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Off-shell generalized parton distributions and form factors of the pion

Wojciech Broniowski^{a,b,*}, Vanamali Shastry^b, Enrique Ruiz Arriola^c

^a H. Niewodniczański Institute of Nuclear Physics PAN, 31-342, Cracow, Poland

^b Institute of Physics, Jan Kochanowski University, 25-406, Kielce, Poland

^c Departamento de Física Atómica, Molecular y Nuclear and Instituto Carlos I de Física Teórica y Computacional, Universidad de Granada, E-18071, Granada, Spain

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ABSTRACT

Off-shell effects in generalized parton distributions (GPDs) of the pion, appearing, e.g., in the Sullivan process, are considered. Due to the lack of crossing symmetry, the moments of GPDs involve also odd powers of the skewness (longitudinal momentum transfer) parameter, which results in emergence of new off-shell form factors. With current-algebra techniques, we derive exact relations between the four off-shell gravitational form factors of the pion, in analogy to the electromagnetic case. Our results place stringent constraints on the off-shell GPDs of the pion. We provide an explicit realization in terms of a chiral quark model, where we show that the off-shell effects in GPDs are potentially significant in modeling physical processes and should not be neglected.

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In recent papers [1,2], accessibility of the pion GPDs [3,4] via the Sullivan process [5] in future electron-ion colliders has been studied, with the conclusion that it may soon fall within experimental reach. Since the corresponding amplitude involves an off-shell pion (cf. Fig. 1), one needs to care about the possible off-shellness issues in such processes and in the GPDs themselves. Whereas these admittedly unmeasurable effects would cancel in an ultimate complete calculation of $e^+p \rightarrow e^+\pi^+n\gamma$,¹ they *unavoidably* do show up in phenomenological approaches which treat the building blocks $p \rightarrow \pi^+n$ and $\gamma^*\pi^+ \rightarrow \gamma\pi^+$ as independent subprocesses.

The pion, being a pseudo-Goldstone boson of the spontaneously broken chiral symmetry, is by far the simplest hadron. Yet, its non-perturbative structure is rich, as can be revealed with the methods involving GPDs (the 3D hadronic tomography [6]). In this Letter we show that relations between various off-shell form factors of the pion (electromagnetic, gravitational) provide highly non-trivial constraints for the structure of the off-shell GPDs. These relations,

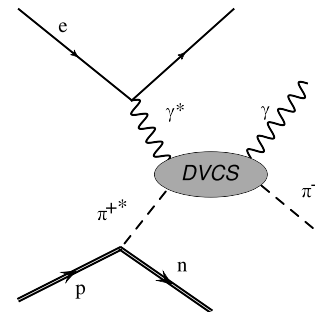


Fig. 1. Sullivan process for the pion electroproduction off the proton, containing the deeply virtual Compton scattering (DVCS) amplitude involving GPDs. Asterisks indicate off-shellness.

which we extend to the case of the four off-shell gravitational form factors, follow from the Ward-Takahashi identities (WTI).

The off-shell quark and gluon GPDs of the pion are defined, in the notation of [7], via the matrix element of bilocal fields

$$\delta_{ab}\delta_{\alpha\beta}H^0(x, \xi, t, p_i^2, p_f^2) + i\epsilon^{abc}\tau_{\alpha\beta}^c H^1(x, \xi, t, p_i^2, p_f^2) = \int \frac{dz^-}{4\pi} e^{ix^+z^-} \langle \pi^b(p_f) | \bar{\psi}_\alpha(-\frac{z}{2}) \gamma^+ \psi_\beta(\frac{z}{2}) | \pi^a(p_i) \rangle \Big|_{z^+=0, z^\perp=0},$$

$$\delta_{ab}H^g(x, \xi, t, p_i^2, p_f^2) = \int \frac{dz^-}{2\pi P^+} e^{ix^+z^-} \langle \pi^b(p_f) | F^{+\mu}(-\frac{z}{2}) F_{\mu^+}(\frac{z}{2}) | \pi^a(p_i) \rangle \Big|_{z^+=0, z^\perp=0}, \tag{1}$$

* Corresponding author.

E-mail addresses: Wojciech.Broniowski@ifj.edu.pl (W. Broniowski), vanamalishastry@gmail.com (V. Shastry), earriola@ugr.es (E. Ruiz Arriola).

¹ Such a calculation, involving all possible QCD diagrams, is not conceivable in a phenomenological approach, but could be hoped for in lattice simulations in distant future.

where ψ indicates the quark field, $F^{\mu\nu}$ is the gluon field tensor, a , b , and c are the isospin indices of the pion, α and β are the quark flavors, and summation over color is implicit. The subscripts 0, 1 denote the isospin of the quark GPDs. The light-cone indices are $v^\pm = (v^0 \pm v^3)/\sqrt{2}$. In the assumed light-cone gauge the link operators do not appear. The adopted symmetric notation for the kinematic variables is

$$P^\mu = \frac{1}{2}(p_f^\mu + p_i^\mu), \quad q^\mu = p_f^\mu - p_i^\mu, \quad \xi = -\frac{q^+}{2p^+}, \quad t = q^2. \quad (2)$$

In the partonic interpretation (in the on-shell case) $p_f^2 = p_i^2 = m_\pi^2$, while $(x + \xi)P^+$ is the longitudinal momentum carried by the struck parton. The GPDs H^0, H^1 and H^g are scale dependent objects which follow the DGLAP-ERBL QCD evolution equations [8,7]. We note that off-shellness of the initial and final hadronic states (pions) does not affect the QCD evolution kernel in the assumed Bjorken limit.

For $p_f^2 = p_i^2$ the crossing symmetry (time-reversal) makes the amplitudes in Eq. (1) even functions of the skewness parameter ξ . This feature no longer holds with general off-shellness, when $p_f^2 \neq p_i^2$, as happens in the Sullivan process of Fig. 1. In particular, in that case the x -moments of the GPDs involve also odd powers of ξ ,

$$\int_{-1}^1 dx x^j H^s(x, \xi, t, p_i^2, p_f^2) = \sum_{i=0}^{j+1} A_{j,i}^s(t, p_i^2, p_f^2) \xi^i, \quad (3)$$

where $s = 0, 1, g$. Among the generalized (off-shell) form factors $A_{j,i}^s$, those related to the conserved electromagnetic and energy-stress tensor currents (the two lowest x -moments) are particularly important. They do not depend on the factorization scale, and therefore are independent of the QCD evolution. Explicitly,

$$\int_{-1}^1 dx H^1 = 2(F - G\xi), \quad (4)$$

$$\int_{-1}^1 dx x[H^0 + H^g] = \theta_2 - \theta_3\xi - \theta_1\xi^2,$$

where the form factors are functions of (t, p_i^2, p_f^2) . Thus the above conditions depend nontrivially on t and the off-shellness parameters.

From a dynamical point of view, the complementary role of the electromagnetic and gravitational form factors at zero momentum transfer, ensuring a proper normalization of the Bethe-Salpeter bound state equation for on-shell states, was recognized long ago [9]. Here we consider a general off-shell case. First, we recall for completeness the results obtained for the off-shell effects in the pion charge form factors [10,11]. The (one-particle irreducible, renormalized) pion-photon vertex (we take the positively charged pion for definiteness) has the general covariant structure

$$\Gamma^\mu(p_i, p_f) = 2P^\mu F(t, p_i^2, p_f^2) + q^\mu G(t, p_i^2, p_f^2). \quad (5)$$

Next, one considers the WTI for the full $\pi\pi\gamma$ vertex (with unamputated external pion propagators):

$$(2\pi)^4 \delta^{(4)}(p_f - p_i - q) G^\mu(p_i, p_f) = \int d^4x d^4y d^4z \times e^{i(p_f \cdot x - p_i \cdot y - q \cdot z)} \langle 0 | T(\phi^+(x) \phi^-(y) J^\mu(z)) | 0 \rangle, \quad (6)$$

where the standard use of the covariantized time order product (or the T^* product), with the time derivatives pulled outside, is

understood from now on. Using the current algebra of the vector and axial currents [12], one finds

$$(2\pi)^4 \delta^{(4)}(p_f - p_i - q) q_\mu G^\mu(p_i, p_f) = i \int d^4x d^4y \times \left(e^{ip_f \cdot (x-y)} - e^{ip_i \cdot (x-y)} \right) \langle 0 | T(\phi^+(x) \phi^-(y)) | 0 \rangle, \quad (7)$$

which yields $q_\mu G^\mu(p_i, p_f) = \Delta(p_f^2) - \Delta(p_i^2)$, where $\Delta(p^2)$ denotes the pion propagator. Next, one passes to the irreducible vertex Γ^μ by the standard leg amputation procedure, namely

$$\Gamma^\mu(p_i, p_f) = \left(i\Delta(p_f^2) \right)^{-1} G^\mu(p_i, p_f) \left(i\Delta(p_i^2) \right)^{-1}, \quad (8)$$

which finally gives the WTI of [11],

$$q_\mu \Gamma^\mu(p_i, p_f) = \Delta^{-1}(p_f^2) - \Delta^{-1}(p_i^2), \quad (9)$$

with $\Delta(p^2)$ denoting the pion propagator. From Eq. (5) one finds that $q_\mu \Gamma^\mu = (p_f^2 - p_i^2)F + tG$, hence the relation

$$(p_f^2 - p_i^2)F(t, p_i^2, p_f^2) + tG(t, p_i^2, p_f^2) = \Delta^{-1}(p_f^2) - \Delta^{-1}(p_i^2) \quad (10)$$

follows. At $t = 0$ (under the natural assumption that $G(t, p_i^2, p_f^2)$ is not singular) one obtains

$$\Delta^{-1}(p_f^2) - \Delta^{-1}(p_i^2) = (p_f^2 - p_i^2)F(0, p_i^2, p_f^2), \quad (11)$$

therefore

$$G(t, p_i^2, p_f^2) = \frac{(p_f^2 - p_i^2)}{t} \left[F(0, p_i^2, p_f^2) - F(t, p_i^2, p_f^2) \right] \quad (12)$$

and $G(0, p_i^2, p_f^2) = (p_i^2 - p_f^2) dF(t, p_i^2, p_f^2)/dt|_{t=0}$. The simplicity of the result should not cover up its depth, namely, the off-shell G form factor is completely expressible via the off-shell F form factor. Further, at the pion pole $\Delta^{-1}(m_\pi^2) = 0$, hence one finds from Eq. (11) that the half-off shell form factors at $t = 0$ are

$$F(0, m_\pi^2, p^2) = F(0, p^2, m_\pi^2) = \frac{\Delta^{-1}(p^2)}{(p^2 - m_\pi^2)}. \quad (13)$$

Taking the limit $p^2 \rightarrow m_\pi^2$ one gets $F(0, m_\pi^2, m_\pi^2) = 1$, which is the charge normalization of the pion. For equal off-shellness of the initial and final pion, Eq. (12) yields immediately $G(t, p^2, p^2) = 0$, which is a manifestation of the crossing symmetry. The form factor $G(t, p^2, m_\pi^2)/p^2$ has been recently studied phenomenologically in a quark model in [13].

Now we pass to novel results for the off-shell gravitational form factors. The gravitational vertex has the general tensorial structure

$$\Gamma^{\mu\nu} = \frac{1}{2}[(q^2 g^{\mu\nu} - q^\mu q^\nu)\theta_1 + 4P^\mu P^\nu \theta_2 + 2(q^\mu P^\nu + q^\nu P^\mu)\theta_3 - g^{\mu\nu} \theta_4], \quad (14)$$

The form factors θ_1 and θ_2 are even under the crossing symmetry, whereas θ_3 and θ_4 are odd. The WTI for the gravitational vertex can be derived as follows: the full vertex is defined as

$$(2\pi)^4 \delta^{(4)}(p_f - p_i - q) G^{\mu\nu}(p_i, p_f) = \int d^4x d^4y d^4z \times e^{i(p_f \cdot x - p_i \cdot y - q \cdot z)} \langle 0 | T(\phi^+(x) \phi^-(y) \Theta^{\mu\nu}(z)) | 0 \rangle, \quad (15)$$

where $\Theta^{\mu\nu}$ is the energy-stress tensor (involving quarks and gluons), obtained by differentiating the action with respect to the metric tensor. It is conserved, $\partial_\mu \Theta^{\mu\nu} = 0$. Current algebra yields the relation (holding for the PCAC pion, not necessary an elementary field) [14]

$$(2\pi)^4 \delta^{(4)}(p_f - p_i - q) q_\mu G^{\mu\nu}(p_i, p_f) = i \int d^4x d^4y \times$$

$$\left(p_i^\nu e^{ip_i \cdot (x-y)} - p_f^\nu e^{ip_f \cdot (x-y)} \right) \langle 0 | T (\phi^+(x) \phi^-(y)) | 0 \rangle, \quad (16)$$

hence $q_\mu G^{\mu\nu}(p_i, p_f) = p_i^\nu \Delta(p_f^2) - p_f^\nu \Delta(p_i^2)$. Finally, the irreducible gravitational vertex becomes

$$q_\mu \Gamma^{\mu\nu}(p_i, p_f) = p_i^\nu \Delta^{-1}(p_f^2) - p_f^\nu \Delta^{-1}(p_i^2) =$$

$$P^\nu [\Delta^{-1}(p_f^2) - \Delta^{-1}(p_i^2)] - \frac{1}{2} q^\nu [\Delta^{-1}(p_f^2) + \Delta^{-1}(p_i^2)]. \quad (17)$$

Remarkably, this relation was first obtained by Brout and Englert [15] using just the general gravitational covariance. On the other hand, from Eq. (14) we get

$$q_\mu \Gamma^{\mu\nu} = (p_f^2 - p_i^2) P^\nu \theta_2 + [t P^\nu + \frac{1}{2}(p_f^2 - p_i^2) q^\nu] \theta_3 - \frac{1}{2} q^\nu \theta_4. \quad (18)$$

Since the four-vectors P and q are linearly independent, comparing their coefficients in Eqs. (17) and (18) we arrive at two relations:

$$(p_f^2 - p_i^2) \theta_2 + t \theta_3 = \Delta^{-1}(p_f^2) - \Delta^{-1}(p_i^2), \quad (19)$$

$$(p_f^2 - p_i^2) \theta_3 - \theta_4 = -[\Delta^{-1}(p_f^2) + \Delta^{-1}(p_i^2)], \quad (20)$$

which is our key result.

Next, we carry out the procedure presented earlier for the charge form factors, now for the case of Eq. (19). At $t = 0$ we have $(p_f^2 - p_i^2) \theta_2(0, p_i^2, p_f^2) = \Delta^{-1}(p_f^2) - \Delta^{-1}(p_i^2)$, therefore

$$\theta_3(t, p_i^2, p_f^2) = \frac{(p_f^2 - p_i^2)}{t} \left[\theta_2(0, p_i^2, p_f^2) - \theta_2(t, p_i^2, p_f^2) \right], \quad (21)$$

with $\theta_3(0, p_i^2, p_f^2) = (p_i^2 - p_f^2) d\theta_2(t, p_i^2, p_f^2)/dt|_{t=0}$. Moreover, comparing to Eq. (11), we find a relation between the off-shell gravitational and charge form factors at $t = 0$,

$$\theta_2(0, p_i^2, p_f^2) = F(0, p_i^2, p_f^2), \quad (22)$$

while for the half-off shell case

$$\theta_2(0, m_\pi^2, p^2) = \theta_2(0, p^2, m_\pi^2) = \frac{\Delta^{-1}(p^2)}{(p^2 - m_\pi^2)}, \quad (23)$$

with $\theta_2(0, m_\pi^2, m_\pi^2) = 1$ expressing the momentum sum rule. From Eq. (20) we find that

$$\theta_4(t, p_i^2, p_f^2) = (p_f^2 - p_i^2) \theta_3(t, p_i^2, p_f^2) + \Delta^{-1}(p_f^2) + \Delta^{-1}(p_i^2). \quad (24)$$

We note that θ_4 vanishes if both the initial and final pions are on mass shell, $\theta_4(t, m_\pi^2, m_\pi^2) = 0$, but in general is non-zero if either pion is off mass shell. The right-hand side can be expressed via θ_2 only,

$$\theta_4(t, p_i^2, p_f^2) = \frac{(p_f^2 - p_i^2)^2}{t} \left[\theta_2(0, p_i^2, p_f^2) - \theta_2(t, p_i^2, p_f^2) \right]$$

$$+ (p_i^2 - m_\pi^2) \theta_2(0, p_i^2, m_\pi^2) + (p_f^2 - m_\pi^2) \theta_2(0, m_\pi^2, p_f^2). \quad (25)$$

We remark that θ_4 does not contribute to the moment in Eq. (3) upon the light-cone projection, as $n_\mu g^{\mu\nu} n_\nu = n^2 = 0$, where $n^\mu = (1, 0, 0, -1)/P^+$. Note that θ_1 , which corresponds to a transverse tensor, does not enter into any constraints from the current conservation. In the chiral limit and the on-shell case of $m_\pi^2 = 0$ one has the low-energy theorem $\theta_1(0, 0, 0) = \theta_2(0, 0, 0)$ [16].

In the last part of this Letter, we illustrate the above general results in a quark model with spontaneously broken chiral symmetry,

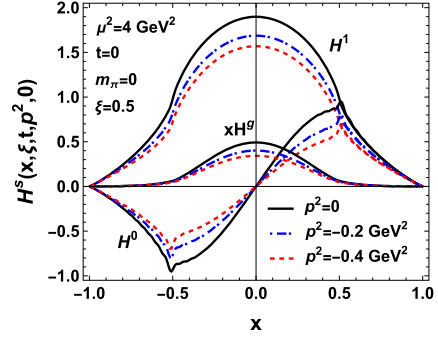


Fig. 2. Half-off-shell effective GPDs of the pion obtained in SQM in the chiral limit for $\xi = 0.5$, $t = 0$, and evolved to $\mu^2 = 4 \text{ GeV}^2$. Line types distinguish the off-shellness, whereas the three bundles correspond to the quark isoscalar GPD, quark isoscalar GPD, and the gluon GPD multiplied with x .

treated at the one-loop (leading- N_c) level. One can straightforwardly obtain expressions for the off-shell charge and gravitational form factors in terms of the appropriate Passarino-Veltman functions and explicitly verify that they comply to all the relations provided above. The formulas become particularly simple in the chiral limit in the Spectral Quark Model (SQM) [17], which is a one-loop chiral quark model with the spectral function chosen in a way that enforces the vector meson dominance of the pion charge form factor. The model is consistent with the chiral, gauge and Lorentz invariance. In the chiral limit, $m_\pi = 0$, manageably short expressions emerge for the half-off-shell case:

$$F(t, p^2, 0) = \frac{M_V^4}{(M_V^2 - p^2)(M_V^2 - t)}, \quad (26)$$

$$G(t, p^2, 0) = \frac{p^2 M_V^2}{(M_V^2 - p^2)(M_V^2 - t)},$$

$$\theta_1(t, p^2, 0) = \frac{M_V^2 \left[\frac{p^2(t-p^2)}{M_V^2 - p^2} + (t - 2p^2)L \right]}{(t - p^2)^2},$$

$$\theta_2(t, p^2, 0) = \frac{M_V^2 \left[\frac{p^2(p^2-t)}{M_V^2 - p^2} + tL \right]}{(t - p^2)^2},$$

$$\theta_3(t, p^2, 0) = \frac{p^2 M_V^2 [p^2 - t + (M_V^2 - p^2)L]}{(t - p^2)^2 (M_V^2 - p^2)},$$

$$\theta_4(t, p^2, 0) = \frac{p^2 M_V^2 [(p^2 - t)(2p^2 - t) + p^2(M_V^2 - p^2)L]}{(t - p^2)^2 (M_V^2 - p^2)},$$

with $L = \log \frac{M_V^2 - p^2}{M_V^2 - t}$ and M_V denoting the ρ meson mass. We note that while in this model, where the inverse pion propagator is $\Delta^{-1}(p^2) = M_V^2 p^2 / (M_V^2 - p^2)$, the charged form factors exhibit a factorized form, this is not the case of the gravitational form factors. The above formulas satisfy all the general relations above, namely Eqs. (12), (21), (22), (24) and (25). It is thus tempting to make a first estimate of the off-shell effects in the pion GPDs in a model implementing these new constraints.

The on-shell GPDs are obtained in SQM at the quark model scale [18], μ_0 , where the valence quarks carry 100% of the momentum, and are subsequently evolved to a higher scale μ with the leading-order DGLAP-ERBL equations [19]. The half-off-shell GPDs at μ_0 involve rather lengthy analytic formulas (not shown here for brevity) and display a lack of factorization in x , t , or p^2 . A sample result (with $t = 0$ and $\xi = 0.5$) at $\mu = 2 \text{ GeV}$ is presented

in Fig. 2. We note a significant dependence on the (space-like) off-shell parameter p^2 . For the difference at the maxima of the curves at $p^2 = -0.2 \text{ GeV}^2$ and $p^2 = 0$ we note the relative effect of 10% for the isovector GPD, and somewhat larger 18% for the isoscalar GPDs. For $p^2 = -0.4 \text{ GeV}^2$ the effects, are, correspondingly, 20% and 35%. As expected, the size depends on p^2 , which is controlled by the kinematics of the Sullivan process. The effect reflects qualitatively the change of normalization with p^2 according to Eq. (4). This feature becomes exact at $\mu \rightarrow \infty$, as then the GPDs become localized in the ERBL region $|x| \leq \xi$ (see e.g. [18]), with the normalization given by Eq. (4), and the relative normalization of xH^0 and xH^g given by the ratio $3N_f/16$, where $N_f = 3$ is the number of active flavors.

The sources of model uncertainty to absolute (not relative) values of the GPDs include the value of the vector meson mass in SQM (which attributes a roughly 10-15% effect to form factors [20] and to parton distributions), the approximation of the exact chiral limit (about 5% [21]), and the uncertainty in the value of the quark-model scale (about 10% [18]).

Finally, we wish to digress on a relevant methodological point, along the lines of [22], concerning evaluation of amplitudes such as in Fig. 1. The off-shellness affects, in general, all the components of the diagram. In our case, it influences the GPD (as discussed above), hence the DVCS amplitude, but also the pion form factor (as well as the pion nucleon form factor, not discussed here). The leg amputation procedure of Eq. (8) presumes that in the propagator attached to the vertex is the full pion propagator, with off-shell effects, and not its pole approximation. Hence we should just use $\Delta(p^2)$ to maintain consistency. If however, as is typically done phenomenologically, one admitted the pion pole term only, $1/(p^2 - m_\pi^2)$, one would miss the factor $\Delta(p^2)(p^2 - m_\pi^2) = 1/F(0, p^2, m_\pi^2)$. This factor could be conventionally attributed to the half-off-shell vertex, by introducing $\Gamma^{*\mu}(t, p^2, m_\pi^2) \equiv \Gamma^\mu(t, p^2, m_\pi^2)/F(0, p^2, m_\pi^2)$, to be used in calculations with the attached pion propagators taken as a pole term. In our case, for the charge form factor we have

$$\Gamma^{*\mu}(t, p^2, m_\pi^2) = 2P^\mu \frac{F(t, p^2, m_\pi^2)}{F(0, p^2, m_\pi^2)} - q^\mu \frac{p^2}{t} \left[1 - \frac{F(t, p^2, m_\pi^2)}{F(0, p^2, m_\pi^2)} \right]. \quad (27)$$

If the dependence on t and p^2 in $F(t, p^2, m_\pi^2)$ factorizes (as is the case of SQM in the chiral limit but not in general), then the only dependence on p^2 sits in front of the factor associated with the q^μ part, which is always present for virtual photons (for a real photon it may be removed by the choice of gauge [22]). This would result in a p^2 independent form factor G/p^2 , as considered in [13]. Similarly to Eq. (27), one could attribute the propagator correction $1/F(0, p^2, m_\pi^2)$ to the half-off-shell GPDs.

Finally, we remind that as discussed in [23], the deeply virtual Compton scattering (DVCS) amplitude, involving the GPDs, enters the cross section formula for the Sullivan process via interference with the Bethe-Heitler amplitude, thus uncertainties in the GPDs carry over linearly to the cross section. That way, the off-shellness contributes to the uncertainties encountered in the modeling of the Sullivan process, together with such quantities as the parton distributions (taken on shell), pion propagator (possibility of inclusion of the excited states) or the pion-nucleon form factor.

To summarize, we have considered, on a general footing, the off-shell GPDs of the pion and the related electromagnetic and gravitational form factors. We have shown that the WTI for the energy-stress tensor results in relations between the four off-shell gravitational form factors, in analogy to the case of the two off-shell electromagnetic form factors. These relations may serve as

consistency constraints in constructing phenomenological off-shell GPDs. We have employed a simple chiral quark model to illustrate the general formalism, as well as to assess the actual size of the effects after the QCD evolution to the scale $\mu = 2 \text{ GeV}$. We find a non-negligible (roughly, 10%) influence already at off-shellness of the order of -0.2 GeV^2 , especially when ξ is not close to 0. We finally note that our analysis can be straightforwardly extended to the other members of the pseudoscalar nonet, in particular the kaons, for which the Sullivan process at the EIC is also currently being considered [24].

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Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

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Data availability

No data was used for the research described in the article.

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