}$	$P_{\bar{A}\bar{B}}$	$\cdots$	$P_{\bar{A}\bar{N}}$	$AP(\bar{A})$
$\bar{B}$	$P_{\bar{B}\bar{A}}$	$P_{\bar{B}\bar{B}}$	$\cdots$	$P_{\bar{B}\bar{N}}$	$AP(\bar{B})$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
$\bar{N}$	$P_{\bar{N}\bar{A}}$	$P_{\bar{N}\bar{B}}$	$\cdots$	$P_{\bar{N}\bar{N}}$	$AP(\bar{N})$

The aggregated preference $AP(\varphi)$ is the sum of the relative preferences $P_{\varphi\theta}$ in each row of Table I.3:

$$AP(\varphi) = \sum_{\theta=\bar{A}}^{\bar{N}} P_{\varphi\theta} \tag{I.7}$$

The maximum and minimum values of the set of interval values $\{\bar{A}, \bar{B}, \ldots, \bar{N}\}$ as well as the descending and ascending rankings of each interval value are obtained as:

$$\max\,\{\bar{A}, \bar{B}, \ldots, \bar{N}\} = \arg\max_{\varphi}\; AP(\varphi) \tag{I.8}$$

$$\min\,\{\bar{A}, \bar{B}, \ldots, \bar{N}\} = \arg\min_{\varphi}\; AP(\varphi) \tag{I.9}$$

$$\begin{aligned} r_d(\varphi) &= r\left(\varphi, \{\bar{A}, \bar{B}, \ldots, \bar{N}\}, [\text{decending order}]\right) = \\ & r\left(AP(\varphi), \{AP(\bar{A}), AP(\bar{B}), \ldots, AP(\bar{N})\}, [\text{decending order}]\right) \end{aligned} \tag{I.10}$$

$$\begin{aligned} r_a(\varphi) &= r\left(\varphi, \{\bar{A}, \bar{B}, \ldots, \bar{N}\}, [\text{ascending order}]\right) = \\ & r\left(AP(\varphi), \{AP(\bar{A}), AP(\bar{B}), \ldots, AP(\bar{N})\}, [\text{ascending order}]\right) \end{aligned} \tag{I.11}$$

2.2. Trapezoidal fuzzy numbers

A trapezoidal fuzzy number is a prevalent form of representing fuzzy numbers [5]. Every fuzzy number is defined through a membership function. A trapezoidal fuzzy number $\tilde{A} = (A_1, A_2, A_3, A_4)$ is defined by a membership function as introduced in Eq. (I.12):

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < A_1, \\ \dfrac{x - A_1}{A_2 - A_1}, & A_1 \le x \le A_2, \\ 1, & A_2 \le x \le A_3, \\ \dfrac{x - A_4}{A_3 - A_4}, & A_3 \le x \le A_4, \\ 0, & x > A_4. \end{cases} \tag{I.12}$$

The important mathematical concepts of trapezoidal fuzzy numbers include (i) basic mathematics, (ii) distance measure, and (iii) extremum and rankings. The following points presents the three items:

- ***Basic mathematics of trapezoidal fuzzy numbers***

A trapezoidal fuzzy number $\tilde{A}=(A_1,A_2,A_3,A_4)$ is defuzzified based on the concept of centroid obtains as [26]:

$$\overline{A} = \frac{1}{3}\left[A_1 + A_2 + A_3 + A_4 - \frac{A_4A_3 - A_1A_2}{(A_4 + A_3) - (A_1 + A_2)} \right]. \tag{I.13}$$

The arithmetic operations for two positive trapezoidal fuzzy numbers $\tilde{A} = (A_1, A_2, A_3, A_4)$ and $\tilde{B} = (B_1, B_2, B_3, B_4)$ as well as a positive real number τ are [26]:

$$\begin{aligned} &\tilde{A} \oplus \tilde{B} = \left(A_1 + B_1, A_2 + B_2, A_3 + B_3, A_4 + B_4\right), \\ &\tilde{A} \ominus \tilde{B} = \left(A_1 - B_4, A_2 - B_3, A_3 - B_2, A_4 - B_1\right), \\ &\tilde{A} \otimes \tilde{B} \cong \left(A_1B_1, A_2B_2, A_3B_3, A_4B_4\right), \\ &\tilde{A} \oslash \tilde{B} \cong \left(A_1/B_4, A_2/B_3, A_3/B_2, A_4/B_1\right), \\ &\tilde{A} \odot \tau = \left(A_1\tau, A_2\tau, A_3\tau, A_4\tau\right). \end{aligned} \tag{I.14}$$

- ***Distance measures of trapezoidal fuzzy numbers***

Based on the vertex method, the traditional crisp distance between positive trapezoidal fuzzy numbers is represented as follows [26]:

$$d_{\mathrm{CR}}(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{6}\left[\left(A_1 - B_1\right)^2 + 2\left(A_2 - B_2\right)^2 + 2\left(A_3 - B_3\right)^2 + \left(A_4 - B_4\right)^2\right]}. \tag{I.15}$$

Interval distance measure for two positive trapezoidal fuzzy numbers is defined as [27]:

$$\overline{d}_{\mathrm{IN}}(\tilde{A}, \tilde{B}) = \left[d_{IN}^L, d_{IN}^U\right], \tag{I.16}$$

where d_{IN}^L and d_{IN}^U are the lower and upper bounds given in Appendix in the supplementary file. $\overline{d}_{\mathrm{IN}}$ is computed based on α-cut with a weighting factor for different values of α. d_{IN}^L is obtained by assigning more weight to the intervals with higher values of α, while d_{IN}^U indicates the equal weights for the intervals at various levels of α.

In the literature of fuzzy computation, different types of distance measures of trapezoidal fuzzy numbers exist, but the introduced distances are crisp or parametric (e.g., based on α-cut)[27]–[29]. In contrast, we propose a novel fuzzy distance measure of positive trapezoidal fuzzy numbers as defined in Eq. (I.17):

$$\tilde{d}(\tilde{A}, \tilde{B}) = \Big\{\left(\varphi_1, \varphi_2, \varphi_3, \varphi_4\right), \quad \text{for Non-Intersection or Partly-Intersection,} \\ \left(0, \varphi_1, \varphi_2, \varphi_5\right), \quad \text{for Fully-Intersection (Inclusion or Equality Modes)}\Big\}, \tag{I.17}$$

$\tilde{d}(\tilde{A}, \tilde{B})$ is also a trapezoidal fuzzy number where $\varphi_1 = \min\theta$, $\varphi_2 = \min\{\theta \setminus \varphi_1\}$, $\varphi_3 = \max\theta$, $\varphi_4 = \max\{\theta \setminus \varphi_3\}$, and $\varphi_5 = \min\{\theta \setminus \{\varphi_1, \varphi_2\}\}$ (operator "\" denotes the extraction of its right side term from set θ). Set θ is defined as follows:

$$\theta = \left\{z_0, \left\{z_{11}, \ldots, z_{ef}, \ldots, z_{44}\right\}\right\}. \tag{I.18}$$

where z_0 and z_{ef} are defined as:

$$z_0 = \left|\overline{A} - \overline{B}\right|, \; z_{ef} = \left|A_e - B_f\right|, \; e = 1, 2, 3, \text{and}, 4, \; f = 1, 2, 3, \text{and}, 4. \tag{I.19}$$

In Table I.4, we have compared the results of the proposed distance measure of trapezoidal fuzzy numbers with those of two measures previously introduced in the literature by providing some numerical examples. Four relative conditions of trapezoidal fuzzy numbers, i.e., Non-Intersection, Partly-Intersection, Fully-Intersection (Inclusion Mode), and Fully-

Intersection (Equality Mode), are considered in Table I.4. $d_{\mathrm{CR}}(\tilde{A},\tilde{B})$ and $\bar{d}_{\mathrm{IN}}(\tilde{A},\tilde{B})$ provide crisp and interval distances obtained based on Eqs. (I.15) and (I.16), respectively. In Table I.4, the two crisp and interval distance measures are followed by the proposed fuzzy distance (obtained based on Eq. (I.17)) and the last column shows the defuzzified values of the fuzzy distance. Table I.4 indicates that $\bar{d}_{\mathrm{IN}}(\tilde{A},\tilde{B})$ has a shortcoming in Equality Mode as it finds a value for the distance between a trapezoidal fuzzy number and itself. However, in Equality Mode, our measure, i.e., $\tilde{d}(\tilde{A},\tilde{B})$, obtains zero which is reasonable. The defuzzified values of the fuzzy distances are consistent with the values of the crisp distances (i.e., $d_{\mathrm{CR}}(\tilde{A},\tilde{B})$). This issue implies the validity of our proposed fuzzy distance measure.

Table I.4. Comparison between the proposed fuzzy distance and other distance measures of trapezoidal fuzzy numbers.

$\tilde{A}$	$\tilde{B}$	$\tilde{A}$ vs. $\tilde{B}$	Relation	$d_{\mathrm{CR}}(\tilde{A},\tilde{B})$[26]	$\bar{d}_{\mathrm{IN}}(\tilde{A},\tilde{B})$[27]	$\tilde{d}(\tilde{A},\tilde{B})$ (our proposed measure)	Defuzzified value of $\tilde{d}(\tilde{A},\tilde{B})$
(1, 2, 3, 4)	(5, 6, 7, 8)	$\bar{A}<\bar{B}\|_{\bar{A}\cap\bar{B}=\varnothing}$	Non-Intersection	4	[4.08, 4.09]	(1, 2, 6, 7)	4
(1, 2, 3, 4)	(3.5, 4.5, 5.5, 6.5)	$\bar{A}<\bar{B}\|_{\bar{A}\cap\bar{B}\neq\varnothing}$	Partly-Intersection	2.5	[2.63, 2.64]	(0.5, 0.5, 3.5, 4.5)	2.26
(1, 2, 3, 4)	(1.5, 2, 2.5, 3)	$\bar{A}\subset\bar{B}$	Fully-Intersection (Inclusion Mode)	0.54	[0.69, 0.72]	(0, 0, 1, 1)	0.5
(1, 2, 3, 4)	(1, 2, 3, 4)	$\bar{A}=\bar{B}$	Fully-Intersection (Equality Mode)	0	[0.82, 0.85]	(0, 0, 0, 0)	0

- ***Extremum and rankings of trapezoidal fuzzy sets***

Several previous studies have presented some techniques for computing extremum and rankings of trapezoidal fuzzy numbers [27], [30]. In this paper, we use a preference matrix for pairwise comparison of trapezoidal fuzzy numbers. First, the trapezoidal fuzzy numbers are converted into interval numbers based on the α-cut ($\alpha = 0.5$); then, the maximum and minimum values of the set of resultant intervals besides the descending and ascending rankings of the set are determined based on the preference matrix. For briefness, we avoid mentioning the details of the related computation which can be found in [18].

3. Risk theory based on prospect theory

In this part, the principles of prospect theory are discussed, initially. Then, the proposed Fuzzy-Rationality-Based Prospect (FRFP) theory is described.

3.1. Principles of prospect theory

Kahneman and Tversky [31] proposed the prospect theory. Daniel Kahneman won a “Nobel Memorial Prize in Economics” for his outstanding work concerning the development of prospect theory. Risky decision-making cannot be tackled by the expected utility theory; however, the prospect theory considers the risk of decisions to provide a more realistic modeling of human behaviors. In prospect theory, some pervasive effects of risky decisions are contemplated which are inconsistent with the expected utility theory. The main pervasive effects in prospect theory are the certainty and isolation effects. The certainty effect relates to the tendency of people to underestimate the probable outcomes compared to the certain ones. That is, the certainty effect matches the tendency of people to be risk-averse toward the selections with sure gains while to be risk-seeking as for the selections associated with sure losses. To streamline the selection process from a set of alternatives, people may neglect the components which the alternatives share while pay attention to the components that differentiate them. This tendency of people is entitled the isolation effect which may result in inconsistent preferences[31].

In prospect theory, a value function denotes the gain or loss of a decision (relative to the reference point) represented as follows[4]:

$$v(x)=\begin{cases} x^{\gamma}, & x \geq 0, \\ -\delta(-x)^{\beta}, & x<0, \end{cases} \tag{I.20}$$

where x represents the deviation of a decision from the reference point dependent on the subjective viewpoint of the decision-maker. x^{γ} and $-\delta(-x)^{\beta}$ are the terms for the gain or loss where γ and β $(0 \leq \gamma, \beta \leq 1)$ denote the concave and convex degree of the terms, respectively[32]. That is, γ and β represent the sensitive degree of the decision-maker on gain or loss, respectively. δ $(\delta>1)$ indicates the loss aversion degree of the decision-maker. In prospect theory, the utility of decision-makers on losses is considered to be higher than the utility on gains[33]. Based on the study of Kahneman and Tversky[31], the values $\gamma=\beta=0.88$ and $\beta=2.25$ could show the real behavior of a decision-maker.

Eq. (I.20) could be reformulated based on the distance to the reference points. Eq. (I.21) shows the distance-based formulation where x is the decision as a positive value, and R^{+} and R^{-} stand for positive and negative reference points, respectively:

$$v(x)=\begin{cases} \left|x-R^{-}\right|^{\gamma}, & x \geq R^{-}, \\ -\delta\left(\left|x-R^{+}\right|\right)^{\beta}, & x<R^{+}. \end{cases} \tag{I.21}$$

Dai et al.[4] introduced a fuzzy prospect theory based on crisp distance of triangular fuzzy numbers. In a similar way, fuzzy prospect theory based on crisp distance of trapezoidal fuzzy numbers can be obtained as:

$$v_{\mathrm{CR}}(\tilde{x})=\begin{cases} \left[d_{\mathrm{CR}}\left(\tilde{x}, \tilde{R}^{-}\right)\right]^{\gamma}, & \tilde{x} \geq \tilde{R}^{-}, \\ -\delta\left[d_{\mathrm{CR}}\left(\tilde{x}, \tilde{R}^{+}\right)\right]^{\beta}, & \tilde{x}<\tilde{R}^{+}, \end{cases} \tag{I.22}$$

where d_{CR} represents crisp distance of trapezoidal fuzzy numbers defined in Eq. (I.15).

3.2. Proposed Fuzzy-Rationality-Based Prospect (FRFP) theory

In this section, we introduce the Fuzzy-Rationality-Based Prospect (FRFP) theory. The theory is based on the cognitive theory of decision-choice as illustrated in Fig. I.1.

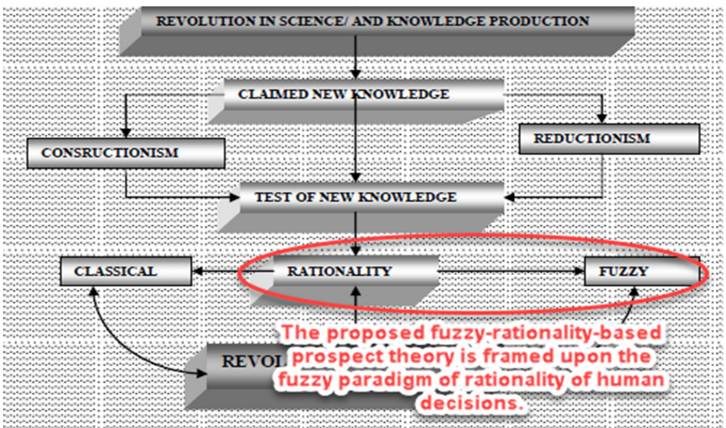

Fig. I.1. The relation of the proposed FRFP theory and the cognitive theory of decision-choice (the flowchart in the figure extracted form Ref. [34]).

For the problems based on the FRFP theory, a fuzzy form of the value function of prospect theory is needed. Some researchers introduced such fuzzy extensions of the value function of prospect theory[4],[35]. They have extended the traditional value function of prospect theory (i.e., Eq. (I.20)) into fuzzy representation or presented fuzzy form of the distance-based value function (i.e., Eq. (I.21)) supported on the crisp distance of fuzzy numbers. However, we suggest a novel derivation by developing the distance-based value function of the prospect theory (i.e., Eq. (I.21)) employing the proposed measure of fuzzy distance (i.e., Eq. (I.17)), as:

$$\tilde{v}(\tilde{x}) = \begin{cases} \left[\tilde{d}\left(\tilde{x}, \tilde{R}^{-}\right)\right]^{\gamma}, & \tilde{x} \geq \tilde{R}^{-}, \\ -\delta\left[\tilde{d}\left(\tilde{x}, \tilde{R}^{+}\right)\right]^{\beta}, & \tilde{x} < \tilde{R}^{+}, \end{cases} \tag{I.23}$$

where $\tilde{x}$ is the fuzzy decision of a decision-maker as a positive value and γ, β, and δ are defined similar to Eq. (I.20). If we consider $\tilde{x}$, $\tilde{R}^{+}$, and $\tilde{R}^{-}$ as trapezoidal fuzzy values, the resultant value function would be also a trapezoidal fuzzy rating (the distance measure defined in Eq. (I.17) provides trapezoidal fuzzy outputs).

Chapter II

Overview on MULTIMOORA for risky uncertain MpMcDM and its applications

MULTIMOORA is a useful multi-criteria decision-making technique. The output of the MULTIMOORA is a ranking obtained by aggregating the results of the ternary ranking methods: Ratio System, Reference Point Approach, and Full Multiplicative Form. In the literature of MULTIMOORA, there is not a comprehensive review study. In this chapter, we conduct an overview of MULTIMOORA by categorizing and analyzing main researches, theoretically and practically.

First, we go through an theoretical survey of MULTIMOORA in terms of the subordinate ranking methods, and the robustness of the method. We analyze MpMcDM with MULTIMOORA. We scrutinize the developments of MULTIMOORA based on uncertainty theories.

Practical problems of MULTIMOORA are discussed mainly sectors concerning industries and healthcare management.

1. Introduction

MpMcDM methods tackle the problem of finding the best solution from a set of candidate alternatives in respect of multiple criteria. Often, there is no alternative which dominates the others on all criteria; thus, decision-makers usually look for the satisfactory solution [36]. The decision-making pproaches can be categorized into three groups: ① Value Measurement Methods, like SAW (Simple Additive Weighting) [37] and WASPAS (Weighted Aggregated Sum Product Assessment) [38]; ② Goal or Reference Level Models, such as TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) [39] and VIKOR (VIse Kriterijumska Optimizacija kompromisno Resenje, in Serbian, Multiple Criteria Optimization Compromise Solution) [40]; and ③ Outranking Techniques, like PROMETHEE (Preference Ranking Organization METHod for Enrichment of Evaluations) [41], ELECTRE (ELimination Et Choix Traduisant la REalité, in French, ELimination and Choice Expressing the Reality) [42], ORESTE (Organísation, Rangement Et SynThèse de donnéEs relarionnelles, in French, Organization, Arrangement and Synthesis of Relational Data) [43], and GLDS (Gained and Lost Dominance Score) method [44].

In 2006, Brauers and Zavadskas [45] introduced MOORA (Multi-Objective Optimization on the basis of a Ratio Analysis) combining Ratio System and Reference Point Approach. In 2010, Brauers and Zavadskas [46] improved MOORA to MULTIMOORA (Multi-Objective Optimization on the basis of a Ratio Analysis plus the full MULTIplicative form) by adding Full Multiplicative Form and employing Dominance Theory to obtain a final integrative ranking based on the results of these triple subordinate methods. Ratio System and Full Multiplicative Form belong to the first group of decision-making approaches (i.e., Value Measurement Methods) while Reference Point Approach falls in the second group of decision-making approaches (i.e., Goal or Reference Level Models).

As Ratio system employs arithmetic weighted aggregation operator, it is useful in applications like student selection in which “independent” criteria exist in the problem. Suppose, we compare two students based on their exam marks. As the exams are independent on each other, arithmetic operator works fine for the case. That is, it is not important that in which exams the student has better performance. Thus, the overall performance in all exams (which are independent) is significant. However, Ratio system has defects in the cases where “dependent” criteria appear in a decision-making problem. Suppose, we compare two investment companies based on their portfolios in different years. The performance of an investment company in each year is “dependent” on the other years, that is, investment in a particular year influences the status of the following years. For example, if the portfolio return in one year were very poor, in reality, this issue should affect the overall performance dramatically. Geometric operator could consider the dependency of performances of each year while arithmetic operator neglects the issue. In the cases where “dependent” criteria exist in decision-making problems, Full Multiplicative Form can be helpful as it applies geometric weighted aggregation operator. Reference Point Approach which utilizes Min-Max Metric is a “conservative” method useful for the cases where the optimal choice for decision-makers is the alternative that does not have a very bad performance on none of the criteria.

To integrate the outcomes of the three subordinate parts, a variety of ranking aggregation techniques can be deployed. In this regard, the most common ranking aggregation tool in the literature of MULTIMOORA is Dominance Theory which is also the concept adopted in the original MULTIMOORA suggested by Brauers and Zavadskas [46]. Other ranking aggregation tools such as Dominance-Directed Graph, Rank Position Method, Technique of Precise Order Preference, Borda Rule, Improved Borda Rule, ORESTE Method, and Optimization Model have also been applied to generate the final ranking of the MULTIMOORA approach.

Only one survey study was previously conducted on MULTIMOORA, by Baležentis and Baležentis [36] in 2014. The work is limited to a few models regarding Group Decision-Making, Fuzzy Set Theory, and practical applications. In the current overview, we discuss MULTIMOORA models not only based on Group Decision-Making, Fuzzy Set Theory and applications, but also evaluate multiple theoretical features, various uncertainty theories, and applications in different fields besides provide bibliometric analysis, and identify significant theoretical and practical challenges. In this regard, this overview can be presented as the following itemized list:

1) We highlight the features of MULTIMOORA by discussing the ternary subordinate utilities and clarifying the robustness of the decision-making method. Besides, we analyze multi-person decision-making structures, and the models used for combination.
2) We present the developments of MULTIMOORA based on uncertainty theories including interval numbers, and fuzzy set theories as well as their combinations. The formulations of the significant uncertain extensions are also provided and all developments are evaluated statistically.
3) We present the applications of MULTIMOORA in the sectors of industries and healthcare management. Also, the important applications are evaluated statistically.

This overview is organized as follows. Section 2 focuses on Theory of MULTIMOORA method. We present applications of MULTIMOORA method in Section 3.

2. Theory of MULTIMOORA method

Fundamentals of MULTIMOORA, MpMcDM with MULTIMOORA, and its uncertain developments are discussed in Sections 2.1, 2.2, and 2.3, respectively.

2.1. Fundamentals of MULTIMOORA

MULTIMOORA exploits the vector normalization technique for generating comparable ratings and three subordinate ranking methods entitled Ratio System, Reference Point Approach, and Full Multiplicative Form. Each of the three ranking methods has some privileges but suffers from shortcomings; thus, MULTIMOORA uses more than one approach. In this subsection, we make a description about these three subordinate ranking method to facilitate the understanding of the MULTIMOORA method.

The first step in an decision-making problem is constructing a decision matrix and weight vector. Thus, for MULTIMOORA, decision matrix composed of the ratings x_{ij} of m candidate alternatives of the problem with respect to n criteria is first constructed, as follows [47]:

$$\begin{array}{c} \qquad\qquad \begin{array}{ccccc} c_1 & \cdots & c_j & \cdots & c_n \end{array} \\ \mathbf{X} = \left[\begin{array}{ccccc} x_{11} & \cdots & x_{1j} & \cdots & x_{1n} \\ \vdots & & \vdots & & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{in} \\ \vdots & & \vdots & & \vdots \\ x_{m1} & \cdots & x_{mj} & \cdots & x_{mn} \end{array} \right] \begin{array}{c} A_1 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{array} \\ \mathbf{W} = \left[\begin{array}{ccccc} w_1 & \cdots & w_j & \cdots & w_n \end{array} \right] \end{array} \tag{II.1}$$

Because the ratings of alternatives on the multiple criteria of the problem may have different dimensions, the ratings should be normalized before utilization in a decision-making model. Different normalization schemes have been employed in MpMcDM methods [45], [48]. Liao et al. [49] made a comparison over different normalization schemes. Brauers et al.

[50] claimed that Van Delft and Nijkamp (i.e., Vector) Normalization is the most robust choice for application in MULTIMOORA. Vector Normalization is represented as follows [45]:

$$x_{ij}^{*} = x_{ij} \Big/ \sqrt{\sum\nolimits_{i=1}^{m} \left(x_{ij} \right)^{2}} \quad \text{(II.2)}$$

Ratio System, as a fully compensatory model, is useful when "independent" criteria exist in the problem. For cases with the existence of "dependent" criteria, Full Multiplicative Form, as an incompletely-compensatory model, is a beneficial tool. Reference Point Approach, as a non-compensatory model, is a "conservative" method comparing Ratio System and Full Multiplicative Form. Ratio System and Full Multiplicative Form both provide the opportunity to compensate the poor performance of an alternative on one criterion by the performances on other criteria (the degree of compensation related to the two techniques is not equal); however, Reference Point Approach does not allow such an opportunity. As "dependent" and "independent" criteria may exist simultaneously in the problem and for the sake of having a "conservative" result, MULTIMOORA integrates the triple methods to exploit the advantages of each of them and reach a final outcome that is more robust than the individual results [51]. We discuss the derivation of the triple subordinate ranking methods besides the connection of the methods with other decision-making approaches, as follows:

- ***Ratio System***

Ratio System which uses the arithmetic weighted aggregation operator is a fully compensatory model. It means that small normalized values of an alternative could be completely compensated by the same degree of large values. In other words, an alternative with poor performance in respect to some criteria and fine performance in respect to the remained criteria can be substituted by an alternative with moderate performance in respect to all criteria [52]. To compute the utility of Ratio System, the weighted normalized ratings are added for beneficial criteria and deducted for non-beneficial criteria as follows [53]:

$$y_i = \sum\nolimits_{j=1}^{g} w_j x_{ij}^{*} - \sum\nolimits_{j=g+1}^{n} w_j x_{ij}^{*}, \quad \text{(II.3)}$$

where g is the number of beneficial criteria and $(n-g)$ is the number of non-beneficial criteria. The best alternative based on Ratio System has the maximum utility y_i and the ranking of this method is obtained in descending order as:

$$\mathbf{R}_{\mathrm{RS}} = \left\{ A_{i|\max\limits_{i} y_i} \succ \cdots \succ A_{i|\min\limits_{i} y_i} \right\}. \quad \text{(II.4)}$$

Ratio System, is inspired by SAW. In SAW, same as Ratio System, the utility is obtained by aggregation of the weighted normalized alternatives ratings; however, there is only one term for sum (i.e., no term exists for subtraction) because SAW's normalization is based on a linear ratio. For beneficial criteria, each alternative rating is divided by the maximum value of ratings per criterion and for non-beneficial criteria, minimum value of ratings per criterion is divided by each alternative rating. The concept of Ratio System can be also found in other decision-making methods like WASPAS and (Multi-Objective Optimization by Simple Ratio Analysis) (MOOSRA). The first term of WASPAS utility is inspired by Ratio System. In MOOSRA, the beneficial sum is divided by the non-beneficial sum while in Ratio System, the non-beneficial sum is subtracted from the beneficial sum.

- ***Reference Point Approach***

In Reference Point Approach, the best alternative is the one that its worst value in respect of all criteria is not very bad [52]. This approach, as a non-compensatory model, first finds the alternatives ratings with the worst performance with respect to each criterion and finally selects the overall best value (i.e., the minimum value) from these worst ratings. Reference Point Approach is based on Tchebycheff Min-Max Metric [45]. Tchebycheff Min-Max Metric is originated from the general theory of Murkowski Metric which is the source of several decision analysis approaches in literature such as Goal Programming. To obtain the utility, first, Maximal Objective Reference Point (MORP) Vector is defined as [45]:

$$r_j = \left\{ \max_i x_{ij}^*, \quad j \le g; \quad \min_i x_{ij}^*, \quad j > g \right\}. \tag{II.5}$$

The distance between the weighted value of each member of MORP Vector and the weighted alternative rating is obtained as [54]:

$$d_{ij} = \left| w_j r_j - w_j x_{ij}^* \right|. \tag{II.6}$$

The utility of Reference Point Approach is obtained by maximizing the distance introduced in Eq. (II.6) as follows [54]:

$$z_i = \max_j d_{ij}. \tag{II.7}$$

The best alternative based on Reference Point Approach has the minimum utility z_i and the ranking of the approach is produced in ascending order as:

$$\mathbf{R}_{\text{RPA}} = \left\{ A_{i|\min_i z_i} \succ \cdots \succ A_{i|\max_i z_i} \right\}. \tag{II.8}$$

In Reference Point Approach, the distance of each alternative rating from MORP Vector is obtained. There are other forms of Reference Point Vectors in the literature, including:

– Utopian Objective Reference Point (UORP) Vector: In this vector, higher values are targeted not the maximum values, necessarily;
– Aspiration Objective Reference Point (AORP) Vector: This vector tries to moderate aspirations as finding the maximum distance from the target values; that is, finding the alternatives with the worst performance.

TOPSIS and VIKOR also fall into the group of "Goal or Reference Level Models". Both of them are based on L_P-*Metric*. TOPSIS is supported on L_2 while VIKOR is formulated on the basis of L_1 and L_∞. In TOPSIS, there exist two Reference Points, including the Positive-Ideal Solution (PIS) inspired by MORP and the Negative-Ideal Solution (NIS) inspired by AORP. In Classical Reference Point Approach, only MORP Vector is considered without paying attention to AORP Vector, but in Extended Reference Point Approach suggested by Eghbali-Zarch et al. [55], AORP Vector is also taken into account. Reference Point Approach sometimes cannot differ on two or more alternatives; that is, the approach leads to same rankings [56]. Thus, Reference Point Approach is often integrated with other decision-making tools to remedy the defect.

- ***Full Multiplicative Form***

Full Multiplicative Form, which uses the geometric weighted aggregation operator, is an incompletely-compensatory model. In this technique, small normalized values of an alternative could not be completely compensated by the same degree of large values. Thus, the issue leads to the perception that an alternative with moderate performance may be superior to an alternative which has both good and bad performances with respect to different criteria [52]. To obtain the utility of Full Multiplicative Form, the product of weighted normalized alternatives ratings on beneficial criteria are divided by the product of weighted normalized alternatives ratings on non-beneficial criteria [53]:

$$u_i = \prod_{j=1}^{g} \left(x_{ij}^*\right)^{w_j} \Big/ \prod_{j=g+1}^{n} \left(x_{ij}^*\right)^{w_j} \quad \text{(II.9)}$$

In utility formula of Full Multiplicative Form, multiplying normalized ratings with weights leads to the same result as the situation in which no weights are considered. Thus, weights should be considered as exponent in utility equation of Full Multiplicative Form. The best alternative based on Full Multiplicative Form has the maximum utility u_i and the ranking of this technique is generated in descending order as:

$$\mathbf{R}_{\text{FMF}} = \left\{ A_{i|\max_i u_i} \succ \cdots \succ A_{i|\min_i u_i} \right\} \quad \text{(II.10)}$$

The concept of Full Multiplicative Form can be observed in other decision-making techniques like WASPAS. That is, the second term of WASPAS utility index is similar to Full Multiplicative Form. However, WASPAS uses a linear ratio for normalization considering the maximum and minimum values of alternatives ratings.

- ***Dominance Theory***

Dominance Theory was used in the original MULTIMOORA method. This theory is supported on some principles including Dominance (Absolute Dominance and Partial Dominance), Equality (Absolute Equality, Partial Equality, and Equality according to Circular Reasoning), and Transitiveness [51]. There are some drawbacks to utilizing Dominance Theory: ① obtaining ranks of alternatives is hard as the theory is not yet automated [6]; ② the theory only uses ordinal values by neglecting the relative importance of alternatives; and ③ circular reasoning happens in some cases which leads to identical ranks which is not satisfactory [52].

- ***Borda and Improved Borda Rules***

Borda Rule, also named Borda Count, is an easy but effective technique from the group of single-winner election methods in which the number of votes equals to the number of alternatives [57]. In this method, if there are t alternatives, the first-ranked alternative gets t votes and the second-ranked gets one vote less, and so on. The final score of Borda Rule is computed by the summation of the scores of the subordinate methods. The highest value of Borda Rule score shows the best alternative.

Improved Borda Rule is based on Borda Count [58]; however, it integrates both cardinal and ordinal values (i.e., utilities and rankings, respectively) of each subordinate methods of MULTIMOORA. In this sense, the Improved Borda Rule is superior to Dominance Theory. To employ the Improved Borda Rule, first, the subordinate utilities are normalized based on Vector Normalization to produce y_i^*, z_i^*, and u_i^*. The assessment value of Improved Borda Rule, i.e., $IMB\left(A_i\right)$, is obtained using the following equation [58]:

$$IMB\left(A_i\right) = y_i^* \frac{m - r(y_i) + 1}{m\,(m+1)/2} - z_i^* \frac{r(z_i)}{m\,(m+1)/2} + u_i^* \frac{m - r(u_i) + 1}{m\,(m+1)/2}, \quad \text{(II.11)}$$

where $r(y_i)$, $r(z_i)$, and $r(u_i)$ are the rankings of Ratio System, Reference Point approach, and Full Multiplicative Form, respectively. The best alternative based on Improved Borda Rule has the maximum value of $IMB(A_i)$.

Remark: Dominance Theory is complicated due to pairwise comparisons and probable occurrence of circular reasoning. The case would be more confusing for decision-makers when the number of alternatives and criteria are large because Dominance Theory is based on manual comparison. Nevertheless, Improved Borda Rule neither needs any manual comparison, nor has special conditions.

- ***Robustness of MULTIMOORA***

In Table II.1, the performance of MOORA which is a part of MULTIMOORA is compared with other decision-making methods. As we can find from Table II.1, MOORA is simple and reliable. Original MULTIMOORA combines MOORA with the full multiplicative form using the dominance theory. Brauers and Zavadskas [51] claimed that "use of two different methods of multi-objective optimization is more robust than the use of a single method; the use of three methods is more robust than the use of two, and so on;" thus, "MULTIMOORA is more robust than MOORA".

Table II.1. Performance of MOORA regarding other decision-making methods [51].

MCDM method	Computational time	Simplicity	Mathematical calculations	Stability	Information type
MOORA	Very less	Very simple	Minimum	Good	Quantitative
AHP	Very less	Very critical	Maximum	Poor	Mixed
TOPSIS	Moderate	Moderately critical	Moderate	Medium	Quantitative
VIKOR	Less	Simple	Moderate	Medium	Quantitative
ELECTRE	High	Moderately critical	Moderate	Medium	Mixed
PROMETHEE	High	Moderately critical	Moderate	Medium	Mixed

Generally, the advantages of MULTIMOORA include: ① simple mathematics, ② low computational time, ③ straightforwardness for decision-makers, ④ using three different methods for determining subordinate rankings, and ⑤ employing ranking aggregation tools for integrating the subordinate rankings. To clarify item ⑤, it is worthwhile to mention that many decision-making methods have only one utility function; however, MULTIMOORA produces an integrative outcome by combining three utility values employing a ranking aggregation tool.

The three subordinate parts of MULTIMOORA are based on the fully compensatory, non-compensatory, and incompletely-compensatory models. As discussed in Subsections 2.2, each of the approaches may have some shortcomings, in practice. Therefore, integration of their outcomes would lead to a more robust final result comparing to the individual outcomes by curing the existing defects.

- ***Graphical summary of MULTIMOORA theory***

The concepts used in Subsections 2.2 and 2.3 to derive the model of MULTIMOORA can be summarized into five phases as illustrated in Fig. II.1. Decision matrix and weight vector are constructed in Phase 1. The decision matrix is normalized in Phase 2. The utilities of subordinate parts of MULTIMOORA, i.e., Ratio System, Reference Point Approach, and Full Multiplicative Form, are computed in Phase 3. Rankings of subordinate methods are produced in Phase 4. Eventually, the subordinate rankings are combined into final outcomes of MULTIMOORA in Phase 5.

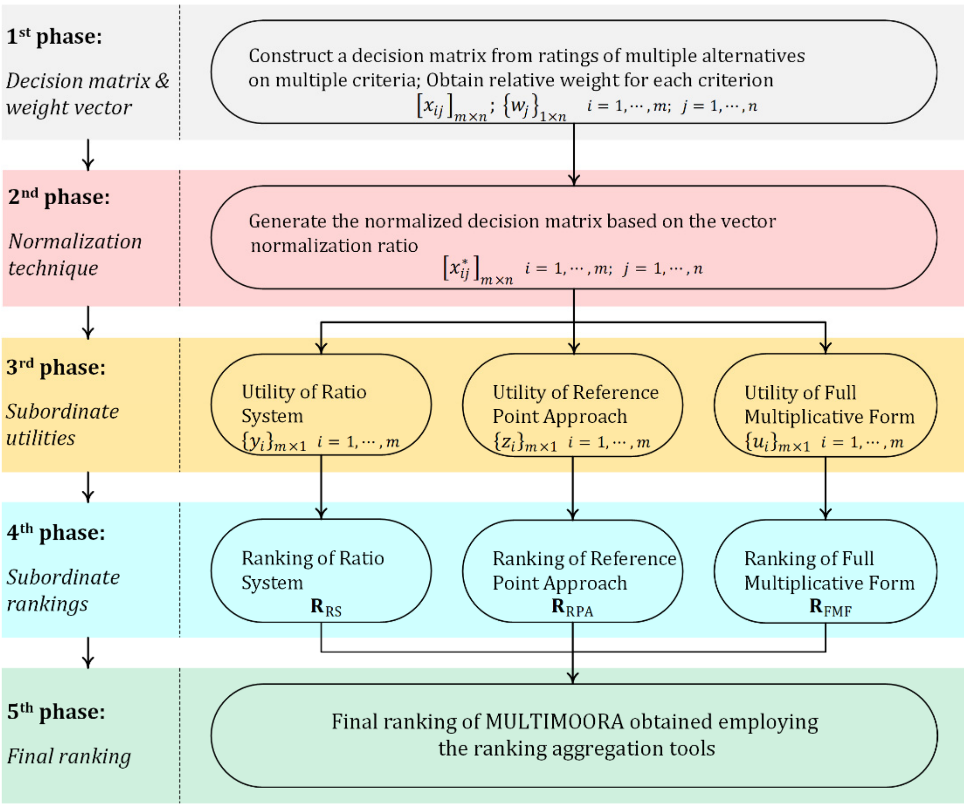

Fig. II.1. Flowchart of MULTIMOORA phases.

2.2. MpMcDM with MULTIMOORA method

In real life, many significant decisions are made through a group of elites and experts rather than considering an individual decision-maker. In industries and factories, technical expert panel takes the crucial decisions on identifying plans and strategies, selecting staff, and exploiting available resources. In practical problems like legal systems, healthcare management, and social services, the significant decisions are usually made based on collective opinions of multiple advisors and experts. Sometimes, experts may have different fields and levels of expertise; therefore, in such cases, collective decisions are analyzed to handle the conflicts among various opinions [2].

Multiple-person decision-making can be done in cooperative or non-cooperative styles. Cooperative group decisions are significant in engineering, medical, and scientific fields while non-cooperative group decisions are common in economic and political areas. Even by considering cooperative group decisions, reaching a complete consensus among all members of the group on the eventual solution is nearly infeasible because decision-makers, who are supposed to have identical goals, may have some opinion conflicts, in practice [59].

Near a half of total studies on MULTIMOORA has a multiple-person decision-making structure, which shows the importance of multiple-person decision-making. In some studies on MULTIMOORA, multiple-person decision-making structure is employed to generate criteria weights and alternatives ratings on the criteria; however, in the others, multiple decision-makers only participate in criteria weighting procedure.

2.3. Developments of MULTIMOORA method under risk and uncertainty

- ***Uncertain developments based on interval number theory***

Interval number theory is a simple but applicable concept of considering vagueness in decision-making problems. Interval numbers can be defined as: ① an extension of a real number; ② a degenerate flat fuzzy number without membership function; and ③ an α-cut of a fuzzy number. INs are important in decision-making problems, because: ① INs require the minimum amount of data; ② decision-makers could easily present the range of available data as interval numbers; and ③ INs are very practical as many data in real problems are essentially reported in the form of ranges.

There are four important MULTIMOORA extensions based on interval theory. Kracka et al. [60] proposed interval MULTIMOORA utilizing arithmetic of interval numbers (MOORE rule), the crisp distance of interval numbers, and comparison based on arithmetic average. Hafezalkotob and Hafezalkotob [11] suggested a new model of interval MULTIMOORA by using preference matrix without degeneration of interval numbers. Hafezalkotob and Hafezalkotob [23] presented interval target-based MULTIMOORA employing MOORE rule, interval distance of interval numbers, and the preference matrix. Hafezalkotob and Hafezalkotob [47] developed interval target-based MULTIMOORA by adding preference-based rankings of interval numbers.

- ***Uncertain developments based on fuzzy set theory***

Fuzzy set theory, introduced by Zadeh [61] in 1965, is an important theory of uncertainty which models the vagueness or imprecision of the human cognitive process. A fuzzy set is generally introduced by a membership function that maps elements to degrees of membership in a certain interval [62]. The theory is very applicable in various fields such as decision making, artificial intelligence, expert systems, control theory, and neural networks. There are different types of fuzzy sets like interval-valued fuzzy number, intuitionistic fuzzy number, and interval type-2 fuzzy set [7].

As fuzzy theory is one of most important concepts of uncertainty, there are many extensions of MULTIMOORA based on this theory. Triangular fuzzy number is the simplest form of representing the fuzziness of data. Triangular fuzzy number with mathematical features such as Vertex method for crisp distance and centroid-based method for defuzzification has combined with MULTIMOORA in several studies [6], [55], [71]–[74], [63]–[70]. However, Tian et al. [62] employed graded mean integration as defuzzification technique to generate triangular fuzzy MULTIMOORA. Trapezoidal fuzzy number with concepts of Vertex method for crisp distance and centroid-based method for defuzzification is used for three developments [3], [5], [26]. Liu et al. [75] applied the integral of area for defuzzification to derive Trapezoidal Fuzzy MULTIMOORA. Stanujkic et al. [76] suggested interval-valued fuzzy MULTIMOORA based on the weighted averaging operator and the geometric averaging operator of interval-valued fuzzy numbers. Dorfesh et al. [77] suggested Interval type-2 fuzzy MULTIMOORA. Generalized interval-valued fuzzy number is a basis for four developments [78]–[81]. In these studies, centroid-based method is used for crisp distance of Generalized interval-valued fuzzy numbers and defuzzification is also based on the crisp distance. Baležentis and Baležentis [82] introduced intuitionistic fuzzy MULTIMOORA based on the power ordered weighted average operator and the power ordered weighted geometric operator as well as Euclidean distance and expected values of interval-valued fuzzy numbers. Baležentis et al. [83] presented another version of intuitionistic fuzzy MULTIMOORA using negation operator, the power ordered weighted average operator, the power ordered weighted geometric operator, comparison rule, and crisp distance of interval-valued fuzzy numbers. Interval-valued intuitionistic fuzzy MULTIMOORA has been developed considering the weighted average operator, the weighted geometric operator, and score function of interval-valued fuzzy numbers [84], [85]. Hesitant fuzzy set was exploited in three studies [86]–[88] for new developments.

3. Application of MULTIMOORA method

The applications of MULTIMOORA in the sector of Industries are divided into the following subsectors: Construction, Automotive, Agricultural, Mining, Entertainment, Logistics, Steel, Aviation, Beverage, Carpentry, Energy, Ship-Building, and Textile Industries, besides Manufacturing System. In Construction Industry subsector, there are several case studies related to Buildings Revitalization Appraisal [84], Project Management [77], [89], and Ranking Countries/Cities/Regions [90] besides the selection of Investment [84], Component [91], [92], Design [60], [93], [94], Material [92], Supplier [95], and Technology [96]. In Automotive Industry subsector, there are multiple case studies related to Battery Recycling Mode Selection [97] and Location Planning [98] as well as the selection of Material [26], [53], Robot [99], Supplier [56], and Vehicle [57], [100], [101]. In Agricultural Industry subsector, the case studies include Farming Efficiency Estimation [68] and the selection of Crop [73], Machine [102], and Supplier [75]. In Mining Industry subsector, there exist four case studies related to Design Selection [76], [103], Mining Technique Selection [104], and Personnel Management [105]. In Entertainment Industry subsector, two case studies exist concerning Company/Industrial Group Selection [58] and Device Selection [106]. In Logistics Industry subsector, two case studies have considered the problems regarding Partner Selection [6] and Transportation Efficiency Evaluation [107]. In Manufacturing System subsector, the practical cases are Enterprise Resource Planning [108] and the selection of Design [109], Machine [23], [80], and Material [11], [53]. In Steel Industry subsector, two researches exist in respect to Risk Evaluation [69], [85].

For other subsectors of Industries sector, there is only one case study. Dorfeshan et al. [77] evaluated a project management problem in the area aircraft component development planning. Çebi and Otay [74] tackled a supplier selection problem in a company operating in beverage industry. Stojić et al. [38] assessed selection process of supplier for a PVC carpentry manufacturing company. Hafezalkotob and Hafezalkotob [3] handled material selection process for the blades of industrial gas turbine. Qin and Liu [110] chose a suitable supplier for purchasing components of ship equipment. Brauers and Zavadskas [111] undertook a project management problem for Tunisian textile industry.

The applications of MULTIMOORA in the sector of Medical/Healthcare Management are divided into the following subsectors: Medical Service, Biomedical Service, and Health-Care Management. In Medical Service subsector, there is one case study related to pharmacological selection of type 2 diabetes [55]. In Biomedical Service subsector, two studies has conducted on the selection process of biomaterials for hip and knee surgical prostheses [47], [54]. In Health-Care Management subsector, three case studies have handled Risk Evaluation and Waste Management. Liu et al. [5] used the concept of failure mode and effects analysis to prevent infant abduction from hospitals. Two researched analyzed the treatment technologies regarding health-care waste management in Shanghai, China [63], [112].

Chapter III

I_S-MpMcDM methodology: Industrial application

In this chapter, we present the I_S-MpMcDM methodology. The methodology is based on interval numbers and modeled by the BWM-MULTIMOORA approach. The method with complete interval computation in which interval distance of interval numbers and preference matrix are used. In addition, we propose a multi-person interval best-worst method with interval preference degree. The multi-person interval best-worst method has two levels of experts.

We introduce the interval Borda rule as an aggregation function which does not have the defects of the dominance theory. We calculate objective interval weights of criteria based on interval entropy method, which are integrated by the subjective weights computed by the multi-person interval best-worst method.

The I_S-MpMcDM methodology is employed to tackle a real-world engineering selection problem of hybrid vehicle engine based on real data and opinions of engineering design experts of automotive industry of Iran.

1. Introduction

In many practical decision-making problems, the available data are associated with some degrees of vagueness. In this regard, different kinds of uncertain sets are applied to tackle the vagueness of data in the context of MpMcDM, including fuzzy sets and interval numbers and their extensions [7], [113]. Interval numbers are the simple form of embodying uncertainty in decision-making problems, which can be defined as: (1) an extension of a real number; (2) a degenerate flat fuzzy number without membership function; (3) an α-cut of a fuzzy number. The merits of interval numbers in MpMcDM problems can be denoted as [3]: (1) interval numbers need the minimum amount of data; (2) decision-maker can easily obtain the range of available data as interval numbers; (3) interval numbers can be very practical. In many real-life problems particularly industrial cases such as material selection [3] and structural systems selection [4], the input data is based on measurements reported as interval numbers. Many MpMcDM methods were extended using the theory of interval numbers such as ELECTRE [4], TOPSIS [115], and VIKOR [6].

The MULTIMOORA technique is a widely used MpMcDM method due to its simple mathematics, low computational time and straightforwardness for decision-makers [46]. It aggregates ranks obtained from three subordinate models, namely, the reference point model, the ratio model, and the full multiplicative model. The MULTIMOORA method has been developed based on various uncertainty theories, including interval numbers [3], fuzzy sets [8], interval-valued fuzzy numbers [9], double hierarchy hesitant fuzzy linguistic term set [117] and the probabilistic linguistic term set [58]. The method is utilized in some practical applications, including mining method selection [12], performance appraisal method assessment [13], and logistics service provider selection [6]. Hafezalkotob et al. [16] reviewed and discussed the important studies on the MULTIMOORA.

Determining the weights of criteria is a critical step in MpMcDM problems [119]. For subjective weighting methods based on the judgments of decision-makers, the best-worst method (BWM) [15] which compares each criterion with the best and worst criteria is more effective than analytic hierarchy process (AHP) which makes pairwise comparisons. In the BWM, the weights of criteria are produced by solving a max-min problem. The method has been developed using uncertainty theories, including intuitionistic fuzzy sets [120], intuitionistic fuzzy multiplicative numbers [121], and Z-numbers [122]. For objective weighting methods based on the decision matrix, Shannon entropy is a powerful tool to calculate weights according to the contrast of information [3]. Shannon entropy measures diversity of data in a decision matrix. There are some extensions of the entropy method based on interval and fuzzy numbers [3], [26]. The entropy-based weighting model has been integrated with the MULTIMOORA method in a few studies [26], [118].

The I_S-MpMcDM methodology is motivated by the interval target-based MULTIMOORA [47], the fuzzy BWM based on the combination of group and individual decisions [2] and the enhanced Borda rule [123]. To tackle some open problems detected in the literature analysis and compared with the three aforementioned studies, the main contributions of this work are discussed in the following five items:

We suggest an interval MULTIMOORA with entire interval computation. That is, no degradation of interval data is performed in the proposed interval MULTIMOORA model. For this purpose, interval distances between interval numbers and preference matrix are used. Hafezalkotob and Hafezalkotob [47] also proposed the interval distances between interval numbers and preference matrix; however, they only introduced a special form of interval MULTIMOORA with target-based criteria. In this chapter, we enhance the original MULTIMOORA [7], which is based on beneficial and non-beneficial criteria, by considering the multiple theories of interval numbers.

- We present a multi-person interval BWM. In Ref. [2], fuzzy triangular preference degree was considered in the multi-person fuzzy BWM model. By contrast, we use interval preference degree with a different multi-person decision-making structure.
- We develop the interval Borda rule to integrate ranks. We propose an interval form of the enhanced Borda rule [123] to integrate the assessment values and ranks of subordinate parts of interval MULTIMOORA to a final assessment values.
- We use interval entropy method to determine the objective weights of criteria. The interval objective weights are then integrated with the subjective weights obtained from the multi-person interval BWM.
- We apply the proposed model in a practical case of selecting hybrid vehicle engines. In many engineering problems like this case study, the ratings of alternatives on their criteria are intrinsically given as interval numbers. Thus, the developed interval decision-making method can be utilized in such significant real-world engineering problems.

The remainder of this chapter is arranged as follows. The derivation of the proposed methodology including the interval BWM, interval entropy, interval MULTIMOORA, and its subordinate parts are presented in Section 2. We present a real-world industrial decision-making problem in Section 3.

2. Theory of I_S-MpMcDM methodology

The decision matrix with interval ratings $\bar{x}_{ij} = \left[x_{ij}^L, x_{ij}^U\right]$ of alternatives A_i, $i = 1, \ldots, m$, on criteria C_j, $j = 1, \ldots, n$, can be represented as follows [124]:

$$\bar{\mathbf{X}} = \begin{array}{c} \begin{matrix} C_1 & \cdots & C_j & \cdots & C_n \end{matrix} \\ \begin{bmatrix} \bar{x}_{11} & \cdots & \bar{x}_{1j} & \cdots & \bar{x}_{1n} \\ \vdots & & \vdots & & \vdots \\ \bar{x}_{i1} & \cdots & \bar{x}_{ij} & \cdots & \bar{x}_{in} \\ \vdots & & \vdots & & \vdots \\ \bar{x}_{m1} & \cdots & \bar{x}_{mj} & \cdots & \bar{x}_{mn} \end{bmatrix} \begin{matrix} A_1 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{matrix} \end{array} \tag{III.1}$$

The flowchart of the proposed methodology is illustrated in Fig. III.1:

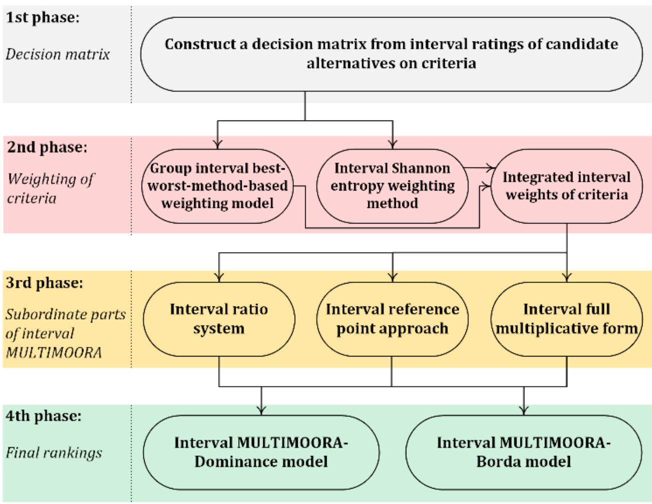

Fig. III.1. Flowchart of the I_S-MpMcDM methodology.

2.1. BWM-based model for weighting process

- ***Subjective weights: Multi-person interval best-worst-method-based weighting model***

Hafezalkotob and Hafezalkotob [2] suggested a group fuzzy BWM. They proposed that director evaluates both the expertise degrees of the panel members and the relative importance of criteria. We propose a group interval BWM. The decision-making structure of the proposed group interval BWM is different from the study of Hafezalkotob and Hafezalkotob [2]. The steps of group interval BWM are as follows:

Step 1. Consider a "director" and expert panel $\{\text{Expert}_1, \text{Expert}_1, \ldots, \text{Expert}_k, \ldots, \text{Expert}_t\}$. In the multi-person interval BWM, the director evaluates the relative importance of the members of the panel in interval numbers, and the expert panel determines the relative importance of the criteria $\{C_1, C_2, ..., C_j, ..., C_n\}$ in interval numbers as well (see Fig. III.2).

Step 2. Determine the best and worst experts by the director.

Step 3. Assess expertise degrees of the panel by the director:

Step 3.1. Specify the interval preference degree $\overline{p}_{Bk} = \left[p_{Bk}^L, p_{Bk}^U\right]$ of the best expert B who has the highest expertise and knowledge over each expert k in the panel. The interval best-to-others vector of the expertise degrees of the panel is: $\overline{P}_B = \left(\overline{p}_{B1}, \overline{p}_{B2}, \ldots, \overline{p}_{Bt}\right)$.

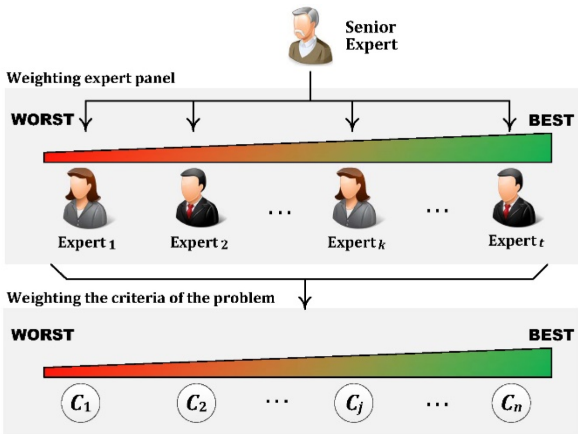

Fig. III.2. Supervised multi-person structure in the I_S-MpMcDM methodology.

Step 3.2. Specify the interval preference degree $\overline{p}_{kW} = \left[p_{kW}^L, p_{kW}^U\right]$ of expert k over the worst expert W who has the lowest expertise and knowledge in the panel. The interval others-to-worst vector of the expertise degrees of the panel is: $\overline{P}_W = \left(\overline{p}_{1W}, \overline{p}_{2W}, \ldots, \overline{p}_{tW}\right)^T$.

Step 3.3. Calculate the optimal weights vector $\left(\lambda_1^*, \lambda_2^*, \ldots, \lambda_t^*\right)$ for the expert panel using the following min-max model:

$$\begin{aligned} &\min \max_k \left\{\left|\lambda_B - \overline{p}_{Bk}\lambda_k\right|, \left|\lambda_k - \overline{p}_{kW}\lambda_W\right|\right\} \\ &\text{s.t.,} \quad \sum_{k=1}^{t} \lambda_k = 1, \quad \lambda_k \geq 0, \quad \text{for all } k \end{aligned} \tag{III.2}$$

Model (III.2) can be converted into a linear programming form as follows:

$$\begin{aligned} &\min \varepsilon, \\ &\text{s.t.}, \left|\lambda_B - \overline{p}_{Bk}\lambda_k\right| \lesssim \varepsilon, \quad \text{for all } k \\ &\quad \left|\lambda_k - \overline{p}_{Wk}\lambda_W\right| \lesssim \varepsilon, \quad \text{for all } k \\ &\quad \sum_{k=1}^{t}\lambda_k = 1, \quad \lambda_k \geq 0, \quad \text{for all } k \end{aligned} \tag{III.3}$$

In Model (III.3), symbol $\lesssim$ denotes 'almost lesser than' constraint and leads to some degree of "smaller" condition considering the interval numbers $\overline{p}_{Bk}$ and $\overline{p}_{Wk}$. Model (III.3) is equivalent to the following form based on the definition of absolute values of $\left|\lambda_B - \overline{p}_{Bk}\lambda_k\right|$ and $\left|\lambda_k - \overline{p}_{Wk}\lambda_W\right|$:

$$\begin{aligned} &\min \varepsilon \\ &\text{s.t.}, \lambda_B - \varepsilon \lesssim \overline{p}_{Bk}\lambda_k, \quad \text{for all } k \\ &\quad \lambda_B + \varepsilon \gtrsim \overline{p}_{Bk}\lambda_k, \quad \text{for all } k \\ &\quad \lambda_k - \varepsilon \lesssim \overline{p}_{Wk}\lambda_W, \quad \text{for all } k \\ &\quad \lambda_k + \varepsilon \gtrsim \overline{p}_{Wk}\lambda_W, \quad \text{for all } k \\ &\quad \sum_{k=1}^{t}\lambda_k = 1, \lambda_k \geq 0, \text{ for all } k \end{aligned} \tag{III.4}$$

According to the crisp equivalents of the interval constraints in Model (III.4), the model can be transformed into the following form:

$$\begin{aligned} &\min \varepsilon \\ &\text{s.t.}, \ \lambda_B - \varepsilon \leq \left[p_{Bk}^{L} + \left(p_{Bk}^{U} - p_{Bk}^{L}\right)\alpha\right]\lambda_k, \quad \text{for all } k \\ &\quad \lambda_B + \varepsilon \geq \left[p_{Bk}^{U} - \left(p_{Bk}^{U} - p_{Bk}^{L}\right)\alpha\right]\lambda_k, \quad \text{for all } k \\ &\quad \lambda_k - \varepsilon \leq \left[p_{Wk}^{L} + \left(p_{Wk}^{U} - p_{Wk}^{L}\right)\alpha\right]\lambda_W, \quad \text{for all } k \\ &\quad \lambda_k + \varepsilon \geq \left[p_{Wk}^{U} - \left(p_{Wk}^{U} - p_{Wk}^{L}\right)\alpha\right]\lambda_W, \quad \text{for all } k \\ &\quad \sum_{k=1}^{t}\lambda_k = 1, \quad \lambda_k \geq 0, \quad \text{for all } k \end{aligned} \tag{III.5}$$

where α $(0 \leq \alpha \leq 1)$ denotes a possibility level which can be defined by the experts. Model (III.5) is linear; thus, solving the problem for any given possibility level α leads to a unique optimal weight vector $\left(\lambda_1^*, \lambda_2^*, \ldots, \lambda_t^*\right)$ and a corresponding objective value ε^*. ε^* denotes the consistency level of the interval preference relation. If $\varepsilon^* = 0$ for all α, we have $\lambda_B/\lambda_j = \overline{p}_{Bk}$ and $\lambda_j/\lambda_W = \overline{p}_{Bk}$, and correspondingly $\overline{p}_{Bk} \times \overline{p}_{kW} = \overline{p}_{BW}$ for all k. Thus, the interval preference relation $\overline{P}_B$ and $\overline{P}_W$ are fully consistent.

Step 4. Multi-person decision-making process: Each expert specifies the best and worst criteria from his/her attitude, and the interval preference degree $\overline{q}_{Bj}^{[k]} = \left[q_{Bj}^{[k],L}, q_{Bj}^{[k],U}\right]$ of the best attribute B over each attribute j. The interval best-to-others vector of criteria determined by expert k is $\overline{Q}_B^{[k]} = (\overline{q}_{B1}^{[k]}, \overline{q}_{B2}^{[k]}, \ldots, \overline{q}_{Bn}^{[k]})$. In analogous, each expert specifies the interval preference degree $\bar{q}_{jW}^{[k]} = \left[q_{jW}^{[k],L}, q_{jW}^{[k],U} \right]$ of each attribute j over the worst attribute W and establishes the interval others-to-worst vector of criteria as $\bar{Q}_W^{[k]} = \left(\bar{q}_{1W}^{[k]}, \bar{q}_{2W}^{[k]}, \ldots, \bar{q}_{nW}^{[k]} \right)^T$.

Step 5. Obtain the optimal weight vector of criteria $\left(w_1^*, w_2^*, \ldots, w_n^* \right)$ by the following model:

$$\begin{aligned} &\min \sum_{k=1}^{t} \lambda_k^* \max_j \left\{ \left| w_B - \bar{q}_{Bj}^{[k]} w_j \right|, \left| w_j - \bar{q}_{jW}^{[k]} w_W \right| \right\} \\ &\text{s.t.,} \quad \sum_{j=1}^{n} w_j = 1, \quad w_j \geq 0, \quad \text{for all } j \end{aligned} \tag{III.6}$$

where λ_k^* is the optimal weight of expert k obtained from Step 3.3. Model (III.6) can be converted to the following form:

$$\begin{aligned} &\min \Psi = \sum_{k=1}^{t} \lambda_k^* \varepsilon_k \\ &\text{s.t.,} \left. \begin{aligned} &\left| w_B - \bar{q}_{Bj}^{[k]} w_j \right| \leq \varepsilon_k, \text{ for all } j \\ &\left| w_j - \bar{q}_{jW}^{[k]} w_W \right| \leq \varepsilon_k, \text{ for all } j \end{aligned} \right\} \text{ for all } k \\ &\quad \sum_{j=1}^{n} w_j = 1, \ w_j \geq 0, \ \text{for all } j \end{aligned} \tag{III.7}$$

Similar to Model (III.5), a crisp equivalent can be formulated for Model (III.7); however, we ignore the detail explanation here for briefness. The crisp equivalent of Model (III.7) has a linear form; thus, its solution leads to a unique optimal weight vector $\left(w_1^*, w_2^*, \ldots, w_n^* \right)$ and the consistency levels $\left(\varepsilon_1^*, \varepsilon_1^*, \ldots, \varepsilon_t^* \right)$ for any given possibility level α. $\Psi = 0$ means the full consistency of interval preference relations of the experts panel, i.e., $\bar{Q}_B^{[k]}$ and $\bar{Q}_W^{[k]}$. However, the interval preference relation may not be so precise to reach the full consistency in practice. In $\bar{Q}_B^{[k]}$ and $\bar{Q}_W^{[k]}$, small deviations from the full consistency situation lead to small positive value of Ψ. Thus, the magnitude of Ψ is a measure of the consistency level of group decisions.

- ***Objective interval weights: Interval Shannon entropy weighting method***

The following steps can be considered to calculate objective interval weights based on Shannon entropy [125] from the interval ratings.

Step 1. Normalize the interval rating $\bar{x}_{ij}$ to obtain the interval project outcome $\bar{p}_{ij} = \left[p_{ij}^L, p_{ij}^U \right]$ based on linear normalization ratio:

$$p_{ij}^L = \frac{x_{ij}^L}{\sum_{i=1}^{m} x_{ij}^U}, \quad p_{ij}^U = \frac{x_{ij}^U}{\sum_{i=1}^{m} x_{ij}^U} \tag{III.8}$$

Step 2. Obtain interval Shannon entropy measure $\bar{E}_j = \left[E_j^L, E_j^U \right]$ based on the interval project outcomes:

$$E_j^L = \min\left\{-E_0\sum_{i=1}^{m}\left(p_{ij}^L \ \ln p_{ij}^L\right),\ -E_0\sum_{i=1}^{m}\left(p_{ij}^U \ \ln p_{ij}^U\right)\right\}$$
$$E_j^U = \max\left\{-E_0\sum_{i=1}^{m}\left(p_{ij}^L \ \ln p_{ij}^L\right),\ -E_0\sum_{i=1}^{m}\left(p_{ij}^U \ \ln p_{ij}^U\right)\right\} \qquad \text{(III.9)}$$

where $E_0 = 1/\ln(m)$.

Step 3. Compute the interval diversification $\overline{\delta}_j = \left[\delta_j^L,\ \delta_j^U\right]$, where

$$\delta_j^L = 1 - E_j^U,\quad \delta_j^U = 1 - E_j^L \qquad \text{(III.10)}$$

Step 4. Specify the objective interval weight $\overline{w}_j^o = \left[w_j^{o,L},\ w_j^{o,U}\right]$, where

$$w_j^{o,L} = \frac{\delta_j^L}{\sum_{j=1}^{n}\delta_j^U},\qquad w_j^{o,U} = \frac{\delta_j^U}{\sum_{j=1}^{n}\delta_j^U} \qquad \text{(III.11)}$$

The higher Shannon entropy measure $\overline{E}_j$ of an attribute is, the smaller variation degree of interval ratings on attribute j will be, and the existent information on the attribute is less. Then its resultant objective interval weight $\overline{w}_j^o$ will be lower [3].

- ***Integrating interval weights of criteria***

The subjective weight w_j^* determined from Model (III.7) and the objective interval weight $\overline{w}_j^o$ determined from Eq. (III.11) can be combined to produce the integrated interval weight $\overline{s}_j = \left[s_j^L,\ s_j^U\right]$, where

$$s_j^L = \frac{w_j^* w_j^{o,L}}{\sum_{j=1}^{n} w_j^* w_j^{o,U}},\qquad s_j^U = \frac{w_j^* w_j^{o,U}}{\sum_{j=1}^{n} w_j^* w_j^{o,U}} \qquad \text{(III.12)}$$

2.2. MULTIMOORA-based model for ranking process

Interval ratings of alternative $\overline{x}_{ij}$ are converted into normalized interval value $\overline{x}_{ij}^* = \left[x_{ij}^{*,L}, x_{ij}^{*,U}\right]$; thus, $x_{ij}^{*,L}$ and $x_{ij}^{*,U}$ are within the range [0, 1]. Stanujkic *et al.* [126] introduced a robust normalization formula for $\overline{x}_{ij}^* = \left[x_{ij}^{*,L}, x_{ij}^{*,U}\right]$ as:

$$x_{ij}^{*,L} = x_{ij}^L \Big/ \sqrt{\frac{1}{2}\sum_{i=1}^{m}\left[\left(x_{ij}^L\right)^2 + \left(x_{ij}^U\right)^2\right]}$$
$$x_{ij}^{*,U} = x_{ij}^U \Big/ \sqrt{\frac{1}{2}\sum_{i=1}^{m}\left[\left(x_{ij}^L\right)^2 + \left(x_{ij}^U\right)^2\right]} \qquad \text{(III.13)}$$

- ***The interval ratio system***

Based on arithmetic operations of interval numbers given in Eq. (I.2), and by considering the integrated interval weight $\overline{s}_j$ determined by Eq. (III.13), the interval assessment value $\overline{y}_i$ is calculated as:

$$\overline{y}_i = \sum_{j=1}^{g}\overline{s}_j\overline{x}_{ij}^* - \sum_{j=g+1}^{n}\overline{s}_j\overline{x}_{ij}^* \qquad \text{(III.14)}$$

The best alternative $A^{*}_{\text{I-RS}}$ has the maximum value of $\bar{y}_i$ which can be found using Eq. (I.8):

$$A^{*}_{\text{I-RS}} = \left\{ A_{i \mid \max_i \bar{y}_i} \right\} \tag{III.15}$$

- ***The interval reference point approach***

Employing Eqs. (I.8) and (I.9), the interval reference point is:

$$\bar{r}_j = \left\{ \max_i \bar{x}^{*}_{ij},\ \ j \le g; \qquad \min_i \bar{x}^{*}_{ij},\ \ j > g \right\} \tag{III.16}$$

The deviation between the weighted normalized interval rating $s_j \bar{x}^{*}_{ij}$ and the interval weighted reference point $w_j \bar{r}_j$ is:

$$\bar{d}^{*}_{ij} = \bar{d}^{*}\left(\bar{s}_j \bar{r}_j, \bar{s}_j \bar{x}^{*}_{ij} \right) \tag{III.17}$$

where $\bar{d}^{*}$ is an interval distance computed by Eq. (I.4). The interval assessment value of the interval reference point $\bar{z}_i$ is determined as follows:

$$\bar{z}_i = \max_j \bar{d}^{*}_{ij} \tag{III.18}$$

Based on Eq. (I.9), the optimal alternative $A^{*}_{\text{I-RP}}$ has the minimum interval assessment value:

$$A^{*}_{\text{I-RP}} = \left\{ A_{i \mid \min_i \bar{z}_i} \right\} \tag{III.19}$$

- ***The interval full multiplicative form***

Based on the arithmetic operations of interval numbers shown as Eq. (I.2) as well as the integrated interval weights $\bar{s}_j$, the interval assessment value $\bar{u}_i$ of the interval full multiplicative form is calculated as follows:

$$\bar{u}_i = \frac{\prod_{j=1}^{g} \left(\bar{x}^{*} \right)^{\bar{s}_j}}{\prod_{j=g+1}^{n} \left(\bar{x}^{*} \right)^{\bar{s}_j}} \tag{III.20}$$

Because all the elements $\bar{x}^{*}$ and $\bar{s}_j$ are between 0 and 1, it is clear that:

$$\left(\bar{x}^{*} \right)^{\bar{s}_j} = \left[\left(x^{L}_{ij} \right)^{s^{U}_{j}} ,\ \left(x^{U}_{ij} \right)^{s^{L}_{j}} \right] \tag{III.21}$$

The best alternative $A^{*}_{\text{I-MF}}$ has the maximum value of $\bar{u}_i$, which can be found using Eq. (I.8):

$$A^{*}_{\text{I-MF}} = \left\{ A_{i \mid \max_i \bar{u}_i} \right\} \tag{III.22}$$

- ***The rankings of the interval MULTIMOORA***

Benshan [123] presented the enhanced Borda rule by consolidating the rankings and assessment values into aggregating assessment values. In this study, we propose the interval Borda rule to integrate the interval assessment values and rankings derived from the three models of interval MULTIMOORA. The interval Borda rule is superior to the dominance theory from the perspectives of mathematics and straightforwardness for decision-makers. Mathematically speaking, the dominance theory only employs the ordinal rankings for aggregation; however, the interval Borda rule considers both cardinal interval assessment values and the ordinal rankings. From the aspect of application, the dominance theory is complicated because of pairwise comparisons and probable occurrence of circular reasoning in particular. The case would be confusing for decision-makers when the number of alternatives and criteria are large. In contrast, the interval Borda rule does not require any manual comparison and has special conditions.

To use the interval Borda rule, the first step is to normalize the subordinate interval assessment values, i.e., Eqs. (III.14), (III.18), and (III.20), to obtain $\overline{y}_i^* = \left[y_i^{*,L}, y_i^{*,U} \right]$, $\overline{z}_i^* = \left[z_i^{*,L}, z_i^{*,U} \right]$, and $\overline{u}_i^* = \left[u_i^{*,L}, u_i^{*,U} \right]$. The method to compute $\overline{y}_i^*$, $\overline{z}_i^*$, and $\overline{u}_i^*$ is identical to Eq. (III.13).

We define the interval Borda rule to obtain the interval assessment values of the interval MULTIMOORA as follows:

$$\overline{IMB}_i = \overline{y}_i^* \frac{m - r(\overline{y}_i) + 1}{m\,(m+1)/2} - \overline{z}_i^* \frac{r(\overline{z}_i)}{m\,(m+1)/2} + \overline{u}_i^* \frac{m - r(\overline{u}_i) + 1}{m\,(m+1)/2} \tag{III.23}$$

The best alternative based on the interval MULTIMOORA- Borda has the maximum value of $\overline{IMB_i}$ that can be found by Eq. (I.8):

$$A^*_{\text{I-MULTIMOORA-Borda}} = \left\{ A_{i \,|\, \max_i \overline{IMB_i}} \right\} \tag{III.24}$$

3. Application of I_S-MpMcDM methodology in industrial sector

This section discusses a practical problem in industrial sector. The case study concerns the selection of appropriate hybrid vehicle engine selection tackled using the proposed I_S-MpMcDM methodology as shown in Fig. III.3. Fig. III.4 illustrates the supervised multi-person structure for the case study.

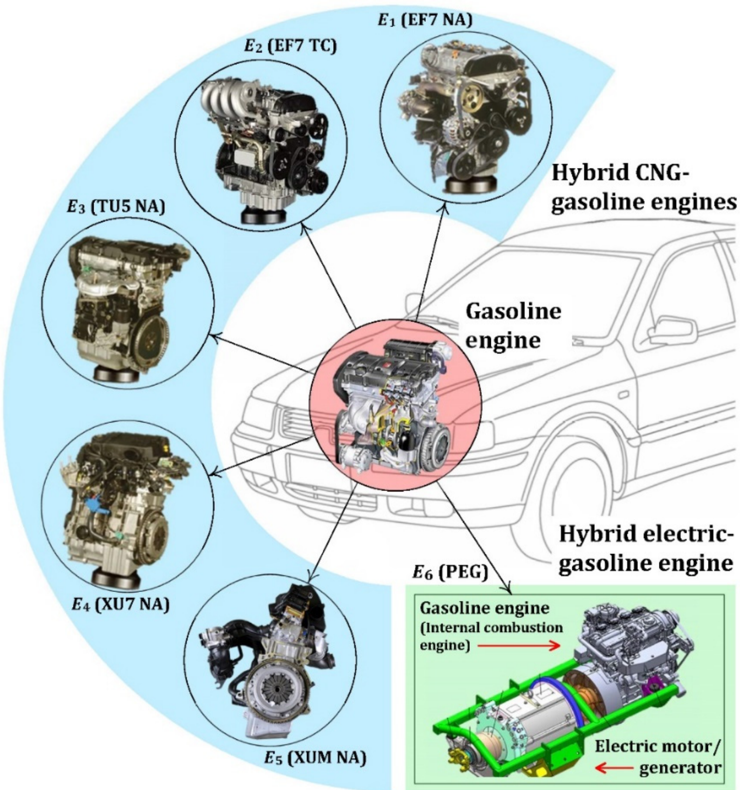

Fig. III.3. The case study of the I_S-MpMcDM methodology: Hybrid vehicle engine selection.

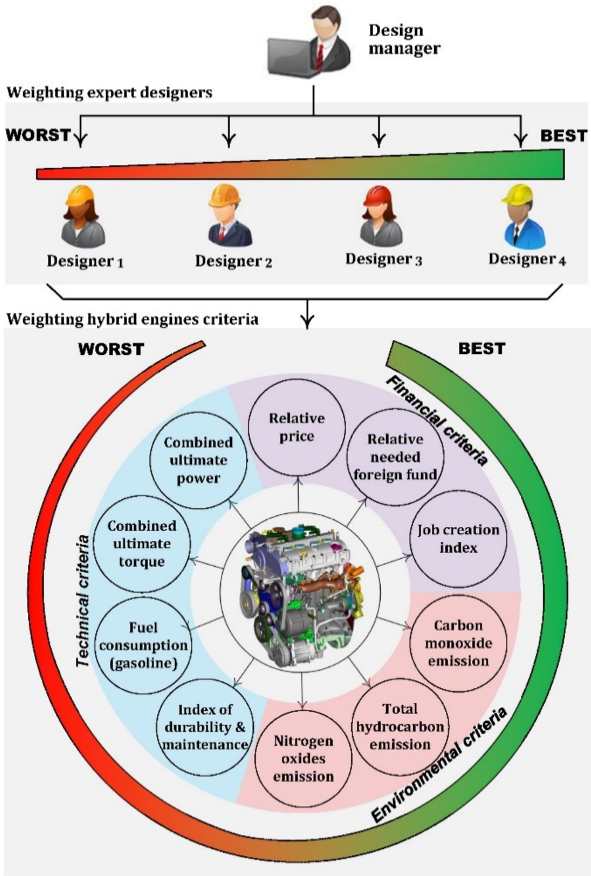

Fig. III.4. Supervised multi-person structure for the case study of the I_S-MpMcDM methodology.

Vehicles are producing a major part of carbon emissions in the world, which has resulted in climate change. The global warming issue and poor air quality are the results of traditional internal combustion engines (ICEs) which are based on fossil fuels, such as gasoline and diesel fuels [127]. In recent decades, however, the efforts for changing the fuel blend and technology of vehicle engines have been markedly increased [128]. Besides, the pressure from social communities have stimulated the development of cleaner and more efficient vehicles and the production of hybrid vehicles. Hybrid vehicles have much lower deleterious effects on environment.

There are different kinds of technologies for hybrid vehicle engines such as compact natural gas (CNG)-gasoline, plug-in electric-gasoline, regenerative electric-gasoline, fuel cell-gasoline, and hydrogen-gasoline. Penetration rates of the category of hybrid electric cars are still low in big cities, mostly due to the high cost of batteries and the low number of existing charging stations [129]. Apart from the USA, Europe, Japan, and India also help in commercializing of hybrid electric cars [130]. In recent years, many leading automotive companies, such as Toyota, Ford, BMW, Honda, Chevrolet, Porsche, and McLaren, have been investing in hybrid vehicles. In addition to the firms, “Tesla” is a major company that both produces high-tech hybrid electric vehicles and batteries as well as solar panels. In late 2017, Tesla released its state-of-the-art electric truck named “Tesla Semi”. The company claimed that “Semi has maximum power and acceleration and requires the lowest energy cost per mile (See tesla.com/semi)”. The mentioned issues clarify that both heavy and light hybrid vehicles will get much more attention worldwide and by the improvements in the technology of batteries as well as optimizing the engine technology and electrical and electronic systems besides investing in charging stations, hybrid vehicles would definitely get a substantial market share of automotive industry in near future. Moreover, government policies including consumer incentives, taxation, investment in research and development, as well as incentives for automotive companies and consumers play a key role in the progress of growing industry of hybrid vehicles [131].

This case study is conducted on the basis of the collected information and comments of engineering design experts of Iran automotive industry (as shown Fig. III.4). Major automotive firms in Iran are researching into the possibility of improving the engine technology from traditional to hybrid engines. Six hybrid engines are taken into account as candidate alternatives in this case study, which include five items based on the type of CNG-gasoline (EF7 naturally aspirated, EF7 Turbo charged, TU5 naturally aspirated, XU7 naturally aspirated, and XUM naturally aspirated) besides one engine with the technology of electric-gasoline which is planned to be produced and assembled in near future. The prospective electric-gasoline (PEG) engine is a serial coupling of two engines, i.e., an internal combustion engine (ICE) based on gasoline fuel and an electric engine (motor/generator). Both ICE and electric engine (as motor) can drive the vehicle separately or simultaneously as coupled by clutch. Electric engine can also act as a generator and charges the batteries. Table III.**1** shows the criteria of the hybrid engines and their units as well as the objective related to each criterion. The criteria of the hybrid engines are classified into three categories: technical, financial, and environmental criteria. Each category has items that may be beneficial or non-beneficial criteria.

Table III.1. Criteria of candidate hybrid vehicle engines.

Type	Criterion name	Abb.*	Unit	Objective
Technical criteria	Combined ultimate power	CUP	hp	Maximum
	Combined ultimate torque	CUT	N.m	Maximum
	Fuel consumption (gasoline)	FCG	L/(100 km)	Minimum
	Index of durability and maintenance	IDM	–	Maximum
Financial criteria	Relative price	RPC	-	Minimum
	Relative needed foreign fund	RNF	-	Minimum
	Job creation index	JCI	-	Maximum
Environmental criteria	Carbon monoxide emission	CME	g/km	Minimum
	Total hydrocarbon emission	THE	g/km	Minimum
	Nitrogen oxides emission	NOE	g/km	Minimum

*Abb.: Abbreviation

The linguistic terms are obtained for IDM, RPC, RNF, and JCI based on the comments of design experts. The corresponding interval numbers for linguistic terms related to the four linguistic criteria are defined as shown in Table III.2. The decision matrix consists of the interval ratings of the hybrid engines on the ten criteria (i.e., the performances of the engines on each specifications) are given in Table III.3. The data of Table III.3 is collected based on the references of related engines.

Table III.2. Corresponding interval numbers for linguistic terms.

Linguistic terms	Corresponding interval number
Very high (VH)	[7, 9]
High (H)	[6, 8]
Medium high (MH)	[5, 7]
Medium (M)	[4, 6]
Medium low (ML)	[3, 5]
Low (L)	[2, 4]
Very low (VL)	[1, 3]

Table III.3. Decision matrix.

Eng. ID[1]	Engine name	Technical criteria				Financial criteria			Environmental criteria		
		CUP	CUT	FCG	IDM	RPC	RNF	JCI	CME	THE	NOE
E_1	EF7 TC[2]	[122, 150]	[171, 209]	[6.5, 7.9]	[6, 8]	[6, 8]	[6, 8]	[2, 4]	[0.51, 0.55]	[0.06, 0.07]	[0.08, 0.09]
E_2	EF7 NA[3]	[96, 117]	[131, 160]	[6.8, 8.3]	[6, 8]	[6, 8]	[6, 8]	[3, 5]	[0.50, 0.53]	[0.07, 0.08]	[0.07, 0.08]
E_3	TU5 NA	[84, 102]	[109, 134]	[6.7, 8.1]	[5, 7]	[5, 7]	[4, 6]	[4, 6]	[0.54, 0.56]	[0.06, 0.07]	[0.10, 0.11]
E_4	XU7 NA	[79, 96]	[116, 142]	[8.3, 10.1]	[4, 6]	[4, 6]	[5, 7]	[6, 8]	[0.60, 0.63]	[0.08, 0.09]	[0.10, 0.11]
E_5	XUM NA	[84, 102]	[117, 143]	[7.9, 9.7]	[3, 5]	[5, 7]	[3, 5]	[6, 8]	[0.61, 0.64]	[0.08, 0.09]	[0.09, 0.10]
E_6	PEG[4]	[110, 125]	[114, 174]	[4.7, 5.8]	[7, 9]	[7, 9]	[7, 9]	[4, 6]	[0.45, 0.48]	[0.05, 0.06]	[0.07, 0.08]

[1] Eng. ID: Engine ID; [2]TC: Turbo-charged; [3]NA: Naturally-aspirated; [4]PEG: Prospective electric-gasoline.

Table III.4 represents the linguistic preferences and the corresponding interval preferences. The design manager evaluates the expertise degrees of the designers by linguistic preferences. The corresponding interval preferences of the linguistic terms considered by the design manager are shown in Table III.5. Indeed, Table III.5 lists the interval best-to-others and others-to-worst vectors of the designers' expertise degrees.

Table III.4. Linguistic preferences versus the corresponding intervals.

Linguistic preferences	Corresponding interval preferences
Equally importance	[1, 1]
Weakly importance	[1, 3]
Fairly importance	[3, 5]
Very importance	[5, 7]
Absolutely importance	[7, 9]

Table III.5. Expertise degrees of designers.

DM	Best & worst designers	Interval preferences obtained from linguistic terms				
			Designer $_1$	Designer $_2$	Designer $_3$	Designer $_4$
Design manager	*Best*: Designer $_2$	$\overline{P}_B$	[3, 5]	[1, 1]	[1, 3]	[7, 9]
	Worst: Designer $_4$	$\overline{P}_W$	[1, 3]	[7, 9]	[1, 3]	[1, 1]

Based on group interval BWM, each designer rates the preferences of the engines' criteria regarding to the best and worst criteria. The corresponding interval preferences of the linguistic terms considered by the designers are shown in Table III.6. Table III.6 presents the interval best-to-others and others-to-worst vectors of preference degrees of the engines criteria.

Table III.6. Preference degrees of the engines criteria.

DMs	Best criterion	Worst criterion	Interval preferences obtained from linguistic terms										
				CUP	CUT	FCG	IDM	RPC	RNF	JCI	CME	THE	NOE
Designer $_1$	JCI	THE	$\overline{Q}_B^{[1]}$	[3, 5]	[5, 7]	[1, 3]	[1, 3]	[5, 7]	[5, 7]	[1, 1]	[5, 7]	[7, 9]	[1, 3]
			$\overline{Q}_W^{[1]}$	[1, 3]	[5, 7]	[1, 3]	[3, 5]	[5, 7]	[1, 3]	[7, 9]	[1, 3]	[1, 1]	[5, 7]
Designer $_2$	RNF	IDM	$\overline{Q}_B^{[2]}$	[5, 7]	[3, 5]	[5, 7]	[7, 9]	[1, 3]	[1, 1]	[1, 3]	[5, 7]	[5, 7]	[1, 3]
			$\overline{Q}_W^{[2]}$	[3, 5]	[5, 7]	[1, 3]	[1, 1]	[5, 7]	[7, 9]	[5, 7]	[1, 3]	[1, 3]	[5, 7]
Designer $_3$	JCI	IDM	$\overline{Q}_B^{[3]}$	[1, 3]	[5, 7]	[3, 5]	[7, 9]	[5, 7]	[1, 3]	[1, 1]	[5, 7]	[3, 5]	[1, 3]
			$\overline{Q}_W^{[3]}$	[3, 5]	[5, 7]	[1, 3]	[1, 1]	[3, 5]	[3, 5]	[7, 9]	[1, 3]	[5, 7]	[5, 7]
Designer $_4$	RPC	CUT	$\overline{Q}_B^{[4]}$	[3, 5]	[7, 9]	[3, 5]	[3, 5]	[1, 1]	[5, 7]	[1, 3]	[3, 5]	[5, 7]	[1, 3]
			$\overline{Q}_W^{[4]}$	[3, 5]	[1, 1]	[3, 5]	[1, 3]	[7, 9]	[5, 7]	[5, 7]	[3, 5]	[1, 3]	[5, 7]

For the subordinate parts and the interval MULTIMOORA-Borda, the best alternative is obtained based on Eqs. (III.15), (III.22), (III.24), and (III.19), shown as follows: $A^*_{\text{I-RS}} = A^*_{\text{I-MF}} = A^*_{\text{I-MULTIMOORA-Borda}} = E_5$ and $A^*_{\text{I-RP}} = E_4$.

The normalized interval ratings of the engines for this problem are calculated using Eq. (III.13). Based on Eq. (III.17), the deviations of the weighted normalized interval ratings are generated. Table III.**7** lists the assessment values for the interval ratio system, the interval reference point model, the interval full multiplicative model, and the interval MULTIMOORA-Borda model computed based on Eqs. (III.14), (III.18), (III.20), and (III.23), respectively, as well as the rankings for the interval ratio system, the interval reference point model, the interval full multiplicative model, and interval MULTIMOORA-Borda.

Table III.7. Assessment values and rankings of the interval MULTIMOORA.

Eng. ID	I-RS		I-RP		I-MF		I-MULTIMOORA-Borda[1]	
	$\overline{y}_i$	$r(\overline{y}_i)$	$\overline{z}_i$	$r(\overline{z}_i)$	$\overline{u}_i$	$r(\overline{u}_i)$	$\overline{IMB_i}$	$r\left(\overline{IMB_i}\right)$
E_1	[-0.205, 0.207]	6	[0.000, 0.047]	6	[0.485, 1.549]	6	[-0.284, 0.044]	6
E_2	[-0.205, 0.210]	5	[0.000, 0.035]	5	[0.525, 1.554]	5	[-0.191, 0.088]	5
E_3	[-0.187, 0.216]	4	[0.000, 0.023]	3	[0.549, 1.648]	2	[-0.075, 0.192]	3
E_4	[-0.200, 0.253]	2	[0.000, 0.010]	1	[0.603, 1.592]	3	[-0.061, 0.218]	2
E_5	[-0.184, 0.250]	1	[0.000, 0.013]	2	[0.592, 1.694]	1	[-0.067, 0.300]	1
E_6	[-0.203, 0.243]	3	[0.000, 0.023]	3	[0.579, 1.572]	4	[-0.114, 0.164]	4

[1]I-MULTIMOORA-Borda: Interval MULTIMOORA-Borda.

Chapter IV

RF_S-MpMcDM_FRFP methodology: Biomedical application

In this chapter, we present the RF_S-MpMcDM_FRFP methodology. The methodology is based on fuzzy-rationality-based fuzzy prospect theory and trapezoidal fuzzy numbers and modeled by the BWM-MULTIMOORA approach.

In the real-world decision-making problems, the judgements of experts are often expressed as uncertain values. Such uncertain judgements may also be associated with some degrees of risk posed by irrationality of experts. Fuzzy prospect theory is a significant approach to tackle risky uncertain problems. Accordingly, we suggest a fuzzy-rationality-based fuzzy prospect theory based on a novel fuzzy distance measure and utilize the theory to develop a risky fuzzy multi-person best-worst method. In the proposed methodology, a director manages an expert panel. The risky fuzzy preferences of the experts are utilized in a fuzzy MULTIMOORA model with target-based normalization technique to generate the final results.

We discuss a practical case on biomedical problem regarding spinal prosthesis material selection. The proposed methodology is beneficial for the problem as multiple biomedical engineers are often involved in sensitive orthopedic treatments and their judgments are subjected to risk of irrationality entailed from work pressures.

1. Introduction

In real-life, rationality of human decisions is imperfect. Thus, considering the risk of the degree of rationality is an interesting topic in the decision-making process. Risky Fuzzy Multi-person Multi-criteria Decision-Making supported on Fuzzy Prospect theory (RF_MpMcDM_FP) deals with the risk of rationality degree of decision-makers' judgments in uncertain circumstances. The RF_MpMcDM_FP approaches have grabbed attention of researchers in the recent years[4], [132]–[134]. Liu and Zhang[132] introduced a hesitant fuzzy MABAC method on the basis of prospect theory. Liu et al.[133] developed a trapezoidal intuitionistic fuzzy Choquet integral operator to obtain the overall prospect value related to each alternatives. Huang et al.[134] put forward an extension of quality function deployment using hesitant fuzzy linguistic sets integrated with prospect theory. Dai et al.[4] presented a dynamic fuzzy MULTIMOORA based on prospect theory.

As philosophers of science and methodologists criticize, human knowledge is subjected to cognitive imperfections leads to "classical and bounded rationalities" evaluated in the "classical paradigm of decision-choice". However, according to "the nature of human cognitive process" which needs flexibility, a "paradigm shift" is inevitable. The new paradigm is regarded as the "fuzzy paradigm of decision-choice". "Fuzzy rationality" is evaluated in this new paradigm[34].

In classical paradigm of decision-choice theory, rationality of decision has been supposed as crisp values[34]. In real-life, decisions are associated with uncertainty and there is doubt on rationality of these decisions. Knowledge vagueness places us under conditions of fuzzy rationality[135]. Fuzzy rationality of decisions is an important issue from the perspective of realistic problems such as Political Economy[136].

Psychologically speaking, human decision always include some degrees of "irrationality" based on Sigmund Freud's opinion. Even if a seller seemingly has a perfect solution for a buyer that fits all their needs, buyers often make irrational decisions and choose alternatives[137]. In real life, the decisions of humans are often associated with uncertainty[138]. The risk entailed from the irrationality of decisions can also have uncertain nature. In healthcare service, biomedical engineers may make irrational decisions because of their critical work and job pressure and their irrational decisions can have some degrees of uncertainty because of the uncertain nature of biomedical problems.

In the context of RF-MpMcDM-FP, irrationality of fuzzy decisions is supposed to be a crisp value (irrationality is considered through fuzzy prospect theory). However, the question is *"how to derive fuzzy irrationality of fuzzy decisions (in the context of RF-MpMcDM-FP)?"* Fuzzy irrationality fits to real-life problems and also leads to a computation without degeneration of fuzzy data. Thus, a research gap exist in the context to tackle fuzzy irrationality needed for realistic risky fuzzy problems with critical situation.

To plug the gap related to "fuzzy rationality in the RF-MpMcDM-FP context", we introduce the fuzzy-rationality-based fuzzy prospect theory in the context of RF-MpMcDM-FP. In the proposed theory, a fuzzy value function can represent the fuzzy rationality of experts' judgments. Thus, the fuzzy-distance-based prospect theory structure allows us to Risky Fuzzy Supervised Multi-person Multi-criteria Decision-Making methodology supported on Fuzzy-Rationality-based Fuzzy Prospect Theory (RF_S-MpMcDM_FRFP) applying the BWM-MULTIMOORA approach. The suggested methodology is derived by introducing the following decision-making models:

1) *Risky fuzzy BWM-based model supported on the FRFP for weighting process of experts and criteria:* We introduce a fuzzy prospect theory supported on a novel fuzzy distance measure of trapezoidal fuzzy numbers applicable for risky fuzzy decision-making models. In the proposed prospect theory, a fuzzy-distance-based value function has been defined. As we have used the fuzzy distance, our development ensures fully uncertain

computation without degeneration of fuzzy information. We employ the proposed fuzzy-distance-based prospect theory to develop a risky fuzzy multi-person best–worst method. In this model, a director determines the relative preference (i.e., competence) of the members of an expert panel. The director and the expert panel both participate in obtaining preferences of problem criteria. A self-reliance coefficient is considered to combine the preferences of the director and expert panel. All preferences are in the form of risky fuzzy values based on the suggested fuzzy-distance-based prospect theory. The risky fuzzy multi-person best–worst method results in subjective weights of problem criteria. We combine the subjective weights with the objective weights obtained supported on a fuzzy TOPSIS-inspired method to reach the integrated weights of problem criteria. The proposed fuzzy distance measure is employed in the fuzzy TOPSIS-inspired method.

2) *Fuzzy MULTIMOORA-based model for ranking process of alternatives*: To obtain the subordinate rankings of alternatives in the risky fuzzy decision-making model, we utilize the fuzzy MULTIMOORA method in which the aforementioned integrated weights of problem criteria are exploited. In the formulation of the fuzzy MULTIMOORA, we use a target-based normalization technique employing the proposed fuzzy distance measure. Eventually, to generate the final rankings of alternatives, the subordinate outcomes of the fuzzy MULTIMOORA are aggregated based on a fuzzy distance matrix. The fuzzy distance matrix outweighs the dominance theory used for rank aggregation in the traditional MULTIMOORA. In the formulation of the fuzzy distance matrix, we have also employed the introduced fuzzy distance measure.

To perform a practical analysis of the proposed methodology, we assess a real-life problem in the area of biomedical engineering application. The purpose of problem is finding appropriate biomaterial for a fixator used in treatment of spinal disorders by considering risk of irrational decisions of biomedical engineers. Besides, we implement sensitivity analyses on the consistency of subjective weights of problem criteria and the variation of the final rankings. The results of the introduced model are compared with two other decision-making methods.

The remander of this chapter is devised as follows: The proposed RF_S-MpMcDM_FRFP methodology is introduced in Section 2. We evaluate a practical problem of surgical implants in Section 3.

2. Theory of RF_S-MpMcDM_FRFP methodology

In this section, we present the RF_S-MpMcDM_FRFP methodology as illustrated in Fig. IV.1. The methodology is derived in two phases: (1) we develop the risky fuzzy weighting process of criteria comprising subjective, objective, and integrated approaches. Subjective weights of problem criteria are obtained using the proposed risky fuzzy multi-person best–worst method. The objective weights are computed using a TOPSIS-inspired method which are then combined with the subjective weights (see Section 2.1); and (2) we utilize a fuzzy ranking process of alternatives including the three subordinate parts of the fuzzy MULTMOORA model and the ranking aggregation technique based on fuzzy distance matrix (see Section 2.2).

The risky fuzzy decision-making problem based on trapezoidal fuzzy numbers has a decision matrix represented as:

$$\tilde{\mathbf{X}} = \left[\tilde{x}_{ij}\right]_{m\times n}, \tag{IV.1}$$

where $\tilde{x}_{ij} = \left(x_{ij,1},\, x_{ij,2},\, x_{ij,3},\, x_{ij,4}\right)$ is the fuzzy rating of alternative A_i, $i = 1, \ldots, m,$ on criterion C_j, $j = 1, \ldots, n.$ For each criterion, the fuzzy target $\tilde{t}_j$ is defined as:

$$\tilde{t}_j = \left\{ \max_i \tilde{x}_{ij}, \text{ if } j \in I; \quad \min_i \tilde{x}_{ij}, \text{ if } j \in J; \quad \tilde{g}_j, \text{ if } j \in K \right\}, \tag{IV.2}$$

where I and J are related to beneficial and cost criteria, respectively. The maximum and minimum of fuzzy ratings on beneficial and cost criteria can be determined utilizing the preference matrix introduced in Section I.2.1. K represents the criteria for which a given value (i.e., $\tilde{g}_j$) is preferred. The value of $\tilde{g}_j$ can be specified by decision-makers or obtained through experiments or extracted from handbooks based on the nature of practical problems.

Fuzzy decision matrix $\tilde{\mathbf{X}}$ should be normalized before exploitation in the decision-making model as the ratings of alternatives on criteria are often not comparable due to various dimensions[16]. We propose a fuzzy target-based normalization technique, as follows:

$$\tilde{f}_{ij} = e^{-\tilde{\rho}_{ij}}, \tag{IV.3}$$

where the exponent value is defined as:

$$\tilde{\rho}_{ij} = \tilde{d}\left(\tilde{x}_{ij}, \tilde{t}_j\right) \Big/ \max_i \bar{d}\left(\tilde{x}_{ij}, \tilde{t}_j\right). \tag{IV.4}$$

In Eq. (IV.4), $\tilde{x}_{ij}$ represents the fuzzy rating (i.e., each array of the fuzzy decision matrix $\tilde{\mathbf{X}}$) and $\tilde{t}_j$ is the fuzzy target for each criterion defined in Eq. (IV.2). The fuzzy distance in the numerator of Eq. (IV.4) is obtained based on Eq. (I.17) and the crisp distance in the denominator is the defuzzified value of the fuzzy distance which could be computed by Eq. (I.13).

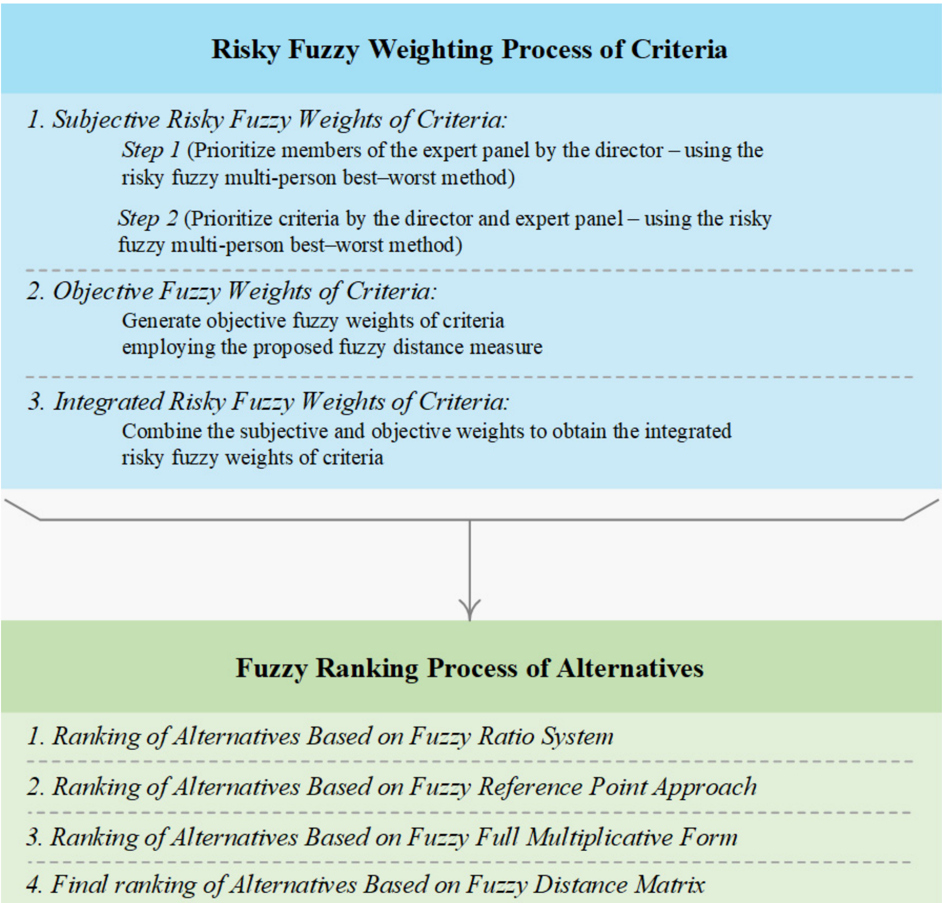

Fig. IV.1. Flowchart of the RF_S-MpMcDM_FRFP methodology.

2.1. BWM-based model for weighting process

In this study, we use an integrative approach for weighting of criteria and experts' competence. First, the subjective weights of criteria are computed based on the comments of a director and an expert panel by utilizing the risky fuzzy multi-person best–worst method. Expertise level of the members of the panel are also determined in the proposed best–worst method by the director. Second, the objective weights are calculated exploiting the fuzzy TOPSIS-inspired method. Third, the aforementioned subjective and objective weights are consolidated to be further used in the fuzzy ranking process of alternatives.

- ***Subjective risky fuzzy weights of criteria***

We propose a risky fuzzy multi-person best–worst method to obtain subjective weights of problem criteria. The risk of decisions are supported on the FRFP theory introduced in Section I.3.2. Best–worst method has exclusive privileges for generating subjective weights in a decision-making problem: (i) it requires less pairwise evaluation in comparison to AHP to obtain criteria weights; (ii) best–worst method is more consistent than AHP method; (iii) the consistency percentage of BWM also denotes the confidence level; and (iv) in contrast with AHP, best–worst method only employs integers to consider preferences.

The supervised multi-person decision-making structure has two levels of decision-making as illustrated in Fig. IV.2. To obtain the subjective weights of criteria using the model, the following procedure is considered (the judgments are made in the form of linguistic terms then converted into trapezoidal fuzzy numbers):

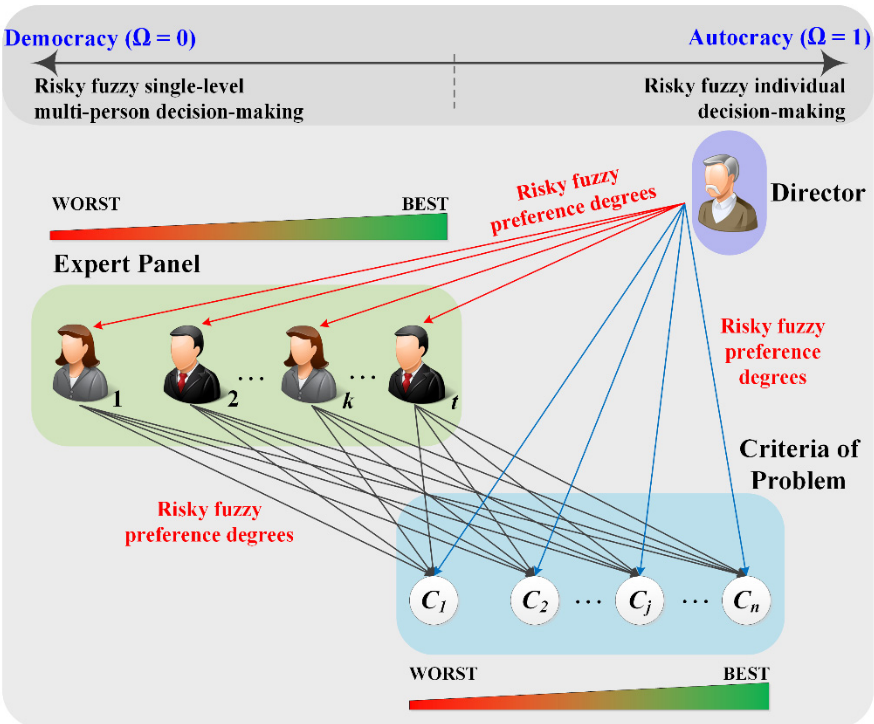

Fig. IV.2. Supervised multi-person structure in the RF_S-MpMcDM_FRFP methodology (Ω denotes self-reliance coefficient considered in Model (IV.16)).

Consider a "director" and a "expert panel" $\{\text{Expert}_1,\ \ldots,\ \text{Expert}_k,\ \ldots,\ \text{Expert}_t\}$. In the risky fuzzy multi-person best–worst method, the director assesses the priority of the expert panel, and both the director and expert panel specify the priority of the criteria $\{C_1,\ \ldots,\ C_j,\ \ldots,\ C_n\}$.

Step 1 (Prioritize members of the expert panel by the director – using the risky fuzzy multi-person best–worst method): The director ascertains the best and worst experts and evaluates the risky fuzzy preference degrees of the expert panel regarding the best and worst experts. The director determines the fuzzy preference degree $\tilde{h}_{Bk} = \left(h_{Bk,1},\ h_{Bk,2},\ h_{Bk,3},\ h_{Bk,4}\right)$ of the best expert B over expert k in the panel. $\tilde{h}_{Bk}$ is a trapezoidal fuzzy value ranging from $(1, 1, 1, 1)$ to $(9, 9, 9, 9)$. Then, based on the value function of the FRFP theory proposed in Section I.3.2, i.e., Eq. (I.23), the risky fuzzy best-to-others experts' competence is calculated as:

$$\tilde{h}^r_{Bk} = \exp\left(-\delta\left[\tilde{d}^*\left(\tilde{h}_{Bk}, \tilde{h}_{BB}\right)\right]^{\beta}\right). \tag{IV.5}$$

In Eq. (IV.5), we use the normalized value of fuzzy distance, i.e., $\tilde{d}^*$, to keep the value of $\tilde{h}^r_{Bk}$ in the same range of $\tilde{h}_{Bk}$, i.e., $(1, 1, 1, 1)$ to $(9, 9, 9, 9)$. Also, we employ the exponential function to avoid zero values. The normalized values of fuzzy distance are computed as $\tilde{d}^* = (d_1/\kappa,\ d_2/\kappa,\ d_3/\kappa,\ d_4/\kappa)$ where d_1, d_2, d_3, and d_4 are the base values of the fuzzy distance measure, and $\kappa = \sqrt{\frac{1}{4}\sum_{j=1}^{n}\left(d_1^2 + d_2^2 + d_3^2 + d_4^2\right)}$. To generate results consistent with the empirical data, we set $\theta = 2.25$ and $\beta = 0.88$ [4]. $\tilde{H}^r_B = \left(\tilde{h}^r_{B1}, \ldots, \tilde{h}^r_{Bk}, \ldots, \tilde{h}^r_{Bt}\right)$ denotes the risky fuzzy best-to-others vector of the preference degrees of the expert panel. The director determines the fuzzy preference degree $\tilde{h}_{kW} = \left(h_{kW,1},\ h_{kW,2},\ h_{kW,3},\ h_{kW,4}\right)$ of expert k over the worst expert W. Then, based on the value function of the fuzzy prospect theory introduced in Eq. (I.23), the risky fuzzy others-to-worst experts' competence is calculated as:

$$\tilde{h}^r_{kW} = \exp\left(\left[\tilde{d}^*\left(\tilde{h}_{kW}, \tilde{h}_{WW}\right)\right]^{\gamma}\right). \tag{IV.6}$$

The reason of employing the normalized values of fuzzy distance and exponential function is similar to the explanation aforementioned in Step 3.1 for obtaining $\tilde{h}^r_{Bk}$. To achieve consistent results with the empirical data, we set $\gamma = 0.88$ [4]. $\tilde{H}^r_W = \left(\tilde{h}^r_{1W}, \ldots, \tilde{h}^r_{kW}, \ldots, \tilde{h}^r_{tW}\right)^T$ indicates the risky fuzzy others-to-worst vector of the preference degrees of the members of the expert panel. The optimal weights vector $\left(\lambda_1^*, \lambda_2^*, \ldots, \lambda_t^*\right)^T$ for the expert panel is computed utilizing a min-max model as follows:

$$\begin{aligned} &\min \max_k \left\{\left|\lambda_B - \tilde{h}^r_{Bk}\lambda_k\right|, \left|\lambda_k - \tilde{h}^r_{kW}\lambda_W\right|\right\}, \\ &\text{s.t.:} \quad \sum_{k=1}^{t}\lambda_k = 1, \quad \lambda_k \geq 0, \quad \text{for all } k. \end{aligned} \tag{IV.7}$$

Model (IV.7) could be linearized as follows:

$$\begin{aligned}
&\min\ \xi,\\
&\text{s.t.:}\quad \left|\lambda_B - \tilde{h}^r_{Bk}\lambda_k\right| \lesssim \xi, \quad \text{for all } k,\\
&\qquad\quad \left|\lambda_k - \tilde{h}^r_{kW}\lambda_W\right| \lesssim \xi, \quad \text{for all } k,\\
&\qquad\quad \sum_{k=1}^{t}\lambda_k = 1, \quad \lambda_k \geq 0, \text{for all } k.
\end{aligned} \tag{IV.8}$$

In Model (IV.8), the symbol $\lesssim$ shows '*almost lesser than*' constraint. An equivalent form of Model (IV.8) could be formulated taking account of the absolute values definitions of $\left|\lambda_B - \tilde{h}^r_{Bk}\lambda_k\right|$ and $\left|\lambda_k - \tilde{h}^r_{kW}\lambda_W\right|$, as follows:

$$\begin{aligned}
&\min\ \xi,\\
&\text{s.t.:}\quad \lambda_B - \xi \lesssim \tilde{h}^r_{Bk}\lambda_k, \quad \text{for all } k,\\
&\qquad\quad \lambda_B + \xi \gtrsim \tilde{h}^r_{Bk}\lambda_k, \quad \text{for all } k,\\
&\qquad\quad \lambda_k - \xi \lesssim \tilde{h}^r_{kW}\lambda_W, \quad \text{for all } k,\\
&\qquad\quad \lambda_k + \xi \gtrsim \tilde{h}^r_{kW}\lambda_W, \quad \text{for all } k,\\
&\qquad\quad \sum_{k=1}^{t}\lambda_k = 1, \ \lambda_k \geq 0, \text{ for all } k.
\end{aligned} \tag{IV.9}$$

By considering the crisp equivalents of fuzzy constraints in Model (IV.9) besides the *α*-cut (with $\alpha = 0.5$) of $\tilde{h}^r_{Bk}$ and $\tilde{h}^r_{kW}$, the model is transformed as follows:

$$\begin{aligned}
&\min\ \xi,\\
&\text{s.t.:}\quad \lambda_B - \xi \leq \left[\frac{\left(h^r_{Bk,1}+h^r_{Bk,2}\right)}{2} + \frac{\left(h^r_{Bk,3}+h^r_{Bk,4}-h^r_{Bk,1}-h^r_{Bk,2}\right)}{2}\sigma\right]\lambda_k, \quad \text{for all } k,\\
&\qquad\quad \lambda_B + \xi \geq \left[\frac{\left(h^r_{Bk,1}+h^r_{Bk,2}\right)}{2} - \frac{\left(h^r_{Bk,3}+h^r_{Bk,4}-h^r_{Bk,1}-h^r_{Bk,2}\right)}{2}\sigma\right]\lambda_k, \quad \text{for all } k,\\
&\qquad\quad \lambda_k - \xi \leq \left[\frac{\left(h^r_{kW,1}+h^r_{kW,2}\right)}{2} + \frac{\left(h^r_{kW,3}+h^r_{kW,4}-h^r_{kW,1}-h^r_{kW,2}\right)}{2}\sigma\right]\lambda_W, \quad \text{for all } k,\\
&\qquad\quad \lambda_k + \xi \geq \left[\frac{\left(h^r_{kW,1}+h^r_{kW,2}\right)}{2} - \frac{\left(h^r_{kW,3}+h^r_{kW,4}-h^r_{kW,1}-h^r_{kW,2}\right)}{2}\sigma\right]\lambda_W, \quad \text{for all } k,\\
&\qquad\quad \sum_{k=1}^{t}\lambda_k = 1, \quad \lambda_k \geq 0, \qquad \text{for all } k,
\end{aligned} \tag{IV.10}$$

where σ $(0 \leq \sigma \leq 1)$ is a possibility level assigned by the decision-makers. As Model (IV.10) has a linear form, it results in a unique optimal weight vector $\left(\lambda^*_1, \lambda^*_2, \ldots, \lambda^*_t\right)^T$ and an objective value ξ^* for any given possibility level σ. ξ^* represents the consistency level of the fuzzy risky preferences. If $\varepsilon^* = 0$ for all σ, we have $\lambda_B / \lambda_j = \tilde{h}^r_{Bk}$ and $\lambda_j / \lambda_W = \tilde{h}^r_{kW}$, and correspondingly $\tilde{h}^r_{Bk} \times \tilde{h}^r_{kW} = \tilde{h}^r_{BW}$ for all k. Hence, the fuzzy risky preferences vectors $\tilde{H}^r_B$ and $\tilde{H}^r_W$ are fully consistent.

Step 2 (Prioritize criteria by the director and expert panel – using the risky fuzzy multi-person best–worst method): The director determines the best and worst criteria and considers the fuzzy preference degree $\tilde{p}_{Bj} = \left(p_{Bj,1},\ p_{Bj,2},\ p_{Bj,3},\ p_{Bj,4}\right)$ of the best criterion B over criterion j. Then, according to the fuzzy-distance-based prospect theory, the risky fuzzy form of $\tilde{p}_{Bj}$ is obtained as:

$$\tilde{p}^{r}_{Bj} = \exp\left(-\theta\left[\tilde{d}^{*}\left(\tilde{p}_{Bj},\tilde{p}_{BB}\right)\right]^{\beta}\right). \tag{IV.11}$$

The risky fuzzy best-to-others vector of the preference degrees of criteria determined by the director is $\tilde{P}^{r}_{B} = \left(\tilde{p}^{r}_{B1},\ldots,\tilde{p}^{r}_{Bj},\ldots,\tilde{p}^{r}_{Bn}\right)^{T}$. In analogous, the director allocates the fuzzy preference degree $\tilde{p}_{jW} = \left(p_{jW,1},\ p_{jW,2},\ p_{jW,3},\ p_{jW,4}\right)$ of criterion C_j over the worst criterion W. The risky fuzzy form of $\tilde{p}_{jW}$ is calculated as:

$$\tilde{p}^{r}_{jW} = \exp\left(\left[\tilde{d}^{*}\left(\tilde{p}_{jW},\tilde{p}_{WW}\right)\right]^{\gamma}\right). \tag{IV.12}$$

The risky fuzzy others-to-worst vector of the preference degrees of criteria determined by the director is obtained as $\tilde{P}^{r}_{W} = \left(\tilde{p}^{r}_{1W},\ldots,\tilde{p}^{r}_{jW},\ldots,\tilde{p}^{r}_{nW}\right)^{T}$. Each expert determines the best and worst criteria and considers the fuzzy preference degree $\tilde{q}^{[k]}_{Bj} = \left(q^{[k]}_{Bj,1},\ q^{[k]}_{Bj,2},\ q^{[k]}_{Bj,3}, q^{[k]}_{Bj,4}\right)$ of the best criterion B over each criterion C_j. Then, according to the fuzzy-distance-based prospect theory, the risky fuzzy form of $\tilde{q}^{[k]}_{Bj}$ is computed as:

$$\tilde{q}^{[k],r}_{Bj} = \exp\left(-\theta\left[\tilde{d}^{*}\left(\tilde{q}^{[k]}_{Bj},\tilde{q}^{[k]}_{BB}\right)\right]^{\beta}\right). \tag{IV.13}$$

The risky fuzzy best-to-others vector of the preference degrees of criteria evaluated by expert k is obtained as $\tilde{Q}^{[k],r}_{B} = (\tilde{q}^{[k],r}_{B1},\ldots,\tilde{q}^{[k],r}_{Bj},\ldots,\tilde{q}^{[k],r}_{Bn})^{T}$. Each expert also ascertains the fuzzy preference degree $\tilde{q}^{[k]}_{jW} = \left(q^{[k]}_{jW,1},\ q^{[k]}_{jW,2},\ q^{[k]}_{jW,3}, q^{[k]}_{jW,4}\right)$ of criterion C_j over the worst criterion W. The risky fuzzy form of $\tilde{q}^{[k]}_{jW}$ is calculated as:

$$\tilde{q}^{[k],r}_{jW} = \exp\left(\left[\tilde{d}^{*}\left(\tilde{q}^{[k]}_{jW},\tilde{q}^{[k]}_{WW}\right)\right]^{\gamma}\right). \tag{IV.14}$$

The risky fuzzy others-to-worst vector of the preference degrees of criteria specified by expert k is obtained as $\tilde{Q}^{[k],r}_{W} = (\tilde{q}^{[k],r}_{1W},\ldots,\tilde{q}^{[k],r}_{jW},\ldots,\tilde{q}^{[k],r}_{nW})^{T}$. The optimal weight vector of criteria $\left(w^{*}_{1}, w^{*}_{2},\ldots, w^{*}_{n}\right)^{T}$ is generated using the following programming model:

$$\begin{aligned}&\min\left\{\Omega\max_{j}\left\{\left|w_{B}-\tilde{p}^{r}_{Bj}w_{j}\right|,\left|w_{j}-\tilde{p}^{r}_{jW}w_{W}\right|\right\}+\left(1-\Omega\right)\sum_{k=1}^{t}\lambda^{*}_{k}\max_{j}\left\{\left|w_{B}-\tilde{q}^{[k],r}_{Bj}w_{j}\right|,\left|w_{j}-\tilde{q}^{[k],r}_{jW}w_{W}\right|\right\}\right\},\\&\text{s.t.: }\sum_{j=1}^{n}w_{j}=1,\ \ w_{j}\geq 0,\ \ \text{for all } j,\end{aligned} \tag{IV.15}$$

where λ^{*}_{k} is the optimal weight of expert k calculated based on Model (IV.10). Model (IV.15) combines the individual and multi-person decisions of the director and expert panel. The director makes a tradeoff between his decision and those of the expert panel by considering a self-reliance coefficient Ω $(0 \le \Omega \le 1)$. $\Omega=0$ means a democracy situation where the director does not interfere in criteria weighting (i.e., s/he only judges the priorities of the expert panel), while $\Omega=1$ indicates an autocracy situation where the criteria weighting of the expert panel are neglected (*i.e.*, the director only evaluates the importance of criteria). Model (IV.15) can be reformulated as:

$$\begin{aligned} &\min \Psi = \Omega\varepsilon + (1-\Omega)\sum_{k=1}^{t}\lambda_k^*\varepsilon_k, \\ &\text{s.t.:} \quad \left|w_B - \tilde{p}_{Bj}^r w_j\right| \le \varepsilon, \quad \text{for all } j, \\ &\qquad \left|w_j - \tilde{p}_{jW}^r w_W\right| \le \varepsilon, \quad \text{for all } j, \\ &\qquad \left.\begin{aligned}&\left|w_B - \tilde{q}_{Bj}^{[k],r} w_j\right| \le \varepsilon_k, \text{ for all } j \\ &\left|w_j - \tilde{q}_{jW}^{[k],r} w_W\right| \le \varepsilon_k, \text{ for all } j\end{aligned}\right\} \text{ for all } k, \\ &\qquad \sum_{j=1}^{n} w_j = 1, \ w_j \ge 0, \quad \text{for all } j. \end{aligned} \tag{IV.16}$$

Model (IV.16) could be transformed to an equivalent crisp model similar to the aforementioned procedure of Model (IV.10). As the resultant crisp formulation is a linear programming model, its solution, for any given possibility level σ, is in the form of an unique optimal weight vector $\left(w_1^*, w_2^*, \ldots, w_n^*\right)^T$, ε^*, and $\left(\varepsilon_1^*, \varepsilon_1^*, \ldots, \varepsilon_t^*\right)^T$. The objective function value Ψ^* implies the consistency of the risky fuzzy preference degrees of the director and expert panel. $\Psi=0$ indicates the full consistency. Nevertheless, the preferences are subjected to variance because of the probable irrationality of the director and expert panel or other reasons. Thus, the full consistency may be not feasible.

- ***Objective weights of criteria***

We employ an objective weighting model supported on the theory of TOPSIS method. The proposed approach exploits the fuzzy distance measure, i.e., Eq. (I.17), and also utilizes the normalization technique, i.e., Eq. (IV.3). To derive the fuzzy objective weights, the optimistic and pessimistic values of the fuzzy normalized ratings of each alternative are first computed as:

$$\tilde{f}_i^+ = \max_j \tilde{f}_{ij}, \quad \tilde{f}_i^- = \min_j \tilde{f}_{ij}. \tag{IV.17}$$

The overall distances between the fuzzy target-based normalized ratings and the optimistic/pessimistic values are obtained:

$$\tilde{D}_j^+ = \sqrt{\sum_{i=1}^{m}\left(\tilde{d}\left(\tilde{f}_i^+, \tilde{f}_{ij}\right)\right)^2}, \quad \tilde{D}_j^- = \sqrt{\sum_{i=1}^{m}\left(\tilde{d}\left(\tilde{f}_i^-, \tilde{f}_{ij}\right)\right)^2}. \tag{IV.18}$$

The objective fuzzy weights are determined as follows:

$$\tilde{w}_j^o = \tilde{D}_j^- \oslash \left(\tilde{D}_j^+ \oplus \tilde{D}_j^-\right). \tag{IV.19}$$

- ***Integrated risky fuzzy weights of criteria***

The combination of the subjective weight w_j^* obtained by Model (IV.16) and the objective fuzzy weight $\tilde{w}_j^o$ determined by Eq. (IV.19) generates the integrated fuzzy weight $\tilde{s}_j$:

$$\tilde{s}_j = \left(w_j^* \otimes \tilde{w}_j^o\right) \oslash \sum_{j=1}^{n}\left(w_j^* \otimes \tilde{w}_j^o\right) \quad \text{(IV.20)}$$

2.2. MULTIMOORA-based model for ranking process

In this part, we obtain the three ranking lists based on subordinate models of fuzzy MULTIMOORA, *i.e.*, the fuzzy ratio system, fuzzy reference point approach, and fuzzy full multiplicative form. The results of each method are then combined into the final ranking using a fuzzy distance matrix.

MULTIMOORA is an integrative decision-making approach based on three subordinate parts: ratio system, reference point approach, and full multiplicative form. Ratio system, as a fully compensatory approach, takes account of independent criteria, whereas full multiplicative form considers dependent criteria and has an incompletely compensatory algorithm. Reference point approach is a non-compensatory technique tends to provide a conservative solution. In reference point approach, imperfections of ratings of an alternative on one criterion is not compensated by the satisfactory performance on another criterion. Thus, by exploiting the advantages of three approaches, MULTIMOORA can be a key tool in the problems where there are independent and dependent criteria while a conservative result is also important[16].

- ***Ranking of alternatives based on fuzzy ratio system***

Using arithmetic operations of trapezoidal fuzzy numbers presented in Eq. (I.14) and considering the integrated fuzzy weights obtained by Eq. (IV.20) as well as the normalized fuzzy ratings by Eq. (IV.3), the assessment value $\tilde{Y}_i$ is generated as follows:

$$\tilde{Y}_i = \sum_{j=1}^{g}\left(\tilde{s}_j \otimes \tilde{f}_{ij}\right) \quad \text{(IV.21)}$$

The best alternative in the fuzzy ratio system has the maximum value of $\tilde{Y}_i$, obtained supported on the mathematics of trapezoidal fuzzy numbers given in Section I.2.2:

$$A^*_{\text{F-RS}} = \left\{ A_{i'} \,\middle|\, i' = \arg\max_i \tilde{Y}_i \right\} \quad \text{(IV.22)}$$

The ranking of alternatives based on the fuzzy ratio system is obtained by ordering the assessment values descendingly.

- ***Ranking of alternatives based on fuzzy reference point approach***

For this approach, first, the deviation between the normalized fuzzy rating $\tilde{f}_{ij}$ and the reference point $\tilde{f}_j^+$ is defined as:

$$\tilde{\Delta}_{ij} = \tilde{d}\left(\tilde{f}_j^+, \tilde{f}_{ij}\right), \quad \text{(IV.23)}$$

where $\tilde{d}$ denotes the fuzzy distance obtained by Eq. (I.17) and $\tilde{f}_j^+ = \max_i \tilde{f}_{ij}$. To generate the assessment value of the fuzzy target-based reference point, we have:

$$\tilde{Z}_i = \max_j \left(\tilde{s}_j \otimes \tilde{\Delta}_{ij}\right) \quad \text{(IV.24)}$$

The best alternative obt(IV.21)ained by the fuzzy reference point approach has the minimum assessment value, where:

$$A^{*}_{\text{F-RP}} = \left\{ A_{i' \mid i' = \arg\min_{i} \tilde{Z}_i} \right\}. \tag{IV.25}$$

The ranking of alternatives in the fuzzy reference point approach is generated by ordering the assessment values ascendingly.

- ***Ranking of alternatives based on fuzzy full multiplicative form***

The assessment value of the fuzzy full multiplicative form is generated as:

$$\tilde{U}_i = \prod_{j=1}^{n} \left(\tilde{f}_{ij} \right)^{\tilde{s}_j}. \tag{IV.26}$$

As all the elements of $\tilde{f}_{ij}$ and $\tilde{s}_j$ are between 0 and 1, it can be shown that:

$$\left(\tilde{f}_{ij} \right)^{\tilde{s}_j} = \left(\left(f_{ij,1} \right)^{s_{j,4}}, \left(f_{ij,2} \right)^{s_{j,3}}, \left(f_{ij,3} \right)^{s_{j,2}}, \left(f_{ij,4} \right)^{s_{j,1}} \right). \tag{IV.27}$$

The optimal alternative with the fuzzy full multiplicative form is calculated as follows:

$$A^{*}_{\text{F-MF}} = \left\{ A_{i' \mid i' = \arg\max_{i} \tilde{U}_i} \right\}. \tag{IV.28}$$

The ranking of alternatives based on the fuzzy full multiplicative is determined by ordering the assessment values descendingly.

- ***Final ranking of alternatives based on fuzzy distance matrix***

We employ a fuzzy distance matrix supported on the proposed fuzzy distance measure to obtain the final ranking of alternatives.

In the original MULTIMOORA method, the dominance theory was employed to consolidate the subordinate ranks of alternatives; however, the theory reveals some disadvantages, such as circular reasoning [18]. The ranking aggregation technique based on fuzzy distance matrix does not have the drawbacks of the dominance theory. The fuzzy distance matrix is defined by considering the assessment values of the fuzzy ratio system, fuzzy reference point approach, and fuzzy full multiplicative form as shown in Table IV.1. In Table IV.1, $\tilde{\pi}_{il}$ equals to:

$$\tilde{\pi}_{il} = \begin{cases} 0, & i = l, \ i,l = 1, \ldots, m, \\ \tilde{I}^{y}_{il} \ominus \tilde{I}^{z}_{il} \oplus \tilde{I}^{u}_{il}, & i \neq l, \ i,l = 1, \ldots, m, \end{cases} \tag{IV.29}$$

where indices i and l represent the position of alternatives. $\tilde{I}^{y}_{il}$, $\tilde{I}^{z}_{il}$, and $\tilde{I}^{u}_{il}$ are respectively defined as:

$$\begin{aligned} \tilde{I}^{y}_{il} &= \left\{ \tilde{d}\left(\tilde{Y}_i, \tilde{Y}_l\right), \ \text{if } \overline{Y}_i \geq \overline{Y}_l; \ -\tilde{d}\left(\tilde{Y}_i, \tilde{Y}_l\right), \ \text{if } \overline{Y}_i < \overline{Y}_l \right\}, \\ \tilde{I}^{z}_{il} &= \left\{ \tilde{d}\left(\tilde{Z}_i, \tilde{Z}_l\right), \ \text{if } \overline{Z}_i \geq \overline{Z}_l; \ -\tilde{d}\left(\tilde{Z}_i, \tilde{Z}_l\right), \ \text{if } \overline{Z}_i < \overline{Z}_l \right\}, \\ \tilde{I}^{u}_{il} &= \left\{ \tilde{d}\left(\tilde{U}_i, \tilde{U}_l\right), \ \text{if } \overline{U}_i \geq \overline{U}_l; \ -\tilde{d}\left(\tilde{U}_i, \tilde{U}_l\right), \ \text{if } \overline{U}_i < \overline{U}_l \right\}, \end{aligned} \tag{IV.30}$$

where $\tilde{Y}_i$, $\tilde{Z}_i$, and $\tilde{U}_i$ are the assessment values of the fuzzy ratio system, reference point approach, and fuzzy full multiplicative form obtained by Eqs. (IV.21), (IV.24), and (IV.26), respectively. $\overline{Y}_i$, $\overline{Z}_i$, and $\overline{U}_i$ are the defuzzified values of their correspondent assessment indices calculated by Eq. (I.13). $\tilde{d}$ denotes the fuzzy distance measure introduced in Eq. Eq. (I.17). In Eq. (IV.30), the negative terms are obtained based on the following formula (for a positive trapezoidal fuzzy number $\tilde{A} = (A_1, A_2, A_3, A_4)$ and a negative real number η):

$$\tilde{A} \odot \eta = \left(A_4\eta, A_3\eta, A_2\eta, A_1\eta \right) . \tag{IV.31}$$

Table IV.1. The fuzzy distance matrix for obtaining final ranking.

Relative fuzzy distances by considering sign of the measure						Aggregated distances
	A_1	$\cdots$	A_l	$\cdots$	A_m	
A_1	$\tilde{\pi}_{11}$	$\cdots$	$\tilde{\pi}_{1l}$	$\cdots$	$\tilde{\pi}_{1m}$	$\tilde{\Gamma}_1 = \sum_{l=1}^{m} \tilde{\pi}_{1l}$
$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
A_i	$\tilde{\pi}_{i1}$	$\cdots$	$\tilde{\pi}_{il}$	$\cdots$	$\tilde{\pi}_{im}$	$\tilde{\Gamma}_i = \sum_{l=1}^{m} \tilde{\pi}_{il}$
$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
A_m	$\tilde{\pi}_{m1}$	$\cdots$	$\tilde{\pi}_{ml}$	$\cdots$	$\tilde{\pi}_{mm}$	$\tilde{\Gamma}_{ml} = \sum_{l=1}^{m} \pi_{ml}$

The best alternative of the fuzzy MULTIMOORA model supported on the fuzzy distance matrix is determined as:

$$A^{*}_{\text{F-MULTIMOORA-FDM}} = \left\{ A_{i'} \,\middle|\, i' = \arg\max_{i} \tilde{\Gamma}_i \right\} . \tag{IV.32}$$

3. Application of RF_S-MpMcDM_FRFP methodology in biomedical sector

In this part, we utilize the decision-making methodology introduced in Section 2 for material selection in a practical biomedical engineering problem. First, the motivation and specifications of the practical problem are depicted. Second, the solution of the biomaterial selection problem based on the RF_S-MpMcDM_FRFP methodology is provided

The biomedical decision-making problems are sensitive cases as they deal with the health of human-beings. The physicians must decide about the situation of their patients with precision and quickness (where applicable) to present satisfactory diagnose and treatment. The precision and quickness lead to prevent fatal or permanent effects on the health of patients. To reach a final decision, the cases should often be evaluated by multiple experts. Additionally, biomedical engineers may make irrational decisions because of their critical work and job pressure. Thus, a systemic multi-person decision-making approach considering risk and consistent with the requirements of biomedical problems can be an effective tool for the experts.

This practical case is related to biomaterial selection for a spinal fixator. A biomaterial is entitled to any substance engineered to be used in biological systems for the aim of diagnostic or therapeutic applications. A number of biomaterials can often be candidate solutions for orthopedic disorders. The proposed methodology helps in finding an appropriate alternative for biomaterial selection problems.

The data of this case study was gathered based on several references of biomaterials [139]–[142] and also by consulting with the orthopedic experts deal with designing implants and surgical treatments. To collect the comments, biomedical engineers have been provided with specific forms to present their preferences about the performance of the candidate biomaterials.

Biocompatibility is a significant feature needed for biomaterials utilized for implants and prostheses[143]. Biocompatibility means the capability of a biomaterial to act suitably without showing serious defects; however, this property is rather case-sensitive which means biocompatibility in a particular usage does not guarantee suitability for another application[144]. Usually, to ensure biocompatibility, the value of the elastic modulus related to biomaterial and that of the tissue should be close as possible. This issue leads to decreasing stress concentration of biomaterial[145]. Thus, elastic modulus is supposed as a target-based criterion for selection of biomaterials. That is, the target value of elastic modulus of a biomaterial should be very close to elastic modulus of the adjacent tissue. Biomaterial properties are ordinarily given as uncertain information in the handbooks of materials science.

Generally, spinal implants are produced to increase the stability of spine and lessen the pressure on the vertebral disc [146]. A sample of spine fixator is shown in Fig. IV.3 in the supplementary file.

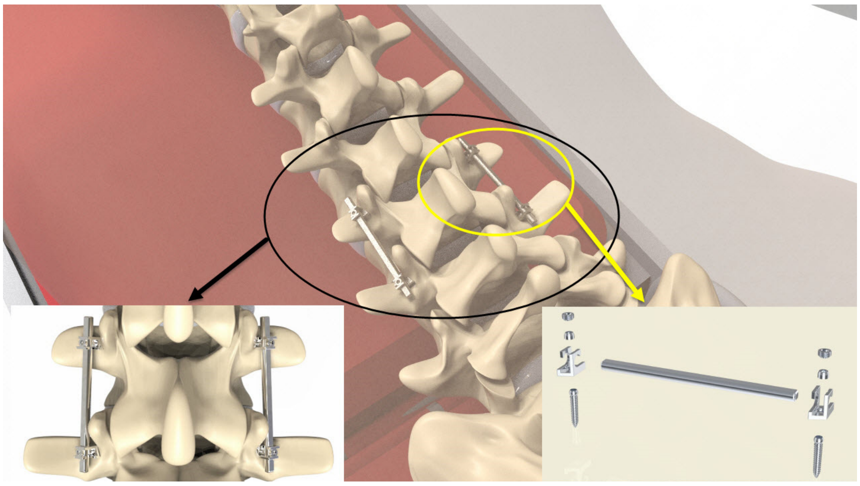

Fig. IV.3. The case study of the RF_S-MpMcDM_FRFP methodology: A typical spinal fixator.

We follow the phases of the proposed methodology (shown in Fig. IV.1), i.e., (i) risky fuzzy weighting process of criteria, (ii) fuzzy ranking process of alternatives, to reach the outcome of the biomaterial selection problem.

Six criteria exist in the problem: elastic modulus, density, tensile strength, fatigue strength, yield strength, and elongation. Elastic modulus and density are target-based criteria and the rest are beneficial. Table IV.2 denotes units and the fuzzy target values for the criteria. Table IV.3 represents the decision matrix of the practical case study. The decision matrix provides the fuzzy performances (i.e., fuzzy values of biomaterials properties) of eight biomaterials preselected as candidates in response to the criteria of the problem. The fuzzy ratings of the biomaterials are normalized by Eq. (IV.3).

Table IV.2. Criteria of candidate biomaterials.

Criterion name	Unit	Type
Elastic modulus	GPa	Target-based $\{\tilde{g}_1 = (12.8,\ 13.1,\ 14.9,\ 15.2)\}$
Density	g/cm^3	Target-based $\{\tilde{g}_2 = (1.9,\ 2.0,\ 2.2,\ 2.3)\}$
Tensile strength	MPa	Beneficial
Fatigue strength	MPa	Beneficial
Yield strength	MPa	Beneficial
Elongation	%	Beneficial

Table IV.3. Decision matrix constructed form the performance of the candidate biomaterials on their criteria.

BioM. ID*	Biomaterial name	Target-based criteria		Beneficial criteria			
		Elastic modulus	Density	Tensile strength	Fatigue strength	Yield strength	Elongation
M_1	Stainless Steel 316L (cold-worked)	(177, 180, 206, 209)	(7.2, 7.3, 8.5, 8.6)	(826, 834, 886, 894)	(298, 301, 319, 322)	(662, 669, 711, 718)	(10.9, 11.2, 12.8, 13.1)
M_2	Stainless Steel 316L (annealed)	(177, 180, 206, 209)	(7.2, 7.3, 8.5, 8.6)	(496, 501, 533, 538)	(252, 254, 270, 272)	(318, 321, 341, 344)	(36.4, 37.2, 42.8, 43.6)

M_3	Wrought CoNiCrMo	(208, 212, 242, 246)	(8.4, 8.6, 9.8, 10.0)	(1721, 1739, 1847, 1865)	(480, 485, 515, 520)	(1523, 1538, 1634, 1649)	(7.3, 7.4, 8.6, 8.7)
M_4	Pure Titanium	(92, 94, 107, 109)	(4.1, 4.2, 4.8, 4.9)	(528, 534, 567, 572)	(230, 233, 247, 250)	(354, 358, 380, 384)	(13.7, 14.0, 16.1, 16.4)
M_5	Ti–6Al–4V	(97, 99, 113, 115)	(4.1, 4.2, 4.8, 4.9)	(826, 834, 886, 894)	(499, 504, 536, 541)	(763, 771, 819, 827)	(9.1, 9.3, 10.7, 10.9)
M_6	Ti-6Al-7Nb	(101, 103, 117, 119)	(4.1, 4.2, 4.8, 4.9)	(1008, 1019, 1082, 1092)	(431, 436, 462, 467)	(768, 776, 824, 832)	(11.8, 12.1, 13.9, 14.2)
M_7	Co–Cr alloy (castable)	(220, 224, 256, 260)	(7.6, 7.7, 8.9, 9.0)	(629, 635, 675, 681)	(408, 412, 438, 442)	(432, 437, 464, 468)	(18.2, 18.6, 21.4, 21.8)
M_8	Co–Cr alloy (wrought)	(220, 224, 256, 260)	(8.3, 8.5, 9.7, 9.9)	(860, 869, 923, 932)	(576, 582, 618, 624)	(624, 631, 670, 676)	(18.2, 18.6, 21.4, 21.8)

* BioM. ID: Biomaterial identification code.

As illustrated in Fig. IV.4, the decision-making structure comprises the manager of biomedical engineering department as the director and four biomedical engineers as the expert panel (the manager of biomedical engineering department is only referred as "manager" in the remainder of this study). To calculate subjective weights employing the proposed risky fuzzy multi-person best–worst method, first we need to obtain risky fuzzy preferences.

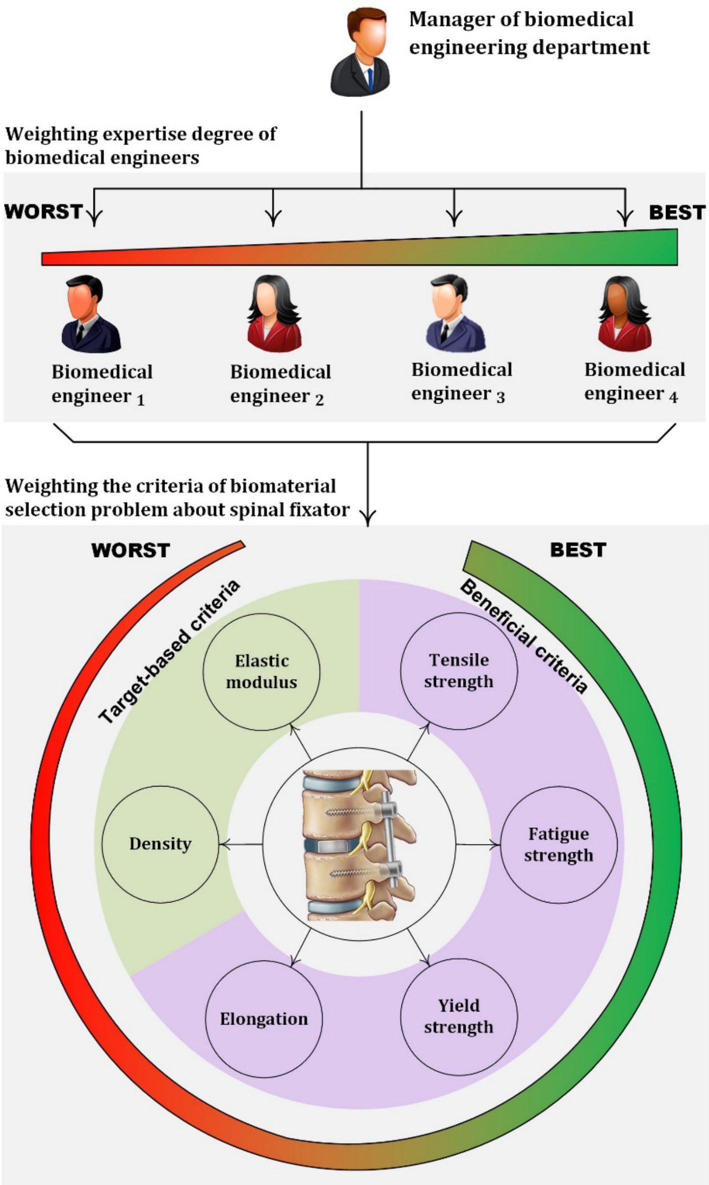

Fig. IV.4. Supervised multi-person structure for the case study of the RF_S-MpMcDM_FRFP methodology.

Table IV.4 lists the terms used for assigning linguistic preferences and the related trapezoidal fuzzy preferences. The manager estimates the relative preferences (i.e., the experts' competence) of the biomedical engineers using linguistic terms. The risky fuzzy preferences are then calculated based on the linguistic terms and Eqs. (IV.5) and (IV.6). Table IV.5 gives the best-to-others and others-to-worst vectors of the biomedical engineers' risky fuzzy preferences degrees from the viewpoint of the manager.

Table IV.4. Linguistic preferences and corresponding trapezoidal fuzzy values.

Linguistic preferences	Corresponding trapezoidal fuzzy preferences
Equal importance	(1, 1, 1, 1)
Minor importance	(2, 2.75, 3.5, 4.25)
Fair importance	(4.25, 4.75, 5.25, 5.75)
Major importance	(5.75, 6.5, 7.25, 8)
Extreme importance	(9, 9, 9, 9)

Table IV.5. Expertise degrees in the form of risky fuzzy preferences.

DM*	Best & worst biomedical engineers	Risky fuzzy preferences based on the proposed fuzzy prospect theory				
			Biomedical engineer 1	Biomedical engineer 2	Biomedical engineer 3	Biomedical engineer 4
Manager	*Best*: Biomedical engineer 3	$\tilde{H}_B^r$	(0.20, 0.20, 0.20, 0.20)	(0.20, 0.48, 0.48, 0.48)	(0.2, 1.00, 1.00, 1.00)	(0.20, 0.36, 0.36, 0.36)
	Worst: Biomedical engineer 1	$\tilde{H}_W^r$	(1.00, 1.00, 1.00, 1.00)	(1.72, 1.72, 1.72, 1.86)	(2.36, 2.36, 2.36, 2.36)	(1.15, 1.15, 1.15, 1.25)

* DM: Decision-maker.

The manager and all biomedical engineers identify the best and worst criteria and then measure the preferences of the biomaterials' criteria with reference to the pre-assigned best and worst criteria. The linguistic judgments of the manager and biomedical engineers are converted into fuzzy values based on Table IV.4 and then the resultant values are transformed into risky fuzzy preference degrees employing Eqs. (IV.11) and (IV.12) besides Eqs. (IV.13) and (IV.14), respectively. Table IV.6 lists the risky fuzzy preference degrees of the criteria from the viewpoints of the manager and biomedical engineers as best-to-others and others-to-worst vectors.

Table IV.6. Risky fuzzy preference degrees of the criteria of the biomaterial selection problem.

DMs	Best criterion	Worst criterion	Risky fuzzy preferences based on the proposed fuzzy prospect theory						
				C_1 (Elastic modulus)	C_2 (Density)	C_3 (Tensile strength)	C_4 (Fatigue strength)	C_5 (Yield strength)	C_6 (Elongation)
Manager	C_1	C_6	$\tilde{P}_B^r$	(1.00, 1.00, 1.00, 1.00)	(0.31, 0.31, 0.31, 0.31)	(0.74, 0.74, 0.74, 0.74)	(0.43, 0.43, 0.43, 0.43)	(0.74, 0.74, 0.74, 0.74)	(0.16, 0.16, 0.16, 0.16)
			$\tilde{P}_W^r$	(2.25, 2.25, 2.25, 2.25)	(1.14, 1.14, 1.14, 1.24)	(1.44, 1.44, 1.44, 1.52)	(1.14, 1.14, 1.14, 1.24)	(1.67, 1.67, 1.67, 1.79)	(1.00, 1.00, 1.00, 1.00)
Biomedical engineer 1	C_4	C_3	$\tilde{Q}_B^{[1],r}$	(0.32, 0.32, 0.32, 0.32)	(0.45, 0.45, 0.45, 0.45)	(0.17, 0.17, 0.17, 0.17)	(1.00, 1.00, 1.00, 1.00)	(0.75, 0.75, 0.75, 0.75)	(0.45, 0.45, 0.45, 0.45)
			$\tilde{Q}_W^{[1],r}$	(1.15, 1.15, 1.15, 1.15)	(1.15, 1.15, 1.15, 1.15)	(1.00, 1.00, 1.00, 1.00)	(2.37, 2.37, 2.37, 2.37)	(1.72, 1.72, 1.72, 1.72)	(1.15, 1.15, 1.15, 1.15)
Biomedical engineer 2	C_5	C_6	$\tilde{Q}_B^{[2],r}$	(0.73, 0.73, 0.73, 0.73)	(0.41, 0.41, 0.41, 0.41)	(0.41, 0.41, 0.41, 0.41)	(0.73, 0.73, 0.73, 0.73)	(1.00, 1.00, 1.00, 1.00)	(0.14, 0.14, 0.14, 0.14)
			$\tilde{Q}_W^{[2],r}$	(1.64, 1.64, 1.64, 1.64)	(1.13, 1.13, 1.13, 1.13)	(1.13, 1.13, 1.13, 1.13)	(1.64, 1.64, 1.64, 1.64)	(2.19, 2.19, 2.19, 2.19)	(1.00, 1.00, 1.00, 1.00)
Biomedical engineer 3	C_4	C_3	$\tilde{Q}_B^{[3],r}$	(0.39, 0.39, 0.39, 0.39)	(0.72, 0.72, 0.72, 0.72)	(0.13, 0.13, 0.13, 0.13)	(1.00, 1.00, 1.00, 1.00)	(0.72, 0.72, 0.72, 0.72)	(0.72, 0.72, 0.72, 0.72)
			$\tilde{Q}_W^{[3],r}$	(1.12, 1.12, 1.12, 1.21)	(1.57, 1.57, 1.57, 1.67)	(1.00, 1.00, 1.00, 1.00)	(2.04, 2.04, 2.04, 2.04)	(1.57, 1.57, 1.57, 1.67)	(1.57, 1.57, 1.57, 1.67)
Biomedical engineer 4	C_1	C_6	$\tilde{Q}_B^{[4],r}$	(1.00, 1.00, 1.00, 1.00)	(0.43, 0.43, 0.43, 0.43)	(0.74, 0.74, 0.74, 0.74)	(0.74, 0.74, 0.74, 0.74)	(0.31, 0.31, 0.31, 0.31)	(0.16, 0.16, 0.16, 0.16)
			$\tilde{Q}_W^{[4],r}$	(2.28, 2.28, 2.28, 2.28)	(1.14, 1.14, 1.14, 1.14)	(1.68, 1.68, 1.68, 1.68)	(1.45, 1.45, 1.45, 1.45)	(1.14, 1.14, 1.14, 1.14)	(1.00, 1.00, 1.00, 1.00)

The values of deviations of the normalized fuzzy ratings are obtained by Eq. (IV.23). Table IV.7 provides the values of assessment indices for the fuzzy ratio system, fuzzy reference point, and fuzzy full multiplicative form, generated utilizing Eqs. (IV.21), (IV.24), and (IV.26), respectively. Moreover, the subordinate and two aggregate rankings lists are provided in Table IV.7. The first aggregate ranking list is obtained based on the dominance theory. To generate the second aggregate ranking list, the fuzzy distance matix introduced in Section 2.2 is used. The related assessment values are computed based on Table IV.1.

Table IV.7. Outcomes of the fuzzy MULTIMOORA with risky fuzzy weights supported on the fuzzy decision matrix.

BioM. ID	RF-RS[1]		RF-RP[2]		RF-MF[3]		RF-MULTIMOORA-Dominance[4]	RF-MULTIMOORA-FDM[5]	
	$\tilde{Y}_i$	$r(\tilde{Y}_i)$	$\tilde{Z}_i$	$r(\tilde{Z}_i)$	$\tilde{U}_i$	$r(\tilde{U}_i)$		$\tilde{L}_i$	$r(\tilde{L}_i)$
M_1	(0.419, 0.427, 0.437, 0.445)	8	(0.156, 0.157, 0.159, 0.161)	6	(0.486, 0.485, 0.486, 0.486)	7	6	(-1.705, -1.654, -1.632, -1.583)	6
M_2	(0.444, 0.450, 0.460, 0.467)	6	(0.175, 0.176, 0.178, 0.180)	7	(0.482, 0.481, 0.481, 0.479)	8	6	(-1.750, -1.717, -1.688, -1.644)	7
M_3	(0.628, 0.638, 0.649, 0.660)	1	(0.082, 0.082, 0.083, 0.084)	3	(0.647, 0.648, 0.649, 0.650)	2	2	(1.580, 1.642, 1.660, 1.717)	1
M_4	(0.438, 0.445, 0.456, 0.464)	7	(0.182, 0.184, 0.186, 0.188)	8	(0.486, 0.485, 0.486, 0.486)	6	6	(-1.793, -1.755, -1.732, -1.681)	8
M_5	(0.607, 0.620, 0.632, 0.646)	2	(0.078, 0.078, 0.079, 0.080)	2	(0.659, 0.662, 0.664, 0.667)	1	1	(1.577, 1.622, 1.658, 1.696)	2
M_6	(0.576, 0.587, 0.599, 0.611)	4	(0.082, 0.082, 0.083, 0.084)	4	(0.642, 0.645, 0.646, 0.649)	3	4	(1.258, 1.309, 1.335, 1.389)	3
M_7	(0.463, 0.472, 0.483, 0.493)	5	(0.097, 0.098, 0.099, 0.100)	5	(0.521, 0.521, 0.522, 0.523)	5	5	(-0.703, -0.648, -0.627, -0.572)	5
M_8	(0.580, 0.588, 0.599, 0.608)	3	(0.073, 0.074, 0.074, 0.075)	1	(0.608, 0.608, 0.609, 0.608)	4	3	(1.056, 1.106, 1.122, 1.166)	4

[1] RF-RS: Fuzzy Ratio System with Risky fuzzy weights; [2] RF-RP: Fuzzy Reference Point approach with Risky fuzzy weights; [3] RF-MF: Fuzzy full Multiplicative Form with Risky fuzzy weights; [4] RF-MULTIMOORA-Dominance: Fuzzy MULTIMOORA supported on Dominance theory with Risky fuzzy weights; [5] RF-MULTIMOORA-FDM: Fuzzy MULTIMOORA supported on Fuzzy Distance Matrix with Risky fuzzy weights.

The best alternative based on the fuzzy ratio system, fuzzy reference point, fuzzy full multiplicative form, and fuzzy target-based MULTIMOORA model with the fuzzy distance matrix are computed by Eqs. (IV.22), (IV.25), (IV.28), and (IV.32), respectively (as risky weights are used to compute subordinate and final rankings, we consider a prefix "R" to indicate the weights):

$$A^*_{\text{RF-RS}} = A^*_{\text{RF-MULTIMOORA-FDM}} = M_3,\ A^*_{\text{RF-RP}} = M_8,\ \text{and}\ A^*_{\text{RF-MF}} = A^*_{\text{RF-MULTIMOORA-Dominance}} = M_5\ .$$

Chapter V

DI_S-MpMcDM_SoESs methodology: Energy application

In this chapter, we present the DI_S-MpMcDM_SoESs methodology. The methodology is based interval numbers and modeled by the BWM-MULTIMOORA approach. The supervised multi-person structure is with subject-oriented expert segments. It is applied in an case study on renewable energy investment project.

Most socio-economic and industrial decision-making problems are interdisciplinary, needing different expertise areas for handling. Considering Subject-oriented Expert Segments (SoESs) can streamline the decision-making process for such problems. The decisions of SoESs are dynamically improved by gaining experience. Accordingly, we propose a methodology to tackle SoESs sturcture. In this methodology, a director evaluates the relative preferences of experts and problem subjects. The methodology is derived through hybrid modeling based on BWM and MULTIMOORA method.

Finally, we evaluate a real-life decision-making problem regarding investment in renewable energy sources. According to the consistency of the BWM-based models and the robustness of the MULTIMOORA-based model, the proposed methodology can be efficient in solving practical DI_S-MpMcDM_SoESs problems.

1. Introduction

Dynamic Multi-person Multi-criteria Decision-Making (D_MpMcDM) deals with the significance of time in decision-making area [147]. In D_MpMcDM problems, multiple experts are responsible for evaluating alternatives according to multiple criteria in a changeable situation by passing time.

The situation is mainly affected by dynamic changes in experts' relative importance and judgments. Consequently, experts' judgments progressively become more reliable based on their gained experience [13], [148]. The D_MpMcDM approaches have been utilized in various practical applications, including safety management, construction industry, and reverse logistics [149]–[151].

In real-life, many socio-economic and industrial institutes tackle interdisciplinary dynamic decision-making problems. In this regard, multiple groups of experts with special competence in each subject are often required [1]. United Nations Economic and Social Council includes several groups of experts focused on information management, accounting, and transport dynamically directed by government officials. Industrially speaking, the Operations Engineering Department of Qatar Petroleum comprises several groups (e.g., related to inspection, process, systems, and planning) that enable the corporation to fulfill dynamic customer requirements [152], [153].

In the context of D_MpMcDM, experts have not been segmented according to their competence [154]–[156]. That is, all experts are supposed to be competent in all problem subjects. The question is: "*Do all experts have competence in each subject?*" In real-life, experts may not have all-inclusive competence [157]. Thus, a research gap exists in the context to tackle the segmentation of experts needed for realistic interdisciplinary problems.

To plug the gap related to "*the segmentation of experts according to experts' competence*," we introduce the Subject-oriented Expert Segments (SoESs) structure in the context of (D_MpMcDM). The SoESs structure is a multi-person decision-making framework in which experts are segmented based on their competence in the problem subjects and supervised by a director. The relative importance of subjects is also assessed by the director. SoESs provide the preferences of the problem alternatives based on their subject-oriented competence.

The introduced SoESs structure can enhance the decision-making process in (D_MpMcDM) problems as: (1) the interdisciplinary nature of practical decision-making problems can be efficiently handled by considering a SoES for each subject; (2) precise evaluation is reached because each subject of the problem is assessed by the experts competent in the subject; (3) the supervision of SoESs can improve the whole decision-making process as the competence of experts is appreciated.

The SoESs structure allows us to propose a decision-making methodology for D_MpMcDM with Subject-oriented Expert Segments (DI_S-MpMcDM_SoESs methodology) applying the BWM-MULTIMOORA approach with interval computation. The suggested methodology is derived by introducing the following decision-making models:

1) *BWM-Based Models Supported on the SoESs Structure for Obtaining Dynamic Weights of Subjects, Experts, and Alternatives*: We develop three stepwise interval BWM-based models to calculate dynamic weights for the decision-making problem. The first model is an interval BWM to calculate the dynamic weights of problem subjects (according to the competence of each SoES). The second model is also an interval BWM to obtain the dynamic weights of SoESs' experts. The third model is an interval multi-person BWM based on the global criterion method to calculate the dynamic weights of alternatives (based on the preferences of SoESs' experts).
2) *MULTIMOORA-Based Model for Obtaining Final Ranking of Alternatives*: We introduce a MULTIMOORA-based model to integrate dynamic weights of alternatives obtained for each time period. First, the integrated significance

coefficients are calculated based on the consistency of the interval multi-person BWM and information entropy method. Second, the dynamic weights of alternatives are aggregated based on the MULTIMOORA-Borda approach to reach the final ranking of alternatives.

We discuss a case study on a renewable energy investment project utilizing the proposed DI_S-MpMcDM_SoESs methodology. Segmentation of experts according to experts' competences is essential in this case study as different competences are required to examine the problem's financial, technological, environmental, and social subjects.

The remainder of this chapter is arranged as follows. Section II presents the theory of the proposed DI_S-MpMcDM_SoESs methodology. A real-life decision-making problem regarding investment in renewable energy sources is assessed in Section 3.

2. Theory of DI_S-MpMcDM_SoES methodology

This section introduces the derivation of the DI_S-MpMcDM_SoESs methodology, allowing segmentation of experts according to experts' competence. Two phases exist for developing this methodology: (1) we compute the dynamic weights of subjects, experts, and alternatives using BWM-based models in which the preferences of director and experts of SoESs are considered (see Section 2.1); (2) we calculate the final ranking of alternatives using a MULTIMOORA-based model by considering the dynamic weights of alternatives and their integrated significance coefficients (see Section 2.2). The flowchart of the proposed decision-making is illustrated in Fig. V.1.

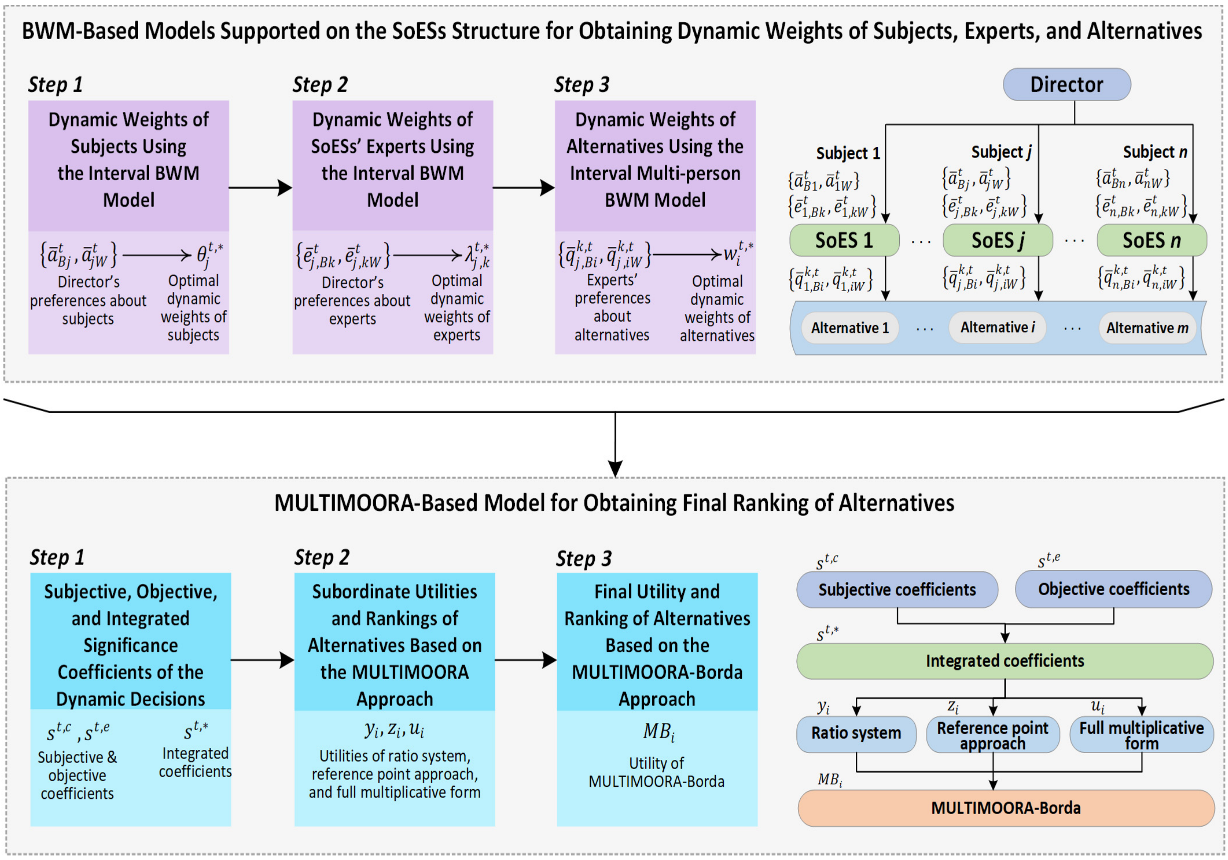

Fig. V.1. Flowchart of the DI_S-MpMcDM_SoES methodology.

2.1. BWM-based models for weighting process

In this section, we develop three BWM-based models supported on the SoESs structure to process experts' evaluations in each time period. The sequence of the three models and the notation (*i.e.,* preferences and weights) are illustrated in Fig. V.1. All preferences are in the form of interval numbers. The purpose of the modeling is to generate dynamic weights of alternatives (obtained by the third model). The first and second models lead to dynamic weights of subjects and experts, respectively. The results of the first and second models are employed in the third model.

The SoESs structure is shown with details in Fig. V.2. The subjects of the problem are represented as $\{S_1, \ldots, S_j, \ldots, S_n\}$, $j = 1, 2 \ldots, n$. For each subject, an SoES is allocated: $\{\text{SoES}_1, \ldots, \text{SoES}_j, \ldots, \text{SoES}_n\}$. Each SoES includes h_j experts, represented by $\{\text{Expert}_1^j, \ldots, \text{Expert}_k^j, \ldots, \text{Expert}_{h_j}^j\}$ in which $k = 1, 2 \ldots, h_j$. The director supervises SoESs and evaluates the relative importance of the subjects.

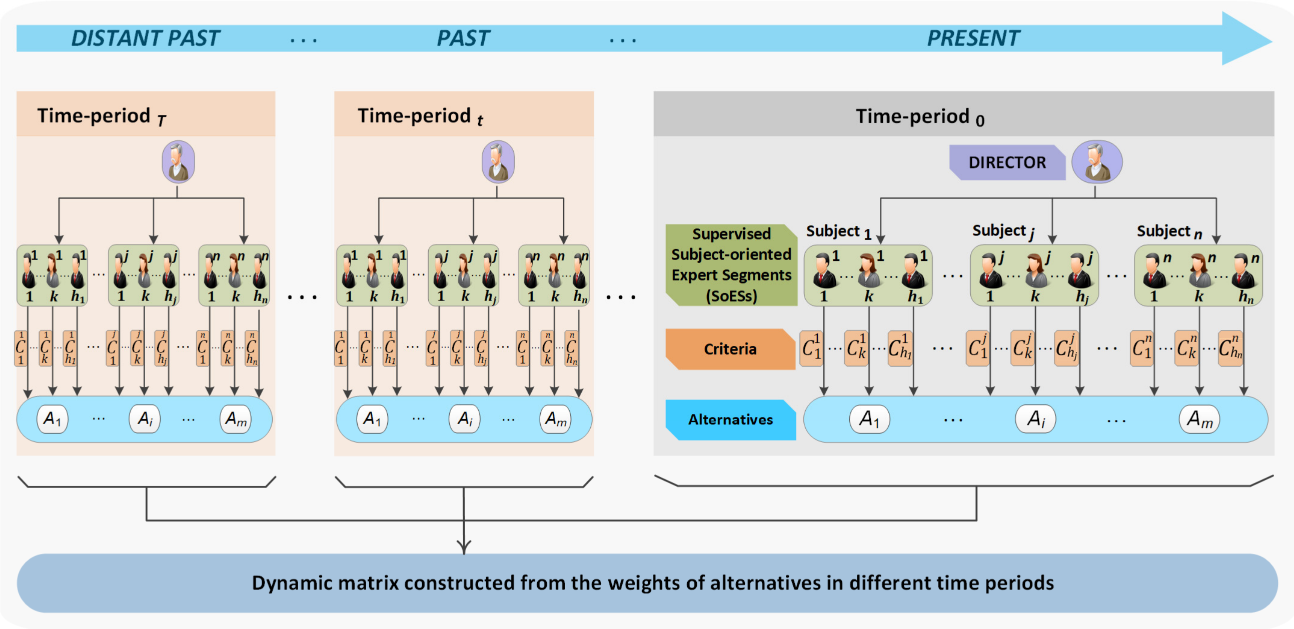

Fig. V.2. Supervised multi-person structure in the DI_S-MpMcDM_SoES methodology.

The criteria of the problem are categorized based on the subjects. For each subject, the set of criteria is represented as $\{C_1^j, \ldots, C_k^j, \ldots, C_{h_j}^j\}$. Each expert of SoES takes decision based on his/her competence in one criterion. Thus, Expert_k^j (*i.e.,* the k th expert in the j th SoES) evaluates all alternatives $(\text{i.e.,} \{A_1, \ldots, A_i, \ldots, A_m\}, i = 1, 2 \ldots, m)$ according to his/her competence in C_k^j (*i.e.,* the k th criterion in the category of the j th subject).

We consider a set of time periods for dynamic decision-making, i.e., $\{P^0, \ldots, P^t, \ldots, P^T\}$ where $t = 0, 1 \ldots, T$ as shown in Fig. V.2. P^0 denotes the present time and the set $\{P^1, \ldots, P^t, \ldots, P^T\}$ represents the historical records. For each time period, the same SoESs structure is assigned.

To obtain dynamic weights of subjects, experts, and alternatives, a step-by-step procedure is adopted as follows:

Step 1 (Dynamic Weights of Subjects Using the Interval BWM Model): For each time period, the director assesses the priority of the problem subjects according to the best and worst items by considering best-to-others and others-to-worst preferences. After designation of the best and worst subjects, the director assesses the interval preferences of all subjects, considering the best and worst subjects.

$\overline{a}_{Bj}^{t} = \left[a_{Bj}^{t,L}, a_{Bj}^{t,U} \right]$ is the interval preference of S_B (the best subject) over S_j for time period t. $\overline{a}_{jW}^{t} = \left[a_{jW}^{t,L}, a_{jW}^{t,U} \right]$ is the interval preference of subject j over S_W (the worst subject) for time period t. The best-to-others and others-to-worst interval preference vectors are obtained for the subjects, respectively, as $\overline{A}_{B}^{t} = \left(\overline{a}_{B1}^{t}, \overline{a}_{B2}^{t}, \ldots, \overline{a}_{Bn}^{t} \right)$ and $\overline{A}_{W}^{t} = \left(\overline{a}_{1W}^{t}, \overline{a}_{2W}^{t}, \ldots, \overline{a}_{nW}^{t} \right)$.

For time period t, the weights of subjects are determined by solving the following interval BWM model:

$$
\begin{aligned}
&\min \varepsilon^{t}, \text{ s.t.} \\
&\left| \theta_{B}^{t} - \overline{a}_{Bj}^{t} \theta_{j}^{t} \right| \lesssim \varepsilon^{t}, \ \forall j, t, \\
&\left| \theta_{j}^{t} - \overline{a}_{jW}^{t} \theta_{W}^{t} \right| \lesssim \varepsilon^{t}, \ \forall j, t, \\
&\textstyle\sum_{j=1}^{n} \theta_{j}^{t} = 1, \quad \forall t; \quad \theta_{j}^{t} \geq 0, \ \forall j, t.
\end{aligned}
\tag{V.1}
$$

Model (V.1) can be converted into the following form by considering crisp equivalents for the constraints:

$$
\begin{aligned}
&\min \varepsilon^{t}, \quad \text{s.t.} \\
&\theta_{B}^{t} - \varepsilon^{t} \leq \left[a_{Bj}^{t,L} + \left(a_{Bj}^{t,U} - a_{Bj}^{t,L} \right) \alpha \right] \theta_{j}^{t}, \ \forall j, t, \\
&\theta_{B}^{t} + \varepsilon^{t} \geq \left[a_{Bj}^{t,L} - \left(a_{Bj}^{t,U} - a_{Bj}^{t,L} \right) \alpha \right] \theta_{j}^{t}, \ \forall j, t, \\
&\theta_{j}^{t} - \varepsilon^{t} \leq \left[a_{jW}^{t,L} + \left(a_{jW}^{t,U} - a_{jW}^{t,L} \right) \alpha \right] \theta_{W}^{t}, \ \forall j, t, \\
&\theta_{j}^{t} + \varepsilon^{t} \geq \left[a_{jW}^{t,L} - \left(a_{jW}^{t,U} - a_{jW}^{t,L} \right) \alpha \right] \theta_{W}^{t}, \ \forall j, t, \\
&\textstyle\sum_{j=1}^{n} \theta_{j}^{t} = 1, \quad \forall t; \quad \theta_{j}^{t} \geq 0, \ \forall j, t.
\end{aligned}
\tag{V.2}
$$

where α $(0 \leq \alpha \leq 1)$ is the possibility level of preferences determined by the director. Model (V.2) is linear; thus, for each time period, solving the problem for any given possibility level α leads to a unique optimal weight vector $\left(\theta_{1}^{t,*}, \theta_{2}^{t,*}, \ldots, \theta_{n}^{t,*} \right)$ and a corresponding objective value $\varepsilon^{t,*}$.

$\varepsilon^{t,*}$ denotes the consistency level of the interval preference relation. If $\varepsilon^{t,*} = 0$ for all α, we have $\theta_{B}^{t} / \theta_{j}^{t} = \overline{a}_{Bj}^{t}$ and $\theta_{j}^{t} / \theta_{W}^{t} = \overline{a}_{jW}^{t}$, and correspondingly $\overline{a}_{Bj}^{t} \times \overline{a}_{jW}^{t} = \overline{a}_{BW}^{t}$ for all j and t. Thus, the interval preference relations $\overline{A}_{B}^{t}$ and $\overline{A}_{W}^{t}$ are entirely consistent.

Step 2 (Dynamic Weights of SoESs' Experts Using the Interval BWM Model): For each time period, the director distinguishes the best and worst experts in each SoES. Afterward, s/he provides the interval preferences (i.e., competence degrees) of all experts of the SoES according to the best and worst experts. $\overline{e}_{j,Bk}^{t} = \left[e_{j,Bk}^{t,L}, e_{j,Bk}^{t,U} \right]$ is the interval preference of Expert_{B}^{j} (the best expert in SoES j) over Expert_{k}^{j} $(k = 1, 2, \ldots, h_j)$ for time period t. $\overline{e}_{j,kW}^{t} = \left[e_{j,kW}^{t,L}, e_{j,kW}^{t,U} \right]$ is the interval preference of Expert_{k}^{j} $(k = 1, 2, \ldots, h_j)$ over Expert_{W}^{j} (the worst expert in SoES j) for time period t.

For SoES j, two interval preference vectors are obtained according to the best and worst experts, respectively, as $\overline{E}_{j,B}^{t} = \left(\overline{e}_{j,B1}^{t}, \overline{e}_{j,B2}^{t}, \ldots, \overline{e}_{j,Bh}^{t} \right)$ and $\overline{E}_{j,W}^{t} = \left(\overline{e}_{j,1W}^{t}, \overline{e}_{j,2W}^{t}, \ldots, \overline{e}_{j,hW}^{t} \right)$.

For time period t, we consider the following interval BWM model to obtain optimal weights of the experts of n SoESs:

$$\begin{aligned}
&\min\ \Delta^t = \sum\nolimits_{j=1}^{n} \theta_j^{t,*} \varepsilon_j^t, \\
&\text{s.t.} \\
&\left.\begin{aligned}
&\left|\lambda_{j,B}^t - \overline{e}_{j,Bk}^t \lambda_{j,k}^t\right| \lesssim \varepsilon_j^t,\ \ \forall k, \\
&\left|\lambda_{j,k}^t - \overline{e}_{j,kW}^t \lambda_{j,W}^t\right| \lesssim \varepsilon_j^t,\ \ \forall k,
\end{aligned}\right\},\ \forall j,t, \\
&\sum\nolimits_{k=1}^{h_j} \lambda_{j,k}^t = 1,\ \ \forall j,t;\ \ \ \lambda_{j,k}^t \geq 0,\ \ \forall k,j,t,
\end{aligned} \tag{V.3}$$

where $\theta_j^{t,*}$ represents the optimal weight of each subject obtained for each time period by solving Model (V.2) in Step 1. Similar to Model (V.2), a crisp equivalent can be formulated for Model (V.3).

The crisp equivalent of Model (V.3) has a linear form; thus, for each time period, its solution leads to a unique optimal weight vector $\left(\lambda_{j,1}^{t,*}, \lambda_{j,2}^{t,*}, \ldots, \lambda_{j,h}^{t,*}\right)$ for each expert and the consistency levels $\left(\varepsilon_1^{t,*}, \varepsilon_1^{t,*}, \ldots, \varepsilon_n^{t,*}\right)$ for any given possibility level α.

$\Delta^{t,*} = 0$ means the complete consistency of interval competence degrees of all experts of the SoESs for time period t, i.e., $\overline{E}_{j,B}^t$ and $\overline{E}_{j,W}^t$. However, the interval preference relation may not be so precise to reach the complete consistency in practice. In $\overline{E}_{j,B}^t$ and $\overline{E}_{j,W}^t$, small deviations from the complete consistency situation lead to small positive values of $\Delta^{t,*}$. Thus, the magnitude of $\Delta^{t,*}$ represents the consistency level of multi-person decisions.

Step 3 (Dynamic Weights of Alternatives Using the Interval Multi-Person BWM Model): For time period t, Expert_k^j selects the best and worst alternatives according to his/her competence in C_k^j (*i.e.,* the k th criterion in the category of the j th subject). Afterward, the expert considers interval preferences of all alternatives regarding the best and worst alternatives.

$\overline{q}_{j,Bi}^{k,t} = \left[q_{j,Bi}^{k,t,L}, q_{j,Bi}^{k,t,U}\right]$ is the interval preference of A_B (the best alternative) over A_i considered by Expert_k^j for time period t. $\overline{q}_{j,iW}^{k,t} = \left[q_{j,iW}^{k,t,L}, q_{j,iW}^{k,t,U}\right]$ is the interval preference of A_i over A_W (the worst alternative) considered by Expert_k^j for time period t. For SoES j and time period t, two interval preference vectors are obtained according to the best and worst alternatives, respectively, as $\overline{Q}_{j,B}^{k,t} = (\overline{q}_{j,B1}^{k,t}, \overline{q}_{j,B2}^{k,t}, \ldots, \overline{q}_{j,Bm}^{k,t})$ and $\overline{Q}_{j,W}^{k,t} = \left(\overline{q}_{j,1W}^{k,t}, \overline{q}_{j,2W}^{k,t}, \ldots, \overline{q}_{j,mW}^{k,t}\right)$.

For each time period, the following interval multi-person BWM model (as a multi-objective programming problem) is derived as follows:

$$\begin{aligned}
\min\ \mathbf{F}^t &= \left[f_1^t,\ \ldots,\ f_j^t,\ \ldots,\ f_n^t\right] \\
&= \left[\sum\nolimits_{k=1}^{h_1} \lambda_{1,k}^{t,*}\varepsilon_{1,k}^t,\ \ldots, \sum\nolimits_{k=1}^{h_j} \lambda_{j,k}^{t,*}\varepsilon_{j,k}^t,\ \ldots,\right. \\
&\qquad\qquad\qquad \left.\sum\nolimits_{k=1}^{h_n} \lambda_{n,k}^{t,*}\varepsilon_{n,k}^t\right], \\
&\text{s.t.:} \\
&\left.\begin{array}{l} \left|w_B^t - \bar{q}_{j,Bi}^{k,t} w_i^t\right| \lesssim \varepsilon_{j,k}^t,\ \ \forall\, i,k, \\ \left|w_i^t - \bar{q}_{j,iW}^{k,t} w_W^t\right| \lesssim \varepsilon_{j,k}^t,\ \ \forall\, i,k, \end{array}\right\},\ \forall\, j,t, \\
&\sum\nolimits_{i=1}^{m} w_i^t = 1,\quad \forall\, t;\ \ w_i^t \geq 0,\ \ \forall\, i,t,
\end{aligned} \tag{V.4}$$

where $\lambda_{j,k}^{t,*}$ is the optimal weight of each expert of SoESs in time period t computed based on Model (V.3) in Step 2. $\mathbf{F}^t = \left[f_1^t, \ldots, f_j^t, \ldots, f_n^t\right]$ is the objective function vector including the set of objective functions of each SoES in time period t, i.e., $f_j^t = \sum_{k=1}^{h_j} \lambda_{j,k}^{t,*}\varepsilon_{j,k}^t$.

Model (V.4) can be solved using multi-objective programming methods such as Global Criterion Method, which is supported on L*p*-metric. Based on this method, an optimal vector is obtained that minimizes a global criterion [158]. The transformed form of Model (V.4) based on the global criterion method is as follows:

$$\begin{aligned}
&\min\ \hat{F}^t = \sum\nolimits_{j=1}^{n} \theta_j^{t,*} \left(\frac{\left|f_j^{t,*} - f_j^t\right|}{f_j^{t,*}}\right)^p, \\
&\text{s.t.:} \\
&\left.\begin{array}{l} \left|w_B^t - \bar{q}_{j,Bi}^{k,t} w_i^t\right| \lesssim \varepsilon_{j,k}^t,\ \ \forall\, i,k, \\ \left|w_i^t - \bar{q}_{j,iW}^{k,t} w_W^t\right| \lesssim \varepsilon_{j,k}^t,\ \ \forall\, i,k, \end{array}\right\},\ \forall\, j,t, \\
&\sum\nolimits_{i=1}^{m} w_i^t = 1,\quad \forall\, t;\ \ w_i^t \geq 0,\ \ \forall\, i,t,
\end{aligned} \tag{V.5}$$

where $\theta_j^{t,*}$ is the optimal weight of each subject in time period t attained through solving Model (V.2) in Step 1. $f_j^{t,*}$ is the ideal objective function of each SoES in time period t. The set of $f_j^{t,*}$ for different values of j represents as $\mathbf{F}^{t,*} = \left[f_1^{t,*},\ \ldots,\ f_j^{t,*},\ \ldots,\ f_n^{t,*}\right]$.

Based on the concept of the global criterion method [159], the aim of Model (V.5) is how close we can get to ideal objective function vector $\mathbf{F}^{t,*}$. The exponent of the objective in Model (V.5), i.e., p, is an integer considered from the set $\{1,\ 2,\ \ldots,\ \infty\}$.

The value of p, considered by the director, denotes the type of distance. For $p = 1$, the deviation terms with lower values are considered more significant. As the value of p increases from 2 to ∞, the greater deviation terms gain more importance [159]. For $p = \infty$, only the greatest deviation term is important and the summation operator converts to maximization of expression $\theta_j^{t,*}\left(\left|f_j^{t,*} - f_j^t\right| / f_j^{t,*}\right)$ for different values of j.

Similar to Model (V.2), a crisp equivalent can be formulated for Model (V.5). The crisp equivalent of Model (V.5) has a linear form; thus, its solution leads to a unique optimal weight vector $\left(w_1^{t,*},\, w_2^{t,*},\, \ldots,\, w_m^{t,*}\right)$ and the consistency levels $\left(\varepsilon_{j,1}^{t,*},\, \varepsilon_{j,2}^{t,*},\, \ldots,\, \varepsilon_{j,h_j}^{t,*}\right)$ for each time period and subject j and any given possibility level α.

$\widehat{F}^{t,*} = 0$ means the complete consistency of interval preference relations of all SoESs for time period t, i.e., $\bar{Q}_{j,B}^{k,t}$ and $\bar{Q}_{j,W}^{k,t}$. However, the interval preference relation may not be so precise to reach the complete consistency in practice. In $\bar{Q}_{j,B}^{k,t}$ and $\bar{Q}_{j,W}^{k,t}$, slight deviations from the complete consistency situation lead to small positive values of $\widehat{F}^{t,*}$. Thus, the magnitude of $\widehat{F}^{t,*}$ is a measure of the consistency level of group decisions in each time period.

As illustrated in Fig. V.2, a dynamic matrix of overall assessments of all experts can be constructed from the results obtained for each time period, where each column is the output of Model (V.5) for the given time period. We show the dynamic matrix in Eq. (V.6). The decision-making process is the same in each time period; however, the judgment of experts may differ based on the variation of their judgments by passing time and changes in the conditions of the problem based on the essential varying dynamic feature of realistic problems.

$$\begin{array}{c} \\ \mathbf{D} = \end{array} \begin{array}{c} \begin{array}{ccccc} P^0 & \cdots & P^t & \cdots & P^T \end{array} \\ \begin{bmatrix} w_1^{0,*} & \cdots & w_1^{t,*} & \cdots & w_1^{T,*} \\ \vdots & & \vdots & & \vdots \\ w_i^{0,*} & \cdots & w_i^{t,*} & \cdots & w_i^{T,*} \\ \vdots & & \vdots & & \vdots \\ w_m^{0,*} & \cdots & w_m^{t,*} & \cdots & w_m^{T,*} \end{bmatrix} \end{array} \begin{array}{c} \\ A_1 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{array}. \tag{V.6}$$

2.2. MULTIMOORA-based model for ranking process

In this section, we develop a MULTIMOORA-based model to aggregate the judgments in multiple time periods to reach an integrative outcome of experts' assessment. First, we must compute the integrated significance coefficients of the dynamic decisions to use in the MULTIMOORA-based model.

The integrated significance coefficients of each time period are obtained based on the concept of consistency of the interval multi-person BWM model and Information Entropy Method.

Step 1 (Subjective, Objective, and Integrated Significance Coefficients of the Dynamic Decisions): The optimal values of the objective function of Model (V.5) are considered to calculate subjective significance coefficients of each time period, i.e., $\widehat{F}^{t,*}$. As $\widehat{F}^{t,*}$ represents the consistency index of the interval multi-person BWM model, it can reflect the importance of the ranking of alternatives for each time period.

The normalized values of $\widehat{F}^{t,*}$ are considered as subjective significance coefficients of each time period:

$$s^{t,c} = \widehat{F}^{t,*} \Big/ \sum_{t=0}^{T} \widehat{F}^{t,*}. \tag{V.7}$$

Information Entropy, also called Shannon Entropy, is a powerful tool to compute objective significance coefficients in D-MpMcDM problems. In the theory of Information Entropy, contrasts of data sets are applied to help distinguish more significant sets [160].

To compute the objective significance coefficients, first, the dynamic matrix, introduced in Eq. (V.6), has to be normalized as:

$$v_i^t = w_i^{t,*} \Big/ \sum_{i=1}^{m} w_i^{t,*} \tag{V.8}$$

Second, Information Entropy Measure is computed using the normalized dynamic matrix as follows:

$$E^t = -k \sum_{i=1}^{m} \left[v_i^t \ln\left(v_i^t\right) \right] \tag{V.9}$$

where $k = 1/\ln(m)$. Objective significance coefficients are computed based on Information Entropy as:

$$s^{t,e} = \left(1 - E^t\right) \Big/ \sum_{t=0}^{T} \left(1 - E^t\right). \tag{V.10}$$

Subjective and objective significance coefficients, i.e., $s^{t,c}$ and $s^{t,e}$, respectively, are consolidated to form integrated significance coefficients as:

$$s^{t,*} = \left(s^{t,c}.s^{t,e}\right) \Big/ \sum_{t=0}^{T} \left(s^{t,c}.s^{t,e}\right). \tag{V.11}$$

Step 2 (Subordinate Utilities and Rankings of Alternatives Based on MULTIMOORA Approach): In this step, we present the derivation of MULTIMOORA subordinate parts, i.e., Ratio System, Reference Point Approach, and Full Multiplicative Form.

The utility of Ratio System is calculated as follows:

$$y_i = \sum_{t=0}^{T} \left(s^{t,*} w_i^{t,*}\right), \tag{V.12}$$

where $s^{t,*}$ and $w_i^{t,*}$ are the integrated significance coefficients of each time period and dynamic weights of alternatives based on Eqs. (V.11) and (V.12), respectively. The utilities of y_i are ordered descendingly to reach the ranking of alternatives based on Ratio System.

The utility of Reference Point Approach is obtained as follows:

$$z_i = \max_t \left| \left(s^{t,*} r^t\right) - \left(s^{t,*} w_i^{t,*}\right) \right|, \tag{V.13}$$

where the maximal objective reference point, i.e., r^t, equals to $\max_i w_i^{t,*}$. The utilities of z_i are ordered ascendingly to find the ranking of alternatives based on Reference Point Approach.

The utility of Full Multiplicative Form is generated as:

$$u_i = \prod_{t=0}^{T} \left(w_i^{t,*}\right)^{s^{t,*}}. \tag{V.14}$$

The utilities of u_i are arranged descendingly to obtain the ranking of alternatives based on Full Multiplicative Form.

Step 3 (Final Utility and Ranking of Alternatives Based on MULTIMOORA-Borda Approach): We use the concept of Borda Rule to integrate the subordinate results of MULTIMOORA.

Borda Rule is a technique from the category of voting theory. From the viewpoint of computation and straightforwardness for decision-makers, Borda Rule outweighs the dominance theory traditionally used in MULTIMOORA method [18].

The formula for generating the final utility of MULTIMOORA-Borda approach is:

$$MB_i = y_i^* \frac{m - r(y_i) + 1}{m\,(m+1)/2} - z_i^* \frac{r(z_i)}{m\,(m+1)/2} + u_i^* \frac{m - r(u_i) + 1}{m\,(m+1)/2}, \quad \text{(V.15)}$$

where y_i^*, z_i^*, and u_i^* are the normalized subordinate utilities computed similar to Eq. (V.8). $r(y_i)$, $r(z_i)$, and $r(u_i)$ are the rankings obtained for Ratio System, Reference Point Approach, and Full Multiplicative Form, respectively. m is the number of alternatives.

The utilities of MB_i are arranged descendingly to obtain the final ranking of alternatives based on MULTIMOORA-Borda approach.

3. Application of DI_S-MpMcDM_SoES methodology in energy sector

This section discusses a practical problem regarding an energy investment project. The case study concerns investment in renewable energy sources (see Fig. V.3) tackled using the proposed D-MpMcDM-SoESs methodology.

As illustrated in Fig. V.4, a SoESs structure is considered because different competences are needed for the problem. The decision-making process is handled by four SoESs focused on financial, technological, environmental, and social subjects. As the director, a project manager supervises them. Each SoESs includes four experts. A project manager assesses the relative importance of the problem subjects and evaluates the relative competence degree of the experts of each SoES. Each expert of a SoES appraises the weights of the candidate renewable energy sources based on their specific competence in a single criterion.

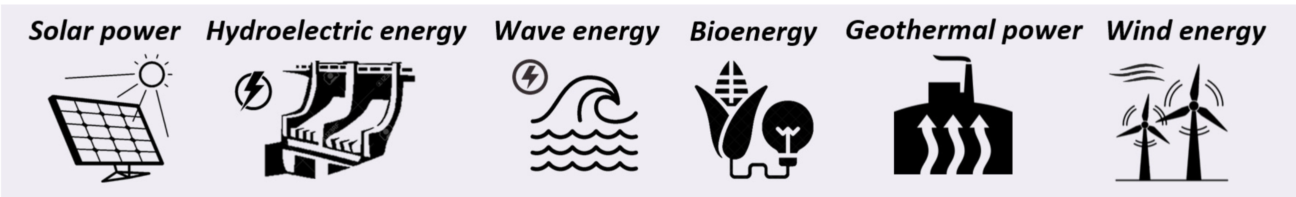

Fig. V.3. The case study of the DI_S-MpMcDM_SoES methodology: Renewable energy investment project.

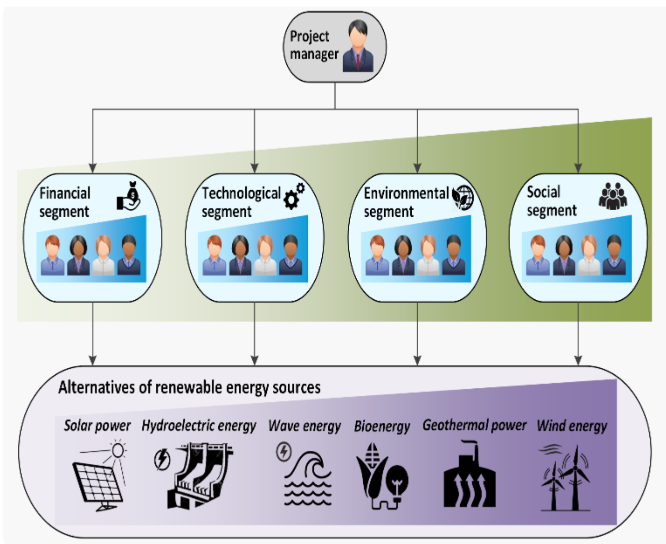

Fig. V.4. Supervised multi-person structure for the case study of the RF_S-MpMcDM_FRFP methodology.

The alternatives of renewable energy sources are six items: solar power, hydroelectric energy, wave energy, bioenergy, geothermal energy, and wind energy.

Solar power can be collected through photovoltaic cells, specially-designed power plants, or water heating systems operated based on sunshine. Hydroelectric energy is a traditional form of harnessing clean energy. The water flowing off the dam powers a turbine to produce electricity. Wave energy is captured utilizing different devices such as absorbers or inverted-pendulum systems. Bioenergy can be absorbed from any organic matter called biomass, which can be directly fired or processed into gas (biogas) or liquid (biofuel). Geothermal energy is obtained by pumping water down into the underground, specifically near the boundaries of Earth's tectonic plates. Wind energy is gathered by turbines established on windy lands [161].

First, we solve the decision-making problem for the present time $(t=0)$, i.e., the data of Oct-2021, based on Models (V.1)–(V.5). Linguistic preferences for all judgments made in Models (V.1)–(V.5) are the same values listed previously in Table III.4. The table also shows the equivalent interval values for the linguistic preferences.

Interval preference vectors considered for obtaining weights of subjects by the project manager are gathered in Table V.1. Besides, optimal weights of the subjects obtained from Model (V.2), i.e., $\theta_j^{0,*}$, for possibility level 0.5 are listed in the table. The value of the objective function of Model (V.2), i.e., $\varepsilon^{0,*}$, is equal to 0.085. Table V.2 gives the interval preference vectors considered by the project manager for comparing experts of SoESs.

Table V.1. Interval preferences of subject of SoESs assessed by the project manager (obtained for the present time $(t=0)$).

Interval preferences and optimal weights	Finance	Technology	Environment	Society
$\overline{A}_B^0$ (Best-to-Others vector) *Best subject*: Environment	[3, 5]	[7, 9]	[1, 1]	[3, 5]
$\overline{A}_W^0$ (Others-to-Worst vector) *Worst subject*: Technology	[3, 5]	[1, 1]	[7, 9]	[3, 5]
$\theta_j^{0,*}$ (Optimal weights)	0.170	0.064	0.596	0.170

Table V.2. Competence degrees of the experts of SoESs assessed by project manager (obtained for the present time $(t=0)$).

SoES	Best expert	Worst expert	Interval preferences transformed from linguistic terms				
				Expert 1	Expert 2	Expert 3	Expert 4
Financial segment	Expert 1	Expert 3	$\overline{E}_{1,B}^0$	[1, 1]	[1, 3]	[7, 9]	[3, 5]
			$\overline{E}_{1,W}^0$	[7, 9]	[5, 7]	[1, 1]	[1, 3]
Technological segment	Expert 2	Expert 1	$\overline{E}_{2,B}^0$	[7, 9]	[1, 1]	[1, 3]	[3, 5]
			$\overline{E}_{2,W}^0$	[1, 1]	[7, 9]	[5, 7]	[3, 5]
Environmental segment	Expert 3	Expert 4	$\overline{E}_{3,B}^0$	[3, 5]	[3, 5]	[1, 1]	[7, 9]
			$\overline{E}_{3,W}^0$	[3, 5]	[1, 3]	[7, 9]	[1, 1]
Social segment	Expert 4	Expert 2	$\overline{E}_{4,B}^0$	[1, 3]	[7, 9]	[1, 3]	[1, 1]
			$\overline{E}_{4,W}^0$	[5, 7]	[1, 1]	[5, 7]	[7, 9]

The six alternatives of renewable energy sources are assessed according to the related criterion based on the comments of each expert of the four SoESs. Table V.3 lists the relative interval preference degrees of the renewable energy sources provided by financial, technological, environmental, and social segments.

Table V.3. Interval preference degrees of the alternatives of renewable energy sources assessed by SoESs (obtained for the present time $(t=0)$) .

SoES	Expert of SoES	Criterion	Best alternative	Worst alternative	Interval preferences transformed from linguistic terms						
						Solar power	Hydroelectric energy	Wave energy	Bioenergy	Geothermal energy	Wind energy
Financial segment	Expert 1	Investment cost	Solar power	Geothermal energy	$\overline{Q}^{1,0}_{1,B}$	[1, 1]	[1, 3]	[3, 5]	[3, 5]	[7, 9]	[5, 7]
					$\overline{Q}^{1,0}_{1,W}$	[7, 9]	[5, 7]	[3, 5]	[1, 3]	[1, 1]	[1, 3]
	Expert 2	Hardware cost	Bioenergy	Geothermal energy	$\overline{Q}^{2,0}_{1,B}$	[5, 7]	[3, 5]	[1, 3]	[1, 1]	[7, 9]	[5, 7]
					$\overline{Q}^{2,0}_{1,W}$	[1, 3]	[3, 5]	[3, 5]	[7, 9]	[1, 1]	[1, 3]
	Expert 3	Maintenance cost	Solar power	Geothermal energy	$\overline{Q}^{3,0}_{1,B}$	[1, 1]	[1, 3]	[1, 3]	[1, 3]	[7, 9]	[5, 7]
					$\overline{Q}^{3,0}_{1,W}$	[7, 9]	[5, 7]	[3, 5]	[5, 7]	[1, 1]	[1, 3]
	Expert 4	Payback period	Solar power	Wave energy	$\overline{Q}^{4,0}_{1,B}$	[1, 1]	[3, 5]	[7, 9]	[1, 3]	[3, 5]	[3, 5]
					$\overline{Q}^{4,0}_{1,W}$	[7, 9]	[1, 3]	[1, 1]	[5, 7]	[3, 5]	[1, 3]
Technological segment	Expert 1	Efficiency	Wind energy	Hydroelectric energy	$\overline{Q}^{1,0}_{2,B}$	[5, 7]	[7, 9]	[1, 3]	[5, 7]	[3, 5]	[1, 1]
					$\overline{Q}^{1,0}_{2,W}$	[1, 3]	[1, 1]	[3, 5]	[1, 3]	[3, 5]	[7, 9]
	Expert 2	Capacity factor	Wind energy	Wave energy	$\overline{Q}^{2,0}_{2,B}$	[3, 5]	[5, 7]	[7, 9]	[1, 3]	[5, 7]	[1, 1]
					$\overline{Q}^{2,0}_{2,W}$	[1, 3]	[1, 3]	[1, 1]	[5, 7]	[1, 3]	[7, 9]
	Expert 3	Technical maturity	Wind energy	Hydroelectric energy	$\overline{Q}^{3,0}_{2,B}$	[3, 5]	[7, 9]	[5, 7]	[1, 3]	[1, 3]	[1, 1]
					$\overline{Q}^{3,0}_{2,W}$	[1, 3]	[1, 1]	[1, 3]	[5, 7]	[3, 5]	[7, 9]
	Expert 4	Feasibility	Solar power	Hydroelectric energy	$\overline{Q}^{4,0}_{2,B}$	[1, 1]	[7, 9]	[1, 3]	[5, 7]	[1, 3]	[5, 7]
					$\overline{Q}^{4,0}_{2,W}$	[7, 9]	[1, 1]	[3, 5]	[1, 3]	[3, 5]	[1, 3]
Environmental segment	Expert 1	Pollution emission	Wave energy	Bioenergy	$\overline{Q}^{1,0}_{3,B}$	[3, 5]	[3, 5]	[1, 1]	[7, 9]	[5, 7]	[1, 3]
					$\overline{Q}^{1,0}_{3,W}$	[1, 3]	[1, 3]	[7, 9]	[1, 1]	[1, 3]	[3, 5]
	Expert 2	Land use	Wind energy	Geothermal energy	$\overline{Q}^{2,0}_{3,B}$	[5, 7]	[3, 5]	[1, 3]	[5, 7]	[7, 9]	[1, 1]
					$\overline{Q}^{2,0}_{3,W}$	[1, 3]	[3, 5]	[5, 7]	[1, 3]	[1, 1]	[7, 9]
	Expert 3	Water use	Wind energy	Bioenergy	$\overline{Q}^{3,0}_{3,B}$	[3, 5]	[5, 7]	[1, 3]	[7, 9]	[5, 7]	[1, 1]
					$\overline{Q}^{3,0}_{3,W}$	[1, 3]	[1, 3]	[3, 5]	[1, 1]	[1, 3]	[7, 9]
	Expert 4	Noise emission	Wind energy	Bioenergy	$\overline{Q}^{4,0}_{3,B}$	[1, 3]	[5, 7]	[1, 3]	[7, 9]	[3, 5]	[1, 1]
					$\overline{Q}^{4,0}_{3,W}$	[5, 7]	[1, 3]	[5, 7]	[1, 1]	[3, 5]	[7, 9]
Social segment	Expert 1	Job creation	Solar power	Bioenergy	$\overline{Q}^{1,0}_{4,B}$	[1, 1]	[5, 7]	[3, 5]	[7, 9]	[1, 3]	[3, 5]
					$\overline{Q}^{1,0}_{4,W}$	[7, 9]	[1, 3]	[3, 5]	[1, 1]	[5, 7]	[1, 3]
	Expert 2	Social acceptance	Hydroelectric energy	Bioenergy	$\overline{Q}^{2,0}_{4,B}$	[1, 3]	[1, 1]	[5, 7]	[7, 9]	[3, 5]	[3, 5]
					$\overline{Q}^{2,0}_{4,W}$	[3, 5]	[7, 9]	[1, 3]	[1, 1]	[3, 5]	[1, 3]
	Expert 3	Market maturity	Solar power	Bioenergy	$\overline{Q}^{3,0}_{4,B}$	[1, 1]	[1, 3]	[5, 7]	[7, 9]	[5, 7]	[1, 3]
					$\overline{Q}^{3,0}_{4,W}$	[7, 9]	[3, 5]	[1, 3]	[1, 1]	[1, 3]	[5, 7]
	Expert 4	Fatal accidents	Solar power	Geothermal energy	$\overline{Q}^{4,0}_{4,B}$	[1, 1]	[1, 3]	[3, 5]	[3, 5]	[7, 9]	[1, 3]
					$\overline{Q}^{4,0}_{4,W}$	[7, 9]	[5, 7]	[1, 3]	[3, 5]	[1, 1]	[5, 7]

For each time, the computation procedure is similar to the present time. For briefness, we ignore to present the preferences tables and only report the final results, i.e., the optimal weights obtained based on Model (V.5). Utilizing the outcomes of Oct-2020, Apr-2021, and Oct-2021, besides the present time results, i.e., Apr-2022, we can construct a dynamic matrix as presented in Table V.**4** (mathematically defined previously in Eq. (V.6)). The data of Table V.**4** is obtained for the possibility level 0.5 of interval preferences and the exponent value 2 of the global criterion method.

Table V.4. Dynamic matrix of the results based on the data of the four time periods.

Renewable energy sources	Results of the interval multi-person BWM model for the four time periods			
	Oct-2020 ($t = -3$)	Apr-2021 ($t = -2$)	Oct-2021 ($t = -1$)	Apr-2022 ($t = 0$)
Solar power	0.148	0.116	0.151	0.153
Hydroelectric energy	0.103	0.082	0.093	0.102
Wave energy	0.205	0.219	0.199	0.191
Bioenergy	0.079	0.063	0.067	0.076
Geothermal energy	0.103	0.082	0.088	0.096
Wind energy	0.362	0.438	0.402	0.382

Table V.5 put forward utilities and rankings of MULTIMOORA-Borda for the case study on renewable energy investment. For Reference Point Approach, the rankings are calculated by arranging the related utilities ascendingly, while for the two other subordinate methods, rankings are obtained based on descending order.

Final utilities are obtained based on Borda Rule The final rankings are calculated by arranging the related utilities descendingly. The best alternative based on subordinate and final results of MULTIMOORA-Borda is similar, *i.e.*, Alternative 6 (wind energy).

The ranking lists of Ratio System and Full Multiplicative Form precisely match. The ranks of Reference Point Approach are very close to those of the two other techniques, i.e., they only differ in one rank (related to geothermal energy).

Table V.5. Utilities and rankings of MULTIMOORA-Borda for the renewable energy investment project.

Renewable energy sources	Ratio system	Reference point approach	Full multiplicative form	MULTIMOORA-Borda
Solar power	0.142 (3)	0.081 (3)	0.141 (3)	0.028 (3)
Hydroelectric energy	0.096 (4)	0.098 (4)	0.096 (4)	-0.015 (4)
Wave energy	0.204 (2)	0.059 (2)	0.204 (2)	0.085 (2)
Bioenergy	0.073 (6)	0.107 (6)	0.073 (6)	-0.062 (6)
Geothermal energy	0.094 (5)	0.098 (4)	0.094 (5)	-0.024 (5)
Wind energy	0.391 (1)	0 (1)	0.389 (1)	0.223 (1)

Chapter VI

Conclusions, publications, and future works

In this chapter, we present the conclusions of the overview and methodologies as well as the published research works based on the thesis and future works in Sections 1, 2, and I.2, respectively.

1. Conclusions

In this section, we present the concluding remarks regarding the three proposed methodologies (i.e., I_S-MpMcDM, RF_S-MpMcDM_FRFP, and DI_S-MpMcDM_SoESs) as well as the points realized by analyzing the previous studies based on the overview on MULTIMOORA.

The following points can be concluded from the MULTIMOORA overview:

- Among uncertain sets, triangular fuzzy number, as a simple fuzzy number, is mostly applied to develop extensions.
- some uncertain developments need more mathematical concepts for generating the models; however, there are several developments which do not require uncertain arithmetic because they simply use score functions which only need crisp arithmetic

The following points can be concluded from the I_S-MpMcDM methodology:

- The formulation of the proposed methodology is based on total interval computation. That is, the proposed theory of interval numbers lead to interval assessment values for subordinate parts of the interval MULTIMOORA-based model. Interval data is usually the outcome of laboratory experiments performed in many industries. Thus, in contrast to the most fuzzy/interval MpMcDM techniques existed in the literature, the model based on I_S-MpMcDM methodology does not need any degradation of the interval numbers. As a result, no data is missed in the modeling and the results are more reliable.
- The integrated weights of criteria used in this methodology combine the comments of experts, based the interval BWM-based model, and the disorder of interval data of decision matrix, based on interval entropy. Thus, the consolidation of the two concepts results in more robust weights of criteria.

The following points can be concluded from the RF_S-MpMcDM_FRFP methodology:

- The risky fuzzy BWM-based model without considering director's decision theoretically leads to more consistent criteria weights, but considering decisions of the expert panel is important in practice.

- Different values of possibility level of the risky fuzzy preferences nearly do not influence the consistency index of the risky fuzzy BWM-based model.
- This risky fuzzy decision-making methodology fits well to the biomaterial selection problem because the work stresses of biomedical engineers may cause irrational decisions.

The following points can be concluded from the DI_S-MpMcDM_SoESs methodology:

- The triple interval BWM-based models are efficient for obtaining dynamic weights in the DI_S-MpMcDM_SoESs problem as the obtained values for consistency levels are high.
- The MULIMOORA-based model is effective for the problem as the values of the resultant subordinate and final rankings closely coincide.
- Considering various values for possibility levels of interval experts' preferences has a negligible impact on final ranking and does not change the best alternative.

2. Publications

The two publications based on the thesis are as follows:

MULTIMOORA overview was published in *Information Fusion* (2019). The details of the publication is:

- *Arian Hafezalkotob, Ashkan Hafezalkotob, Huchang Liao, and Francisco Herrera, "An overview of MULTIMOORA for multi-criteria decision-making: Theory, developments, applications, and challenges," Information Fusion, vol. 51, pp. 145–177, 2019.*

The I_S-MpMcDM methodology was published in *IEEE Transactions on Cybernetics* (2020). The details of the publication is:

- *Arian Hafezalkotob, Ashkan Hafezalkotob, Huchang Liao, and Francisco Herrera. Interval MULTIMOORA method integrating interval Borda rule and interval best-worst-method-based weighting model: Case study on hybrid vehicle engine selection. IEEE Transactions on Cybernetics, vol. 50 (3), pp. 1157–1169, 2020.*

3. Future works

In this section, we give several guidelines for future research in the regard of the theory and application of the three proposed methodologies as well as the challenges for development of the MULTIMOORA.

The following directions are suggested for future works in the area of the MULTIMOORA:

- Prospective researchers may focus on cooperative and non-cooperative multi-person MULTIMOORA models. The comparative analysis of the cooperative and non-cooperative multi-person decision-making models can also be interesting.
- Risk management and data mining approaches are less worked in the studies on MULTIMOORA. Measuring multifarious risks and specifying the acceptability degree of each risk and analysis of costs and advantages of considering risks are the challenges at the heart of real-world decision-making problems.

The following directions are suggested for future works following the I_S-MpMcDM methodology:

- We considered that director only assesses the relative importance of the experts. The methodology can be enhanced to a more general form when director is responsible for evaluating the relative importance of the criteria of the problem.

- The interval BWM-based model can be extended by calculating the lower and upper limits for the subjective optimal weights. That is, the subjective crisp optimal weights obtained in this study can be converted to interval numbers by finding extreme values.

The following directions are suggested for future works following the RF_S-MpMcDM_FRFP methodology:

- The risks of irrational decisions of director and the expert panel can be regarded with different attitudes. Such assumption can be reasonable as director probably has more experience and knowledge about the problem than expert panel.
- The RF_S-MpMcDM_FRFP methodology can be employed in many real-world problems in which risky fuzzy decisions are significant such as practical cases on healthcare management and socioeconomic policy-making.

The following directions are suggested for future works following the DI_S-MpMcDM_SoESs methodology:

- The proposed DI_S-MpMcDM_SoESs methodology may have defects in tackling problems with interdependent subjects. For such problems, an extension of the DI_S-MpMcDM_SoESs methodology considering multiple segments with different competence of experts oriented on each subject can be a feasible solution.
- The behavior assessment of SoESs regarding the risk aversion of the experts' judgments can be appealing future research for which the anticipated regret theory is beneficial.
- Application of realistic fuzzy and linguistic representations to compute uncertain preferences of experts can be another extension of the proposed methodology [162], [163].

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