Exploring flavour at the energy frontier in the Little Higgs paradigm

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«The Higgs mechanism is just a reincarnation of the Communist Party: it controls the masses [1].»

Lenin, apocryphal.

Abstract

The Standard Model of Particle Physics is one of the most predictive and elegant theories of the History of Physics. It explains what are the fundamental constituents of matter and how they interact. This model has been tested in a wide variety of scenarios and essentially all the experimental measurements seem to agree with its predictions.

However, in this framework the Higgs boson is an elementary particle. This makes the size of its mass quite unnatural. The Higgs mass, not protected by any symmetry, receives quadratically divergent contributions coming from arbitrarily large energies and thus there is no reason that justifies why the Higgs boson should be light. This is the so called hierarchy problem.

One of the most elegant proposals to face this problem consists of assuming that the Higgs boson, rather than an elementary particle, is a composite state of unknown heavy fermions bounded by a new strong interacting sector. This is motivated by the treatment of mesons in Quantum Chromodynamics. As a consequence, at high energies there is no Higgs because those heavy fermions would be the fundamental constituents of a new theory that extends the Standard Model. If this idea is realized in Nature it would end, once and for all, with the hierarchy problem.

The composite Higgs paradigm can be implemented in many ways, giving rise to a vast family of models. One of the frameworks that have received more attention is the *Littlest Higgs model with T-parity*. In this model the Higgs mass does not receive quadratically divergent contributions. Hence the Higgs is naturally light. Furthermore, the T-parity is a discrete symmetry under which the Standard Model particles are even and most of the new particles are odd. As a consequence, the contributions of these particles to precision observables are one-loop suppressed and thus under control.

Within this framework we will study flavour-changing transitions. In particular, we are interested in the contributions of a heavy fermion singlet that can be either T-even or T-odd under the discrete symmetry. We will show that the contributions of the T-odd singlet to lepton flavour-changing Higgs decays and to neutrino masses do not decouple in the limit of a heavy singlet mass. These issues are not present in the T-even singlet case.

Motivated by the anomalous behaviour of the singlet, we will prove that the Littlest Higgs model with T-parity is not invariant under its gauge group. As a consequence, we will develop a new Littlest Higgs model with T-parity compatible with gauge invariance. For that purpose, the global symmetry group will be minimally enlarged with respect to the original model and new fermion and scalar degrees of freedom will be introduced.

To show explicitly the viability of the model we will impose current constraints on exotic quarks; we will consider that the usual dark photon, the lightest T-odd particle, accounts for all the dark matter relic density of the Universe; and we will demand that the masses of the new scalar fields do not exceed the TeV. This fixes the value of certain parameters while others get correlated, so the particle spectrum gets bounded from below and above, keeping the model viable. Finally, we will study the main decay channels of the new scalar fields to show that their decay rate is comparable to that of the Higgs. In terms of production rates, they are relatively heavy and generated by an electroweak interaction so they are not significantly produced at the LHC.

Resumen

El Modelo Estándar de la Física de Partículas es una de las teorías más predictivas y elegantes de la historia de la Física. Explica cuáles son los constituyentes fundamentales de la materia y cómo son sus interacciones. Ha sido puesto a prueba en una gran variedad de escenarios y prácticamente todas las medidas experimentales están de acuerdo con sus predicciones.

Sin embargo, en este marco teórico el bosón de Higgs es una partícula elemental. Esto hace que su masa sea muy antinatural ya que, no estando protegida por ninguna simetría, recibe correcciones cuadráticas de energías arbitrariamente altas y no hay razón teórica que justifique que el Higgs sea ligero. Es lo que se conoce como el problema de las jerarquías.

Una de las soluciones más elegantes a este problema consiste en asumir que el bosón de Higgs, en lugar de ser elemental, es un estado compuesto de fermiones pesados y desconocidos ligados por un nuevo sector fuertemente interactuante. Esto está motivado por el tratamiento de los mesones en Cromodinámica Cuántica. Como consecuencia, a energías altas no hay bosón de Higgs ya que dichos fermiones pesados serían los constituyentes fundamentales de una nueva teoría que extendería al Modelo Estándar. Si esta idea se lleva a cabo en la Naturaleza se acabaría de raíz con el problema de las jerarquías.

El paradigma de un Higgs compuesto puede ser llevado a cabo de múltiples maneras, dando lugar a una vasta familia de modelos. Uno de los marcos teóricos que más atención ha recibido ha sido el "Littlest Higgs model with T-parity". En este modelo, la masa del Higgs no recibe correcciones cuadráticamente divergentes, lo que hace que el Higgs sea ligero de forma natural. Por otro lado, la T-paridad es una simetría discreta bajo la cual las partículas del Modelo Estándar son pares mientras que la mayor parte de las nuevas partículas son impares. Como consecuencia, las contribuciones de las nuevas partículas a observables de precisión están controladas.

Dentro de este marco teórico estudiaremos transiciones fermiónicas con cambio de sabor. En particular, estaremos interesados en las contribuciones de un singlete leptónico pesado que puede ser tanto par como impar bajo la simetría discreta. Veremos que las contribuciones del singlete impar a desintegraciones del Higgs con leptones de diferente sabor en el estado final, y también a la masa de los neutrinos, no desacoplan en el límite en que la masa del singlete es muy pesada. Estas patologías no están presentes en el caso en que el singlete es par.

Motivados por el comportamiento anómalo de este singlete, demostraremos que el "Littlest Higgs model with T-parity" no es invariante bajo el grupo local de simetrías. Como consecuencia, desarrollaremos un nuevo "Littlest Higgs model" que respeta la invariancia gauge. Para ello el grupo global de simetrías deberá ser extendido mínimamente con respecto al modelo original y será necesario introducir nuevos grados de libertad, tanto escalares como fermiónicos.

Para mostrar explícitamente la viabilidad de este modelo impondremos cotas actuales a la masa de quarks exóticos; supondremos que el fotón oscuro, la partícula impar más ligera, da cuenta de toda la densidad de materia oscura del Universo; y que la masa de los nuevos escalares no supera el TeV. Esto fija el valor de ciertos parámetros del modelo y correlaciona otros, de modo que el espectro de partículas queda acotado inferior y superiormente, haciendo el modelo viable. Finalmente, estudiaremos los principales canales de desintegración de los nuevos escalares y veremos que su ritmo de desintegración es aproximadamente igual al del Higgs. Estos escalares son relativamente pesados y producidos por una interacción electrodébil por lo que no se espera que den lugar a señales apreciables en el LHC.

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Contents

Abstract VII

Resumen IX

Agradecimientos/Acknowledgements XI

1. Introduction 1
- 1.1. Brief description of the Standard Model of Particle Physics . . . 1
- 1.2. The SM as an effective field theory . . . 3
- 1.3. The hierarchy problem and a natural electroweak scale . . . 5
- 1.4. The Higgs as a composite particle . . . 7
  - 1.4.1. Motivation: mesons as low energy QCD bound states . . . 7
    - QCD and confinement . . . 7
    - Effective field theory for the light mesons . . . 8
  - 1.4.2. The composite Higgs idea and a Little Higgs . . . 8
- 1.5. Structure of the Thesis . . . 10

2. General formalism for Little Higgs models 13
- 2.1. Symmetries and the CCWZ formalism . . . 13
  - 2.1.1. Global symmetries and Goldstone bosons . . . 13
  - 2.1.2. Gauge symmetries . . . 17
- 2.2. Vacuum misalignment . . . 18
- 2.3. Collective symmetry breaking and Little Higgs . . . 19
- 2.4. T-parity . . . 21
- 2.5. A toy model . . . 21
  - 2.5.1. Global symmetries . . . 22
  - 2.5.2. Gauge symmetries . . . 23
  - 2.5.3. Lagrangian . . . 24
    - Gauge sector . . . 24
    - Scalar sector . . . 24
    - Fermion sector . . . 24
  - 2.5.4. T-parity implementation . . . 27
    - Gauge sector . . . 27
    - Scalar sector . . . 27
    - Fermion sector . . . 28
- 2.6. Chapter summary . . . 34

3. The Littlest Higgs model with T-parity 37
- 3.1. Littlest Higgs model with T-parity setup . . . 37
  - 3.1.1. Requirements . . . 37
  - 3.1.2. Global symmetries . . . 38
  - 3.1.3. Gauge symmetry . . . 39
  - 3.1.4. Lagrangian . . . 40

Gauge sector . . . 40
Scalar sector . . . 41
Fermion sector . . . 42
3.1.5. Mass eigenfields . . . 49
Gauge fields . . . 49
Scalar fields after gauge fixing . . . 50
Fermion fields . . . 51
3.2. Phenomenology of the LHT . . . 54
3.2.1. Lepton flavour violating Higgs decays . . . 55
T-even heavy singlet . . . 59
T-odd heavy singlet . . . 64
3.2.2. Neutrino masses in the LHT . . . 67
T-even heavy singlet . . . 67
T-odd heavy singlet . . . 73
3.3. Non gauge invariance of the LHT . . . 75
3.4. Chapter summary . . . 79

4. The New and gauge invariant Littlest Higgs model with T-parity 81
4.1. The New Littlest Higgs model with T-parity setup . . . 81
4.1.1. A minimal extension of the global symmetry . . . 81
4.1.2. The additional Goldstone fields . . . 83
4.1.3. Extra fermions and their interactions . . . 84
A minimal setup that introduces quadratic Higgs mass corrections . . . 85
A viable model with consistent fermion representations . . . 86
4.1.4. Physical fields . . . 90
Gauge bosons . . . 90
Scalars after gauge fixing . . . 91
Fermion masses and mixings in the NLHT . . . 93
4.1.5. New lepton contributions to LFV Higgs decays . . . 96
4.2. The Background Field Method . . . 97
4.2.1. Deriving the master formula for the background field method . . . 97
4.2.2. Quadratic and logarithmic corrections to the scalar potential . . . 103
Gauge boson contribution to the scalar potential . . . 104
Fermion contributions to the scalar potential . . . 105
Physical scalar masses and Higgs potential . . . 109
4.2.3. LFV Higgs decays with a T-odd singlet in the LHT revisited . . . 111
4.3. Phenomenology of the NLHT . . . 114
4.3.1. Parameter space . . . 114
4.3.2. A simplified model . . . 116
4.3.3. Particle spectrum and scalar decays . . . 117
4.4. Chapter summary . . . 121

5. Conclusions 125

6. Conclusiones 131

A. SU(5) generators 137
A.1. Unbroken SO(5) generators . . . 137
A.2. Rest of SU(5) generators . . . 137
A.3. Building SU(3) subalgebras . . . 138
A.3.1. Upper-left SU(3) . . . 138

A.3.2. Lower-right SU(3) 139
A.3.3. Associated SO(5) transformations 140

B. Higgs mass dependence on the Yukawa couplings in the LHT **141**
B.1. T-odd case 144
B.2. T-even case 144
B.3. Appendix summary 145

C. A more economical top quark sector in the NLHT **147**
C.1. Simplification of the quark sector 147
C.2. Top and top partner masses 148
C.3. Physical scalar masses and Higgs potential 149

D. NLHT Feynrules/FeynArts/FormCalc model file **153**

List of Figures **159**

List of Tables **161**

Bibliography **163**

Dedicado in memoriam *a mi abuelo Manuel Poyatos Pedrosa.*

Chapter 1

Introduction

1.1. Brief description of the Standard Model of Particle Physics

The deep understanding of the nature of matter and its interactions that the physicists achieved during the XX century is collected in the *Standard Model* (SM). It is one of the most predictive and successful theories of all times and even today most of the experiments seem to agree with its predictions. It has been tested in a wide range of energies allowing us to find what today we consider to be the building blocks of the Universe.

The SM is a quantum field theory. This means that it is invariant under global transformations of the Poincaré group, that includes the Lorentz group together with space-time translations, and respects the principles of quantum mechanics.[1] On the other hand, all the particles we know have internal quantum numbers and thus live in representations of the gauge (local) group

$$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y. \tag{1.1}$$

The $\mathrm{SU}(3)_c$ factor describes the theory of strong interactions or Quantum Chromodynamics (QCD). Its associated quantum number is the color. The rest of the group $G_{\mathrm{EW}} \equiv \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ defines the electroweak (EW) interactions, the unification of electromagnetism (EM) with the weak interaction. The quantum numbers associated to $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ are the weak isospin and hypercharge, respectively. The quantum number associated to electromagnetism is the electric charge.

The matter fields are described by fermions, fields with semi-integer spin under the Lorentz group. They are cast into quarks (q) and leptons (l) depending on whether they experience the strong force or not. Under $\mathrm{SU}(3)_c$, quarks transform in the fundamental representation while leptons are color singlets. On the other hand, under the Lorentz group fermions decompose into their left (L) and right-handed (R) chiralities. The left-handed fermions are $\mathrm{SU}(2)_L$ doublets while the right-handed are singlets. All of them have different hypercharge under $\mathrm{U}(1)_Y$ as shown in table 1.1. Their explicit form is given by

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad l_L = \begin{pmatrix} e_L \\ \nu_L \end{pmatrix}, \quad u_R, \quad d_R, \quad e_R. \tag{1.2}$$

Notice the absence of a right-handed neutrino ν_R. These fermions are collected into families, that is, different replicas with the same quantum numbers differing only in their mass. On the other hand, the force mediators are described by vector gauge bosons, Lorentz vector fields that under the internal symmetries transform as Lie algebra-valued connections. They are responsible for the invariance of the theory under local transformations of the group (1.1). There

[1] The Lorentz group $\mathrm{SO}(3,1)$ does not allow representations with semi-integer spin and thus it cannot describe fermions. Indeed what we call Lorentz group in this context is the universal cover $\mathrm{SL}(2,\mathbf{C})$, the group of complex 2×2 matrices with unit determinant.

is the same number of vector bosons that generators in the Lie algebra of the gauge group: eight gluons g^a_μ, associated to $\mathrm{SU}(3)_c$, three weak gauge bosons W^i_μ associated to $\mathrm{SU}(2)_L$ and the hypercharge gauge boson B_μ. Their quantum numbers are gathered in table. 1.1.

However, the gauge symmetry does not allow to introduce massive vector fields and fermions. But in nature it turns out that the mediators of the weak force $W^\pm_\mu$ (orthogonal complex combinations of W^1_μ and W^2_μ) and Z_μ (a combination of W^3_μ and B_μ), and most of the fermions are massive and thus some mechanism must be implemented to provide them with a mass while keeping the photon A_μ (the remaining orthogonal combination of W^3_μ and B_μ) and the gluons massless.[2]

According to the ideas of Nambu [2], Goldstone, Salam and Weinberg [3, 4], when a theory has a continuous global symmetry G that gets spontaneously broken to one of its continuous subgroups H, that is, the vacuum solution is only invariant under H, a set of massless scalar fields (*Goldstone bosons*) appear in the spectrum. If the continuous symmetry is local, Brout, Englert, Higgs, Guralnik, Hagen and Kibble [5–8] realized that some of these Goldstone bosons (the would-be Goldstone bosons) disappear from the physical spectrum turning into the longitudinal modes of the gauge bosons. As a consequence the gauge bosons become massive. This is the so called *Higgs mechanism*.

Following the above procedure, the electroweak gauge group $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gets spontaneously broken to the electromagnetic group $\mathrm{U}(1)_Q$ by the *vev* of an $\mathrm{SU}(2)_L$ doublet with four degrees of freedom: the Higgs doublet

$$H = \begin{pmatrix} i\pi^+ \\ \frac{v+h+i\pi^0}{\sqrt{2}} \end{pmatrix}. \tag{1.3}$$

Three of these scalars provide the longitudinal modes of the $W^\pm_\mu$ and Z_μ, which acquire a mass proportional to the Higgs *vev*, while the photon and the gluons remain massless. Only one scalar field remains in the physical spectrum: the Higgs boson.

To provide masses for the fermions, except for the neutrinos that were assumed to be massless in the original SM, different Yukawa Lagrangians that couple the fermions to the Higgs doublet are introduced. Their masses are thus proportional to the corresponding Yukawa coupling and the Higgs *vev* .

Finally, the requirement of renormalizability, that is, that all divergences generated at any order in perturbation theory can be absorbed by a finite set of parameters, fixes the operator content of the SM. Only operators with mass dimension $d \leq 4$ are allowed. Thus the renormalizable SM Lagrangian with the field content and symmetries above and the implementation of the Higgs mechanism reads

$$\begin{aligned} \mathcal{L}_{SM} = &-\frac{1}{2}\mathrm{tr}\left(\widetilde{G}_{\mu\nu}\widetilde{G}^{\mu\nu}\right) - \frac{1}{2}\mathrm{tr}\left(\widetilde{W}_{\mu\nu}\widetilde{W}^{\mu\nu}\right) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &+ \overline{q}^\alpha_L i\not{D} q^\alpha_L + \overline{l}^\alpha_L i\not{D} l^\alpha_L + \overline{u}^\alpha_R i\not{D} u^\alpha_R + \overline{d}^\alpha_R i\not{D} d^\alpha_R + \overline{e}^\alpha_R i\not{D} e^\alpha_R \\ &+ (D_\mu H)^\dagger D^\mu H - \mu^2 H^\dagger H - \lambda(H^\dagger H)^2 - \left[y^u_{\alpha,\beta}\overline{q}^\alpha_L \widetilde{H} u^\beta_R + y^d_{\alpha,\beta}\overline{q}^\alpha_L H d^\beta_R + y^e_{\alpha,\beta}\overline{l}^\alpha_L H e^\beta_R + \mathrm{h.c.}\right], \end{aligned} \tag{1.4}$$

where α, β are flavour indices, $\widetilde{H} = -i\sigma^2 H^*$ and $y^f_{\alpha\beta}$ are Yukawa couplings. The field strength tensors are given by

$$\widetilde{G}_\mu = G^a_\mu \frac{\lambda^a}{2} \tag{1.5}$$

[2]We will consider massless neutrinos in this discussion.

SM field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
g_μ^a	**8**	**1**	0
W_μ^i	**1**	**3**	0
B_μ	**1**	**1**	0
$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	**3**	**2**	$\frac{1}{6}$
u_R	**3**	**1**	$\frac{2}{3}$
d_R	**3**	**1**	$-\frac{1}{3}$
$l_L = \begin{pmatrix} e_L \\ \nu_L \end{pmatrix}$	**1**	**2**	$-\frac{1}{2}$
e_R	**1**	**1**	-1
$H = \begin{pmatrix} i\pi^+ \\ \frac{v+h+i\pi^0}{\sqrt{2}} \end{pmatrix}$	**1**	**2**	$\frac{1}{2}$

TABLE 1.1: Quantum numbers of the different SM constituents in the weak eigenbasis.

$$\widetilde{G}_{\mu\nu} = \partial_\mu \widetilde{G}_\nu - \partial_\nu \widetilde{G}_\mu - ig_s \left[\widetilde{G}_\mu, \widetilde{G}_\nu\right] \tag{1.6}$$

$$\widetilde{W}_\mu = W_\mu^i \frac{\sigma^i}{2} \tag{1.7}$$

$$\widetilde{W}_{\mu\nu} = \partial_\mu \widetilde{W}_\nu - \partial_\nu \widetilde{W}_\mu - ig \left[\widetilde{W}_\mu, \widetilde{W}_\nu\right] \tag{1.8}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{1.9}$$

where $\lambda^a/2$, g_S and $\sigma^i/2$, g are the $SU(3)_c$ and $SU(2)_L$ generators and gauge couplings, respectively. The covariant derivative $\not{D} \equiv i\gamma^\mu D_\mu$ is given by

$$D_\mu = \partial_\mu - ig_S \frac{\lambda^a}{2} G_\mu^a - ig \frac{\sigma^i}{2} W_\mu^i + ig' Y B_\mu \tag{1.10}$$

where g' is the $U(1)_Y$ gauge coupling.

1.2. The SM as an effective field theory

The discovery of the Higgs boson in 2012 by the ATLAS and CMS experiments [9, 10] finally completed the SM puzzle. However there still remains open questions that the SM cannot address satisfactorily. Among others,

1. The underlying mechanism of *electroweak symmetry breaking* (EWSB) [11–13].
2. The flavour puzzle, that is, the hierarchy between the different SM fermion masses and mixings [14–17].
3. Related to the previous one, the origin of neutrino masses and mixings [18–26].
4. The nature of dark matter [27–33].
5. The origin of dark energy [34–37].

6. The quantum behaviour of gravity, which requires a quantum theory of the space-time itself [38–42].

As a consequence, the SM should not be regarded as the ultimate explanation of nature but as an *effective field theory* (EFT), just a partial description of the world that is valid only below an energy scale Λ_{SM}. Above that scale, a new theory with extra symmetries and degrees of freedom, with typical masses of size Λ_{SM}, should clarify some of these open questions and provide a microscopic explanation for the observed values of the SM input parameters. Following a top-down approach, the *SM effective field theory* (SMEFT) arises as the low energy limit of a more fundamental theory after integrating out the new heavy degrees of freedom at the scale Λ_{SM}.[3] The SMEFT consists of the SM Lagrangian plus an infinite series of local operators invariant under the SM gauge group with arbitrary energy dimension $d > 4$ suppressed by powers of Λ_{SM}

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{d>4} c_i^d \frac{\mathcal{O}_d^i}{\Lambda_{\text{SM}}^{d-4}}, \tag{1.11}$$

where the coefficients c_i^d are functions of the parameters of the underlying new theory. This suppression is an enormous benefit because the higher dimension operators break accidental symmetries of the SM such as lepton and baryon number or custodial symmetry. Thus the impact of these operators is small, which explains why we have not (yet) observed transitions in nature that violate these symmetries. On the other hand, the SMEFT is not a renormalizable theory in the usual sense, conversely to the SM. However it is renormalizable order by order in loops and in energy dimension.

Nowadays, the SM cutoff Λ_{SM} remains unknown, since most experiments seem to agree with the SM predictions.[4] One of its tentative values is the Planck mass $M_{Pl} \approx 1.2 \times 10^{19}$ GeV [44], that it is the scale in which gravitational effects start to become important and a quantum theory of gravity is required. Other models like *Grand Unification theories* (GUT's) predict that there is a scale at which the strong and electroweak gauge couplings unify [45, 46]. This scale is closely related to the proton decay, whose non observation sets a lower bound for the GUT scale $\Lambda_{\text{GUT}} \sim 10^{16}$ GeV. Finally, oscillation experiments reveal that neutrinos have a small mass $m_\nu \lesssim 0.1$ eV. Within the SM framework, neutrinos receive masses from the dimension-five Weinberg operator[5] [47]

$$\mathcal{O}_{d=5} = (\overline{l_L} H^c)(H^c l_L^c), \tag{1.12}$$

This operator must be suppressed by $\Lambda \approx 10^{14}$ GeV in order to reproduce the observed order of magnitude of neutrino masses. In any case, there is a huge gap between these tentative values of Λ_{SM} and the electroweak scale set by the Higgs *vev* $v \approx 250$ GeV. However, in the following section we will motivate why new physics should play an important role already at the TeV scale.

[3]The SMEFT is also an end in itself. It is an extremely useful tool to parametrize, in a model independent fashion, deviations from the SM predictions. Their unknown coefficients are fixed by the *matching procedure* to any UV theory allowing to test its IR predictions and eventually discard theories.

[4]However, there are current experimental measurements related to lepton flavour non-universality, the electron and muon anomalous magnetic moments $(g-2)$ and the W boson mass that challenge the SM and push towards the direction of New Physics around the TeV scale [43].

[5]The Weinberg operator is the only dimension 5 operator that can be built using the SM gauge symmetry and fields.

1.3. The hierarchy problem and a natural electroweak scale

Previously we have pointed out that operators with energy dimension $d > 4$ are suppressed by powers of the cutoff scale $\Lambda_{\rm SM}^{d-4}$. Then the same argument states that the Higgs mass parameter should be naturally expected to be of size

$$c\Lambda_{\rm SM}^2 H^\dagger H, \tag{1.13}$$

where c is a constant. Comparing with the measured value of the Higgs mass, $m_h = 125$ GeV, implies that $c \sim 10^{-28} \lll 1$ for $\Lambda_{\rm SM} = \Lambda_{\rm GUT}$. Thus the small value of c is a measure of the huge hierarchy between the Higgs mass and $\Lambda_{\rm SM}$.

According to the *naturalness criterion* [1], mass hierarchies of this type are naturally understood if some symmetry is restored in the limit of vanishing field mass. In that case, the naive dimensional analysis above does not work to estimate the size of the field mass. For instance, the lightness of the weak gauge bosons $W^\pm, Z$ is explained due to the restoration of gauge symmetry when their masses are turned off since all of them depend on the Higgs *vev*. The same happens with fermions. However, this is not the case for the Higgs field since the SM does not enhance its symmetry by turning off the Higgs mass. The absence of such a symmetry that protects the Higgs mass squared against quadratically divergent contributions from arbitrarily high energy scales is the seed of the so called *hierarchy problem*. This is typical in theories with elementary scalar fields.

Let us reformulate the hierarchy problem in a more suitable form. This will clarify why one should expect new physics at the TeV scale. Suppose that we knew the UV theory that extends the SM beyond $\Lambda_{\rm SM}$. Thus the loop corrections to the Higgs mass could be fully computed evaluating the integral

$$\delta m_h^2 = \int_0^\infty \frac{dm_h^2}{dE} dE = \int_0^{\Lambda_{\rm SM}} \frac{dm_h^2}{dE}({\rm SM}) dE + \int_{\Lambda_{\rm SM}}^\infty \frac{dm_h^2}{dE}({\rm UV}) dE \equiv \delta_{\rm SM} m_h^2 + \delta_{\rm UV} m_h^2 \tag{1.14}$$

where we split the contributions coming from the SM fields bellow $\Lambda_{\rm UV}$ and those coming from the new degrees of freedom of the UV theory above $\Lambda_{\rm UV}$. The SM contribution is well known and can be computed evaluating the Feynman diagrams in fig. 1.1

$$\delta_{\rm SM} m_h^2 \approx \frac{3\lambda_t^2}{8\pi^2}\Lambda_{\rm SM}^2 - \frac{\Lambda_{\rm SM}^2}{16\pi^2}\left(3g^2 + g'^2\right) - \frac{3\lambda}{16\pi^2}\Lambda_{\rm SM}^2, \tag{1.15}$$

where $\lambda_t \approx 0.956$ is the top quark Yukawa coupling, $g \approx 0.641$ and $g' \approx 0.344$ are the electroweak coupling constants and $\lambda \approx 0.13$ is the Higgs quartic coupling. Numerically, the top quark quadratic divergence is the most important due to its large Yukawa coupling and color multiplicity as we show in fig. 1.2. As a first approximation let us consider only this contribution. Then we can define the degree of fine-tuning as

$$\Delta = \sqrt{\frac{\delta_{\rm SM} m_h^2}{m_h^2}} \approx \sqrt{\frac{3\lambda_t^2}{8\pi^2}}\frac{\Lambda}{m_h} \approx \frac{\Lambda}{640~{\rm GeV}}. \tag{1.16}$$

This allows to talk about fine-tuning in a more quantitative way. For a cutoff of size $\Lambda_{\rm GUT}$ we have a fine-tuning $\Delta \approx 10^{12}$. This means that the results of the two integrals in eq. (1.14) must have different sign and agree in the first 12 digits to predict the experimental value of the Higgs mass. These large cancellations are not natural since both contributions are a priori completely unrelated. Besides, we will never manage to measure a quantity with such an accuracy. This implies that we would not be able to unveil the dynamics of EWSB and the Higgs mass would always be an input parameter of the theory. However, if new physics enter around the TeV

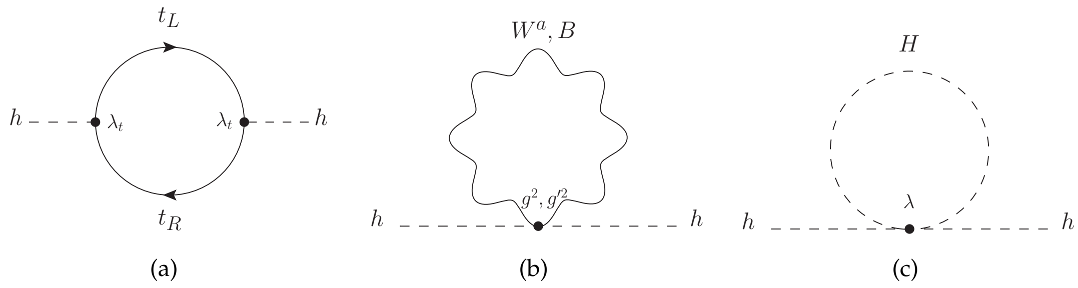


FIGURE 1.1: Most relevant one-loop quadratically divergent contributions to the Higgs mass squared in the SM in the gauge eigenbasis.

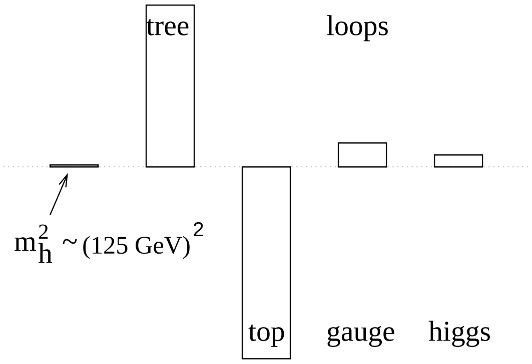


FIGURE 1.2: Fine tuning required to obtain the experimental value of the Higgs mass in the SM for $\Lambda_{\text{SM}} \approx 10$ TeV. Figure taken from [49].

scale, Δ can be reasonably small to generate a natural electroweak scale with only a small amount of fine-tuning.

The above discussion is mainly focused on quadratic divergences that are generated because we regularized the theory with a cutoff. One may argue that such divergences are not physical. Indeed they are not even present in other regularization schemes such as dimensional regularization and we would not have to worry about them. However, even in this kind of schemes a quadratic sensitivity to high energy scales is still there [48]. For instance, consider that at high energies the Higgs is coupled to another scalar field Φ with $M_\Phi \gg m_h$ through a term of the kind $\sim \lambda_\Phi H^2 \Phi^2$. This is what happens for instance in GUT models. At one loop, after subtracting the pole in $1/\epsilon$ where $\epsilon = 4 - d$ and using the $\overline{\text{MS}}$ renormalization scheme, one finds

$$\delta m_h^2 \sim \lambda_\Phi M_\Phi^2 \log\left(\frac{\mu^2}{M_\phi^2}\right) , \tag{1.17}$$

where μ is the renormalization scale. Thus, in the form of eq. (1.13) or eq. (1.17) the Higgs mass always receives quadratic contributions from any high energy scale of the theory. As a consequence, to keep a light Higgs one needs to fine-tune the parameters of the theory if the masses involved are large. This fine-tuning is, in general, unstable under quantum corrections. From the theoretical point of view this is quite unsatisfactory, what justifies the theoretical effort in the construction of models to address the hierarchy problem in order to generate a natural electroweak scale.

1.4. The Higgs as a composite particle

In the previous section we pointed out that in the SM the Higgs suffers from a hierarchy problem due to its a priori unexpected light mass according to the naturalness criterion. However there are other scalar fields in nature such as mesons in QCD that are naturally light compared to the rest of hadronic resonances, with masses of a few hundred MeV. This leads us to the following question: *what is the main difference between the Higgs and the QCD mesons?* The answer to this question is that mesons are not fundamental particles but composite states of quarks with a finite size and bounded by the strong force. Being composite, their masses are insensitive to energies beyond the inverse of the typical meson size. But in the SM the Higgs field is an elementary particle and thus receives mass corrections from arbitrarily large energies. If we could realize the Higgs as a composite state of some kind of strong force analogous to QCD (but not QCD, or otherwise it would be too light) at the TeV scale we would solve the hierarchy problem. These ideas were first proposed in the 80's [50–52] and have received and increasing attention in recent years [19, 53–69] mainly due to the lack of signals of supersymmetry.

In this section we will review some of the basic aspects of QCD that are responsible for generating light masses for the mesons. These characteristics guide the construction of models that realize the Higgs boson as a composite particle.

1.4.1. Motivation: mesons as low energy QCD bound states

QCD and confinement

The high and low energy behaviour of QCD is encoded in the renormalization group evolution of the strong coupling constant g_S. Neglecting the quark masses, at one-loop the running of g_S with the energy reads

$$\frac{dg_S}{d\log\mu} = -\frac{\beta_0}{(4\pi)^2} g_S^3 \quad \text{with} \quad \beta_0 = 11 - \frac{2}{3} n_F, \tag{1.18}$$

where μ is the renormalization scale and n_F is the number of active flavours.[6] Since only six quark flavours seem to exist in nature and β_0 is always positive for $n_F \leq 15$, the r.h.s. of eq. (1.18) is negative, so QCD has a stable fixed point in the UV and it is an asymptotically free theory [70]. The study of its low energy behaviour is more interesting. For that purpose, let us integrate from $\mu = \Lambda_{\text{UV}}$ to an arbitrary energy scale $E < \Lambda_{\text{UV}}$

$$g_S^2(E) = \frac{g_S^2(\Lambda_{\text{UV}})}{1 - g_S^2(\Lambda_{\text{UV}}) \frac{2\beta_0}{(4\pi)^2} \log\left(\frac{\Lambda_{\text{UV}}}{E}\right)}. \tag{1.19}$$

Due to the negative sign in eq. (1.18) there exists an energy scale $m_* < \Lambda_{\text{UV}}$ such that the strong coupling constant becomes non-perturbative[7]

$$\log\left(\frac{\Lambda_{\text{UV}}}{m_*}\right) = \frac{1}{2\beta_0}\left(\frac{4\pi}{g_S(\Lambda_{\text{UV}})}\right)^2. \tag{1.20}$$

[6] A quark flavour is active if its mass $m_q \ll E$, with E the available energy in the process. For all other purposes, m_q can be set to zero regardless whether it is active or not.

[7] The value of m_* is independent of the chosen Λ_{UV}. This is because g_S verifies eq. (1.18) and hences physical observables such as masses do not depend on the energy.

In the context of QCD, the new physical scale m_* is called Λ_{QCD} and it is exponentially suppressed with respect to Λ_{UV} allowing for a large separation between both scales.[8] Based on perturbative arguments, one usually takes the UV cutoff satisfying $\Lambda_{\text{UV}} \lesssim 4\pi m_*$. This mechanism that generates a physical dimensionful parameter through the running of dimensionless coupling constants at the UV is called *dimensional transmutation*. This is possible because at Λ_{UV} we have neglected the quark masses and the only relevant coupling is g_S.

Below Λ_{QCD}, QCD is a strongly coupled theory that confines quarks into baryons and mesons. Consequently a perturbative treatment of QCD below 1 GeV is not consistent. The way to deal analytically with QCD in this low energy regime is to use an EFT approach [72, 73] with cutoff $\Lambda_{\text{UV}} \sim 1$ GeV. This is based on the fact that a description of the strong interaction in terms of the physical degrees of freedom at this scale, the pseudoscalar mesons, is possible.

Effective field theory for the light mesons

The link between the QCD Lagrangian and the low-energy EFT's at energies below Λ_{QCD} is built from the symmetries of the light-quark sector which appear if the masses of the active quarks vanish [44, 70, 72, 74]. This is called the *chiral limit*. Considering only the three lightest quark flavours u, d and s, the QCD Lagrangian is invariant under global transformations of the group $\text{SU}(3)_L \times \text{SU}(3)_R$, acting on the left-handed and right-handed chiralities independently.[9] However, the QCD vacuum is non-trivial and the global symmetry group gets spontaneously broken to the diagonal $\text{SU}(3)_V$ at the scale Λ_{QCD} by the quark condensate

$$\langle 0 \mid \bar{q}_{Li} q_{Rj} \mid 0 \rangle = \Lambda^3_{\text{QCD}} \delta_{ij}. \tag{1.21}$$

As a consequence, the Goldstone theorem ensures the existence of eight massless scalar fields, the *Goldstone bosons* [4]. Physically they represent the octet of pseudoscalar mesons. Their dynamics are parametrized by a non-linear sigma model using the Callan-Coleman-Wess-Zumino formalism (CCWZ) [75, 76] and are fixed by symmetry. Being Goldstone bosons, as far as the global symmetry remains unbroken, the mesons are massless, what justifies their lightness.

However the global symmetry is not exact but broken at the UV by the different quark masses. The scalar fields are coupled to this source of breaking of the global symmetry acquiring a tree-level mass of typical size Λ_{QCD}. At low energies the electroweak interactions also break the global symmetry giving one-loop corrections to the meson masses. These corrections are controlled since the loops are cutoff at the scale $\Lambda_{\text{UV}} \lesssim 4\pi\Lambda_{\text{QCD}}$. The loop contributions are proportional to Λ_{QCD} times an electroweak coupling and hence small with respect to the tree-level contribution.

1.4.2. The composite Higgs idea and a Little Higgs

Once we have described the main features of QCD, let us apply the above mechanisms to generate a composite Higgs and a natural electroweak scale. Since the Higgs boson is color blind, we will omit the $\text{SU}(3)_c$ factor of the SM gauge group in the remaining of this Thesis. As

[8] The value of Λ_{QCD} depends on the number of active flavours encoded in the parameter β_0 in eq. (1.18). This is because the strong coupling constant g_S must vary smoothly when passing a quark threshold $g_S\left(E = m_q, n_F\right) = g_S\left(E = m_q, n_F - 1\right)$. This implies [71]

$$\Lambda_{\text{QCD}}\left(n_F\right) = \Lambda_{\text{QCD}}\left(n_F - 1\right)\left(\frac{\Lambda_{\text{QCD}}\left(n_F - 1\right)}{m_q}\right)^{2/(33-2n_F)}.$$

For three active flavours $\Lambda_{\text{QCD}} \approx 250$ MeV.

[9] The global symmetry group also includes an $\text{U}(1)_L \times \text{U}(1)_R \cong \text{U}(1)_V \times \text{U}(1)_A$ factor. The $\text{U}(1)_V$ is associated to baryon number while the axial $\text{U}(1)_A$ is anomalous and thus is not a symmetry of the quantized Lagrangian. We will ignore this factor in what follows.

a consequence, when we refer to the SM gauge group it will be implicitly assumed that we are only considering the electroweak subgroup $G_{\rm EW} = {\rm SU}(2)_L \times {\rm U}(1)_Y$.

First of all we need a new strongly interacting sector that at $\Lambda_{\rm UV}$ is close to a free fixed point and confines at the scale m_*. We assume that this confinement or *compositeness* scale is generated by the mechanism of dimensional transmutation at the UV, allowing for a natural separation between $\Lambda_{\rm UV}$ and m_* of typically $\Lambda_{\rm UV} \lesssim 4\pi m_*$. But unlike QCD, the UV theory is unknown, we ignore its fundamental constituents, the global and gauge symmetries and the pattern of spontaneous symmetry breaking. However, from the low energy point of view, the EFT theory is built postulating a global symmetry group G that gets spontaneously broken to one of its continuous subgroups H by the vacuum at the scale m_*. This generates a set of Goldstone bosons living in the G/H coset, among them the Higgs field, whose dynamics are parametrized à la CCWZ by a non-linear sigma model. Since G and H do not come from first principles, the composite Higgs idea admits multiple realizations giving rise to a vast family of models.

To generate a mass for the scalar fields, they must be coupled to the sources of explicit breaking of the global symmetry. In contrast with QCD, in most composite Higgs models the global symmetry is not broken at the UV, that is, there is no analogous to the quark masses.[10] Hence, there are no tree-level contributions to the composite scalar masses. Instead, their masses are generated from loop effects of terms that at low energies break the global symmetries. Those are the electroweak interactions, as for the mesons, and the Yukawa couplings. The latter need to be incorporated because the Higgs is responsible for providing masses to the SM fermions.

In this family of models, the quadratically divergent contributions to the Higgs field coming from the explicit breaking terms of the global symmetry are controlled. The low energy quanta have a too large wavelength and cannot resolve the Higgs size $l_h \sim 1/m_*$. Therefore the Higgs behaves as an elementary particle and the loop integral in eq. (1.14) grows linearly with the energy, thus resulting in a quadratic divergence (1.16) like in the SM. However, this growth gets cancelled by the finite size effects when the energy approaches the compositeness scale m_*. The linear SM behaviour is then replaced by a peak at the compositeness scale followed by a steep fall. The Higgs mass generation gets localized around m_* being insensitive to much higher energies. Being a composite particle there is no Higgs field at the UV and no dimension 2 mass terms as in eq. (1.13), solving the hierarchy problem.

The earlier attempts to implement this mechanism during the 80's [50–52] assumed that the compositeness scale m_* was not far away from the electroweak scale, and thus $m_* \gtrsim v$. However new exotic particles are expected at these new scale that have not been observed so far at colliders. This has pushed $m_* \gtrsim 1$ TeV which is already quite large to be compatible with the measured Higgs mass $m_h \approx 125$ GeV so certain amount of fine-tuning is required to generate a Higgs lighter than the rest of new resonances and stabilize the electroweak scale.

There is another idea that is implemented in *Little Higgs models* in which we will focus in this Thesis. This subset of composite Higgs models has a number of global symmetries that act non linearly on the Higgs ensuring its Goldstone nature when just one gauge or Yukawa coupling is non-vanishing. However, the combination of all the gauge and Yukawa couplings breaks all these global symmetries, generating a Higgs mass that is nevertheless not quadratically sensitive to $\Lambda_{\rm UV}$ at the one-loop level. This is the so called *collective symmetry breaking mechanism*. In terms of Feynman diagrams this can be qualitatively explained as follows. In order to generate a contribution to the Higgs mass parameter, loop diagrams require insertions of all the gauge or Yukawa couplings that break the global symmetry, relaxing their degree of divergence. Otherwise the global symmetry is enough to forbid any mass. Hence the contributions are at most logarithmically divergent of size $\sim m_*/4\pi$, ensuring a natural light Higgs

[10] This assumption can be relaxed allowing some terms breaking the global symmetry at the UV. However this does not introduce new radically different phenomena and thus we will disregard this possibility in our discussion.

without fine-tuning for $m_* \sim 1$ TeV, despite having gauge and Yukawa couplings of order one and thus stabilizing the electroweak scale. These light pseudo-Goldstone bosons are dubbed *Little Higgses* [77, 78].

1.5. Structure of the Thesis

This Thesis is organized as follows:

- In Chapter 2 we introduce the mathematical formalism that allows to realize the Higgs as a composite state of a strong interacting sector: the *Callan-Coleman-Wess-Zumino (CCWZ) formalism*. This characterizes the family of composite Higgs models. Within this vast family of models we highlight that Little Higgs models are less fine-tuned than other composite Higgs models because they introduce the *collective symmetry breaking mechanism*. We also define a discrete Z_2 symmetry, T-parity, to alleviate direct and indirect constraints from EWPD due to the new heavy particles that these models include. We end this chapter by providing an explicit example of a Little Higgs model.

- In Chapter 3 we focus on a particular realization of the Little Higgs paradigm: the *Littlest Higgs model with T-parity* (LHT). This is the minimal setup that implements all the properties of a Little Higgs model from a simple global group. Within this framework we study the contributions of an SU(2) lepton singlet, either T-even or T-odd, to different lepton flavour violating observables. Concerning lepton flavour violating Higgs decays, we find that the contributions of a T-even singlet are finite and decouple while the contributions of a T-odd singlet are UV divergent and there is no available counterterm. Next, we provide a small Majorana mass to the left-handed components of the SU(2) singlet to generate neutrino masses. While a T-even singlet provides neutrino masses that decouple in the limit of a large singlet mass, the T-odd singlet generates neutrino masses that are independent of the mass of the singlet. Within the T-even singlet scenario we also study the contributions of the singlet to the relevant process $\mu \to e\gamma$. This provides an upper bound for the masses of the T-odd mirror fermions. Motivated by the anomalous behaviour of the T-odd singlet, at the end of this chapter we show that the LHT is non gauge invariant due to the non-trivial interplay between the non-linear realization of the global symmetry and the implementation of the discrete T-parity.

- In Chapter 4 we build explicitly a new and gauge invariant Littlest Higgs model with T-parity (NLHT) that cures the afflictions we find in Chapter 3. For that purpose the global group is minimally enlarged with respect to the LHT introducing additional scalar fields and fermions. To evaluate the masses of the physical scalar fields, we introduce the *Background Field Method* (BFM). We also use the BFM to compute the counterterm for lepton flavour violating Higgs decays in the LHT. In the last part of this chapter we explicitly verify the viability of our model imposing current electroweak precision data and cosmological constraints. As a consequence, some parameters get fixed while others get correlated and the particle spectrum is bounded from below and above. Finally, we evaluate the main decay channels and lifetime of the new scalar fields.

- Chapter 5 is finally devoted to the main conclusions of the Thesis.

Most of original results presented in this Thesis have been published:

– The calculation of several lepton flavour violating observables and the generation of neutrino masses (Chapter 3), in refs. [65, 66].

– The proof of the non gauge invariance of the LHT (Chapter 3) and the construction of a new and gauge invariant Littlest Higgs model with T-parity (Chapter 4), in ref. [68].

– The phenomenological study within the NLHT framework (Chapter 4), in ref. [69].

Auxiliary material and some unpublished results appear in several appendices:

- Appendix A gives explicit expressions for the SU(5) generators. We also construct the Lie algebra of the two different SU(3) subgroups of SU(5).
- Appendix B is devoted to the analysis of the cancellation of quadratic and logarithmic divergences to the Higgs mass in the LHT.
- Appendix C provides an alternative version of the NLHT. We identify the usual top quark partners, responsible for the cancellation of the quadratic divergence of the Higgs mass from the top quark, with the new heavy singlets required by gauge invariance of the NLHT.
- Appendix D shows an application of the FeynRules model file for the NLHT that computes the masses of the new scalar fields using FeynArts and FormCalc.

Chapter 2

General formalism for Little Higgs models

In this Chapter we introduce the Callan, Coleman, Wess and Zumino (CCWZ) [75, 76] formalism that allows to describe the low energy dynamics of the Goldstone bosons associated to the spontaneous breaking of a strong interacting sector at the scale f. This formalism characterizes the Composite Higgs paradigm. The Higgs boson, being part of the set of Goldstone bosons resulting from this spontaneous breaking, is massless. To provide it with a mass at the one loop level we will discuss the notion of vacuum misalignment. In a general Composite Higgs model, quadratic divergences to the Higgs mass proportional to the cutoff $\Lambda \sim 4\pi f$ arise. However, in these models a plethora of new particles with masses of size f are present whose non observation pushes f to the multi-TeV range. As a consequence, the contributions to the Higgs mass get be large and certain amount of fine-tuning is necessary to generate a natural electroweak scale. To avoid an excessive fine-tuning one implements the Collective symmetry breaking mechanism, which avoids quadratic divergences and gives rise to the Little Higgs paradigm. As a result of this, extra degrees of freedom are required that are responsible for the cancellation of the unwanted quadratic divergences. However, current collider bounds put strong constraints on these particles. To reconcile these models with data it is necessary the introduction of an extra Z_2 symmetry: T-parity. Finally we will provide a toy model in which all these features are implemented.

2.1. Symmetries and the CCWZ formalism

2.1.1. Global symmetries and Goldstone bosons

Let G be a compact, connected and semi-simple Lie group of dimension n. This gets spontaneously broken by the *vev* Σ_0 of a field transforming in a linear and unitary representation R_Σ of G to one of its continuous subgroups H of dimension d. The particular form of the reference vacuum Σ_0 is arbitrary since a global transformation relates all possible vacua. However, we will consider that the embedding of H in G is such that contains the electroweak SM group $\mathrm{SU}(2) \times \mathrm{U}(1)$ as a subgroup. Let us denote with T^b ($b = 1, ..., d$) the generators of H and X^a ($a = 1, ..., n-d$) the remaining generators needed to obtain a basis of the Lie algebra of G. We assume without loss of generality that they are orthogonal with respect to the Cartan inner product.

The spontaneous breaking $G \to H$ due to the *vev* Σ_0 implies that taking an arbitrary element $U \in H$,

$$R_\Sigma(U)\Sigma_0 = \Sigma_0, \tag{2.1}$$

that is, Σ_0 is invariant (a singlet) under the action of the subgroup H. Choosing the exponential map, the transformation U can be written as $U = e^{i\beta^b T^b}$ with real parameters β^b. Taking the derivative with respect to the parameters and evaluating in the neutral element $\beta^b = 0$, one arrives from the group representation (2.1) to a representation $\mathcal{R}$ of the Lie algebra fulfilling

$$\frac{\partial}{\partial \beta^b}\Big|_{\beta^b=0} R_\Sigma(U)\Sigma_0 = \mathcal{R}_\Sigma(T^b)\Sigma_0 = 0. \tag{2.2}$$

In other words, the unbroken generators annihilate the vacuum.

According to the Goldstone theorem [4], the spontaneous breaking of G to H leads to $n-d$ Goldstone bosons $\pi^a(x)$ along the direction of the broken generators. Choosing for the Goldstone fields the exponential parametrization (see for instance [79]), one can define the non linear field

$$\xi = e^{i\pi^a(x)X^a/f} \equiv e^{i\Pi/f}, \tag{2.3}$$

where f is the decay constant introduced to keep the argument of the exponential dimensionless and we defined the Goldstone matrix $\Pi = \pi^a X^a$. This decay constant is nothing but the confinement scale m_* of the model.

Let us derive the transformation properties of the non linear field ξ under G. Close enough to the identity element, every group element $V \in G$ has a unique decomposition of the form

$$V = e^{i\alpha^a X^a} e^{i\beta^b T^b}, \tag{2.4}$$

where $e^{i\beta^b T^b} \in H$ and α^a, β^b are real parameters. Since ξ is spanned by the broken generators one can take any element $V \in G$ and the unique element $U \in H$ such that

$$V\xi = V' \equiv \xi' U, \tag{2.5}$$

where

$$\xi' = \xi(\Pi'), \quad \Pi' = \Pi'(V,\Pi), \quad U = U(V,\Pi) \tag{2.6}$$

are determined by the particular structure of the Lie group and then

$$\xi' = V\xi U^{-1}. \tag{2.7}$$

Let us further define

$$U : \Psi \to R_\Psi(U)\Psi, \tag{2.8}$$

where $R_\Psi(U)$ is a linear and unitary representation of the subgroup H and Ψ is an element of some Hilbert space. Then the following transformations,

$$V : \xi \to \xi', \quad \Psi \to R_\Psi(U)\Psi, \tag{2.9}$$

give a (non linear) realization of G. To verify it, let us observe that taking $V_1, V_2 \in G$,

$$V_2 V_1 \xi = \xi'' \widetilde{U}. \tag{2.10}$$

On the other hand, using that $V_1\xi = \xi' U_1$ and $V_2\xi' = \xi'' U_2$ and comparing with the previous expression, we find

$$\xi'' U_2 U_1 = \xi'' \widetilde{U}, \tag{2.11}$$

and hence $\widetilde{U} = U_2 U_1$. Moreover, since $R(U)$ is a representation then

$$R(\widetilde{U}) = R(U_2)\, R(U_1)\,. \tag{2.12}$$

The transformation of the non linear field ξ or, equivalently, on the Goldstone fields $\pi^a(x)$ is a group realization by itself. The transformation on Ψ, on the other hand, is meaningful only together with that of ξ, since U is a function of Π and V according to eq. (2.6) so is $R_\Psi(U)$ as well. If $R_\Psi(U)$ is reducible, the field Ψ decomposes into a set of fields that do not mix.

If V belongs to the subgroup H, then $V = U$, and one can write

$$U\xi = U\xi U^{-1} U \equiv \xi^{'} U \tag{2.13}$$

and comparing with eq. (2.7),

$$\xi^{'} = U\xi U^{-1}. \tag{2.14}$$

In this case, the transformation $\xi \to \xi^{'}$ is a linear transformation independent of Π and therefore the transformation of eq. (2.8) is also linear. This implies that, when restricted to the subgroup H, the group realization in eq. (2.9) becomes a linear representation. Notice that

$$U\Pi^n U^{-1} = \left(U\Pi U^{-1}\right)^n \tag{2.15}$$

and thus the Goldstone matrix transforms linearly in the adjoint representantion of H,

$$\Pi^{'} = U\Pi U^{-1}. \tag{2.16}$$

Considering now transformations only along the direction of the broken generators $V = e^{i\alpha^a X^a} \approx \mathbb{1} + i\alpha^a X^a$, at leading order in Π and α^a one finds, using eq. (2.5) with $U \approx \mathbb{1}$,

$$\Pi^{'} = \Pi + f\alpha^a X^a + \dots\,. \tag{2.17}$$

This is the shift symmetry that forbids mass terms for the Goldstone fields.

In the special case in which the group G admits the Lie algebra automorphism

$$T^b \to T^b, \quad X^a \to -X^a, \tag{2.18}$$

the transformation on ξ can be simplified. This is the case of *symmetric cosets*, where the commutation relations between the generators take the schematic form

$$[T,T] \sim T, \quad [T,X] \sim X, \quad [X,X] \sim T \tag{2.19}$$

where the first equation holds because H is a subgroup, the second one because the structure constants are antisymmetric and the last one only holds for symmetric cosets, so it depends in general on all the generators. Applying the automorphism (2.18) on eq. (2.5) we find

$$\xi D(V^{-1}) = U^{-1}\xi^{'}, \tag{2.20}$$

where $D(V)$ is the image of the group element V by the automorphism and U remains invariant. Substituting U from eq. (2.7) the transformation of ξ reads

$$\xi^{'2} = V\xi^2 D(V^{-1}). \tag{2.21}$$

Equivalently if ξ' is obtained from eq. (2.7) one finds

$$V\xi U^{-1} = U\xi D(V^{-1}). \tag{2.22}$$

This expression can be interpreted as a definition of the matrix U and depends on the particular action of the automorphism D on the group.

To construct non-trivial invariants it is useful to introduce the *Maurer-Cartan* form that can be decomposed in the Lie algebra of G [80]

$$i\xi^{-1}\partial_\mu\xi = d_{\mu,a}X^a + e_{\mu,b}T^b \equiv d_\mu + e_\mu, \tag{2.23}$$

where $d_{\mu,a}$ has an index along the broken generators and $e_{\mu,a}$ along the unbroken ones. Both are functions of the Goldstone fields in Π. Under G, the Maurer-Cartan form transforms as

$$i\xi^{-1}\partial_\mu\xi \xrightarrow{G} U\left(i\xi^{-1}\partial_\mu\xi\right)U^{-1} + iU\partial_\mu U^{-1}. \tag{2.24}$$

Notice that the shift term is the Maurer-Cartan form associated to the transformation $U \in H$. Therefore it decomposes on the Lie algebra of H and it does not have components along the broken generators. Since we know that the Goldstone bosons (with components along the broken generators) transform linearly under H, the shift is carried entirely by the e symbol while d transforms linearly with U

$$d_\mu \xrightarrow{G} Ud_\mu U^{-1} \tag{2.25}$$

$$e_\mu \xrightarrow{G} U\left(e_\mu + i\partial_\mu\right)U^{-1}. \tag{2.26}$$

Let us point out that d_μ, unlike Π transforms under H even if we perform a transformation under the full G. Inspecting the form of d_μ at leading order in the Goldstone fields and using that our basis of generators is orthogonal, we have

$$d^a_\mu = i\mathrm{tr}\left[\xi^{-1}\partial_\mu\xi X^a\right] \approx -\frac{1}{f}\partial_\mu\pi^a + \dots, \tag{2.27}$$

that can be used, for instance, to construct the kinetic term for the Goldstone fields. On the other hand, the e symbol transforms as a connection associated with local H invariance. In general, these two objects can be employed to construct covariant derivatives and field-strengths.

Another important piece to construct invariants is the field Σ that transforms linearly under the full global group. It is built from the Goldstone fields ξ and the *vev* Σ_0 as follows,

$$\Sigma \equiv R_\Sigma(\xi)\Sigma_0 \xrightarrow{G} R_\Sigma\left(V\xi U^{-1}\right)\Sigma_0 = R_\Sigma(V)R_\Sigma(\xi)\Sigma_0 = R_\Sigma(V)\Sigma, \tag{2.28}$$

where in the last step we have used that the *vev* Σ_0 is invariant under the action of H. We will use this field to build the kinetic term of the Goldstone fields. For that purpose the derivative, that transforms as Σ under the global group, is simply given by

$$\partial_\mu\Sigma \xrightarrow{G} R_\Sigma(V)\partial_\mu\Sigma, \tag{2.29}$$

which allows to write the kinetic term and self interactions for the Goldstone fields

$$\mathcal{L}_{S,\mathrm{kin}} = cf^2\mathrm{tr}\left[(\partial^\mu\Sigma)^\dagger\partial_\mu\Sigma\right], \tag{2.30}$$

where c is a constant to be fixed imposing canonically normalized kinetic terms.

As already emphasized, the transformation of Ψ is only meaningful with that of ξ since

$R_\Psi(U)$ is in general a non linear function of the Goldstone fields for a given $V \in G$. Thus from the transformation of Ψ in eq. (2.8) the derivative $\partial_\mu \Psi$ does not transform as Ψ. To find the appropriate covariant derivative let us construct the object $R_\Psi(\xi)\Psi$ that under the global group transforms

$$R_\Psi(\xi)\Psi \xrightarrow{G} R_\Psi(V)\left(R_\Psi(\xi)\Psi\right). \tag{2.31}$$

Taking the derivative

$$\partial_\mu \left[R_\Psi(\xi)\Psi\right] = \partial_\mu R_\Psi(\xi)\Psi + R_\Psi(\xi)\partial_\mu \Psi, \tag{2.32}$$

that transforms as $R_\Psi(\xi)\Psi$ under G,

$$\partial_\mu \left[R_\Psi(\xi)\Psi\right] \xrightarrow{G} R_\Psi(V)\partial_\mu \left[R_\Psi(\xi)\Psi\right], \tag{2.33}$$

where we used the transformation of ξ in eq. (2.8). Applying $R_\Psi(\xi^{-1})$, we finally obtain the covariant derivative that transforms as Ψ under G

$$D_\mu \Psi \equiv R_\Psi(\xi^{-1})\partial_\mu \left[R_\Psi(\xi)\Psi\right] = \partial_\mu \Psi + R_\Psi(\xi^{-1}\partial_\mu \xi)\Psi, \tag{2.34}$$

$$D_\mu \Psi \xrightarrow{G} R_\Psi(U) D_\mu \Psi, \tag{2.35}$$

where in the second term of (2.34) we can recognize the Maurer-Cartan form in the representation Ψ that compensates the non linear dependence on the Goldstone fields of $\partial_\mu \Psi$.

2.1.2. Gauge symmetries

We wish to extend the previous construction considering that a subgroup $G_g \subset G$, with generators λ^A and coupling constant g, is made local by introducing the corresponding set of gauge fields A^A_μ, which act as components of a Lie-algebra-valued connection. They transform

$$A_\mu \equiv A^A_\mu \lambda^A \xrightarrow{G_g} V_g A_\mu V_g^{-1} - \frac{i}{g} V_g \partial_\mu V_g^{-1}, \tag{2.36}$$

under a local transformation $V_g(x) \in G_g$. The first term corresponds to the transformation in the adjoint representation of G_g while the second term is the corresponding shift due to the connection nature of A_μ.

To introduce gauge interactions in the Lagrangian one has to add couplings to the conserved currents $J^{\mu,A}$ associated with the global symmetry generators

$$\mathcal{L} \to \mathcal{L} + A^A_\mu J^{\mu,A} \tag{2.37}$$

to compensate for the variation of the original $\mathcal{L}$ under space-time dependent transformations and thus leading to a locally invariant theory under G_g. For that purpose, one promotes the derivative of the linear field Σ (2.29) to a covariant derivative

$$\partial_\mu \Sigma \to D_\mu \Sigma \equiv \partial_\mu \Sigma + ig\mathcal{R}_\Sigma\left(A_\mu\right)\Sigma, \tag{2.38}$$

that transforms properly under the gauged subgroup

$$D_\mu \Sigma \xrightarrow{G_g} R_\Sigma\left(V_g\right) D_\mu \Sigma \tag{2.39}$$

using eq. (2.36) in the representation R_Σ. The covariant derivative (2.38) is valid for any field transforming in a linear representation of G. With this covariant derivative one can introduce gauge interactions for the Goldstone fields

$$\mathcal{L}_S = cf^2 \text{tr}\left[(D^\mu \Sigma)^\dagger D_\mu \Sigma \right]. \tag{2.40}$$

On the other hand, one has also to add kinetic terms for the gauge bosons. Introducing the covariant derivative for the gauge fields,

$$D_\mu A_\nu = \partial_\mu A_\nu + ig\mathcal{R}^{\text{adj}}\left(A_\mu\right) A_\nu = \partial_\mu A_\nu + ig\left[A_\mu, A_\nu\right], \tag{2.41}$$

one can build the corresponding field-strength tensor

$$F_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu \tag{2.42}$$

and the kinetic term reads

$$\mathcal{L}_G = -\frac{1}{2}\text{tr}\left(F_{\mu\nu}F^{\mu\nu}\right). \tag{2.43}$$

To obtain the covariant derivative $D_\mu \Psi$ compatible with local transformations $V_g \in G_g$, one promotes eq. (2.31) to

$$\partial_\mu \left[R_\Psi(\xi)\Psi\right] \to D_\mu \left[R_\Psi(\xi)\Psi\right] = \partial_\mu \left[R_\Psi(\xi)\Psi\right] + igR_\Psi(A_\mu)R_\Psi(\xi)\Psi, \tag{2.44}$$

that transforms properly under the gauge subgroup

$$D_\mu \left[R_\Psi(\xi)\Psi\right] \xrightarrow{G_g} R_\Psi(V_g)R_\Psi(\xi)\Psi. \tag{2.45}$$

Finally, applying $R_\Psi(\xi^{-1})$ one obtains

$$D_\mu \Psi \equiv R_\Psi(\xi^{-1})D_\mu \left[R_\Psi(\xi)\Psi\right] = \partial_\mu \Psi + R_\Psi(\xi^{-1})D_\mu R_\psi(\xi)\Psi, \tag{2.46}$$

where we have defined the covariant derivative for the ξ field in the representation of the Ψ field

$$D_\mu R_\Psi(\xi) = \partial_\mu R_\Psi(\xi) + igR_\Psi(A_\mu)R_\Psi(\xi). \tag{2.47}$$

2.2. Vacuum misalignment

In the previous section, we have chosen a reference vacuum Σ_0 transforming in a certain linear representation R_Σ of the global group G. This *vev* breaks spontaneously the global group into one of its subgroups H and thus it is invariant under the action of H. This vacuum choice is arbitrary but for our construction we will impose that H contains the SM electroweak group G_{EW} as a subgroup. On the other hand, the coset G/H must contain some generators transforming as doublets under the SM generators because the Higgs field is one of the Goldstone bosons resulting from the $G \to H$ spontaneous symmetry breaking. In order to be identified with the SM Higgs field, it must take a *vev* to trigger EWSB. However, the *vev* of a Goldstone field is not physical since it can be eliminated by means of a global transformation.

This is no longer true when some G breaking terms are introduced in the theory, because the (pseudo)-Goldstone bosons now dynamically develop a physical potential through loops and their *vev* cannot be eliminated since the global symmetry is not exact [52]. One can interpret this in a geometrical way (see fig. 2.1). Taking the field Σ transforming in the linear representation

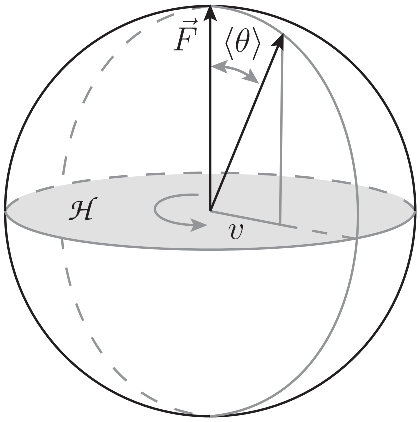


FIGURE 2.1: Illustration of the vacuum misalignment mechanism for the case of $G = SO(3)$ and $H = SO(2)$, where $\vec{F} = \Sigma_0$ and $\theta = \Pi$. (Figure extracted from [80]).

R_Σ of the global group in eq. (2.28) and replacing the Higgs field by its *vev*, v, one can define $\langle\Sigma\rangle \equiv R_\Sigma(\langle\xi\rangle)\Sigma_0$, parametrizing the misalignment between the chosen reference vacuum and the actual vacuum.[1] The misalignment angle is defined by $\sin\langle\theta\rangle = v/f$. In Composite Higgs models one is interested in a large separation between the EWSB scale and the compositness scale, that is, $\sin\langle\theta\rangle \ll 1$. This angle governs the departure from the SM predictions. The limit $\sin\langle\theta\rangle \to 0$ for fixed v, corresponds to decoupling the composite sector from the low-energy physics by sending to infinity the scale f. In this limit, only the Higgs boson remains in the spectrum while all the other new bound states and resonances decouple. The theory thus reduces to the SM and the composite Higgs becomes elementary. This mechanism is called *vacuum misalignment* [51, 52, 81]. This is what distinguishes a Composite Higgs model from Technicolor in which $f \sim v$. The Higgs *vev* in this case is generated directly by dimensional transmutation at the UV without an intermediate scale [82].

In order to ensure a small misalignment angle, one can assume a certain degree of fine-tuning taking place in the scalar potential ensuring $\sin\langle\theta\rangle \ll 1$. However this option is not completely satisfactory because the idea behind a Composite Higgs model is precisely to avoid the Hierarchy Problem by means of some symmetry. This is why in Little Higgs models an extra mechanism called *collective symmetry breaking* is introduced to ensure a small mass term in the potential relative to the quartic coupling and thus leading to a naturally small *vev*.

2.3. Collective symmetry breaking and Little Higgs

In a general Composite Higgs scenario, the pseudo-Goldstone masses are generated at one-loop and are proportional to the sources of explicit breaking of the global symmetry. The loops are cut off at the scale $\Lambda_{UV} \lesssim 4\pi f$ with $f \sim 1$ TeV and, in general, the scalar masses are proportional to $m_s \approx cf$, with c a constant. However this is not enough to naturally generate a light Higgs compared to the rest of the resonances because the scale of new physics is being pushed to the multi-TeV regime due to the non observation of new heavy particles. This implies that certain amount of fine-tuning on the different parameters of the model is required ensuring that $c \ll 1$. From the theoretical point of view this is very unlikely since we are dealing again with a sort of "Little" Hierarchy problem. But we can explore another possibility. Let us suppose that instead of a one-loop quadratically divergent contribution this is substituted by a logarithmically divergent contribution proportional to the scale f instead of Λ_{UV} [83]. In that

[1] In most of this Thesis we will consider that the only field that gets a *vev* is the Higgs field.

case, the leading one loop contribution to the Higgs mass parameter can be roughly estimated as

$$\mu^2 \sim \frac{\lambda^2}{16\pi^2} f^2 \log \frac{\Lambda_{\rm UV}^2}{f^2} \sim \frac{\lambda^2}{8\pi^2} f^2 \log(4\pi), \tag{2.48}$$

which is of the right order of magnitude to generate the correct EWSB scale if $f \sim 1$ TeV, with $\lambda \lesssim 1$ any dimensionless coupling. Notice that the quadratic divergences at two-loop level and beyond do not need to be cut off at the scale f. The two-loop contribution with cut off Λ reads

$$\delta\mu^2 \sim \frac{\lambda^4}{(16\pi^2)^2} \Lambda_{\rm UV}^2 \sim \frac{\lambda^4}{16\pi^2} f^2, \tag{2.49}$$

and it is subdominant with respect to the logarithmically divergent contribution in eq. (2.48). Let us discuss how to implement gauge and Yukawa interactions for the Goldstone fields that can fulfill the above requirements.

To introduce gauge interactions for the Higgs and the rest of scalar resonances, a subgroup $G_g \in G$ is weakly gauged. This is one source of explicit breaking of the global symmetry and could potentially generate quadratically divergent contributions to the Higgs mass parameter. However, in Little Higgs models, the gauge group is the product of at least two factors $G_1 \times G_2 \times \ldots$, each of which contains an $SU(2) \times U(1)$ subgroup. This gets spontaneously broken to the SM gauge group at the scale f by the same *vev* that breaks $G \to H$. The gauged subgroup is embedded in G in such a way that the each of the G_i factors commute with a subgroup of G that acts non linearly with the Higgs. This implies that if only one of the G_i is gauged, the remaining part of the global symmetry of the theory is sufficient to ensure that the Higgs is an exact Goldstone boson, and is therefore massless to all orders in perturbation theory and even non-perturbatively [83]. Only when the full $G_1 \times G_2 \times \ldots$ group is gauged the Higgs ceases to be an exact Goldstone boson and acquires a potential. This structure is called collective symmetry breaking by gauge interactions. It implies that any non-vanishing quantum contribution to the Higgs mass parameter must be necessarily proportional to the product of *all* the gauge couplings constants corresponding to the different G_i factors relaxing the degree of divergence to be, at most, logarithmic. If one the coupling constant is set to zero, this mechanism ensures a vanishing Higgs mass. As a result of the enlarged gauge group, these models contain additional gauge bosons at the TeV scale responsible for the cancellation of the quadratically divergent contributions. This is an important difference with respect to supersymmetric models where such cancellations are due to the contributions of particles with different statistics.

In addition to the gauge couplings of the Higgs, the model also needs to incorporate Yukawa interactions. In a generic model with cutoff $\Lambda_{\rm UV}$ where the Yukawa interactions explicitly break the global symmetries, they may generate quadratically divergent contributions to the Higgs mass parameter of size

$$\mu^2 \sim \frac{y_i^2 N_i^c}{16\pi^2} \Lambda^2 \sim y_i^2 N_i^c f^2, \tag{2.50}$$

where y_i is the Yukawa coupling of the corresponding fermion running in the loop and N_i^c the number of colors, 1 for leptons and 3 for quarks. With the exception of the top quark, all SM fermions have small Yukawa couplings, $y_i \lesssim 0.03$ and their contribution does not induce any fine tuning in the Higgs mass for $\Lambda_{\rm UV} \sim 10$ TeV. However, the top quark has a Yukawa of order one, and the quadratic divergence induced by top loops needs to be eliminated to avoid fine tuning. Inspired by the mechanism employed for gauge interactions, several Yukawa couplings are introduced in the Lagrangian, each one preserving by itself enough of the global symmetry to ensure exact vanishing of the Higgs mass. Again, quantum corrections to the Higgs mass must involve all Yukawa couplings, and no quadratically divergent contributions are generated. This requires the introduction of *top partners* at the TeV scale.

2.4. T-parity

This class of models typically predicts new particles with masses of size f due to the enlarged symmetry group required to implement the collective symmetry breaking. They are responsible for the stabilization of the electroweak scale, cancelling the SM quadratically divergent contributions to the Higgs mass parameter. However, the exchange of these heavy particles results in large corrections to precision electroweak observables [84–86]. This is because earlier implementations of Little Higgs models, such the Littlest Higgs model [87], do not contain the custodial $\mathrm{SU}(2)_C$ global symmetry [88–91], and thus weak isospin is violated. As a consequence, the mass of the heavy particles must be raised to be compatible with EWPD, imposing strong lower bounds on the scale $f > 4-5$ TeV, hence generating a tension with naturalness and again destabilizing the electroweak scale, that is proportional to f according to eq. (2.48).

One approach to solve this issue is precisely the incorporation of an approximately exact $\mathrm{SU}(2)_C$ global symmetry acting on the Higgs sector. For instance, in [89] a model based in the global group $[\mathrm{SO}(5)]^8 = [\mathrm{SO}(5)_L]^4 \times [\mathrm{SO}(5)_R]^4$ spontaneously broken to $[\mathrm{SO}(5)_{L+R}]^4$ by the *vev* of four different tensor fields each one transforming in the bifundamental of $\mathrm{SO}(5)_{Li} \times \mathrm{SO}(5)_{Ri}$, $i = 1-4$ is proposed. This gets explicitly broken by gauging a subgroup $\mathrm{SO}(5) \times [\mathrm{SU}(2) \times \mathrm{U}(1)]$. An alternative and simpler model is presented in [91], based on the global SO(9) that gets spontaneously broken to $\mathrm{SO}(5) \times \mathrm{SO}(4)$ by the *vev* of only one symmetric tensor. The gauge group is given by $\mathrm{SO}(4) \times [\mathrm{SU}(2) \times \mathrm{U}(1)]$. However, these models with extended global and gauge groups are artificially complicated.

On the other hand, a different approach based on the introduction of a new Z_2 discrete symmetry also improves the consistency with precision electroweak observables while keeping the minimal structure of the minimal Little Higgs models. This is called T-parity and was introduced by Cheng and Low in [92, 93] in analogy to the R-parity in Supersymmetry. This can be implemented in any Little Higgs model based on a product gauge group. While the SM particles are T-even, most of the new heavy particles are T-odd and thus pair-produced forbidding most of the tree-level contributions from heavy particles to observables involving only SM particles as external states. The corrections to precision electroweak observables thus enter at the one loop level, relaxing all the constraints and allowing to lower the scale f, reconciling these models with naturalness.

Due to T-parity, the new T-odd particles cannot be singly produced, which implies that direct searches rely on pair-production. Once they are produced, they will eventually decay into the lightest T-odd particle (LTP), that is stable, since T-parity forbids it from decaying into lighter particles that are all T-even. Usually the LTP is the heavy partner of the hypercharge gauge boson due to the smallness of the $\mathrm{U}(1)_Y$ gauge coupling and the normalization of the different U(1) generators. The LTP is thus electrically neutral, providing a potential dark matter candidate and giving rise to missing energy signals in collider experiments.

2.5. A toy model

In this section we illustrate how these mechanisms work in a toy model. This model already contains all the features that we will encounter in more realistic models. In particular, the presence of global SU(3) factors [49, 86, 93–96] that act non linearly on the Higgs when a single gauge or Yukawa coupling is turned on. This ensures that no quadratically divergent contributions to the Higgs mass parameter are generated as we will show explicitly in our toy model. Another feature is the addition of external U(1) factors in order to accommodate all the fermion hypercharges.

2.5.1. Global symmetries

The toy model is based on the global $G = \mathrm{SU}(3)_1 \times \mathrm{SU}(3)_2$ that gets spontaneously broken to the diagonal $H = \mathrm{SU}(3)_V$ at the scale f by the *vev* of a 6×6 tensor transforming in the bifundamental representation of G

$$\Sigma_0 = \begin{pmatrix} 0_{3\times3} & \mathbf{1}_{3\times3} \\ \mathbf{1}_{3\times3} & 0_{3\times3} \end{pmatrix}, \tag{2.51}$$

leaving $16 - 8 = 8$ Goldstone bosons. This *vev* is preserved by the 8 $\mathrm{SU}(3)_V$ generators T^a satisfying

$$T^a \Sigma_0 - \Sigma_0 T^a = 0, \tag{2.52}$$

that can be written in block diagonal form

$$T^a = \begin{pmatrix} \lambda^a & 0_{3\times3} \\ 0_{3\times3} & \lambda^a \end{pmatrix}, \tag{2.53}$$

where λ^a, $a = 1 - 8$ are the Gell-Mann matrices. The coset $\mathrm{SU}(3)_1 \times \mathrm{SU}(3)_2 \to \mathrm{SU}(3)_V$ is symmetric. Hence to characterize the broken generators one can define an outer automorphism in the Lie algebra such that the unbroken generators satisfy $T^a \xrightarrow{\text{aut}} \Sigma_0 T^a \Sigma_0 = T^a$ while the broken generators $X^a \xrightarrow{\text{aut}} \Sigma_0 X^a \Sigma_0 = -X^a$ and thus

$$X^a \Sigma_0 + \Sigma_0 X^a = 0. \tag{2.54}$$

They can also be written in block diagonal form

$$X^a = \begin{pmatrix} \lambda^a & 0_{3\times3} \\ 0_{3\times3} & -\lambda^a \end{pmatrix}. \tag{2.55}$$

Following the CCWZ formalism, the 8 Goldstone bosons are conveniently collected in the Goldstone matrix $\pi^a(x) X^a$ that allows to define the non linear field Ξ

$$\Xi = e^{i\pi^a X^a/f} = \begin{pmatrix} \xi & 0_{3\times3} \\ 0_{3\times3} & \xi^\dagger \end{pmatrix}, \quad \Xi \xrightarrow{G} V \Xi V_V^\dagger \tag{2.56}$$

where V is an $\mathrm{SU}(3)_1 \times \mathrm{SU}(3)_2$ transformation and $V_V(\Pi, V)$ is the compensating $\mathrm{SU}(3)_V$ transformation that depends on the Goldstone fields and V. They can also be written in block diagonal form

$$V = \begin{pmatrix} V_1 & 0_{3\times3} \\ 0_{3\times3} & V_2 \end{pmatrix}, \quad V_V = \begin{pmatrix} U & 0_{3\times3} \\ 0_{3\times3} & U \end{pmatrix}, \tag{2.57}$$

where V_i and U belongs to $\mathrm{SU}(3)_i$ and $\mathrm{SU}(3)_V$, respectively. We have also introduced the field ξ which is one of the pieces we need to construct invariants

$$\xi = e^{i\Pi/f}, \quad \xi \xrightarrow{G} V_1 \xi U^\dagger = U \xi V_2^\dagger, \tag{2.58}$$

with $\Pi = \pi^a(x)\lambda^a$ and the transformation properties follows straightforwardly from eq. (2.56). From ξ we can build the field that transforms linearly under G,

$$\Sigma = \xi^2, \quad \Sigma \xrightarrow{G} V_1 \Sigma V_2^\dagger, \tag{2.59}$$

that will allow us to write the kinetic term and gauge interactions for the scalar fields. From now on we will use the notation of matrices 3×3 and we will work with ξ and Σ.

2.5.2. Gauge symmetries

In order to break the global symmetries and implement the collective symmetry breaking mechanism, the subgroup $G_g = [\mathrm{SU}(2) \times \mathrm{U}(1)]_1 \times [\mathrm{SU}(2) \times \mathrm{U}(1)]_2$ is weakly gauged. Each $G_i = [\mathrm{SU}(2) \times \mathrm{U}(1)]_i$ factor commutes with a global factor $\mathrm{SU}(3)_j$ with $i \neq j$. The gauge group is spanned by the Hermitian and traceless generators

$$Q_i^a = \frac{1}{2}\begin{pmatrix} \sigma^a & 0 \\ 0 & 0 \end{pmatrix}, \qquad\qquad Y_i = \frac{1}{6}\mathrm{diag}\,(1, 1, -2)\,, \tag{2.60}$$

with σ^a the three Pauli matrices. The normalization of the hypercharge generator is chosen to find a Higgs boson among the Goldstone fields. This can be seen as follows. The fundamental representation $\mathbf{3}$ of SU(3) decomposes under $\mathrm{SU}(2) \times \mathrm{U}(1)$ [97]

$$\mathbf{3} = \mathbf{2}_y \oplus \mathbf{1}_{-2y}, \tag{2.61}$$

because the generators of the global group, in particular the hypercharge generator, are traceless. To fix the hypercharge y one uses that the Goldstone matrix Π transforms in the adjoint representation $\mathbf{8}$ of $\mathrm{SU}(3)_V$. This can be found in the product $\mathbf{3}_V \otimes \mathbf{3}_V^* = \mathbf{8} \oplus \mathbf{1}$ and under $\mathrm{SU}(2) \times \mathrm{U}(1)$ decomposes

$$\mathbf{8} = \mathbf{3}_0 \oplus \mathbf{1}_0 \oplus \mathbf{2}_{3y}, \tag{2.62}$$

where we have used eq. (2.61). To identify the Higgs boson with the doublet, one fixes $y = 1/6$. As a consequence, the fundamental representation $\mathbf{3}$ of SU(3) decomposes into an SU(2) doublet with hypercharge $1/6$ and a singlet with hypercharge $-1/3$ while the Goldstone matrix decomposes into a neutral SU(2) singlet, a real triplet and a complex doublet. In the SM symmetric phase it takes the form

$$\Pi = \begin{pmatrix} \dfrac{\omega^0}{2} + \dfrac{\eta}{2\sqrt{3}} & \dfrac{\omega^+}{\sqrt{2}} & \dfrac{\pi^+}{\sqrt{2}} \\ \dfrac{\omega^-}{\sqrt{2}} & -\dfrac{\omega^0}{2} + \dfrac{\eta}{2\sqrt{3}} & \dfrac{h + i\pi^0}{2} \\ \dfrac{\pi^-}{\sqrt{2}} & \dfrac{h - i\pi^0}{2} & -\dfrac{1}{\sqrt{3}}\eta \end{pmatrix}. \tag{2.63}$$

When the gauge group gets spontaneously broken by Σ_0 to the diagonal $\mathrm{SU}(2) \times \mathrm{U}(1)$, the triplet and the singlet becomes the longitudinal modes of the new heavy gauge fields. Finally, the Higgs takes a *vev* that spontaneously breaks the SM gauge group to $\mathrm{U}(1)_{\mathrm{em}}$ and three of the four components of the complex doublet become the longitudinal modes of the SM gauge bosons. The only remaining scalar in the physical spectrum is the Higgs boson.

2.5.3. Lagrangian

Gauge sector

First one adds the gauge boson kinetic term and self interactions

$$\mathcal{L}_G = \sum_{j=1}^{2}\left[-\frac{1}{2}\mathrm{tr}\left(\widetilde{W}_{j\mu\nu}\widetilde{W}_j^{\mu\nu}\right) - \frac{1}{4}B_{j\mu\nu}B_j^{\mu\nu}\right], \tag{2.64}$$

in terms of fields and field strength tensors,

$$\widetilde{W}_{j\mu} = W^a_{j\mu}Q^a_j, \tag{2.65}$$

$$\widetilde{W}_{j\mu\nu} = \partial_\mu\widetilde{W}_{j\nu} - \partial_\nu\widetilde{W}_{j\mu} - ig_j\left[\widetilde{W}_{j\mu}, \widetilde{W}_{j\nu}\right], \tag{2.66}$$

$$B_{j\mu\nu} = \partial_\mu B_{j\nu} - \partial_\nu B_{j\mu}. \tag{2.67}$$

Scalar sector

To build up kinetic terms and gauge interactions for the scalar fields, and eventually obtain gauge boson mass terms, one defines the covariant derivative for the linear field Σ,

$$D_\mu\Sigma = \partial_\mu\Sigma + ig_1W^a_{1\mu}Q^a_1\Sigma - ig_2W^a_{2\mu}\Sigma Q^a_2 - ig'_1B_{1\mu}Y_1\Sigma + ig'_2B_{2\mu}\Sigma Y_2, \tag{2.68}$$

where we have used that the tensor Σ transforms according to eq. (2.59). Let us point out that switching off the coupling constants of $[\mathrm{SU}(2)\times\mathrm{U}(1)]_1$ ($[\mathrm{SU}(2)\times\mathrm{U}(1)]_2$), the covariant derivative transforms covariantly under the global $\mathrm{SU}(3)_2$ ($\mathrm{SU}(3)_1$). Then the scalar Lagrangian given by

$$\mathcal{L}_S = \frac{f^2}{4}\mathrm{tr}\left[\left(D_\mu\Sigma\right)^\dagger D^\mu\Sigma\right] \tag{2.69}$$

is invariant under the global $\mathrm{SU}(3)_2$ ($\mathrm{SU}(3)_1$) and the Higgs cannot develop a mass due to the interactions of only one of the factors of the gauge group. However, when both couplings are non vanishing at the same time, all the global SU(3)'s are explicitly broken. Since both gauge couplings are needed to generate a non vanishing contribution to the Higgs mass, the divergence is, at most, logarithmic. After evaluating the Feynman diagrams depicted in fig. 2.2, the leading order contribution reads

$$\delta\mu^2_{\mathrm{gauge}} = \frac{f^2}{16\pi^2}\left(\frac{3}{4}g_1^2g_2^2 + \frac{1}{12}g_1'^2g_2'^2\right)\log\Lambda^2, \tag{2.70}$$

as a consequence of the collective symmetry breaking mechanism.

Fermion sector

In the fermionic sector we will focus on the top quark that iis the heaviest SM particle and generates quadratically divergent contributions to the Higgs mass. Again, the idea is to introduce operators that exactly preserve one of the global SU(3) factors while softly breaking the other. For that purpose the SM top quark doublet $q_L = (t_L, b_L)$ is promoted to a complete SU(3) multiplet containing a heavy top partner that can cancel the quadratic divergence of the top quark. However, we showed that the fundamental representation of SU(3) contains an SU(2) doublet with hypercharge $1/6$ and a singlet of hypercharge $-1/3$. The doublet has the appropriate quantum numbers to be identified with the SM left-handed quark doublet, but the

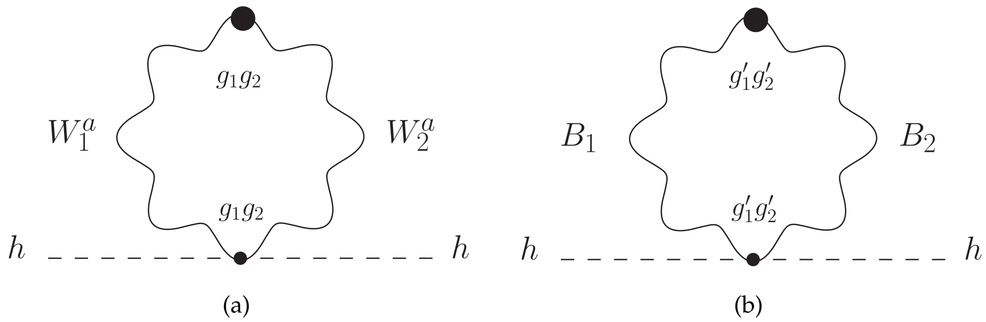


FIGURE 2.2: One-loop contribution to the Higgs mass squared from gauge bosons in the gauge eigenbasis. The big blobs represent off-diagonal mass insertions proportional to $g_1 g_2$ and $g_1' g_2'$.

singlet is a down-type quark and thus cannot compensate for the quadratic divergence of the top quark. To address this issue let us instead consider the antifundamental representation $\mathbf{3}^*$. This contains an SU(2) doublet of hypercharge $-1/6$ and a singlet of hypercharge $1/3$. Nevertheless, extending the global symmetry group to

$$\mathrm{SU}(3)_1 \times \mathrm{SU}(3)_2 \times \mathrm{U}(1)_1 \times \mathrm{U}(1)_2 \tag{2.71}$$

one can assign the extra hypercharge of $1/3$ for the doublet and the singlet to the external U(1) factors according to table 2.1. As a consequence, the gauge hypercharge is now the sum of those in $\mathrm{SU}(3)_1 \times \mathrm{SU}(3)_2$ and $\mathrm{U}(1)_1 \times \mathrm{U}(1)_2$, preserving the number of U(1) gauge bosons. This allows us to define the *royal* triplet

$$\chi_L = \begin{pmatrix} -i\sigma^2 q_L \\ iU_L \end{pmatrix}, \quad \chi_L \xrightarrow{G} V_1^* \chi_L \tag{2.72}$$

transforming in the $\mathbf{3}_1^*$ representation of $\mathrm{SU}(3)_1$.

To account for the right-handed counterparts of the top quark and the top partner, we also introduce the incomplete right-handed multiplets

$$\chi_R = \begin{pmatrix} 0_2 \\ it_R \end{pmatrix}, \quad \chi_R' = \begin{pmatrix} 0_2 \\ iU_R \end{pmatrix}, \quad \chi_R \xrightarrow{G} V_2^* \chi_R, \quad \chi_R' \xrightarrow{G} V_1^* \chi_R' \tag{2.73}$$

transforming in the $\mathbf{3}_2^*$ and $\mathbf{3}_1^*$ of the global group, respectively. The fields t_R and U_R are SU(2) singlets and they also receive the required extra hypercharge from the external U(1) factors according to table 2.1.

To provide a mass for the top quark and the top partner in this toy model there are two possible implementations of a Yukawa Lagrangian. The first one is given by

$$\mathcal{L}_t^1 = -\frac{\lambda_1 f}{\sqrt{2}} \overline{\chi}_{Li} \Sigma_{ij}^* \chi_{Rj} - \frac{\lambda_2 f}{\sqrt{2}} \overline{\chi}_L \chi_R' + \text{h.c.} = -i \frac{\lambda_1}{\sqrt{2}} \overline{\chi}_{Li} \Sigma_{i3}^* t_R - \frac{\lambda_2}{\sqrt{2}} \overline{U}_L U_R + \text{h.c.}, \tag{2.74}$$

where in the last equality we use that the right-handed multiplets are incomplete. Let us carefully inspect the symmetries of this implementation of the top quark Lagrangian, since the use of incomplete multiplets breaks the global symmetries. It is clear that due to the introduction of the $\mathrm{SU}(3)_1$ left-handed royal triplet, the first term is invariant under a global $\mathrm{SU}(3)_1$ transformation. However, it only preserves the upper $\mathrm{SU}(2)_2$ block of $\mathrm{SU}(3)_2$ due to the incomplete $\mathrm{SU}(3)_2$ right-handed multiplet χ_R. On the other hand, the second term is invariant under

	$[SU(2)\times U(1)]^2 \subset SU(3)^2$	$[U(1)']^2$	$[SU(2)\times U(1)]^2_{\text{gauge}}$
q_L	$(2,1)_{(-\frac{1}{6},0)}$	$(\frac{1}{3},0)$	$(2,1)_{(\frac{1}{6},0)}$
t_R	$(1,1)_{(0,\frac{1}{3})}$	$(\frac{1}{3},0)$	$(1,1)_{(\frac{1}{3},\frac{1}{3})}$
U_L, U_R	$(1,1)_{(\frac{1}{3},0)}$	$(\frac{1}{3},0)$	$(1,1)_{(\frac{2}{3},0)}$

TABLE 2.1: Quantum numbers for quarks. The gauged hypercharge $U(1)_i$ is the sum of that in $SU(3)_i$ and the one in the extra $U(1)'_i$.

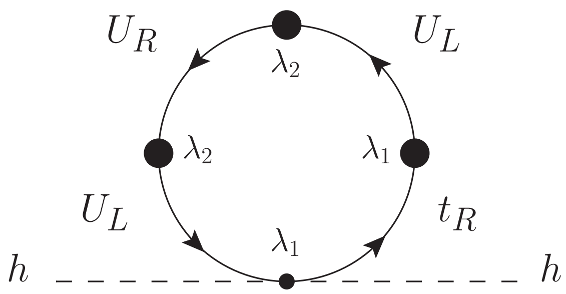


FIGURE 2.3: One-loop contribution to the Higgs mass squared from the top quark and top partners in the gauge eigenbasis. The big blobs are mass insertions proportional to the λ_1, λ_2 Yukawa couplings in $\mathcal{L}_t^1$ and $\mathcal{L}_t^2$ in eqs. (2.74) and (2.75), respectively. Diagrams with only t_L and t_R are absent because they are $SU(3)_1$ symmetric.

$SU(3)_2$ but it only preserves the upper $SU(2)_1$ of $SU(3)_1$. Therefore, this Lagrangian properly implements the collective symmetry breaking mechanism and no quadratically divergent contributions to the Higgs mass are generated.

Another proposal for the top quark Yukawa Lagrangian is the following. Consider the product $\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}$ of $SU(3)$. This contains an antisymmetric singlet that can be built with the totally antisymmetric Levi-Civita tensor. Applying this argument to the indices of both SU(3) factors, one can construct

$$\begin{aligned}\mathcal{L}_t^2 &= -\frac{\lambda_1 f}{\sqrt{2}}\epsilon_{ijk}\epsilon_{xyz}\overline{\chi}_{Li}\Sigma_{jx}\Sigma_{ky}\chi_{Rz} - \frac{\lambda_2 f}{\sqrt{2}}\overline{\chi}_L\chi'_R + \text{h.c.}\\ &= -i\frac{\lambda_1 f}{\sqrt{2}}\epsilon_{ijk}\epsilon_{xy}\overline{\chi}_{Li}\Sigma_{jx}\Sigma_{ky}t_R - \frac{\lambda_2 f}{\sqrt{2}}\overline{U}_L U_R + \text{h.c.},\end{aligned} \tag{2.75}$$

where in the last expression ϵ_{ijk} with $\{i,j,k\} = 1,2,3$ and ϵ_{xy} with $\{x,y\} = 1,2$ are the $SU(3)_1$ and $SU(2)_2$ Levi-Civita tensors, respectively. The global symmetries of this Lagrangian are the same than in the previous case justifying that again no quadratically divergent contributions are generated. The advantage of this apparently more sophisticated proposal is that it can also be applied to simple groups that contain different SU(3) factors that mix, as it is the case of the *Littlest Higgs model* [87].

On the other hand, the logarithmically divergent contribution to the Higgs mass is non vanishing and gives a *negative* correction

$$\delta\mu^2_{\text{top}} = -\frac{3}{16\pi^2}f^2\lambda_1^2\lambda_2^2\log\Lambda^2 \tag{2.76}$$

in both implementations of the top quark Yukawa Lagrangian in eqs. (2.74) and (2.75) from the diagram in fig. 2.3. This term can compete with the positive gauge boson contribution in eq. (2.70), giving rise to a negative Higgs mass squared and thus triggering the EWSB.

Finally, we add the kinetic term and gauge interactions for fermions taking into account their quantun numbers, given in table 2.1,

$$\mathcal{L}_F = i\overline{\chi}_L\gamma^\mu D^*_\mu\chi_L + i\overline{u}_R\gamma^\mu\left[\partial_\mu + ig'_1\frac{1}{3}B_{1\mu} + ig'_2\frac{1}{3}B_{2\mu}\right]t_R + i\overline{U}_R\gamma^\mu\left[\partial_\mu + ig'_1\frac{2}{3}B_{1\mu}\right]U_R, \quad (2.77)$$

where the covariant derivative is given by

$$D_\mu = \partial_\mu - ig_1 W^a_{1\mu}Q^a_1 + ig'_1\left(Y_1 - \frac{1}{3}\mathbb{1}_{3\times 3}\right)B_{1\mu}. \quad (2.78)$$

Then, the full Lagrangian of this theory is

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_S + \mathcal{L}_t^{1(2)} + \mathcal{L}_F. \quad (2.79)$$

This toy model contains all the features of a Little Higgs model. In particular it is free of quadratically divergent contributions to the Higgs mass thanks to the use of the collective symmetry breaking mechanism. However the new heavy particles can generate dangerous contributions to observables that are very constrained by EWPD as we explained in sec. 2.4. For that reason, in the following we will show how to implement the discrete T-parity symmetry in our toy model.

2.5.4. T-parity implementation

Gauge sector

The T-parity leaves invariant the SM fields (T-even) and adds a minus sign to the new (heavy) fields (T-odd). Thus the latter are pair-produced, relaxing direct and indirect constraints from EWPD. The action of T-parity interchanges the two gauge groups

$$G_1 \stackrel{T}{\longleftrightarrow} G_2, \quad (2.80)$$

which requires that the coupling constants of both copies must be equal $g_1 = g_2 = \sqrt{2}g$ and $g'_1 = g'_2 = \sqrt{2}g'$. With this condition the Lagrangian in eq. (2.64) is not only gauge but also T-parity invariant. The T-odd and T-even combinations of W^a_1, W^a_2 and B_1, B_2 are identified with the new and the SM gauge bosons, respectively.

Scalar sector

To define T-parity consistently in the scalar sector, the would-be Goldstone bosons η and $\omega^\pm$, ω^0 must be T-odd while the Higgs remains T-even. For that purpose, we introduce the element $\Omega \in \mathrm{SU}(3)$ such that

$$\Pi \xrightarrow{T} -\Omega\Pi\Omega, \quad \Omega = \mathrm{diag}\,(-1,-1,1)\,. \quad (2.81)$$

It is important to notice that Ω commutes with the gauge generators but does not commute with the full set of SU(3) generators. The T-parity transformation of the Goldstone matrix implies

$$\xi \xrightarrow{T} \Omega\xi^\dagger\Omega, \quad \Sigma \xrightarrow{T} \Omega\Sigma^\dagger\Omega. \quad (2.82)$$

The scalar Lagrangian in eq. (2.69) with $g_1 = g_2 = \sqrt{2}g$ and $g'_1 = g'_2 = \sqrt{2}g'$ is T-parity and gauge invariant and provides the gauge boson mass terms. However, the action of the collective symmetry breaking mechanism is now hidden due to the lost of the explicit dependence on gauge couplings of the different gauge groups.

Regarding to the gauge boson contributions to the Higgs mass term, these are given by the same diagrams of fig. 2.2 giving rise to the same result of eq. (2.70) with $g_1 = g_2 = \sqrt{2}g$ and $g'_1 = g'_2 = \sqrt{2}g'$.

Fermion sector

Introducing T-parity in the fermion sector is less straightforward. Notice that either of the proposals for the top quark Yukawa Lagrangian in eqs. (2.74) and (2.75) are not invariant under the T-parity transformations (2.82) assuming that χ_L and t_R are T-even states. For that reason, keeping t_R T-even, one introduces two left-handed royal triplets in the 3_1^* and 3_2^* representations of the global group

$$\chi_{1L} = \begin{pmatrix} -i\sigma^2 q_{1L} \\ iU_{1L} \end{pmatrix}, \quad \chi_{2L} = \begin{pmatrix} -i\sigma^2 q_{2L} \\ iU_{2L} \end{pmatrix}, \quad \chi_{1L} \xrightarrow{G} V_1^* \chi_{1L}, \quad \chi_{2L} \xrightarrow{G} V_2^* \chi_{2L}. \tag{2.83}$$

For the T-parity transformation of these multiplets one can contemplate two different options,

$$a) \quad \chi_{1L} \overset{\mathrm{T}}{\longleftrightarrow} \Omega \chi_{2L}, \tag{2.84}$$

$$b) \quad \chi_{1L} \overset{\mathrm{T}}{\longleftrightarrow} -\chi_{2L}, \tag{2.85}$$

that differ in the T-parity transformation of the left-handed top partner fields $U_{1L} \overset{\mathrm{T}}{\longleftrightarrow} \pm U_{2L}$.

To provide a mass for the top partners, one adds their corresponding right-handed counterparts, U_{1R} and U_{2R}, in incomplete right-handed multiplets transforming in the 3_1^* and 3_2^* representations of the global group, respectively [2]

$$\chi'_{1L} = \begin{pmatrix} 0_2 \\ iU_{1R} \end{pmatrix}, \quad \chi'_{2L} = \begin{pmatrix} 0_2 \\ iU_{2R} \end{pmatrix}, \quad \chi'_{1R} \xrightarrow{G} V_1^* \chi'_{1R}, \quad \chi'_{2R} \xrightarrow{G} V_2^* \chi_{2R}. \tag{2.86}$$

transforming consistently under T-parity as

$$a) \quad U_{1R} \overset{\mathrm{T}}{\longleftrightarrow} U_{2R}, \tag{2.87}$$

$$b) \quad U_{1R} \overset{\mathrm{T}}{\longleftrightarrow} -U_{2R}. \tag{2.88}$$

Then the first proposal for the top quark Yukawa Lagrangian in eq. (2.74), invariant under the T-parity realization in eqs. (2.84) and (2.87), reads

$$\mathcal{L}_t^{1a} = -i\frac{\lambda_1 f}{2}\left[\overline{\chi}_{1Li}\Sigma_{i3}^* + \overline{\chi}_{2Li}\Sigma_{i3}^T\right] u_R - \frac{\lambda_2 f}{\sqrt{2}}\left[\overline{U}_{1L}U_{1R} + \overline{U}_{2L}U_{2R}\right] + \text{h.c.}, \tag{2.89}$$

providing a mass for the top quark contained in the T-even combination of q_{1L} and q_{2L}. The dependence on $\Omega \in SU(3)$ is eliminated using unitarity and $\Omega_{33} = 1$. To inspect the global symmetries of this Lagrangian, let us first focus on the terms proportional to λ_1. Due to the transformation properties of Σ and treating t_R as an spurious field, the first term requires t_R transforming as a 3_2^* like in the case without T-parity in eq. (2.73). On the other hand, the second term would require t_R transforming as a 3_1^*. Hence, an embedding similar to that in eq. (2.73) is

[2] They could also be singlets under the full $SU(3)_1 \times SU(3)_2$ and everything would still be consistent. However we prefer to keep the theory as symmetric as possible.

	$[\mathrm{SU}(2)\times\mathrm{U}(1)]^2\subset\mathrm{SU}(3)^2$	$[\mathrm{U}(1)']^2$	$[\mathrm{SU}(2)\times\mathrm{U}(1)]^2_{\text{gauge}}$
q_{1L}	$(2,1)_{(-\frac{1}{6},0)}$	$(\frac{1}{3},0)$	$(2,1)_{(\frac{1}{6},0)}$
q_{2L}	$(1,2)_{(0,-\frac{1}{6})}$	$(0,\frac{1}{3})$	$(1,2)_{(0,\frac{1}{6})}$
t_R	$(1,1)_{(0,0)}$	$(\frac{1}{3},\frac{1}{3})$	$(1,1)_{(\frac{1}{3},\frac{1}{3})}$
U_{1L},U_{1R}	$(1,1)_{(\frac{1}{3},0)}$	$(\frac{1}{3},0)$	$(1,1)_{(\frac{2}{3},0)}$
U_{2L},U_{2R}	$(1,1)_{(0,\frac{1}{3})}$	$(0,\frac{1}{3})$	$(1,1)_{(0,\frac{2}{3})}$

TABLE 2.2: Quantum numbers for quarks transforming in a linear representation of the global group. The gauged hypercharge $\mathrm{U}(1)_i$ is the sum of that inside $\mathrm{SU}(3)_i$ and the one in the extra $\mathrm{U}(1)'_i$.

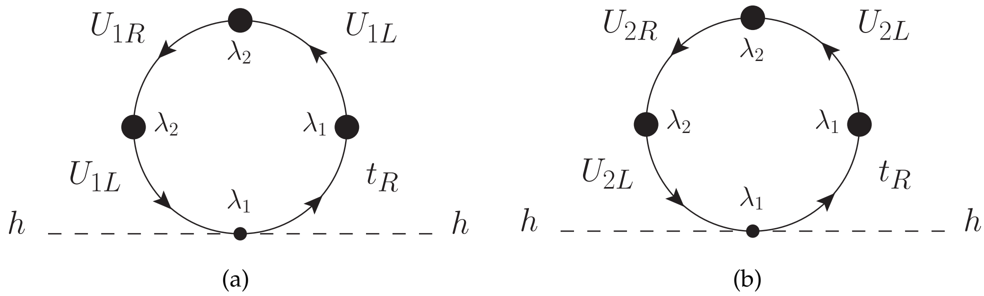


FIGURE 2.4: One-loop contribution to the Higgs mass squared from the top quark and top partners in the gauge eigenbasis in the toy model with T-parity. The big blobs are mass insertions proportional to the Yukawa couplings λ_1 and λ_2.

not consistent and t_R must be a singlet under $\mathrm{SU}(3)_1\times\mathrm{SU}(3)_2$ with all its gauge hypercharge laying in the external $\mathrm{U}(1)$ factors. As a consequence, the first term is invariant under $\mathrm{SU}(3)_1\times\mathrm{SU}(2)_2$ transformations while the second term is invariant under $\mathrm{SU}(2)_1\times\mathrm{SU}(3)_2$, leading to a potential logarithmically divergent contribution to the Higgs mass proportional to λ_1^4 from the integration of χ_{1L} and χ_{2L}. However, this is not the case because Σ^*_{i3} has only a free index in $\mathrm{SU}(3)_1$ while Σ^T_{i3} has a free index in $\mathrm{SU}(3)_2$ and the only possible invariant operator that can be generated is

$$\mathcal{O}^{\log}_{1a}=\left(\Sigma^*_{3i}\Sigma^T_{i3}\right)^2 \tag{2.90}$$

which, from unitarity and complete multiplets, does not depend on the Goldstone fields. Regarding the term proportional to λ_2, the first one preserves $\mathrm{SU}(2)_1\times\mathrm{SU}(3)_2$ while the second one preserves $\mathrm{SU}(3)_1\times\mathrm{SU}(2)_2$. So the first term proportional to λ_1 and the first proportional to λ_2 can combine in exactly the same fashion as in the case without T-parity, and the same for the two other terms. As a result of this, the implementation of T-parity in this Lagrangian leads to two copies of the case without T-parity, hence contributing to the Higgs mass with just a logarithmic dependence on the cutoff proportional to $\lambda_1^2\lambda_2^2$ as one can check evaluating the Feynman diagrams in fig. 2.4.

The second proposal for the top quark Lagrangian in eq. (2.75) invariant under the T-parity transformations in eqs. (2.84) and (2.87) is given by

$$\mathcal{L}^{2a}_t=-i\frac{\lambda_1 f}{2}\epsilon_{ijk}\epsilon_{xy}\left[\overline{\chi}_{1Li}\Sigma_{jx}\Sigma_{ky}+\overline{\chi}_{2Li}\Sigma^\dagger_{jx}\Sigma^\dagger_{ky}\right]t_R-\frac{\lambda_2 f}{\sqrt{2}}\left[\overline{U}_{1L}U_{1R}+\overline{U}_{2L}U_{2R}\right]+\text{h.c.}, \tag{2.91}$$

	$SU(2)\times U(1)\subset SU(3)_V$	$U(1)'_{1+2}$	$[SU(2)\times U(1)]_{\rm gauge}$
q_{HR}	$2_{-\frac{1}{6}}$	$\frac{1}{3}$	$2_{\frac{1}{6}}$
x_{HR}, x_{HL}	$1_{\frac{1}{3}}$	$\frac{1}{3}$	$1_{\frac{2}{3}}$

TABLE 2.3: Quantum numbers for quarks transforming in a non linear representation of the global group. The gauged hypercharge is the sum of that inside $SU(3)_V$ and the one in the extra $U(1)'_{1+2}$.

with $\{i,j,k\}=1,2,3$ and $\{x,y\}=4,5$. In order to eliminate the explicit dependence on $\Omega\in SU(3)$ we used that the Levi-Civita tensor ϵ_{ijk} is invariant under SU(3) transformations and, on the other hand, the upper 2×2 block of Ω is minus the identity and thus leaving ϵ_{xy} invariant. Notice that each of the terms proportional to λ_1 should have its own Levi-Civita tensors since in the first term the i,j,k are $SU(3)_1$ indices and k,l are $SU(2)_2$ indices while for the second term is just the opposite. However, they have the same mathematical expression which allows to write them as a common factor for both terms. This in turn implies that the global symmetries are exactly the same as in eq. (2.89), justifying that the Higgs mass squared only receives a logarithmically divergent contribution proportional to $\lambda_1^2\lambda_2^2$. One can explicitly check that the potential logarithmic contribution proportional to λ_1^4 coming from the integration of χ_{1L} and χ_{2L} leads to

$$\mathcal{O}_{2a}^{\rm log}=\left(\epsilon_{ijk}\epsilon_{xy}\epsilon_{ij'k'}\epsilon_{x'y'}\Sigma_{jx}\Sigma_{ky}\Sigma^\dagger_{x'j'}\Sigma^\dagger_{y'k'}\right)^2, \tag{2.92}$$

that does not depend on the Goldstone fields after using $\epsilon_{ijk}\epsilon_{ij'k'}=\delta_{jj'}\delta_{kk'}-\delta_{jk'}\delta_{kj'}$ and the unitarity of Σ.

On the other hand, implementing the T-parity transformations in eqs. (2.85) and (2.88) in both proposals for the top quark Yukawa Lagrangian leads to

$$\mathcal{L}_t^{1b}=-i\frac{\lambda_1}{2}\left[\overline{\chi}_{1Li}\Sigma^*_{i3}-\overline{\chi}_{2Li}(\Omega\Sigma^T\Omega)_{i3}\right]t_R-\frac{\lambda_2}{\sqrt{2}}\left[\overline{U}_{1L}U_{1R}+\overline{U}_{2L}U_{2R}\right]+\text{h.c.}. \tag{2.93}$$

and

$$\begin{aligned}\mathcal{L}_t^{2b}=&-i\frac{\lambda_1}{2}\epsilon_{ijk}\epsilon_{xy}\left[\overline{\chi}_{1Li}\Sigma_{jx}\Sigma_{ky}-\overline{\chi}_{2Li}(\Omega\Sigma^\dagger\Omega)_{jx}(\Omega\Sigma^\dagger\Omega)_{ky}\right]t_R\\&-\frac{\lambda_2}{\sqrt{2}}\left[\overline{U}_{1L}U_{1R}+\overline{U}_{2L}U_{2R}\right]+\text{h.c.}\end{aligned} \tag{2.94}$$

These Lagrangians also share the same global symmetries. Out of the two terms proportional to λ_1, the first one is invariant under $SU(3)_1$. However, contrary to the Yukawa Lagrangians (2.74) and (2.75), the second term depends explicitly on Ω, that cannot be eliminated, and thus it is not invariant under global $SU(3)_2$ transformations. In $\mathcal{L}_t^{1b}$ the element Ω cannot be eliminated using unitarity because the transformation under T-parity of χ_{1L} in eq. (2.85) does not involve Ω. On the other hand, in $\mathcal{L}_t^{2b}$, one cannot apply the SU(3) invariance of ϵ_{ijk} because there is not the appropriate number of Ω's to contract with each index of the Levi-Civita tensors. This may generate a quadratically divergent contribution to the Higgs mass proportional to λ_1^2. However, the unitarity of Ω is enough to forbid this potential quadratic divergence. This can be explicitly shown in each case inspecting the only operators that arise at one loop proportional to λ_1^2 after integrating χ_{2L} and t_R,

$$\mathcal{O}_{1b}^{\rm quad}=(\Omega\Sigma^*\Omega)_{3i}(\Omega\Sigma^T\Omega)_{i3}, \tag{2.95}$$

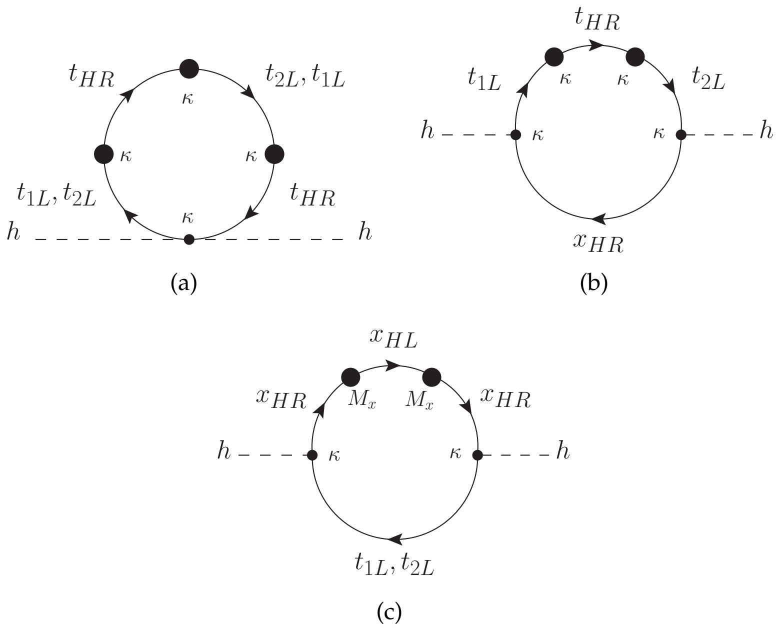


FIGURE 2.5: One-loop contribution to the Higgs mass squared from $\mathcal{L}_H$ in the gauge eigenbasis. The big blobs represent off-diagonal mass insertions proportional to κ and mass insertions proportional to M_x.

$$\mathcal{O}_{2b}^{\text{quad}} = \epsilon_{ijk}\epsilon_{ij'k'}\epsilon_{xy}\epsilon_{x'y'}(\Omega\Sigma^\dagger\Omega)_{jx}(\Omega\Sigma^\dagger\Omega)_{ky}(\Omega\Sigma\Omega)_{x'j'}(\Omega\Sigma\Omega)_{y'k'}. \tag{2.96}$$

The first operator does not depend on the Goldstone fields because Ω and Σ are unitary. For the second operator, after applying $\epsilon_{ijk}\epsilon_{ij'k'} = \delta_{jj'}\delta_{kk'} - \delta_{jk'}\delta_{kj'}$ and unitarity, it does not depend on the scalar fields either. Regarding the logarithmically divergent contributions proportional to λ_1^4, the integration of χ_{2L} leads to the same operators $\mathcal{O}_{1b}^{\text{quad}}$ and $\mathcal{O}_{2b}^{\text{quad}}$ but squared. On the other hand, the integration of χ_{1L}, χ_{2L} and t_R, combining properly the different SU(3) indices leads to

$$\mathcal{O}_{1b}^{\log} = \left[\Sigma_{3i}^*\Sigma_{i3}^T\right]\left[(\Omega\Sigma^T\Omega)_{3i}(\Omega\Sigma^*\Omega)_{i3}\right] = \left[\Sigma_{3i}^*\Sigma_{i3}^T\right]^2 \tag{2.97}$$

$$\begin{aligned}\mathcal{O}_{2b}^{\log} &= \left[\epsilon_{ijk}\epsilon_{xy}\epsilon_{ij'k'}\epsilon_{x'y'}\Sigma_{jx}\Sigma_{ky}\Sigma^\dagger_{x'j'}\Sigma^\dagger_{y'k'}\right] \\ &\times \left[\epsilon_{ijk}\epsilon_{xy}\epsilon_{ij'k'}\epsilon_{x'y'}(\Omega\Sigma\Omega)_{jx}(\Omega\Sigma\Omega)_{ky}(\Omega\Sigma^\dagger\Omega)_{x'j'}(\Omega\Sigma^\dagger\Omega)_{y'k'}\right] \\ &= \left[\epsilon_{ijk}\epsilon_{xy}\epsilon_{ij'k'}\epsilon_{x'y'}\Sigma_{jx}\Sigma_{ky}\Sigma^\dagger_{x'j'}\Sigma^\dagger_{y'k'}\right]^2, \end{aligned} \tag{2.98}$$

that due to the unitarity of Ω do not depend on the Goldstone fields. Therefore this T-parity realization also provides a contribution to the Higgs mass squared proportional to $\lambda_1^2\lambda_2^2$ from the diagrams in fig. 2.4.

One may think that depending on the T-parity implementation, the diagrams in fig. 2.4 would give a different result. However, the two different T-parity realizations only differ in the sign of the couplings of U_{2L} to t_R. Since there are two of those couplings in the second diagram of fig. 2.4, the same contribution is generated in both cases. Another crucial feature is that there is no diagram mixing U_1 and U_2 as we discussed above.

Contrary to the case without T-parity, this is not the end of the story. Due to the introduction of two different doublets q_{1L} and q_{2L}, their T-odd combination dubbed *mirror* quarks, remains massless at this stage. To provide it with a heavy vector-like mass proportional to f we compose two incomplete left-handed multiplets in the anti-fundamental representation of $SU(3)_1$ and $SU(3)_2$ respectively

$$\Psi_1 = \begin{pmatrix} -i\sigma^2 q_{1L} \\ 0 \end{pmatrix}, \quad \Psi_2 = \begin{pmatrix} -i\sigma^2 q_{2L} \\ 0 \end{pmatrix}, \tag{2.99}$$

with the same transformation properties under the global group and T-parity as those for χ_{1L}, χ_{2L} in eqs. (2.83), (2.84) and (2.85). They are coupled through the ξ field to a *complete* right-handed multiplet transforming in the 3^* of $SU(3)_V$

$$\Psi_R = \begin{pmatrix} -i\sigma^2 q_{HR} \\ i x_{HR} \end{pmatrix}, \quad \Psi_R \xrightarrow{G} U^* \Psi_R, \tag{2.100}$$

where U is the non linear transformation depending on V_1, V_2 and Π, that becomes linear when $V_1 = V_2 = U \in SU(3)_V$. Under the SM gauge group Ψ_R decomposes in a doublet with hypercharge $1/6$ and a singlet of hypercharge $2/3$ with quantum numbers under the different factors of the global symmetry group given in table 2.3. Notice that, as a consequence of T-parity and the complete Ψ_R multiplet, the fermion content of the model is, at least, doubled with respect to the case without T-parity. Under the different T-parity implementations Ψ_R transforms

$$a) \quad \Psi_R \overset{\mathrm{T}}{\longleftrightarrow} \Omega \Psi_R \tag{2.101}$$

$$b) \quad \Psi_R \overset{\mathrm{T}}{\longleftrightarrow} -\Psi_R, \tag{2.102}$$

that differ on the T-parity assignation of the right-handed field x_{HR}. Consequently one can construct two different versions of a Yukawa Lagrangian

$$a) \quad \mathcal{L}_H^a = -\kappa f \left(\overline{\Psi}_1 \xi^* + \overline{\Psi}_2 \xi^T \right) \Psi_R + \text{h.c.} \tag{2.103}$$

$$b) \quad \mathcal{L}_H^b = -\kappa f \left(\overline{\Psi}_1 \xi^* + \overline{\Psi}_2 \Omega \xi^T \Omega \right) \Psi_R + \text{h.c.}, \tag{2.104}$$

depending on the T-parity realization. Let us analyse each option. In option $a)$ the matter content of the right-handed multiplet ensures that the Lagrangian respects the collective symmetry breaking mechanism. The first term is invariant under $SU(3)_2$ preserving an $SU(2)_1 \subset SU(3)_1$ and the second term is invariant under $SU(3)_1$ preserving an $SU(2)_2 \subset SU(3)_2$ and hence the Higgs mass can only receive a logarithmically divergent contribution from these interactions proportional to κ^4 from the two first diagrams in fig. 2.5. However, in option $b)$ the second term is not invariant under $SU(3)_1$ global transformations due to the presence of Ω. This could yield again to a potential quadratically divergent contribution to the Higgs mass from the integration of Ψ_2 and Ψ_R. However, the only operator that can contribute to this quadratic divergence has the form

$$\mathcal{O}_{Rb}^{\text{quad}} = \text{tr}(A^\dagger \Omega \xi^T \Omega B B^\dagger \Omega \xi^* \Omega A), \tag{2.105}$$

where

$$A = \begin{pmatrix} -i\sigma^2 & \\ & 0 \end{pmatrix}, \quad B = \begin{pmatrix} -i\sigma^2 & \\ & i \end{pmatrix} \tag{2.106}$$

take into account that Ψ_2 and Ψ_R are incomplete and complete multiplets, respectively. This operator does not depend on the Goldstone fields due to the unitarity of Ω and the complete Ψ_R in eq. (2.100) that implies $BB^\dagger = \mathbb{1}_{3\times 3}$.

On the other hand, to provide a heavy mass for the right-handed field x_{HR} we introduce a left-handed singlet x_{HL} to build

$$\mathcal{L}_x = -M_x \overline{x}_{HL} x_{HR} + \text{h.c.}. \tag{2.107}$$

The mass term for the fermion singlet is not invariant under the global $SU(3)_V$, but without involving the Higgs, it is unable to reintroduce a quadratically divergent contribution to the Higgs mass. However, logarithmically divergent contributions are generated from the Feynman diagrams in fig. 2.5. Assuming a single family of quarks those read

$$\delta\mu^2_{H,a} = \frac{3}{4\pi^2} \log\Lambda^2 \left(-\kappa^4 f^2 + \frac{\kappa^2}{2} M_x^2\right), \tag{2.108}$$

$$\delta\mu^2_{H,b} = \frac{3}{4\pi^2} \log\Lambda^2 f^2 + \frac{\kappa^2}{2} M_x^2, \tag{2.109}$$

for the T-even and T-odd realization, respectively. This kind of contributions are typical in Little Higgs model with T-parity. Contrary to the top sector, they differ because the second Feynman diagram in fig. 2.5 involves couplings of x_{HR} with a t_{1L} and a t_{2L} and those with t_{2L} have opposite sign in the different T-parity implementations. For the Littlest Higgs model with T-parity these are discussed in more detail in Appendix A.

Finally we add the kinetic term and gauge interactions for all the fermions using the quantum numbers given in tables 2.2 and 2.3. On the one hand, for the fermions transforming linearly

$$\begin{aligned}\mathcal{L}_F = {} & i\overline{\chi}_{1L}\gamma^\mu D^{1*}_\mu \chi_{1L} + i\overline{\chi}_{2L}\gamma^\mu D^{2*}_\mu \chi_{2L} + i\overline{u}_R \left[\partial_\mu + i\sqrt{2}g'\left(\frac{1}{3}B_{1\mu} + \frac{1}{3}B_{2\mu}\right)\right] t_R \\ & + i\overline{U}_{1R}\left(\partial_\mu + i\sqrt{2}g'\frac{2}{3}B_{1\mu}\right)U_{1R} + i\overline{U}_{2R}\left(\partial_\mu + i\sqrt{2}g'\frac{2}{3}B_{2\mu}\right)U_{2R},\end{aligned} \tag{2.110}$$

where the covariant derivatives read

$$D^1_\mu = \partial_\mu - i\sqrt{2}gW^a_{1\mu}Q^a_1 + i\sqrt{2}g'\left(Y_1 - \frac{1}{3}\mathbb{1}_{3\times3}\right)B_{1\mu} \tag{2.111}$$

$$D^2_\mu = \partial_\mu - i\sqrt{2}gW^a_{2\mu}Q^a_2 + i\sqrt{2}g'\left(Y_2 - \frac{1}{3}\mathbb{1}_{3\times3}\right)B_{2\mu}, \tag{2.112}$$

independently on the action of T-parity. On he other hand, the CCWZ formalism (see eq. (2.46)) provides the kinetic term and gauge interactions for the right-handed fermions transforming non linearly under the global group in eq. (2.100). Applying invariance under the realization (2.101) of the discrete T-parity symmetry leads to

$$\mathcal{L}^a_{F'} = i\overline{\Psi}_R\gamma^\mu\left(\partial_\mu + \frac{1}{2}\xi^T(D^1_\mu\xi)^* + \frac{1}{2}\xi^*(D^2_\mu\xi)^T + \frac{1}{6}B_{1\mu}\mathbb{1}_{3\times3} + \frac{1}{6}B_{2\mu}\mathbb{1}_{3\times3}\right)\Psi_R \tag{2.113}$$

or under the alternative realization (2.102)

$$\mathcal{L}^b_{F'} = i\overline{\Psi}_R\gamma^\mu\left(\partial_\mu + \frac{1}{2}\xi^T(D^1_\mu\xi)^* + \frac{1}{2}\Omega\xi^*(D^2_\mu\xi)^T\Omega + \frac{1}{6}B_{1\mu}\mathbb{1}_{3\times3} + \frac{1}{6}B_{2\mu}\mathbb{1}_{3\times3}\right)\Psi_R, \tag{2.114}$$

where we used that Ω commutes with the gauge generators. The covariant derivatives of the ξ field under $SU(3)_i$ are defined as

$$D^1_\mu\xi = \partial_\mu\xi - \sqrt{2}igW^a_{1\mu}Q^a_1\xi + \sqrt{2}ig'B_{1\mu}Y_1\xi \tag{2.115}$$

$$D^2_\mu \xi = \partial_\mu \xi + \sqrt{2} i g W^a_{2\mu} \xi Q^a_2 - \sqrt{2} i g' B_{2\mu} \xi Y_2. \tag{2.116}$$

The full Lagrangian invariant under T-parity that implements the collective symmetry breaking mechanism is given by

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_S + \mathcal{L}_t + \mathcal{L}_H + \mathcal{L}_x + \mathcal{L}_F + \mathcal{L}_{F'}. \tag{2.117}$$

2.6. Chapter summary

In this chapter we have introduced the general formalism to build a consistent Little Higgs model that addresses the Hierarchy problem.

The first part has been devoted to the CCWZ formalism that allows to consider the Higgs as one of the Goldstone bosons resulting from the spontaneous breaking of a global symmetry. As far as this global symmetry remains exact, none of the Goldstone fields develops a potential but only derivative interactions.

In the following step, the addition of different gauge and Yukawa interactions for the Goldstone fields breaks the global symmetries and a potential for the Goldstone fields is generated through loops. However, it is customary a mechanism that controls how this breaking propagates to the scalar fields, eliminating the quadratically divergent contributions to the Higgs mass and avoiding the reintroduction of the fine-tuning we try to eliminate. Such a mechanism is called collective symmetry breaking. Each of the breaking terms preserves a different subgroup of the global symmetry that acts non linearly on the Higgs and thus, individually, any of them can generate a mass. However, the combination of several of these terms break all the global symmetries protecting the Higgs, generating a squared mass that is not quadratically sensitive to the cutoff scale.

As consequence of the extended global and gauge symmetries, there are new heavy particles at the TeV scale. To avoid unacceptable large contributions to the electroweak precision observables, a new discrete symmetry called T-parity is implemented in such a way that the SM particles are T-even and most of the new heavy particles are T-odd and pair-produced, relaxing direct and indirect constraints.

In the final part we have built a toy model implementing all the previous features. This is based on the global symmetry group $SU(3)_1 \times SU(3)_2$ spontaneously broken to $SU(3)_V$ at a scale $f \sim$ TeV leading to 8 Goldstone bosons. To implement the collective symmetry breaking, a subgroup $[SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2$ of the global symmetry is weakly gauged and gets spontaneously broken to the the diagonal $SU(2) \times U(1) \subset SU(3)_V$ identified as the SM gauge group giving rise to 4 heavy gauge fields. Once the SM gets spontaneously broken, only the Higgs field remains as a physical Goldstone boson. Since each factor of the gauge group leaves an exact SU(3) factor of the global symmetry untouched, the Higgs mass does not develop a quadratically divergent contribution proportional to the gauge couplings of just one of the factors. However, the combination of the gauge couplings of both factors generate just a logarithmically divergent contribution to the Higgs mass.

Regarding fermions, the top quark has the largest Yukawa coupling in the SM and generates a quadratically divergent contribution to the Higgs mass. To implement the collective symmetry breaking in this sector, one needs to enforce an SU(3) global symmetry promoting the SM left-handed quark doublet to a complete SU(3) triplet, containing a heavy top partner. However, the fundamental representation of SU(3) does not contain an extra up-type quark, but a down-type quark. Hence, to accommodate the quantum numbers of the top partner, the global symmetry group needs to be enlarged with an extra $U(1)_1 \times U(1)_2$. The gauged hypercharge Y_i is thus the sum of the hypercharge inside of the $SU(3)_i$ factor and the one laying in the extra $U(1)_i$ factor, preserving the number of U(1) gauge bosons. Using the extended global symmetry group, two different kinds of Yukawa Lagrangians are built. They provide the top quark with a a mass and couplings to the Higgs as well as a heavy mass of size f for the top partner, without generating a quadratically divergent contribution to the Higgs mass at one loop. On the other hand, the logarithmically divergent contribution has a negative sign and can compete with the gauge boson contribution, hence leading to the SM spontaneous symmetry breaking.

In order to avoid direct and indirect constraints from EWPD due to the heavy new particles, the discrete T-parity symmetry is implemented in the toy model. T-parity interchanges the factors of the gauge group and so the couplings constants to both of them must be equal. As a consequence the new (heavy) gauge bosons are T-odd while the SM gauge bosons are T-even. To be consistent, all would-be Goldstone bosons must be T-odd except for the Higgs doublet. For that purpose, one introduces the matrix $\Omega \in SU(3)$ that commutes with the gauge generators but not with the full global symmetry. The implementation of T-parity in the top sector is less straightforward. It requires two different SU(3) left-handed multiplets that include two top partners that can be related through T-parity in two different ways. The T-even combination contains the T-even left-handed quark doublet that gets a mass from the T-invariant version of the Yukawa Lagrangians introduced in the case without T-parity, together with the T-even and T-odd heavy top partners. However, there still remains a massless T-odd combination of left-handed quark doublets (mirror quarks). To provide them with a vector-like mass proportional to f one introduces a complete right-handed multiplet transforming non linearly under $SU(3)_V$ that contains the right-handed counterpart of the mirror quark doublet and an extra singlet than can be either T-even or T-odd. Left and right-handed multiplets are coupled through the non linear-field ξ to build a new Yukawa Lagrangian. Finally, one introduces the left-handed counterpart of the singlet to to provide it with a vector-like mass. This singlet is essential to cancel the quadratically divergent contribution to the Higgs mass squared generated by the mirror quark doublets. As a consequence of the introduction of the T-parity, the fermion spectrum at least doubles with respect to the case without T-parity.

All these features will be implemented in the *Littlest Higgs model with T-parity* in the next chapter.

Chapter 3

The Littlest Higgs model with T-parity

This chapter reviews the Littlest Higgs model with T-parity (LHT) as a paradigmatic example of a Composite Higgs model. Its prominent feature is the implementation of the discrete T-parity symmetry to alleviate direct and indirect constraints from EWPD. To that end, the SM particles are T-even and (most of) the new particles are T-odd and pair-produced. This has an impact on the phenomenology that we will show. We will study in particular how the usual matter content together with the non trivial interplay between the non linear realization of the global symmetry and T-parity break the gauge invariance of the original model. This chapter ellaborates on earlier studies [61, 64] and is based on original work published in refs. [65, 68].

3.1. Littlest Higgs model with T-parity setup

3.1.1. Requirements

Here we closely follow the discussion in [87]. The goal is to realize the Higgs as the pseudo-Goldstone boson whose low energy dynamics comes from an approximate global symmetry in a way that its mass is not quadratically sensitive to the cutoff at one loop. As a consequence the model will be weakly coupled up to energies one loop factor above the electroweak scale.

We assume that the Higgs is part of a pseudo-Goldstone multiplet parametrizing a coset space G/H, with the decay constant f of the order of a TeV. As previously emphasized, the origin of this symmetry breaking pattern is irrelevant since the construction will ensure that the electroweak scale is insensitive to it. The sigma model model does not spontaneously break the SM gauge group at the scale f, so the subgroup H should contain an $\mathrm{SU}(2)\times\mathrm{U}(1)$ subgroup. As in the SM, the electroweak interactions will induce at one loop a quadratically divergent mass for the Higgs. To avoid this we use the collective symmetry breaking mechanism: we assume that G contains a weakly gauge subgroup consisting of two copies of $\mathrm{SU}(2)\times\mathrm{U}(1)$: $G\supset G_1\times G_2=[\mathrm{SU}(2)\times\mathrm{U}(1)]^2$. Each G_i must commute with a different subgroup of G that acts non linearly on the Higgs, hence protecting it from developing a mass. The combination of both types of weak gauge interactions breaks all the global symmetries that act on the Higgs, and then it ceases to be an exact Goldstone boson. The quadratically divergent contributions to the Higgs mass from gauge interactions must involve both types of couplings, and hence appear first at two loops. In this case the Higgs mass squared is raditatively stable with a cutoff of order 10 TeV, when the model becomes strongly coupled and this effective description looses its validity.

Let us now look for the minimal implementation of the above requirements. Since G contains the subgroup $[\mathrm{SU}(2)\times\mathrm{U}(1)]^2$ it must be at least of rank 4. Also G must contain two different subgroups of the form $G_i\times X_i, i=1,2$, where X_i acts non linearly on the Higgs. Furthermore each X_i must contain an $\mathrm{SU}(2)\times\mathrm{U}(1)$ subgroup with some generators transforming like doublets. Taking $X_i=\mathrm{SU}(3)_i$ and $G=\mathrm{SU}(5)$ we fulfil the aforementioned requirements.

A candidate for the subgroup H is SO(5), which contains the diagonal part of the product $G_1 \times G_2$. Then we will consider the coset $SU(5)/SO(5)$.

Since each gauge group G_i commutes with a different $SU(3)$ global symmetry subgroups of SU(5), none of them alone can generate a potential for the Higgs. The two gauge groups together, however, break all global symmetries protecting the Higgs. As a consequence, the Higgs mass is proportional to both gauge couplings and a quadratic divergence cannot be generated at one loop.

3.1.2. Global symmetries

Here we will describe the implementation of the global symmetries. The model is based on the global symmetry group $G = SU(5)$ broken spontaneously to $H = SO(5)$ by the *vev* of a symmetric tensor,[1]

$$\Sigma_0 = \begin{pmatrix} 0_{2\times2} & 0 & \mathbf{1}_{2\times2} \\ 0 & 1 & 0 \\ \mathbf{1}_{2\times2} & 0 & 0_{2\times2} \end{pmatrix}, \tag{3.1}$$

leaving $24 - 10 = 14$ broken generators. This spontaneous breaking direction fixes the embedding of SO(5) in SU(5) with the fundamental representation of the latter reduced to the defining (real) representation of the former. The vacuum is preserved by the unbroken generators fulfilling the relation

$$T^a\Sigma_0 + \Sigma_0 T^{aT} = 0. \tag{3.2}$$

Since SU(5)/SO(5) is a symmetric coset there is an inherited automorphism in the Lie algebra [97]. The above expression suggests the definition of the inner automorphism

$$T^a \xrightarrow{\text{aut}} -\Sigma_0 T^{aT}\Sigma_0 = T^a, \tag{3.3}$$

where the last equality follows from eq. (3.2). Thus the unbroken generators commute with the automorphism. The set of broken generators anticommute with the automorphism and thus are orthogonal to the previous set

$$X^a \xrightarrow{\text{aut}} -\Sigma_0 X^{aT}\Sigma_0 = -X^a. \tag{3.4}$$

This characterizes the broken generators as the set that verifies

$$X^a\Sigma_0 - \Sigma_0 X^{aT} = 0. \tag{3.5}$$

As a result of this automorphism, broken and unbroken generators verify the following schematic commutation relations

$$[T,T] \sim T, \quad [T,X] \sim X, \quad [X,X] \sim T, \tag{3.6}$$

The first two equations are general for any subgroup of a group. However the third one is only valid for a symmetric coset. Without loss of generality, for the rest of the work we will take an orthonormal and hermitian basis of generators. Following the CCWZ formalism, one defines the Goldstone matrix $\Pi = \pi^a X^a$. This is the key ingredient to introduce the nonlinear field ξ

[1] The global group only contains the electroweak part of the SM. One must add an external SU(3) color factor to include strong interactions.

that under the global group transforms,

$$\xi = e^{i\Pi/f}, \quad \xi \xrightarrow{G} V\xi U^\dagger \tag{3.7}$$

where f is the scale of spontaneous symmetry breaking of the global group, V is an SU(5) transformation and $U = U(V, \Pi)$ is the compensating SO(5) non linear transformation, depending on V and the Goldstone fields encoded in Π. This non linear transformation preserves the exponential form of the transformed ξ. At leading order in the f expansion, the transformation of ξ along the direction of the broken generators corresponds to a shift transformation on the Goldstone fields as pointed out in chapter 2.

Let us derive one of the most relevant properties that the non linear transformation U verifies. The characterization of the broken generators in eq. (3.5) leads to $\Pi = \Sigma_0 \Pi^T \Sigma_0$ and using that $\Sigma_0^2 = \mathbb{1}_{5\times 5}$ one obtains

$$\xi = \Sigma_0 \xi^T \Sigma_0, \tag{3.8}$$

independent of the chosen basis to parametrize the Goldstone fields. Applying the transformation given by the CCWZ formalism in eq. (3.7) to the previous expression and using eq. (3.2) together with the hermiticity of the generators that implies $U\Sigma_0 = \Sigma_0 U^*$ one gets

$$V\xi U^\dagger = \Sigma_0 (V\xi U^\dagger)^T \Sigma_0 = U\xi\Sigma_0 V^T \Sigma_0. \tag{3.9}$$

Eq. (3.9) can be interpreted as a definition of the non linear transformation U. It is a consequence of the particular embedding of SO(5) in SU(5) and the action of the inner automorphism over the Lie algebra.

This formalism allows to define a tensor field Σ that transforms linearly under the global symmetry group

$$\Sigma = \xi\Sigma_0\xi^T = \xi^2\Sigma_0, \quad \Sigma \xrightarrow{G} V\Sigma V^T, \tag{3.10}$$

where we have used eq. (3.5) to commute ξ with Σ_0. This field will allow to build the kinetic terms and gauge interactions for the Goldstone fields.

3.1.3. Gauge symmetry

The gauge subgroup is generated by the hermitian and traceless generators

$$Q_1^a = \frac{1}{2}\begin{pmatrix} \sigma^a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0_{2\times 2} \end{pmatrix}, \qquad Y_1 = \frac{1}{10}\mathrm{diag}\,(3,3,-2,-2,-2)\,, \tag{3.11}$$

$$Q_2^a = \frac{1}{2}\begin{pmatrix} 0_{2\times 2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\sigma^{a*} \end{pmatrix}, \qquad Y_2 = \frac{1}{10}\mathrm{diag}\,(2,2,2,-3,-3)\,, \tag{3.12}$$

with σ^a the three Pauli matrices. For the gauge generators the normalization used is $\mathrm{tr}\left(Q_j^a Q_k^b\right) = \frac{1}{2}\delta^{ab}\delta_{jk}$ and $\mathrm{tr}\left(Y_j Y_k\right) = \frac{1}{10}\delta_{jk} + \frac{1}{5}$ and the rest of traces vanish. Thanks to the automorphism defined in eq. (3.3), one can relate the generators of both copies of $SU(2)\times U(1)$ through the expressions

$$Q_1^a = -\Sigma_0 Q_2^{aT}\Sigma_0, \quad Y_1 = -\Sigma_0 Y_2^T \Sigma_0, \tag{3.13}$$

that allows a consistent definition of the discrete T-parity symmetry. The *vev* along the direction of Σ_0 also breaks spontaneously the gauge group down to the diagonal subgroup $SU(2)_L \times U(1)_Y$ identified as the SM gauge group, generated by the combinations $\{Q_1^a + Q_2^a, Y_1 + Y_2\} \subset \{T^a\}$, while the unbroken combinations is a subset of the broken generators $\{Q_1^a - Q_2^a, Y_1 - Y_2\} \subset \{X^a\}$. The set of broken generators span the Goldstone matrix

$$\Pi = \begin{pmatrix} -\frac{\omega^0}{2} - \frac{\eta}{\sqrt{20}} & -\frac{\omega^+}{\sqrt{2}} & -i\frac{\pi^+}{\sqrt{2}} & -i\Phi^{++} & -i\frac{\Phi^+}{\sqrt{2}} \\ -\frac{\omega^-}{\sqrt{2}} & \frac{\omega^0}{2} - \frac{\eta}{\sqrt{20}} & \frac{v+h+i\pi^0}{2} & -i\frac{\Phi^+}{\sqrt{2}} & \frac{-i\Phi^0+\Phi^P}{\sqrt{2}} \\ i\frac{\pi^-}{\sqrt{2}} & \frac{v+h-i\pi^0}{2} & \sqrt{\frac{4}{5}}\eta & -i\frac{\pi^+}{\sqrt{2}} & \frac{v+h+i\pi^0}{2} \\ i\Phi^{--} & i\frac{\Phi^-}{\sqrt{2}} & i\frac{\pi^-}{\sqrt{2}} & -\frac{\omega^0}{2} - \frac{\eta}{\sqrt{20}} & -\frac{\omega^-}{\sqrt{2}} \\ i\frac{\Phi^-}{\sqrt{2}} & \frac{i\Phi^0+\Phi^P}{\sqrt{2}} & \frac{v+h-i\pi^0}{2} & -\frac{\omega^+}{\sqrt{2}} & \frac{\omega^0}{2} - \frac{\eta}{\sqrt{20}} \end{pmatrix}, \tag{3.14}$$

where v is the Higgs *vev*. Under the SM gauge group the Goldstone matrix decomposes as

$$\Pi : 1_0 \oplus 3_0 \oplus 2_{1/2} \oplus 3_1, \tag{3.15}$$

including a complex symmetric $SU(2)$ triplet and its hermitian conjugate

$$\Phi = \begin{pmatrix} -i\Phi^{++} & -i\frac{\Phi^+}{\sqrt{2}} \\ -i\frac{\Phi^+}{\sqrt{2}} & \frac{-i\Phi^0+\Phi^P}{\sqrt{2}} \end{pmatrix}, \quad \Phi^\dagger = \begin{pmatrix} i\Phi^{--} & i\frac{\Phi^-}{\sqrt{2}} \\ i\frac{\Phi^-}{\sqrt{2}} & \frac{i\Phi^0+\Phi^P}{\sqrt{2}} \end{pmatrix}, \tag{3.16}$$

the SM Higgs doublet

$$H = \begin{pmatrix} i\pi^+ \\ \frac{v+h+i\pi^0}{\sqrt{2}} \end{pmatrix}, \tag{3.17}$$

plus an $SU(2)$ triplet

$$\omega = \begin{pmatrix} -\frac{\omega^0}{2} & -\frac{\omega^+}{\sqrt{2}} \\ -\frac{\omega^-}{\sqrt{2}} & \frac{\omega^0}{2} \end{pmatrix} \tag{3.18}$$

and a singlet, η. After SSB, the latter two will become the longitudinal modes of the heavy gauge fields.

3.1.4. Lagrangian

Gauge sector

In the construction of the Lagrangian we take into account the action of the discrete T-parity symmetry. This is introduced to keep the SM gauge bosons T-even while the new ones are T-odd and thus pair-produced, relaxing direct and indirect constraints from EWPD [93, 98]. After the spontaneous breaking of the global symmetry, the set of T-odd gauge bosons are naturally heavy while the SM gauge bosons remain massless. The action of T-parity on the gauge sector consists of an interchange of the two gauge groups

$$G_1 \overset{\mathrm{T}}{\longleftrightarrow} G_2. \tag{3.19}$$

This requires that the coupling constants of both copies must be equal $g_1 = g_2 = \sqrt{2}g$, $g_1' = g_2' = \sqrt{2}g'$, with the first set of couplings referring to SU(2) and the second to U(1). The gauge Lagrangian then takes the form

$$\mathcal{L}_G = \sum_{j=1}^{2} \left[-\frac{1}{2}\mathrm{tr}\left(\widetilde{W}_{j\mu\nu}\widetilde{W}_j^{\mu\nu} \right) - \frac{1}{4}B_{j\mu\nu}B_j^{\mu\nu} \right], \tag{3.20}$$

in terms of fields and field strength tensors,

$$\widetilde{W}_{j\mu} = W^a_{j\mu}Q^a_j, \quad \widetilde{W}_{j\mu\nu} = \partial_\mu \widetilde{W}_{j\nu} - \partial_\nu \widetilde{W}_{j\mu} - i\sqrt{2}g\left[\widetilde{W}_{j\mu}, \widetilde{W}_{j\nu} \right], \quad B_{j\mu\nu} = \partial_\mu B_{j\nu} - \partial_\nu B_{j\mu}, \tag{3.21}$$

where in the first expression the index j is fixed. Before the electroweak SSB, the SM gauge bosons come from the T-even combinations

$$W^\pm = \frac{1}{2}\left[\left(W_1^1 + W_2^1 \right) \mp i \left(W_1^2 + W_2^2 \right) \right], \quad W^3 = \frac{W_1^3 + W_2^3}{\sqrt{2}}, \quad B = \frac{B_1 + B_2}{\sqrt{2}}, \tag{3.22}$$

while the remaining T-odd combinations will define the heavy fields

$$W_H^\pm = \frac{1}{2}\left[\left(W_1^1 - W_2^1 \right) \mp i \left(W_1^2 - W_2^2 \right) \right], \quad W_H^3 = \frac{W_1^3 - W_2^3}{\sqrt{2}}, \quad B_H = \frac{B_1 - B_2}{\sqrt{2}}. \tag{3.23}$$

Scalar sector

In order to assign a T-even parity to the SM Higgs boson and T-odd parities to the rest of scalar fields, one defines

$$\Pi \overset{\mathrm{T}}{\rightarrow} -\Omega\Pi\Omega, \quad \Omega = \mathrm{diag}\left(-1,-1,1,-1,-1\right). \tag{3.24}$$

It is important to remark that the element Ω belongs to the center of the gauge group and consequently commutes with the gauge generators but not with the full global symmetry. This fact will be crucial when discussing the non gauge invariance of the model. One can also check that Ω belongs to SO(5) since it leaves Σ_0 invariant. The T-parity transformation of the Goldstone fields implies

$$\xi \overset{\mathrm{T}}{\rightarrow} \Omega\xi^\dagger\Omega, \quad \Sigma \overset{\mathrm{T}}{\rightarrow} \widetilde{\Sigma} \equiv \Omega\Sigma_0\Sigma^\dagger\Sigma_0\Omega. \tag{3.25}$$

With these ingredients one builds the scalar Lagrangian which is gauge and T-parity invariant using eq. (3.13),

$$\mathcal{L}_S = \frac{f^2}{8}\mathrm{tr}\left[\left(D^\mu\Sigma\right)^\dagger D_\mu\Sigma \right], \tag{3.26}$$

where the covariant derivative is defined as

$$D_\mu\Sigma = \partial_\mu\Sigma - \sqrt{2}i\sum_{j=1}^{2}\left[gW^a_{j\mu}\left(Q^a_j\Sigma + \Sigma Q^{aT}_j \right) - g'B_{j\mu}\left(Y_j\Sigma + \Sigma Y_j^T \right) \right]. \tag{3.27}$$

Notice that, as in the toy model in Chapter 2, switching off the gauge couplings of one of the $SU(2) \times U(1)$ factors of the gauge group, the covariant derivative transforms covariantly under global transformations of the SU(3) containing that factor. This is because the global SU(3) and the remaining gauge subgroup commute. As a consequence the Higgs cannot develop a mass term that involves couplings of only one of the gauge factors.

Fermion sector

Implementing T-parity and providing masses to all fermions is less straightforward. In fact, this is the main source of the non gauge invariance of the model.

First of all, in the usual procedure [93, 98] one introduces two left-handed SU(5) quintuplets in the anti-fundamental and fundamental representations, respectively, for leptons and quarks,

$$\Psi_1 = \begin{pmatrix} -i\sigma^2 l_{1L} \\ 0 \\ 0_2 \end{pmatrix}, \quad \Psi_2 = \begin{pmatrix} 0_2 \\ 0 \\ -i\sigma^2 l_{2L} \end{pmatrix}, \quad \Psi_1^q = \begin{pmatrix} -i\sigma^2 q_{1L} \\ 0 \\ 0_2 \end{pmatrix}, \quad \Psi_2^q = \begin{pmatrix} 0_2 \\ 0 \\ -i\sigma^2 q_{2L} \end{pmatrix}. \tag{3.28}$$

These incomplete multiplets break explicitly the global symmetry, but the gauge group is preserved. Under a gauge transformation these multiplets transform

$$\Psi_1^{(q)} \xrightarrow{G_g} V_g^* \Psi_1^{(q)}, \quad \Psi_2^{(q)} \xrightarrow{G_g} V_g \Psi_2^{(q)}. \tag{3.29}$$

From these transformation properties, it is clear that leptons and quarks would receive the same hyperchages under the gauge group since those are fixed by the form of the generators in eqs. (3.11) and (3.12). For that reason one needs to enlarge the global symmetry group with two extra factors $U(1)_1''$ and $U(1)_2''$ hence preserving gauge invariance and the collective symmetry breaking mechanism [61]

$$SU(5) \times U(1)_1'' \times U(1)_2''. \tag{3.30}$$

Then for any field the actual hypercharges under the gauged $U(1)_1$ and $U(1)_2$ will be the sum of those under the $U(1)_1'$ and $U(1)_2'$ present in SU(5) plus the extra ones.

For a T-parity transformation it is common to contemplate two options [61, 65, 93, 98]:

$$a) \quad \Psi_1^{(q)} \xrightarrow{T} \Omega\Sigma_0\Psi_2^{(q)}, \tag{3.31}$$

$$b) \quad \Psi_1^{(q)} \xrightarrow{T} -\Sigma_0\Psi_2^{(q)}. \tag{3.32}$$

Then one can define T-even and T-odd combinations given respectively by

$$a) \quad \Psi_+^{(q)} = \frac{\Psi_1^{(q)} + \Omega\Sigma_0\Psi_2^{(q)}}{\sqrt{2}}, \quad \Psi_-^{(q)} = \frac{\Psi_1^{(q)} - \Omega\Sigma_0\Psi_2^{(q)}}{\sqrt{2}}, \tag{3.33}$$

$$b) \quad \Psi_+^{(q)} = \frac{\Psi_1^{(q)} - \Sigma_0\Psi_2^{(q)}}{\sqrt{2}}, \quad \Psi_-^{(q)} = \frac{\Psi_1^{(q)} + \Sigma_0\Psi_2^{(q)}}{\sqrt{2}}. \tag{3.34}$$

The T-odd combination of left-handed lepton and quarks $l_{HL} = (l_{1L} + l_{2L})/\sqrt{2}$ and $q_{HL} = (q_{1L} + q_{2L})/\sqrt{2}$ needs to be paired with right-handed doublets l_{HR} and q_{HR} so that the 'mirror' fermions l_H, q_H get a vector-like mass. To that end, a right-handed SO(5) quintuplet is introduced

$$\Psi_R = \begin{pmatrix} -i\sigma^2(\tilde{l}^c_-)_R \\ i\chi_R \\ -i\sigma^2 l_{HR} \end{pmatrix}, \quad \Psi_R^q = \begin{pmatrix} -i\sigma^2(\tilde{q}^c_-)_R \\ i\chi_R^q \\ -i\sigma^2 q_{HR} \end{pmatrix}. \tag{3.35}$$

We will denote with a subscript $\pm$ the T-parity assignment to be defined below. The T-odd doublets $(\tilde{l}^c_-)_R$ and $(\tilde{q}^c_-)_R$ describe the 'mirror-partner' fermions and $\chi_R^{(q)}$ are SU(2) singlets, that in principle can be taken either $(\chi_+^{(q)})_R$ or $(\chi_-^{(q)})_R$. Some authors [99–102] leave these quintuplets incomplete, assuming that the mirror-partner fermions and the singlets decouple, so they just include in them the doublets l_{HR}, q_{HR}.

The transformation under the gauge subgroup reads

$$\Psi_R^{(q)} \xrightarrow{G_g} U_g \Psi_R^{(q)}, \tag{3.36}$$

where U_g is the SO(5) non linear transformation that satisfies eq. (3.9) for a given V_g. Consequently with the T-parity transformations for the left-handed quintuplets defined above, there are also two different T-parity actions over the right-handed quintuplets

$$a) \quad \Psi_R \xrightarrow{\mathrm{T}} \Omega\Psi_R, \tag{3.37}$$

$$b) \quad \Psi_R \xrightarrow{\mathrm{T}} -\Psi_R. \tag{3.38}$$

The first one differs from the second in that $\chi_R^{(q)}$ are T-even; the rest of the fields are all T-odd. It is important to mention that the cancellation of the quadratic divergence caused by the mirror fermions to the Higgs mass is independent on the T-parity assignation of $\chi_R^{(q)}$. With this in mind, one can correspondingly construct two versions of a Yukawa Lagrangian,

$$\mathcal{L}_{Y_H}^{(a)} = -\kappa_l f \left(\overline{\Psi}_2\xi + \overline{\Psi}_1\Sigma_0\xi^\dagger\right)\Psi_R + \text{h.c.}, \tag{3.39}$$

$$\mathcal{L}_{Y_H}^{(b)} = -\kappa_l f \left(\overline{\Psi}_2\xi + \overline{\Psi}_1\Sigma_0\Omega\xi^\dagger\Omega\right)\Psi_R + \text{h.c.} \tag{3.40}$$

for leptons and similarly for quarks

$$\mathcal{L}_{Y_{qH}}^{(a)} = -\kappa_q f \left(\overline{\Psi}_2^q\xi + \overline{\Psi}_1^q\Sigma_0\xi^\dagger\right)\Psi_R^q + \text{h.c.}, \tag{3.41}$$

$$\mathcal{L}_{Y_{qH}}^{(b)} = -\kappa_q f \left(\overline{\Psi}_2^q\xi + \overline{\Psi}_1^q\Sigma_0\Omega\xi^\dagger\Omega\right)\Psi_R^q + \text{h.c.} \tag{3.42}$$

Those are tailored to provide the mirror fermions with a mass order κf.

Let us show how the collective symmetry breaking works in this sector and justify why, in principle, the SO(5) quintuplet must be complete. For concreteness let us focus on leptons since quarks can be worked out in the same fashion. As in the gauge sector, the key ingredient is the presence of an exact global SU(3) acting non linearly on the Higgs when a single term that breaks the global SU(5) is turned on. This means that each of the terms in eqs. (3.39) and (3.40) should be invariant under a different SU(3) factor that we construct explicitly in Appendix A. Notice that due to the field content in Ψ_2, this quintuplet is invariant under the action of the SU(3) located in the upper-left corner of SU(5). These SU(5) transformations, when acting on the left on ξ, leave invariant its last two rows that couple to the non vanishing components of Ψ_2. However, the associated SO(5) transformations U, that at the infinitesimal level can be written as

$$U \approx \mathbb{1} + i\beta^b T^b + ..., \tag{3.43}$$

involve SO(5) generators (see Appendix A) that acting on Ψ_R mix the mirror and mirror partner leptons with the singlet. Consequently the SU(3) invariance tells us that the mirror partner leptons and the singlet cannot be ignored. Analogously, the second term in eq. (3.39) is invariant under the SU(3) located in the lower-right corner of the SU(5) matrices if Ψ_R is complete. Consequently the collective symmetry breaking mechanism ensures that no quadratic contributions to the Higgs mass proportional to κ_l are generated. On the other hand, the second term in eq. (3.40) is not invariant under the lower-right SU(3) because of the presence of Ω that does not commute with any of the SU(3)'s. This could potentially lead to a quadratic divergence to the Higgs mass coming from the operator

$$\mathcal{O} = \mathrm{tr}(B^\dagger \Sigma_0 \Omega \xi^\dagger \Omega C C^\dagger \Omega \xi \Omega \Sigma_0 B) = \mathrm{tr}(B^\dagger B), \tag{3.44}$$

where

$$B = \begin{pmatrix} -i\sigma^2 & & \\ & 0 & \\ & & 0_2 \end{pmatrix}, \quad C = \begin{pmatrix} -i\sigma^2 & & \\ & i & \\ & & -i\sigma^2 \end{pmatrix} \tag{3.45}$$

take into account the Ψ_1 and Ψ_R are incomplete and complete multiplets, respectively. But this operator does not depend on the Goldstone fields due to the unitarity of Ω and the complete Ψ_R quintuplet. Nevertheless, not all the fields in the right-handed quintuplet have the appropriate quantum numbers to participate in the Higgs self-energy. As we show in Appendix B, only the mirror leptons and the singlet contribute and thus the singlet cannot be ignored in a consistent Little Higgs model. This justifies why some authors do not include the mirror partner leptons in the SO(5) quintuplet.

In order to give a mass of order λv to the SM leptons and down-type quarks after the electroweak SSB, the following Yukawa Lagrangian has been proposed [103–108],

$$\mathcal{L}_Y = -i\frac{\lambda_l f}{4}\epsilon_{xyz}\epsilon_{rs}\left[(\overline{\Psi_2^{X^*}})_x \Sigma_{ry}\Sigma_{sz} + (\overline{\Psi_1^X}\Sigma_0\Omega)_x \widetilde{\Sigma}_{ry}\widetilde{\Sigma}_{sz}\right]\ell_R + \mathrm{h.c.}, \tag{3.46}$$

$$\mathcal{L}_{Y_d} = -i\frac{\lambda_d f}{4}\epsilon_{xyz}\epsilon_{rs}\left[(\overline{\Psi_2^{qX^*}})_x \Sigma_{ry}\Sigma_{sz} + (\overline{\Psi_1^{qX}}\Sigma_0\Omega)_x \widetilde{\Sigma}_{ry}\widetilde{\Sigma}_{sz}\right] d_R + \mathrm{h.c.}, \tag{3.47}$$

where $\{x, y, z\} = 3, 4, 5$ and $\{r, s\} = 1, 2$. Here the left-handed fermions are embedded in incomplete SU(5) quintuplets,

$$\Psi_1^X = \begin{pmatrix} X l_{1L} \\ 0 \\ 0_2 \end{pmatrix}, \quad \Psi_2^{X^*} = \begin{pmatrix} 0_2 \\ 0 \\ X^* l_{2L} \end{pmatrix}, \quad \Psi_1^{qX} = \begin{pmatrix} X q_{1L} \\ 0 \\ 0_2 \end{pmatrix}, \quad \Psi_2^{qX^*} = \begin{pmatrix} 0_2 \\ 0 \\ X^* q_{2L} \end{pmatrix} \tag{3.48}$$

transforming under the global group as

$$\Psi_1^{(q)X} \xrightarrow{G} V\Psi_1^{(q)X}, \quad \Psi_2^{(q)X^*} \xrightarrow{G} V^*\Psi_2^{(q)X^*} \tag{3.49}$$

and under T-parity as

$$\Psi_1^{(q)X} \xrightarrow{T} \Omega\Sigma_0\Psi_2^{(q)X^*} = -\Sigma_0\Psi_2^{(q)X^*}, \tag{3.50}$$

where the last equality follows from the field content of these multiplets. This is because the

	Y'_1	Y'_2
Σ_{13}	$\frac{1}{10}$	$\frac{2}{5}$
Σ_{14}	$\frac{1}{10}$	$-\frac{1}{10}$
Σ_{15}	$\frac{1}{10}$	$-\frac{1}{10}$
Σ_{23}	$\frac{1}{10}$	$\frac{2}{5}$
Σ_{24}	$\frac{1}{10}$	$-\frac{1}{10}$
Σ_{25}	$\frac{1}{10}$	$-\frac{1}{10}$
$\Sigma_{13}\Sigma_{24}$	$\frac{1}{5}$	$\frac{3}{10}$
$\Sigma_{13}\Sigma_{25}$	$\frac{1}{5}$	$\frac{3}{10}$
$\Sigma_{23}\Sigma_{14}$	$\frac{1}{5}$	$\frac{3}{10}$
$\Sigma_{23}\Sigma_{15}$	$\frac{1}{5}$	$\frac{3}{10}$

TABLE 3.1: Hypercharges under $U(1)'_1 \times U(1)'_2 \subset SU(5)$ of the Σ components in eqs. (3.46) and (3.47).

element Ω changes the T-parity assignation of the field in the middle of the multiplets with respect to the others and in this case there is no field in this position. As a consequence both T-parity realizations in these sectors are equivalent. Although ℓ_R and d_R are SU(5) singlets and all indices are contracted, this Lagrangian is not invariant under the global SU(5) symmetry, broken by Σ_0 and the incomplete multiplets. Nevertheless we just need that the gauge symmetry G_g is preserved, and this requires the introduction of $\Psi_1^{(q)X}$ ($\Psi_2^{(q)X^*}$) transforming *opposite* to $\Psi_1^{(q)}$ ($\Psi_2^{(q)}$) so that the SM charged leptons and down-type quarks, with left-handed components in the T-even doublets $l_L = (l_{1L} - l_{2L})/\sqrt{2}$ and $q_L = (q_{1L} - q_{2L})/\sqrt{2}$, get a mass. On the other hand, ℓ_R and d_R inherit no hypercharge from the global SU(5) symmetry group. Therefore their hypercharges must lie in the external $U(1)''_1 \times U(1)''_2$ introduced above to get the proper SM hypercharge, $Y = -1$ and $Y = -1/3$, respectively.

The auxiliary field X and its complex conjugate X^* are introduced in eq. (3.48) in order to reverse the U(1) charges of the left-handed components and at the same time compensate for the hypercharge assignment of the right-handed leptons and quarks. From the hypercharges of the Σ components in table 3.1 (note that all relevant products have the same values) and the requirements above one derives the charge assignments for the fermion fields of table 3.2. A particular realization of the scalar X can be constructed with the fields already present in the model [99, 103, 107]. The element Σ_{33} has hypercharges $(Y_1, Y_2) = (-\frac{2}{5}, \frac{2}{5})$, and it is a SU(2) singlet, so we can identify $X = \Sigma_{33}^{-\frac{1}{4}}$ and $X^* = (\Sigma_{33}^\dagger)^{-\frac{1}{4}}$, which also verifies the right transformation under T-parity $X \xrightarrow{T} X^*$.[2]

To provide masses to the SM up-type quarks we must take into account that the top quark is the heaviest SM particle and introduces a quadratic divergence to the Higgs mass proportional to its Yukawa coupling. The rest of the SM quarks also introduce such a divergence, but their Yukawa couplings are much smaller. For that reason, in the LHT one implements the collective symmetry breaking mechanism only in the top sector, assuming that the rest of quadratic divergences introduced by other SM quarks are negligible. To that end, the up-type quarks of

[2]In view of the charge assignments in table 3.2, it is clear that X^* is not the complex conjugate of X, since they do not have opposite hypercharges under the U(1) factors. However only gauged hypercharges matter and they are actually opposite, so we prefer to keep this notation. Indeed, the particular realization for these fields shows that one is the hermitian conjugate of the other.

	Y'_1	Y'_2	Y''_1	Y''_2	Y_1	Y_2
ℓ_R	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
d_R	0	0	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$
u_R, t_R	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
T_{2R}	0	0	$\frac{2}{15}$	$\frac{8}{15}$	$\frac{2}{15}$	$\frac{8}{15}$
T_{1R}	0	0	$\frac{8}{15}$	$\frac{2}{15}$	$\frac{8}{15}$	$\frac{2}{15}$
l_{2L}	$-\frac{1}{5}$	$-\frac{3}{10}$	0	0	$-\frac{1}{5}$	$-\frac{3}{10}$
l_{1L}	$-\frac{3}{10}$	$-\frac{1}{5}$	0	0	$-\frac{3}{10}$	$-\frac{1}{5}$
$X^* l_{2L}$	$\frac{1}{5}$	$\frac{3}{10}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{10}$	$-\frac{1}{5}$
$X l_{1L}$	$\frac{3}{10}$	$\frac{1}{5}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{5}$	$-\frac{3}{10}$
$q_{2L}, \mathcal{T}_{2L}$	$-\frac{1}{5}$	$-\frac{3}{10}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{15}$	$\frac{1}{30}$
$q_{1L}, \mathcal{T}_{1L}$	$-\frac{3}{10}$	$-\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{30}$	$\frac{2}{15}$
$X^* q_{2L}$	$\frac{1}{5}$	$\frac{3}{10}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{30}$	$\frac{2}{15}$
$X q_{1L}$	$\frac{3}{10}$	$\frac{1}{5}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{2}{15}$	$\frac{1}{30}$
T_{2L}	$-\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{15}$	$\frac{8}{15}$
T_{1L}	$\frac{1}{5}$	$-\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{8}{15}$	$\frac{2}{15}$
X^*	$\frac{2}{5}$	$\frac{3}{5}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{10}$	$\frac{1}{10}$
X	$\frac{3}{5}$	$\frac{2}{5}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{10}$	$-\frac{1}{10}$

TABLE 3.2: Hypercharge assignments under $U(1)'_1 \times U(1)'_2 \subset SU(5)$ and $U(1)''_1 \times U(1)''_2$ for fermions transforming in a linear representation of the gauge group. The hypercharges under the gauge group $U(1)_1 \times U(1)_2$ come from $Y_j = Y'_j + Y''_j,\ j = 1, 2$.

	Y'	Y''	Y
$\Psi_R = \begin{pmatrix} -i\sigma^2 (\widetilde{l^c_-})_R \\ i(\chi_+)_R \\ -i\sigma^2 l_{HR} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ 0 \\ -\frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ 0 \\ -\frac{1}{2} \end{pmatrix}$
$\Psi^q_R = \begin{pmatrix} -i\sigma^2 (\widetilde{q^c_-})_R \\ i(\chi^q_+)_R \\ -i\sigma^2 q_{HR} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ 0 \\ -\frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} \frac{2}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{pmatrix}$	$\begin{pmatrix} \frac{5}{6} \\ \frac{2}{3} \\ \frac{1}{6} \end{pmatrix}$
$(\widetilde{l^c_-})_L$	$\frac{1}{2}$	0	$\frac{1}{2}$
$(\chi_\pm)_L$	0	0	0
$(\widetilde{q^c_-})_L$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$
$(\chi^q_\pm)_L$	0	$\frac{2}{2}$	$\frac{2}{3}$

TABLE 3.3: Hypercharge assignments under the diagonal $U(1)' \subset SO(5)$ and $U(1)''$ for fermions transforming in a non linear representation of the gauge group.

the first two generations receive a mass from

$$\mathcal{L}_{Y_u} = -i\frac{\lambda_u}{4} f \epsilon_{ijk}\epsilon_{xy} \left[\left(\overline{Q}_1\right)_i \Sigma_{jx}\Sigma_{ky} + \left(\overline{Q}_2\Sigma_0\Omega\right)_i \widetilde{\Sigma}_{jx}\widetilde{\Sigma}_{ky} \right] u_R + \text{h.c.} \tag{3.51}$$

$\{i,j,k\} = 1,2,3$ and $\{x,y\} = 4,5$ and the new multiplets multiplets are defined as follows

$$Q_1 = \begin{pmatrix} -i\sigma^2 q_{1L} \\ 0 \\ 0_2 \end{pmatrix}, \quad Q_2 = \begin{pmatrix} 0_2 \\ 0 \\ -i\sigma^2 q_{2L} \end{pmatrix}. \tag{3.52}$$

For the top quark one implements the collective symmetry breaking mechanism in [87, 98, 103, 109]

$$\begin{aligned} \mathcal{L}_{Y_t} = & -i\frac{\lambda_1}{4} f \epsilon_{ijk}\epsilon_{xy} \left[\left(\overline{Q}_1^t\right)_i \Sigma_{jx}\Sigma_{ky} + \left(\overline{Q}_2^t\Sigma_0\Omega\right)_i \widetilde{\Sigma}_{jx}\widetilde{\Sigma}_{ky} \right] t_R \\ & - \frac{\lambda_2 f}{\sqrt{2}} \left(\overline{T}_{1L}T_{1R} + \overline{T}_{2L}T_{2R}\right) + \text{h.c.}, \end{aligned} \tag{3.53}$$

by promoting the top quark left-handed doublet to a triplet of the different SU(3) global factors contained in SU(5). Those are embedded in the quintuplets

$$Q_1^t = \begin{pmatrix} -i\sigma^2 \mathcal{T}_{1L} \\ iT_{1L} \\ 0_2 \end{pmatrix}, \quad Q_2^t = \begin{pmatrix} 0_2 \\ iT_{2L} \\ -i\sigma^2 \mathcal{T}_{2L} \end{pmatrix}, \tag{3.54}$$

with

$$\mathcal{T}_{rL} = \begin{pmatrix} t_{rL} \\ b_{rL} \end{pmatrix}, \quad r = 1,2. \tag{3.55}$$

The transformation properties under the gauge group and T-parity for the multiplets in eqs. (3.52) and (3.54) are given by

$$Q_1^{(t)} \xrightarrow{G_g} V_g^* Q_1^{(t)}, \quad Q_2^{(t)} \xrightarrow{G_g} V_g Q_2^{(t)}, \quad Q_1^{(t)} \xrightarrow{\text{T}} \Omega\Sigma_0 Q_2^{(t)} \tag{3.56}$$

and for the SU(2) singlets are consistently given by

$$T_{1R} \xrightarrow{\text{T}} T_{2R}, \quad t_R \xrightarrow{\text{T}} t_R. \tag{3.57}$$

The other possible realization of T-parity in these sectors

$$Q_1^{(t)} \xrightarrow{\text{T}} -\Sigma_0 Q_2^{(t)}, \quad T_{1R} \xrightarrow{\text{T}} -T_{2R}, \tag{3.58}$$

is completely equivalent. This is because the Lagrangian for the quarks of the first two families in eq. (3.51) has the same structure as the those for SM leptons and down-type quarks and the same argument applies. On the other hand, the multiplets in eq. (3.54) include a set of 'cancelon' fields or top partners T_{1L}, T_{2L} and their corresponding right-handed counterparts T_{1R}, T_{2R}. The different transformations under T-parity only changes the combinations that are defined as T-even and T-odd, having no impact on the phenomenology.

The collective symmetry breaking in the top quark sector works similarly as in the toy model in Chapter 2. In both realizations of T-parity, the first term proportional to λ_1 is invariant under the SU(3) in the upper-left corner of the SU(5) matrices. The second term is invariant under the SU(3) in lower-right corner of SU(5) in the first realization of T-parity because Ω

can be eliminated using that the Levi-Civita tensors ϵ_{ijk} and ϵ_{xy} are invariant under the action of Ω.In the second realization of T-parity, there are not enough Ω's to eliminate them using the invariance of the Levi-Civita tensors. However the unitarity of Ω and its explicit form is enough to forbid a quadratic divergence coming from the term

$$\mathcal{O}_{\Lambda^2} = \epsilon_{ijk}\epsilon_{xy}\epsilon_{ij'k'}\epsilon_{x'y'}\widetilde{\Sigma}_{jx}\widetilde{\Sigma}_{ky}\widetilde{\Sigma}^\dagger_{jx}\widetilde{\Sigma}^\dagger_{ky} \tag{3.59}$$

that is invariant under the lower-right SU(3). Concerning a possible logarithmically divergent contribution proportional to λ_1^4 from the first two terms in brackets, the only operator that can arise is

$$\mathcal{O}_{\log\Lambda^2} = \left(\epsilon_{ijk}\epsilon_{xy}\Sigma_{jx}\Sigma_{ky}\epsilon_{ij'k'}\epsilon_{x'y'}\Sigma^\dagger_{j'x'}\Sigma^\dagger_{k'y'}\right)\left(\epsilon_{ijk}\epsilon_{xy}\widetilde{\Sigma}_{jx}\widetilde{\Sigma}_{ky}\epsilon_{ij'k'}\epsilon_{x'y'}\widetilde{\Sigma}^\dagger_{j'x'}\widetilde{\Sigma}^\dagger_{k'y'}\right), \tag{3.60}$$

independently of the chosen realization of T-parity. In the last step we applied the properties of Ω. Notice that it factorizes in two independent pieces that do not mix the different SU(3) factors and thus no logarithmically divergent contribution to the Higgs mass can be generated.

The terms proportional to λ_2 are not invariant under any of the SU(3) factors but they have no couplings to the Higgs. Thus both couplings are required to generate just a logarithmically divergent contribution to the Higgs mass at one loop. This structure leads to a mixing between the top quark and the corresponding T-even partner as we will show later.

The mirror-partner fermions $(\widetilde{l}^c_-)$, $(\widetilde{q}^c_-)$ and the gauge SU(2) singlets $\chi_\pm, \chi^q_\pm$ must be heavy. It is customary [93, 98] to give them a large vector-like mass introducing the left-handed SU(2) doublets $(\widetilde{l}^c_-)_L, (\widetilde{q}^c_-)_L$ and singlets $(\chi_\pm)_L, (\chi^q_\pm)_L$ in incomplete SO(5) multiplets,

$$\Psi_L = \begin{pmatrix} (\widetilde{l}^c_-)_L \\ 0 \\ 0_2 \end{pmatrix}, \quad \Psi^q_L = \begin{pmatrix} (\widetilde{q}^c_-)_L \\ 0 \\ 0_2 \end{pmatrix}, \quad \Psi^\chi_L = \begin{pmatrix} 0 \\ (\chi_\pm)_L \\ 0_2 \end{pmatrix}, \quad \Psi^{\chi_q}_L = \begin{pmatrix} 0 \\ (\chi^q_\pm)_L \\ 0_2 \end{pmatrix}. \tag{3.61}$$

Their direct mass terms are assumed to be a *soft* breaking of the SO(5) global symmetry and have the form

$$\mathcal{L}_{\widetilde{M},M_\chi} = -\widetilde{M}_l\overline{(\widetilde{l}^c_-)}_L(\widetilde{l}^c_-)_R - M_\chi\overline{(\chi_\pm)}_L(\chi_\pm)_R - \widetilde{M}_q\overline{(\widetilde{q}^c_-)}_L(\widetilde{q}^c_-)_R - M_{\chi^q}\overline{(\chi^q_\pm)}_L(\chi^q_\pm)_R + \text{h.c.} \tag{3.62}$$

Finally, the CCWZ formalism provides us with the kinetic terms and gauge interactions of all fermions transforming in a non linear representation,

$$\mathcal{L}_F = \mathcal{L}_{F_L} + \mathcal{L}_{F_R} + (\Psi_R \to \Psi_L) + (\Psi_R \to \Psi^\chi_L) \tag{3.63}$$

where

$$\mathcal{L}_{F_L} = i\overline{\Psi}_1\gamma^\mu D^*_\mu\Psi_1 + i\overline{\Psi}_2\gamma^\mu D_\mu\Psi_2 + i\overline{\Psi}^q_1\gamma^\mu D^{q*}_\mu\Psi^q_1 + i\overline{\Psi}^q_2\gamma^\mu D^q_\mu\Psi^q_2 \tag{3.64}$$

and depending on the T-parity implementation,

$$\begin{aligned}\mathcal{L}^{(a)}_{F_R} &= i\overline{\Psi}_R\gamma^\mu\left[\partial_\mu + \frac{1}{2}\xi^\dagger\left(D_\mu\xi\right) + \frac{1}{2}\xi\Sigma_0 D^*_\mu\left(\Sigma_0\xi^\dagger\right)\right]\Psi_R \\ &+ i\overline{\Psi}^q_R\gamma^\mu\left[\partial_\mu + \frac{1}{2}\xi^\dagger\left(D^q_\mu\xi\right) + \frac{1}{2}\xi\Sigma_0 D^{q*}_\mu\left(\Sigma_0\xi^\dagger\right)\right]\Psi^q_R, \end{aligned}\tag{3.65}$$

$$\begin{aligned}\mathcal{L}^{(b)}_{F_R} &= i\overline{\Psi}_R\gamma^\mu\left[\partial_\mu + \frac{1}{2}\xi^\dagger\left(D_\mu\xi\right) + \frac{1}{2}\Omega\xi\Sigma_0 D^*_\mu\left(\Sigma_0\xi^\dagger\right)\Omega\right]\Psi_R \\ &+ i\overline{\Psi}^q_R\gamma^\mu\left[\partial_\mu + \frac{1}{2}\xi^\dagger\left(D^q_\mu\xi\right) + \frac{1}{2}\Omega\xi\Sigma_0 D^{q*}_\mu\left(\Sigma_0\xi^\dagger\right)\Omega\right]\Psi^q_R, \end{aligned}\tag{3.66}$$

with the covariant derivative for leptons defined as

$$D_\mu = \partial_\mu - \sqrt{2}ig\left(W^a_{1\mu}Q^a_1 + W^a_{2\mu}Q^a_2\right) + \sqrt{2}ig'\left(B_{1\mu}Y_1 + B_{2\mu}Y_2\right). \tag{3.67}$$

and for quarks

$$D^q_\mu = \partial_\mu - \sqrt{2}ig\left(W^a_{1\mu}Q^a_1 + W^a_{2\mu}Q^a_2\right) + \sqrt{2}ig'\left[B_{1\mu}\left(Y_1 + \frac{1}{3}\mathbb{1}_{5\times5}\right) + B_{2\mu}\left(Y_2 + \frac{1}{3}\mathbb{1}_{5\times5}\right)\right], \tag{3.68}$$

$$D^{q*}_\mu = \partial_\mu + \sqrt{2}ig\left(W^a_{1\mu}Q^{aT}_1 + W^a_{2\mu}Q^{aT}_2\right) - \sqrt{2}ig'\left[B_{1\mu}\left(Y_1 - \frac{1}{3}\mathbb{1}_{5\times5}\right) + B_{2\mu}\left(Y_2 - \frac{1}{3}\mathbb{1}_{5\times5}\right)\right], \tag{3.69}$$

where we took into account the extra hypercharge assigned in the external $U(1)''_1 \times U(1)''_2$ as shown in table 3.2 and the property in eq. (3.13). Finally for the SM right-handed fermions

$$\mathcal{L}_{F'} = i\bar{\ell}_R\left(\partial_\mu + ig'B_\mu\right)\ell_R + i\bar{d}_R\left(\partial_\mu + i\frac{1}{3}g'B_\mu\right)d_R + i\bar{u}_R\left(\partial_\mu - i\frac{2}{3}g'B_\mu\right)u_R. \tag{3.70}$$

and the top quark-partners

$$\begin{aligned}\mathcal{L}_{F'_{T_1,T_2}} =& i\bar{T}_{1L}\left[\partial_\mu - \sqrt{2}ig'\left(\frac{8}{15}B_{1\mu} + \frac{2}{15}B_{2\mu}\right)\right]T_{1L} + i\bar{T}_{2L}\left[\partial_\mu - \sqrt{2}ig'\left(\frac{2}{15}B_{1\mu} + \frac{8}{15}B_{2\mu}\right)\right]T_{2L} \\ &+ L \to R.\end{aligned} \tag{3.71}$$

3.1.5. Mass eigenfields

Gauge fields

After electroweak SSB, the T-even SM gauge bosons eigenstates are obtained diagonalizing $\mathcal{L}_S$ in eq. (3.26)

$$W^\pm = \frac{1}{\sqrt{2}}\left(W^1 \mp iW^2\right), \quad \begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix}\begin{pmatrix} W^3 \\ B \end{pmatrix} \tag{3.72}$$

with

$$W^a = \frac{W^a_1 + W^a_2}{\sqrt{2}}, \qquad B = \frac{B_1 + B_2}{\sqrt{2}}; \tag{3.73}$$

whereas the T-odd mass eigenstates, expanding $\mathcal{L}_S$ up to order v^2/f^2 are

$$W^\pm_H = \frac{1}{\sqrt{2}}\left(W^1_H \mp iW^2_H\right), \quad \begin{pmatrix} Z_H \\ A_H \end{pmatrix} = \begin{pmatrix} 1 & -x_H\frac{v^2}{f^2} \\ x_H\frac{v^2}{f^2} & 1 \end{pmatrix}\begin{pmatrix} W^3_H \\ B_H \end{pmatrix} \tag{3.74}$$

with

$$W^a_H = \frac{W^a_1 - W^a_2}{\sqrt{2}}, \quad B_H = \frac{B_1 - B_2}{\sqrt{2}}, \quad x_H = \frac{5gg'}{4\left(5g^2 - g'^2\right)}. \tag{3.75}$$

Their corresponding masses to order v^2/f^2 are

$$M_W = \frac{gv}{2}\left(1-\frac{v^2}{12f^2}\right), \quad M_Z = M_W/c_W, \quad e = g s_W = g' c_W,$$
$$M_{W_H} = M_{Z_H} = gf\left(1-\frac{v^2}{8f^2}\right), \quad M_{A_H} = \frac{g'}{\sqrt{5}}f\left(1-\frac{5v^2}{8f^2}\right), \tag{3.76}$$

where e is the unit of electric charge and s_W, c_W are respectively the sine and cosine of the Weinberg angle θ_W.

Scalar fields after gauge fixing

The spontaneous breaking of gauge symmetries leads to kinetic mixing between gauge bosons and would-be Goldstone boson fields. In the mass eigenbasis, these unwanted mixing terms can be removed, up to an irrelevant total derivative, by introducing the appropriate gauge-fixing Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{gf}} = &-\frac{1}{2\xi_\gamma}(\partial_\mu A^\mu)^2 - \frac{1}{2\xi_Z}(\partial_\mu Z^\mu - \xi_Z M_Z \pi^0)^2 - \frac{1}{\xi_W}|\partial_\mu W^\mu + i\xi_W M_W \pi^-|^2 \\ &- \frac{1}{2\xi_{A_H}}(\partial_\mu A_H^\mu + \xi_{A_H} M_{A_H}\eta)^2 - \frac{1}{2\xi_{Z_H}}(\partial_\mu Z_H^\mu - \xi_{Z_H} M_{Z_H}\omega^0)^2 \\ &- \frac{1}{\xi_{W_H}}|\partial_\mu W_H^\mu + i\xi_{W_H} M_{W_H}\omega^-|^2,\end{aligned} \tag{3.77}$$

defining which Goldstone fields are unphysical and can be absorbed.

After the SSB, the kinetic terms of the scalar fields we have introduced are neither diagonal nor canonically normalized. In order to define the physical scalars and identify the actual would-be-Goldstone fields we will perform the following redefinitions [106]

$$\begin{aligned}\pi^0 &\to \pi^0\left(1+\frac{v^2}{12f^2}\right), \\ \pi^\pm &\to \pi^\pm\left(1+\frac{v^2}{12f^2}\right), \\ h &\to h, \\ \Phi^0 &\to \Phi^0\left(1+\frac{v^2}{12f^2}\right), \\ \Phi^P &\to \Phi^P + \left(\sqrt{10}\eta - \sqrt{2}\omega^0 + \Phi^P\right)\frac{v^2}{12f^2}, \\ \Phi^\pm &\to \Phi^\pm\left(1+\frac{v^2}{24f^2}\right) \pm i\omega^\pm\frac{v^2}{12f^2}, \\ \Phi^{++} &\to \Phi^{++}, \\ \eta &\to \eta + \frac{5g'\eta - 4\sqrt{5}[g'(\omega^0+\sqrt{2}\Phi^P) - 6gx_H\omega^0]}{24g'}\frac{v^2}{f^2}, \\ \omega^0 &\to \omega^0 + \frac{5g(\omega^0+4\sqrt{2}\Phi^P) - 4\sqrt{5}\eta(5g+6g'x_H)}{120g}\frac{v^2}{f^2}, \\ \omega^\pm &\to \omega^\pm\left(1+\frac{v^2}{24f^2}\right) \pm i\Phi^\pm\frac{v^2}{6f^2}.\end{aligned} \tag{3.78}$$

After these redefinitions, the scalars η, ω^0 and $\omega^\pm$ are the would-be-Goldstone bosons of the SSB of the gauge group down to the SM gauge group, eaten by A_H, Z_H and $W_H^\pm$. Similarly, π^0 and $\pi^\pm$ are the would-be-Goldstone bosons of the SSB of the SM gauge group down to $\mathrm{U}(1)_{\mathrm{em}}$, eaten by Z and $W^\pm$. The physical pseudo-Goldstone bosons include the Higgs boson and the complex scalar triplet Φ composed of $\Phi^{\pm\pm}$, $\Phi^\pm$, Φ^0 and Φ^P.

Fermion fields

Fermion masses and mass eigenvectors are obtained from the diagonalization of the 3×3 matrices κ_l, κ_q, λ_l, λ_u, λ_d, M_χ, M_{χ_q}, $\widetilde{M}_l$, $\widetilde{M}_q$ in and the mixing between the top quark and its corresponding T-even partner coming from eq. (3.53).

Omitting flavour indices, for each of the three SM (T-even) left-handed fermion doublets (l_L, q_L) there is a vector-like doublet of heavy T-odd mirror fermions (l_H, q_H)

$$l_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} = \frac{l_{1L} - l_{2L}}{\sqrt{2}}, \qquad l_{HL} = \begin{pmatrix} \nu_{HL} \\ \ell_{HL} \end{pmatrix} = \frac{l_{1L} + l_{2L}}{\sqrt{2}}, \quad l_{HR} = \begin{pmatrix} \nu_{HR} \\ \ell_{HR} \end{pmatrix}, \tag{3.79}$$

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = \frac{q_{1L} - q_{2L}}{\sqrt{2}}, \qquad q_{HL} = \begin{pmatrix} u_{HL} \\ d_{HL} \end{pmatrix} = \frac{q_{1L} + q_{2L}}{\sqrt{2}}, \quad q_{HR} = \begin{pmatrix} u_{HR} \\ d_{HR} \end{pmatrix} \tag{3.80}$$

where

$$l_{rL} = \begin{pmatrix} \nu_{rL} \\ \ell_{rL} \end{pmatrix}, \quad q_{rL} = \begin{pmatrix} u_{rL} \\ d_{rL} \end{pmatrix}, \quad r = 1,2 \tag{3.81}$$

are part of the SU(5) multiplets $\Psi_r^{(q)}$ in eq. (3.28) and l_{HR}, q_{HR} are part of the SO(5) multiplets $\Psi_R^{(q)}$ in eq. (3.35). The SM right-handed charged leptons ℓ_R and quarks d_R, u_R are singlets under the full SU(5) but have hypercharges under the external $\mathrm{U}(1)''$ groups.

In addition there is a heavy right-handed doublet of mirror-partner fermions,

$$(\widetilde{l}^c_-)_R = \begin{pmatrix} (\widetilde{\nu}^c_-)_R \\ (\widetilde{\ell}^c_-)_R \end{pmatrix}, \quad (\widetilde{q}^c_-)_R = \begin{pmatrix} (\widetilde{u}^c_-)_R \\ (\widetilde{d}^c_-)_R \end{pmatrix} \tag{3.82}$$

together with a right-handed SU(2) singlet $(\chi_\pm)_R$ to complete the SO(5) right-handed quintuplet in eq. 3.35 and their corresponding left-handed counterparts,

$$(\widetilde{l}^c_-)_L = \begin{pmatrix} (\widetilde{\nu}^c_-)_L \\ (\widetilde{\ell}^c_-)_L \end{pmatrix}, \quad (\widetilde{q}^c_-)_L = \begin{pmatrix} (\widetilde{u}^c_-)_L \\ (\widetilde{d}^c_-)_L \end{pmatrix}. \tag{3.83}$$

and $\left(\chi_\pm^{(q)}\right)_L$ living in the incomplete SO(5) left-handed quintuplets in eq. (3.61).

Now we work out the top quark sector $\mathcal{L}_{Y_t}$ in eq. (3.53), that requires extra couplings and extra fermion fields T_{1L} and T_{2L}, belonging to Q_1^t and Q_2^t respectively, and their right-handed counterparts T_{1R}, T_{2R} required to implement the collective symmetry breaking mechanism and avoid dangerous quadratically divergent contributions to the Higgs mass. One can define the T-even and T-odd combinations of top partners

$$(T_+)_{L,R} = \frac{(T_1)_{L,R} + (T_2)_{L,R}}{\sqrt{2}}, \quad (T_-)_{L,R} = \frac{(T_1)_{L,R} - (T_2)_{L,R}}{\sqrt{2}}. \tag{3.84}$$

The T-odd combination does not mix with any other T-odd quark since it is isolated in this sector. However, the T-even combination mixes with t_R. Expanding $\mathcal{L}_Y^t$ at leading order in v/f

we find that the top quark matrix is given by

$$-\mathcal{L}_{Y_t} \supset \left(\bar{t}_L, (\overline{T}_+)_L\right) \begin{pmatrix} \frac{\lambda_1 v}{\sqrt{2}} & 0 \\ \frac{\lambda_1 f}{\sqrt{2}} & \frac{\lambda_2 f}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} t_R \\ (T_+)_R \end{pmatrix} + \text{h.c.}\,. \tag{3.85}$$

This matrix can be diagonalized at leading order redefining the right-handed fields

$$\begin{pmatrix} t_R \\ (T_+)_R \end{pmatrix} \to \begin{pmatrix} c_R & s_R \\ -s_R & c_R \end{pmatrix} \begin{pmatrix} t_R \\ (T_+)_R \end{pmatrix}, \quad c_R = \frac{\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}, \quad s_R = \frac{\lambda_1}{\sqrt{\lambda_1^2+\lambda_2^2}}. \tag{3.86}$$

Then the physical mass of the SM top quark in the LHT is given by[3]

$$m_t = \frac{1}{\sqrt{2}} \frac{\lambda_1 \lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}} v. \tag{3.87}$$

Notice that according to the previous expression, the Yukawa couplings λ_1 and λ_2 are not independent but verify

$$\frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} = \left(\frac{v}{\sqrt{2} m_t}\right)^2, \tag{3.88}$$

where the top quark mass is $m_t \approx 173$ GeV [110]. For the T-even and T-odd partners one finds[4]

$$M_{T_+} = \sqrt{\frac{\lambda_1^2+\lambda_2^2}{2}} f, \quad M_{T_-} = \frac{\lambda_2}{\sqrt{2}} f. \tag{3.89}$$

Next we introduce flavour indices and derive the mass eigenstates. Since T-parity is exact, the SM (T-even) fermions do not mix with the heavy T-odd combinations. However, the SM neutral leptons and up-type quarks could mix, depending on the T-parity realization, with χ_+, χ_+^q (see table 3.5). Therefore, at leading order, the SM mass eigenstates result from the diagonalization of the matrices λ_l, λ_d and λ_u in eqs. (3.46), (3.47) and (3.51). Thus in general

$$\ell_L \to V_L^\ell \ell_L, \quad \ell_R \to V_R^\ell \ell_R, \tag{3.90}$$
$$d_L \to V_L^d d_L, \quad d_R \to V_R^d d_R, \tag{3.91}$$
$$u_L \to V_L^u u_L, \quad u_R \to V_R^u u_R, \tag{3.92}$$

and the masses of SM charged leptons, down-type quarks and up-type quarks read from

$$\frac{\lambda_l v}{\sqrt{2}} \left(1 - \frac{v^2}{12 f^2}\right) = V_L^\ell m_\ell V_R^{\ell\dagger}, \tag{3.93}$$
$$\frac{\lambda_d v}{\sqrt{2}} \left(1 - \frac{v^2}{12 f^2}\right) = V_L^d m_d V_R^{d\dagger}, \tag{3.94}$$
$$\frac{\lambda_u v}{\sqrt{2}} \left(1 - \frac{v^2}{3 f^2}\right) = V_L^u m_u V_R^{u\dagger}, \tag{3.95}$$

[3]The fact that the top mass is proportional to the product of the Yukawa couplings λ_1 and λ_2 is a consequence of the collective symmetry breaking mechanism. Since the mass of the top is nothing but its coupling to the Higgs boson, its contribution to the Higgs mass at one loop is just logarithmically divergent and proportional to the product $\lambda_1\lambda_2$.

[4]We have an extra factor $\sqrt{2}$ multiplying the top and top partner masses with respect to [111] due to the different definitions of the top Yukawa couplings λ_1 and λ_2 in eq. (3.53).

	t_L	$(T_+)_L$	$(\chi_+^q)_L$
t_R	•	–	–
$(T_+)_R$	v	•	–
$(\chi_+^q)_R$	v	–	•

TABLE 3.4: Order of the mixing between the top quark and the rest of T-even up-type quarks once the diagonalization in eq. (3.86) is performed. A dot means that they are connected by the mass term and a dash indicates that no mixing term is generated to order v^2.

where $V^{\ell}_{L,R}$, $V^{d}_{L,R}$, $V^{u}_{L,R}$ are unitary matrices in flavour space. Likewise, the heavy charged lepton and down-type quark mass eigenstates are obtained by the replacements

$$\ell_{HL} \to V_L^{l_H}\ell_{HL}, \quad \ell_{HR} \to V_R^{l_H}\ell_{HR}, \tag{3.96}$$
$$d_{HL} \to V_L^{d_H}d_{HL}, \quad d_{HR} \to V_R^{d_H}d_{HR}, \tag{3.97}$$
$$\left(\widetilde{\ell}^c_-\right)_L \to \widetilde{V}_L^l\left(\widetilde{\ell}^c_-\right)_L, \quad \left(\widetilde{\ell}^c_-\right)_R \to \widetilde{V}_R^l\left(\widetilde{\ell}^c_-\right)_R, \tag{3.98}$$
$$\left(\widetilde{d}^c_-\right)_L \to \widetilde{V}_L^q\left(\widetilde{d}^c_-\right)_L, \quad \left(\widetilde{d}^c_-\right)_R \to \widetilde{V}_R^q\left(\widetilde{d}^c_-\right)_R, \tag{3.99}$$

with

$$\sqrt{2}\kappa_l f = V_L^{l_H} m_{\ell_H} V_R^{l_H\dagger}, \tag{3.100}$$
$$\sqrt{2}\kappa_q f = V_L^{d_H} m_{d_H} V_R^{d_H\dagger}, \tag{3.101}$$
$$\widetilde{M}_l = \widetilde{V}_L^l \widetilde{m}_l \widetilde{V}_R^{l\dagger}, \tag{3.102}$$
$$\widetilde{M}_q = \widetilde{V}_L^q \widetilde{m}_q \widetilde{V}_R^{q\dagger}, \tag{3.103}$$

where $V^{l_H}_{L,R}$, $V^{d_H}_{L,R}$, $\widetilde{V}^{l}_{L,R}$, $\widetilde{V}^{q}_{L,R}$ are also unitary matrices in flavour space.

For the neutral lepton sector and up-type quarks the fields have to be redefined at leading order as follows,[5]

$$\nu_L \to V_L^{\ell}\nu_L, \tag{3.104}$$
$$\nu_{HL} \to V_L^{l_H}\nu_{HL}, \quad \nu_{HR} \to V_R^{l_H}\nu_{HR}, \tag{3.105}$$
$$u_{HL} \to V_L^{q_H}u_{HL}, \quad u_{HR} \to V_R^{q_H}u_{HR}, \tag{3.106}$$
$$(\widetilde{\nu}^c_-)_L \to \widetilde{V}_L^l(\widetilde{\nu}^c_-)_L, \quad (\widetilde{\nu}^c_-)_R \to \widetilde{V}_R^l(\widetilde{\nu}^c_-)_R, \tag{3.107}$$
$$(\widetilde{u}^c_-)_L \to \widetilde{V}_L^q(\widetilde{u}^c_-)_L, \quad (\widetilde{u}^c_-)_R \to \widetilde{V}_R^q(\widetilde{u}^c_-)_R, \tag{3.108}$$
$$(\chi_\pm)_L \to V_L^{\chi}(\chi_\pm)_L, \quad (\chi_\pm)_R \to V_R^{\chi}(\chi_\pm)_R, \tag{3.109}$$
$$(\chi_\pm^q)_L \to V_L^{\chi^q}(\chi_\pm^q)_L, \quad (\chi_\pm^q)_R \to V_R^{\chi^q}(\chi_\pm^q)_R. \tag{3.110}$$

[5]In [61] the partner lepton fields $\widetilde{l}_-$ (equivalently the mirror-partner quarks) are rotated with matrices $\widetilde{V}^l_{L,R}$. Here we adopt the convention of rotating their conjugates $\widetilde{l}^c_-$, which seems more natural as these are the ones embedded in the SO(5) quintuplet. To relate both conventions, $\widetilde{V}^l_L \equiv \left(\widetilde{V}^l_R\right)^*$ and $\widetilde{V}^l_R \equiv \left(\widetilde{V}^l_L\right)^*$.

	$(\chi_+)_L, (\chi_+^q)_L$	$(\chi_-)_L, (\chi_-^q)_L$	ν_L, u_L	ν_{HL}, u_{HL}	$(\tilde{\nu}_-^c)_L, (\tilde{u}_-^c)_L$
$(\chi_+)_R, (\chi_+^q)_R$	•	–	v	–	–
$(\chi_-)_R, (\chi_-^q)_R$	–	•	–	v	–
ν_R, u_R	–	–	•	–	–
ν_{HR}, u_{HR}	–	–	–	•	–
$(\tilde{\nu}_-^c)_R, (\tilde{u}_-^c)_R$	–	–	–	v^2	•

TABLE 3.5: As in table 3.4 but for the neutral lepton fields, SM up-type quarks of the first two generations and rest of the new LHT up-type quarks.

Note that the SM neutrinos rotate in flavour space with the same matrix as the charged leptons, V_L^ℓ. Their corresponding masses up to corrections from the mixing in table 3.5 are

$$m_{\nu_H} = m_{\ell_H}\left(1 - \frac{v^2}{8f^2}\right), \tag{3.111}$$
$$m_{u_H} = m_{d_H}\left(1 - \frac{v^2}{8f^2}\right), \tag{3.112}$$
$$M_\chi^{\text{diag}} = V_L^\chi M_\chi V_R^{\chi\dagger}, \tag{3.113}$$
$$M_{\chi^q}^{\text{diag}} = V_L^{\chi^q} M_{\chi^q} V_R^{\chi^q\dagger}. \tag{3.114}$$

The neutral mirror-partner leptons and up-type mirror-partner quarks masses verify the same relation as the charged mirror-partner leptons and down-type mirror-partner quarks respectively in eqs. (3.102) and (3.103).

The misalignment between different fermion sectors will be a source of flavour violation. Thus, parametrizing these misalignments we define the following unitary matrices

$$V = V_L^{l_H\dagger} V_L^\ell, \quad W = \tilde{V}_L^\dagger V_L^{l_H}, \quad Z = V_R^{\chi\dagger} V_R^{l_H}, \tag{3.115}$$

for leptons and similarly for quarks

$$V^u = V_L^{q_H\dagger} V_L^u, \quad V^d = V_L^{q_H\dagger} V_L^d, \quad W^q = \tilde{V}_L^{q\dagger} V_L^{q_H}, \quad Z^q = V_R^{\chi^q\dagger} V_R^{q_H}, \quad V^{CKM} = V_L^{u\dagger} V_L^d. \tag{3.116}$$

However, since we have separated the third generation of quarks to implement the collective symmetry breaking mechanism for the top sector, it is useful and we will consider that the Yukawa coupling λ_u is diagonal in flavour space from the very beginning. This implies that the rotation matrices $V_L^u = V_R^u = \mathbb{1}$ and the misalignment matrices V^u and V^{CKM} take the simpler form $V^u = V_L^{q_H\dagger}$ and $V^{CKM} = V_L^d$. This completes the derivation of the full Lagrangian.

3.2. Phenomenology of the LHT

In this section we will study the phenomenology of the LHT with special emphasis on the contributions of the heavy singlet $\chi_\pm$ to different physical observables. In particular we will focus on Higgs decays to charged leptons of different flavour, the generation of neutrino masses and finally the lepton anomalous magnetic dipole moment factor $(g-2)_\ell$.

3.2.1. Lepton flavour violating Higgs decays

First of all it is worthwhile to discuss briefly the case without the singlet. The global symmetries of the LHT prevent tree level lepton flavour violating (LFV) Higgs decays, but they are generated at one loop via the new T-odd particles. In particular, the key ingredient is the misalignment between the Yukawa sectors in eq. (3.46) and eq. (3.39) (or equivalently eq. (3.40) without the singlet) and the mass matrix of the mirror-partner leptons in eq. (3.62). Without loss of generality the SM charged lepton Yukawa coupling, λ_l, is assumed to be diagonal. Let us comment the parametric dependence of the amplitude of this process. All the contributions to this observable come from the new particles and thus they are one loop suppressed. So there is a universal factor of $1/(16\pi^2)$. On the other hand, since the Yukawa couplings of the SM charged leptons are diagonal, the LFV Higgs decay amplitude must proceed via higher dimensional operators, that are suppressed by a high energy scale that we will call generically M, and scale at leading order like v^2/M^2. As we will show later, any contribution of order 1 to this process can be reabsorbed through a rotation of the charged lepton fields. Moreover, the amplitude of this decay must be proportional to the SM Yukawa couplings λ_l and to the Yukawa couplings κ_l squared through couplings and/or masses.[6] Hence, the amplitude for the Higgs decay $h \to \bar{\ell}\ell'$ scales as

$$\mathcal{M} \propto \frac{1}{16\pi^2}\frac{v^2}{M^2}\lambda\Delta\kappa^2 \sin 2\theta, \tag{3.117}$$

where we call generically θ to any of the misalignment angles between the different leptonic sectors, and $\Delta\kappa^2$ are splittings of Yukawa couplings squared.

In [61] it was shown that only including the contributions of the mirror and mirror-partner leptons, l_H and $\tilde{l}^c_-$ respectively, one can obtain a finite amplitude for lepton flavour violating Higgs decays in the LHT model. It was also shown that the mirror-partner leptons cannot be decoupled in these decays and furthermore, they introduce a new source of lepton flavour violation via their coupling to the physical pseudo-Goldstone triplet Φ through the Yukawa Lagrangian in eq. (3.39) or equivalently eq. (3.40). The divergent part of the amplitude can be parametrized as

$$\mathcal{M}_{\text{div}}(h \to \bar{\ell}\ell') = \frac{1}{16\pi^2}\left(C_{UV}^{(1)} + \frac{v^2}{f^2}C_{UV}^{\left(\frac{v^2}{f^2}\right)}\right)\frac{1}{\epsilon}\sum V^\dagger_{\ell' i}V_{i\ell}\frac{m^2_{\ell Hi}}{f^2}\bar{u}\left(p', m_{\ell'}\right)\left(\frac{m_{\ell'}}{v}P_L + \frac{m_\ell}{v}P_R\right)v\left(p, m_\ell\right), \tag{3.118}$$

since the infinite parts of the Passarino-Veltman functions do not depend on the internal masses running in the loop.[7] The different topologies that contribute to this process are listed in fig. 3.1. The cancellations between different contributions are shown in tables 3.6 and 3.7. The infinities of order 1 cancel without introducing the mirror-partner leptons, and the same for the finite parts. This implies that the mixing of order v^2 in table 3.5 between the mirror and mirror-partner neutral leptons contributes to higher order in the expansion v/f and thus it is negligible at the order we work. However, for the infinities of order v^2/f^2 both sets of leptons are necessary to give a finite result. On the other hand, to show the non decoupling behaviour of the mirror-partner leptons when their mass is taken large, we parametrize the finite part of the amplitude

$$\mathcal{M}\left(h \to \bar{\ell}\ell'\right) = \frac{1}{16\pi^2}\bar{u}\left(p', m_{\ell'}\right)\left(\frac{m_{\ell'}}{v}c_L P_L + \frac{m_\ell}{v}c_R P_R\right)v\left(p, m_\ell\right) \tag{3.119}$$

[6] We neglect higher powers of the Yukawa coupling λ_l because ist entries are small.

[7] This in turn implies that due to the unitarity of the missalignment matrix W, between the mirror and mirror-partner leptons, it will not appear in the divergent parts of the Passarino-Veltman functions.

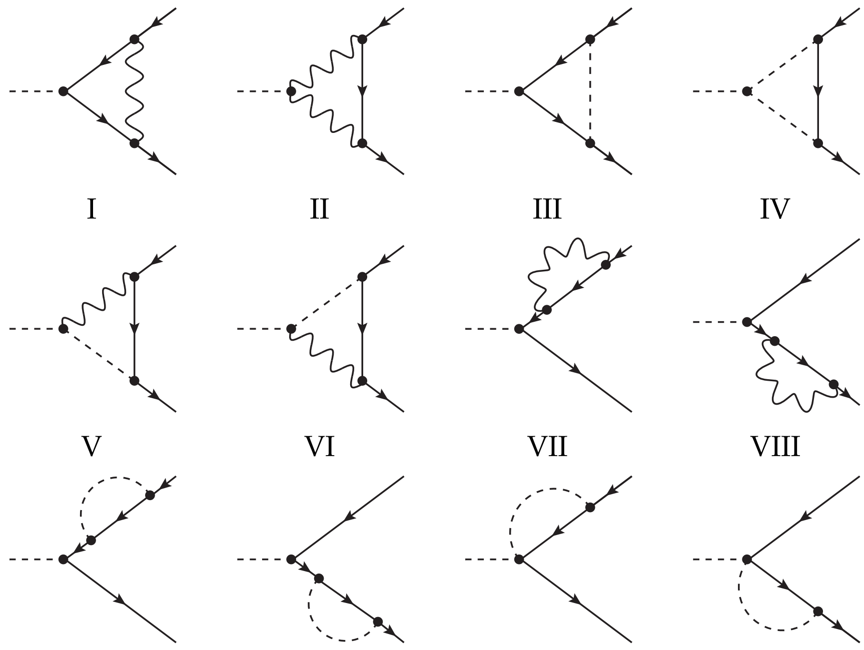


FIGURE 3.1: Topologies contributing to $h \to \bar{\ell}\ell'$.

with

$$c_{L,(R)} = g^2 \frac{v^2}{f^2} \left[\sum_{i=1}^{3} V^{\dagger}_{\ell' i} V_{i\ell} F\left(m_{\ell_{Hi}}, M_{W_H}, M_{A_H}, M_\Phi\right) \right. \\ \left. + \sum_{i,j,k=1}^{3} V^{\dagger}_{\ell' i} \frac{m_{\ell_{Hi}}}{M_{W_H}} W^{\dagger}_{ij} W_{jk} \frac{m_{\ell_{Hk}}}{M_{W_H}} V_{k\ell} G\left(\widetilde{M}_{lj}, m_{\ell_{Hk(i)}}, M_\Phi\right) \right]. \tag{3.120}$$

The first term depends on the function F that can be found in the Appendix of ref. [61] and only involves mirror leptons. As a consequence, this first source of lepton flavour violation comes from the matrix V parametrizing the misalignment between the SM leptons and the mirror leptons. The second term is the contribution of the mirror-partner leptons and depends on the matrices V and W, with the latter parametrizing the misalignment between the mirror and mirror-partner leptons, giving rise to a new source of lepton flavour violation. The non decoupling behaviour of the mirror-partner leptons is encoded in the function G

$$G\left(\widetilde{M}_{lj}, m_{\ell_{Hk(i)}}, M_\Phi\right) = \frac{1}{16} - \frac{1}{2} C_{00}\left(0, m_h^2, 0; M_\Phi^2, \widetilde{M}_{lj}^2, m_{\ell_{Hk}}^2\right) - \frac{1}{8} \widetilde{M}_{lj} m_{\ell_{Hk}} C_0\left(0, m_h^2, 0; M_\Phi^2, \widetilde{M}_{lj}^2, m_{\ell_{Hk}}^2\right) \\ - \frac{1}{12} M_\Phi^2 C_1\left(0, m_h^2, 0; M_\Phi^2, \widetilde{M}_{lj}^2, m_{\ell_{Hk}}^2\right). \tag{3.121}$$

In particular, the term proportional to C_{00} scales with $\log \widetilde{M}_j$ and there is no decoupling. This is a consequence of the *soft* breaking of the SO(5) global symmetry by these mass terms.

To estimate the corresponding branching ratio for the most interesting channel, $h \to \tau^{\pm}\mu^{\mp}$, the SM charged leptons have to be rotated in flavour space. This is because the process $h \to \bar{\ell}\ell'$ contributes to the charged leptons mass matrix when the Higgs field is replaced by the *vev*. This rotation results in an extra factor of 2/3 for the final amplitude. To understand this factor we can parametrize the SM charged lepton masses and their couplings to the Higgs boson by

$C_{UV}^{(1)}$	I	II	III	IV	V+VI	VII+VIII	IX+X	XI+XII	Sum
ω, ν_H	–	–	•	•	–	–	$\frac{1}{2}$	$-\frac{1}{2}$	•
ω^0, ℓ_H	–	–	•	•	–	–	$\frac{1}{4}$	$-\frac{1}{4}$	•
η, ℓ_H	–	–	•	•	–	–	$\frac{1}{20}$	$-\frac{1}{20}$	•
Total	–	–	•	•	–	–	$\frac{4}{5}$	$-\frac{4}{5}$	•

TABLE 3.6: Divergent contributions proportional to $\frac{1}{\epsilon}$, with $\epsilon = 4 - d$ the extra dimensions in dimensional regularization, of each particle set running in the loop and topology in fig. 3.1 contributing at $\mathcal{O}(1)$. A dash means that the field set does not run in the diagram, whereas a dot indicates that infinite and finite parts vanish.

$C_{UV}^{(\frac{v^2}{f^2})}$	I	II	III	IV	V+VI	VII+VIII	IX+X	XI+XII	Sum
W_H, ν_H	0	0	–	–	–	•	–	–	0
W_H, ω, ν_H	–	–	–	–	0	–	–	–	0
ω, ν_H	–	–	$\frac{1}{8}$	$-\frac{1}{16}$	–	–	$-\frac{1}{12}$	$\frac{5}{48}$	$\frac{1}{12}$
Z_H, ℓ_H	•	0	–	–	–	•	–	–	0
Z_H, ω^0, ℓ_H	–	–	–	–	0	–	–	–	0
ω^0, ℓ_H	–	–	•	$-\frac{1}{32}$	–	–	$-\frac{5}{48} + \frac{1}{2} x_H \frac{c_W}{s_W}$	$\frac{3}{16} - \frac{1}{2} x_H \frac{c_W}{s_W}$	$\frac{5}{96}$
A_H, ℓ_H	•	0	–	–	–	•	–	–	0
A_H, η, ℓ_H	–	–	–	–	0	–	–	–	0
η, ℓ_H	–	–	•	$-\frac{1}{32}$	–	–	$-\frac{17}{240} - x_H \frac{s_W}{10 c_W}$	$-\frac{1}{80} + x_H \frac{s_W}{10 c_W}$	$-\frac{11}{96}$
Z_H, A_H, ℓ_H	–	0	–	–	–	–	–	–	0
ω^0, η, ℓ_H	–	–	–	$\frac{1}{16}$	–	–	–	–	$\frac{1}{16}$
W_H, Φ, ν_H	–	–	–	–	0	–	–	–	0
Φ, ν_H	–	–	•	•	–	–	$-\frac{1}{16}$	$\frac{1}{48}$	$-\frac{1}{24}$
ω, Φ, ν_H	–	–	–	$\frac{1}{12}$	–	–	–	–	$\frac{1}{12}$
ω^0, Φ^P, ℓ_H	–	–	–	$\frac{1}{48}$	–	–	–	–	$\frac{1}{48}$
η, Φ^P, ℓ_H	–	–	–	$-\frac{1}{48}$	–	–	–	–	$-\frac{1}{48}$
$\Phi, \tilde{\nu}^c$	–	–	$-\frac{1}{8}$	$\frac{1}{48}$	–	–	•	$-\frac{1}{48}$	$-\frac{1}{8}$
Total	0	0	0	$\frac{1}{24}$	0	•	$-\frac{47}{240}$	$\frac{37}{240}$	0

TABLE 3.7: As in table 3.6 but to $\mathcal{O}(v^2/f^2)$.

the following effective Lagrangian in the LHT,

$$\mathcal{L}_{\text{eff}} = -\lambda_{li}\bar{l}_{Li}H\ell_{Ri} + \frac{\lambda_{li}}{6f^2}\left(H^\dagger H\right)\bar{l}_{Li}H\ell_{Ri} + \frac{c_{ij}}{M^2}\left(H^\dagger H\right)\bar{l}_{Li}H\ell_{Rj} + \text{h.c.} \tag{3.122}$$

$$\supset \left[\left(-m_{\ell_i}\delta_{ij} + \frac{1}{2\sqrt{2}}\frac{v^3}{M^2}c_{ij}\right) + \frac{h}{v}\left(-m_{\ell_i}\delta_{ij} + m_{\ell_i}\delta_{ij}\frac{v^2}{6f^2} + \frac{3}{2\sqrt{2}}\frac{v^3}{M^2}c_{ij}\right)\right]\bar{\ell}_{Li}\ell_{Rj} + \text{h.c.}, \tag{3.123}$$

where c_{ij} are the corresponding one-loop Wilson coefficients. In the second step we have used eq. (3.93) to relate the Yukawa couplings of the SM leptons with their actual mass. The key point is the relative factor of 3 between the Yukawa coupling and the mass term at order v^2/f^2, originating from the expansion $(v+h)^3 = v^2 + 3v^2h + \ldots$. Diagonalizing the first term in parentheses in eq. (3.123) does not diagonalize the second. We can rotate the charged leptons to express them in the physical basis,

$$\ell_{Li} \to \left[\delta_{ij} + \frac{v^2}{M^2}\left(A_L\right)_{ij}\right]\ell_{Lj}, \tag{3.124}$$

$$\ell_{Ri} \to \left[\delta_{ij} + \frac{v^2}{M^2}\left(A_R\right)_{ij}\right]\ell_{Rj}, \tag{3.125}$$

where A_L and A_R are order 1-loop anti-unitary matrices ($A^\dagger_{L,R} = -A_{L,R}$) satisfying

$$m_{\ell_i}\left(A_R\right)_{ij} - \left(A_L\right)_{ij}m_{\ell_j} = \frac{c_{ij}}{2\sqrt{2}}v \quad (i \neq j, \text{ physical basis}). \tag{3.126}$$

Then

$$\mathcal{L}_{\text{eff}} \supset \frac{1}{\sqrt{2}}\frac{v^2}{M^2}c_{ij}h\bar{\ell}_{Li}\ell_{Rj} + \text{h.c.} \quad (i \neq j, \text{ physical basis}) \tag{3.127}$$

and comparing with eq. (3.123) we see that the effect of going to the physical basis just amounts to a simple re-scaling of the off-diagonal Yukawa couplings by a factor 2/3. The LFV partial width can be written as

$$\Gamma\left(h \to \tau^+\mu^- + \tau^-\mu^+\right) = \frac{m_h}{16\pi}\frac{m_\tau^2 + m_\mu^2}{v^2}\frac{4}{9}\left(|c_L^{\tau\mu}|^2 + |c_R^{\tau\mu}|^2\right), \tag{3.128}$$

and its branching ratio

$$\text{Br}\left(h \to \tau\mu\right) = \text{Br}\left(h \to b\bar{b}\right)\frac{\Gamma\left(h \to \tau^+\mu^- + \tau^-\mu^+\right)}{\Gamma\left(h \to b\bar{b}\right)} \approx 0.6\frac{m_\tau^2}{6m_b^2}\frac{4}{9}\left(|c_L^{\tau\mu}|^2 + |c_R^{\tau\mu}|^2\right). \tag{3.129}$$

Now assuming that only the second and third families of leptons mix, we parametrize the matrices V, W

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{pmatrix}, \quad W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_2 & \sin\theta_2 \\ 0 & -\sin\theta_2 & \cos\theta_2 \end{pmatrix}. \tag{3.130}$$

Choosing a benchmark point $f = 1$ TeV, $m_{\ell_{H}2,3} = 1.0, 8.1$ TeV, $\widetilde{M}_{2,3} = 10, 50$ TeV, $\theta_{1,2} = \frac{\pi}{3}, \frac{\pi}{25}$ and the relation $M_\Phi \approx \sqrt{2}m_h f/v$ [111], we obtain

$$\text{Br}\left(h \to \tau\mu\right) \approx 0.2 \times 10^{-6}. \tag{3.131}$$

To obtain this result it is important the non decoupling behaviour of the mirror-partner leptons. In general this branching ratio tends to be smaller when all the T-odd masses are similar and there are often large cancellations. In the following we will investigate how the different T-parity assignations of the SU(2) leptonic singlet affect the previous result.

T-even heavy singlet

In this part we consider a T-even singlet, χ_+. Hence we will use the Yukawa Lagrangian in eq. (3.39) to obtain its corresponding couplings to scalar fields. The purpose of this section is to first prove that the contributions of the singlet to the process $h \to \ell_{Li}\ell_{Rj}$ are UV finite. The finite part can be split into order 1 and order v^2/M^2 contributions. However we will show that these order 1 contributions can be absorbed through a rotation of the charged SM lepton fields as in eq. (3.126) leaving an order v^2/f^2 remnant. Finally we will illustrate that the singlet can be decoupled from the spectrum.

Being T-even, the contributions of the singlet to $h \to \bar{\ell}_{Li}\ell_{Rj}$ can be worked out separately from those of the T-odd sector. Through the Yukawa Lagrangian in eq. (3.39), this singlet couples to the SM Higgs doublet and the T-even left-handed leptons. Their quantum numbers are appropriate to generate order 1 vertices and consequently order 1 contributions to this LFV process. In view of the mixing between the neutral lepton fields in table 3.5, there is a term that mixes the right-handed singlet with the SM neutrino of order v. Due to the, a priori, non vanishing order 1 contributions to $h \to \bar{\ell}_{Li}\ell_{Rj}$, it is mandatory a diagonalization of the corresponding mass matrix up to order v^3. After performing the rotations in flavour space of the neutral leptons in eq. (3.104) the mass matrix takes the form

$$\mathcal{L} \supset -\begin{pmatrix} \bar{\nu}_L & \overline{(\chi_+)_L} \end{pmatrix} \begin{pmatrix} 0 & \frac{v}{2f}\left(1-\frac{v^2}{12f^2}\right) V^\dagger m_{\ell_H} Z^\dagger \\ 0 & M_\chi^{\text{diag}} \end{pmatrix} \begin{pmatrix} \nu_R \\ (\chi_+)_R \end{pmatrix} + \text{h.c.}. \tag{3.132}$$

In view of the entries of the mass matrix in eq. (3.132) one can notice that at leading order it does not depend on the scale f since $m_{\ell_H} = \sqrt{2}\kappa_l f$. Thus the largest scale are the diagonal entries of M_χ and one can consider a perturbative diagonalization expanding in powers of v/M_χ. The first step corresponds to block-diagonalize the mass matrix in eq. (3.132) rotating the left-handed fields by the perturbative unitary matrix

$$\begin{pmatrix} \nu_L \\ (\chi_+)_L \end{pmatrix} \to \begin{pmatrix} \mathbb{1}-\frac{1}{2}\theta^\dagger\theta & \theta^\dagger \\ -\theta & \mathbb{1}-\frac{1}{2}\theta\theta^\dagger \end{pmatrix} \begin{pmatrix} \nu_L \\ (\chi_+)_L \end{pmatrix} \tag{3.133}$$

fulfilling[8]

$$\begin{pmatrix} \mathbb{1}-\frac{1}{2}\theta^\dagger\theta & -\theta^\dagger \\ \theta & \mathbb{1}-\frac{1}{2}\theta\theta^\dagger \end{pmatrix} \begin{pmatrix} 0 & \frac{v}{2f}\left(1-\frac{v^2}{12f^2}\right) V^\dagger m_{\ell_H} Z^\dagger \\ 0 & M_\chi^{\text{diag}} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & m_\chi \end{pmatrix}. \tag{3.134}$$

The solution of the above matrix equation gives the matrix θ and the non diagonal mass matrix m_χ,

$$\theta = \frac{v}{2f}\left(1-\frac{v^2}{12f^2}\right)\left(m_\chi^{-1}\right)^\dagger Z m_{\ell_H} V, \quad m_\chi = \left[\mathbb{1}+\frac{1}{2}\theta\theta^\dagger\right] M_\chi^{\text{diag}}. \tag{3.135}$$

[8]The full rotation matrix is unitary but each block is not.

Finally, to diagonalize m_χ up to order v^2, the new left-handed $(\chi_+)_L$ and right-handed $(\chi_+)_R$ fields must be rotated

$$(\chi_+)_{L,R} \to (\mathbb{1} + B_{L,R})\,(\chi_+)_{L,R} \tag{3.136}$$

where $B_{L,R}$ are anti-hermitian matrices ($B^\dagger_{L,R} = -B_{L,R}$) of order v^2/m^2_χ verifying the diagonalization equation

$$\widetilde{m}_\chi = (\mathbb{1} - B_L)\, m_\chi\, (\mathbb{1} + B_R) \tag{3.137}$$

and $\widetilde{m}_\chi$ is diagonal at the order we work. The expressions of $B_{L,R}$ are not important for the final result. However, at the order we work and assuming that the diagonal entries of m_χ are different, one can apply successive 2×2 SU(2) infinitesimal rotations to m_χ obtaining

$$(B_L)_{ij} = -\frac{1}{2}\left(\theta\theta^\dagger\right)_{ij}\frac{\widetilde{m}_i^2 + \widetilde{m}_j^2}{\widetilde{m}_i^2 - \widetilde{m}_j^2}; \quad i \neq j, \qquad (B_L)_{ii} = 0, \tag{3.138}$$

$$(B_R)_{ij} = -\left(\theta\theta^\dagger\right)_{ij}\frac{\widetilde{m}_i\widetilde{m}_j}{\widetilde{m}_i^2 - \widetilde{m}_j^2}; \quad i \neq j, \qquad (B_R)_{ii} = 0. \tag{3.139}$$

Thus the expression for the matrix θ in terms of the physical mass of the singlet is given by

$$\theta = \frac{v}{2f}\left(1 - \frac{v^2}{12f^2}\right)(\mathbb{1} + B_L)\,\widetilde{m}_\chi^{-1}\widetilde{Z} m_{\ell_H} V, \tag{3.140}$$

where we have defined $\widetilde{Z} := (\mathbb{1} - B_R)\, Z$ that takes into account the physical misalignment between the Yukawa coupling κ and the physical mass of the singlet $\widetilde{m}_\chi$. Like the full mass matrix, θ does not depend on f at leading order and it is of order v/m_χ. In view of the first entry of the unitary matrix in eq. (3.133) it is clear that the neutrino eigenfields contribute to the LFV process through the combination $\theta^\dagger\theta$. Hence our Feynman rules include both the singlet and the neutrino. The relevant Feynman rules in the mass eigenbasis are collected in tables 3.8, 3.9, 3.10 and 3.11 in terms of generic couplings for the following general vertices involving scalars (S), fermions (F) and/or gauge bosons (V):

$$\begin{aligned}
[\text{(S...S)SFF}] &= i(c_L P_L + c_R P_R)\,, \\
[\text{SV}_\mu\text{V}_\nu] &= iKg^{\mu\nu}\,, \\
[\text{V}_\mu\text{FF}] &= i\gamma^\mu(g_L P_L + g_R P_R)\,, \\
[\text{S}(p_1)\text{S}(p_2)\text{V}_\mu] &= iG(p_1 - p_2)^\mu\,, \\
[\text{SS}(p_1)\text{S}(p_2)] &= iJ\left(p_1^2 + p_2^2 + 4p_1\cdot p_2\right)\,,
\end{aligned}$$

where all momenta are assumed incoming. The conjugate vertices are obtained by replacing

$$c_{L,R} \leftrightarrow c^*_{R,L}\,, \quad K \leftrightarrow K^*\,, \quad g_{L,R} \leftrightarrow g^*_{R,L}\,, \quad G \leftrightarrow -G^*\,, \quad J \leftrightarrow J^*\,. \tag{3.141}$$

The Feynman diagrams that contribute to this process are those in fig. 3.1 with the neutrino, the singlet and SM charged gauge bosons and unphysical charged scalars running in the loop. Not all the topologies contribute to the LVF observable at the order we work. For instance, topology II is of next order. This is because the coupling hW^+W^- is order v and to produce lepton flavour changing one needs the $\theta^\dagger\theta$ part of the coupling $W^+\overline{v}_L\ell_L$ when the neutrino runs in the loop. Since the order of magnitude of θ is v/m_χ, this contribution is at least of order v^4 when the $\not{p}$ of the internal fermion propagator acts over the external legs giving the mass of the

[SFF]	c_L	c_R
$h\,\overline{\chi_+}\,\nu$	$-\left(1-\frac{v^2}{6f^2}\right)\widetilde{m}_\chi\frac{\theta}{v}+\frac{1}{2}\widetilde{m}_\chi\theta\theta^\dagger\theta+\widetilde{m}_\chi B_L\frac{\theta}{v}$	0
$h\,\overline{\chi_+}\,\chi_+$	$-\widetilde{m}_\chi\frac{\theta\theta^\dagger}{v}$	$-\frac{\theta\theta^\dagger}{v}\widetilde{m}_\chi$
$\pi^+\,\overline{\nu}\,\ell$	0	$i\sqrt{2}\frac{m_\ell}{v}\left(1+\frac{v^2}{12f^2}\right)$
$\pi^+\,\overline{\chi_+}\,\ell$	$-\sqrt{2}i\widetilde{m}_\chi\frac{\theta}{v}\left(1+\frac{v^2}{12f^2}\right)+i\sqrt{2}\widetilde{m}_\chi B_L\frac{\theta}{v}$	$i\sqrt{2}\theta\frac{m_\ell}{v}\left(1+\frac{v^2}{12f^2}\right)-i\sqrt{2}B_L\theta\frac{m_\ell}{v}$
$h\,\overline{\ell}\,\ell$	$-\frac{m_\ell}{v}\left(1-\frac{v^2}{6f^2}\right)$	$-\frac{m_\ell}{v}\left(1-\frac{v^2}{6f^2}\right)$

TABLE 3.8: Scalar-Fermion-Fermion couplings at $\mathcal{O}(v^2/f^2)$.

[SSFF]	c_L	c_R
$h\,\pi^+\,\overline{\nu}\,\ell$	0	$\frac{-i}{3\sqrt{2}f^2}m_\ell+\frac{i}{6\sqrt{2}f^2}\theta^\dagger\theta m_\ell$
$h\,\pi^+\,\overline{\chi_+}\,\ell$	$\frac{i}{3\sqrt{2}f^2}\widetilde{m}_\chi\theta$	$\frac{-i}{3\sqrt{2}f^2}\theta m_\ell$

TABLE 3.9: Scalar-Scalar-Fermion-Fermion couplings at $\mathcal{O}(v^2/f^2)$.

[S$V_\mu V_\nu$]	K
$h\,W^+\,W^-$	$g^2\frac{v}{2}\left(1-\frac{v^2}{3f^2}\right)$

[V_μFF]	g_L
$W^+\,\overline{\nu}\,\ell$	$\frac{g}{\sqrt{2}}\mathbb{1}-\frac{g}{2\sqrt{2}}\theta^\dagger\theta$
$W^+\overline{\chi_+}\ell$	$\frac{g}{\sqrt{2}}\theta$

TABLE 3.10: Scalar-Vector-Vector and Vector-Fermion-Fermion couplings at $\mathcal{O}(v^2/f^2)$. The right-handed Vector-Fermion-Fermion couplings g_R vanish.

[S(p_1)S(p_2) V_μ]	G
$h\pi^+W^-$	$\frac{ig}{2}$

[SS(p_1)S(p_2)]	J
$h\,\pi^+\,\pi^-$	$\frac{v}{6f^2}$

TABLE 3.11: Scalar-Scalar-Vector and Scalar-Scalar-Scalar couplings at $\mathcal{O}(v^2/f^2)$.

corresponding charged lepton. On the other hand, if the left-handed part of the singlet runs in the loop, the coupling $W^+\overline{(\chi_+)}_L \ell_L$ is of order θ and after applying the equation of motion on the external legs this other contribution is again of order v^4. The sum of the contributions of order v^2/f^2 coming from topologies VII and VIII cancel exactly as it happened in the case without the singlet (see tables 3.6, 3.7).

Parametrizing the amplitude as in eq. (3.119) where we now split the Wilson coefficients $c_{L,R}$ into order 1 and order v^2, $c_{L,R} = c^{(1)}_{L,R} + c^{(v^2)}_{L,R}$ and using the definition of θ in eq. (3.140), the non trivial order 1 contributions come from topologies III and IX+X and those are

$$c_L^{(1)}\Big|_{\rm III} = -2\left(V^\dagger \frac{m_{\ell_H}}{2f}\widetilde{Z}^\dagger\right)_{2k}\left(\widetilde{Z}\frac{m_{\ell_H}}{2f}V\right)_{k1}\left[B_0\left(m_h^2; m_{\chi_k}^2, 0\right) + M_W^2 C_0\left(0, m_h^2, 0; M_W^2, m_{\chi_k}^2, 0\right)\right] \tag{3.142}$$

$$c_L^{(1)}\Big|_{\rm IX+X} = 2\left(V^\dagger \frac{m_{\ell_H}}{2f}\widetilde{Z}^\dagger\right)_{2k}\left(\widetilde{Z}\frac{m_{\ell_H}}{2f}V\right)_{k1} B_0\left(0; m_{\chi_k}^2, 0\right), \tag{3.143}$$

while the rest are

$$\begin{aligned} c_L^{(v^2)}\Big|_{\rm I} =& g^2 v^2 \left(V^\dagger \frac{m_{\ell_H}}{2f}\widetilde{Z}^\dagger\right)_{2k}\left(\widetilde{Z}\frac{m_{\ell_H}}{2f}V\right)_{k1} \times \\ & \left[C_0\left(0, m_h^2, 0; M_W^2, m_{\chi_k}^2, 0\right) + C_2\left(0, m_h^2, 0; M_W^2, m_{\chi_k}^2, 0\right) + C_2\left(0, m_h^2, 0; M_W^2, 0, m_{\chi_k}^2\right)\right] \end{aligned} \tag{3.144}$$

$$\begin{aligned} c_L^{(v^2)}\Big|_{\rm III} =& \frac{v^2}{3f^2}\left(V^\dagger \frac{m_{\ell_H}}{2f}\widetilde{Z}^\dagger\right)_{2k}\left(\widetilde{Z}\frac{m_{\ell_H}}{2f}V\right)_{k1}\left[B_0\left(m_h^2; m_{\chi_k}^2, 0\right) + M_W^2 C_0\left(0, m_h^2, 0; M_W^2, m_{\chi_k}^2, 0\right)\right] \\ & + 2\left(V^\dagger \frac{m_{\ell_H}}{2f}\widetilde{Z}^\dagger\right)_{2k}\left(\widetilde{Z}\frac{m_{\ell_H}}{2f}V\right)_{km}\left(V^\dagger \frac{m_{\ell_H}}{2f}\widetilde{Z}^\dagger\right)_{ml}\left(\widetilde{Z}\frac{m_{\ell_H}}{2f}V\right)_{1}\frac{v^2}{m_{\chi_k}^2}\times \\ & \left[B_0\left(m_h^2; m_{\chi_l}^2, 0\right) + M_W^2 C_0\left(0, m_h^2, 0; M_W^2, m_{\chi_l}^2, 0\right)\right. \\ & \left. - B_0\left(m_h^2; m_{\chi_l}^2, m_{\chi_k}^2\right) - M_W^2 C_0\left(0, m_h^2, 0; M_W^2, m_{\chi_l}^2, m_{\chi_k}^2\right) + 2m_{\chi_k}^2 C_2\left(0, m_h^2, 0; M_W^2, m_{\chi_l}^2, m_{\chi_k}^2\right)\right] \end{aligned} \tag{3.145}$$

$$\begin{aligned} c_L^{(v^2)}\Big|_{\rm IV} =& -\frac{v^2}{3f^2}\left(V^\dagger \frac{m_{\ell_H}}{2f}\widetilde{Z}^\dagger\right)_{2k}\left(\widetilde{Z}\frac{m_{\ell_H}}{2f}V\right)_{k1}\left[B_1\left(0; m_{\chi_k}^2, M_W^2\right) + 2B_0\left(0, m_{\chi_k}^2, M_W^2\right)\right. \\ & \left. -2\left(m_h^2 - M_W^2\right)\left(C_2\left(0, m_h^2, 0; m_{\chi_k}^2, M_W^2, M_W^2\right) + C_0\left(0, m_h^2, 0; m_{\chi_k}^2, M_W^2, M_W^2\right)\right)\right] \end{aligned} \tag{3.146}$$

$$\begin{aligned} c_L^{(v^2)}\Big|_{\rm V+VI} =& g^2\left(V^\dagger \frac{m_{\ell_H}}{2f}\widetilde{Z}^\dagger\right)_{2k}\left(\widetilde{Z}\frac{m_{\ell_H}}{2f}V\right)_{k1}\frac{v^2}{m_{\chi_k}^2}\left[B_0\left(0; 0, M_W^2\right) - B_0\left(0; m_{\chi_k}^2, M_W^2\right)\right. \\ & + M_W^2\left(C_0\left(0, m_h^2, 0; 0, M_W^2, M_W^2\right) - C_0\left(0, m_h^2, 0; m_{\chi_k}^2, M_W^2, M_W^2\right)\right) \\ & \left. - m_{\chi_k}^2 C_2\left(0, m_h^2, 0; m_{\chi_k}^2, M_W^2, M_W^2\right)\right] \end{aligned} \tag{3.147}$$

$$c_L^{(v^2)}\Big|_{\rm IX+X} = -\frac{v^2}{3f^2}\left(V^\dagger \frac{m_{\ell_H}}{2f}\widetilde{Z}^\dagger\right)_{2k}\left(\widetilde{Z}\frac{m_{\ell_H}}{2f}V\right)_{k1} B_0\left(0, M_W^2, m_{\chi_k}^2\right) \tag{3.148}$$

$$c_L^{(v^2)}\Big|_{\rm XI+XII} = \frac{v^2}{3f^2}\left(V^\dagger \frac{m_{\ell_H}}{2f}\widetilde{Z}^\dagger\right)_{2k}\left(\widetilde{Z}\frac{m_{\ell_H}}{2f}V\right)_{k1}\left[B_0\left(0; M_W^2, m_{\chi_k}^2\right) - B_1\left(0; M_W^2, m_{\chi_k}^2\right)\right]. \tag{3.149}$$

The corresponding Wilson coefficients c_R are obtained from c_L just by the substitution $C_2 \to C_1$. The calculation has been explicitly checked using the Mathematica package *Package X* [112]. As one can check in table 3.12, the order 1 UV divergences coming from III + IX+X cancel but not the finite parts as it happened in the case of the mirror leptons. On the other hand, the contributions order v^2 coming from III + IX+X and IV + XI+XII also cancels as we show in

$C_{\rm UV}^{(1)}$	I	II	III	IV	V+VI	VII+VIII	IX+X	XI+XII	Sum
π, χ_+, ν	–	–	-1	–	–	–	1	–	0

TABLE 3.12: Order 1 singlet and neutrino divergent contributions proportional to $\frac{1}{\epsilon}$, with $\epsilon = 4 - d$ the extra dimensions in dimensional regularization. A dash means that the field set does not run in the diagram, whereas a 0 indicates that only the infinite part vanishes.

$C_{\rm UV}^{(\frac{v^2}{f^2})}$	I	II	III	IV	V+VI	VII+VIII	IX+X	XI+XII	Sum
π, χ_+, ν	0	–	$\frac{1}{6}$	$-\frac{1}{4}$	0	•	$-\frac{1}{6}$	$\frac{1}{4}$	0

TABLE 3.13: As in table 3.12 but to order v^2/f^2 singlet and neutrino divergent contributions proportional to $\frac{1}{\epsilon}$, with $\epsilon = 4 - d$ the extra dimensions in dimensional regularization. A dash means that the field set does not run in the diagram, whereas a 0 indicates that only the infinite part vanishes.

table 3.13. Thus the contribution of the singlet to this LFV process is finite if it is chosen to be T-even. It is important to mention that the result does not depend on the anti-hermitian matrix B_L as previously advertised but depends on B_R through $\widetilde{Z}$. The matrices B_R and the original Z are not physical, being $\widetilde{Z}$ the physical combination and the new source of flavour violation. In this sense, our Wilson coefficients are a function of physical parameters only. As one can notice, not all terms order v^2 are suppressed by f^2 but some of them are suppressed by m_χ^2. These come from order 1 vertices with the neutrino in the non rotated basis that generate terms with θ order v^2 due to the mixing between the left-handed neutrino and the left-handed singlet.

As happened in the case without the singlet, the previous results must be re-scaled because they contribute to the SM charged lepton matrix when the Higgs is replaced by its *vev*. The effective Lagrangian that parametrizes the couplings of the Higgs boson to the SM charged leptons including the one-loop contributions reads in this case

$$\mathcal{L}_{\rm eff} = -\lambda_{li}\bar{l}_{Li}H\ell_{Ri} + \frac{\lambda_{li}}{6f^2}\left(H^\dagger H\right)\bar{l}_{Li}H\ell_{Ri} + \alpha_{ij}\bar{l}_{Li}H\ell_{Rj} + \frac{\beta_{ij}}{M^2}\left(H^\dagger H\right)\bar{l}_{Li}H\ell_{Rj} + \text{h.c.} \quad (3.150)$$

$$\supset \left(-m_{\ell_i}\delta_{ij} + \frac{\alpha_{ij}}{\sqrt{2}}v + \frac{1}{2\sqrt{2}}\frac{v^3}{M^2}\beta_{ij}\right)\bar{\ell}_{Li}\ell_{Rj}$$
$$+\frac{h}{v}\left(-m_{\ell_i}\delta_{ij} + m_{\ell_i}\delta_{ij}\frac{v^2}{6f^2} + \frac{\alpha_{ij}}{\sqrt{2}}v + \frac{3}{2\sqrt{2}}\frac{v^3}{M^2}\beta_{ij}\right)\bar{\ell}_{Li}\ell_{Rj} + \text{h.c.}, \quad (3.151)$$

where we include now a new term with the order 1 Wilson coefficient. As we did above, the SM leptons are rotated by

$$\ell_{Li} \to \left[\delta_{ij} + (A_L)_{ij}\right]\ell_{Lj}, \quad (3.152)$$
$$\ell_{Ri} \to \left[\delta_{ij} + (A_R)_{ij}\right]\ell_{Rj}, \quad (3.153)$$

where $A_{L,R}$ are again order one-loop anti-hermitian matrices. In this case we do not factor out any scale from the matrices since they include order 1 and v^2/M^2 contributions as showed below. The diagonalization of the charged SM leptons imposes

$$m_{\ell_i}(A_R)_{ij} - (A_L)_{ij}m_{\ell_j} = \frac{\alpha_{ij}}{\sqrt{2}}v + \frac{\beta_{ij}}{2\sqrt{2}M^2}v^3 \quad (i \neq j, \text{physical basis}). \quad (3.154)$$

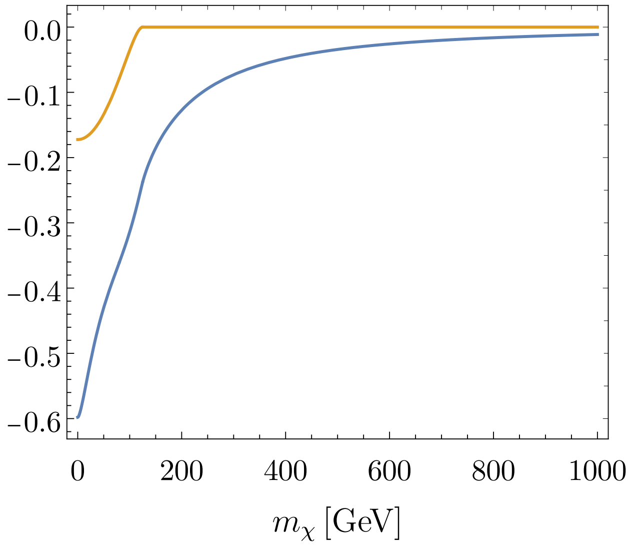


FIGURE 3.2: In blue and orange respectively, real and imaginary parts of the one loop function in $c_L^{\rm phys}$. The singlet χ_+ decouples.

Thus, substituting the rotation and using the previous condition we find

$$\mathcal{L}_{\rm eff} \supset \left(\frac{\alpha_{ij}}{\sqrt{2}} \frac{v^2}{6f^2} + \frac{\beta_{ij}}{\sqrt{2}} \frac{v^2}{M^2} \right) h \bar{\ell}_{Li} \ell_{Rj} + \text{h.c.}. \tag{3.155}$$

Comparing with eq. (3.150), the order 1 contribution is multiplied by a factor $\frac{v^2}{6f^2}$ and the order v^2/M^2 is re-scaled again by a factor $2/3$ and the physical c_L is

$$c_L^{\rm phys} = \frac{v^2}{6f^2} c_L^{(1)} + \frac{2}{3} c_L^{(v^2)}. \tag{3.156}$$

We would like to highlight the importance of the term $\left(H^\dagger H\right) \bar{l}_{Li} H \ell_{Ri}$ in the effective Lagrangian. Due to its presence, it is impossible to completely reabsorb the order 1 contribution in the diagonalization of the SM charged leptons mass matrix. Thus all the physical contribution to this LVF observable is order v^2/M^2 with $M = f, m_\chi$. With this in mind we can show that the T-even singlet decouples in this observable. In fig. 3.2 we plot the leading contributions that are precisely those of order v^2/f^2 while the contributions order v^2/m_χ^2 coming from Topology I and Topology III are subleading in comparison. This behaviour is independent of the mechanism that provides a mass to the singlet.

T-odd heavy singlet

In this section we will assume that the singlet is T-odd, χ_-. This implies that now, instead of coupling to the SM Higgs doublet, it couples to the T-odd triplets ω and Φ through the Yukawa Lagrangian in eq. (3.40). The purpose of this section is to show that the contributions of a heavy T-odd singlet are UV divergent.

Through the Yukawa Lagrangian in eq. (3.40), the right-handed singlet couples to the triplets ω and Φ with hypercharges 0 and 1, respectively, and to the T-even (SM) left-handed leptons. This implies that there are no order 1 vertices with the singlet and the SM charged leptons, contrary to the T-even case. An addtional Higgs doublet is necessary to obtain a singlet from the combination of all these fields. As a consequence, any vertex with these scalar fields and the right-handed $(\chi_-)_R$ is at least order v/f. Thus there is no need to diagonalize the mixing

[SFF]	c_L	c_R
$h\,\overline{\chi_-}\,\nu_H$	$Z\frac{m_{\ell_H}}{2f}\left(1-\frac{v^2}{4f^2}\right)$	0
$\omega^+\,\overline{\nu}_H\,\ell$	$-i\frac{m_{\ell_H}}{\sqrt{2}f}V$	$iV\frac{m_\ell}{\sqrt{2}}\left(1+\frac{v^2}{8f^2}\right)$
$\omega^+\,\overline{\chi_-}\,\ell$	$iZ\frac{m_{\ell_H}}{\sqrt{2}f}V\frac{v}{4f}$	0
$\phi^+\,\overline{\nu}_H\,\ell$	$\frac{m_{\ell_H}}{\sqrt{2}f}V\frac{v^2}{8f^2}$	$V\frac{m_\ell}{\sqrt{2}}\left(1-\frac{v^2}{8f^2}\right)$
$\phi^+\,\overline{\chi_-}\,\ell$	$-Z\frac{m_{\ell_H}}{\sqrt{2}f}V\frac{v}{4f}$	0

TABLE 3.14: Scalar-Fermion-Fermion couplings at $\mathcal{O}(v^2/f^2)$ involving ν_H and a T-odd singlet.

[SSFF]	c_L	c_R
$h\,\omega^+\,\overline{\chi_-}\,\ell$	$iZ\frac{m_{\ell_H}}{4\sqrt{2}f^2}V\left(1-\frac{v^2}{6f^2}\right)$	0
$h\,\phi^+\,\overline{\chi_-}\,\ell$	$-Z\frac{m_{\ell_H}}{4\sqrt{2}f^2}V\left(1-\frac{v^2}{12f^2}\right)$	0

TABLE 3.15: Scalar-Scalar-Fermion-Fermion couplings at $\mathcal{O}(v^2/f^2)$ involving a T-odd singlet.

between the T-odd singlet, ν_H and $\widetilde{\nu}^c_-$ in table 3.5 and it is ignored everywhere. The leading contributions of the T-odd singlet to this process are of order v^2/f^2.

To extract the UV divergent part of this process coming from the introduction of the T-odd singlet, we can use part of the information previously learned from the case without the singlet and the case with a T-even singlet. The topologies one has to evaluate are again those depicted in fig. 3.1. Since the left-handed singlet does not couple to the charged gauge bosons, Topology I requires an insertion of the mixing between the singlet and ν_H to give a new contribution. However at order 1 this topology is finite by its own as one can read from table 3.6. Thus no divergences order v^2/f^2 are generated through mixing. The same argument applies for Topology II but in this case two mixing insertions are necessary. Topology III has a potential divergence of order v^2/f^2 when the right-handed components of the singlet run in the loop. Topology IV may generate a divergence but this is of next order since the couplings between the singlet and the scalar T-odd triplets are of order v/f as was discussed above. Furthermore the couplings between the Higgs doublet and the triplets are of order v/f^2 as one can infer from the structure of the $\mathcal{L}_S$ Lagrangian in eq. (3.26) and the SU(2) quantum numbers of the involved scalar fields. Besides, any insertion of the mixing would require a ν_{HL} in the loop and a right-handed SM charged lepton in an external leg, leading again to negligible corrections at the order we work. Topology V+VI and VII+VII do not contribute either, by the same arguments as Topologies I and II. Topology IX+X with only the right-handed components of the singlet running in the loop vanishes as in the T-even case. Thus since the left-handed components of the singlet do not have Yukawa couplings, an insertion of the mixing with ν_H is needed. However, since the order 1 divergences of these diagrams cancel with Topology XI+XII, the corresponding diagrams with mixing insertions also cancel and the only remaining possibility is Topology XI+XII with only the right-handed components of the singlet running in the loop. Thus to evaluate Topology III and XI+XII the Feynman rules are listed in tables 3.14 and 3.15

$C_{UV}^{(\frac{v^2}{f^2})}$	III	XI+XII	Sum
ω, ν_H, χ_-	$-\frac{1}{8}$	–	$-\frac{1}{8}$
ω, χ_-	–	$\frac{1}{32}$	$\frac{1}{32}$
Φ, ν_H, χ_-	$\frac{1}{8}$	–	$\frac{1}{8}$
Φ, χ_-	–	$\frac{1}{32}$	$\frac{1}{32}$
Total	0	$\frac{1}{32}$	$\frac{1}{32}$

TABLE 3.16: Order $\frac{v^2}{f^2}$ T-odd singlet contributions to the UV divergent part of $h \to \bar{\ell}_i \ell_j$.

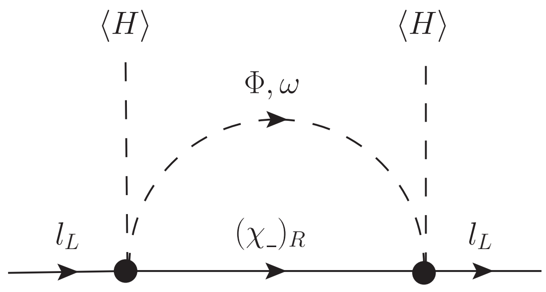


FIGURE 3.3: Feynman diagrams that generate the divergent operators in eq. (3.158) in LFV Higgs decays in the T-odd χ scenario.

As in the case without the singlet, the divergent part of the amplitude can be parametrized by the following expression

$$\mathcal{M}_{\text{div}}^{\chi_-}(h \to \bar{\ell}\ell') = \frac{1}{16\pi^2} \frac{v^2}{f^2} C_{UV}^{(\frac{v^2}{f^2})} \frac{1}{\epsilon} \sum V_{\ell' i}^{\dagger} V_{i\ell} \frac{m_{\ell Hi}^2}{f^2} \bar{u}\,(p', m_{\ell'}) \left(\frac{m_{\ell'}}{v} P_L + \frac{m_\ell}{v} P_R \right) v\,(p, m_\ell)\,, \quad (3.157)$$

where all the new contributions are of order v^2/f^2 and the UV divergences do not depend on the mass of the particles running in the loop. The results are collected in table 3.16.

As one can notice, the contributions coming from Topology III cancel since they have opposite signs. On the other hand, the contributions from Topology XI+XII have the same sign and they do not cancel. Since only the right-handed components of χ_- run in the loop, the divergence comes exclusively from the κ sector in eq. (3.40). Thus one has to disregard the T-odd possibility to have a one-loop predictive model and adopt the T-even realization for the singlet within the minimal LHT scenario. If one still insists in the T-odd realization, one needs to generate the tree level operator in the SM symmetric phase (see fig. 3.3)

$$\mathcal{O} = \left(\bar{l}_{iL}\sigma^a H\right) i\partial\!\!\!/ \left(H^\dagger \sigma^a l_{jL}\right) + \frac{1}{2}\left(\bar{l}_{iL}\sigma^a \widetilde{H}\right) i\partial\!\!\!/ \left(\widetilde{H}^\dagger \sigma^a l_{jL}\right), \quad (3.158)$$

in a $[\mathrm{SU}(2) \times \mathrm{U}(1)]^2$ gauge invariant fashion and with this relation between the coefficients to renormalize the divergent contribution. In the above equation $\widetilde{H} = i\sigma^2 H^*$, σ^a are the three Pauli matrices and i, j are flavour indices. The first term comes from integrating out the singlet and the triplet Φ and the second term comes from the integration of the singlet and the triplet

	Y_1'	Y_2'	Y_1''	Y_2''	Y_1	Y_2
χ_{1L}	$\frac{1}{5}$	$-\frac{1}{5}$	0	0	$\frac{1}{5}$	$-\frac{1}{5}$
χ_{2L}	$-\frac{1}{5}$	$\frac{1}{5}$	0	0	$-\frac{1}{5}$	$\frac{1}{5}$
$x\chi_{1L}$	$-\frac{1}{5}$	$\frac{1}{5}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{7}{10}$	$-\frac{3}{10}$
$\widetilde{x}\chi_{2L}$	$\frac{1}{5}$	$-\frac{1}{5}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{10}$	$-\frac{7}{10}$
x	$-\frac{2}{5}$	$\frac{2}{5}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{9}{10}$	$-\frac{1}{10}$
$\widetilde{x}$	$\frac{2}{5}$	$-\frac{2}{5}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{10}$	$-\frac{9}{10}$

TABLE 3.17: As in table 3.2 for the extra χ_{1L} and χ_{2L} left-handed fields and the new scalar fields x, $\widetilde{x}$ required for the left-handed couplings of χ_- to ℓ.

ω.[9] This option has the pathology of LFV operators at tree level and one would need to find a mechanism to explain why the corresponding coefficients should be small to be compatible with the current LHC constraints [113]. On the other hand this operator cannot be generated in a gauge invariant manner. We will show that precisely the Yukawa Lagrangian (3.40) is not invariant under global transformations along the direction of the gauge generators. As a consequence these operators do not respect gauge invariance under the full gauge group $[\mathrm{SU}(2)\times\mathrm{U}(1)]^2$ but preserve the diagonal SM gauge group.

Another possibility one could explore is trying to involve the λ sector in eq. (3.46) to cancel this divergence through couplings of the form $(h+v)^2\,\omega^+\overline{(\chi_-)_L}\ell_R$ and $(h+v)^2\,\phi^+\overline{(\chi_-)_L}\ell_R$. To do so, one would need to introduce the left-handed fields χ_{1L}, χ_{2L} in the SU(5) multiplets in eq. (3.28). The T-odd combination would pair with $(\chi_-)_R$ and receive a mass of order κf from eq. (3.40) whereas the remaining T-even combination would remain massless unless another mechanism is provided to generate a mass. Those fields would have hypercharges $\left(\frac{1}{5},-\frac{1}{5}\right)$ and $\left(-\frac{1}{5},\frac{1}{5}\right)$ under the gauge group according to the transformation properties of the SU(5) multiplets in eq. (3.28). However, it is not possible to introduce such fields in the multiplets of eq. (3.48) since they live in the opposite representation of the gauge group as those in eq. (3.28). There are no available scalar fields x, $\widetilde{x}$ to reverse the corresponding hypercharges of the new fermions fields and at the same time take into account the extra hypercharge required to identify ℓ_R with the SM right-handed charged leptons as it is shown in table 3.17. Thus we would have to assign ℓ_R to larger representations as in [114, 115].

3.2.2. Neutrino masses in the LHT

T-even heavy singlet

In this part we will assume that the lepton field χ is T-even (χ_+). The left-handed χ_L living in the incomplete SO(5) multiplet in eq. (3.61) is an SU(2) singlet. Therefore it is natural to include a small Majorana mass for it. We will assume that lepton number (LN) is only broken by small Majorana masses μ by a term of the form

$$\mathcal{L}_\mu = -\frac{\mu}{2}\overline{(\chi_+^c)_L}\,(\chi_+)_L + \text{h.c.}, \tag{3.159}$$

where $(\chi_+^c)_L = C\overline{(\chi_+)}_L^T$ with C the particle-antiparticle conjugation operator defined in terms of the Dirac matrices, $C = i\gamma^0\gamma^2$ (see for instance ref. [116]). This term is also assumed as a new

[9] There is another operator arising from the integration of the singlet η, $(\bar{l}_L\widetilde{H})i\partial\!\!\!/(\widetilde{H}^\dagger l_L)$. However this operator does not contribute to LFV Higgs decays.

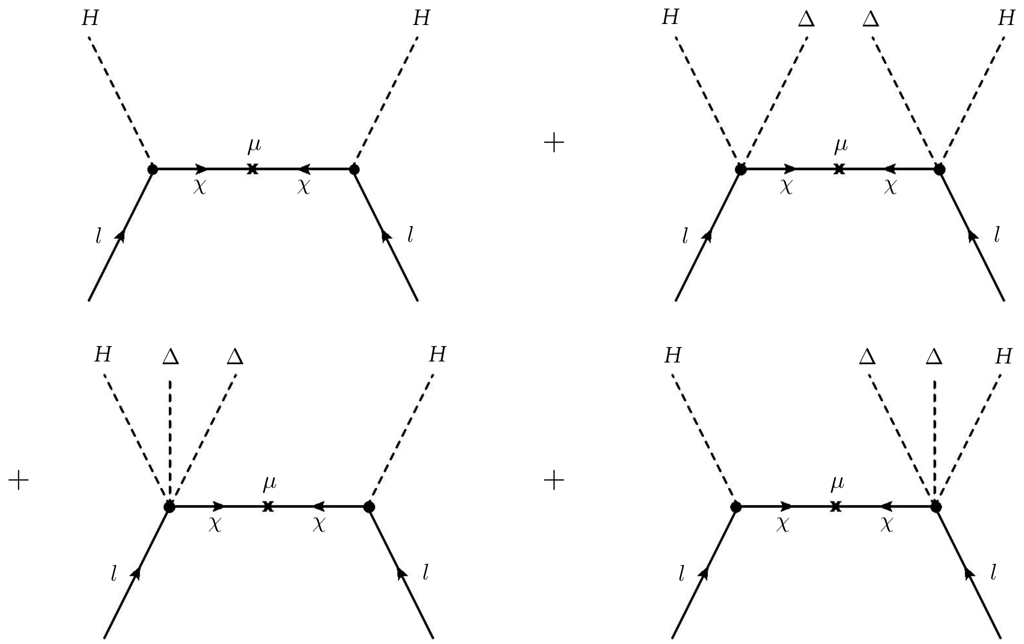


FIGURE 3.4: Diagrammatic expansion of the tree-level integration out of χ_+ in eq. (3.165).

source of *soft* breaking of the SO(5) global symmetry. To generate the neutrino mass matrix we use the inverse seesaw mechanism and thus we integrate out at tree-level the quasi-Dirac χ_+ using the corresponding equations of motion. Ignoring contributions from gauge bosons and derivatives of Goldstone bosons in the kinetic term for the non-linear right-handed fermions in eq. (3.65) we obtain

$$i\partial\!\!\!/ \, (\chi_+)_L - M_\chi \, (\chi_+)_R - \mu \, (\chi_+)_L^c = 0 \tag{3.160}$$

$$i\partial\!\!\!/ \, (\chi_+)_R - M_\chi^\dagger \, (\chi_+)_L + i\kappa_l^\dagger f \left(\xi^\dagger \Psi_2 + \xi \Sigma_0 \Psi_1\right)_3 + (...) = 0, \tag{3.161}$$

where (...) includes the gauge boson interactions and derivative interactions of Goldstone bosons with leptons. Combining these two equations one can obtain second order differential equations for the left and right-handed components of the singlet ignoring derivatives of Goldstones and fermions in $\Psi_{1,2}$. Since the mass of the quasi-Dirac singlet is much larger than the momentum transfer, one can consider an expansion of the inverse of the operator $\left(\partial^2 + M_\chi^2\right)^{-1} \approx M_\chi^{-2}$ obtaining at leading order in μ

$$(\chi_+)_L \approx \left(M_\chi^\dagger\right)^{-1} i\kappa_l^\dagger f \left(\xi^\dagger \Psi_2 + \xi \Sigma_0 \Psi_1\right)_3 \tag{3.162}$$

$$(\chi_+)_R \approx i M_\chi^{-1} \mu \left(M_\chi^{-1}\right)^T \kappa_l^T f \left(\xi^T \Psi_2^c + \xi^* \Sigma_0 \Psi_1^c\right)_3 . \tag{3.163}$$

Substituting the expressions for $(\chi_+)_L$ and $(\chi_+)_R$ we find

$$\mathcal{L} \supset \frac{1}{2} \left(\kappa f M_\chi^{-1}\right)^* \mu \left(\kappa_l f M_\chi^{-1}\right)^\dagger \left(\mathcal{O}_\chi + \mathcal{O}_\chi'\right) + \text{h.c.}, \tag{3.164}$$

where omitting family indices

$$\mathcal{O}_\chi = \overline{\Psi}_1^c \xi \frac{\mathbb{1}+\Omega}{2} \xi^T \Psi_1 + \overline{\Psi}_2^c \xi^* \frac{\mathbb{1}+\Omega}{2} \xi^\dagger \Psi_2, \tag{3.165}$$

$$\mathcal{O}'_\chi = \overline{\Psi}_1^c \Sigma_0 \xi^T \frac{\mathbb{1}+\Omega}{2} \xi^\dagger \Psi_2 + \overline{\Psi}_2^c \xi^* \frac{\mathbb{1}+\Omega}{2} \xi \Sigma_0 \Psi_1. \tag{3.166}$$

These operators contain the dimension 5 Weinberg operator

$$\mathcal{L} \supset -\frac{1}{2f^2} \left(\kappa_l f M_\chi^{-1}\right)^* \mu \left(\kappa_l f M_\chi^{-1}\right)^\dagger \left(\bar{l}_L^c \widetilde{H}^*\right) \left(\widetilde{H}^\dagger l_L\right) + \text{h.c.}, \tag{3.167}$$

where $\widetilde{H} = -i\sigma^2 H^*$ and also contributions from dimension 7 operators including the triplet $\Delta \equiv -i\sigma^2\Phi$ (that absorbes the prefactor $-i\sigma^2$ in the definition of Ψ_R in eq. (3.35))

$$\mathcal{L} \supset \frac{1}{2f^4} \left(\kappa_l f M_\chi^{-1}\right)^* \mu \left(\kappa_l f M_\chi^{-1}\right)^\dagger \left[\frac{1}{6}\left(\bar{l}_L^c \Delta^T \Delta^* \widetilde{H}^*\right)\left(\widetilde{H}^\dagger l_L\right) + \frac{1}{6}\left(\bar{l}_L^c \widetilde{H}^*\right)\left(\widetilde{H}^\dagger \Delta^\dagger \Delta\, l_L\right)\right] + \text{h.c.}. \tag{3.168}$$

These latter operators would contribute to the neutrino mass matrix once T-parity gets broken by the *vev* of the neutral component of the triplet $\langle \Phi^0 \rangle \neq 0$. Notice that there is no dimension 4 and 5 operators involving only triplets. The dimension 4 operator is forbidden by T-parity since it should come from a dimension 5 T-parity preserving operator. This is also the reason why the type II seesaw is not allowed in the LHT but it is in the model without T-parity [104]. The dimension 5 operator with two triplets is allowed by T-parity but two extra Higgses are required to compensate for the hypercharge of the two lepton doublets. Thus the only possibility is the dimension 7 operators above that are subleading with respect to the Weinberg operator by a factor $-\langle \Phi^0 \rangle / 6f^2$. Hence keeping the leading order contribution, the tree-level integration of the singlet gives the neutrino mass matrix when the Higgs is replaced by the *vev*

$$\mathcal{M}_\nu^{\text{even}} = \theta^T \mu \theta, \tag{3.169}$$

where the Yukawa coupling κ_l and the mass matrix M_χ have been rotated at leading order in flavour space according to eqs. (3.100) and (3.111), we have redefined without loss of generality $\mu \to V_L^{\chi *} \mu V_L^{\chi \dagger}$ and the SM neutrinos have been rotated according to eq. (3.104) to introduce the definition of θ in eq. (3.140). For consistency of the model we will assume that the natural size of the eigenvalues for M_χ are ~ 10 TeV, of the order of $4\pi f$ with $f \sim$ TeV, as required by current EWPD with the κ_l eigenvalues of order 1. The μ eigenvalues that parametrize the LN violation will be much smaller than the GeV. The predictions of the SM neutrino masses and the lepton flavour violation (LFV) contributions of the quasi-Dirac singlets χ_+ are those of the inverse seesaw [117–119] as we will review in the following.

Inverse seesaw masses and mixings The neutrino mass matrix $\mathcal{M}_\nu^{\text{even}}$ is not diagonal in the basis where the charged leptons mass matrix is diagonal. In fact,

$$\mathcal{M}_\nu^{\text{even}} = U_{\text{PMNS}}^* \mathcal{D}_\nu^{\text{even}} U_{\text{PMNS}}^\dagger \tag{3.170}$$

where U_{PMNS} is the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix [120–123] and $\mathcal{D}_\nu$ is the diagonal neutrino mass matrix. Then, solving eq. (3.169),

$$\mu = (\theta^T)^{-1} U_{\text{PMNS}}^* \mathcal{D}_\nu^{\text{even}} U_{\text{PMNS}}^\dagger (\theta)^{-1}. \tag{3.171}$$

Therefore, for an invertible matrix θ, μ can always be adjusted to fit light neutrino masses and mixings. For instance, for a single family, if $f > 1$ TeV, $\kappa = 1$ and $M_\chi = 10$ TeV, we find that $\mu \sim 0.3$ keV for neutrino masses of order 0.1 eV. These values of μ are natural, since they parametrize the explicit breaking of LN [13].

The experimental limits on θ can always be satisfied without implementing flavour symmetries on the model but LFV constraints set stringent limits on the high energy scale f as well as on the mixing between light and heavy leptons.

LFV limits The θ matrix elements give the mixing between the light and heavy quasi-Dirac neutrinos, ν and χ_+, respectively, according to eq. (3.133),

$$(U_{\mathrm{PMNS}})_{ij}\,\nu_{Lj} \to \left[\mathbb{1} - \frac{1}{2}\theta^\dagger\theta\right]_{ij} \nu_{Lj} + \theta^\dagger\,(\chi_+)_{Lj}\,, \quad (\chi_+)_{Li} \to \left[\mathbb{1} - \frac{1}{2}\theta\theta^\dagger\right]_{ij} (\chi_+)_{Lj} - \theta_{ij}\nu_{Lj}, \tag{3.172}$$

where we neglect the contributions of the LN violating parameter μ to the rotations. θ is also constrained by lepton flavour conserving processes at tree level because they modify the SM charged and neutral currents. Using the notation introduced in [124, 125],

$$\begin{aligned}
\mathcal{L}_W^\nu &= \frac{g}{\sqrt{2}}\overline{\nu}_{Li} W_{ij}\gamma^\mu \ell_{Lj} W_\mu^+ + \text{h.c.}, \quad \text{with} \quad W_{ij} = \left\{U_{\mathrm{PMNS}}^\dagger \left[\mathbb{1} - \frac{1}{2}\theta^\dagger\theta\right]\right\}_{ij}\,, \\
\mathcal{L}_Z^\nu &= \frac{g}{2c_W}\overline{\nu}_{Li} X_{ij}\gamma^\mu \nu_{Lj} Z_\mu\,, \quad \text{with} \quad X_{ij} = \left\{U_{\mathrm{PMNS}}^\dagger \left[\mathbb{1} - \theta^\dagger\theta\right] U_{\mathrm{PMNS}}\right\}_{ij}\,,
\end{aligned} \tag{3.173}$$

where X is hermitian verifying $X = WW^\dagger$ at leading order. Hence X can be diagonalized by a unitary matrix. Since $U_{\mathrm{PMNS}}^\dagger\theta^\dagger\theta U_{\mathrm{PMNS}}$ is hermitian, all its eigenvalues are positive and, at leading order, proportional to v^2/f^2. This implies that all the eigenvalues of X are less than 1. In these conditions, X verifies the following positivity constraints [124, 125]

$$|X_{ij}|^2 \le X_{ii}X_{jj} \quad \text{and} \quad |\delta_{ij} - X_{ij}|^2 \le (1 - X_{ii})(1 - X_{jj}). \tag{3.174}$$

This latter equation reduces to the Schwarz identity $|\left(\theta^\dagger\theta\right)_{ij}|^2 \le \left(\theta^\dagger\theta\right)_{ii}\left(\theta^\dagger\theta\right)_{jj}$.

More stringent are the constraints coming from one loop LVF processes like $(g-2)_\ell$ and, at higher order, the Electric Dipole Moment of the electron (EDM$_e$).[10] Even though they are suppressed by the corresponding loop factor $1/16\pi^2$, they significantly restrict the θ matrix elements and the mass of the heavy quasi-Dirac neutrinos, M_χ. The couplings between the SM leptons and χ_+ are given by

$$\mathcal{L}_W^{\chi_+} = \frac{g}{\sqrt{2}}\overline{(\chi_+)}_{Li}\theta_{ij}\gamma^\mu \ell_j W_\mu^+ + \text{h.c.}, \quad \mathcal{L}_Z^{\nu\chi_+} = \frac{g}{2c_w}\overline{(\chi_+)}_{Li}\left(\theta U_{\mathrm{PMNS}}\right)_{ij}\gamma^\mu \nu_{Lj}^l Z_\mu + \text{h.c.}. \tag{3.175}$$

Finally, the SFF coupling in table 3.8 plays a crucial role in Higgs decays. At leading order,

$$\mathcal{L}_\chi^\nu = -\overline{\nu}_{Li}\left(U_{\mathrm{PMNS}}^\dagger\theta\right)_{ij}\frac{m_{\chi_j}}{v}(\chi_+)_R\, h + \text{h.c.}. \tag{3.176}$$

We can now derive conservative bounds for the LFV parameter θ. In the upper part of table 3.18 we collect the limits from EWPD obtained assuming that each neutrino mixes only with one light neutrino of a given flavour and that only one mixing is non-vanishing at a time [126, 129]. This means that only $\theta_{ii} \neq 0$ in the basis where the charged leptons are diagonal. On the other

[10] The addition of heavy neutrinos does not modify the SM neutral currents for charged leptons at tree level and then, they remain lepton flavour conserving and universal.

EWPD (only one $\theta_{ii} \neq 0$, at 95 % C.L. [126])		
$\lvert\theta_{e1}\rvert < 0.04$	$\lvert\theta_{\mu 2}\rvert < 0.03$	$\lvert\theta_{\tau 3}\rvert < 0.09$
LFV at 90 % C.L. ($m_{\chi_k} = 10$ TeV)		
Br$(\mu \to e\,\gamma) < 4.2 \times 10^{-13}$ [127] $\lvert\theta^*_{je}\theta_{j\mu}\rvert < 0.26 \times 10^{-4}$	Br$(\tau \to e\,\gamma) < 3.3 \times 10^{-8}$ [128] $\lvert\theta^*_{je}\theta_{j\tau}\rvert < 0.62 \times 10^{-2}$	Br$(\tau \to \mu\,\gamma) < 4.4 \times 10^{-8}$ [128] $\lvert\theta^*_{j\mu}\theta_{j\tau}\rvert < 0.71 \times 10^{-2}$

TABLE 3.18: Limits on the mixing between the SM and the heavy quasi-Dirac neutrinos from electroweak precision data (top) and from lepton flavour violating processes (bottom). The sum on the repeated index $j = 1, 2, 3$ is understood.

$[\mathrm{V}_\mu(p_1)\,\mathrm{V}_\nu(p_2)\,\mathrm{V}_\rho(p_3)]$	C
$\gamma W^+ W^-$	$-e$

$[\mathrm{S}(p_1)\,\mathrm{S}(p_2)\,\mathrm{V}_\mu]$	G
$\pi^+\pi^-\gamma$	$-e$

$[\mathrm{S}\mathrm{V}_\mu \mathrm{V}_\nu]$	K
$\pi^+ W^-\gamma$	$i\frac{e^2}{s_W}v$

TABLE 3.19: Vector-Vector-Vector, Scalar-Scalar-Vector couplings and Scalar-Vector-Vector couplings for $\gamma \to \bar{\ell}\ell'$.

hand, if one assumes universality, implying that the three diagonal mixings θ_{ii} are equal, their absolute value is found to be $|\theta_{ii}| < 0.03$ at 95% C.L [130]. This implies that using the definition of θ in eq. (3.140) at leading order and $v \approx 246$ GeV, one finds

$$|\widetilde{Z}\kappa_l^{\text{diag}} V|_{ii} < 0.17 \left(\frac{m_{\chi_i}}{\text{TeV}}\right). \tag{3.177}$$

This expression can be translated into information about the diagonal components of the Yukawa coupling κ_l under the following assumptions. Taking the Yukawa coupling λ_l in eq. (3.46) diagonal implies $V_L^l = V_R^l = \mathbb{1}$ and thus $V = V_L^{H\dagger}$. On the other hand, taking also the mass matrix of the (χ_+) diagonal at leading order implies that $V_L^\chi = V_R^\chi = \mathbb{1}$ and thus $Z \approx \widetilde{Z} = V_R^H$. Hence $\widetilde{Z}\kappa_l^{\text{diag}} V = \kappa_l$. This in turn translates into an upper bound on the mass of the T-odd mirror leptons $m_{\ell_H} = \sqrt{2}\kappa_l f$.

To further constraint the θ matrix, one can consider the most stringent bounds coming from the non observation of the radiative decays $\ell \to \ell'\gamma$. The contribution of (χ_+) can be evaluated in the 't Hooft-Feynman gauge in a similar way as its corresponding contribution to LFV Higgs decays. The potential one-loop topologies for a vector field coupled to leptons are listed in fig. 3.5. Not all of them give a contribution for the photon since it does not couple to neutral leptons and thus I and III are zero. On the other hand, gauge invariance reduces the vertex for an on-shell photon to a dipole transition

$$i\Gamma^\mu_\gamma(p_\ell, p_{\ell'}) = ie\left[iF_M^\gamma\left(Q^2\right) + F_E^\gamma\left(Q^2\right)\gamma_5\right]\sigma^{\mu\nu}Q_\nu, \tag{3.178}$$

where $Q_\nu = (p_{\ell'} - p_\ell)_\nu$. The topologies that according to *Package X* [112] give a contribution to this operator are II, IV and VI. Thus to the Feynman rules given above in subsection 3.2.1 we have to add those in table 3.19,

$$[\mathrm{V}_\mu(p_1)\,\mathrm{V}_\nu(p_2)\,\mathrm{V}_\rho(p_3)] = iJ\left[g^{\mu\nu}(p_2 - p_1)^\rho + g^{\nu\rho}(p_3 - p_2)^\mu + g^{\rho\mu}(p_1 - p_3)^\nu\right],$$

where all momenta are assumed incoming.

The decay width of the process is, neglecting $m_{\ell'} \ll m_\ell$,

$$\Gamma\left(\ell \to \ell'\gamma\right) = \frac{\alpha}{2}m_\ell^3\left(|F_M^\gamma|^2 + |F_E^\gamma|^2\right), \tag{3.179}$$

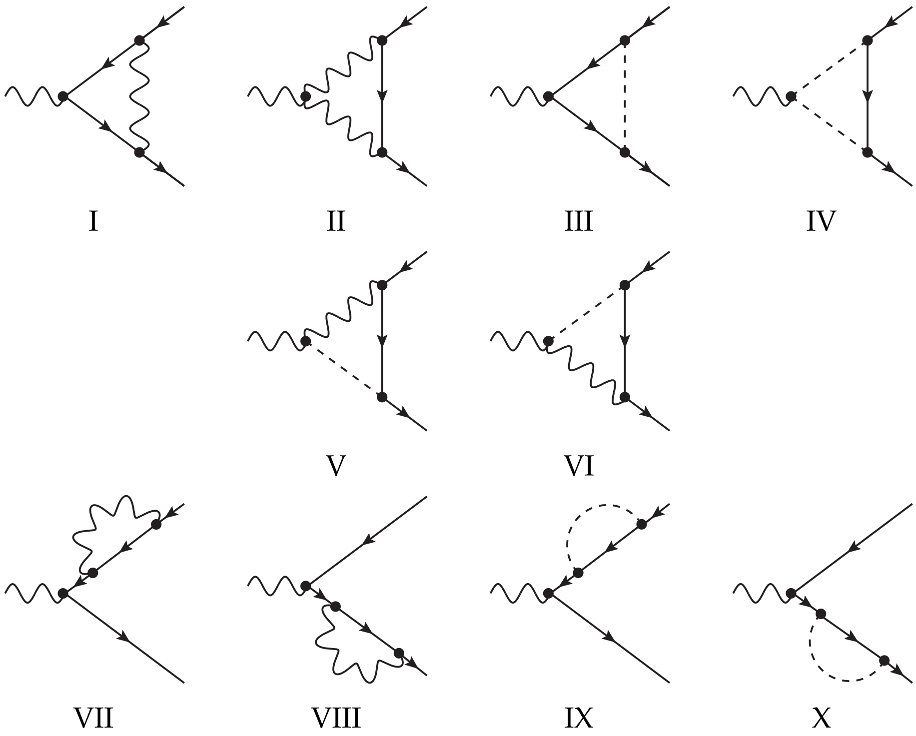


FIGURE 3.5: Topologies contributing to $\gamma \to \bar{\ell}\ell'$.

with $\alpha = e^2/4\pi$, and the form factor

$$F_M^\gamma = \theta^*_{j\ell'}\theta_{j\ell}\frac{\alpha_W}{16\pi}\frac{m_\ell}{M_W^2}F_M^{\chi,\nu}\left(\frac{M_W^2}{m_{\chi_j}^2}\right), \tag{3.180}$$

where $\alpha_W = \alpha/s_W^2$ and

$$F^{\chi,\nu}(x) = -\frac{2+5x-x^2}{4(1-x)^3} - \frac{3x}{2(1-x)^4}\log x \xrightarrow{x\to 0} -\frac{1}{2}, \tag{3.181}$$

with $x = M_W^2/m_{\chi_j}^2$. The form factor F_E^γ verifies $F_E^\gamma = iF_M^\gamma$. The corresponding branching ratio reads

$$\text{Br}\left(\ell \to \ell'\gamma\right) = \frac{3\alpha}{2\pi}\left|\theta^*_{j\ell'}\theta_{j\ell}F_M^{\chi,\nu}\left(\frac{M_W^2}{m_{\chi_j}^2}\right)\right|^2. \tag{3.182}$$

Then, to estimate the bounds on the mixing, one can substitute $F_M^{\chi,\nu}$ by its limit $-1/2$ for $x \to 0$ which implies $m_{\chi_j}^2 \gg M_W^2$ in eq. (3.181). This results in the bounds for $|\theta^*_{j\ell'}\theta_{j\ell}|$ in table 3.18.

The mixing matrix $\theta \simeq v/(2f)m_{\ell_H}m_\chi^{-1}$ between the light and heavy neutrinos delimits the $m_\chi - m_{\ell_H}$ region allowed by the bound $\theta < 0.03$. This region is also slightly restricted by the non observation of heavy lepton production since no bound can be set searching for the direct production of ℓ_H [131]. In fig. 3.6 we draw these regions for $f = 2.0$ and 2.5 TeV for $\kappa_l = 0.2$. In both cases, quasi-Dirac neutrino masses below 500 GeV are excluded. We must emphasize that the ℓ_H production limit depends on f dramatically because pair production of new vector-like leptons decaying into a SM lepton and the lightest T-odd gauge boson, A_H (missing energy), at the LHC is very much suppressed for $f > 2$ TeV [131], then drastically relaxing the lower

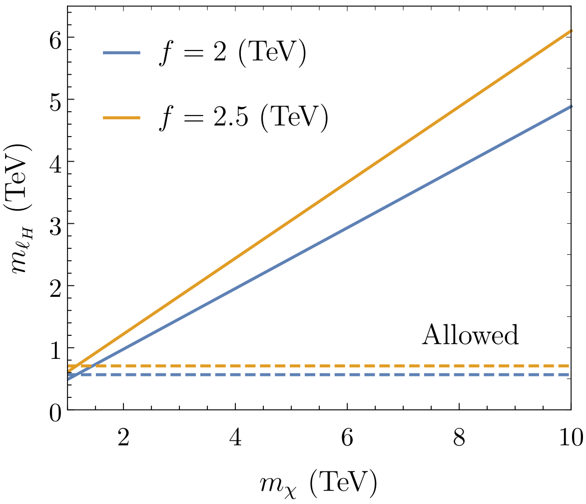


FIGURE 3.6: Allowed mass region for the mirror leptons mass m_{ℓ_H} versus the quasi-Dirac neutrino mass m_χ for different values of the NP scale f. Solid lines are fixed by the upper bound of 0.03 on the mixing between the SM and heavy neutrinos for $f = 2$ TeV and 2.5 TeV. The dashed lines correspond to a fixed value of $\kappa_l = 0.2$ according to [131].

bound on m_χ. The limit from neutrino mixing will improve with a more precise determination of the constraints from EWPD while the improvement of the bound on lepton-pair production will mainly require a higher colliding energy. Both will cut down the allowed mass region in the LHT as a function of the NP scale f, mainly fixed by the non-observation of the heavy quarks (with masses above 2 TeV [131, 132]) and the T-odd gauge bosons (with $M_{Z_H} \gtrsim 1.25$ TeV and thus $f \gtrsim 1.5$ TeV), mostly independent of the heavy quarks Yukawa coupling κ_q. The chosen value of κ_l in [131] will be compatible with the T-odd A_H gauge boson as dark matter candidate in the context of a new and gauge invariant LHT (see chapter 4).

On the other hand, although it is flavour conserving ($\ell' = \ell$), we can also compute at this stage the contribution of the singlet to the muon magnetic moment $a_\mu = 2m_\mu F_M^\gamma$ (the contribution of the the full set of T-odd leptons can be found in [64]), whose current experimental value is $a_\mu^{\text{exp}} = (116592061 \pm 41) \times 10^{-11}$ [110]. Assuming universality, $\delta_\mu^{\text{T-even}} = -1.1 \times 10^{-9} \theta_{j\mu}^* \theta_{j\mu}$ and then equal to -9.9×10^{-13} for $\theta_{j\mu}^* \theta_{j\mu} = 0.03^2$. This result is too small and negative to explain a significant departure from the SM prediction $a_\mu^{\text{SM}} = (116591810 \pm 59) \times 10^{-11}$ [110].

The contribution of the heavy quasi-Dirac (χ_+) to the EDM$_e$ vanishes at one loop. This is because F_M^γ for $\ell' = \ell$ is real. The loop functions are real since the masses of the particles running in the loop are not close to any physical threshold and the mixing matrix elements as well as their complex conjugates enter symmetrically. Thus due to the relation $F_E^\gamma = iF_M^\gamma$, F_E^γ is purely imaginary at one loop.

T-odd heavy singlet

In this part we will briefly consider a mechanism for neutrino masses with the T-odd realization for the singlet (χ_-). We already proved that this case presents pathologies in LFV Higgs decays, giving an UV divergent contribution, and neutrino masses are not an exception as we will show below. However it presents the advantage of being one loop suppressed. Since it is an SU(2) singlet, it is natural to include a small Majorana mass for the left-handed component of the singlet

$$\mathcal{L}_\mu = -\frac{\mu}{2} \overline{(\chi_-)_L^c} (\chi_-)_L + \text{h.c.} \tag{3.183}$$

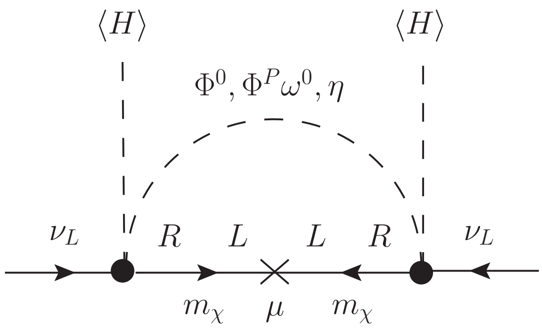


FIGURE 3.7: Feynman diagrams that lead to neutrino mass generation in the radiative inverse seesaw scenario with a T-odd singlet (χ_{-}).

as in the T-even case for the inverse seesaw. However, in this case we have to use the couplings derived from the Yukawa Lagrangian in eq. (3.40). Since the singlet is T-odd, it does not have renormalizable couplings with the Higgs and the SM neutrino of the form $h\overline{\nu}_L\left(\chi_{-}\right)_R$. However we can find couplings with the neutral components of the T-odd triplets ω, Φ and with the scalar singlet η coming from dimension 5 operators,

$$\begin{aligned}\mathcal{L}_\kappa \supset\, & iZ_{ik}\frac{m_{\ell_{Hk}}}{f}V_{kj}\frac{1}{8f}h\omega^0\overline{(\chi_{-})}_{Ri}\nu_{Lj} + Z_{ik}\frac{m_{\ell_{Hk}}}{f}V_{kj}\frac{1}{4\sqrt{2}f}h\phi^0\overline{(\chi_{-})}_{Ri}\nu_{Lj} \\ & + iZ_{ik}\frac{m_{\ell_{Hk}}}{f}V_{kj}\frac{1}{4\sqrt{2}f}h\omega^0\overline{(\chi_{-})}_{Ri}\nu_{Lj} + iZ_{ik}\frac{m_{\ell_{Hk}}}{f}V_{kj}\frac{3}{8\sqrt{5}f}h\eta\overline{(\chi_{-})}_{Ri}\nu_{Lj} + \text{h.c.}. \end{aligned} \quad (3.184)$$

Thus to obtain neutrino masses the quasi-Dirac (χ_{-}) as well as the neutral components of the triplets and the scalar singlet must run in the diagrams at one loop if T-parity is preserved. This is the so called radiative inverse seesaw mechanism [133]. On the other hand, if T-parity gets broken in the scalar sector by a tiny *vev* along the direction of the CP-even neutral components of the triplets and the singlet, we would generate a tree-level operator for neutrino masses with χ_{-} running in the diagrams. However, as in the T-even case, we will not consider such a T-parity violating scenario.

Notice that except for the coupling with the scalar singlet η these couplings are the analogous to those in table 3.15 that enter in Topology XI+XII and give the UV divergent contribution to LFV Higgs decays but with the neutral SM leptons. To obtain the neutrino mass matrix one has to evaluate the one-loop diagrams in fig. 3.7 when the Higgs is replaced by its *vev*.

In this case evaluating the diagrams we obtain the mass matrix for the SM neutrinos

$$\begin{aligned}\left(\mathcal{M}_\nu^{\text{odd}}\right)_{ij} = \frac{1}{16\pi^2}\left(Z\frac{m_{\ell_H}}{f}V\right)^T_{ik} m_{\chi_k}\mu_{kl}m_{\chi_l}\left(Z\frac{m_{\ell_H}}{f}V\right)_{lj}\frac{v^2}{f^2}\times \\ \left(\frac{1}{64}F_{kl}\left(M_{W_H}\right) + \frac{9}{320}F_{kl}\left(M_{A_H}\right) + \frac{1}{16}F_{kl}\left(M_\Phi\right)\right), \end{aligned} \quad (3.185)$$

where the one-loop function $F_{kl}\left(m_s\right)$ is defined by

$$F_{kl}\left(m_s\right) = \frac{m_{\chi_k}^2\log\left(\frac{m_{\chi_l}^2}{m_{\chi_k}^2}\right)}{\left(m_{\chi_k}^2 - m_{\chi_l}^2\right)\left(m_{\chi_k}^2 - m_s^2\right)} + \frac{m_s^2\log\left(\frac{m_{\chi_l}^2}{m_s^2}\right)}{\left(m_{\chi_k}^2 - m_s^2\right)\left(m_{\chi_l}^2 - m_s^2\right)} \quad (3.186)$$

and after some algebra it is straightforward to show that it is symmetric under $k \leftrightarrow l$. However, the neutrino masses do not go to zero in the limit $m_\chi \to \infty$. Assuming degenerate masses m_χ

for the different flavours of χ_- in the one-loop function in eq. (3.186) one gets

$$\lim_{m_{\chi_k}^2 \to m_\chi} F_{kl}\left(m_s\right) = \frac{1}{m_\chi^2 - m_s^2} + \frac{m_s^2 \log\left(\frac{m_\chi^2}{m_s^2}\right)}{\left(m_\chi^2 - m_s^2\right)^2}, \tag{3.187}$$

and finally taking the limit $m_\chi \to \infty$ in the neutrino mass matrix $\mathcal{M}_\nu^{\text{odd}}$ with $m_{\chi_k} = m_\chi$ yields

$$\lim_{m_\chi \to \infty} \left(\mathcal{M}_\nu^{\text{odd}}\right)_{ij} = \frac{1}{16\pi^2} \left(Z \frac{m_{\ell_H}}{f} V\right)^T_{ik} \mu_{kl} \left(Z \frac{m_{\ell_H}}{f} V\right)_{lj} \frac{17}{160} \frac{v^2}{f^2}. \tag{3.188}$$

Hence the neutrino masses in the LHT with a T-odd realization for the lepton singlet with the radiative inverse seesaw mechanism go like $1/f^2$ instead of $1/m_\chi^2$ and the χ_- fields do not decouple as we already knew from LFV Higgs decays. This can be expected a priori. By power counting, the vertices between the SM neutrino, the T-odd singlet χ_- and the T-odd scalar fields, once the Higgs is replaced by its *vev* are of order $\sim v/f$, and thus the neutrino mass is proportional to $\mu v^2/f^2$ times a one loop function that must be dimensionless since the prefactor has already dimensions of mass. Inside the loop-function, the numerators of the fermion propagators give a m_χ^2. Since m_χ is the highest scale in the integral, one can take the limit of vanishing scalar masses and thus the integral of the rest of the loop-function must give $\sim 1/m_\chi^2$. Therefore the result goes like $\sim 1/(16\pi^2) \times m_\chi^2/m_\chi^2 \sim 1/(16\pi^2)$, independent of the singlet mass.

One last comment regarding the T-parity violating scenario. If the T-odd scalar fields are replaced by a tiny *vev* , v', then the vertices between the Higgs, the singlet and the SM neutrino are order $\sim \kappa v v'/f$, and thus the neutrino mass would be proportional to $\mu\kappa^2 v^2 v'^2/f^2$ times the contribution coming from the intermediate singlet propagators that go like $\sim 1/m_\chi^2$ and the neutrino masses of size $\mu\kappa^2(v/f)^2(v'/m_\chi)^2$ that is suppressed by v^2/f^2 as the T-parity preserving case and by v'^2/m_χ^2.

Finally, as in the T-even case, the neutrino mass matrix is not flavour diagonal in the basis where the charged leptons are diagonal. Hence introducing the PMNS rotation matrix,

$$\mathcal{M}_\nu^{\text{odd}} = U_{\text{PMNS}}^* \mathcal{D}_\nu^{\text{odd}} U_{\text{PMNS}}^\dagger, \tag{3.189}$$

one can adjust μ to fit light neutrino masses and mixings. Considering only one family and taking $f \approx 1$ TeV and $\kappa = 1$, we find that $\mu \approx 1.2$ keV. That is a factor of 3 greater than in the T-even case with the same set of data. One can notice that the one loop suppression is not enough to increase the μ parameter in a significant amount if the result does not depend explicitly on the mass of the singlet.

3.3. Non gauge invariance of the LHT

This section is devoted to one of the main results of this Thesis. Motivated by the unexpected pathologies we found in the T-odd realization of the fermion singlet (χ_-) in LFV Higgs decays and neutrino masses due to interactions of the right-handed singlet through the Yukawa Lagrangian in eq. (3.40), we will show that this sector does not preserve gauge invariance under $[\text{SU}(2) \times \text{U}(1)]^2$. The same happens for the quark sector in eqs. (3.41), (3.42) and so we will focus on the lepton sector. In particular, the Lagrangian is not gauge invariant under global transformations along the direction of the gauge generators and, as a consequence, gauge invariance is explicitly broken in the T-odd realization. On the other hand, we will also show that any SO(5) quintuplet should be completed and all its components must get the same mass. This also affects the T-even realization.

Let us start from eq. (3.9), that defines the non linear SO(5) transformation matrices in terms of a fixed SU(5) matrix V and the Goldstone bosons encoded in ξ. This equation is consequence of the particular embedding of SO(5) in SU(5) coming from the definition of the vacuum Σ_0 in eq. (3.1) and the action of the inner automorphism in the Lie algebra. Consider an arbitrary global SU(5) transformation in a neighbourhood of the identity and let us find the corresponding SO(5) transformation

$$V = e^{i\alpha^a X^a + i\beta^b T^b}, \quad U = e^{i\sigma^b T^b}, \tag{3.190}$$

where we used the exponential parametrization for the transformation matrices in a neighbourhood of the identity. U depends only on the unbroken SO(5) generators T^b whereas V depends in general on all the set of SU(5) generators. The matrix U depends on the Goldstone fields and the SU(5) matrix V through the parameters σ^b which are a function of Π, α^a and β^b. To derive the explicit form of these coefficients we take an infinitesimal transformation V and U in eq. (3.9). This means that we will keep just the first order in the transformation parameters α^a and β^b and to obtain

$$\begin{aligned}
&\left(\mathbb{1} + i\alpha^a X^a + i\beta^b T^b\right)\xi\left(1 - i\sigma^b T^b\right) = \left(\mathbb{1} + i\sigma^b T^b\right)\xi\Sigma_0\left(\mathbb{1} + i\alpha^a X^{aT} + i\beta^b T^{bT}\right)\Sigma_0 \\
&\Rightarrow \sigma^b\left\{T^b, \xi\right\} = \beta^b\left\{T^b, \xi\right\} + \alpha^a\left[X^a, \xi\right],
\end{aligned} \tag{3.191}$$

where $[A, B]$ and $\{A, B\}$ are the commutator and anti-commutator, respectively. In the last step we used the properties that unbroken and broken generators satisfy with respect to Σ_0 in eqs. (3.2) and (3.5) to eliminate it from the equation. On the other hand, motivated by the Taylor expansion on the NP scale f of the matrix ξ,

$$\xi = \sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{i\Pi}{f}\right)^n, \tag{3.192}$$

we parametrize the dependence on f (and consequently on the Goldstone fields) of the parameters σ^b,

$$\sigma^b = \sum_{n=0}^{\infty} \frac{\sigma_n^b}{f^n}. \tag{3.193}$$

Let us substitute order by order in $1/f$ in eq. (3.191). The zeroth order gives $\sigma_0^b = i\beta^b$, which is the linear part of the transformation. Particularizing to a transformation along the direction of the gauge generators, neglecting higher order corrections, the matrix U_g would depend only on the SM gauge generators and then the Yukawa Lagrangians in eqs. (3.39) and (3.40) would be both gauge invariant, with or without completing the SO(5) quintuplet Ψ_R with the singlet χ and the mirror-partner leptons. Besides, if for phenomenological interests one decided to include all the fields, each component could get unrelated masses. This is because due to the form of the SM generators, they do not mix the upper and lower components of the quintuplet. However, including the Π-dependent effects of the transformation, one must solve eq. (3.191) for higher orders in $1/f$,

$$\sum_{n,m=0}^{\infty} \sigma_m^b \frac{1}{f^{n+m}} \frac{i^n}{n!}\left\{T^b, \Pi^n\right\} = \beta^b \sum_{n=0}^{\infty} \frac{1}{f^n} \frac{i^n}{n!}\left\{T^b, \Pi^n\right\} + \alpha^a \sum_{n=0}^{\infty} \frac{1}{f^n} \frac{i^n}{n!}\left[X^a, \Pi^n\right]. \tag{3.194}$$

Renaming the indices of the term in the l.h.s one obtains

$$\sum_{n,m=0}^{\infty} \sigma_m^b \frac{1}{f^{n+m}} \frac{i^n}{n!} \left\{T^b, \Pi^n\right\} = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \sigma_m^b \frac{1}{f^n} \frac{i^{n-m}}{(n-m)!} \left\{T^b, \Pi^{n-m}\right\} \tag{3.195}$$

and then identifying term by term of the sum,

$$\sum_{m=0}^{n} \sigma_m^b \frac{i^{n-m}}{(n-m)!} \left\{T^b, \Pi^{n-m}\right\} = \beta^b \frac{i^n}{n!} \left\{T^b, \Pi^n\right\} + \alpha^a \frac{i^n}{n!} \left[X^a, \Pi^n\right], \quad n \geq 0. \tag{3.196}$$

From the previous expression and using the information we obtained from the zeroth-order $\sigma_0^b = \beta^b$ we get

$$\sum_{m=1}^{n} \sigma_m^b \frac{i^{n-m}}{(n-m)!} \left\{T^b, \Pi^{n-m}\right\} = \alpha^a \frac{i^n}{n!} \left[X^a, \Pi^n\right], \quad n \geq 1. \tag{3.197}$$

This expression reveals that the non linear part of the infinitesimal SO(5) transformation only has a dependence on the coefficients that parametrize transformations along the direction of the broken generators. From this we can obtain the coefficient σ_1^b using that the basis of generators we chose is orthogonal,

$$2\sigma_1^b = i\alpha^a \mathrm{tr}\left(\left[X^a, \Pi\right] T^b\right). \tag{3.198}$$

This trace is different from zero because the automorphism in the Lie algebra implies that the commutator between broken generators[11] implies $\left[X^a, X^b\right] \sim T^c$ (see eq. (3.6)). The fact that in this expression appears the commutator between the Goldstone matrix and the broken generators in short implies that $\sigma_1^b T^b$ is a linear combination of all SO(5) generators. Coming back to eq. (3.197), separating the term $m = n$ from the rest and taking traces on both sides of the equation, we obtain a recursive formula for the n-th coefficient for $n > 1$,

$$2\sigma_n^b T^b = -\sum_{m=1}^{n-1} \sigma_m^b \frac{i^{n-m}}{(n-m)!} \left\{T^b, \Pi^{n-m}\right\} + \alpha^a \frac{i^n}{n!} \left[X^a, \Pi^n\right]. \tag{3.199}$$

In general, the right-handed side will depend on all the SO(5) generators. Using our orthogonal basis, we can multiply both sides of the previous equation by T^c and take the trace to find

$$2\sigma_n^c = \mathrm{tr}\left[\left(-\sum_{m=1}^{n-1} \sigma_m^b \frac{i^{n-m}}{(n-m)!} \left\{T^b, \Pi^{n-m}\right\} + \alpha^a \frac{i^n}{n!} \left[X^a, \Pi^n\right]\right) T^c\right], \quad n > 1. \tag{3.200}$$

Just for completeness, we also derive σ_2^c that turns out to be zero,

$$\begin{aligned} 2\sigma_2^c =& \mathrm{tr}\left[\left(-i\sigma_1^b \left\{T^b, \Pi\right\} - \frac{1}{2}\alpha^a \left[X^a, \Pi^2\right]\right) T^c\right] \\ =& \frac{1}{2}\mathrm{tr}\left[\left(\alpha^a \mathrm{tr}\left(\left[X^a, \Pi\right] T^b\right) \left\{T^b, \Pi\right\} - \alpha^a \left[X^a, \Pi^2\right]\right) T^c\right] \\ =& \frac{1}{2}\mathrm{tr}\left[\left(\alpha^a \left\{\left[X^a, \Pi\right], \Pi\right\} - \alpha^a \left[X^a, \Pi^2\right]\right) T^c\right] \\ =& \frac{1}{2}\mathrm{tr}\left[\left(\alpha^a \left[X^a, \Pi^2\right] - \alpha^a \left[X^a, \Pi^2\right]\right) T^c\right] = 0, \end{aligned} \tag{3.201}$$

[11] Notice that the Goldstone bosons are the excitations along the direction of the broken generators.

where we have substituted σ_1^b calculated above, then we have replaced $A = \frac{1}{2}[X^a, \Pi] = \mathrm{tr}\left(AT^b\right)T^b$ in $\{A, \Pi\} = \mathrm{tr}\left(AT^b\right)\left\{T^b, \Pi\right\}$ and finally we have used $\{[X^a, \Pi], \Pi\} = [X^a, \Pi]\,\Pi + \Pi\,[X^a, \Pi] = \left[X^a, \Pi^2\right]$. Therefore, given an infinitesimal SU(5) transformation

$$V \approx \mathbb{1} + i\alpha^a X^a + i\beta^b T^b, \tag{3.202}$$

the corresponding infinitesimal SO(5) transformation reads

$$U \approx \mathbb{1} + i\beta^b T^b - \frac{1}{2f}\alpha^a\,[X^a, \Pi] + \mathcal{O}\left(\frac{\Pi^3}{f^3}\right), \tag{3.203}$$

which is the result of the particular embedding of SO(5) into SU(5) and the automorphism defined in the Lie algebra.

Now let us focus on the particular case of a global SU(5) transformation belonging to the gauged subgroup $[\mathrm{SU}(2)\times\mathrm{U}(1)]_1 \times [\mathrm{SU}(2)\times\mathrm{U}(1)]_2$ that is spontaneously broken to the SM subgroup $[\mathrm{SU}(2)\times\mathrm{U}(1)] \subset \mathrm{SO}(5)$. Let us denote this kind of transformations with the subindex g, V_g and U_g respectively. V_g is expanded by no more than the union of the set $\left\{X_g^a\right\} = \{Q_1^a - Q_2^a, Y_1 - Y_2\}$ of broken generators and the set $\left\{T_g^b\right\} = \{Q_1^a + Q_2^a, Y_1 + Y_2\}$ of unbroken ones. However, from eq. (3.203) it is clear that, due to the non linearity of the SO(5) transformation matrices U, restricting ourselves to transformations V_g along the directions of the gauge generators does not imply that the matrix U_g depends only on the diagonal gauge subgroup generators. The key point is the presence of the commutator $\left[X_g^a, \Pi\right]$ that cannot be expanded in general in terms of just the SM gauge generators since $\left[X_g^a, \Pi\right] \neq \mathrm{tr}\left(\left[X_g^a, \Pi\right] T_g^b\right) T_g^b$, rather requiring the full set of SO(5) generators. One of the consequences of this fact is that the Lagrangians $\mathcal{L}_{Y_H}^{(b)}$ and $\mathcal{L}_{Y_{qH}}^{(b)}$ in eqs. (3.40) and (3.42), whose second term (the T-parity transformed of the first one) depend explicitly on the center of the gauge group element Ω (hence commuting with gauge generators) is not invariant under a global transformation along the direction of the gauge generators and consequently is not gauge invariant. Taking for instance eq. (3.40):

$$-\kappa_l f\left(\overline{\Psi}_2\xi\Psi_R + \overline{\Psi}_1\Sigma_0\Omega\xi^\dagger\Omega\Psi_R\right) \xrightarrow{G_g} -\kappa_l f\left(\Psi_2\xi\Psi_R + \overline{\Psi}_1\Sigma_0\Omega\xi^\dagger U_g^\dagger\Omega U_g\Psi_R\right), \tag{3.204}$$

where U_g and $U_g^\dagger$ cannot be simplified because in general they do not commute with Ω.

This has two important implications. First, it is clear that this implementation of the discrete T-parity in the fermionic sector with a T-odd χ must be discarded because it is incompatible with gauge invariance. Thus one does not have to worry about the pathologies we found related to the UV divergent contributions of the singlet found in Higgs decays in § 3.2.1 and the non dependence of neutrino masses on the mass of the singlet in § 3.2.2 in the T-odd case because, in particular, the result is gauge dependent: all the contributions to these processes come from the Yukawa Lagrangian in eq. (3.40) and the operators generated are not invariant under global transformations along the direction of broken generators and thus under gauge transformations. On the other hand, regardless of the T-parity realization, it results apparent that the SO(5) multiplets must be complete (as for instance Ψ_R with the mirror fermions and the singlet) because a non-linear transformation U_g mixes all its components and not just those laying in the invariant subspaces under the linear part of (3.203). In particular, the incomplete SO(5) representations $\Psi_L^{(q)}$ and $\Psi_L^{\chi(q)}$ in eq. (3.61), introduced to give a vector-like mass to the mirror fermions and the singlets through eq. (3.62) by coupling them with their corresponding right-handed counterparts in $\Psi_R^{(q)}$, do not only break the global SO(5) but also the gauge invariance, as we have just shown.

In contrast to the case with a T-odd singlet (χ_{-}), the amplitudes for LFV Higgs decays are finite at one loop if the gauge singlet is chosen to be T-even as we showed in § 3.2.1, and the contribution decouples when its mass is taken infinitely large. The same happens to its contribution to neutrino masses. However, LFV Higgs decays exhibit a non-decoupling behaviour proportional to the logarithm of the mirror-partner masses and to the misalignment between the mass matrices of mirror and mirror-partner leptons in eq. (3.120). This new source of flavour violation can be interpreted as a vestige of the broken gauge invariance, that it is restored when partners, mirror-partner fermions and singlet share a complete multiplet and hence get their masses from the same coupling. Recall that, as already emphasized, the mirror-partner fermions could not be ignored to obtain a finite LFV Higgs decay amplitude, since they cancel the divergence of the mirror fermions. The contribution of χ_{+} is finite on its own as we showed in subsec 3.2.1.

At any rate, we need to provide a new mechanism to give masses at least of order f to the mirror-partner fermions and the χ, since they should be heavy enough to fulfill the EWPD constraints [103, 106]. A way to proceed that at the same time is compatible with the gauge symmetry and T-parity is the object of the next chapter where we will also study its phenomenological implications.

3.4. Chapter summary

In the first part of this chapter we have introduced the Littlest Higgs model with T-parity as a paradigmatic example of a Little Higgs model. The Higgs and new scalar degrees of freedom arise as the pseudo Nambu-Goldstone bosons of an approximate global symmetry explicitly broken by gauge and Yukawa interactions. The advantage of this model with respect to the model without T-parity is that the new particles must be pair-produced to preserve T-parity. This symmetry significantly relaxes direct and indirect constraints coming from EWPD. The full Lagrangian was derived and we discussed the two different implementations of the discrete T-parity symmetry in the fermionic sector affecting the gauge singlets χ.

Once the Lagrangian was built, we explored the phenomenology of the model. First we reviewed the calculation of the LFV Higgs decays including the set of mirror-partner leptons whose contribution together with the mirror leptons leads to a finite result. We noticed that the amplitude exhibits a logarithmic non decoupling behaviour in the mass of the mirror-partner leptons. Thus they cannot be ignored as in previous phenomenological studies. Later on, we explore the gauge singlet contribution to this process. It turns out that if it is chosen to be T-even the contributions to LFV Higgs decays are UV finite and decouple. On the other hand if it is chosen to be T-odd, the corresponding amplitude is UV divergent. The origin of this divergence are the couplings of the right-handed components of the singlet with the T-odd scalar triplets coming from the Yukawa Lagrangian $\mathcal{L}^{(b)}_{Y_H}$ in eq. (3.40). Within this minimal LHT setup we introduced it is not possible to cure the divergence since the quantum numbers of the multiplets in eq. (3.48) do not allow to introduce the left-handed χ_{-} and involve the SM leptons Yukawa Lagrangian in eq. (3.46) to provide the proper couplings in order to cancel the UV divergence.

On the other hand, it is natural to consider a small Majorana mass μ for the left-handed counterpart of the singlet and provide a mechanism for neutrino masses. In the T-even case, the SM neutrinos receive masses at tree-level through the inverse seesaw mechanism. The singlet is integrated out using the tree-level equations of motion generating neutrino masses of size $\sim \kappa^2 \mu v^2 / m_\chi^2$ that also decouples in the limit $m_\chi \to \infty$. On the contrary, taking a T-odd singlet implies that neutrino masses are generated at one loop due to T-parity. However the integration at one loop of the singlet results in a non decoupling contribution to neutrino masses of size $\sim 1/(16\pi^2)\kappa^2\mu v^2/f^2$ independent of the singlet mass. This is because the one-loop function is dimensionless and taking the limit of vanishing scalar masses, being m_χ the highest scale in the integral, the result is a constant independent of any mass. As for LFV Higgs decays, the involved couplings come from the Yukawa Lagrangian in eq. (3.40).

Finally, motivated by the anomalous behaviour of the T-odd singlet in LFV Higgs decays and neutrino masses, we showed that the Yukawa Lagrangian in eq. (3.40) responsible of providing masses to the mirror fermions in the T-odd case is not gauge invariant. The scalar fields are part of the non linear field ξ transforming with an SO(5) matrix. These transformations, even particularizing for a transformation in the gauge group depend in general on all SO(5) generators and not only on those associated to the SM subgroup. Since $\mathcal{L}^b_{Y_H}$ depends explicitly on the element of the center of the gauge group Ω, that commutes with the gauge generators but not with the full set of SO(5) or SU(5) generators, this Lagrangian is not gauge invariant. This forces us to rule out the T-odd realization of the singlet. On the other hand, the form of the SO(5) transformations has also a consequence in the matter content of the model transforming in a non linear SO(5) representation. Since under a gauge transformation there are no invariant subspaces in these non linear representations, the SO(5) multiplets must be completed and thus the mirror-partner leptons and the T-even singlet cannot be ignored from the spectrum. However, their mass terms are not allowed by gauge invariance since the components of a SO(5) multiplet cannot be separated to receive different masses. Consequently a new mechanism to provide them with a mass, at least of order f, is required since they must be heavy enough to fulfill the EWPD constraints.

Chapter 4

The New and gauge invariant Littlest Higgs model with T-parity

In the last part of chapter 3 we showed that the original Littlest Higgs model with T-parity with the minimal setup is not gauge invariant. The non linear SO(5) transformations depends on the full set of SO(5) generators even if we restrict to transformations along the directions of the gauge generators. As a consequence, the T-odd realization of T-parity in which the singlet in the center of the SO(5) multiplets is T-odd was discarded because the Lagrangian depends explicitly on the element of the gauge group Ω*. This has also consequences on the T-even realization: the SO(5) multiplets must be completed since SO(5) generators mix all their components and thus all of them must receive the same mass. Throughout this chapter we will build the Lagrangian of a full gauge invariant Littlest Higgs model with T-parity. This model will require an enlarged global group and a new set of scalar fields. In the fermion sector we will include the minimal set that provides gauge invariant masses to all everyone. Later on we will derive the Coleman-Weinberg potential using the Background Field Method, that will allow us to obtain the masses of all the scalar fields of our theory. Finally we will study part of the rich phenomenology of this model. We will show that our model has enough room to accommodate a viable candidate for dark matter and at the same time be compatible with current EWPD and cosmology. This chapter is based on original work published in refs. [68, 69].*

4.1. The New Littlest Higgs model with T-parity setup

In this section we construct in full detail the Lagrangian of the New Littlest Higgs model with T-parity (NLHT) with explicit compatibility between gauge invariance and T-parity in order to address the problems we found at the end of the previous chapter. The guiding line is the necessity of giving a vector-like mass term compatible with gauge invariance to the mirror-partner fermions and the T-even singlet (χ_+), without introducing their left-handed counterparts in additional SO(5) multiplets transforming like Ψ_R, which would then be incomplete and hence at odds with the gauge symmetry.

4.1.1. A minimal extension of the global symmetry

To introduce the minimal set of fermion fields we will assume that left-handed components of χ and the mirror-partner fermions transform only under an external $\mathrm{SU}(2) \times \mathrm{U}(1)$. Then they will not mix with the others, as it would happen if they belonged to the same SO(5) multiplets, since the SM gauge generators do not mix different subspaces. This requires the minimal enlargement of the original global symmetry group SU(5) to[1]

$$G = \mathrm{SU}(5) \times [\mathrm{SU}(2) \times \mathrm{U}(1)]''_1 \times [\mathrm{SU}(2) \times \mathrm{U}(1)]''_2 \,. \tag{4.1}$$

This larger global group gets broken spontaneously when *two* non-linear tensor fields, Σ and $\widehat{\Sigma}$, acquire a *vev* at an energy scale f,[2]

$$\mathrm{SU}(5) \times [\mathrm{SU}(2) \times \mathrm{U}(1)]''_1 \times [\mathrm{SU}(2) \times \mathrm{U}(1)]''_2 \xrightarrow{\Sigma_0, \widehat{\Sigma}_0} \mathrm{SO}(5) \times [\mathrm{SU}(2) \times \mathrm{U}(1)]'' \,, \tag{4.2}$$

where Σ_0 in eq. (3.1) breaks spontaneously SU(5) down to SO(5) as before, and $\widehat{\Sigma}_0 = \Sigma_0$ breaks the extra piece to its diagonal subgroup $[\mathrm{SU}(2) \times \mathrm{U}(1)]''$, leaving $14 + 4 = 18$ Goldstone bosons. This particular breaking direction allows to take advantage of all the properties we showed in § 3.1.2.

Throughout the rest of this Thesis, the notation we follow for the extra fields and their corresponding transformation properties under the global and gauge group consist of putting a hat over their symbols. In our construction the new tensor field $\widehat{\Sigma}$ transforms only under $[\mathrm{SU}(2) \times \mathrm{U}(1)]''_1 \times [\mathrm{SU}(2) \times \mathrm{U}(1)]''_2$ piece of the global group. To be consistent, for the scalar and fermionic sector we take the representation of the extra piece of the global group over a 5-dimensional complex space generated by the same set of matrices of eqs. (3.11) and (3.12). This in particular means that we will be able to keep the structure of multiplets we have been using in the fermion sector.

Regarding to the gauged subgroup, it is still $[\mathrm{SU}(2) \times \mathrm{U}(1)]_1 \times [\mathrm{SU}(2) \times \mathrm{U}(1)]_2$ but now it is contained in the product

$$\begin{aligned}[\mathrm{SU}(2) \times \mathrm{U}(1)]_1 \times [\mathrm{SU}(2) \times \mathrm{U}(1)]_2 \subset \Big([\mathrm{SU}(2) \times \mathrm{U}(1)]'_1 \times [\mathrm{SU}(2) \times \mathrm{U}(1)]'_2 \subset \mathrm{SU}(5)\Big) \times \\ [\mathrm{SU}(2) \times \mathrm{U}(1)]''_1 \times [\mathrm{SU}(2) \times \mathrm{U}(1)]''_2 \,.\end{aligned} \tag{4.3}$$

This in particular allows us to preserve the number of gauge bosons. Likewise, the diagonal SM gauge group is

$$[\mathrm{SU}(2) \times \mathrm{U}(1)] \subset \Big([\mathrm{SU}(2) \times \mathrm{U}(1)]' \subset \mathrm{SO}(5)\Big) \times [\mathrm{SU}(2) \times \mathrm{U}(1)]'' \,. \tag{4.4}$$

In this way, we can introduce fermions whose corresponding non linear transformations involve only the SM gauge generators and thus alleviating the aforementioned difficulties. The Lagrangian for the gauge fields and their self-interactions is as shown before in eq. (3.20).

[1] We omit the extra $\mathrm{U}(1)'''_j$, $j = 1, 2$ factors required to assign the proper hypercharges to the right-handed SM leptons and all quarks. We will assume that this factor is always implicit. Notice also that the double prime has been replaced by a triple prime. The double prime is now reserved for the extra piece $\big([\mathrm{SU}(2) \times \mathrm{U}(1)]''\big)^2$ of the global group.

[2] One could take different SSB scales in the different sectors. However, for simplicity we will assume that the SSB of both pieces of the global group occurs at the same energy scale f.

4.1.2. The additional Goldstone fields

According to the SSB pattern defined above, the additional Goldstone matrix is expanded by the set of broken generators of the extra group, namely $\{Q_1^a - Q_2^a, Y_1 - Y_2\}$. It reads

$$\widehat{\Pi} = \begin{pmatrix} -\frac{\widehat{\omega}^0}{2} - \frac{\widehat{\eta}}{\sqrt{20}} & -\frac{\widehat{\omega}^+}{\sqrt{2}} & 0 & 0 & 0 \\ -\frac{\widehat{\omega}^-}{\sqrt{2}} & \frac{\widehat{\omega}^0}{2} - \frac{\widehat{\eta}}{\sqrt{20}} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\frac{4}{5}}\widehat{\eta} & 0 & 0 \\ 0 & 0 & 0 & -\frac{\widehat{\omega}^0}{2} - \frac{\widehat{\eta}}{\sqrt{20}} & -\frac{\widehat{\omega}^-}{\sqrt{2}} \\ 0 & 0 & 0 & -\frac{\widehat{\omega}^+}{\sqrt{2}} & \frac{\widehat{\omega}^0}{2} - \frac{\widehat{\eta}}{\sqrt{20}} \end{pmatrix}. \tag{4.5}$$

These Goldstone fields are charged only under the extra piece of the global group $[\mathrm{SU}(2) \times \mathrm{U}(1)]_1'' \times [\mathrm{SU}(2) \times \mathrm{U}(1)]_2''$. Under the SM gauge group, it decomposes in

$$\widehat{\Pi} : 1_0 \oplus 3_0, \tag{4.6}$$

including a new SU(2) triplet

$$\widehat{\omega} = \begin{pmatrix} -\frac{\widehat{\omega}^0}{2} & -\frac{\widehat{\omega}^+}{\sqrt{2}} \\ -\frac{\widehat{\omega}^-}{\sqrt{2}} & \frac{\widehat{\omega}^0}{2} \end{pmatrix} \tag{4.7}$$

and a singlet, $\widehat{\eta}$. One can notice that these scalar have the same quantum numbers as the corresponding would-be Goldstone bosons of the original LHT. They will actually mix with them and we will have to redefine the scalar fields to define the actual would-be Goldstone bosons of the NLHT. From the Goldstone matrix $\widehat{\Pi}$ and following the CCWZ formalism we define the non-linear sigma field

$$\widehat{\xi} = e^{i\widehat{\Pi}/f} \xrightarrow{G} \widehat{V}\widehat{\xi}\widehat{U}^\dagger = \widehat{U}\widehat{\xi}\Sigma_0\widehat{V}^T\Sigma_0, \tag{4.8}$$

where $\widehat{V}$, $\widehat{U}$ are transformations of the extra piece of the global group. We have also used eq. 3.5 to write the right-handed side of the last equality. It is important to notice that in the particular case of a gauge transformation V_g and $\widehat{V}_g$ coincide. However, from eqs. (3.9) and (4.8) it turns out that U_g and $\widehat{U}_g$ are different as we wish, since the former depends on V_g and Π, involving all SO(5) generators according to eq. (3.203), while the latter depends on V_g and $\widehat{\Pi}$, requiring just the SM generators. This can be derived straightforwardly from eqs. (3.203) and (4.8) since U and $\widehat{U}$ verify similar equations at the infinitesimal level with the substitutions mentioned above.

We also introduce the tensor field transforming linearly under the extra piece of the global group,

$$\widehat{\Sigma} = \widehat{\xi}\Sigma_0\widehat{\xi}^T = \widehat{\xi}^2\Sigma_0, \quad \widehat{\Sigma} \xrightarrow{G} \widehat{V}\widehat{\Sigma}\widehat{V}^T. \tag{4.9}$$

We assign a T-odd parity to all the new scalar fields defining

$$\widehat{\Pi} \xrightarrow{T} -\widehat{\Pi} \tag{4.10}$$

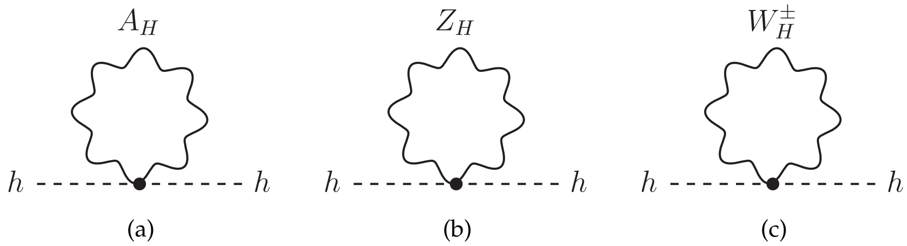


FIGURE 4.1: One-loop Feynman diagrams that would lead to unacceptable quadratic divergences in the Higgs mass if the mixed Lagrangian of eq. (4.13) did not vanish.

and therefore

$$\widehat{\xi} \xrightarrow{T} \widehat{\xi}^\dagger, \quad \widehat{\Sigma} \xrightarrow{T} \Sigma_0 \widehat{\Sigma}^\dagger \Sigma_0. \tag{4.11}$$

One can build a T-parity and gauge invariant Lagrangian for the kinetic terms and self-interactions of the new scalars similarly to $\mathcal{L}_S$ in eq. (3.26) according to the transformation properties of the tensor field $\widehat{\Sigma}$ under the gauge group taking $\widehat{V} = V_g$ in eq. (4.9),

$$\mathcal{L}_{\widehat{S}} = \frac{f^2}{8} \mathrm{tr} \left[\left(D^\mu \widehat{\Sigma} \right)^\dagger D_\mu \widehat{\Sigma} \right], \tag{4.12}$$

where the covariant derivative for $\widehat{\Sigma}$ is defined similarly to that in eq. (3.27),

$$D_\mu \widehat{\Sigma} = \partial_\mu \widehat{\Sigma} - \sqrt{2} i \sum_{j=1}^{2} \left[g W_{j\mu}^a \left(Q_j^a \widehat{\Sigma} + \widehat{\Sigma} Q_j^{aT} \right) - g' B_{j\mu} \left(Y_j \widehat{\Sigma} + \widehat{\Sigma} Y_j^T \right) \right]. \tag{4.13}$$

At this point one could think on considering an additional term mixing both scalar sectors,

$$\mathcal{L}_{S\widehat{S}} = \alpha_{S\widehat{S}} f^2 \mathrm{tr} \left[\left(D^\mu \Sigma \right)^\dagger D_\mu \widehat{\Sigma} \right] + \text{h.c.}, \tag{4.14}$$

allowed by both gauge symmetry and T-parity and hence should be included in the Lagrangian. However this Lagrangian involves couplings of heavy gauge bosons to scalar fields that lead to unacceptable quadratically divergent contributions to the Higgs mass from the diagrams of fig. 4.1. Therefore, one must take $\alpha_{S\widehat{S}} = 0$.[3]

4.1.3. Extra fermions and their interactions

The original LHT allows for masses of SM and mirror fermions through Yukawa interactions but, in order to be consistent with gauge and T-parity invariance, the right-handed components of the mirror fermions must share a complete SO(5) quintuplet $\Psi_R^{(q)}$ with the T-odd mirror-partner fermions $(\widetilde{\ell}^c_-)_R$, $(\widetilde{q}^c_-)_R$ and the T-even singlet $(\chi_+^{(q)})_R$ according to eq. (3.204). To provide these fields with a heavy mass one needs to introduce their left-handed components as well, but they cannot live in SO(5) multiplets transforming non linearly like $\Psi_R^{(q)}$ (as the $\Psi_L^{(q)}$

[3]The operator mixing both scalar sectors in eq. (4.13) can be generated at one loop for instance integrating out gauge bosons. This might translate into quadratically divergent contributions to the Higgs mass at two loops. However, since we are interested in a one-loop predictive model with at most logarithmically divergent contributions to the Higgs, we will not pay attention to this issue.

and $\Psi_L^{\chi_{(q)}}$ of eq. (3.61) usually introduced) because then they would be incomplete and their vector-like mass terms would break gauge invariance.

We will show below that it is possible to provide mass terms to *all* fermions in a gauge and T-parity invariant fashion in the context of the extended global symmetry with the new SSB pattern described in the previous section. We will consider two different scenarios and, in order to explore their implications, we will focus on the leptonic sector since the quark sector will be a copy of the former. In a first proposal we include the left-handed $(\widetilde{\ell}^c_-)_L$ and $(\chi_+)_L$ in a quintuplet charged only under the external $[\mathrm{SU}(2)\times\mathrm{U}(1)]''$ subgroup of the enlarged global symmetry, but this minimal model will generate undesired quadratic contributions to the Higgs mass. Then, as a viable solution, we will be forced to further extend the fermion content with additional T-even mirror-partners $\widetilde{l}^c_+$ and a T-odd (χ_-) embedding their left and right-handed components in appropriate representations of SU(5) and $[\mathrm{SU}(2)\times\mathrm{U}(1)]''$, respectively.

A minimal setup that introduces quadratic Higgs mass corrections

Let us introduce the left-handed components of $\widetilde{l}^c$ and (χ_+) in such a way that they transform under the SM gauge group but do not mix under an SO(5) transformation. Then, taking a representation of the extra group that acts over the 5-dimensional space, we compose the following multiplet

$$\widehat{\Psi}_L = \begin{pmatrix} -i\sigma^2(\widetilde{l}^c_-)_L \\ i(\chi_+)_L \\ 0 \end{pmatrix}, \quad \widehat{\Psi}_L \xrightarrow{G_g} \widehat{U}_g\widehat{\Psi}_L, \tag{4.15}$$

emphasizing that it transforms non linearly, and not under SO(5) but just under the diagonal subgroup of the extra group. For simplicity, we have chosen that both fields lay in the same multiplet. However they could be split into two and our conclusions would not change since the gauge generators in eqs. (3.11) and (3.12) do not mix different subspaces. Under the discrete T-parity symmetry we define

$$\widehat{\Psi}_L \xrightarrow{T} \Omega\widehat{\Psi}_L, \tag{4.16}$$

in order to assign the proper parities. The right-handed fields form the SO(5) quintuplet Ψ_R of eq. (3.35) with

$$\Psi_R = \begin{pmatrix} -i\sigma^2(\widetilde{l}^c_-)_R \\ i(\chi_+)_R \\ -i\sigma^2 l_{HR} \end{pmatrix}, \quad \Psi_R \xrightarrow{G_g} U_g\Psi_R, \quad \Psi_R \xrightarrow{\mathrm{T}} \Omega\Psi_R. \tag{4.17}$$

We may now pair $\widehat{\Psi}_L$ and Ψ_R in the following Yukawa Lagrangian

$$\mathcal{L}^{\widehat{\xi}\xi} = -\kappa' f\overline{\widehat{\Psi}}_L\left(\widehat{\xi}^\dagger\xi + \widehat{\xi}\xi^\dagger\right)\Psi_R + \mathrm{h.c.} \tag{4.18}$$

to give $\widetilde{l}^c_-$ and χ_+ a mass of order $\kappa' f$. This is compatible with gauge invariance and T-parity since the combinations

$$\widehat{\xi}^\dagger\xi \xrightarrow{G_g} \widehat{U}_g\widehat{\xi}^\dagger\xi U_g^\dagger, \tag{4.19}$$

$$\widehat{\xi}^\dagger\xi \xrightarrow{\mathrm{T}} \Omega\widehat{\xi}\xi^\dagger\Omega, \tag{4.20}$$

transform properly under the gauge group and T-parity. In the last equation we have used that Ω commutes with $\widehat{\xi}$ because only gauge generators appear in the Taylor expansion of $\widehat{\xi}$.

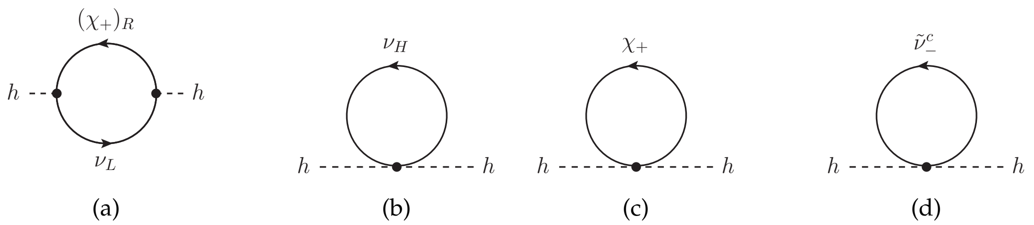


FIGURE 4.2: One-loop Feynman diagrams contributing to the quadratic divergences of the Higgs self-energy from the fermion sector in the LHT extended with $\widehat{\Psi}_L$ (4.15). Diagrams (a) and (b) from $\mathcal{L}_{Y_H}$ cancel each other, whereas (c) and (d) from $\mathcal{L}_Y^{\hat{\xi}\xi}$ (4.18) are proportional to κ'^2 and do *not* cancel.

Unfortunately, this minimal setup leads to unacceptable quadratic divergences to the Higgs boson mass from the diagrams in fig. 4.2. The quadratic divergences coming from diagrams (a) and (b), already in the original LHT, cancel each other (see § B.2). However diagrams (c) and (d), from the new interaction Lagrangian in eq. (4.18) *do not cancel* and add up to yield

$$\delta\mu^2 = \frac{3}{4\pi^2}\mathrm{tr}\left(\kappa'_l\kappa'^\dagger_l\right)\Lambda^2 \tag{4.21}$$

where μ^2 is the parameter of the quadratic term parameter in the Higgs potential.[4] Thus, in order to prevent such a quadratic divergence, an alternative mechanism to give masses to the singlet χ_+ and the mirror-partner leptons is required.

A viable model with consistent fermion representations

Instead of including the new left-handed fields in quintuplets transforming non linearly under the extra piece of the global group as above, we proceed to complete the SU(5) multiplets as follows

$$\Psi_1 = \begin{pmatrix} -i\sigma^2 l_{1L} \\ i\chi_{1L} \\ -i\sigma^2\widetilde{l}^c_{1L}\end{pmatrix}, \quad \Psi_2 = \begin{pmatrix} -i\sigma^2\widetilde{l}^c_{2L} \\ i\chi_{2L} \\ -i\sigma^2 l_{2L}\end{pmatrix}, \tag{4.22}$$

with the usual transformation properties under the gauge group

$$\Psi_1 \xrightarrow{G_g} V_g^*\Psi_1, \quad \Psi_2 \xrightarrow{G_g} V_g\Psi_2, \quad \Psi_1 \xrightarrow{\mathrm{T}} \Omega\Sigma_0\Psi_2, \tag{4.23}$$

so that $(\chi_+)_R$ and the $(\widetilde{l}^c_-)_R$ inside the SO(5) quintuplet Ψ_R of eq. (4.17) can couple to the combination with the right T-parity and quantum numbers and get a mass proportional to κf through the same $\mathcal{L}_{Y_H}$ in eq. (3.39) as the mirror leptons do.

But still the T-odd combination $(\chi_-)_L = (\chi_{1L} - \chi_{2L})/\sqrt{2}$ and the T-even combination of the mirror-partner $(\widetilde{l}^c_+)_L = (\widetilde{l}^c_{1L} - \widetilde{l}^c_{2L})$ remain massless.[5] At this stage, barring the explicit breaking of the global symmetry due to the gauge interactions of $[\mathrm{SU}(2)\times\mathrm{U}(1)]^2$, the theory would be SU(5) invariant because now the fermion multiplets are complete. This implies that the Higgs would be an exact Goldstone boson with no mass corrections. To give a mass to this additional

[4]The analogous to Lagrangian in eq. (4.13) for quarks would give the same contribution multiplied by the corresponding color factor.

[5]Notice that although the T-parities of these fields are different, the relative sign in their corresponding definitions is the same due to the presence of Ω that changes the sign of the field of the middle of the multiplets in eq. (3.33).

combination of fields we are forced to break the global group introducing their corresponding right-handed components in an incomplete multiplet that transforms under the extra piece of the global group (analogous to eq. (4.15) but for opposite chiralities and T-parities),

$$\widehat{\Psi}_R = \begin{pmatrix} -i\sigma^2(\widetilde{l}^c_+)_R \\ i(\chi_-)_R \\ 0 \end{pmatrix}, \quad \widehat{\Psi}_R \xrightarrow{G_g} \widehat{U}_g \widehat{\Psi}_R, \quad \widehat{\Psi}_R \xrightarrow{\mathrm{T}} -\Omega \widehat{\Psi}_R, \tag{4.24}$$

where we emphasize one more time that the new T-even partner lepton doublet and the new T-odd singlet do not mix under a gauge group transformation what allows them to be separated in different multiplets. Finally, we couple this multiplet to Ψ_1 and Ψ_2 through the non linear field $\widehat{\xi}$,

$$\mathcal{L}_{\widehat{Y}_H} = -\widehat{\kappa} f \left(\overline{\Psi}_2 \widehat{\xi} - \overline{\Psi}_1 \Sigma_0 \widehat{\xi}^\dagger \right) \widehat{\Psi}_R + \text{h.c.}, \tag{4.25}$$

using again the fact that Ω commutes with $\widehat{\xi}$. Thus $\widetilde{l}^c_+$ and χ_- get a mass of order $\widehat{\kappa} f$.

We can justify that this construction does not lead to unwanted quadratically divergent contributions to the Higgs mass, in contrast to $\mathcal{L}^{\widehat{\xi}\xi}$ in eq. (4.18). On the one hand, the new sector in eq. (4.25) does not have a direct coupling to the Higgs field. On the other hand, as emphasized above, the Yukawa Lagrangian in eq. (3.39) with complete SU(5) and SO(5) multiplets is fully SU(5) invariant. Thus both couplings κ and $\widehat{\kappa}$ are needed to generate a contribution to the Higgs boson mass just proportional to the logarithm of the cutoff. As one may notice, this is exactly the same philosophy used to build the top quark Lagrangian in eq. (3.53): the *collective symmetry breaking* mechanism. In order to explicitly check this, one has to evaluate the one loop-diagrams showed in fig. 4.3 that add up to yield

$$\delta\mu^2 = \frac{3f^2}{4\pi^2} \mathrm{tr}\left(\kappa_l \kappa_l^\dagger \widehat{\kappa}_l \widehat{\kappa}_l^\dagger \right) \log \Lambda^2. \tag{4.26}$$

This result obtained diagrammatically will be reproduced in section 4.2 from the calculation of the Coleman-Weinberg potential using the background field method. The SM right-handed leptons receive their masses from the same Yukawa Lagrangian as in eq. 3.46.

There remains the introduction of kinetic terms and gauge interactions for all fermion fields in this viable version of the model. The new left-handed fields $\widetilde{l}^c_{rL}$ and χ_{rL} that make up the left-handed components of $\widetilde{l}^c_\pm$ and $\chi_\pm$ belong to the SU(5) quintuplets Ψ_r ($r = 1, 2$) so they will get their kinetic terms and gauge interactions from the same Lagrangian $\mathcal{L}_{F_L}$ of eq. (3.64). The right-handed components of $\widetilde{l}^c_-$ and χ_+ where already in the SO(5) quintuplet Ψ_R with kinetic terms and interactions from $\mathcal{L}_{F_R}$ in eq. (3.65). For the right-handed fields $(\widetilde{l}^c_+)_R$ and $(\chi_-)_R$ in the new quintuplet $\widehat{\Psi}_R$, following the CCWZ formalism, we introduce

$$\mathcal{L}_{\widehat{F}_R} \supset i\overline{\widehat{\Psi}}_R \gamma^\mu \left[\partial_\mu + \frac{1}{2} \widehat{\xi}^\dagger \left(D_\mu \widehat{\xi} \right) + \frac{1}{2} \widehat{\xi} \Sigma_0 D^*_\mu \left(\Sigma_0 \widehat{\xi}^\dagger \right) \right] \widehat{\Psi}_R, \tag{4.27}$$

with the covariant derivative as in eq. (3.67). Notice that under a T-parity transformation we have

$$\overline{\widehat{\Psi}}_R \hat{\xi}^\dagger D_\mu \hat{\xi} \widehat{\Psi}_R \xrightarrow{\mathrm{T}} \overline{\widehat{\Psi}}_R \hat{\xi} \Sigma_0 D^*_\mu \left(\Sigma_0 \hat{\xi}^\dagger \right) \widehat{\Psi}_R, \tag{4.28}$$

since again Ω commutes with $\widehat{\xi}$ and the gauge generators, where we used the property in eq. (3.13). The SM right-handed leptons receive their hypercharge from the extra U(1) factors as in the original LHT model and their corresponding gauge interactions come from $\mathcal{L}_{F'}$ in

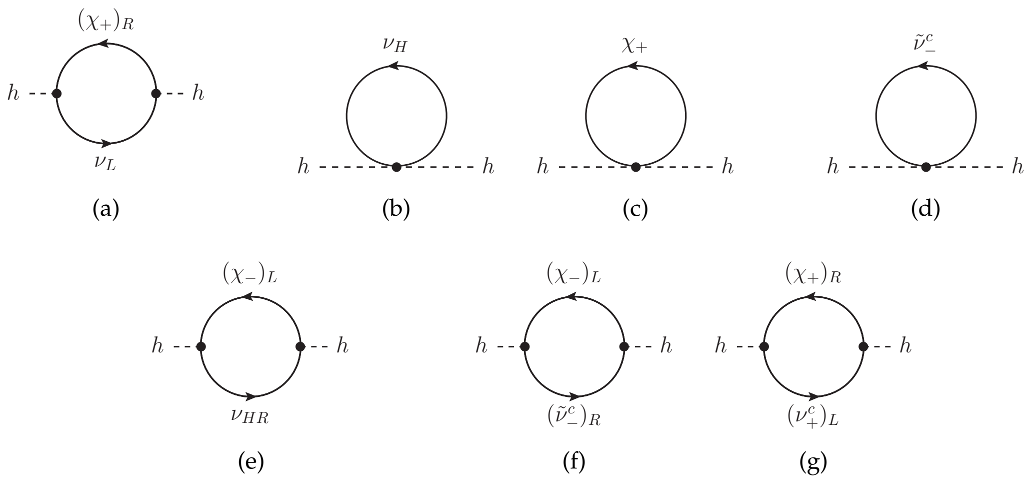


FIGURE 4.3: One-loop Feynman diagrams contributing to the quadratic divergences of the Higgs self-energy from the fermion sector in the NLHT. Diagrams (a) and (b) arise from $\mathcal{L}_{Y_H}$ and cancel each other, as in fig. 4.2. The rest stem from $\mathcal{L}_{\widehat{Y}_H}$ in eq. (4.25) and also cancel among themselves.

eq. (3.70).

Now let us briefly comment on the quark sector. The SU(5) multiplets in the Yukawa Lagrangian in eq. (3.41) need also to be completed by

$$\Psi_1^q = \begin{pmatrix} -i\sigma^2 q_{1L} \\ i\chi_{1L}^q \\ -i\sigma^2 \widetilde{q}_{1L}^c \end{pmatrix}, \quad \Psi_2^q = \begin{pmatrix} -i\sigma^2 \widetilde{q}_{2L}^c \\ i\chi_{2L}^q \\ -i\sigma^2 q_{2L} \end{pmatrix}, \tag{4.29}$$

with the same transformation properties under the gauge group and T-parity as those for leptons in eq. (4.23). They couple to the fields in the SO(5) and $[\mathrm{SU}(2)\times\mathrm{U}(1)]''$ quintuplets

$$\Psi_R^q = \begin{pmatrix} -i\sigma^2 (\widetilde{q}_-^c)_R \\ i\,(\chi_+^q)_R \\ -i\sigma^2 q_{HR} \end{pmatrix}, \quad \widehat{\Psi}_R^q = \begin{pmatrix} -i\sigma^2 (\widetilde{q}_+^c)_R \\ i\,(\chi_-^q)_R \\ 0_2 \end{pmatrix} \tag{4.30}$$

with transformation properties given in eqs. (4.17) and (4.24), through the Yukawa Lagrangian in eq. (3.41) and the new Lagrangian

$$\mathcal{L}_{\widehat{Y}_H}^q = -\widehat{\kappa}_q f \left(\overline{\Psi}_2^q \widehat{\xi} - \overline{\Psi}_1^q \Sigma_0 \widehat{\xi}^\dagger\right) \widehat{\Psi}_R^q + \text{h.c.}, \tag{4.31}$$

respectively.

Except for the top quark, the rest of SM quarks, receive their masses from the Yukawa Lagrangians in eqs. (3.47) and (3.51). For the top quark sector, we introduce a different Yukawa Lagrangian respecting the collective symmetry breaking mechanism,

$$\begin{aligned} \mathcal{L}_Y^t = &-i\frac{\lambda_1 f}{4}\epsilon_{ijk}\epsilon_{xy}\left[\left(\overline{Q}_1^t\right)_i \Sigma_{jx}\Sigma_{ky} + \left(\overline{Q}_2^t \Sigma_0 \Omega\right)_i \widetilde{\Sigma}_{jx}\widetilde{\Sigma}_{ky}\right] t_R \\ &- \frac{\lambda_2 f}{\sqrt{2}}\left(\overline{T}_{1L}\widehat{X}T_{1R} + \overline{T}_{2L}\widehat{X}^* T_{2R}\right) + \text{h.c.}, \end{aligned} \tag{4.32}$$

where $\{i,j,k\} = 1,2,3$ and $\{x,y\} = 4,5$, t_R is a T-even right-handed singlet and $(T_{1,2})_R$ are also SU(2) right-handed singlets. The multiplets that appear in the previous Lagrangian are those

	$[\mathrm{SU}(2)' \times \mathrm{U}(1)']^2 \subset \mathrm{SU}(5)$	$[\mathrm{U}(1)''']^2$	$[\mathrm{SU}(2) \times \mathrm{U}(1)]^2_{\mathrm{gauge}}$
ℓ_R	$(0,0)_{(0,0)}$	$(-\frac{1}{2},-\frac{1}{2})$	$(0,0)_{(-\frac{1}{2},-\frac{1}{2})}$
d_R	$(0,0)_{(0,0)}$	$(-\frac{1}{6},-\frac{1}{6})$	$(0,0)_{(-\frac{1}{6},-\frac{1}{6})}$
u_R, t_R, T_{1R}, T_{2R}	$(0,0)_{(0,0)}$	$(\frac{1}{3},\frac{1}{3})$	$(0,0)_{(\frac{1}{3},\frac{1}{3})}$
l_{2L}	$(1,2)_{(-\frac{1}{5},-\frac{3}{10})}$	$(0,0)$	$(1,2)_{(-\frac{1}{5},-\frac{3}{10})}$
l_{1L}	$(2,1)_{(-\frac{3}{10},-\frac{1}{5})}$	$(0,0)$	$(2,1)_{(-\frac{3}{10},-\frac{1}{5})}$
$q_{2L}, \mathcal{T}_{2L}$	$(1,2)_{(-\frac{1}{5},-\frac{3}{10})}$	$(\frac{1}{3},\frac{1}{3})$	$(1,2)_{(\frac{2}{15},\frac{1}{30})}$
$q_{1L}, \mathcal{T}_{1L}$	$(2,1)_{(-\frac{3}{10},-\frac{1}{5})}$	$(\frac{1}{3},\frac{1}{3})$	$(2,1)_{(\frac{1}{30},\frac{2}{15})}$
T_{2L}	$(1,1)_{(-\frac{1}{5},\frac{1}{5})}$	$(\frac{1}{3},\frac{1}{3})$	$(1,1)_{(\frac{2}{15},\frac{8}{15})}$
T_{1L}	$(1,1)_{(\frac{1}{5},-\frac{1}{5})}$	$(\frac{1}{3},\frac{1}{3})$	$(1,1)_{(\frac{8}{15},\frac{2}{15})}$
χ_{2L}	$(1,1)_{(-\frac{1}{5},\frac{1}{5})}$	$(0,0)$	$(1,1)_{(-\frac{1}{5},\frac{1}{5})}$
χ_{1L}	$(1,1)_{(\frac{1}{5},-\frac{1}{5})}$	$(0,0)$	$(1,1)_{(\frac{1}{5},-\frac{1}{5})}$
$\widetilde{l}^c_{2L}$	$(2,1)_{(\frac{3}{10},\frac{1}{5})}$	$(0,0)$	$(2,1)_{(\frac{3}{10},\frac{1}{5})}$
$\widetilde{l}^c_{1L}$	$(1,2)_{(\frac{1}{5},\frac{3}{10})}$	$(0,0)$	$(1,2)_{(\frac{1}{5},\frac{3}{10})}$
χ^q_{2L}	$(1,1)_{(-\frac{1}{5},\frac{1}{5})}$	$(\frac{1}{3},\frac{1}{3})$	$(1,1)_{(\frac{2}{15},\frac{8}{15})}$
χ^q_{1L}	$(1,1)_{(\frac{1}{5},-\frac{1}{5})}$	$(\frac{1}{3},\frac{1}{3})$	$(1,1)_{(\frac{8}{15},\frac{2}{15})}$
$\widetilde{q}^c_{2L}$	$(2,1)_{(\frac{3}{10},\frac{1}{5})}$	$(\frac{1}{3},\frac{1}{3})$	$(2,1)_{(\frac{19}{30},\frac{8}{15})}$
$\widetilde{q}^c_{1L}$	$(1,2)_{(\frac{1}{5},\frac{3}{10})}$	$(\frac{1}{3},\frac{1}{3})$	$(1,2)_{(\frac{8}{15},\frac{19}{30})}$

TABLE 4.1: Quantum numbers for fermions transforming in a linear representation. These fields are singlets under $[\mathrm{SU}(2)'' \times \mathrm{U}(1)'']^2$.

defined in eq. (3.52) and fullfill the transformation properties in eqs. (3.56) and (3.57). The auxiliary field $\widehat{X} = \widehat{\Sigma}_{33}^{-1/2} = e^{-i\sqrt{4/5}\hat{\eta}}$ and its complex conjugate with hypercharges under the gauge group $(Y_1, Y_2) = \left(\frac{1}{5}, -\frac{1}{5}\right)$ and $\left(-\frac{1}{5}, \frac{1}{5}\right)$ respectively, are introduced to change the gauge hypercharges of the right-handed top quark-partners $(T_{1,2})_R$ to those of the right-handed t_R, namely $(Y_1, Y_2) = \left(\frac{1}{3}, \frac{1}{3}\right)$. This new hypercharge assignment seems more natural since all fermions in a linear representation of the global group that require extra hypercharge receive half of it in each of the external $\mathrm{U}(1)_j'''$ factors ($j = 1, 2$). The quarks transforming in a non-linear representation receive the extra hypercharge needed in the diagonal $\mathrm{U}(1)'''$. This is because the hypercharge under the gauged part of the global $\mathrm{SU}(5) \times \left([\mathrm{SU}(2) \times \mathrm{U}(1)]''\right)^2$ is fixed by the form of the hypercharge generators in eqs. (3.11) and (3.12). Thus the quantum number assignments for all the fermions of the NLHT model are depicted in tables 4.1 and 4.2. The fields belonging to the complete SO(5) representation are written in their corresponding multiplet to emphasize that they mix under a gauge transformation (3.203).

According to table 4.2, the left-handed quarks and the right-handed quarks transforming under SU(5) and SO(5) receive their kinetics term and gauge interactions through eqs. (3.64) and (3.65), whereas the right-handed quarks transforming in a non linear representation of the

	$SU(2)' \times U(1)' \subset SO(5)$	$SU(2)'' \times U(1)''$	$[U(1)''']$	$[SU(2) \times U(1)]_{\text{gauge}}$
$\begin{pmatrix} -i\sigma^2(\widetilde{l}^c_-)_R \\ i(\chi_+)_R \\ -i\sigma^2 l_{HR} \end{pmatrix}$	$\begin{pmatrix} 2_{\frac{1}{2}} \\ 1_0 \\ 2_{-\frac{1}{2}} \end{pmatrix}$	1_0	0	$\begin{pmatrix} 2_{\frac{1}{2}} \\ 1_0 \\ 2_{-\frac{1}{2}} \end{pmatrix}$
$\begin{pmatrix} -i\sigma^2(\widetilde{q}^c_-)_R \\ i(\chi^q_+)_R \\ -i\sigma^2 q_{HR} \end{pmatrix}$	$\begin{pmatrix} 2_{\frac{1}{2}} \\ 1_0 \\ 2_{-\frac{1}{2}} \end{pmatrix}$	1_0	$\frac{2}{3}$	$\begin{pmatrix} 2_{\frac{7}{6}} \\ 1_{\frac{2}{3}} \\ 2_{\frac{1}{6}} \end{pmatrix}$
$(\widetilde{l}^c_+)_R$	1_0	$2_{\frac{1}{2}}$	0	$2_{\frac{1}{2}}$
$(\chi_-)_R$	1_0	1_0	0	1_0
$(\widetilde{q}^c_+)_R$	1_0	$2_{\frac{1}{2}}$	$\frac{2}{3}$	$2_{\frac{7}{6}}$
$(\chi^q_-)_R$	1_0	1_0	$\frac{2}{3}$	$1_{\frac{2}{3}}$

TABLE 4.2: Quantum numbers for right-handed fermions transforming in a non linear representation.

extra piece of the global group interact through

$$\mathcal{L}_{\widehat{F}_R} \supset i\overline{\widehat{\Psi^q}}_R\gamma^\mu \left[\partial_\mu + \frac{1}{2}\widehat{\xi}^\dagger \left(D^q_\mu \widehat{\xi}\right) + \frac{1}{2}\widehat{\xi}\Sigma_0 D^{q*}_\mu \left(\Sigma_0 \widehat{\xi}^\dagger\right)\right] \widehat{\Psi^q}_R \tag{4.33}$$

according to the CCWZ formalism, with D^q_μ defined in eq. (3.68). Finally the right-handed SM leptons, down-type quarks and up-type quarks of the first and second generations receive their kinetic term and gauge interactions from eq. (3.70) whereas the top quark-partners kinetic terms are given in the NLHT by

$$\begin{aligned}\mathcal{L}_{F'_{T_1,T_2}} = & i\overline{T}_{1L}\left[\partial_\mu - \sqrt{2}ig'\left(\frac{8}{15}B_{1\mu} + \frac{2}{15}B_{2\mu}\right)\right]T_{1L} + i\overline{T}_{2L}\left[\partial_\mu - \sqrt{2}ig'\left(\frac{2}{15}B_{1\mu} + \frac{8}{15}B_{2\mu}\right)\right]T_{2L} \\ & + i\overline{T}_{1R}\left[\partial_\mu - \sqrt{2}ig'\left(\frac{1}{3}B_{1\mu} + \frac{1}{3}B_{2\mu}\right)\right]T_{1R} + i\overline{T}_{2R}\left[\partial_\mu - \sqrt{2}ig'\left(\frac{1}{3}B_{1\mu} + \frac{1}{3}B_{2\mu}\right)\right]T_{2R}.\end{aligned} \tag{4.34}$$

Now we proceed to get the physical fields.

4.1.4. Physical fields

Gauge bosons

After the electroweak SSB, the SM gauge bosons are obtained from the T-even fields of eq. (3.72) by diagonalizing the Lagrangian $\mathcal{L}_S$ of eq. (3.26). In order to obtain the mass eigenstates of the T-odd gauge bosons one has to expand both $\mathcal{L}_S$ (3.26) and $\mathcal{L}_{\widehat{S}}$ (4.12) up to order v^2/f^2 to get

$$W^\pm_H = \frac{1}{\sqrt{2}}\left(W^1_H \mp iW^2_H\right), \tag{4.35}$$

$$\begin{pmatrix} Z_H \\ A_H \end{pmatrix} = \begin{pmatrix} 1 & -x_H\frac{v^2}{f^2} \\ x_H\frac{v^2}{f^2} & 1 \end{pmatrix}\begin{pmatrix} W^3_H \\ B_H \end{pmatrix} \tag{4.36}$$

with

$$W_H^a = \frac{W_1^a - W_2^a}{\sqrt{2}}, \quad B_H = \frac{B_1 - B_2}{\sqrt{2}}, \quad x_H = \frac{5gg'}{8\,(5g^2 - g'^2)}. \tag{4.37}$$

Notice that the mixing angle x_H between the neutral T-odd gauge bosons in the NLHT and LHT in eq. (3.75) are different. This is because x_H accounts for the mixing v^2/f^2 and in the NLHT only $\mathcal{L}_S$ contains the Higgs boson whereas the $\mathcal{L}_{\widehat{S}}$ sector is absent in the LHT.

The corresponding masses to order v^2/f^2 are given by

$$M_W = \frac{gv}{2}\left(1 - \frac{v^2}{12f^2}\right), \quad M_Z = M_W/c_W, \tag{4.38}$$

$$M_{W_H} = M_{Z_H} = \sqrt{2}gf\left(1 - \frac{v^2}{16f^2}\right), \tag{4.39}$$

$$M_{A_H} = \sqrt{\frac{2}{5}}g'f\left(1 - \frac{5v^2}{16f^2}\right). \tag{4.40}$$

Notice also that even though the gauge boson are the same as in the original model (the gauge group is still $[SU(2)\times U(1)]^2$), the masses of the T-odd combinations are at leading order a factor of $\sqrt{2}$ heavier than in (3.75). The reason is that the new extra scalar sector parametrized by $\widehat{\Sigma}$ also takes the *vev* Σ_0 hence giving an additional contribution to the heavy T-odd gauge boson masses. On the other hand, at leading order the T-even gauge bosons receive a mass proportional to the Higgs *vev* and this only belongs to Σ, so their masses do not change.

Scalars after gauge fixing

The successive spontaneous breaking of the gauge symmetry leads to a kinetic mixing between gauge bosons and the would-be Goldstone bosons fields. In the mass eigenbasis, these unwanted mixing terms can be removed, up to an irrelevant total derivative, by introducing the appropriate gauge-fixing Lagrangian,

$$\begin{aligned}\mathcal{L}_{\rm gf} = &-\frac{1}{2\xi_\gamma}(\partial_\mu A^\mu)^2 - \frac{1}{2\xi_Z}(\partial_\mu Z^\mu - \xi_Z M_Z \pi^0)^2 - \frac{1}{\xi_W}|\partial_\mu W^\mu + i\xi_W M_W \pi^-|^2 \\ &- \frac{1}{2\xi_{A_H}}(\partial_\mu A_H^\mu + \xi_{A_H} M_{A_H}\eta)^2 - \frac{1}{2\xi_{Z_H}}(\partial_\mu Z_H^\mu - \xi_{Z_H} M_{Z_H}\omega^0)^2 - \frac{1}{\xi_{W_H}}|\partial_\mu W_H^\mu + i\xi_{W_H} M_{W_H}\omega^-|^2,\end{aligned} \tag{4.41}$$

defining the unphysical Goldstone fields that can be absorbed from the spectrum in the unitary gauge.

After the SSB, the kinetic terms of the scalar fields we have introduced are neither diagonal nor canonically normalized. Besides, the new set of T-odd scalars belonging to $\widehat{\Sigma}$ resulting from the SSB of the extra piece of the global group mix with the old ones that have the same quantum numbers. In order to define the physical combination of scalar fields and identify the actual would-be Goldstone fields at least at order v^2/f^2, we perform the following field redefinitions in two steps. First we perform a rotation of 45° in the subspace of every scalar pair with the same quantum numbers, so that only one of them (the unhatted) will retain the kinetic mixing with a gauge boson, hence becoming the actual would-be Goldstone field at leading order,

$$\omega^\pm \to \frac{1}{\sqrt{2}}\left(\omega^\pm - \widehat{\omega}^\pm\right), \quad \widehat{\omega}^\pm \to \frac{1}{\sqrt{2}}\left(\omega^\pm + \widehat{\omega}^\pm\right), \tag{4.42}$$

$$\omega^0 \to \frac{1}{\sqrt{2}}\left(\omega^0 - \widehat{\omega}^0\right), \quad \widehat{\omega}^0 \to \frac{1}{\sqrt{2}}\left(\omega^0 + \widehat{\omega}^0\right), \tag{4.43}$$

$$\eta \to \frac{1}{\sqrt{2}}\left(\eta - \widehat{\eta}\right), \quad \widehat{\eta} \to \frac{1}{\sqrt{2}}\left(\eta + \widehat{\eta}\right). \tag{4.44}$$

At this point all kinetic-mixing terms are of order v^2/f^2. In the next step we impose that all kinetic terms are canonically normalized and diagonal so that the actual would-be Goldstone fields can be removed by the gauge fixing (3.77) at order v^2/f^2. To that end we rescale and mix them as follows, according to the method described in [106] for the original LHT,

$$h \to h, \tag{4.45}$$

$$\pi^0 \to \pi^0 \left(1 + \frac{v^2}{12f^2}\right), \tag{4.46}$$

$$\pi^\pm \to \pi^\pm \left(1 + \frac{v^2}{12f^2}\right), \tag{4.47}$$

$$\Phi^0 \to \Phi^0 \left(1 + \frac{v^2}{12f^2}\right), \tag{4.48}$$

$$\Phi^P \to \Phi^P \left(1 + \frac{v^2}{12f^2}\right) + \left(-\omega^0 - \widehat{\omega}^0 + \sqrt{5}\left(\eta + \widehat{\eta}\right)\right)\frac{v^2}{12f^2}, \tag{4.49}$$

$$\Phi^\pm \to \Phi^\pm \left(1 + \frac{v^2}{24f^2}\right) \pm i\left(\omega^\pm + \widehat{\omega}^\pm\right)\frac{v^2}{12\sqrt{2}f^2}, \tag{4.50}$$

$$\Phi^{\pm\pm} \to \Phi^{\pm\pm}, \tag{4.51}$$

$$\eta \to \eta \left(1 + \frac{5v^2}{48f^2}\right) + \frac{-5g'\widehat{\eta} - \sqrt{5}\left[g'\left(\omega^0 - \widehat{\omega}^0 + 2\Phi^P\right) - 12 g x_H \omega^0\right]}{12g'}\frac{v^2}{f^2}, \tag{4.52}$$

$$\omega^0 \to \omega^0 \left(1 + \frac{v^2}{48f^2}\right) + \frac{5g\left(-\widehat{\omega}^0 + 2\Phi^P + \sqrt{5}\widehat{\eta}\right) - \sqrt{5}\left(5g + 12g' x_H\right)\eta}{60g}\frac{v^2}{f^2}, \tag{4.53}$$

$$\omega^\pm \to \omega^\pm \left(1 + \frac{v^2}{48f^2}\right) + \left(\pm i\sqrt{2}\Phi^\pm - \widehat{\omega}^\pm\right)\frac{v^2}{12f^2}, \tag{4.54}$$

$$\widehat{\eta} \to \widehat{\eta}\left(1 + \frac{5v^2}{48f^2}\right) + \left(5\eta - \sqrt{5}\omega^0\right)\frac{v^2}{24f^2}, \tag{4.55}$$

$$\widehat{\omega}^0 \to \widehat{\omega}^0 \left(1 + \frac{v^2}{48f^2}\right) + \left(\omega^0 - \sqrt{5}\widehat{\eta} - \sqrt{5}\eta\right)\frac{v^2}{24f^2}, \tag{4.56}$$

$$\widehat{\omega}^\pm \to \widehat{\omega}^\pm \left(1 + \frac{v^2}{48f^2}\right) + \omega^\pm \frac{v^2}{24f^2}. \tag{4.57}$$

After these redefinitions, the scalars η, ω^0 and $\omega^\pm$ are the would-be Goldstone bosons of the SSB of the gauge group $[\mathrm{SU}(2)\times\mathrm{U}(1)]^2$ to the SM gauge group, eaten by A_H, Z_H and $W_H^\pm$ respectively and thus becoming their longitudinal modes. Similarly, π^0 and $\pi^\pm$ are the corresponding would-be Goldstone bosons of the SSB of the SM gauge group down to $\mathrm{U}(1)_{\mathrm{em}}$ eaten by the Z and $W^\pm$. The remaining scalar fields are all physical. They are the Higgs boson, a T-odd complex triplet of hypercharge $Y = 1$, composed of $\Phi^{\pm\pm}$, $\Phi^\pm$, Φ^0 and Φ^P, and the four new T-odd scalars with the same quantum numbers as the would-be Goldstone fields of the original LHT, a singlet $\widehat{\eta}$ and a real triplet with $Y = 0$ composed of $\widehat{\omega}^0$ and $\widehat{\omega}^\pm$. All of them will get a mass by gauge and Yukawa interactions from the Coleman-Weinberg potential [104, 134]. The triplet Φ will receive a mass of order f from the quadratically divergent contributions to the potential and the rest of physical scalars from the logarithmically divergent contributions.

As a consequence, the masses of $\widehat{\eta}$, $\widehat{\omega}^0$ and $\widehat{\omega}^\pm$ are independent of f, but they could still be large thanks to the interplay of different Yukawa couplings (see § 4.2.2 and § 4.3).

Fermion masses and mixings in the NLHT

When the fermion content of the model is extended beyond one family, the different Yukawa couplings κ, $\widehat{\kappa}$ and λ for leptons and quarks in $\mathcal{L}_{Y_H}$, $\mathcal{L}_{\widehat{Y}_H}$ and $\mathcal{L}_Y$, respectively, must be promoted to 3×3 matrices in flavour space. Omitting flavour indices, for each of the three SM (T-even) left-handed fermion doublets (l_L, q_L) there is a vector-like doublet of heavy T-odd mirror fermions (l_H, q_H) as in the original LHT in eq. (3.79). In the NLHT there are also two heavy right-handed mirror-partner fermion doublets,

$$(\widetilde{l}^c_-)_R = \begin{pmatrix} (\widetilde{\nu}^c_-)_R \\ (\widetilde{\ell}^c_-)_R \end{pmatrix}, \quad (\widetilde{q}^c_-)_R = \begin{pmatrix} (\widetilde{u}^c_-)_R \\ (\widetilde{d}^c_-)_R \end{pmatrix}, \tag{4.58}$$

$$(\widetilde{l}^c_+)_R = \begin{pmatrix} (\widetilde{\nu}^c_+)_R \\ (\widetilde{\ell}^c_+)_R \end{pmatrix}, \quad (\widetilde{q}^c_+)_R = \begin{pmatrix} (\widetilde{u}^c_+)_R \\ (\widetilde{d}^c_+)_R \end{pmatrix}. \tag{4.59}$$

The first set is T-odd and necessary to complete the SO(5) quintuplets Ψ_R and Ψ^q_R together with $(\chi_+)_R$ and $\left(\chi^q_+\right)_R$ while the second is T-even and lives in the incomplete multiplets $\widehat{\Psi}_R$ and $\widehat{\Psi}^q_R$ in eqs. (4.24) and (4.30) along with $(\chi_-)_R$, charged under the external piece of the global group $\left([\mathrm{SU}(2)\times \mathrm{U}(1)]''\right)^2$. Their corresponding left-handed counterparts come from the SU(5) multiplets

$$(\widetilde{l}^c_-)_L = \begin{pmatrix} (\widetilde{\nu}^c_-)_L \\ (\widetilde{\ell}^c_-)_L \end{pmatrix} = \frac{(\widetilde{l}^c_1)_L + (\widetilde{l}^c_2)_L}{\sqrt{2}}, \quad (\widetilde{q}^c_-)_L = \begin{pmatrix} (\widetilde{u}^c_-)_L \\ (\widetilde{d}^c_-)_L \end{pmatrix} = \frac{(\widetilde{q}^c_1)_L + (\widetilde{q}^c_2)_L}{\sqrt{2}} \tag{4.60}$$

$$(\widetilde{l}^c_+)_L = \begin{pmatrix} (\widetilde{\nu}^c_+)_L \\ (\widetilde{\ell}^c_+)_L \end{pmatrix} = \frac{(\widetilde{l}^c_1)_L - (\widetilde{l}^c_2)_L}{\sqrt{2}}, \quad (\widetilde{q}^c_+)_L = \begin{pmatrix} (\widetilde{u}^c_+)_L \\ (\widetilde{d}^c_+)_L \end{pmatrix} = \frac{(\widetilde{q}^c_1)_L - (\widetilde{q}^c_2)_L}{\sqrt{2}}. \tag{4.61}$$

with

$$(\widetilde{l}^c_r)_L = \begin{pmatrix} (\widetilde{\nu}^c_r)_L \\ (\widetilde{\ell}^c_r)_L \end{pmatrix}, \quad (\widetilde{q}^c_r)_L = \begin{pmatrix} (\widetilde{u}^c_r)_L \\ (\widetilde{d}^c_r)_L \end{pmatrix}, \quad r = 1,2. \tag{4.62}$$

Finally, we have the aforementioned SU(2) singlets $(\chi_+)_R$, $\left(\chi^q_+\right)_R$ and $(\chi_-)_R$, $(\chi^q_-)_R$. Their left-handed counterparts are the combinations with proper T-parities of the fields $\chi^{(q)}_{1L}$ and $\chi^{(q)}_{2L}$ completing the SU(5) quintuplets,

$$(\chi^{(q)}_+)_L = \frac{\chi^{(q)}_{1L} + \chi^{(q)}_{2L}}{\sqrt{2}}, \quad (\chi^{(q)}_-)_L = \frac{\chi^{(q)}_{1L} - \chi^{(q)}_{2L}}{\sqrt{2}}. \tag{4.63}$$

The top quark sector includes a set of top partners T_{1L} and T_{2L}, belonging to Q^t_1 and Q^t_2, respectively and their right-handed counterparts T_{1R}, T_{2R}. The corresponding T-parity eigenstates are defined in eq. (3.84). To determine the mass eigenstates one has to diagonalize the top mass matrix in eq. (3.85) as in the original LHT in eq. (3.86).

Next we introduce flavour indices and derive the mass eigenstates. In the NLHT, since T-parity is also exact, the SM fermions do not mix with the heavy T-odd combinations. They cannot mix with the T-even combination of mirror-partner fermions $\widetilde{l}^c_+$, $\widetilde{q}^c_+$ either, because they have an extra hypercharge $\Delta_Y = +1$ and a coupling through the Yukawa Lagrangian $\mathcal{L}_{\widehat{Y}_H}$ would require a T-even compensating scalar field of hypercharge $Y = 1$ acquiring a *vev*. Therefore the

SM charged lepton and down-type quark mass eingenstates are those obtained from the diagonalization of the matrices λ_l and λ_d in eqs. (3.46), (3.47) with the replacements in eqs. (3.90) and (3.91), that leads to the corresponding masses in eq. (3.93). On the other hand, the heavy charged lepton and down-type quark mass eigenstates are obtained by the replacements

$$\ell_{HL} \to V_L^{l_H} \ell_{HL}, \quad \ell_{HR} \to V_R^{l_H} \ell_{HR}, \tag{4.64}$$

$$d_{HL} \to V_L^{d_H} d_{HL}, \quad d_{HR} \to V_R^{d_H} d_{HR}, \tag{4.65}$$

$$(\widetilde{\ell}^c_-)_L \to V_L^{l_H} (\widetilde{\ell}^c_-)_L, \quad (\widetilde{\ell}^c_-)_R \to V_R^{H} (\widetilde{\ell}^c_-)_R, \tag{4.66}$$

$$(\widetilde{d}^c_-)_L \to V_L^{d_H} (\widetilde{d}^c_-)_L, \quad (\widetilde{d}^c_-)_R \to V_R^{d_H} (\widetilde{d}^c_-)_R, \tag{4.67}$$

$$(\widetilde{\ell}^c_+)_L \to V_L^{\widetilde{\ell}^c_+} (\widetilde{\ell}^c_+)_L, \quad (\widetilde{\ell}^c_+)_R \to V_R^{\widetilde{\ell}^c_+} (\widetilde{\ell}^c_+)_R, \tag{4.68}$$

$$(\widetilde{d}^c_+)_L \to V_L^{\widetilde{q}^c_+} (\widetilde{d}^c_+)_L, \quad (\widetilde{d}^c_+)_R \to V_R^{\widetilde{q}^c_+} (\widetilde{d}^c_+)_R, \tag{4.69}$$

with

$$\sqrt{2}\kappa_l f = V_L^{l_H} m_{\ell_H} V_R^{l_H\dagger} = V_L^{l_H} m_{\widetilde{\ell}^c_-} V_R^{l_H\dagger} \tag{4.70}$$

$$\sqrt{2}\kappa_q f = V_L^{q_H} m_{d_H} V_R^{q_H\dagger} = V_L^{q_H} m_{\widetilde{d}^c_-} V_R^{q_H\dagger} \tag{4.71}$$

$$\sqrt{2}\widehat{\kappa}_l f = V_L^{\widetilde{\ell}^c_+} m_{\widetilde{l}^c_+} V_R^{\widetilde{\ell}^c_+\dagger} \tag{4.72}$$

$$\sqrt{2}\widehat{\kappa}_q f = V_L^{\widetilde{d}^c_+} m_{\widetilde{d}^c_+} V_R^{\widetilde{d}^c_+\dagger}. \tag{4.73}$$

Notice that in contrast to the LHT (see for instance [61]), in the NLHT the T-odd mirror fermions doublets l_H and q_H and their partners $\widetilde{l}^c_-$, $\widetilde{q}^c_-$ rotate with the same matrix as the T-even singlets χ_+ and χ^q_+, at leading order, getting a mass proportional to the Yukawa couplings κ_l and κ_q from $\mathcal{L}_{Y_H}$ and $\mathcal{L}^q_{Y_H}$. The new combinations with opposite T-parities, $\widetilde{l}^c_+$, $\widetilde{q}^c_+$ and χ_-, χ^q_-, get masses proportional to $\widehat{\kappa}_l$ and $\widehat{\kappa}_q$ from $\mathcal{L}_{\widehat{Y}_H}$ and $\mathcal{L}^q_{\widehat{Y}_H}$. Then for the neutral lepton sector and up-type quarks the fields have to be redefined at leading order as follows,

$$\nu_{HL} \to V_L^{l_H} \nu_{HL}, \quad \nu_{HR} \to V_R^{l_H} \nu_{HR}, \tag{4.74}$$

$$u_{HL} \to V_L^{q_H} u_{HL}, \quad u_{HR} \to V_R^{q_H} u_{HR}, \tag{4.75}$$

$$(\widetilde{\nu}^c_-)_L \to V_L^{l_H} (\widetilde{\nu}^c_-)_L, \quad (\widetilde{\nu}^c_-)_R \to V_R^{l_H} (\widetilde{\nu}^c_-)_R, \tag{4.76}$$

$$(\widetilde{u}^c_-)_L \to V_L^{q_H} (\widetilde{u}^c_-)_L, \quad (\widetilde{u}^c_-)_R \to V_R^{q_H} (\widetilde{u}^c_-)_R, \tag{4.77}$$

$$(\widetilde{\nu}^c_+)_L \to V_L^{\widetilde{\ell}^c_+} (\widetilde{\nu}^c_+)_L, \quad (\widetilde{\nu}^c_+)_R \to V_R^{\widetilde{\ell}^c_+} (\widetilde{\nu}^c_+)_R, \tag{4.78}$$

$$(\widetilde{u}^c_+)_L \to V_L^{\widetilde{d}^c_+} (\widetilde{u}^c_+)_L, \quad (\widetilde{u}^c_+)_R \to V_R^{\widetilde{d}^c_+} (\widetilde{u}^c_+)_R, \tag{4.79}$$

$$(\chi_+)_L \to V_L^{l_H} (\chi_+)_L, \quad (\chi_+)_R \to V_R^{l_H} (\chi_+)_R, \tag{4.80}$$

$$(\chi^q_+)_L \to V_L^{q_H} (\chi^q_+)_L, \quad (\chi^q_+)_R \to V_R^{q_H} (\chi^q_+)_R, \tag{4.81}$$

$$(\chi_-)_L \to V_L^{\widetilde{\ell}^c_+} (\chi_-)_L, \quad (\chi_-)_R \to V_R^{\widetilde{\ell}^c_+} (\chi_-)_R, \tag{4.82}$$

$$(\chi^q_-)_L \to V_L^{\widetilde{d}^c_+} (\chi^q_-)_L, \quad (\chi^q_-)_R \to V_R^{\widetilde{d}^c_+} (\chi^q_-)_R, \tag{4.83}$$

whereas we will assume that the Yukawa coupling λ_u in eq. (3.51) is diagonal in flavour space as in the LHT and thus $V_L^u = V_R^u = \mathbb{1}$. The corresponding mass matrices are diagonalized at

TABLE 4.3: Order of the mixing between neutral fields and up-type quarks in the NLHT. A dot means that they are connected by the mass term and a dash indicates that no mixing term is generated to order v^2.

	$(\chi_+)_L, (\chi_+^q)_L$	$(\chi_-)_L, (\chi_-^q)_L$	ν_L, u_L	ν_{HL}, u_{HL}	$(\widetilde{\nu}_+^c)_L, (\widetilde{u}_+^c)_L$	$(\widetilde{\nu}_-^c)_L, (\widetilde{u}_-^c)_L$
$(\chi_+)_R, (\chi_+^q)_R$	•	–	v	–	v	–
$(\chi_-)_R, (\chi_-^q)_R$	–	•	–	–	–	–
ν_R, u_R	–	–	•	–	–	–
ν_{HR}, u_{HR}	–	v	–	•	–	v^2
$(\widetilde{\nu}_+^c)_R, (\widetilde{u}_+^c)_R$	–	–	–	–	•	–
$(\widetilde{\nu}_-^c)_R, (\widetilde{u}_-^c)_R$	–	v	–	v^2	–	•

leading order by

$$\sqrt{2}\kappa_l f\left(1-\frac{v^2}{8f^2}\right) = V_L^{l_H} m_{\nu_H} V_R^{l_H\dagger} = V_L^{l_H} m_{\widetilde{\nu}_-^c} V_R^{l_H\dagger}, \tag{4.84}$$

$$\sqrt{2}\kappa_q f\left(1-\frac{v^2}{4f^2}\right) = V_L^{q_H} m_{u_H} V_R^{q_H\dagger} = V_L^{q_H} m_{\widetilde{u}_-^c} V_R^{q_H\dagger}, \tag{4.85}$$

$$\sqrt{2}\kappa_l f = V_L^{l_H} m_{\chi_+} V_R^{l_H\dagger}, \tag{4.86}$$

$$\sqrt{2}\kappa_q f = V_L^{q_H} m_{\chi_+^q} V_R^{q_H\dagger}, \tag{4.87}$$

$$\sqrt{2}\widehat{\kappa}_l f = V_L^{\widetilde{\ell}_+^c} m_{\chi_-} V_R^{\widetilde{\ell}_+^c\dagger} = V_L^{\widetilde{\ell}_+^c} m_{\widetilde{\nu}_+^c} V_R^{\widetilde{\ell}_+^c\dagger}, \tag{4.88}$$

$$\sqrt{2}\widehat{\kappa}_q f = V_L^{\widetilde{d}_+^c} m_{\chi_-^q} V_R^{\widetilde{d}_+^c\dagger} = V_L^{\widetilde{d}_+^c} m_{\widetilde{u}_+^c} V_R^{\widetilde{d}_+^c\dagger}. \tag{4.89}$$

To find the mass eigenstates of the neutral leptons and up-type quarks one also has to take into account that they mix when the Lagrangian is expanded up to order v^2/f^2 (see table 4.3) as in the LHT but now with the extra fields participating in the mixing. The mixing order v/f is the most pressing to include, since it corrects the masses at order v^2/f^2. The mixing of order v^2/f^2 only plays a role in the diagonalization matrix entering in the masses at order v^4/f^4. The mixing in the top quark sector is the same as in the LHT in table 3.4.

The misalignment between the sectors κ_l, $\widehat{\kappa}_l$ and λ_l in the leptonic sector and κ_q, $\widehat{\kappa}_q$, λ_d and λ_u in the leptonic sector is a source of flavour mixing. The matrices parametrizing this misalignment can be defined as follows:

$$V \equiv V_L^{l_H\dagger} V^{\ell_L}, \quad \widehat{W} \equiv V_L^{\widetilde{\ell}_+^c\dagger} V_L^{l_H} \tag{4.90}$$

in the leptonic sector and

$$V^u \equiv V_L^{q_H\dagger}, \quad V^d \equiv V_L^{q_H\dagger} V_L^d, \quad \widehat{W}^q \equiv V_L^{\widetilde{d}_+^c\dagger} V_L^{q_H}, \quad V^{\text{CKM}} = V_L^d, \tag{4.91}$$

in the quark sector, where in the first and second definitions we have assumed that λ_u is diagonal and thus $V_L^u = V_R^u = \mathbb{1}$. One could argue that additional mixing matrices $V_L^{\widetilde{\ell}_+^c} V_L^\ell$ for leptons and $V_L^{\widetilde{d}_+^c} V_L^u$, $V_L^{\widetilde{d}_+^c} V_L^d$ for quarks are needed, but this is not the case because there is no gauge or Yukawa coupling between the SM doublets l_L, q_L and the new fields. On the other hand, the flavour matrices of the LHT $W \equiv V_L^{\widetilde{\ell}_-^c} V_L^{l_H}$, $W^q \equiv V_L^{\widetilde{d}_-^c} V_L^{d_H}$ and $Z \equiv V_R^{\chi\dagger} V_R^{l_H}$, $Z^q \equiv V_R^{\chi^q\dagger} V_R^{q_H}$

introduced in eqs. (3.115) and (3.116) are both the identity in our model, since now the T-odd combination of mirror-partner fermions and the T-even combination of SU(2) singlets χ_+, χ_+^q receive their masses from the same Yukawa Lagrangian $\mathcal{L}_{Y_H}$, $\mathcal{L}_{Y_H}^q$.

This completes the derivation of the full Lagrangian. Now we proceed to a brief discussion about the contribution of the new lepton fields to LFV Higgs decays.

4.1.5. New lepton contributions to LFV Higgs decays

In the original LHT one could define two different implementations of T-parity on the fermion fields, namely T-even and T-odd χ. In § 3.3 we have shown that the T-odd case is incompatible with gauge invariance and leads to an infinite contribution to LFV Higgs decays at order v^2/f^2 (see § 3.2.1). However we have also shown in § 3.2.1 that the T-even option gives a finite result since the contribution of the singlet is finite by itself.

As already emphasized, gauge invariance requires the singlet in the SO(5) quintuplet to be T-even as it is the case in the NLHT. Our $(\chi_+)_R$ has the same couplings to the SM leptons as the T-even singlet of the original LHT, so its contribution to LFV Higgs decays is finite as well. Moreover, to provide gauge invariant mass terms to all fermions in our model, we had enlarge the fermion field content. Firstly we have completed the left-handed SU(5) quintuplets in eq. (4.22). Their combination with well defined T-parity now includes two new singlets $(\chi_\pm)_L$ and two new doublets of mirror-partner leptons $(\widetilde{l}^c_\pm)_L$ apart from the usual doublets l_L and l_{HL} of SM and mirror leptons, respectively. And secondly we have introduced the right-handed quintuplet $\widehat{\Psi}_R$ in eq. (4.24) including a T-even doublet of mirror-partner leptons $(\widehat{l}^c_+)_R$ and a T-odd singlet $(\chi_-)_R$. One may argue that these new fields could reintroduce unwanted divergences in LFV Higgs decays, since in particular there is a new T-odd $(\chi)_-$. To show that this is actually not the case, we will analyze below their couplings to the SM charged leptons. They are needed to compute the divergences of the different classes of one loop diagrams in fig. 3.1. The relevant vertices come from the Lagrangians $\mathcal{L}_{F_L}$, $\mathcal{L}_{Y_H}$, $\mathcal{L}_{\widehat{Y}_H}$ and $\mathcal{L}_Y$ in eqs. (3.64), (3.39), (4.25) and (3.46), respectively.

The couplings between gauge bosons and left-handed fermions in $\mathcal{L}_{F_L}$ involve the gauge generators in eqs. (3.11) and (3.12), that cannot connect the upper and lower components of the quintuplets and thus forbidding any coupling between the SM charged leptons and the new left-handed lepton fields. Concerning the Yukawa Lagrangian $\mathcal{L}_{Y_H}$, the new left-handed fields share the SU(5) quintuplets in eqs. (4.22) with l_{1L} and l_{2L} preventing any coupling to the SM charged leptons. On the other hand, the new Yukawa Lagrangian $\mathcal{L}_{\widehat{Y}_H}$ couples l_{1L} and l_{2L} to the multiplet $\widehat{\Psi}_R$ in eq. (4.24) through the non linear sigma field $\widehat{\xi}$. Since it is built as the exponential of the new Goldstone bosons multiplying the broken combination of gauge generators, the same argument applied in $\mathcal{L}_{F_L}$ is also valid for this sector. In particular, the T-odd right-handed singlet only couples to its T-even and T-odd left-handed counterparts through the new Goldstone fields, hence avoiding also any coupling to the Higgs. Therefore, its interactions are completely different to those of the original model with the T-odd singlet option. Finally, the Yukawa Lagrangian $\mathcal{L}_Y$ only couples the the right-handed SM charged leptons ℓ_R to l_{1L} and l_{2L} because the multiplets Ψ_1^X and $\Psi_2^{X^*}$ in eq. (3.48) are incomplete and as we discussed at the end of § 3.2.1, their quantum numbers under the gauge group do not allow to introduce the left-handed χ_{1L} and χ_{2L} in the center of these multiplets.

Finally, the new neutral fields might still generate a divergent contribution through mixing with other neutral fields that have direct couplings to the Higgs and the SM charged leptons, as ν_{HR} for instance. Restricting ourselves to order v^2/f^2 the possible terms are presented in table 4.3. Among the one-loop diagrams listed in fig. 3.1, one can distinguish two different cases. In the case where the diagram involves a Higgs coupling to two neutral fermions, that can be inferred from eq. (4.84) and table 4.3 by substituting the *vev* by a Higgs boson according to $v \to h$ and $v^2 \to (v+h)^2$, at least a mass or a mixing insertion is required. On the other hand, one may check that before the mixing insertion the degree of divergence of this kind of topologies is at most of order $\log \Lambda$ in the cutoff regularization scheme. Therefore, the contribution of the new fermion fields to this kind of topologies is finite. This is because the mass of the propagator contributes as $\sim M/p^2$ to the loop and each mixing insertion is accompanied by the propagator of the new field that contributes at least as $\sim 1/\not{p}$, with p the corresponding loop momentum. In the remaining subclass of diagrams, in which the Higgs does not couple to two neutral fermions, at least two mixing insertions are required and those topologies are at most divergent as $\sim \Lambda$ before the mixing insertions. Thus the contributions of the new fermion fields to LFV Higgs decays are finite. This stems from two reasons: the NLHT is gauge invariant and we have chosen the appropriate fermion field representations.

4.2. The Background Field Method

In this section we derive a master formula to obtain the divergences of any theory generated at one loop in a gauge invariant fashion. For that purpose we will apply the *background field method* (BFM) with a cutoff in the so called proper time variable. In particular this will allow us to classify the divergences into quadratic and logarithmic. This classification is crucial in a Composite Higgs model since only logarithmically divergent contributions to the Higgs boson mass are admissible as we have emphasized in previous sections. Once the master formula is derived, we will employ it to calculate the Coleman-Weinberg potential for the scalar fields [134], generated by integrating out fermions and gauge bosons, and the divergent operator for the LFV Higgs decays in § 3.2.1, generated by integrating out the T-odd singlet χ_- and scalar fields.

4.2.1. Deriving the master formula for the background field method

The BFM allows to find the divergent terms of a theory in a gauge invariant way, translating the divergences in the space-time integration into those of the functional trace of a heat kernel in a new variable called proper time [135, 136]. This method has been extensively employed in the literature to study the renormalization of the linear realization of the SM [137, 138] and more recently to its non linear realization in refs. [139, 140], where a master formula was derived in the dimensional regularization scheme using super-heat kernel tools [141]. The BFM is also useful when dealing with the Standard Model Effective Field Theory (SMEFT). In particular, in [142] a master formula that includes the effects of bosonic operators up to dimension six is applied to the calculation of the Renormalization Group equations in the context of the SMEFT.

Our starting point is a general Lorentz-invariant four-dimensional action containing real bosonic fields and operators at most bilinear in the fermion fields,

$$S\left[\varphi^i, A^a_\mu, \psi^b, \overline{\psi}^b\right] = \int d^4x\, \mathcal{L}\left(\varphi^i, A^a_\mu, \psi^b, \overline{\psi}^b\right), \tag{4.92}$$

that it is all we need for our case. The latin indices refer to the different species of bosons and fermions in our theory.[6] To obtain the generating functional of Green functions, we couple the fields to external sources

$$Z[j_i, J^\mu_a, \rho_b, \overline{\rho}_b] = \int \left[D\varphi D A_\mu D\psi D\overline{\psi}\right] \exp\left\{i\left(S + \left\langle j_i\varphi^i + J^\mu_a A^a_\mu + \overline{\psi}^b\rho_b + \overline{\rho}_b\psi^b\right\rangle\right)\right\}, \tag{4.93}$$

where $Z = e^{iW}$ with W the generating functional of connected Green functions and $\langle\cdots\rangle$ stands for integration over spacetime. The normalization of the path integral is $Z[0] = 1$. We define

[6] If the bosonic fields are complex they are split into real and imaginary parts.

the classical, or background fields, as the solutions of the classical equations of motion (EoM),

$$\left.\frac{\delta S}{\delta\varphi^i}\right|_{\varphi^i_{\rm cl}} + j_i = 0, \quad \left.\frac{\delta S}{\delta A^a_\mu}\right|_{A^a_{\mu,{\rm cl}}} + J^\mu_a = 0, \tag{4.94}$$

$$\left.\frac{\delta S}{\delta\overline{\psi}^b}\right|_{\psi^b_{\rm cl}} + \rho_b = 0, \quad \left.\frac{\delta S}{\delta\psi^b}\right|_{\overline{\psi}^b_{\rm cl}} - \overline{\rho}_b = 0, \tag{4.95}$$

where the subscript 'cl' stands for classical and we used the Grassmannian character of fermion fields to derive the last two relations.

In order to obtain the one-loop contributions to the scalar potential and the divergent operator generated in LFV Higgs decays after integrating the singlet χ_R in the T-odd χ_- case, we perform a change of variables in the path integral consisting of a linear split of each field in two parts: the background field and the quantum excitations, which will be the new integration variables in the path integral,

$$\varphi^i = \varphi^i_{\rm cl} + \phi^i, \tag{4.96}$$

$$A^a_\mu = A^a_{\mu,{\rm cl}} + {\rm a}^a_\mu, \tag{4.97}$$

$$\psi^b = \psi^b_{\rm cl} + \chi^b, \tag{4.98}$$

$$\overline{\psi}^b = \overline{\psi}^b_{\rm cl} + \overline{\chi}^b, \tag{4.99}$$

The background fields only appear as external legs in the Feynman diagrammatic approach whereas the quantum excitations only occur inside the loops. Substituting in the Lagrangian, neglecting terms higher than quadratic in the quantum fluctuations, one may parametrize the action as

$$S + \left\langle j_i\varphi^i + J^\mu_a A^a_\mu + \overline{\psi}^b\rho_b + \overline{\rho}_b\psi^b \right\rangle = S^{(0)} + \left\langle j_i\varphi^i_{\rm cl} + J^\mu_a A^a_{\mu,{\rm cl}} + \overline{\psi}^b_{\rm cl}\rho_b + \overline{\rho}_b\psi^b_{\rm cl} \right\rangle + S^{(2)}\left[\varphi^i_{\rm cl}, A^a_{\mu,{\rm cl}}, \psi^b_{\rm cl}, \overline{\psi}^b_{\rm cl}; \phi^i, {\rm a}^a_\mu, \chi^b, \overline{\chi}^b\right]. \tag{4.100}$$

The first term on the r.h.s is the classical action evaluated in the background fields and the last is quadratic in the quantum excitations, [7] and takes the general form

$$S^{(2)}\left[\varphi^i_{\rm cl}, A^a_{\mu,{\rm cl}}, \psi^b_{\rm cl}, \overline{\psi}^b_{\rm cl}; \phi^i, {\rm a}^a_\mu, \chi^b, \overline{\chi}^b\right] = \int d^4x\, \mathcal{L}^{(2)}\left(\varphi^i_{\rm cl}, A^a_{\mu,{\rm cl}}, \psi^b_{\rm cl}, \overline{\psi}^b_{\rm cl}; \phi^i, {\rm a}^a_\mu, \chi^b, \overline{\chi}^b\right), \tag{4.101}$$

with

$$\mathcal{L}^{(2)}\left(\varphi^i_{\rm cl}, A^a_{\mu,{\rm cl}}, \psi^b_{\rm cl}, \overline{\psi}^b_{\rm cl}; \phi^i, {\rm a}^a_\mu, \chi^b, \overline{\chi}^b\right) = -\frac{1}{2}\Psi^k A_k^{\ l}\Psi_l + \overline{\chi}^b\left(i\partial\!\!\!/ - G\right)_{bc}\chi^c + \overline{\chi}^c\Gamma^i_c\Psi_i + \Psi^i\overline{\Gamma}_{i,c}\chi^c$$
$$\equiv -\frac{1}{2}\Psi^T A\Psi + \overline{\chi}B\chi + \overline{\chi}\Gamma\Psi + \Psi^T\overline{\Gamma}\chi, \tag{4.102}$$

where Ψ^i defined bellow collects the bosonic fluctuations. The dependence on the background fields $\varphi^i_{\rm cl}$, $A^a_{\mu,{\rm cl}}$ is encoded in the bosonic matrices A and G, with $\{k,l\}$ indices of any type (Lorentz and internal indices). The matrices Γ and $\overline{\Gamma}$ are fermionic and thus contain a linear dependence on the background fermions depending also on the background bosonic fields. After integrating by parts, the interaction amoing bosonic fields is given by second order differential

[7] The part of the action that is linear in the quantum fields is identically zero since it is proportional to the classical EoM $\frac{\delta S}{\delta F} = 0$, with F any background field an those are on shell.

operators

$$A = D^{\mu} D_{\mu} + V, \quad \text{with} \quad D_{\mu} = \partial_{\mu} + N_{\mu}, \tag{4.103}$$

where N_{μ} is antisymmetric. Both N_{μ} and V depend on the background bosonic fields. On the other hand, the most general Lorentz structure of the boson-fermion interaction that it is bilinear in the fermion fields is parametrized through

$$G = \left(r + \rho_{\mu}\gamma^{\mu}\right) P_R + \left(l + \lambda_{\mu}\gamma^{\mu}\right) P_L, \tag{4.104}$$

where $l = r^{\dagger}$ parametrize the interactions between fermions and scalar fields and ρ^{μ}, γ^{μ} parametrize the interactions between fermions and gauge boson fields. To express the bosonic interactions in the canonical form of eq. (4.103) one must introduce appropriate gauge-fixing terms for the fluctuating gauge fields, still preserving the gauge invariance of the one-loop effective action [143]. Now we redefine all the fluctuating gauge fields to have the same sign in the derivatives of time and space components,

$$\widetilde{\mathrm{a}}^a_{\mu} = \left(i\mathrm{a}^a_0, \mathrm{a}^a_i\right) \equiv M_{\mu}^{\ \nu}\mathrm{a}^a_{\nu}, \quad M_{\mu}^{\ \nu} = \mathrm{diag}\left(i, 1, 1, 1\right). \tag{4.105}$$

The contravariant vector $\mathrm{a}^{a\mu} = \left(\mathrm{a}^a_0, -\mathrm{a}^a_i\right)$ transforms with the inverse matrix

$$\widetilde{\mathrm{a}}^{a\mu} = \left(M^{-1}\right)^{\mu}_{\ \nu} \mathrm{a}^{a\nu} = \left(-i\mathrm{a}^a_0, -\mathrm{a}^a_i\right), \tag{4.106}$$

$$\left(M^{-1}\right)^{\mu}_{\ \nu} = \mathrm{diag}\left(-i, 1, 1, 1\right), \tag{4.107}$$

implying that $\widetilde{\mathrm{a}}^a_{\mu} = -\widetilde{\mathrm{a}}^{a\mu}$.[8] In terms of these new gauge fields, their kinetic term reads

$$\mathcal{L}_{\text{kin,g}} = -\frac{1}{2}\widetilde{\mathrm{a}}^a_0\partial^2\widetilde{\mathrm{a}}^a_0 - \frac{1}{2}\widetilde{\mathrm{a}}^a_i\partial^2\widetilde{\mathrm{a}}^a_i = -\frac{1}{2}\left(-\widetilde{\mathrm{a}}^{a\mu}\right)\delta^{\nu}_{\mu}\partial^2\widetilde{\mathrm{a}}^a_{\nu}. \tag{4.108}$$

Thus comparing with eq. (4.102) we collect the quantum excitations of scalar and gauge boson fields in the object $\Psi^k = \left(\phi^i, -\widetilde{\mathrm{a}}^{a\mu}\right)$ and $\Psi_l = \left(\phi^i, \widetilde{\mathrm{a}}^a_{\mu}\right)$ and thus $\Psi^k = \Psi_k$. The advantage of this redefinition of the fluctuating gauge fields is that the functions at both sides of the operator A are the same, which is necessary to later perform a Gaussian integration.

The expansion of the action to second order in the fluctuations is enough to get the generating functional W to one loop,

$$W = W_{L=0} + W_{L=1} + \text{higher order corrections}, \tag{4.109}$$

where L is the number of loops. Then, to this order, the generating functional Z can be written as $Z = Z_{L=0}Z_{L=1}$. The first factor is

$$Z_{L=0} = e^{iW_{L=0}} = e^{i\left(S^{(0)} + \left\langle j_i\varphi^i_{\text{cl}} + J^{\mu}_a A^a_{\mu,\text{cl}} + \overline{\psi}^b_{\text{cl}}\rho_b + \overline{\rho}_b\psi^b_{\text{cl}}\right\rangle\right)}, \tag{4.110}$$

a constant for the path integral, independent of the quantum variables. Taking logarithms,

$$W_{L=0} = S^{(0)} + \left\langle j_i\varphi^i_{\text{cl}} + J^{\mu}_a A^a_{\mu,\text{cl}} + \overline{\psi}^b_{\text{cl}}\rho_b + \overline{\rho}_b\psi^b_{\text{cl}}\right\rangle, \tag{4.111}$$

which after subtracting the source term is nothing but the classical action. The second term $W_{L=1}$ is more interesting and includes the one loop effective Lagrangian. This comes from

[8] The auxiliary matrix M appears in intermediate steps of the calculation but it will cancel at the end.

eq. (4.102),

$$e^{iW_{L=1}} = \int [D\Psi D\chi D\overline{\chi}]\, e^{iS^{(2)}} = \int [D\Psi D\chi D\overline{\chi}]\, e^{i\int d^4x\, \left(-\frac{1}{2}\Psi^T A\Psi + \overline{\chi}B\chi + \overline{\chi}\Gamma\Psi + \Psi^T\overline{\Gamma}\chi\right)}. \tag{4.112}$$

Before the Gaussian integration we must remove the crossed term $\Psi - \chi$ through the field redefinition of the quantum fields

$$\Psi^k \to \Psi^k, \tag{4.113}$$

$$\chi \to \chi - B^{-1}\Gamma^l\Psi_l, \tag{4.114}$$

$$\overline{\chi} \to \overline{\chi} - \Psi^k\overline{\Gamma}_k B^{-1}, \tag{4.115}$$

with Jacobian equal to one. Substituting in the one-loop functional we find

$$e^{iW_{L=1}} = \int [D\Psi D\chi D\overline{\chi}]\, e^{iS^{(2)}} = \int [D\Psi D\chi D\overline{\chi}]\, e^{i\int d^4x\, \left(-\frac{1}{2}\Psi^T\left(A+2\overline{\Gamma}B^{-1}\Gamma\right)\Psi + \overline{\chi}B\chi\right)}. \tag{4.116}$$

Thus, performing the Gaussian integration over the quantum fluctuations in the path integral and taking logarithms, the one-loop generating functional reads

$$W_{L=1} = \frac{i}{2}\log \mathrm{Det}\left(A + 2\overline{\Gamma}B^{-1}\Gamma\right) - i\log \mathrm{Det}\, B \tag{4.117}$$

where Det stands for the functional determinant and capital letters indicate that it involves a spacetime integration. Since it does not depend on the source term, $W_{L=1}$ can be interpreted as the one-loop effective action. Squaring the operator B as in [135, 144], we may write

$$\log \mathrm{Det}\, B = \frac{1}{2}\log \mathrm{Det}\left(BB^c\right), \tag{4.118}$$

where $B^c = -i\not{\partial} - G^c$ with

$$G^c = \left(r - \gamma^\mu\rho_\mu\right)P_L + \left(l - \gamma^\mu\lambda_\mu\right)P_R. \tag{4.119}$$

Notice that B^c is obtained from B by the replacements $\gamma^\mu \to -\gamma^\mu$, $P_{L,R} \to P_{R,L}$. Then we can write the one-loop generating functional as

$$W_{L=1} = \frac{i}{2}\log \mathrm{Sdet}\begin{pmatrix} A & i\sqrt{2}\overline{\Gamma}B^c \\ i\sqrt{2}\Gamma & BB^c \end{pmatrix} \equiv \frac{i}{2}\log \mathrm{Sdet}\,\Delta. \tag{4.120}$$

In this compact notation borrowed from supersymmetry, Sdet$M$ stands for the superdeterminant or Berezinian of a supermatrix M,

$$M = \begin{pmatrix} a & \alpha \\ \beta & b \end{pmatrix}, \quad \mathrm{Sdet}\, M \equiv \mathrm{Det}\left(a - \alpha b^{-1}\beta\right)/\mathrm{Det}\, b, \tag{4.121}$$

where the entries a, b (α, β) are bosonic (fermionic) variables. Notice that Δ can be written in the canonical form,

$$\Delta = \left(\partial^\mu + \Lambda^\mu\right)\left(\partial_\mu + \Lambda_\mu\right) + Y, \tag{4.122}$$

expression which holds in our case because our starting Lagrangian is, at most, bilinear in the fermion fields [139, 142]. Going to the Euclidean spacetime, performing the usual change of variables in the time coordinate, $t = -i\tau$, we can rewrite the one-loop effective action

$$W^E_{L=1} = -iW_{L=1} = \frac{1}{2}\log\mathrm{Det}\left(-\Delta^E\right) = \frac{1}{2}\mathrm{Str}_E\log\left(-\Delta^E\right), \tag{4.123}$$

where Str_E is the supertrace ($\mathrm{Str}M = \mathrm{tr}\,a - \mathrm{tr}\,b$) that also includes an Euclidean spacetime integration.[9] In the last expression, the operator Δ^E is defined as

$$\Delta^E = -\Delta = \left(\partial^E_\mu + \Lambda^E_\mu\right)\left(\partial^E_\mu + \Lambda^E_\mu\right) - Y^E, \tag{4.124}$$

where the Euclidean versions of the matrices above verify

$$\Lambda_\mu(t,\vec{x}) = \begin{cases}\Lambda_0(t,\vec{x}) = i\Lambda^E_0(\tau,\vec{x})\\ \Lambda_i(t,\vec{x}) = \Lambda^E_i(\tau,\vec{x}),\end{cases} \tag{4.125}$$

$$Y(t,\vec{x}) = Y^E(\tau,\vec{x}). \tag{4.126}$$

To obtain the divergences of the one-loop functional, we rewrite it using the proper time or Schwinger representation [145][10]

$$W^E_{L=1} = -\frac{1}{2}\int_0^\infty \frac{ds}{s}\mathrm{Str}_E\, e^{s\Delta^E} = -\frac{1}{2}\int_0^\infty \frac{ds}{s}\int d^4x^E\,\mathrm{str}\,\mathbf{K}(s;x^E;x^E), \tag{4.127}$$

where the integrand is the supertrace of the heat kernel of the elliptic operator Δ^E. The reason to go to Euclidean spacetime is that the heat kernel is better behaved [146]. DeWitt [147] proposes the following ansatz for the heat kernel in the limit $s \to 0$,

$$\mathbf{K}(s;x^E;x^E) = \frac{1}{(4\pi s)^2}\sum_{n=0}^{\infty} a_n\left(x^E,x^E\right)s^n. \tag{4.128}$$

The coefficients a_n are the so called Seeley-DeWitt coefficients [147, 148] which are completely regular when both arguments are equal. As we will show below, the divergences we are looking for are proportional to the first coefficients. The integral converges in the upper limit.

To regularize the integral in the lower limit, let us focus in the units of the proper time variable s. Since the argument of the exponential is dimensionless, s has units of inverse mass squared. Using a proper time cutoff in the lower limit we may write [149]

$$W^E_{L=1,\mathrm{reg}} = -\frac{1}{2}\int_{\Lambda^{-2}}^\infty \frac{ds}{s}\int d^4x^E\,\mathrm{str}\,\mathbf{K}(s;x^E;x^E), \tag{4.129}$$

with $\Lambda \to \infty$ the usual energy cutoff. Then we can use the DeWitt expansion in eq. (4.128) to solve the integral in the proper time variable and find the divergences in the lower limit:

$$\int_{\Lambda^{-2}}^{E_0^{-2}} \frac{ds}{s}s^{n-2} \sim \begin{cases}\frac{1}{2-n}\Lambda^{4-2n} & n<2\\ \log\Lambda^2 & n=2\\ -\frac{1}{n-2}\Lambda^{-(2n-4)} & n>2\end{cases} \tag{4.130}$$

where we have introduced an upper limit E_0^{-2} related to the maximum value of the proper time variable for which the DeWitt expansion of the heat kernel is valid. Actually this energy

[9] The supertrace and the superdeterminant have the same properties as the usual trace and determinant with respect to the logarithms.

[10] The sign in the exponential is appropriate since one has to move to momentum space ($\partial^E_\mu \to -ip^E_\mu$) introducing a plane wave basis and this gives the exponential suppression in the upper limit.

scale should appear in the argument of $\log \Lambda^2$ making it dimensionless, but it will be omitted here and in the following. As previously advertised, the divergences are found just in the first three coefficients.[11] Since we are interested in quadratic and logarithmic divergences, only the expressions of a_1 and a_2 are needed. These can be found in [135] and their values in Euclidean spacetime are

$$a_1\left(x^E, x^E\right) = -Y^E \tag{4.131}$$

$$a_2\left(x^E, x^E\right) = \frac{1}{12} Z^E_{\mu\nu} Z^E_{\mu\nu} + \frac{1}{2} Y^{E2}, \tag{4.132}$$

where

$$Z^E_{\mu\nu} = \partial^E_\mu \Lambda^E_\nu - \partial^E_\nu \Lambda^E_\mu + \left[\Lambda^E_\mu, \Lambda^E_\nu\right]. \tag{4.133}$$

Writing everything together, the divergent part of the effective one-loop action reads

$$W^E_{L=1,\text{reg}} = -\frac{1}{2}\frac{1}{16\pi^2}\int d^4x^E \text{str}\left[-Y^E\Lambda^2 + \left(\frac{1}{12}Z^E_{\mu\nu}Z^E_{\mu\nu} + \frac{1}{2}Y^{E2}\right)\log\Lambda^2\right]. \tag{4.134}$$

Turning back to the Minkowski spacetime using eq. (4.125), the divergent part of the Lagrangian at one loop is

$$\mathcal{L}^{\text{div}}_{L=1} = -\frac{1}{32\pi^2}\text{str}\left[Y\Lambda^2 - \left(\frac{1}{12}Z^{\mu\nu}Z_{\mu\nu} + \frac{1}{2}Y^2\right)\log\Lambda^2\right]. \tag{4.135}$$

Finally, using equations (4.120) and (4.122), the expressions for the matrices Λ^μ and Y read

$$\Lambda^\mu = \begin{pmatrix} N^\mu & \frac{1}{\sqrt{2}}\overline{\Gamma}\gamma^\mu \\ 0 & \frac{i}{2}\left(G\gamma^\mu - \gamma^\mu G^c\right) \end{pmatrix}, \tag{4.136}$$

and

$$Y = \begin{pmatrix} V & -\frac{1}{\sqrt{2}}\overline{\Gamma}\overleftarrow{\not{\partial}} + \frac{1}{\sqrt{2}}\overline{\Gamma}\not{N} - \frac{i}{2\sqrt{2}}\overline{\Gamma}\gamma^\mu G\gamma_\mu \\ i\sqrt{2}\Gamma & -\frac{i}{2}\partial_\mu\left(G\gamma^\mu - \gamma^\mu G^c\right) + \frac{1}{4}\left(G\gamma^\mu G\gamma_\mu - \gamma^\mu G^c G\gamma_\mu + \gamma^\mu G^\dagger \gamma_\mu G^c\right) \end{pmatrix}, \tag{4.137}$$

where we used that N^μ is antisymmetric and the Grassmannian character of $\overline{\Gamma}$ and Γ. Thus substituting in eq. (4.135) we obtain the final expression of the master formula for quadratic divergences

$$\mathcal{L}^{\Lambda^2}_{L=1} = -\frac{\Lambda^2}{32\pi^2}\text{tr}\left(V - 4rl\right) \tag{4.138}$$

and for logarithmic divergences

$$\mathcal{L}^{\log\Lambda^2}_{L=1} = \frac{1}{32\pi^2}\log\Lambda^2\left(\text{tr}\left[\frac{1}{12}N_{\mu\nu}N^{\mu\nu} + \frac{1}{2}V^2 - \frac{1}{3}\rho_{\mu\nu}\rho^{\mu\nu} - \frac{1}{3}\lambda_{\mu\nu}\lambda^{\mu\nu} + 2D_\mu r D^\mu l - 2rlrl\right] + \overline{\Gamma}i\not{D}\Gamma\right), \tag{4.139}$$

where

$$N_{\mu\nu} = \partial_\mu N_\nu - \partial_\nu N_\mu + \left[N_\mu, N_\nu\right] \tag{4.140}$$

[11] The coefficient $a_0 = 1$ is independent of the background fields, being only important if we were dealing with gravity, since it contributes to the vacuum energy.

$$\rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + i\left[\rho_\mu, \rho_\nu\right] \tag{4.141}$$
$$\lambda_{\mu\nu} = \partial_\mu \lambda_\nu - \partial_\nu \lambda_\mu + i\left[\lambda_\mu, \lambda_\nu\right] \tag{4.142}$$
$$D_\mu r = \partial_\mu r + i\lambda_\mu r - ir\rho_\mu \tag{4.143}$$
$$D_\mu l = \partial_\mu l + i\rho_\mu l - il\lambda_\mu \tag{4.144}$$
$$D_\mu \Gamma = \partial_\mu \Gamma + N_\mu \Gamma - \frac{i}{2} G\gamma_\mu . \tag{4.145}$$

Eq. (4.139) for the logarithmic divergences is in agreement with refs. [137–140, 142] with the identification $\log \Lambda^2 \leftrightarrow 2/\epsilon$. From the previous expressions we can obtain the scalar potential and the divergent operator generated in LFV Higgs decays in the T-odd case. To obtain the potential we take $\lambda_\mu = \rho_\mu = 0$ since these matrices only include background gauge bosons. Besides, in our model N^μ, which comes from integrating the gauge bosons quantum excitations, does not include background scalar fields (see eq. 3.20) and thus does not contribute to the scalar potential. Hence only non derivative terms involving l, r and the part of V coming from integrating the quantum excitations of the gauge bosons will contribute.[12] Using that

$$\mathcal{V}^{\text{div}}_{L=1} \subset -\mathcal{L}^{\text{div}}_{L=1}, \tag{4.146}$$

we find that the quadratic and logarithmic parts of the potential from eqs. (4.138) and (4.139) read

$$\mathcal{V}^{\Lambda^2}_{L=1} = \frac{\Lambda^2}{32\pi^2}\text{tr}\, V - \frac{\Lambda^2}{8\pi^2}\text{tr}\,(lr)\,, \tag{4.147}$$

and

$$\mathcal{V}^{\log\Lambda^2}_{L=1} = -\frac{1}{64\pi^2}\log\Lambda^2 \text{tr}\, V^2 + \frac{1}{16\pi^2}\log\Lambda^2 \text{tr}\,(lrlr)\,. \tag{4.148}$$

Finally, to obtain the divergent operator in LFV Higgs decays in the LHT, which comes from the integration of scalar fields and fermions in the T-odd case (see eq. (3.158)), we need the term

$$\mathcal{L}_{\text{div}} = \frac{1}{32\pi^2}\log\Lambda^2\, \overline{\Gamma} i\not{\partial}\Gamma, \tag{4.149}$$

that includes external fermions fields, scalars and one derivative.

4.2.2. Quadratic and logarithmic corrections to the scalar potential

To obtain the quadratic and logarithmic contributions to the scalar one-loop effective potential, we have to evaluate the expression for the matrices V, l and r defined in eqs. (4.102), (4.103), (4.104). In this work we neglect the contribution of the lightest SM lepton Yukawa interactions in eqs. (3.46) as well as those of the lightest SM quarks in eqs. (3.47) and (3.51), since their Yukawa couplings are parametrically small. Thus, the leading contributions come from the interaction between scalars and gauge bosons in eqs. (3.26), (4.12), the Yukawa Lagrangians for the heavy fermions in eqs. (3.39), (3.41), (4.25) and (4.31), with the field content of eqs. (4.17), (4.22), (4.24), (4.30) and (4.29), and the top quark Langrangians in eqs. (4.32) and (3.52).

In what follows, the Higgs potential is parametrized as

$$\mathcal{V}_{\text{Higgs}} = \mu^2\left(H^\dagger H\right) + \lambda\left(H^\dagger H\right)^2, \tag{4.150}$$

[12] In a composite Higgs model, the scalar fields are the pseudo-Nambu Goldstone bosons of the spontaneous breaking of an approximate global symmetry. Thus the self interactions of scalar fields are derivative and they do not generate mass terms.

invariant under the SM gauge group. For the rest of the scalar fields we are only interested in their mass terms since they cannot develop a *vev* respecting the T-parity discrete symmetry. This implies that their mass terms must be positive as we will impose below.

Gauge boson contribution to the scalar potential

Using eq. (4.96) for the fluctuating gauge bosons in eqs. (3.26) and (4.12), we expand to second order in the quantum excitations,

$$\begin{aligned}\mathcal{L}_S^{(2)} + \mathcal{L}_{\widehat{S}}^{(2)} \supset \frac{f^2}{2}\sum_{j,k=1}^{2} \Big[& g^2 \omega^a_{j\mu}\omega^{b\mu}_k \mathrm{tr}\left(Q^a_j Q^b_k + Q^a_j \Sigma Q^{bT}_k \Sigma^\dagger\right) + g'^2 b_{j\mu} b^\mu_k \mathrm{tr}\left(Y_j Y_k + Y_j \Sigma Y_k^T \Sigma^\dagger\right) \\ & - g g' \omega^a_{j\mu} b^\mu_k \mathrm{tr}\left(Q^a_j \Sigma Y^T_k \Sigma^\dagger\right) - g g' b^\mu_j \omega^b_{k\mu} \mathrm{tr}\left(Q^b_k \Sigma Y^T_j \Sigma^\dagger\right)\Big] \\ + \frac{f^2}{2}\sum_{j,k=1}^{2} \Big[& g^2 \omega^a_{j\mu}\omega^{b\mu}_k \mathrm{tr}\left(Q^a_j Q^b_k + Q^a_j \widehat{\Sigma} Q^{bT}_k \widehat{\Sigma}^\dagger\right) + g'^2 b_{j\mu} b^\mu_k \mathrm{tr}\left(Y_j Y_k + Y_j \widehat{\Sigma} Y_k^T \widehat{\Sigma}^\dagger\right) \\ & - g g' \omega^a_{j\mu} b^\mu_k \mathrm{tr}\left(Q^a_j \widehat{\Sigma} Y^T_k \widehat{\Sigma}^\dagger\right) - g g' b^\mu_j \omega^b_{k\mu} \mathrm{tr}\left(Q^b_k \widehat{\Sigma} Y^T_j \widehat{\Sigma}^\dagger\right)\Big], \end{aligned} \tag{4.151}$$

where the background scalars are parametrized in the non linear sigma fields Σ, $\widehat{\Sigma}$ and $\omega^a_{j\mu}$, $b_{j\mu}$ are the quantum excitations of the gauge bosons associated to SU(2) and U(1), respectively. As already mentioned, using the appropriate gauge for these fluctuating fields [143], their kinetic terms take the canonical form in eq. (4.103). The redefinition of the gauge fields as in eqs. (4.105) and (4.106),

$$\omega^a_{j\mu}\omega^{b\mu}_k = -\left(-\tilde{\omega}^{a\mu}_j\right)\delta^\nu_\mu \tilde{\omega}^b_{k\nu}, \tag{4.152}$$

$$b_{j\mu} b^\mu_k = -\left(-\tilde{b}^\mu_j\right)\delta^\nu_\mu \tilde{b}_{k\nu}, \tag{4.153}$$

$$\omega^a_{j\mu} b^\mu_k = -\left(-\tilde{\omega}^{a\mu}_j\right)\delta^\nu_\mu \tilde{b}_{k\nu}, \tag{4.154}$$

allows us to find the corresponding matrix V

$$V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}, \tag{4.155}$$

where

$$V_{11} = f^2 \delta^\nu_\mu g^2 \mathrm{tr}\left(2 Q^a_j Q^b_k + Q^a_j \Sigma Q^{bT}_k \Sigma^\dagger + Q^a_j \widehat{\Sigma} Q^{bT}_k \widehat{\Sigma}^\dagger\right) \tag{4.156}$$

$$V_{12} = -f^2 \delta^\nu_\mu g g' \mathrm{tr}\left(Q^a_j \Sigma Y^T_k \Sigma^\dagger + Q^a_j \widehat{\Sigma} Y^T_k \widehat{\Sigma}^\dagger\right) \tag{4.157}$$

$$V_{21} = -f^2 \delta^\nu_\mu g g' \mathrm{tr}\left(Q^b_k \Sigma Y^T_j \Sigma^\dagger + Q^b_k \widehat{\Sigma} Y^T_j \widehat{\Sigma}^\dagger\right) \tag{4.158}$$

$$V_{22} = f^2 \delta^\nu_\mu g'^2 \mathrm{tr}\left(2 Y_j Y_k + Y_j \Sigma Y^T_k \Sigma^\dagger + Y_j \widehat{\Sigma} Y^T_k \widehat{\Sigma}^\dagger\right). \tag{4.159}$$

According to eq. (4.147), to obtain the quadratic divergences of this sector we must take the trace of the matrix in eq. (4.155). V is a block matrix with gauge group as well as Lorentz indices in each block. Taking the trace implies $a = b$, $j = k$, $\mu = \nu$ in each block and then summming over the diagonal terms of the matrix (summation over repeated indices in their corresponding range is understood),

$$\mathcal{V}^{\Lambda^2}_{L=1,g} = \frac{\Lambda^2}{8\pi^2}\left[g^2 f^2 \mathrm{tr}\left(Q^a_j \Sigma Q^{aT}_j \Sigma^\dagger + Q^a_j \widehat{\Sigma} Q^{aT}_j \widehat{\Sigma}^\dagger\right) + g'^2 f^2 \mathrm{tr}\left(Y_j \Sigma Y^T_j \Sigma^\dagger + Y_j \widehat{\Sigma} Y^T_j \widehat{\Sigma}^\dagger\right)\right] \tag{4.160}$$

where a global facor 4 comes from $\delta^\mu_\mu = 4$ and irrelevant constant terms have been dropped. This part of the potential contains a mass term for the triplet Φ and a quartic Higgs coupling,

$$\mathcal{V}^{\Lambda^2}_{L=1,g} \supset \frac{\Lambda^2}{4\pi^2}\left(g^2+g'^2\right)\mathrm{tr}\left(\Phi^\dagger\Phi\right)+\frac{1}{16\pi^2}\frac{\Lambda^2}{f^2}\left(g^2+g'^2\right)\left(H^\dagger H\right)^2, \tag{4.161}$$

and no mass terms for the rest of the physical scalars. Since the leading order of the mass of the triplet Φ and the Higgs quartic coupling is $\Lambda^2 \lesssim 16\pi^2 f^2$, we will neglect all logarithmic contributions to these operators in the following.

To evaluate the logarithmic divergences, from eq. (4.148) we must take the trace of V^2. To construct this matrix we must pair the indices of both V factors. For the sake of clarity, the left upper block V^2_{11} would contain

$$V^2_{11} \supset f^4\delta^\nu_\mu g^2\mathrm{tr}\left(2Q^a_jQ^b_k+Q^a_j\Sigma Q^{bT}_k\Sigma^\dagger+Q^a_j\widehat{\Sigma}Q^{bT}_k\widehat{\Sigma}^\dagger\right)\times\delta^\mu_\lambda g^2\mathrm{tr}\left(2Q^b_kQ^c_l+Q^b_k\Sigma Q^{cT}_l\Sigma^\dagger+Q^b_k\widehat{\Sigma}Q^{cT}_l\widehat{\Sigma}^\dagger\right). \tag{4.162}$$

Then we repeat the same procedure applied above to take the trace. Again a global factor of 4 will appear from the trace over the Lorentz indices $\delta^\mu_\nu\delta^\nu_\mu = \delta^\mu_\mu = 4$ leading to

$$\begin{aligned}\mathcal{V}^{\log\Lambda^2}_{L=1,g} = &-\frac{1}{16\pi^2}f^4\log\Lambda^2\\ &\times\Big[g^4\mathrm{tr}\left(2Q^a_jQ^b_k+Q^a_j\Sigma Q^{bT}_k\Sigma^\dagger+Q^a_j\widehat{\Sigma}Q^{bT}_k\widehat{\Sigma}^\dagger\right)\times\mathrm{tr}\left(2Q^b_kQ^a_j+Q^b_k\Sigma Q^{aT}_j\Sigma^\dagger+Q^b_k\widehat{\Sigma}Q^{aT}_j\widehat{\Sigma}^\dagger\right)\\ &+2g^2g'^2\mathrm{tr}\left(Q^a_j\Sigma Y^T_k\Sigma^\dagger+Q^a_j\widehat{\Sigma}Y^T_k\widehat{\Sigma}^\dagger\right)\times\mathrm{tr}\left(Q^a_j\Sigma Y^T_k\Sigma^\dagger+Q^a_j\widehat{\Sigma}Y^T_k\widehat{\Sigma}^\dagger\right)\\ &+g'^4\mathrm{tr}\left(2Y_jY_k+Y_j\Sigma Y^T_k\Sigma^\dagger+Y_j\widehat{\Sigma}Y^T_k\widehat{\Sigma}^\dagger\right)\times\mathrm{tr}\left(2Y_kY_j+Y_k\Sigma Y^T_j\Sigma^\dagger+Y_k\widehat{\Sigma}Y^T_j\widehat{\Sigma}^\dagger\right)\Big].\end{aligned} \tag{4.163}$$

This part of the potential contains a contribution to the μ^2 term in the Higgs potential and a mass term for the triplet $\widehat{\omega}$,

$$\begin{aligned}\mathcal{V}^{\log\Lambda^2}_{L=1,g} &\supset f^2\log\Lambda^2\left(\frac{3g^4}{8\pi^2}+\frac{g'^4}{40\pi^2}\right)\left(H^\dagger H\right)+\frac{g^4}{\pi^2}f^2\log\Lambda^2\mathrm{tr}\left(\widehat{\omega}\widehat{\omega}\right)\\ &=\frac{1}{16\pi^2}\log\Lambda^2\left(3g^2M^2_{W_H}+g'^2M^2_{A_H}\right)\left(H^\dagger H\right)+\frac{g^2}{2\pi^2}M^2_{W_H}\log\Lambda^2\mathrm{tr}\left(\widehat{\omega}\widehat{\omega}\right),\end{aligned} \tag{4.164}$$

where we have used that at leading order $M^2_{W_H} = 2g^2f^2$ and $M^2_{A_H} = \frac{2}{5}g'^2f^2$ as well as the field redefinitions in eq. (4.42). These contribution to their masses are naturally smaller than those to the mass of the triplet Φ since they are proportional just to $\log\Lambda^2$ and there is a suppression of $1/(16\pi^2)$ which allows masses of order of the electroweak scale. No mass term is induced for the physical scalar $\widehat{\eta}$ by gauge interactions.

Fermion contributions to the scalar potential

In order to obtain the heavy lepton, heavy quark and top contribution to the potential, the matrices r and l in eq. (4.104) are needed. Since the Lagrangian for quarks is more involved, we illustrate the process with the lepton contribution. For simplicity, from eqs. (4.17), (4.22) and (4.24) we define the new multiplets

$$\Psi_2 = \mathcal{A}\widetilde{\Psi}_2, \quad \Psi_1 = \mathcal{A}\widetilde{\Psi}_1, \quad \Psi_R = \mathcal{A}\widetilde{\Psi}_R, \quad \widehat{\Psi}_R = \mathcal{B}\widetilde{\widehat{\Psi}}_R, \tag{4.165}$$

where

$$\mathcal{A} = \begin{pmatrix} -i\sigma^2 & & \\ & i & \\ & & -i\sigma^2 \end{pmatrix}, \quad \mathcal{B} = \begin{pmatrix} -i\sigma^2 & & \\ & i & \\ & & 0_{2\times 2} \end{pmatrix} \tag{4.166}$$

factor our the matrices from the multiplets. Splitting the fields as in eq. (4.96) we get

$$\mathcal{L}^{(2)}_{Y_H,\widehat{Y}_H} \supset -\begin{pmatrix} \overline{\psi}_2 & \overline{\psi}_1 & \bullet & \bullet \end{pmatrix} \begin{pmatrix} 0 & 0 & \kappa_l f \mathcal{A}^\dagger \xi \mathcal{A} & \widehat{\kappa}_l f \mathcal{A}^\dagger \widehat{\xi} \mathcal{B} \\ 0 & 0 & \kappa_l f \mathcal{A}^\dagger \Sigma_0 \xi^\dagger \mathcal{A} & -\widehat{\kappa}_l f \mathcal{A}^\dagger \Sigma_0 \widehat{\xi}^\dagger \mathcal{B} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \bullet \\ \bullet \\ \psi_R \\ \widehat{\psi}_R \end{pmatrix} + \text{h.c.} \tag{4.167}$$

where ψ_1, ψ_2, ψ_R and $\widehat{\psi}_R$ are the quantum fluctuations of $\widetilde{\Psi}_1$, $\widetilde{\Psi}_2$, $\widetilde{\Psi}_R$ and $\widetilde{\widetilde{\Psi}}_R$, respectively, and the bullet means that the corresponding field is not present in the theory. All the flavour dependence is encoded in the couplings κ and $\widehat{\kappa}$. Comparing with eqs. (4.102) and (4.104), the form of the matrices r and l for leptons is

$$r_l = l_l^\dagger = \begin{pmatrix} 0 & 0 & \kappa_l f \mathcal{A}^\dagger \xi \mathcal{A} & \widehat{\kappa}_l f \mathcal{A}^\dagger \widehat{\xi} \mathcal{B} \\ 0 & 0 & \kappa_l f \mathcal{A}^\dagger \Sigma_0 \xi^\dagger \mathcal{A} & -\widehat{\kappa}_l f \mathcal{A}^\dagger \Sigma_0 \widehat{\xi}^\dagger \mathcal{B} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{4.168}$$

To evaluate the quadratically divergent contribution to the potential arising from this sector, we need the product lr

$$l_l r_l = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2\kappa_l^\dagger \kappa_l f^2 1_{5\times 5} & \kappa_l^\dagger \widehat{\kappa}_l f^2 \mathcal{A}^\dagger \left(\xi^\dagger \widehat{\xi} - \xi \widehat{\xi}^\dagger\right) \mathcal{B} \\ 0 & 0 & \widehat{\kappa}_l^\dagger \kappa_l f^2 \mathcal{B}^\dagger \left(\widehat{\xi}^\dagger \xi - \widehat{\xi} \xi^\dagger\right) \mathcal{A} & 2\widehat{\kappa}_l^\dagger \widehat{\kappa}_l f^2 1_{3\times 3} \end{pmatrix}. \tag{4.169}$$

As in the gauge boson case, the matrix lr is block diagonal and its trace is the sum of the trace of each of its diagonal entries,

$$\text{tr}\left(l_l r_l\right) = 10\text{tr}\left(\kappa_l^\dagger \kappa_l f^2\right) + 6\text{tr}\left(\widehat{\kappa}_l^\dagger \widehat{\kappa}_l f^2\right), \tag{4.170}$$

which is independent of the scalar fields. In this model, all the scalars are protected from quadratic divergences coming from the new sector. Analogously, for the logarithmic divergences we have to evaluate

$$\begin{aligned} \text{tr}\left(l_l r_l l_l r_l\right) &= 2\text{tr}\left(\kappa_l \kappa_l^\dagger \widehat{\kappa}_l \widehat{\kappa}_l^\dagger f^4\right) \times \text{tr}\left[\left(\widehat{\xi}^\dagger \xi - \widehat{\xi} \xi^\dagger\right)\left(\xi^\dagger \widehat{\xi} - \xi \widehat{\xi}^\dagger\right) \mathcal{B}\mathcal{B}^\dagger\right] \\ &\supset -2\text{tr}\left(\kappa_l \kappa_l^\dagger \widehat{\kappa}_l \widehat{\kappa}_l^\dagger f^4\right) \text{tr}\left[\left(\widehat{\Sigma}^\dagger \Sigma + \Sigma^\dagger \widehat{\Sigma}\right) \mathcal{B}\mathcal{B}^\dagger\right] \\ &= -2\text{tr}\left(\kappa_l \kappa_l^\dagger \widehat{\kappa}_l \widehat{\kappa}_l^\dagger f^4\right) \sum_{a=1}^{3} \sum_{b=1}^{5} \left[\widehat{\Sigma}^\dagger_{ab} \Sigma_{ba} + \Sigma^\dagger_{ab} \widehat{\Sigma}_{ba}\right], \end{aligned} \tag{4.171}$$

using that $\hat{\xi}$ commutes with $\mathcal{B}\mathcal{B}^\dagger$ and $\Sigma_0^2 = 1_{5\times 5}$. Then, the logarithmically divergent contribution to the potential is

$$\mathcal{V}_{L=1,l}^{\log\Lambda^2} = -\frac{1}{8\pi^2}\log\Lambda^2 \mathrm{tr}\left(\kappa_l\kappa_l^\dagger\widehat{\kappa}_l\widehat{\kappa}_l^\dagger f^4\right) \times \sum_{a=1}^{3}\sum_{b=1}^{5}\left[\widehat{\Sigma}_{ab}^\dagger\Sigma_{ba} + \Sigma_{ab}^\dagger\widehat{\Sigma}_{ba}\right]. \tag{4.172}$$

This expression contains leading order contributions to the μ^2 parameter of the Higgs potential and to the masses of $\widehat{\omega}$ and $\widehat{\eta}$

$$\mathcal{V}_{L=1,l}^{\log\Lambda^2} \supset \frac{f^2}{8\pi^2}\log\Lambda^2 \mathrm{tr}(\kappa_l\kappa_l^\dagger\widehat{\kappa}_l\widehat{\kappa}_l^\dagger) \times \left(6H^\dagger H + \frac{36\widehat{\eta}^2}{5} + 8\mathrm{tr}(\widehat{\omega}^\dagger\widehat{\omega})\right), \tag{4.173}$$

where we have used the leading order field redefinitions in eq. (4.42). This expression is in agreement with the diagrammatic calculation for the Higgs part in eq. (4.26).

The last contribution comes from the heavy quarks in eqs. (3.41), (4.31) and the top sector in eq. (4.32). In this last equation we notice that due to the presence of the three dimensional Levi-Civita tensor $\epsilon_{i,j,k}$ with $\{i,j,k\} = 1,2,3$, only the three upper components of Q_1 and $\Sigma_0\Omega Q_2$ are relevant. Then, comparing eqs. (4.29) and (3.54) we have that for $i = 1,2$ one could substitute $(Q_{1,2})_i$ by $\left(\Psi_{1,2}^q\right)_i$ and for $i = 3$ we would have $i\,T_{1L}$ and $i\,T_{2L}$. Then, similarly as we did for leptons, we perform the following substitutions

$$\Psi_1^q = \mathcal{A}\widetilde{\Psi}_1^q, \quad \Psi_2^q = \mathcal{A}\widetilde{\Psi}_2^q, \quad Q_1 \to \mathcal{C}\widetilde{\Psi}_1^q, \quad Q_2 \to \mathcal{C}\widetilde{\Psi}_2^q, \tag{4.174}$$

where

$$\mathcal{C} = \begin{pmatrix} -i\sigma^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -i\sigma^2 \end{pmatrix}, \tag{4.175}$$

since only the aforementioned components of those multiplets are relevant. The zero in the middle of $\mathcal{C}$ takes into account that the multiplets $Q_{1,2}$ and $\Psi_{1,2}^q$ differ in the field in its center. Collecting the left-handed and right-handed quantum fields in vectors

$$\begin{aligned} v_L^T &= \left(\psi_2^{qT}, \psi_1^{qT}, \bullet, \bullet, \bullet, t_{2L}^T, t_{1L}^T\right), \\ v_R^T &= \left(\bullet, \bullet, \psi_R^{qT}, \widehat{\psi}_R^{qT}, t_R^T, t_{2R}^T, t_{1R}^T\right), \end{aligned} \tag{4.176}$$

we may write

$$\mathcal{L}_{Y_H,\widehat{Y}_H,t}^{(2,q)} = -\overline{v}_L r_q v_R + \mathrm{h.c.} \tag{4.177}$$

where

$$r_q = \begin{pmatrix} 0 & 0 & (r_{2R})_{mn\beta}^{\ \ \ \alpha} & (r_{2\widehat{R}})_{mn\beta}^{\ \ \ \alpha} & (r_{2t})_{m\beta}^{\ \ \alpha} & 0 & 0 \\ 0 & 0 & (r_{1R})_{mn\beta}^{\ \ \ \alpha} & (r_{1\widehat{R}})_{mn\beta}^{\ \ \ \alpha} & (r_{1t})_{m\beta}^{\ \ \alpha} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (r_{T_2t})_{\beta}^{\ \alpha} & (r_{T_2T_2})_{\beta}^{\ \alpha} & 0 \\ 0 & 0 & 0 & 0 & (r_{T_1t})_{\beta}^{\ \alpha} & 0 & (r_{T_1T_1})_{\beta}^{\ \alpha} \end{pmatrix}, \tag{4.178}$$

the greek indices are the SU(3) color indices and as before $l_q = r_q^\dagger$. As previously advertised, this sector is more involved and it manifests in the size of the matrix r_q. This is because apart of the introduction of the right-handed singlets $T_{1,2R}$ the field in the center of the SU(5) multiplets $\Psi^q_{1,2}$ $\left(\chi^q_{1,2L}\right)$ is not the same as that inside of $Q_{1,2}$ $(T_{1,2L})$. The components of the matrix r_q are defined as

$$(r_{2R})_{mn\beta}{}^{\alpha} = \kappa_q f \left(\mathcal{A}^\dagger \xi \mathcal{A}\right)_{mn} \delta^\alpha_\beta, \tag{4.179}$$

$$\left(r_{2\widehat{R}}\right)_{mn\beta}{}^{\alpha} = \widehat{\kappa}_q f \left(\mathcal{A}^\dagger \widehat{\xi} \mathcal{B}\right)_{mn} \delta^\alpha_\beta, \tag{4.180}$$

$$(r_{2t})_{m\beta}{}^{\alpha} = \frac{i}{4}\lambda_1 f \left(\mathcal{C}^\dagger \Sigma_0 \Omega\right)_{mi} \epsilon_{ijk}\epsilon_{xy}\tilde{\Sigma}_{jx}\tilde{\Sigma}_{ky}\delta^\alpha_\beta, \tag{4.181}$$

$$(r_{1R})_{mn\beta}{}^{\alpha} = \kappa_q f \left(\mathcal{A}^\dagger \Sigma_0 \xi^\dagger \mathcal{A}\right)_{mn} \delta^\alpha_\beta, \tag{4.182}$$

$$\left(r_{1\widehat{R}}\right)_{mn\beta}{}^{\alpha} = -\widehat{\kappa}_q f \left(\mathcal{A}^\dagger \Sigma_0 \widehat{\xi}^\dagger \mathcal{B}\right)_{mn} \delta^\alpha_\beta, \tag{4.183}$$

$$(r_{1t})_{m\beta}{}^{\alpha} = \frac{i}{4}\lambda_1 f \mathcal{C}^\dagger_{mi}\epsilon_{ijk}\epsilon_{xy}\Sigma_{jx}\Sigma_{ky}\delta^\alpha_\beta, \tag{4.184}$$

$$(r_{T_2 t})_{\beta}{}^{\alpha} = \frac{1}{4}\lambda_1 f \epsilon_{3jk}\epsilon_{xy}\tilde{\Sigma}_{jx}\tilde{\Sigma}_{ky}\delta^\alpha_\beta, \tag{4.185}$$

$$(r_{T_1 t})_{\beta}{}^{\alpha} = \frac{1}{4}\lambda_1 f \epsilon_{3jk}\epsilon_{xy}\Sigma_{jx}\Sigma_{ky}\delta^\alpha_\beta, \tag{4.186}$$

$$(r_{T_2 T_2})_{\beta}{}^{\alpha} = \frac{\lambda_2}{\sqrt{2}} f \widehat{X}^* \delta^\alpha_\beta, \tag{4.187}$$

$$(r_{T_1 T_1})_{\beta}{}^{\alpha} = \frac{\lambda_2}{\sqrt{2}} f \widehat{X} \delta^\alpha_\beta. \tag{4.188}$$

Proceeding similarly as for leptons, the quadratically divergent part of the potential due to quarks coming from the product lr reads

$$\mathcal{V}^{\Lambda^2}_{L=1,q} = -\frac{3\Lambda^2}{128\pi^2}\lambda_1^2 f^2 \epsilon_{ijk}\epsilon_{inp}\epsilon_{xy}\epsilon_{qr} \times \left(\Sigma_{jx}\Sigma_{ky}\Sigma^\dagger_{nq}\Sigma^\dagger_{pr} + \widetilde{\Sigma}_{jx}\widetilde{\Sigma}_{ky}\widetilde{\Sigma}^\dagger_{nq}\widetilde{\Sigma}^\dagger_{pr}\right), \tag{4.189}$$

where the factor 3 comes from $\delta^\alpha_\alpha = N_C = 3$. This term contains a contribution to the triplet Φ mass and to the quartic Higgs coupling,

$$\mathcal{V}^{\Lambda^2}_{L=1,q} \supset \frac{3\lambda_1^2}{4\pi^2}\Lambda^2 \mathrm{tr}\left(\Phi^\dagger\Phi\right) + \frac{3\lambda_1^2}{16\pi^2}\frac{\Lambda^2}{f^2}\left(H^\dagger H\right)^2. \tag{4.190}$$

From the product $lrlr$ we get the quark contribution to the logarithmic part of the potential given by

$$\begin{aligned}
\mathcal{V}^{\log\Lambda^2}_{L=1,q} = \frac{3}{16\pi^2}\log\Lambda^2 \Big[& \frac{1}{16}\lambda_1^2\lambda_2^2 f^4 \epsilon_{3jk}\epsilon_{3j'k'}\epsilon_{xy}\epsilon_{x'y'} \times \left(\Sigma_{jx}\Sigma_{ky}\Sigma^\dagger_{j'x'}\Sigma^\dagger_{k'y'} + \widetilde{\Sigma}_{jx}\widetilde{\Sigma}_{ky}\widetilde{\Sigma}^\dagger_{j'x'}\widetilde{\Sigma}^\dagger_{k'y'}\right) \\
&+ \frac{1}{16^2}\lambda_1^4 f^4 \left(\epsilon_{ijk}\epsilon_{ij'x'}\epsilon_{xy}\epsilon_{x'y'}\Sigma^\dagger_{jx}\Sigma^\dagger_{ky}\Sigma_{j'x'}\Sigma_{k'y'} + \epsilon_{ijk}\epsilon_{ij'x'}\epsilon_{xy}\epsilon_{x'y'}\widetilde{\Sigma}^\dagger_{jx}\widetilde{\Sigma}^\dagger_{ky}\widetilde{\Sigma}_{j'x'}\widetilde{\Sigma}_{k'y'}\right)^2 \\
&+ \frac{1}{8}\left(\kappa_q\kappa_q^\dagger\right)_{33}\lambda_1^2 f^4 \epsilon_{rjk}\epsilon_{rj'k'}\epsilon_{xy}\epsilon_{x'y'}\left(\widetilde{\Sigma}_{jx}\widetilde{\Sigma}_{ky}\widetilde{\Sigma}^\dagger_{j'x'}\widetilde{\Sigma}^\dagger_{k'y'} + \Sigma_{jx}\Sigma_{ky}\Sigma^\dagger_{j'x'}\Sigma^\dagger_{k'y'}\right) \\
&+ \frac{1}{8}\left(\kappa_q\kappa_q^\dagger\right)_{33}\lambda_1^2 f^4 \epsilon_{rjk}\epsilon_{r'j'k'}\epsilon_{xy}\epsilon_{x'y'}\Big(\left(\Omega\Sigma_0\Sigma\right)_{rr'}\Sigma_{jx}\Sigma_{ky}\widetilde{\Sigma}^\dagger_{j'x'}\widetilde{\Sigma}^\dagger_{k'y'} + \mathrm{h.c.}\Big) \\
&+ \frac{1}{8}\left(\widehat{\kappa}_q\widehat{\kappa}_q^\dagger\right)_{33}\lambda_1^2 f^4 \epsilon_{rjk}\epsilon_{rj'k'}\epsilon_{xy}\epsilon_{x'y'} \\
&\quad \times \left((\widehat{\xi}\Sigma_0)_{br}\widetilde{\Sigma}_{jx}\widetilde{\Sigma}_{ky}\widetilde{\Sigma}^\dagger_{j'x'}\widetilde{\Sigma}^\dagger_{k'y'}(\Sigma_0\widehat{\xi}^\dagger)_{r'b} + (\widehat{\xi}\Sigma_0\Omega)_{br}\widetilde{\Sigma}_{jx}\widetilde{\Sigma}_{ky}\widetilde{\Sigma}^\dagger_{j'x'}\widetilde{\Sigma}^\dagger_{k'y'}(\Omega\Sigma_0\widehat{\xi})_{r'b}\right)
\end{aligned}$$

$$- \frac{1}{8}\left(\widehat{\kappa}_q\widehat{\kappa}_q^\dagger\right)_{33}\lambda_1^2 f^4 \epsilon_{rjk}\epsilon_{r'j'k'}\epsilon_{xy}\epsilon_{x'y'}\left((\widehat{\xi}\Sigma_0)_{br}\Sigma_{jx}\Sigma_{ky}\widetilde{\Sigma}^\dagger_{j'x'}\widetilde{\Sigma}^\dagger_{k'y'}(\Omega\Sigma_0\widehat{\xi})_{r'b} + \text{h.c.}\right)$$
$$- 2\text{tr}\left(\kappa_q\kappa_q^\dagger\widehat{\kappa}_q\widehat{\kappa}_q^\dagger f^4\right)\sum_{a=1}^{3}\sum_{b=1}^{5}\left(\widehat{\Sigma}^\dagger_{ab}\Sigma_{ba} + \Sigma^\dagger_{ab}\widehat{\Sigma}_{ba}\right)\Big], \tag{4.191}$$

where the factor 3 comes from the number of colors, $r, r' = 1, 2$ because the fields in the middle of the multiplets Ψ_1^q and Ψ_2^q are not the same as those in Q_1^t and Q_2^t anb $b = 1, 2, 3$. This term contains a *negative* contribution to the μ^2 parameter of the Higgs potential from the first term in brackets and a contribution similar to leptons up to a factor of 3 coming from the last term

$$\mathcal{V}^{\log\Lambda^2}_{L=1,q} \supset -\frac{3}{16\pi^2}\log\Lambda^2 f^2\lambda_1^2\lambda_2^2\left(H^\dagger H\right) + \frac{3f^2}{8\pi^2}\log\Lambda^2\text{tr}(\kappa_q\kappa_q^\dagger\widehat{\kappa}_q\widehat{\kappa}_q^\dagger)\times\left(6H^\dagger H + \frac{36\widehat{\eta}^2}{5} + 8\text{tr}(\widehat{\omega}^\dagger\widehat{\omega})\right), \tag{4.192}$$

where again we have used the field redefinitions in eq. (4.42). Notice that even though the singlet scalar field $\widehat{\eta}$ in $\widehat{X}$ is coupled to the top partners, it does not develop a mass term proportional to λ_2. This is because in this sector $\widehat{\eta}$ can be understood as the Goldstone boson of the spontaneous breaking of the gauged $[\text{U}(1)]^2$ to $\text{U}(1)$. This prevents the generation of a new contribution to its mass at one loop coming from this sector of the theory.

Physical scalar masses and Higgs potential

We can finally collect our results for the physical scalar masses and the Higgs potential at one loop. From eqs. (4.161) and (4.190) we find the mass of the heaviest T-odd triplet Φ

$$M_\Phi^2 = \frac{\Lambda^2}{4\pi^2}\left(g^2 + g'^2 + 3\lambda_1^2\right) \tag{4.193}$$

and the Higgs quartic coupling

$$\lambda = \frac{1}{16\pi^2}\frac{\Lambda^2}{f^2}\left(g^2 + g'^2 + 3\lambda_1^2\right), \tag{4.194}$$

that are proportional to the cutoff squared. This last equation provides an explicit relation between the cutoff scale and one of the top quark Yukawa couplings since the experimental value of λ is $\lambda = M_h^2(2v^2) \approx 0.13$. The mass for the new T-odd triplet $\widehat{\omega}$ is given by eqs. (4.164), (4.173) and (4.192),

$$M_{\widehat{\omega}}^2 = \frac{f^2}{\pi^2}\log\Lambda^2\left[g^4 + T_\kappa\right], \tag{4.195}$$

with

$$T_\kappa \equiv \text{tr}(\kappa\kappa_l^\dagger\widehat{\kappa}_l\widehat{\kappa}_l^\dagger) + 3\text{tr}(\kappa_q\kappa_q^\dagger\widehat{\kappa}_q\widehat{\kappa}_q^\dagger). \tag{4.196}$$

The new T-odd scalar $\widehat{\eta}$ only receives a contribution from eqs. (4.173) and (4.192),

$$M_{\widehat{\eta}}^2 = \frac{f^2}{\pi^2}\log\Lambda^2\frac{9}{5}T_\kappa. \tag{4.197}$$

And the μ^2 parameter of the Higgs potential follows from eqs. (4.164), (4.173) and (4.192),

$$\mu^2 = \frac{f^2}{16\pi^2}\log\Lambda^2\left(6g^4 + \frac{2}{5}g'^4 - 3\lambda_1^2\lambda_2^2 + 12T_\kappa\right). \tag{4.198}$$

To obtain the physical mass of the Higgs boson we have to minimize the potential of eq. (4.150). If the contribution of the top sector dominates over the heavy Yukawa and gauge interactions then $\mu^2 < 0$ and the EWSB is triggered,

$$\frac{\partial \mathcal{V}_{\text{Higgs}}}{\partial H^\dagger} = 0 \quad \Rightarrow \quad \mu^2 H + 2\lambda \left(H^\dagger H \right) H = 0 \tag{4.199}$$

when the neutral component of the Higgs doublet gets a *vev*

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = \sqrt{\frac{-\mu^2}{\lambda}}. \tag{4.200}$$

In our case, this expression gives

$$v^2 = \frac{f^4}{\Lambda^2} \log \Lambda^2 \frac{3\lambda_1^2\lambda_2^2 - 6g^4 - \frac{2}{5}g'^4 - 12T_\kappa}{g^2 + g'^2 + 3\lambda_1^2}. \tag{4.201}$$

Then, from $M_h^2 = -2\mu^2 = 2\lambda v^2$, and eqs. (4.194) and (4.201) we have

$$M_h^2 = \frac{f^2}{8\pi^2} \log \Lambda^2 \left(3\lambda_1^2\lambda_2^2 - 6g^4 - \frac{2}{5}g'^4 - 12T_\kappa \right), \tag{4.202}$$

whose experimental value is $M_h \simeq 125$ GeV. Comparing previous expressions, we find the same relation between the masses of the Higgs and the usual triplet of the LHT,

$$M_\Phi^2 = 2\frac{f^2}{v^2} M_h^2. \tag{4.203}$$

Its mass is proportional to the scale f and thus it is naturally heavy. This is a consequence of the absence of any symmetry or further mechanism that protects its mass from quadratically divergent corrections. For the rest of the new T-odd scalars we find

$$M_{\widehat{\omega}}^2 = 8M_h^2 \frac{g^4 + T_\kappa}{3\lambda_1^2\lambda_2^2 - 6g^4 - \frac{2}{5}g'^4 - 12T_\kappa}, \tag{4.204}$$

$$M_{\widehat{\eta}}^2 = \frac{72}{5} M_h^2 \frac{T_\kappa}{3\lambda_1^2\lambda_2^2 - 6g^4 - \frac{2}{5}g'^4 - 12T_\kappa}. \tag{4.205}$$

where $3\lambda_1^2\lambda_2^2 - 6g^4 - \frac{2}{5}g'^4 - 12T_\kappa$ must be positive to be compatible with the SM SSB in eq. (4.198). The masses of the new T-odd scalars are proportional to the Higgs mass and independent of the scale f. This implies that they are naturally light, but they are strictly increasing with T_κ for fixed values of λ_1, λ_2. This is a consequence of the gauge symmetry since these new scalars share the same quantum numbers as the would-be Goldstone bosons and they mix with them at leading order (see eq. (4.42)). However, for the theory to be consistent, their masses must remain below the cutoff scale Λ. Notice that taking the ratio of eqs. (4.204) and (4.204) we obtain

$$M_{\widehat{\eta}}^2 = \frac{9}{5} \frac{T_\kappa}{T_\kappa + g^4} M_{\widehat{\omega}}^2, \tag{4.206}$$

and thus from $g \approx 0.653$ one finds $M_{\widehat{\eta}} > M_{\widehat{\omega}}$ if $T_\kappa \geq \frac{5}{4}g^4 \approx 0.227$.

4.2.3. LFV Higgs decays with a T-odd singlet in the LHT revisited

Using the BFM one can obtain the divergent operator for LFV Higgs decays in the T-odd singlet case (see § 3.2.1) within the original LHT scenario in chapter 3. The divergence comes from topologies XI+XII in fig. 3.1 when the right-handed T-odd singlet χ_R and the scalar fields $\omega^\pm$ and $\phi^\pm$ propagate inside the loop. According to eq. (3.158), the resulting operator in the SM symmetric phase involves two Higgs doublets, two SM left-handed fermion doublets and one derivative. Thus, one uses eq. (4.149) to integrate out the right-handed fermion multiplet Ψ_R, where χ_R lives, and one scalar field in ξ.

The Yukawa Lagrangian responsible of providing the couplings to compute the T-odd singlet contribution to LFV Higgs decays is $\mathcal{L}_{Y_H}^{(b)}$ in eq. (3.40). This Lagrangian depends explicitly on Ω. Consequently, as it was shown in § 3.3, it is not invariant under the gauge group $[\mathrm{SU}(2)\times \mathrm{U}(1)]^2 \subset \mathrm{SU}(5)$. This is because given a gauge transformation V_g, the associated non linear transformation U_g in general depends on all the SO(5) generators and Ω does not commute with all of them. Hence, one would not expect that the resulting counterterm respects the gauge symmetry either. However, $\mathcal{L}_{Y_H}^{(b)}$ is invariant under transformations of the diagonal $\mathrm{SU}(2)\times \mathrm{U}(1) \in \mathrm{SO}(5)$ associated to the SM gauge group. According to the general CCWZ formalism, transformations restricted to the unbroken group becomes linear. As a consequence, since the SM is contained in the unbroken SO(5), $V_g = U_g$ with U_g depending only on the SM gauge generators that commute with Ω. Thus the counterterm would be invariant under the SM as well, since the BFM respects the gauge symmetry.

To start with the derivation of the operator, we need the expression for the matrices Γ and $\overline{\Gamma}$ in eq. (4.102) after expanding the Yukawa Lagrangian $\mathcal{L}_{Y_H}^{(b)}$ in fermion and scalar quantum fluctuations. According to [138, 139, 150], when the scalar fields come parametrized in a non linear sigma field ξ, it is common to perform a multiplicative split instead of a linear one. In our case,

$$\xi \xrightarrow{\text{BFM}} \xi\Xi \approx \xi\left(\mathbb{1} + i\frac{\pi_q^a}{f}X^a\right), \tag{4.207}$$

where $\Xi = e^{i\pi_q^a X^a/f}$ and π_q^a are the quantum fluctuations of the Goldstone fields. The exponential is expanded up to first order since only a single scalar field propagates inside the loop. On the other hand one performs the usual linear split for the right-handed fermion fields in eq. (4.96) and uses eq. (4.165)

$$\mathcal{L}_{Y_H}^{(b,2)} \supset -\kappa_{ij} f\left(\overline{\Psi}_{2i}\xi\frac{i\pi_q^a}{f}X^a + \overline{\Psi}_{1i}\Sigma_0\Omega\frac{-i\pi_q^a}{f}X^a\xi^\dagger\Omega\right)\mathcal{A}\psi_{Rj} + \text{h.c.} \tag{4.208}$$

where i, j are flavour indices and we used the basis of hermitian generators. Thus, the matrices $\overline{\Gamma}$ and Γ read

$$\overline{\Gamma}^a = -i\kappa_{ij}\left(\overline{\Psi}_{2i}\xi X^a - \overline{\Psi}_{1i}\Sigma_0\Omega X^a\xi^\dagger\Omega\right)\mathcal{A}, \tag{4.209}$$

$$\Gamma^a = i\kappa_{ji}^\dagger\mathcal{A}^\dagger\left(X^a\xi^\dagger\Psi_{2i} - \Omega\xi X^a\Omega\Sigma_0\Psi_{1i}\right). \tag{4.210}$$

Substituting in eq. (4.149) and using that $\mathcal{A}$ is unitary due to the complete SO(5) right-handed multiplet Ψ_R yields

$$\mathcal{L}_{\text{c.t.}} = \frac{1}{32\pi^2}\log\Lambda^2\kappa_{ij}\kappa_{jl}^\dagger\left(\overline{\Psi}_{2i}\xi X^a - \overline{\Psi}_{1i}\Sigma_0\Omega X^a\xi^\dagger\Omega\right)i\partial\!\!\!/\left(X^a\xi^\dagger\Psi_{2l} - \Omega\xi X^a\Omega\Sigma_0\Psi_{1l}\right). \tag{4.211}$$

However, a careful inspection of the previous expression reveals that T-parity is not preserved. Using eqs. (3.25) and (3.32) in the operator above results in

$$\begin{aligned}
&\left(\overline{\Psi}_{2i}\xi X^a - \overline{\Psi}_{1i}\Sigma_0\Omega X^a\xi^\dagger\Omega\right) i\partial\!\!\!/ \left(X^a\xi^\dagger\Psi_{2l} - \Omega\xi X^a\Omega\Sigma_0\Psi_{1l}\right)\\
&\xrightarrow{T} \left(\overline{\Psi}_{2i}\Omega X^a\Omega\xi - \overline{\Psi}_{1i}\Sigma_0\Omega\xi^\dagger\Omega X^a\right) i\partial\!\!\!/ \left(\xi^\dagger\Omega X^a\Omega\Psi_{2l} - X^a\Omega\xi\Omega\Sigma_0\Psi_{1l}\right)\\
&= \left(\overline{\Psi}_{2i}X^a\xi - \overline{\Psi}_{1i}\Sigma_0\Omega\xi^\dagger X^a\Omega\right) i\partial\!\!\!/ \left(\xi^\dagger X^a\Psi_{2l} - \Omega X^a\xi\Omega\Sigma_0\Psi_{1l}\right),
\end{aligned} \tag{4.212}$$

where in the last equality we used that Ω belongs to SO(5) because $\Omega\Sigma_0\Omega = \Sigma_0$ and thus leaves invariant the Lie algebra subspace generated by the orthogonal set of broken generators $\Omega X^a\Omega = X^{a'} = c_a^{a'}(\Omega)\, X^a$ with $c_a^{a'}$ are in general a function of the SO(5) transformation verifying the orthogonality condition $c_a^{a'}c_b^{a'} = \delta_{ab}$. This is the action of the adjoint representation of SO(5) in the Lie algebra of SU(5). Notice that the non T-invariance comes from the fact that X^a and ξ are interchanged with respect to the original expression (4.211).

Consequently, the usual multiplicative split for the scalar fields parametrized in a non linear sigma field is not valid in general. To understand why the multiplicative split fails, consider the following scheme

$$\begin{array}{ccc}
\xi & \overset{\text{BFM}}{\rightarrow} & \xi\Xi \\
T\downarrow & & \downarrow T \\
\Omega\xi^\dagger\Omega & \overset{\text{BFM}}{\rightarrow} & \Omega\Xi^\dagger\xi^\dagger\Omega \overset{?}{=} \Omega\xi^\dagger\Xi^\dagger\Omega,
\end{array} \tag{4.213}$$

where the last equality only holds when $\left[\Pi, \Pi_q\right] = 0$ and this is not the case. Then, since the BFM and T-parity do not commute, we are forced to use the linear split for the scalar fields

$$\Pi \xrightarrow{\text{BFM}} \Pi + \Pi_q. \tag{4.214}$$

Again, as only a single scalar field propagates inside the loop, we Taylor-expand up to first order in the quantum fluctuations of the scalar fields. This yields

$$\xi \xrightarrow{\text{BFM}} e^{i(\Pi+\Pi_q)/f} \approx \xi + \xi^a_{(1)}\frac{\pi^a}{f}, \tag{4.215}$$

where $\xi^a_{(1)}$ is the *parametric derivative* of ξ with respect to the Goldstone fields [151]

$$\xi^a_{(1)} = f\frac{\partial\xi}{\partial\pi^a} = f\sum_{n=1}^{\infty}\frac{1}{n!}\left(\frac{i}{f}\right)^n\frac{\partial\Pi^n}{\partial\pi^a}. \tag{4.216}$$

Since the generators X^a do not commute among themselves the derivative of any power of Π with respect to the Goldstone fields can be written as

$$\frac{\partial\Pi^n}{\partial\pi^a} = X^a\Pi^{n-1} + \Pi X^a\Pi^{n-2} + ... + \Pi^{n-2}X^a\Pi + \Pi^{n-1}X^a. \tag{4.217}$$

The parametric derivative of ξ, $\xi^a_{(1)}$, can be expressed in closed form using the integral representation [151, 152]

$$\xi^a_{(1)} = f\int_0^1 d\tau e^{(1-\tau)\Pi/f} iX^a e^{\tau\Pi/f}. \tag{4.218}$$

Substituting eq. (4.215) and the linear split for the fermion field in $\mathcal{L}_{Y_H}^{(b)}$

$$\mathcal{L}_{Y_H}^{(b,2)} \supset -\kappa_{ij} f \left(\overline{\Psi}_{2i} \xi^a_{(1)} \frac{\pi^a_q}{f} + \overline{\Psi}_{1i} \Sigma_0 \Omega \frac{\pi^a_q}{f} \xi^{a\dagger}_{(1)} \Omega \right) \mathcal{A} \psi_{Rj} + \text{h.c.} \tag{4.219}$$

we find the form of the new matrices $\overline{\Gamma}$ and Γ,

$$\overline{\Gamma}^a = -\kappa_{ij} \left(\overline{\Psi}_{2i} \xi^a_{(1)} + \overline{\Psi}_{1i} \Sigma_0 \Omega \xi^{a\dagger}_{(1)} \Omega \right) \mathcal{A}, \tag{4.220}$$

$$\Gamma^a = -\kappa^\dagger_{ji} \mathcal{A}^\dagger \left(\xi^{a\dagger}_{(1)} \Psi_{2i} + \Omega \xi^a_{(1)} \Omega \Sigma_0 \Psi_{1i} \right), \tag{4.221}$$

and the form of the new counterterm,

$$\mathcal{L}_{\text{c.t.}} = \frac{1}{32\pi^2} \log \Lambda^2 \kappa_{ij} \kappa^\dagger_{jl} \left(\overline{\Psi}_{2i} \xi^a_{(1)} + \overline{\Psi}_{1i} \Sigma_0 \Omega \xi^{a\dagger}_{(1)} \Omega \right) i\partial\!\!\!/ \left(\xi^{a\dagger}_{(1)} \Psi_{2l} + \Omega \xi^a_{(1)} \Omega \Sigma_0 \Psi_{1l} \right). \tag{4.222}$$

In order to check whether the new operator is invariant under the SM group and T-parity, one needs to obtain the transformation properties of the parametric derivative $\xi^a_(1)$. To that end, one substitutes $\Pi \xrightarrow{\text{SO(5)}} U \Pi U^\dagger$ and $\Pi \xrightarrow{T} -\Omega \Pi \Omega$ in eq. (4.218),

$$\xi^a_{(1)} \xrightarrow{\text{SO(5)}} c^{a'}_a (U) U \xi^a_{(1)} U^\dagger, \quad \xi^a_{(1)} \xrightarrow{\text{T}} -c^{a'}_a (\Omega) \Omega \xi^{a\dagger}_{(1)} \Omega. \tag{4.223}$$

Thus, particularizing for a SM transformation $U = U_g$ and using the orthogonality property of the coefficients $c^{a'}_a$, the counterterm Lagrangian (4.149) is invariant under the SM group and T-parity. [13]

Since the singlet χ_R lives in the third entry of the fermion multiplet Ψ_R, to isolate its contribution from the operator in eq. (4.222) one needs to select the third entry of the row and column vectors $\overline{\Gamma}$ and Γ. Using the expression for the broken generators in appendix A, the resulting operators involving two Higgs doublets read

$$\mathcal{L}_{\text{c.t.}} \supset \frac{1}{32\pi^2} \log \Lambda^2 \kappa_{ij} \kappa^\dagger_{jl} \frac{1}{8f^2} \left[\left(\bar{l}_{iL} \sigma^a H \right) i\partial\!\!\!/ \left(H^\dagger \sigma^a l_{lL} \right) + \frac{1}{2} \left(\bar{l}_{iL} \sigma^a \widetilde{H} \right) i\partial\!\!\!/ \left(\widetilde{H}^\dagger \sigma^a l_{lL} \right) \right], \tag{4.224}$$

in agreement with eq. (3.158). Using the EoM for the lepton fields, substituting one of the Higgs doublets for their corresponding *vev* and the relation $m_{\ell_H} = \sqrt{2}\kappa f$, one can recover the result for the divergences in LFV in § 3.2.1 (reminding the identification $2/\epsilon \leftrightarrow \log \Lambda^2$),

$$\mathcal{L}_{\text{c.t.}} \supset 2 \times \frac{1}{16\pi^2} \log \Lambda^2 \frac{v^2}{64 f^2} V^\dagger_{\ell' j} \frac{m^2_{\ell_{Hj}}}{f^2} V_{j\ell} h \overline{\ell'} \left(\frac{m_{\ell'}}{v} P_L + \frac{m_\ell}{v} P_R \right) \ell, \tag{4.225}$$

where we have diagonalized the Yukawa coupling κ according to eq. (3.100) and the left-handed leptons are rotated as in eq. (3.96). The factor of two comes from the sum of the two different operators in eq. (4.224).

An important remark is that in the T-even case, that can be obtained straightforwardly from the T-odd case through the substitution $\Omega \to \mathbb{1}$ in eq. (4.222), one obtains an operator similar to that in eq. (4.224) but with the T-odd mirror leptons instead of the SM leptons. This explains why in the T-even case the contributions of the singlet to LFV Higgs decays are finite on their own.

[13] Technically speaking this piece is only invariant under SM global transformations. The full counterterm would include the rest of the interactions coming from the terms including N_μ and $G\gamma_\mu$ in eq. (4.140) leading to the SM gauge invariance.

Finally, the operator in eq. (4.222) should be added to the original LHT Lagrangian with an arbitrary constant in order to renormalize the corresponding divergence. This constant is thus a free parameter that should be measured by the experiment. However, this is very unlikely from the theoretical point of view because apart from being unable to predict the contribution of the singlet to LFV Higgs decays in the T-odd scenario, there is no fundamental reason why the constant should be small in order to reproduce the current measures.

4.3. Phenomenology of the NLHT

In this section we will study the parameter space and the particle spectrum of the NLHT compatible with current EWPD and cosmology constraints paying special attention to the decay channels and lifetime of the new T-odd scalars. We will restrict ourselves to a simplified version of the model. This will lead to interesting correlations between the top quark and the rest of the heavy quarks Yukawa couplings, constraining the interval of allowed values of the NP scale f and fixing the value of the leptons Yukawa coupling.

4.3.1. Parameter space

Before constraining the parameter space of the model, some relations must be taken into account. Let us focus first on the Yukawa couplings λ_1 and λ_2 in eqs. (3.87). They are not independent but related through the top quark mass $m_t \approx 173$ GeV [153] in eq. (3.87). At leading order in v one finds

$$\frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} = \left(\frac{v}{\sqrt{2}m_t}\right)^2 . \tag{4.226}$$

We will choose λ_1 as the independent parameter. On the other hand, it is clear from eq. (3.89), that the T-even top partner T_+ is heavier and we have to impose that its mass must remain below the cutoff scale Λ. Otherwise it should be integrated out and would not be part of the spectrum of the theory. The cutoff scale Λ is related to the Higgs quartic coupling $\lambda = \frac{1}{2}(M_h/v)^2 \approx 0.13$, the Yukawa coupling λ_1 and the gauge couplings $g \approx 0.641$ and $g' \approx 0.344$ through eq. (4.194), where $g = e/s_W$ and $g' = e/c_W$ with $s_W^2 = 1 - M_W^2/M_Z^2$ and $e^2 = 4\pi\alpha$. This condition constrains the value of λ_1 to be in the interval $\lambda_1 \in [1.05, 1.71]$ and equivalently $\lambda_2 \in [1.22, 3.10]$, as shown in fig. 4.4. Since the cutoff scale and the top quark-partner masses are proportional to the NP scale f, fig. 4.4 also shows that $\Lambda/f \in [1.49, 2.32]$. These values of the cutoff are compatible with the perturbative limit $\Lambda \leq 4\pi f$. In addition, current EWPD constraints impose that vector-line quarks must be heavier than about 2 TeV [131, 132]. Therefore, we will force the lighter top partner T_- to have a mass $M_{T_-} > 2$ TeV, implying $f \simeq 900$ GeV.

In the NLHT, T-parity is an exact discrete symmetry. This implies that the lightest T-odd particle (LTP) of the spectrum is stable. To be a viable dark matter, the LTP must be electrically neutral and have weak interactions. In this model, the potentially light and neutral T-odd particles are the A_H gauge boson, the neutral leptons with masses proportional to the free parameters κ_l and $\widehat{\kappa}_l$ in eq. (4.84), the T-odd singlet $\widehat{\eta}$ and the neutral component of the T-odd real triplet $\widehat{\omega}^0$. The neutral components of the original complex triplet Φ, ϕ^0 and ϕ^P, do not play a role here. They are always heavier than A_H since comparing eqs. (4.40) and (4.203) one finds that $M_{A_H}/f = g'\sqrt{2/5} \approx 0.217$ and $M_\Phi/f = \sqrt{2}M_h/v \approx 0.718$. At leading order, the neutral leptons share the same mass as the charged ones and the same happens with the neutral and the charged components of the real triplet $\widehat{\omega}$. This rules them out. Therefore, the possible dark matter candidates in our model are the scalar singlet $\widehat{\eta}$ and the A_H gauge boson.

Let us briefly comment the scenario in which the singlet $\widehat{\eta}$ is the dark matter candidate. As we discussed in § 4.2.2, in the region of the parameter space where the traces T_κ in eq. (4.196)

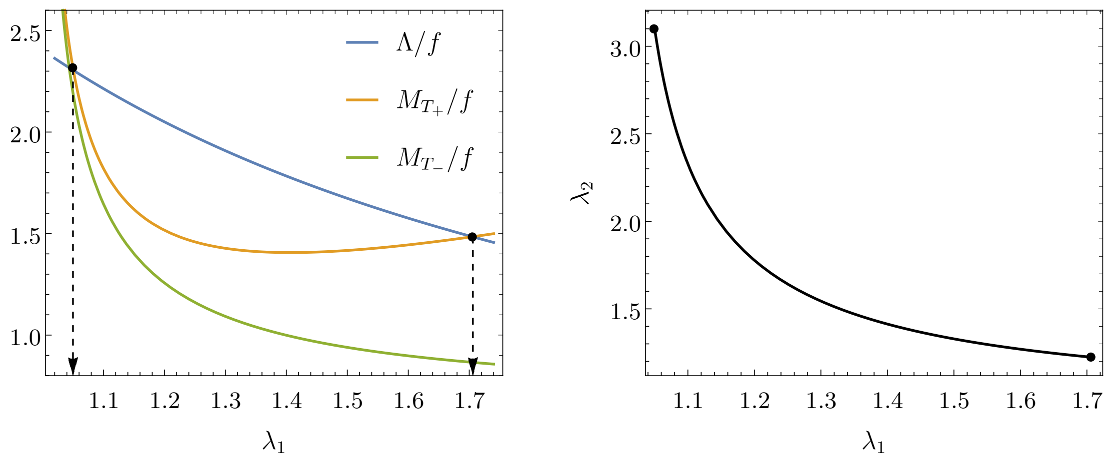


FIGURE 4.4: The interval of $\lambda_1 \in [1.05, 1.71]$ yielding a top quark-partner mass M_{T_+} below the scale Λ on the left-hand-side plot determines the range of possible values of $\lambda_2 \in [1.22, 3.10]$ through eq. (3.87) on the plot of the right-hand side. These intervals are independent of f but the mass scales are proportional to f.

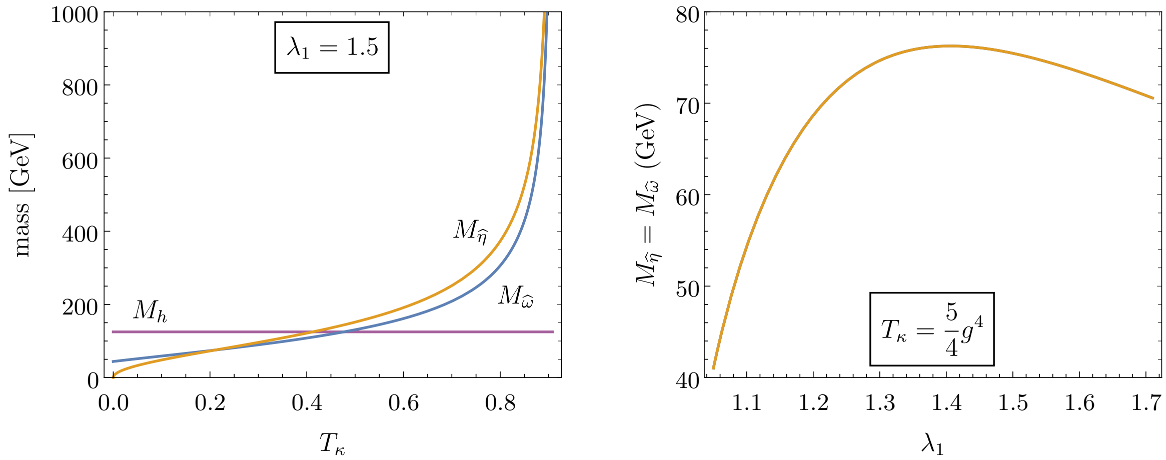


FIGURE 4.5: Left: masses of the Higgs and the new T-odd scalars as a function of T_κ for a chosen value of $\lambda_1 = 1.5$. They are naturally light except for T_κ close to its minimum ($T^{\max}(\lambda_1 = 1.5) \approx 0.90$). Right: mass of the new T-odd singlet scalar field $\hat{\eta}$ as a function of the allowed values of $\lambda_1 \in [1.05, 1.71]$ when $M_{\hat{\eta}} = M_{\hat{\omega}}$.

satisfies the bound $T_\kappa < \frac{5}{4}g^4 \approx 0.211$, the scalar singlet is lighter than the real triplet $\hat{\omega}$ (see eq. (4.206)). According to fig. 4.5, its mass is bounded from above to $M_{\hat{\eta}} \lesssim 80$ GeV, and since $M_{A_H} \gtrsim 200$ GeV for $f \gtrsim 900$ GeV, the singlet would be the LTP if the non SM leptons are heavy enough. This opens a possible window to light scalar dark matter, as already suggested in ref. [154], that in principle has room in the NLHT. However, vector dark matter candidates are less constrained (see for instance [155]) and then we will work assuming the scenario where A_H is the LTP. This will allow us to compare with previous works [103, 156, 157] in the context of the original LHT. Then the new scalars must be heavier than the heavy photon A_H, which also implies that the singlet is heavier than the triplet. For that reason necessarily $T_\kappa > 0.227$ in order to get scalar masses above 200 GeV due to the strictly increasing behaviour of the new scalar masses with T_κ as it is shown in fig. 4.5.

4.3.2. A simplified model

It is well known that, in order to reproduce within the LHT the current dark matter relic density $\Omega h^2 = 0.120 \pm 0.001$ [158], it is necessary the presence of co-annihilators if the LTP has $M_{A_H} \gtrsim 200$ GeV since A_H cannot account by itself all the dark matter relic density. In ref. [156], for instance, the T-odd leptons and quarks share masses and were taken nearly mass-degenerate with the A_H gauge boson. However, according to the current constrains on the vector-like quark masses already mentioned [131, 132], this assumption would lead to a very heavy LTP. In contrast, the NLHT includes more leptonic degrees of freedom than the LHT due to the requirements of gauge invariance and absence of quadratically divergent contributions to the Higgs mass (see § 4.1).

Here we will analyze a simplified NLHT model in which all heavy lepton and quark Yukawa couplings are diagonal and degenerate in flavour space, that is, $\kappa_{l,i} = \widehat{\kappa}_{l,i} \equiv \kappa_l$ for leptons and $\kappa_{q,i} = \widehat{\kappa}_{q,i} \equiv \kappa_q$ for quarks, but they have different masses $m_{\ell_H} = \sqrt{2}\kappa_l f \gtrsim M_{A_H}$ and $m_{q_H} = \sqrt{2}\kappa_q f$. The chosen degeneracy implies that the traces appearing in the expression for the masses of the new T-odd scalars in eqs. (4.204) and (4.205) reduce to $T_\kappa = 3\kappa_l^4 + 9\kappa_q^4$. We will consider that only the T-odd heavy leptons can act as co-annihilators.

One should keep in mind that the contribution of a co-annihilator to the dark matter relic density is exponentially suppressed with the temperature as $\sim e^{-\Delta/T_{\rm fo}}$, where Δ is the mass splitting between the dark matter candidate and the co-annihilator and $T_{\rm fo} \sim 20-30$ GeV is the freeze-out temperature [159]. To prevent that the scalars influence the relic density, we safely impose a lower bound to the mass of the lighter T-odd real triplet $M_{\widehat{\omega}} > M_{A_H} + M_h$ using the fact that the scalar masses can take any value, adjusting T_κ according to fig. (4.5). This condition sets a minimum value for the traces $T^{\rm min}_{\kappa,\widehat{\omega}}$ depending on f, through M_{A_H}, and λ_1.

Implementing this simplified version of our model in FeynRules [160], using micrOMEGAs [161] to evaluate the relic density together with the relevant processes and finally running T3PS [162] to scan the parameter space we obtain the plot in fig. 4.6. The main contributions to the dark matter relic density come from the co-annihilation between T-odd leptons to pair-produce SM gauge bosons $W^\pm$ and Z. In the plot, the narrow orange band covers the region where the relic abundance of A_H is compatible with 100% of the observed dark matter density. This can be approximated by the expression

$$m_{\ell_H} \approx 1.16 M_{A_H}, \tag{4.227}$$

for $M_{A_H} \in [200, 800]$ GeV. From the expressions of M_{A_H} in eq. (4.40) and $m_{\ell_H} = \sqrt{2}\kappa_l f$, the heavy lepton Yukawa coupling is then fixed to $\kappa_l \approx 0.185$ for $f \gtrsim 900$ GeV.

We have already set a lower bound to the lighter T-odd scalar mass, $M_{\widehat{\omega}} > M_{A_H} + M_h$. Using naturalness arguments one can also expect upper bounds for the scalars. As previously emphasized, the new scalar masses are proportional to the Higgs mass and are independent of the NP scale f, so they should remain light. To be definite we impose the upper bound $M_{\widehat{\eta}} < 1$ which provides the maximum trace values $T^{\rm max}_{\kappa,\widehat{\eta}}$ for which $M_{\widehat{\eta}} = 1$ TeV given λ_1 and it is independent of the scale f. Imposing the condition $T^{\rm max}_{\kappa,\widehat{\eta}} > T^{\rm min}_{\kappa,\widehat{\omega}}$ in the $\lambda_1 - f$ plane yields to the upper bound $f \lesssim 3.1$ TeV as shown in fig. 4.7. The lower bound that follows from $M_{T\_} > 2$ TeV is also depicted.

On the rest of heavy quarks, with masses proportional to κ_q, the condition that sets the A_H as the LTP implies $m_{q_H}/f > M_{A_H}/f$, that is $\kappa_q > g'/\sqrt{5} \approx 0.16$. footnoteThis condition is not necessary for the top quark-partners. The lighter $T\_$ has a mass $M_{T\_} = f\lambda_2/\sqrt{2}$ with $\lambda_2 \in [1.22, 3.10]$ and is always heavier than A_H since $\lambda_2/\sqrt{2} > \sqrt{2/5}g'$. However the current bound on the heavy quark masses sets a limit $\kappa_q > \sqrt{2}$ TeV$/f$, condition that is stronger for

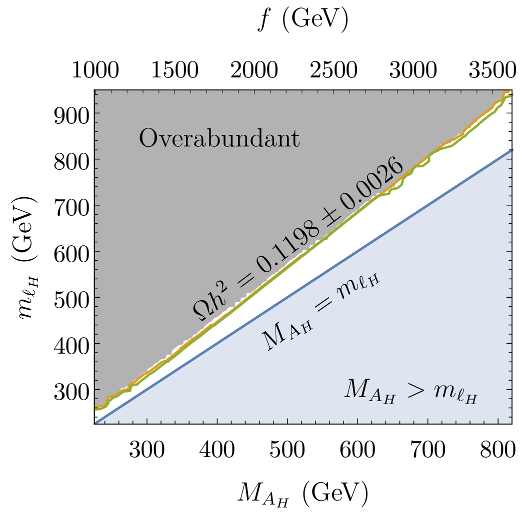


FIGURE 4.6: Region in the $M_{A_H} - m_{\ell_H}$ plane compatible with A_H as dark matter candidate (white). The A_H constitutes 100% (50%) of the dark matter abundance for masses within the orange (green) bands. The grey region is excluded because it would yield dark matter overabundance and the blue region is excluded because A_H would not be the LTP.

values of $f \lesssim 3$ TeV allowed by fig. 4.7. From this and the already constrained value of $\kappa_l \approx 0.185$ we derive the restriction $T_\kappa > T^{\min}_{\kappa,q_H}(f) \equiv 0.0035 + 36\ \text{TeV}^4/f^4$. Imposing the condition $T^{\max}_{\kappa,\hat{\eta}} > T^{\min}_{\kappa,q_H}(f)$ sets a lower bound for f and the definite allowed region in the $\lambda_1 - f$ plane of fig. 4.7, which features a possible window for f between 2.0 and 3.1 TeV. The minimum for the traces is actually given by $T^{\min}_{\kappa,\hat{\omega}}(\lambda_1, f)$.

4.3.3. Particle spectrum and scalar decays

As a consequence of all the previous constraints, the Yukawa couplings of the heavy quarks κ_q and the top quark coupling λ_1 get strongly correlated as we show in fig. 4.8. This correlation is nearly independent of f. This is because the maximum and minimum values of κ_q are extremely close to each other due to the asymptotic behaviour of the new scalar masses with T_κ since they must be heavier than A_H with $M_{A_H} \gtrsim 450$ GeV for $f \approx 2$ TeV and $T^{\max}_{\kappa,\hat{\eta}}$ does not depend on f. However, not all the values of λ_1 are available for every possible value of f, except for $f \in [2.5, 3.0]$, as can be seen in fig. 4.7 and is reflected in fig. 4.8. Comparing figs. 4.4 and 4.8 one can explicitly check that heavy quarks are safely below the cutoff scale because $m_{q_H}/f = \sqrt{2}\kappa_q < \Lambda/f$. Their common masses are in practice a function of λ_1, ranging between $0.8f$ and f.

Regarding the scalar masses, fig. 4.9 shows that they are constrained to the intervals $M_{\hat{\omega}} \in [600, 800]$ GeV and $M_{\hat{\eta}} \in [800, 1000]$ GeV. These intervals only depend on f. The maximum values are fixed by the condition $M_{\hat{\eta}} < 1$ TeV. The minimum values come from the condition $M_{\hat{\omega}} > M_{A_H} + M_h$ depending on f. The allowed values for λ_1 for a given f are determined by the conditions $T^{\max}_{\kappa,\hat{\eta}}(\lambda_1) > T^{\min}_{\kappa,\hat{\omega}}(\lambda_1, f)$ and $T^{\max}_{\kappa,\hat{\eta}}(\lambda_1) > T^{\min}_{\kappa,q_H}(f)$. For completeness we show below the mass ranges of the T-odd gauge bosons, the usual T-odd complex scalar triplet and the heavy leptons for the allowed values of the scale f,

$$M_{A_H} \in [450, 680]\ \text{GeV}, \tag{4.228}$$

$$M_{W_H} = M_{Z_H} \in [1850, 2750]\ \text{GeV}, \tag{4.229}$$

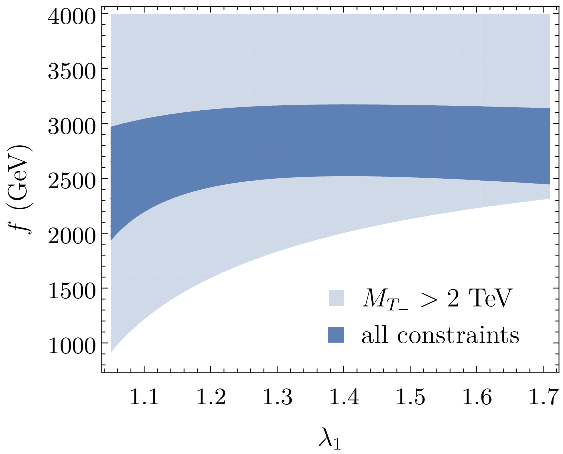


FIGURE 4.7: Values of f compatible with $M_{T_-} > 2$ TeV as a function of λ_1 (light blue). The region is further constrained (darker blue) by the conditions $T^{\max}_{\kappa,\widehat{\eta}}(\lambda_1) > T^{\min}_{\kappa,\widehat{\omega}}(\lambda_1, f)$ (upper bound) and $T^{\max}_{\kappa,\widehat{\eta}}(\lambda_1) > T^{\min}_{\kappa,q_H}(f)$ (lower bound) as explained in the text.

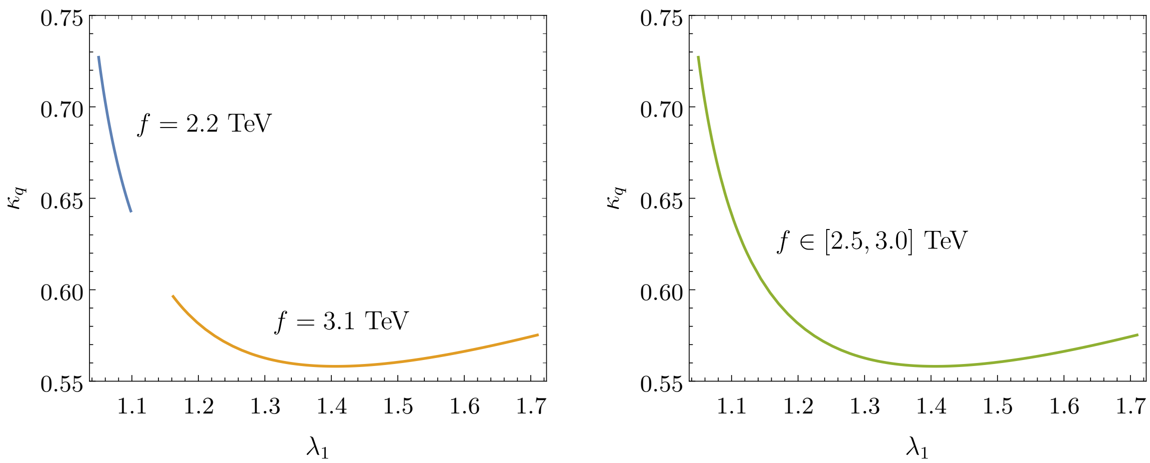


FIGURE 4.8: Correlation between the heavy quarks Yukawa coupling κ_q and the top quark Yukawa coupling λ_1 for different values of the scale f.

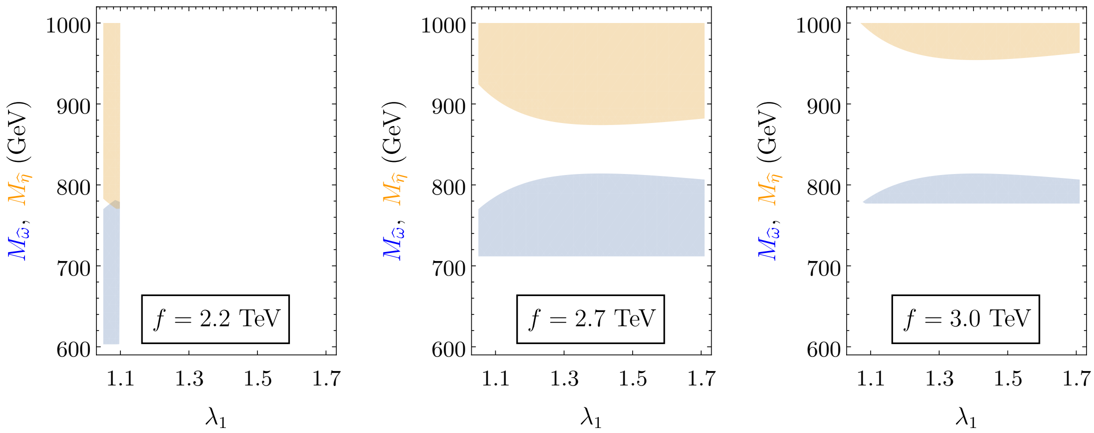


FIGURE 4.9: Ranges of allowed masses of the new T-odd scalars for different values of the scale f as a function of λ_1

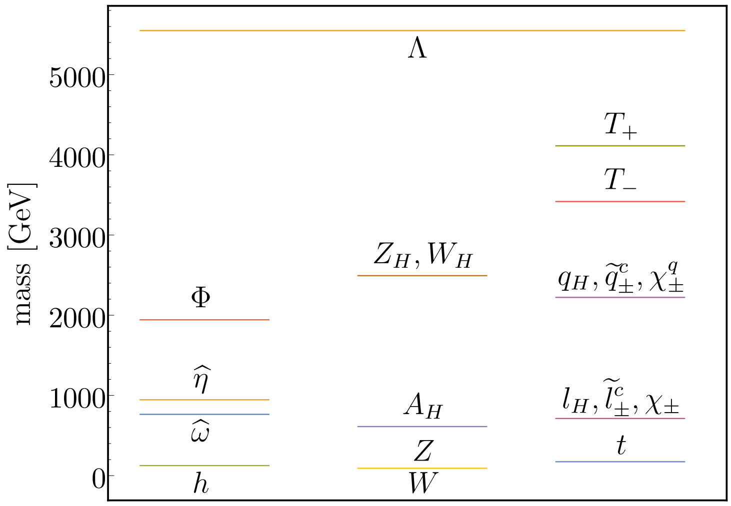


FIGURE 4.10: Typical NLHT spectrum for $f \approx 2.7$ TeV and $\lambda_1 = 1.2$.

$$M_\Phi \in [1450, 2150] \text{ GeV}, \tag{4.230}$$

$$m_{\ell_H} \in [530, 800] \text{ GeV}. \tag{4.231}$$

Our vector-like leptons decay to one standard lepton and a heavy photon, an exotic decay that is not excluded in principle by current LHC searches within these mass ranges [163]. For $f = 2.7$ GeV, the typical situation is shown in fig. 4.10, where we show the scalar fields on the left-hand side, the gauge bosons on the middle and the fermions on the right-hand side. We also show the cutoff scale for these particular values of f and λ_1 for further comparison. In the NLHT, as in the original LHT, the top quark-partners $T_\pm$ with masses proportional to the order one Yukawa couplings λ_1 and λ_2 are the heaviest particles. The rest of heavy T-even and T-odd quarks are lighter, since their masses are proportional to the Yukawa coupling κ_q in our simplified model, with $\kappa_q \in [0.55, 0.73]$. Both types of quarks are heavier than 2 TeV to avoid current EWPD constraints [131, 132]. Moving to the gauge bosons sector, the T-odd W_H and Z_H are the the following heaviest particles after the top quark-partners. There is a gap of more of 1 TeV with the neutral A_H gauge boson, being the LTP and our dark matter candidate. As our model requires co-annihilations to reproduce the current dark matter relic density of the universe, the heavy T-odd leptons, that share masses with the T-even ones, are a few tens of

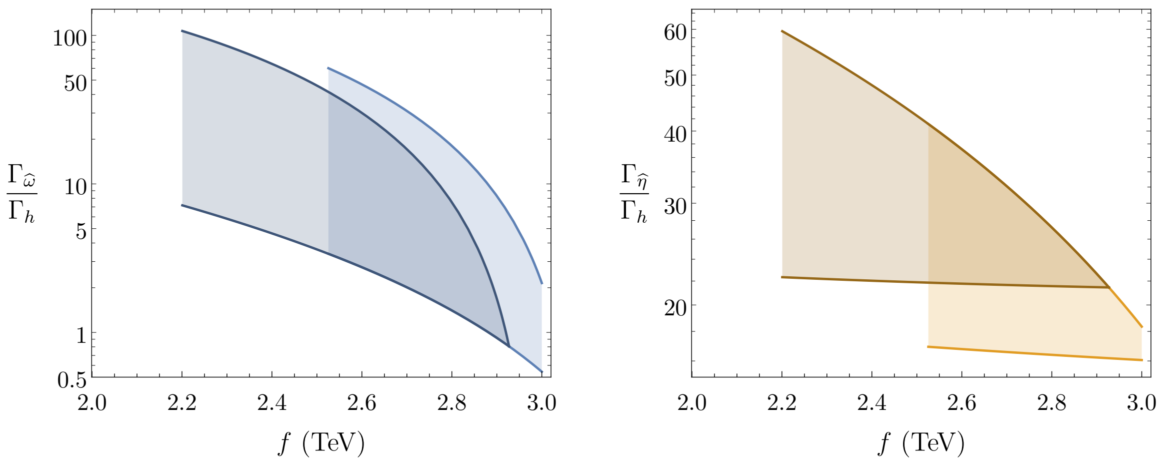


FIGURE 4.11: Decay widths for the triplet and singlet scalar fields normalized to the Higgs width for $\lambda_1 = 1.1$ and $\lambda_1 = 1.5$ (lighter color). The upper and lower bounds correspond to the maximum and minimum values of the scalar masses.

GeV heavier than the LTP. Finally, in the scalar sector, we have fixed the maximum mass of the new T-odd scalars to 1 TeV using a naturalness argument. This implies that the usual LHT complex triplet Φ is the heaviest scalar with a mass 1 TeV above the singlet $\widehat{\eta}$. This is followed by the lighter triplet $\widehat{\omega}$. The SM particles are lighter than the LTP, so the spectrum we show, for comparison, just the heaviest ones: the top quark, the $W^{\pm}$ and Z gauge bosons and the Higgs.

Now that we have studied the full spectrum of the model, let us discuss the decay channels of the new T-odd scalars $\widehat{\omega}$ and $\widehat{\eta}$. At leading order they decay mostly into a left-handed SM lepton and a right-handed T-odd mirror lepton preserving both T-parity and electric charge. The couplings involved are independent of the Higgs *vev* because the SM left-handed leptons are SU(2) doublets and so are their corresponding mirror versions with same hypercharge as the SM leptons. The strength of the interaction is given by the Yukawa coupling κ_l in eq. (3.39) fixed to $\kappa_l \approx 0.185$ and scale as

$$\widehat{\omega}^{\pm} : \widehat{\omega}^{0} : \widehat{\eta} \sim \frac{1}{\sqrt{2}} : \frac{1}{2} : \frac{1}{\sqrt{20}}. \tag{4.232}$$

Thus the quantum numbers of mirror and SM leptons are the appropriate to match those of a triplet for $\widehat{\omega}$ and a singlet for $\widehat{\eta}$. The decays to other leptons (with opposite T-parities) involve couplings suppressed by powers of $v/f < 0.12$ or are kinematically suppressed by the higher masses, of at least $m_{\ell_H} \gtrsim 530$ GeV in our simplified model. In addition, from eqs. (3.26) and (4.12), the charged components of the triplet $\widehat{\omega}^{\pm}$ decay into a gauge boson A_H and a SM $W^{\pm}$ while the neutral component $\widehat{\omega}^{0}$ can decay into an A_H and a Higgs boson. The singlet $\widehat{\eta}$ can decay similarly to the neutral component of the triplet but, being heavier, it can also decay to the neutral component of the triplet and a Higgs boson. However those couplings are also suppressed by powers of the Higgs *vev*. We have explicitly checked that these other channels contribute less than a 3% to the total decay widths.

In fig. 4.11 we show the range of values of the triplet and the singlet decay widths normalized to the Higgs boson width ($\Gamma_h \approx 4$ MeV) for a couple of values of λ_1. The charged and neutral components of the triplet have the same width at this order because the couplings to the neutral component are a factor $1/\sqrt{2}$ smaller than those for the charged components but for the neutral component one can exchange particles for antiparticles in the final state adding a contribution that compensates the factor $(1/\sqrt{2})^2$ in the decay width. Note that the upper bounds for the width of the triplet and singlet are of the same order. This is because the singlet is heavier than the triplet but the couplings of the singlet to leptons are a factor $\sqrt{5}$ smaller than those for the neutral component of the triplet. On the other hand, the lower bound of the triplet decay width is much smaller. This suppression comes from the kinematic factor since the lower bound for the triplet mass is only a few tens of GeV larger than the mass of the T-odd mirror leptons. In any case their lifetimes are small, since in the worst case the triplet could live twice more than the Higgs in the available parameter space.

Nevertheless, these new T-odd scalar particles would not be generated in a significant amount at the LHC. They are produced by an electroweak interaction together with another T-odd particle and the energy threshold for this process is very high considering the usual spectrum in fig. 4.10.

4.4. Chapter summary

In this chapter we have built a new and gauge invariant Littlest Higgs model with T-parity to cure the issues we found in the LHT at the end of Chapter 3. Compared to the original model, the global symmetry group is enlarged with an extra $[\mathrm{SU}(2) \times \mathrm{U}(1)]^2$ factor that gets spontaneously broken to the diagonal $[\mathrm{SU}(2) \times \mathrm{U}(1)]$ by the *vev* of a new non-linear sigma field that contains four scalars. This allows us to introduce fermion fields that only transform in this additional non-linear representation without invoking again incomplete SO(5) multiplets that would be incompatible with gauge invariance. The gauged subgroup is then contained in the product of the two $[\mathrm{SU}(2) \times \mathrm{U}(1)]^2$ factors, the one inside SU(5) and the extra one, hence preserving the number of gauge boson fields.

Once the global and gauged groups were defined we have explored two different options focusing in the leptonic sector since the extension to the quark sector is straightforward. In a first attempt, the left-handed components of the mirror-partner leptons and the SM singlet χ were introduced in a representation that only transforms under the diagonal $\mathrm{SU}(2) \times \mathrm{U}(1)$ of the external $[\mathrm{SU}(2) \times \mathrm{U}(1)]^2$, coupled to their right-handed counterparts through both the original and the new non-linear sigma fields. Then we tried with a model based on the completion of the SU(5) multiplets with new left-handed fields and the introduction of the additional right-handed components in a representation of the aforementioned external $\mathrm{SU}(2) \times \mathrm{U}(1)$. The first proposal, despite of being more economical in terms of fermion fields, had to be discarded because the global symmetry is explicitly broken in a way that the remaining symmetry is not enough to protect the Higgs mass from dangerous quadratically divergent contributions, as we proved by a diagrammatic calculation. It turns out that the quadratic contributions coming from the seagull diagrams with the T-even χ and the mirror-partner neutrinos do not cancel among themselves. However, the model with complete SU(5) multiplets, that includes a new T-odd singlet χ_- and a T-even doublet of mirror-partner leptons $\tilde{l}^c_+$, is viable because it prevents all scalar fields from quadratic divergences: if the coupling giving masses to the extra fermion fields is switched off, the usual Yukawa Lagrangian remains SU(5) invariant. In fact the Higgs mass squared only presents an admissible logarithmic divergences proportional to $\kappa_l^2\hat{\kappa}_l^2$, involving the product of two different couplings giving masses to the non-standard fermions hence respecting the collective symmetry breaking philosophy.

Next we have found the mass eigenfields that diagonalize the Lagrangian up to order v^2/f^2 as well as the fermion masses and flavour mixing matrices parametrizing the misalignment of the different Yukawa couplings $(\kappa, \hat{\kappa}, \lambda)$ in the flavour space of several fermion families. This NLHT model keeps one of the original sources of lepton flavour violation [54, 57, 102] (the mixing matrix V in eq. (4.90)), eliminates those found in [61, 65] (now $W = Z = \mathbb{1}$) and introduces an additional source ($\widehat{W}$) related to the new Yukawa coupling $\hat{\kappa}$ connecting the original to the extra fermion sector.

In addition, we have considered the influence of the new fermion fields in LFV Higgs decays. As we showed in § 3.2.1, the contribution of the T-even right-handed singlet χ_+ is finite by its own and the new model preserves this feature since its left-handed component does not couple to the Higgs field. The contribution of the remaining fields, including the T-odd singlet χ_{-}, is finite. This is because they enter in the loop through two insertions of their mixing term with the original fields of the LHT model (see table 4.3), thus reducing the degree of divergence of the topologies involved in the process.

In a next step, we derived a master formula from the BFM and classify the divergences into quadratic and logarithmic. Using the master formula we calculate the Coleman-Weinberg potential for the scalar fields generated by integrating out at one loop vector bosons and fermions, including both heavy quarks and leptons. We have calculated the one-loop contributions to the masses of the Higgs and the complex triplet of the original LHT model, as well as those of the new scalars. In our model, the Higgs mass is still not sensitive to quadratic divergences coming from the heavy lepton and heavy quark sectors. On the other hand, the relation between the Higgs mass and the complex triplet mass remains the same. Besides, the Higgs quartic coupling generated at leading order from the quadratically divergent terms of the potential does not receive contributions from the new sector. The masses of the new physical scalars, the T-odd singlet and real triplet, are found to be proportional to just the logarithm of the cut-off scale Λ. The reason for this is because, having the same quantum numbers of the original would-be Goldstone bosons of the LHT to be eaten after the SSB at the scale f, they mix with them at leading order, hence inheriting part of the symmetry that protects the actual would-be Goldstone bosons from the NLHT of developing a mass. Parametrically, apart from gauge couplings, the new scalar masses depend on the Yukawa couplings λ_1 and λ_2, that provide masses to the top quark and its corresponding T-even and T-odd partners, and the Yukawa couplings κ and $\widehat{\kappa}$, giving gauge invariant masses to the rest of non standard heavy fermions (in principle different for quarks and leptons).

We also employed the BFM to calculate the counterterm for LFV Higgs decays with a T-odd χ in the LHT integrating out the fermionic singlet and a scalar field. We found that the usual multiplicative split of the Goldstone fields when they come parametrized in a non-linear sigma field is not valid in our case since it does not commute with the discrete T-parity symmetry leading to a T-parity non invariant counterterm. On the other hand, applying a linear split we obtain a T-parity invariant counterterm. For that purpose we had to introduce the object $\xi^{a}_{(1)}$ obtained from ξ taking the derivative with respect to the Goldstone fields. The form of this counterterm and the straightforward extension to the T-even χ case explains why in the former we obtained a logarithmically divergent contribution to LFV Higgs decays and a finite contribution to the latter. In the T-odd case we obtain two different dimension six operators with two Higgs fields, a derivative and two SM leptons whose contributions do not cancel after applying the equations of motion. On the other hand, in the T-even case one obtains a similar operator but with the T-odd mirror leptons instead.

To conclude this chapter, we have studied the parameter space of the NLHT and the decay channels of the new T-odd scalars. As in the original LHT, the top quark Yukawa coupling λ_2 is a function of λ_1 given the top quark mass. The condition that the heavier top quark-partner is below the cutoff scale Λ constrains the value of λ_1 to the interval $[1.05, 1.71]$. The experimental lower bound on vector-like quark masses above 2 TeV [131, 132] pushes the allowed value of $f \gtrsim 900$ GeV.

Since T-parity is exact, the LTP is stable. To have a viable dark matter candidate, the LTP must be electrically neutral. The NLHT contains two potential candidates, the singlet $\widehat{\eta}$ and the usual heavy photon A_H. Although there is enough space to explore the singlet as a light dark matter component (with $M_{\widehat{\eta}} \lesssim 80$ GeV), we have chosen it to be the heavy photon in order to compare with previous works [103, 156, 157]. The aforementioned lower bound to f implies that the LTP has $M_{A_H} \gtrsim 200$ GeV and $M_{\widehat{\omega}} < M_{\widehat{\eta}}$. On the other hand, the scalar masses do not depend on the high energy scale f and thus they must remain light by naturalness arguments.

Therefore, to be conservative, we impose an upper bound of 1 TeV to the mass of the singlet.

In order to reproduce the current relic density with heavy photons, the presence of co-annihilators is necessary unless the LTP is lighter than the lower bound set above [156]. To put vector-like quarks beyond current bounds and at the same time provide a not so heavy dark matter candidate yielding the right relic abundance, we have adopted a simplified NLHT model with degenerate and flavour-diagonal couplings $\kappa = \hat{\kappa}$, different for leptons (κ_l) and quarks (κ_q), such that the corresponding heavy masses are $m_{l_H} = \sqrt{2}\kappa_l f \gtrsim M_{A_H}$ and $m_{q_H} = \sqrt{2}\kappa_q f \gtrsim 2$ TeV, respectively. We also require that only T-odd leptons act as co-annihilators by taking the scalar masses above the limit $M_{\hat{\omega}} > M_{A_H} + M_h$.

As a consequence of all these constraints, the allowed high energy scale f lies approximately in the interval $f \in [2, 3]$ TeV, the common Yukawa coupling of the heavy quarks gets strongly correlated to the top Yukawa coupling λ_1 and, demanding that all dark matter of the universe is made of the NLHT heavy photons, the Yukawa coupling for all heavy leptons is fixed to $\kappa_l \approx 0.185$. A typical spectrum of this scenario is displayed in fig. 4.10.

Finally we studied the dominant decay channels of the new scalar particles. We found that $\hat{\omega}$ and $\hat{\eta}$ decay mostly into a T-odd mirror lepton and a SM lepton with the proper quantum numbers. Other channels are negligible because the involve couplings that suffer from suppressions by powers $v/f < 0.12$. They decay very fast; the (lighter) triplet $\hat{\omega}$ could live at most two times longer than the Higgs boson. In any case, these new scalars are heavier than about 600 GeV and would be generated by an electroweak interaction together with another heavy T-odd particle at the LHC, so not very sizeable production rates are expected.

Chapter 5

Conclusions

In this Chapter we summarize the main conclusions of the Thesis.

- In Chapter 1 we motivated that, in spite of the success of the SM unveiling the nature of matter and its interactions, it should be regarded as an effective field theory valid up to some unknown high energy scale Λ_{SM}. This is because there are still open questions that it cannot address such as the Hierarchy problem. In the SM the Higgs boson mass squared is not protected by any symmetry and thus receives quadratically divergent contributions from arbitrarily high energy scales. As a consequence, quantum corrections pushes the Higgs mass to Λ_{SM}. Thus, an UV theory that extends the SM with an elementary Higgs must provide a contribution to the Higgs mass squared of approximate the same size to compensate for the SM contribution and keep the Higgs light. This is very unnatural, because the EW scale should be blind to effects of very high energy scales.

  Composite Higgs models and, in particular, Little Higgs models offer an elegant and well motivated solution to the Hierarchy problem at the TeV scale. Based on the well known example of the chiral symmetry of QCD, these models postulate a confining strong interacting sector with a global symmetry G. This gets spontaneously broken by the vacuum to a subgroup H at the scale $f \approx 1$ TeV. The Higgs boson is thus one of the Goldstone bosons associated to this spontaneous breaking. In addition to the Higgs, new particles with typical masses of size f arise as a consequence of the enlarged global group.

- In Chapter 2 we introduced the mathematical formalism that allows to realize the Higgs as the Goldstone boson of the spontaneous breaking of a global symmetry G to a continuous subgroup H: the CCWZ formalism. In the Composite Higgs scenario it is assumed that the electroweak symmetry group $G_{EW} = SU(2)_L \times U(1)_Y$ is embedded in H. The Goldstone bosons are massless as far as the global symmetry is exact. But in Nature the Higgs boson is massive and must take a *vev* to trigger the spontaneous breaking of the SM down to the electromagnetic group. To generate a mass and a physical *vev* for the Higgs at the one loop-level we further explained the idea of vacuum misalignment: the global symmetry is broken by gauge and Yukawa interactions and thus the Higgs ceases to be an exact Goldstone boson becoming a pseudo-Goldstone boson. Its mass is proportional to the sources of breaking of the global symmetry times the cutoff scale $\Lambda \approx 4\pi f$ and suppressed by a loop factor. As a consequence, in these models the Higgs mass is typically of size $m_h \approx cf$ with $c < 1$ and $f \approx 1$ TeV to reproduce the observed value $m_h \approx 125$ GeV.

  However, none of the new heavy resonances with masses proportional to f has been observed so far. This fact pushes the scale f to the multi-TeV regime increasing the fine-tuning among the parameters of the theory to ensure $c \ll 1$. For that reason, in Little Higgs models quadratically divergent contributions to the Higgs mass proportional to the cutoff scale are forbidden which leads us to the notion of collective symmetry breaking. Gauge and Yukawa interactions break the global symmetry in such a way that when a single gauge or Yukawa coupling is non vanishing there still remains an exact subgroup of the global symmetry acting non linearly on the Higgs. This ensures the Goldstone nature of the Higgs and a mass term is not generated. However, when all gauge or Yukawa couplings are non vanishing, all the global symmetries that protect the Higgs are broken. As a consequence, the Higgs develops a mass that is proportional to the product of all gauge and Yukawa couplings which is, at most, logarithmically sensitive to the cutoff scale. This mechanism introduces a plethora of new particles responsible of the cancellation of quadratic divergences. To suppress their contribution to observables that are constrained by current precision measurements, a new Z_2 symmetry, T-parity, is introduced under which the SM particles are T-even and most of the new particles are T-odd. As a result the leading order contributions of these new particles are one-loop suppressed, relaxing all the constraints.

Finally we built a toy model based on the global product group $SU(3)_1 \times SU(3)_2 \to SU(3)_V$ in which all the previous features are implemented. This spontaneous breaking leads to 8 Goldstone bosons. The gauge group is $[SU(2) \times U(1)]^2$ that gets spontaneously broken to $[SU(2) \times U(1)]_V$. However, it turns out that extra U(1) factors are required to accommodate the hypercharge of all fermions. With this explicit example we showed that with a global SU(3) global factor protecting the Higgs when a single gauge or Yukawa coupling is non-vanishing is sufficient. We explicitly checked that the Higgs mass does not develop a quadratic sensitivity to the cutoff scale as a result of the collective symmetry breaking mechanism, but just receives logarithmically divergent contributions proportional to the top quark sector Yukawa couplings and gauge couplings. We also showed that the implementation of T-parity at least doubles the fermion spectrum with respect to the model without T-parity. In particular, the T-odd partners of the SM fermions, the so called mirror fermions, need to be heavy. For that purpose, one introduces a complete right-handed $SU(3)_V$ multiplet that includes the right-handed components of the mirror fermions and an extra singlet, that can be either T-even or T-odd. They couple to the left-handed mirror fermions through a new Yukawa Lagrangian tailored to provide the mirror fermions with masses of order κf, respecting the collective symmetry breaking. For the singlet, one introduces its left-handed counterparts in an incomplete $SU(3)_V$ multiplet to provide it with a vector-like mass M_χ without coupling to the Higgs. As a consequence, with respect to the case without T-parity, the Higgs mass squared receives extra logarithmically divergent contributions proportional to the Yukawa coupling of the mirror fermions and the mass of the singlet.

- In Chapter 3 we focus on a particular Little Higgs model: the *Littlest Higgs model with T-parity*. This is based on the coset $SU(5) \to SO(5)$ giving rise to 14 Goldstone bosons: the Higgs doublet, a complex triplet, a real triplet and a singlet. This is the minimal setup that implements all the previous requirements using a global simple group. The gauge group is $[SU(2) \times U(1)]^2$ that gets spontaneously broken to $SU(2) \times U(1)$, that needs to be enlarged by two extra U(1) factors to accommodate all the hypercharges. Within this framework we are particularly interested in the contributions of the new exotic leptons to flavour changing observables. Among these new leptons, one finds a doublet of mirror leptons, an extra doublet of mirror-partner leptons and a singlet. Their right-handed components share a complete SO(5) quintuplet while the left-handed components of the singlet and the mirror leptons live in different incomplete SO(5) quintuplets. Under T-parity, the mirror and mirror partner leptons are T-odd while the singlet can be either T-even or T-odd.

  Within this framework we studied lepton flavour violating Higgs decays. First we reviewed that, contrary to other previous calculations, the contributions of mirror and mirror-partner leptons add up to yield a finite result. Motivated by this result we also computed the contributions of the lepton singlet to this process. In the T-even scenario, the singlet mixes with the SM neutrino at order v. On the other hand, the fields that run in the loops are the SM fields and the singlet. They have the appropriate quantum numbers to generate order 1 contributions to this process. As a consequence, one has to consistently diagonalize the singlet-neutrino mass matrix up to order v^2/f^2. After the diagonalization, the contributions of the neutrino and singlet mass eigenstates are finite and decouple. In the T-odd scenario, the singlet couples at order v with the mirror neutrinos. However, since all the fields inside the loop are T-odd, they do not have the quantum numbers to generate order 1 contributions and a diagonalization of the corresponding singlet-mirror neutrino mass matrix is not necessary at the order we work. Finally, the contributions of the T-odd singlet are UV divergent. We found the SM symmetric operators that contribute to this divergence and it turns out that there is no available counterterm in the model.

Later on, we studied neutrino mass generation. For that purpose we provided a small Majorana mass μ to the left-handed component of the singlet. The tree-level integration of this quasi-Dirac singlet provides neutrino masses of size $\mu v^2/M_\chi^2$, with M_χ the Dirac mass of the singlet, via the inverse seesaw mechanism. Neutrino masses vanish in the limit of large singlet mass. However, in the T-odd scenario, the one-loop integration of the singlet provides neutrino masses of size $\mu v^2/f^2$ that do not vanish in the limit of large singlet mass. Finally, in the T-even scenario we also computed the contributions of the singlet to $\mu \to e\gamma$. Considering that the singlet accounts for all the observed deviation with respect to the SM prediction together with the non observation of mirror leptons constrains the region m_{ℓ_H}-m_χ providing an upper bound for the mirror lepton masses. Although flavour conserving, we also studied the contribution of the singlet to the muon anomalous magnetic moment. This turns out to be small to explain a significant departure from the SM prediction.

Motivated by the anomalous behaviour of the singlet in the T-odd scenario, we showed that the LHT is, in general, non gauge invariant. This is because given a transformation of the gauge group $[\mathrm{SU}(2) \times \mathrm{U}(1)]^2 \in \mathrm{SU}(5)$, the associated SO(5) transformation involves all the SO(5) generators and not only those of its $\mathrm{SU}(2) \times \mathrm{U}(1)$ subgroup. This has several consequences. First of all, one has to disregard the T-odd option for the singlet because the Yukawa Lagrangian that provides masses to the mirror fermions depends on Ω which does not commute with all the SO(5) generators. Secondly, the SO(5) quintuplets must be complete because the SO(5) generators mix all their components. Consequently, the mirror-partner fermions and the singlet cannot be avoided in any case. Finally, for the same reason, the usual mass terms of the singlet and the mirror-partner fermions are not compatible with gauge invariance since one cannot isolate a member of a SO(5) quintuplet to provide it a mass. Thus a mechanism to provide them a heavy vector-like mass compatible with gauge invariance is required.

- In Chapter 4, we explicitly built the Lagrangian of a new and gauge invariant LHT (NLHT) that addresses all the afflictions we encountered in the LHT. For that purpose the global symmetry group SU(5) is enlarged minimally with an extra $[\mathrm{SU}(2) \times \mathrm{U}(1)]^2$ factor that gets spontaneously broken to the diagonal $\mathrm{SU}(2) \times \mathrm{U}(1)$ by the *vev* of a new non linear sigma field with four extra scalars. This allows to introduce fermion fields that only transform in this additional non linear representation without invoking again SO(5) multiplets that would need to be completed by gauge invariance. The gauge group is now contained in the product of the two $[\mathrm{SU}(2) \times \mathrm{U}(1)]^2$, the one inside SU(5) and the extra one, still preserving the number of gauge bosons. Under the gauge group the extra scalar fields decompose into a real triplet and singlet, $\widehat{\omega}$ and $\widehat{\eta}$, respectively.

In the fermion sector to provide a mass to the T-odd mirror-partner fermions and the T-even singlet, we proposed to complete the left-handed SU(5) multiplets and introduce the additional right-handed components in a representation that only transforms under the external $SU(2) \times U(1)$. As a consequence, our model includes T-even and T-odd singlets and doublets of mirror-partner fermions in the lepton and quark sectors. This construction prevents all the scalar fields from quadratic divergences coming from the heavy fermion sector. In fact the Higgs mass squared only presents an admissible logarithmic divergence proportional to $\kappa^2\widehat{\kappa}^2$ involving the Yukawa couplings giving masses to the non-standard fermions (in principle different for quarks and leptons). We also found the fermion masses and flavour mixing matrices parametrizing the misalignment of the Yukawa couplings (κ, $\widehat{\kappa}$ and λ) in the flavour space of several fermion families. This new version of the LHT keeps one of the original sources of lepton flavour violation [54, 57, 102] (the mixing matrix V in eqs. (3.115) and (3.116)), eliminates those found in [61, 65] (now $\mathsf{W} = \mathsf{Z} = 1$) and introduces an additional source ($\widehat{\mathsf{W}}$) related to the new Yukawa coupling $\widehat{\kappa}$ connecting the original to the extra fermion sector. We also considered the contributions of the new fermion fields in LFV Higgs decays that turn to be finite by power counting.

Later we applied the background field method (BFM) to calculate the Coleman-Weinberg potential for the scalar fields and also the counterterm for LFV Higgs decays in the LHT with a T-odd singlet. In our model, the Higgs mass is not sensitive to quadratic divergences. Parametrically it depends on the gauge couplings g and g', the top quark sector Yukawa couplings λ_1 and λ_2 and the Yukawa couplings κ and $\widehat{\kappa}$. The mass of the complex triplet and the Higgs quartic coupling are insensitive to the new heavy fermion sector, depending on λ_1 and g, g'. They come from the quadratic divergent part of the Coleman-Weinberg potential and thus the triplet is naturally heavier than the Higgs, with a mass of order f. On the other hand, the masses of the new scalar fields are found to be proportional to just the logarithm of the cutoff. This is because the new scalar fields and the would-be Goldstone bosons share the same quantum numbers, hence inheriting part of their symmetry. Consequently their masses are proportional to the Higgs mass and naturally light, depending on the same parameters as the Higgs mass.

With respect to the counterterm for LFV Higgs decays in the LHT in the T-odd singlet scenario, we found that the usual multiplicative split of the Goldstone bosons when they come parametrized in a non linear sigma field is not compatible with T-parity. Hence we were forced to apply a linear split also for the scalar fields introducing the parametric derivative of the field ξ, $\xi^a_{(1)}$, with respect to the Goldstone fields. From the counterterm we reproduced the result we found in Chapter 3 for the operators that contribute to the UV divergence. We extended the same methodology to also study the T-even singlet case. We found that the counterterm contains similar operators to those of the T-odd case but with the mirror leptons instead of the SM leptons. This justifies why in the T-even case the contributions of the singlet are finite on their own.

In the last part we studied whether the new model is viable. First, by self-consistency of the model, we impose that no mass exceeds the cutoff scale. Later, to simplify the flavour structure of the model, we further assume mass degenerate heavy leptons and heavy quarks with masses proportional to just two parameters: κ_l and κ_q. Finally, applying current lower bounds on vector-like quarks, $m_q > 2$ TeV, and imposing that the usual heavy photon with the heavy T-odd leptons as co-annihilators reproduces the current dark matter relic density, we find that f gets constrained within the interval between

2 and 3 TeV, the Yukawa coupling of heavy leptons gets fixed to $\kappa_l \approx 0.185$ and the Yukawa coupling of heavy quarks κ_q becomes greatly correlated to the top quark Yukawa couplings λ_1, λ_2. The particle spectrum is then bounded from below and above, with the heavy photon at about 0.5 TeV (the lightest T-odd state), not far from the heavy leptons, the new scalars below 1 TeV, the usual complex scalar triplet close to the heavy weak bosons at about 1.5 to 2.5 TeV, and the heavy quarks and top quark partners between 2 and 5 TeV.

Finally we studied the dominant decay channels of the new scalar particles. We found that $\widehat{\omega}$ and $\widehat{\eta}$ decay mostly into a T-odd mirror lepton and a SM lepton with the proper quantum numbers. They decay very fast; the (lighter) triplet $\widehat{\omega}$ could live at most two times longer than the Higgs boson. In any case, these new scalars are heavier than about 600 GeV and would be generated by and electroweak interaction together with another heavy T-odd particle at the LHC, so not very sizeable production rates are expected.

Capítulo 6

Conclusiones

En este capítulo recopilamos las principales conclusiones de esta Tesis.

- En el Capítulo 1 motivamos que, a pesar del éxito del Modelo Estándar (ME) desvelando la naturaleza de la materia y sus interacciones, éste debe ser entendido como una teoría efectiva que es sólo válida hasta una desconocida y alta escala de energía Λ_{SM}. Esto es debido a que aún existen cuestiones que no puede explicar, como el Problema de las jerarquías. En el ME la masa al cuadrado del Higgs no está protegida por ninguna simetría y por ello recibe contribuciones cuadráticamente divergentes a su masa de escalas de energía arbitrariamente altas. Como consecuencia, las correcciones cuánticas tienden a impulsar la masa del Higgs hacia Λ_{SM}. Esto implica que una teoría ultravioleta que extienda al ME con un Higgs elemental debe producir contribuciones a la masa al cuadrado del Higgs de aproximadamente la misma magnitud para compensar la contribución del ME y mantener al Higgs ligero. Esto es muy antinatural, porque la escala electrodébil debería ser insensible a lo que ocurra a muy altas escalas de energía.

  Los modelos de Higgs compuesto, y en particular los modelos de "Little Higgs", ofrecen una solución elegante y bien motivada al problema de las Jerarquías a la escala del TeV. Basados en el conocido ejemplo de la Cromodinámica Cuántica quiral, en estos modelos se postula la existencia de un sector fuertemente interactuante con una simetría global G. Ésta es rota espontáneamente por el vacío a un subgrupo H a la escala $f \approx 1$ TeV. El bosón de Higgs es uno de los bosones de Goldstone asociados a esta ruptura espontánea. Además del Higgs, aparecen nuevas partículas de masa f como consecuencia del grupo de simetrías extendido.

- En el Capítulo 2 se introduce el formalismo matemático que permite realizar al bosón de Higgs como el bosón de Goldstone asociado a la ruptura espontánea de una simetría global G a uno de sus continuos subgrupos H: el formalismo de CCWZ. En el escenario de un Higgs compuesto se asume que el grupo de simetrías electrodébil $G_{\mathrm{EW}} = \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ está embebido en H. Los bosones de Goldstone no tiene masa mientras la simetría global sea exacta. Pero en la Naturaleza el bosón de Higgs es masivo y debe tomar un *vev* para producir la ruptura espontánea del ME al electromagnetismo. Para generar una masa y un *vev* físico para el Higgs a un loop explicamos la idea del desalineamiento del vacío: la simetría global es rota por las interacciones gauge y de Yukawa haciendo que el Higgs deje de ser un bosón de Goldstone exacto para convertirse en un pseudo bosón de Goldstone. Su masa es proporcional a las fuentes de ruptura de la simetría global y a la escala de cutoff $\Lambda \approx 4\pi f$ y está suprimida por un factor de loop. Como consecuencia, en estos modelos la masa del Higgs tiene un tamaño típico de $m_h \approx cf$ con $c < 1$ y $f \approx 1$ TeV para reproducir el valor observado de $m_h \approx 125$ GeV.

  Sin embargo, ninguna de las nuevas resonancias con masas proporcionales a f han sido observadas aún. Este hecho impulsa la escala f al régimen del multi-TeV, incrementando el ajuste fino entre los parámetros de la teoría para asegurar que $c \ll 1$. Por esta razón, en los modelos de "Little Higgs" las contribuciones cuadráticamente divergentes proporcionales al cutoff a la masa del Higgs están prohibidas. Esto nos lleva a la noción de ruptura colectiva de la simetría. Las interacciones gauge y de Yukawa rompen la simetría global de tal manera que cuando sólo un acoplamiento gauge o Yukawa es distinto de cero existe un subgrupo de la simetría global exacto que actúa no linealmente sobre el Higgs. Esto asegura el carácter de bosón de Goldstone del Higgs y no se genera un término de masas. Sin embargo, cuando todos los acoplamientos gauge o Yukawa son distintos de cero, todas las simetrías globales que protegen al Higgs están rotas. Como consecuencia, el Higgs desarrolla una masa proporcional al producto de todos los acoplamientos gauge y Yukawa que es, a lo sumo, logarítmicamente sensible al cutoff. Este mecanismo introduce una plétora de nuevas partículas que son las responsables de la cancelación de las divergencias cuadráticamente divergentes. Para suprimir la contribución de estas nuevas partículas a observables que están constreñidos por las actuales medidas de precisión, se introduce una nueva simetría discreta, la T-paridad, ante la cual las partículas del ME son pares y muchas de las nuevas partículas son impares. Como resultado las contribuciones de estas partículas están suprimidas por un factor de loop, relajando todas las restricciones.

Finalmente construimos un "toy model" basado en el grupo global producto $SU(3)_1 \times SU(3)_2 \to SU(3)_V$ en el que se implementan todas las propiedades ya mencionadas. Esta ruptura espontánea produce 8 bosones de Goldstone. El grupo gauge es $[SU(2) \times U(1)]^2$ que se rompe espontáneamente a $[SU(2) \times U(1)]_V$. Sin embargo, se necesitan factores U(1) extra para acomodar las hipercargas de todos los fermiones. Con este ejemplo explícito mostramos que es suficiente que un factor global SU(3) proteja al Higgs cuando sólamente un acoplamiento gauge o Yukawa es distinto de cero. Hemos comprobado explícitamente que la masa del Higgs no desarrolla una sensibilidad cuadrática al cutoff como resultado de la ruptura colectiva de la simetría sino que sólamente recibe contribuciones logarítmicamente divergentes proporcionales a los acoplamientos Yukawa del sector del quark top y a los acoplamientos gauge. También hemos mostrado que la implementación de la T-paridad al menos dobla el espectro de fermiones con respecto al modelo sin T-paridad. En particular, los compañeros impares de los fermiones del ME, los "mirror fermions", deben ser pesados. Por ello se introduce un multiplete completo de $SU(3)_V$ de quiralidad derecha que incluye las componentes derecha de los "mirror fermions" y un singlete extra, que puede ser tanto par como impar bajo T-paridad. Estos se acoplan a los "mirror fermions" de quiralidad izquierda a través de un nuevo Lagrangiano de Yukawa que se construye para dotar de una masa κf a los "mirror fermions", respetando la ruptura colectiva de la simetría. Para dotar de una masa M_χ al singlete, sin nuevos acoplamientos con el Higgs, se introducen sus componentes de quiralidad izquierda en un multiplete incompleto de $SU(3)_V$. Como consecuencia, con respecto al caso sin T-paridad, la masa al cuadrado del Higgs recibe nuevas contribuciones logarítmicamente divergentes proporcionales al acoplamiento de Yukawa de los "mirror fermions" y a la masa del singlete.

- En el Capítulo 3 nos centramos en un modelo de "Little Higgs" concreto: el *modelo de "Littlest Higgs" con T-paridad* (LHT). Éste está basado en el coset $SU(5) \to SO(5)$ dando lugar a 14 bosones de Goldstone: el doblete de Higgs, un triplete complejo, un triplete real y un singlete. Ésta es la forma mínima de implementar todos los requerimientos mencionados anteriormente usando un grupo global de simetría simple. El grupo gauge es $[SU(2) \times U(1)]^2$ que es roto espontáneamente a $SU(2) \times U(1)$. El grupo gauge debe ser extendido con dos grupos U(1) extra para acomodar todas las hipercargas. En este marco teórico estamos particularmente interesados en las contribuciones de los leptones exóticos a observables con cambio de sabor. Entre estos nuevos leptones podemos encontrar un doblete de "mirror leptons", un doblete extra de "mirror partner fermions" y un singlete. Sus componentes de quiralidad derecha comparten un multiplete completo de SO(5) mientras que las componentes de quiralidad izquierda del singlete y de los "mirror-partner leptons" viven en diferentes multipletes incompletos de SO(5). Bajo T-paridad, los 'mirror leptons" y los "mirror-partner leptons" son impares mientras que el singlete puede ser tanto par como impar.

En este marco teórico hemos estudiado decaimientos del Higgs con violación de sabor leptónico. Primero hemos probado revisado que, contrariamente a otros cálculos previos, las contribuciones de los "mirror leptons" y "mirror-partner leptons" se suman para dar un resultado finito. Motivados por este resultado, hemos calculado las contribuciones del singlete fermiónico a este proceso. En el escenario de un singlete par, éste se mezcla a orden v con el neutrino del ME. Por otro lado, los campos que recorren el loop son los campos del ME y el singlete. Estos campos tienen los números cuánticos apropiados para generar contribuciones orden 1 a este proceso. Como consecuencia, hay que diagonalizar consistentemente la matriz de masas singlete-neutrino hasta orden v^2/f^2. Tras la diagonalización, las contribuciones de los autoestados de masa del neutrino y del singlete son finitas y desacoplan. En el escenario de un singlete impar, éste se acopla a orden v con los "mirror neutrinos". Sin embargo, como todos los campos en el loop son impares bajo T-paridad, estos no tienen los números cuánticos para generar contribuciones orden 1 y la diagonalización de la matriz de masas no es necesaria al orden en que trabajamos. Finalmente, las contribuciones del singlete impar son divergentes en el ultravioleta. Hemos obtenido los operadores simétricos bajo el ME que contribuyen a esta divergencia y no hay disponible un contratérmino en el modelo.

También hemos estudiado un mecanismo para dotar de masa a los neutrinos. Para ello hemos introducido una pequeña masa de Majorana μ a las componentes de quiralidad izquierda del singlete. La integración a nivel árbol de este "quasi-Dirac singlet" proporciona una masa a los neutrinos de magnitud $\mu v^2/M_\chi^2$, con M_χ la masa de Dirac del singlete, vía el mecanismo de "inverse seesaw". Las masas de los neutrinos se anulan en el límite en el que el singlete es pesado. Sin embargo, en el caso en que el singlete es impar bajo T-paridad, su integración a un loop proporciona una masa a los neutrinos de tamaño $\mu v^2/f^2$ que no se anula en el límite en que el singlete es pesado. Finalmente, en el escenario en el que el singlete es par también hemos calculado las contribuciones de este singlete a $\mu \to e\gamma$. Considerando que el singlete da cuenta de toda la desviación con respecto a la predicción del ME junto con la no observación de los "mirror leptons" restringe la región m_{ℓ_H}-m_χ dando una cota superior a las masas de los "mirror leptons". Aún conservando el sabor, también hemos estudiado las contribuciones del singlete al momento magnético anómalo del muón. Éstas son pequeñas como para explicar una desviación significativa con respecto a la predicción del ME.

Motivados por el comportamiento anómalo del singlete en el escenario en que éste es impar bajo T-paridad, hemos probado que el LHT, en general, no es invariante gauge. Esto es porque dada una transformación del grupo gauge $[\mathrm{SU}(2) \times \mathrm{U}(1)]^2 \in \mathrm{SU}(5)$, la transformación de SO(5) asociada involucra a todos los generadores de SO(5) y no solamente aquellos de su subgrupo $\mathrm{SU}(2) \times \mathrm{U}(1)$. Esto tiene importantes consecuencias. Primero, hay que descartar la opción impar para el singlete porque el Lagrangiano de Yukawa que da masa a los "mirror fermions" depende de Ω que no conmuta con todos los generadores de SO(5). Segundo, los quintupletes de SO(5) tienen que estar completos porque los generadores de SO(5) mezclan todas sus componentes. Consecuentemente, los "mirror-partner fermions" y el singlete deben estar incluidos en cualquier caso. Finalmente, por la misma razón, el término de masas usual del singlete y de los "mirror-partner fermions" no es compatible con la simetría gauge ya que no se puede aislar un miembro de un quintuplete de SO(5) para dotarle de masa. Como consecuencia, se requiere un mecanismo para proveerlos de una masa "vector-like" compatible con la invariancia gauge.

- En el Capítulo 4 construimos explícitamente el Lagrangiano de un nuevo "Littlest Higgs model" invariante gauge (NLHT) que cura las patologías que encontramos en el LHT. Para ello el grupo de simetría global SU(5) es extendido mínimamente con un factor extra $[SU(2) \times U(1)]^2$ que es espontáneamente roto a su subgrupo diagonal $SU(2) \times U(1)$ por el *vev* de un nueva campo sigma no lineal con cuatro escalares extra. Esto permite introducir campos fermiónicos que solamente transformen en esta representación adicional no lineal sin usar de nuevo multipletes de SO(5) que deben estar completos por invariancia gauge. El grupo gauge está ahora contenido en el producto de los dos $[SU(2) \times U(1)]^2$, el que está en SU(5) y el factor externo, preservando el número de bosones de gauge. Ante el grupo gauge los nuevos campos escalares se descomponen en un triplete real y un singlete, $\widehat{\omega}$ and $\widehat{\eta}$, respectivamente.

  En el sector fermiónico, para dotar de masa a los "mirror-partner fermions" y al singlete, proponemos completar los multipletes de quiralidad izquierda de SU(5) e introducir las componentes de quiralidad derecha adicionales en una representación que solamente transforma bajo el $SU(2) \times U(1)$ externo. Como consecuencia, nuestro modelo incluye dos singletes y dos dobletes de "mirror-partner fermions", siendo uno de ellos par y el otro impar, tanto en el sector de quarks como de leptones. Esta construcción previene a todos los campos escalares de recibir correcciones cuadráticamente divergentes provenientes del sector de fermiones pesados. De hecho la masa del Higgs al cuadrado sólo presenta una divergencia logarítmica admisible proporcional a $\kappa^2\widehat{\kappa}^2$ que involucra a los acoplamientos de Yukawa que dan masa a los fermiones no estándar (en principio diferentes para quark y leptones). También hemos encontrado las matrices de masa de los fermiones y las matrices de mezcla de sabor que parametrizan el desalineamiento de los acoplamientos Yukawa (κ, $\widehat{\kappa}$ y λ) en el espacio de sabor de varias familias de fermiones. Esta nueva versión del LHT mantiene una de las fuentes originales de violación de sabor leptónico [54, 57, 102] (la matriz de mixing V en las ecs. (3.115) y (3.116)), elimina las encontradas en [61, 65] (ahora $W = Z = 1$) e introduce una fuente adicional ($\widehat{W}$) relacionada con el nuevo acoplamiento de Yukawa $\widehat{\kappa}$ conectando el sector de fermiones originales con el nuevo sector. También hemos discutido las contribuciones de los nuevos fermiones a desintegraciones del Higgs con cambio de sabor leptónico que resultan ser finitas.

  Luego hemos aplicado el "background field method" (BFM) para calcular el potencial de Coleman-Weinberg para los campos escalares así como el contratérmino para decaimientos del Higgs con violación de sabor con un singlete impar. En nuestro modelo, la masa del Higgs no es sensible a divergencias cuadráticas. Paramétricamente ésta depende de los acoplamientos gauge g and g', de los acoplamientos de Yukawa del sector del quark top λ_1 y λ_2 y de los acoplamientos de Yukawa κ y $\widehat{\kappa}$. La masa del triplete complejo y el acoplamiento cuártico del Higgs son insensibles al nuevo sector de fermiones, dependiendo de λ_1 y g, g'. Ambos parámetros provienen de la parte cuadráticamente divergente del potencial de Coleman-Weinberg por lo que el triplete es naturalmente más pesado que el Higgs, con una masa de orden f. Por otro lado, las masas de los nuevos campos escalares son proporcionales al logaritmo de la escala Λ. Esto es porque los nuevos campos escalares y los "would-be Goldstone bosons" comparten los mismos números cuánticos y por ello heredan parte de su simetría. Consecuentemente sus masas son proporcionales a la masa del Higgs y son naturalmente ligeros, dependiendo de los mismos parámetros que la masa del Higgs.

Con respecto al contratérmino para decaimientos del Higgs a leptones de distinto sabor en el caso del singlete impar, encontramos que la expansión multiplicativa para los bosones de Goldstone que usualmente se lleva a cabo cuando estos vienen parametrizados en un campo sigma no lineal no es compatible con la T-paridad. Esto nos fuerza a usar una expansión aditiva también para los campos escalares para lo cual tuvimos que introducir la derivada paramétrica del campo ξ, $\xi^a_{(1)}$, con respecto a los campos de Goldstone. Del contratérmino hemos reproducido el resultado que encontramos en el Capítulo 3 para los operadores que contribuyen a la divergencia ultravioleta. También extendimos esta misma metodología par estudiar el caso en que el singlete es par. En este caso el contratérmino contiene operadores similares al caso impar pero con los "mirror leptons" en lugar de con los leptones del ME. Esto justifica porqué en el caso del singlete par sus contribuciones son finitas.

En la última parte hemos estudiado si el nuevo modelo es viable. Primero, por la propia autoconsistencia del modelo, imponemos que ninguna masa pueda exceder el cutoff. Después, para simplificar la estructura de sabor del modelo, también asumimos masas degeneradas para leptones y quarks pesados siendo éstas propocionales a sólo dos parámetros: κ_l y κ_q. Finalmente, aplicando las actuales cotas inferiores a la masa de los "vector-like quarks" , $m_q > 2$ TeV, e imponiendo que el fotón pesado junto con los leptones pesados impares actuando como co-aniquiladores reproduce la actual densidad de materia oscura, restringimos f al intervalo entre 2 y 3 TeV, fijamos el acoplamiento de Yukawa de los leptones pesados $\kappa_l \approx 0{,}185$ y el acoplamiento de Yukawa de los quarks pesados κ_q se correlaciona con los acoplamientos de Yukawa del sector del quark top λ_1, λ_2. El espectro de partículas está acotado tanto superior como inferiormente con un fotón pesado de aproximadamente 0.5 TeV (el estado impar más ligero), no muy lejos de los leptones pesados, los nuevos campos escalares por debajo de 1 TeV, el triplete complejo con una masa muy próxima a los bosones de gauge pesados con masas entre 1.5 y 2.5 TeV, y los quarks pesados y los compañeros del quark top con masas comprendidas entre 2 y 5 TeV.

Finalmente estudiamos los principales canales de desintegración de las nuevas partículas escalares. Hemos visto que $\widehat{\omega}$ y $\widehat{\eta}$ decaen en un "mirror lepton" impar y un leptón del ME con los números cuánticos apropiados. Estos escalares decaen muy rápido; el triplete (el más ligero) $\widehat{\omega}$ podría vivir a lo sumo dos veces más que el bosón de Higgs. En cualquier caso, estos nuevos escalares tienen unas masas por encima de unos 600 GeV y serían generados por una interacción electrodébil junto con otra partícula pesada e impar en el LHC, por lo que no se espera que den lugar a señales apreciables.

Appendix A

SU(5) generators

SU(5) is the group of unitary 5×5 matrices with unit determinant. As a consequence, its generators are hermitian and traceless.

A.1. Unbroken SO(5) generators

The 10 generators of the unbroken SO(5) preserve the vacuum Σ_0 satisfying

$$T^a \Sigma_0 + \Sigma_0 T^{aT} = 0. \tag{A.1}$$

Our chosen basis is given by

$$T^1 = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad T^2 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & -\frac{1}{2} & 0 \end{pmatrix}, \quad T^3 = \begin{pmatrix} 0 & \frac{i}{2} & 0 & 0 & 0 \\ -\frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{i}{2} \\ 0 & 0 & 0 & -\frac{i}{2} & 0 \end{pmatrix},$$
$$T^4 = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad T^5 = \begin{pmatrix} 0 & 0 & \frac{i}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -\frac{i}{2} & 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & \frac{i}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad T^6 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 \end{pmatrix},$$
$$T^7 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{i}{2} \\ 0 & 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 & 0 \\ -\frac{i}{2} & 0 & 0 & 0 & 0 \end{pmatrix}, \quad T^8 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} \end{pmatrix}, \quad T^9 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 \end{pmatrix},$$
$$T^{10} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{i}{2} & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 & \frac{i}{2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{i}{2} & 0 & 0 \end{pmatrix}. \tag{A.2}$$

A.2. Rest of SU(5) generators

The 14 SU(5) broken generators are orthogonal to the previous ones satisfying

$$X^a \Sigma - \Sigma_0 X^{aT} = 0. \tag{A.3}$$

They take the form

$$X^1 = \begin{pmatrix} \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{\frac{2}{3}} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad X^2 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}, \quad X^3 = \begin{pmatrix} 0 & \frac{i}{2} & 0 & 0 & 0 \\ -\frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & 0 & \frac{i}{2} & 0 \end{pmatrix}$$

$$X^4 = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad X^5 = \begin{pmatrix} 0 & 0 & \frac{i}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -\frac{i}{2} & 0 & 0 & \frac{i}{2} & 0 \\ 0 & 0 & -\frac{i}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad X^6 = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$X^7 = \begin{pmatrix} 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad X^8 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 \end{pmatrix}, \quad X^9 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{i}{2} \\ 0 & 0 & 0 & \frac{i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{i}{2} & 0 & 0 & 0 \\ -\frac{i}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$X^{10} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}, \quad X^{11} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{i}{2} & 0 & 0 \\ 0 & -\frac{i}{2} & 0 & 0 & \frac{i}{2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{i}{2} & 0 & 0 \end{pmatrix}, \quad X^{12} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix},$$

$$X^{13} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}, \quad X^{14} = \begin{pmatrix} \sqrt{\frac{2}{15}} & 0 & 0 & 0 & 0 \\ 0 & -\sqrt{\frac{3}{10}} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\frac{2}{15}} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\frac{2}{15}} & 0 \\ 0 & 0 & 0 & 0 & -\sqrt{\frac{3}{10}} \end{pmatrix}. \tag{A.4}$$

A.3. Building SU(3) subalgebras

SU(5) contains two different SU(3) factors laying in the upper-left and lower-right corners of the SU(5) matrices. We can build explicitly their corresponding algebras through linear combinations of broken and unbroken generators.

A.3.1. Upper-left SU(3)

We can embed the SU(3) generators (Gell-Mann matrices) in the upper-left corner of SU(3) through the following linear combinations

$$\lambda_1^1 = T^2 + X^2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \lambda_1^2 = -T^3 - X^3 = \begin{pmatrix} 0 & -i & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{A.5}$$

$$\lambda_1^3 = \frac{1}{\sqrt{2}}T^1 - \frac{1}{\sqrt{2}}T^8 + \frac{1}{\sqrt{6}}X^1 + \sqrt{\frac{5}{6}}X^{14} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{A.6}$$

$$\lambda_1^4 = T^4 + X^4 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \lambda_1^5 = -T^5 - X^5 = \begin{pmatrix} 0 & 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{A.7}$$

$$\lambda_1^6 = T^9 + X^{10} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \lambda_1^7 = T^{10} - X^{11} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 \\ 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{A.8}$$

$$\lambda_1^8 = \frac{1}{\sqrt{2}}T^1 + \frac{1}{\sqrt{2}}T^8 + \frac{5}{\sqrt{6}}X^1 - \sqrt{\frac{5}{6}}X^{14} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{A.9}$$

A.3.2. Lower-right SU(3)

The SU(3) generators can be embedded in the lower right-corner of SU(5) through the following linear combinations

$$\lambda_2^1 = -T^4 + X^4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2^2 = -T^5 + X^5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i & 0 \\ 0 & 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{A.10}$$

$$\lambda_2^3 = \frac{1}{\sqrt{2}}T^1 - \sqrt{\frac{3}{2}}X^1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2^4 = -T^9 + X^{10} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \tag{A.11}$$

$$\lambda_2^5 = -T^{10} - X^{11} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i & 0 & 0 \end{pmatrix}, \quad \lambda_2^6 = -T^2 + X^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \tag{A.12}$$

$$\lambda_2^7 = -T^3 + X^3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & i & 0 \end{pmatrix}, \tag{A.13}$$

$$\lambda_2^8 = -\frac{1}{\sqrt{2}}T^1 + \sqrt{2}T^8 - \frac{1}{\sqrt{6}}X^1 + \sqrt{\frac{10}{3}}X^{14} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{pmatrix}. \tag{A.14}$$

A.3.3. Associated SO(5) transformations

Once we have built the generators of the different SU(3) factors contained in SU(5), one may wonder about the corresponding SO(5) transformations provided by the CCWZ formalism.

Remember that, given an infinitesimal SU(5) transformation

$$V \approx \mathbb{1} + i\alpha^a X^a + i\beta^b T^b, \tag{A.15}$$

the part of the infinitesimal SO(5) transformation independent of the Goldstone fields is thus given by

$$U \approx \mathbb{1} + i\beta^b T^b + ... \tag{A.16}$$

As a consequence, the same SO(5) generators used to construct the different SU(3) subalgebras appear in the transformation U, that is, the set T^1, T^2, T^3, T^4, T^5, T^8, T^9, T^{10}. However, this set does not form a subalgebra of SO(5) as one can check by taking the following commutator,

$$\left[T^4, T^{10}\right] = \frac{i}{2}\left(T^2 + T^6\right), \tag{A.17}$$

where T^6 is not included in the previous set.

Appendix B

Higgs mass dependence on the Yukawa couplings in the LHT

In Chapter 4 we introduced the NLHT. In this framework we obtained the explicit expression for the Higgs mass parameter μ^2 using the BFM. It reads

$$\mu^2 = \frac{f^2}{16\pi^2} \log \Lambda^2 \left(6g^4 + \frac{2}{5}g'^4 - 3\lambda_1^2\lambda_2^2 + 12T_\kappa \right). \tag{B.1}$$

The first two terms are the gauge boson contributions, the third term comes from the top sector that includes the top quark and its corresponding T-even and T-odd partners, and the last term proportional to $T_\kappa \equiv \mathrm{tr}(\kappa_l \kappa_l^\dagger \widehat{\kappa}_l \widehat{\kappa}_l^\dagger) + 3\mathrm{tr}(\kappa_q \kappa_q^\dagger \widehat{\kappa}_q \widehat{\kappa}_q^\dagger)$ is the contribution of the rest of heavy fermions with masses proportional to the Yukawa couplings κ_l, $\widehat{\kappa}_l$, κ_q and $\widehat{\kappa}_q$. The gauge boson and top sector contributions to the Higgs mass parameter are well known [93, 98]. On the other hand, contributions proportional to the κ couplings are not discussed in the literature.[1] Thus one may wonder whether the μ^2 dependence on these Yukawa couplings is an exclusive feature of the new model or, on the contrary, it was also present in the LHT. If so we would like to understand under which assumptions this contribution can be avoided.

One may naively think that, since in the LHT there were no $\widehat{\kappa}_l$, $\widehat{\kappa}_q$ Yukawa couplings, turning them to zero in the NLHT one would obtain the LHT result for μ^2. However this is not the case because the LHT is not recovered in the limit of vanishing $\widehat{\kappa}_l$, $\widehat{\kappa}_q$ Yukawa couplings as one can read from table B.1. In the LHT the SU(5) left-handed multiplets are not completed and the SU(2) singlet χ and the doublet of mirror-partner fermions, when included, receive masses that are independent of any κ coupling. This breaks explicitly the SU(5) global symmetry and thus a contribution to the Higgs mass is not forbidden.[2] Besides, there are two different actions of T-parity on the singlet. On the other hand one can justify why in the NLHT turning these Yukawa couplings to zero gives a vanishing contribution. This is due to the restoration of the global SU(5) symmetry in the heavy fermion sector due to the complete SU(5) multiplets that ensures a vanishing contribution to the Higgs mass as a consequence of the collective symmetry breaking mechanism.

To study the possible contributions proportional to the κ Yukawa couplings in the LHT we will focus just on leptons. The quarks could be worked out in the same fashion up to color factors. We will discuss which fields can contribute to the Higgs mass parameter and show the explicit calculations. All the information about the different fields and Yukawa Lagrangian of the heavy lepton sector is gathered in table B.1.

First of all, as we discussed in Chapter 3, in principle all the leptons in the SO(5) quintuplet Ψ_R are required to cancel the quadratically divergent contributions to the Higgs mass. This is because, given an SU(3) transformation of the upper-left or lower-right corner of the

[1] In the Littlest Higgs model without T-parity [87] the Higgs mass parameter has no dependence on κ simply because there are no mirror fermions and thus no $\mathcal{L}_\kappa$ Lagrangian.

[2] It also breaks gauge invariance, as shown in § 3.3.

	LHT	NLHT
Global group	$G = \text{SU}(5)$ $\downarrow$ $H = \text{SO}(5)$	$G = \text{SU}(5) \times [\text{SU}(2) \times \text{U}(1)]^2$ $\downarrow$ $H = \text{SO}(5) \times [\text{SU}(2) \times \text{U}(1)]$
Scalar fields	ξ –	ξ $\widehat{\xi}$
Fermions	$\Psi_1 = \begin{pmatrix} -i\sigma^2 l_{1L} \\ 0 \\ 0_2 \end{pmatrix} \in \mathbf{5}^*$ $\Psi_2 = \begin{pmatrix} 0_2 \\ 0 \\ -i\sigma^2 l_{2L} \end{pmatrix} \in \mathbf{5}$ $\Psi_R = \begin{pmatrix} -i\sigma^2(\widetilde{l}^c_-)_R \\ i(\chi_\pm)_R \\ -i\sigma^2 l_{HR} \end{pmatrix} \in \underline{\mathbf{5}}$ $\Psi_L = \begin{pmatrix} -i\sigma^2(\widetilde{l}^c_-)_L \\ 0 \\ 0_2 \end{pmatrix} \in \underline{\mathbf{5}}$ $\Psi^\chi_L = \begin{pmatrix} 0_2 \\ i(\chi_\pm)_L \\ 0_2 \end{pmatrix} \in \underline{\mathbf{5}}$ –	$\Psi_1 = \begin{pmatrix} -i\sigma^2 l_{1L} \\ i\chi_{1L} \\ -i\sigma^2 \widetilde{l}^c_{1L} \end{pmatrix} \in \mathbf{5}^*$ $\Psi_2 = \begin{pmatrix} -i\sigma^2 \widetilde{l}^c_{2L} \\ i\chi_{2L} \\ -i\sigma^2 l_{2L} \end{pmatrix} \in \mathbf{5}$ $\Psi_R = \begin{pmatrix} -i\sigma^2(\widetilde{l}^c_-)_R \\ i(\chi_+)_R \\ -i\sigma^2 l_{HR} \end{pmatrix} \in \underline{\mathbf{5}}$ – – $\widehat{\Psi}_R = \begin{pmatrix} -i\sigma^2(\widetilde{l}^c_+)_R \\ i(\chi_-)_R \\ 0_2 \end{pmatrix} \in 2_{\frac{1}{2}} \oplus 1_0$
Yukawa Lagrangian	$\mathcal{L}^{a)}_{Y_H} = -\kappa f \left(\overline{\Psi}_2\xi + \overline{\Psi}_2\Sigma_0\xi^\dagger\right)\Psi_R$ $-M\overline{(\widetilde{l}^c_-)}_L(\widetilde{l}^c_-)_R - M_\chi\overline{(\chi_+)}_L(\chi_+)_R + \text{h.c.}$ $\mathcal{L}^{b)}_{Y_H} = -\kappa f \left(\overline{\Psi}_2\xi + \overline{\Psi}_2\Sigma_0\Omega\xi^\dagger\Omega\right)\Psi_R$ $-M\overline{(\widetilde{l}^c_-)}_L(\widetilde{l}^c_-)_R - M_\chi\overline{(\chi_-)}_L(\chi_-)_R + \text{h.c.}$ –	$\mathcal{L}_{Y_H} = -\kappa f \left(\overline{\Psi}_2\xi + \overline{\Psi}_1\Sigma_0\xi^\dagger\right)\Psi_R + \text{h.c.}$ $\mathcal{L}_{\widehat{Y}_H} = -\widehat{\kappa} f \left(\overline{\Psi}_2\widehat{\xi} - \overline{\Psi}_1\Sigma_0\widehat{\xi}^\dagger\right)\widehat{\Psi}_R + \text{h.c.}$

TABLE B.1: Comparison between the LHT and NLHT heavy fermion sector. A dash means that the corresponding fields are absent in one model with respect to the other. In gray color we indicate the fields that are optional in the LHT. **5**, **5*** are the fundamental and antifundamental SU(5) representations while $\underline{\mathbf{5}}$ is the fundamental SO(5) representation.

Fields	c_L	c_R
$h\overline{\nu}_H\chi_-$	0	$\frac{\kappa}{\sqrt{2}}$
$hh\overline{\nu}_H\nu_H$	$\frac{\kappa}{2\sqrt{2}f}$	$\frac{\kappa}{2\sqrt{2}f}$

TABLE B.2: Feynman rules needed to compute the contribution of the heavy leptons to the Higgs mass parameter in the LHT with a T-even SU(2) singlet.

SU(5) matrices, the associated SO(5) transformation acting on the right-handed quintuplet Ψ_R involves SO(5) generators (see Appendix A) that mix the right-handed mirror leptons l_{HR} and the mirror-partner leptons with the SU(2) singlet $\chi_\pm$. With a complete SO(5) multiplet, the term $\overline{\Psi}_2\xi\Psi_R$ is invariant under the upper-left SU(3). For the second term, one has to distinguish between the different T-parity realizations. In option $a)$ the singlet χ_+ is T-even and the term $\overline{\Psi}_1\Sigma_0\xi^\dagger\Psi_R$ is invariant under the lower-right SU(3). On the contrary, in option $b)$ the singlet χ_- is T-odd and the term $\overline{\Psi}_1\Sigma_0\Omega\xi^\dagger\Omega\Psi_R$ is not invariant under the lower-right SU(3) but the unitarity of Ω is enough to avoid a quadratically divergent contribution to the Higgs mass squared from this term. Thus a complete right-handed SO(5) quintuplet ensures the absence of quadratically divergent contributions to the Higgs mass. However, not all these fermions couple to the Higgs in the appropriate way to generate a mass.

The mirror leptons l_H always contribute to the Higss mass parameter. The left-handed mirror leptons live in a combination of Ψ_1 and Ψ_2 that couple to their right-handed counterparts in the SO(5) quintuplet Ψ_R through the field ξ. As a consequence, and independently of the action of T-parity, $\mathcal{L}_{Y_H}^{a)}$ and $\mathcal{L}_{Y_H}^{b)}$ generate the operator $(\bar{l}_{HL}\widetilde{H})(\widetilde{H}^\dagger l_{HR})$ with $\widetilde{H} = -i\sigma^2 H^*$. Thus only the neutral mirror leptons can contribute.

On the other hand, the mirror-partner leptons $\widetilde{l}^c_-$ do not couple to the Higgs in the same fashion. Even though the right-handed mirror-partner and mirror leptons share an SO(5) multiplet, they are embedded with opposite hypercharges and the left-handed mirror-partner leptons do not live in a combination of the SU(5) multiplets but in an isolate SO(5) representation. Hence their contribution to the Higgs mass can only arise via mixing with the left-handed mirror leptons in the SU(5) multiplets. The only invariant operator that can arise from $\mathcal{L}_{Y_H}^{a)}$ or $\mathcal{L}_{Y_H}^{b)}$ is thus $(\bar{l}_{HL}\widetilde{H})(\widetilde{H}^T\widetilde{l}^c_R)$ that cannot contribute to the Higgs mass parameter in the SM symmetric phase. Consequently, the mirror-partner fermions, even necessary for other purposes, do not participate in the cancellation of divergences of the Higgs mass.

For the contributions of the SU(2) singlet $\chi_\pm$ one has to distinguish between the different T-parity definitions. Similarly to the mirror-partner leptons, the right-handed singlet also shares the SO(5) multiplet with the right-handed mirror leptons while its left-handed counterpart lives in an isolate SO(5) multiplet. Thus the only possible contribution to the Higgs mass would come from the mixing with a left-handed field living in a SU(5) representation. Choosing the T-even realization $\mathcal{L}_{Y_H}^{a)}$ provides $\bar{l}_L\widetilde{H}(\chi_+)_R$ while choosing the T-odd realization gives the operator $\bar{l}_{HL}\widetilde{H}(\chi_-)_R$. Both operators contribute to the Higgs mass parameter as we will show in the next two sections with an explicit calculation. Consequently, the singlet cannot be avoided in the aim of preserving a consistent Little Higgs model without quadratically divergent contributions to the Higgs mass.

Let us evaluate the quadratic and logarithmic contributions to the Higgs mass coming from mirror leptons and the singlet to elucidate if the latter can also be avoided.

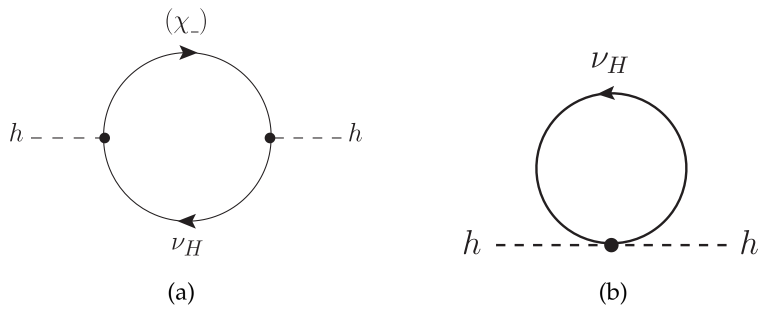


FIGURE B.1: One-loop Feynman diagrams contributing to the quadratic and logarithmic divergences of the Higgs self-energy from the fermion sector in the LHT with a T-odd SU(2) singlet.

B.1. T-odd case

Let us begin with the T-odd scenario since the T-even casecan be read from this as we will justify below. The Feynman rules we need come from the operators $(\bar{l}_{HL}\widetilde{H})(\widetilde{H}^\dagger l_{HR})$ and $\bar{l}_{HL}\widetilde{H}(\chi_-)_R$. For one family these are collected in table B.2. The Feynman diagrams that contribute to the Higgs mass parameter are depicted in fig. B.1. In general they generate quadratic an logarithmic divergent contributions that read

$$\delta\mu_a^2 = \frac{1}{4\pi^2}\left[-\frac{\kappa^2}{2}\Lambda^2 + \frac{\kappa^2}{2}\left(M_\chi^2 + m_{\ell_H}^2\right)\log\Lambda^2\right] = \frac{1}{4\pi^2}\left[-\frac{\kappa^2}{2}\Lambda^2 + \frac{\kappa^2}{2}\left(M_\chi^2 + 2\kappa^2 f^2\right)\log\Lambda^2\right], \tag{B.2}$$

$$\delta\mu_b^2 = \frac{1}{4\pi^2}\left(\frac{\kappa^2}{2}\Lambda^2 - m_{\ell_H}^3\frac{\kappa}{2\sqrt{2}f}\log\Lambda^2\right) = \frac{1}{4\pi^2}\left(\frac{\kappa^2}{2}\Lambda^2 - \kappa^4 f^2\log\Lambda^2\right), \tag{B.3}$$

where in the second step of eqs. (B.2) and (B.3) we used that the mass of the mirror leptons is $m_{\ell_H} = \sqrt{2}\kappa f$. The quadratically divergent contributions to the Higgs mass cancel because the singlet is included in the right-handed SO(5) multiplet and the divergence is independent of the mass of the particles running in the loop.

The logarithmically divergent contributions, after using the relation $m_{\ell_H} = \sqrt{2}\kappa f$ in eq. (B.2), do not vanish but there survives a term of size

$$\delta\mu^2 = \frac{1}{8\pi^2}\kappa^2 M_\chi^2\log\Lambda^2, \tag{B.4}$$

that cannot be avoided unless χ_- is massless.

B.2. T-even case

The T-even singlet scenario can be studied analogously to the T-odd case. The Feynman rules we need come from the operators $(\bar{l}_{HL}\widetilde{H})(\widetilde{H}^\dagger l_{HR})$ and $\bar{l}_L\widetilde{H}(\chi_+)_R$. For one family these are collected in table B.3. In this case the Feynman diagrams that contribute are depicted in fig. B.2 and generate divergent contributions of size

$$\delta\mu_a^2 = \frac{1}{4\pi^2}\left(-\frac{\kappa^2}{2}\Lambda^2 + \frac{\kappa^2}{2}M_\chi^2\log\Lambda^2\right), \tag{B.5}$$

$$\delta\mu_b^2 = \frac{1}{4\pi^2}\left(\frac{\kappa^2}{2}\Lambda^2 - \kappa^4 f^2\log\Lambda^2\right). \tag{B.6}$$

Fields	c_L	c_R
$h\bar{\nu}\chi_+$	0	$-\frac{\kappa}{\sqrt{2}}$
$hh\bar{\nu}_H\nu_H$	$\frac{\kappa}{2\sqrt{2}f}$	$\frac{\kappa}{2\sqrt{2}f}$

TABLE B.3: Feynman rules needed to compute the contribution of the heavy leptons to the Higgs mass parameter in the LHT with a T-even SU(2) singlet.

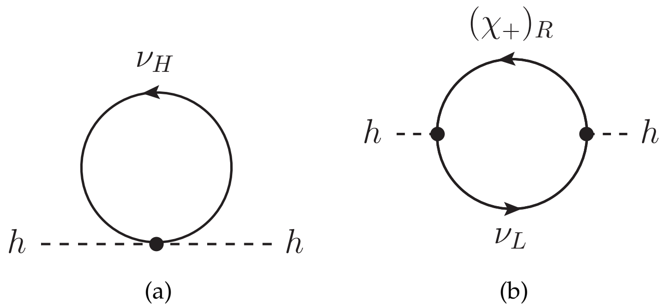


FIGURE B.2: One-loop Feynman diagrams contributing to the quadratic and logarithmic divergences of the Higgs self-energy from the fermion sector in the LHT with a T-even SU(2) singlet.

Notice that the contribution of the mirror leptons $\delta\mu_b^2$ is the same as in the T-odd case since the different implementation of T-parity does not affect the couplings of the mirror leptons to the Higgs as one can realize by comparing tables B.2 and B.3. On the other hand, the singlet coupling to the Higgs has opposite sign with respect to the T-odd case but the coupling enters squared in the first diagram of figs. B.1 and B.2. Thus the contribution of the T-even singlet can also be read from the T-odd case just by setting $m_{\ell_H} = 0$, because the SM neutrinos are massless.

As in the T-odd case the quadratically divergent contributions to the Higgs mass parameter are independent of the mass of the particle running in the loop. They cancel due to the presence of the singlet in the SO(5) right-handed multiplet.

On the other hand, the logarithmically divergent contributions are, in general, non vanishing,

$$\delta\mu^2 = \frac{1}{8\pi^2}\kappa^2\left[M_\chi^2 - \left(\sqrt{2}\kappa f\right)^2\right]\log\Lambda^2, \tag{B.7}$$

unless the mass of the singlet is exactly $M_\chi = \sqrt{2}\kappa f$. This is the condition that allows to neglect a contribution proportional to κ^4 and $\kappa^2 M_\chi^2$ to the Higgs mass parameter in the LHT with a T-even singlet χ_+.

B.3. Appendix summary

In this Appendix we have explored the dependence of the Higgs mass parameter on the Yukawa couplings κ_l and κ_q responsible for providing masses to the the heavy fermions (except for the top partners) within the LHT scenario. This is motivated by the explicit dependence of the Higgs mass on T_κ in the NLHT in eq. (B.1).

Focusing on leptons, the Lagrangian responsible for providing masses to the mirror leptons is $\mathcal{L}_{Y_H}^{a)}$ or $\mathcal{L}_{Y_H}^{b)}$ depending on whether the SU(2) singlet inside the SO(5) right-handed quintuplet is chosen to be T-even or T-odd, respectively. As far as the SO(5) quintuplets are completed, both Lagrangians are compatible with the absence of quadratically divergent contributions to the Higgs mass. This is because given a global transformation of the different SU(3) factors inside SU(5), the corresponding SO(5) transformation mixes the mirror and mirror-partner leptons with the singlet and hence all of them must be included. However, not all the fields participate in the quadratic divergences. Independently of the T-parity definition, the mirror leptons contribute through the operator $(\bar{l}_{HL}\widetilde{H})(\widetilde{H}^\dagger l_{HR})$ and thus only the neutral mirror leptons couple to the Higgs. The singlet contributes through $\bar{l}_L\widetilde{H}(\chi_+)_R$ or $\bar{l}_{HL}\widetilde{H}(\chi_-)_R$ in the T-even and T-odd scenario, respectively. On the other hand, the mirror-partner fermions, even though being necessary for other purposes, do not participate in the cancellation of divergences of the Higgs mass parameter. This is because mirror and mirror-partner leptons have opposite hypecharges and the left-handed mirror-partner leptons do not live in the SU(5) multiplets. As a consequence the only operator that can arise is $(\bar{l}_{HL}\widetilde{H})(\widetilde{H}^T(\widetilde{l}^c_-)_R)$ that cannot contribute to the Higgs mass.

The quadratically divergent contributions to the Higgs mass cancel as far as the the singlet is included in the SO(5) right-handed multiplet, independently of its mass and of the chosen action of T-parity. Thus, by consistency of the model, the singlet cannot be avoided in any case. However, the logarithmically divergent contributions only cancel in the T-even case if the singlet and mirror leptons share the same mass $M_\chi = \sqrt{2}\kappa f$ while in the T-odd scenario there survives a term $\sim \kappa^2 M_\chi^2$. Consequently, the T-even scenario is the only that can accommodate a Higgs mass that is independent of the Yukawa couplings κ and of the mass of the SU(2) singlet.

Appendix C

A more economical top quark sector in the NLHT

C.1. Simplification of the quark sector

In Chapter 4 we built the different Yukawa Lagrangians needed to provide masses for all quarks in the model while preserving gauge invariance. First of all one introduces the completed left-handed SU(5) quintuplets

$$\Psi_1^q = \begin{pmatrix} -i\sigma^2 q_{1L} \\ i\chi_{1L}^q \\ -i\sigma^2 \widetilde{q}_{1L}^c \end{pmatrix}, \quad \Psi_2^q = \begin{pmatrix} -i\sigma^2 \widetilde{q}_{2L}^c \\ i\chi_{2L}^q \\ -i\sigma^2 q_{2L} \end{pmatrix} \tag{C.1}$$

with transformation properties under the global group and T-parity,

$$\Psi_2^q \xrightarrow{G_g} V_g \Psi_2^q, \quad \Psi_1^q \xrightarrow{G_g} V_g^* \Psi_1^q, \quad \Psi_2^q \xrightarrow{\mathrm{T}} \Omega\Sigma_0 \Psi_1^q. \tag{C.2}$$

They couple to the fields in the SO(5) and $[\mathrm{SU}(2)\times \mathrm{U}(1)]''$ quintuplets

$$\Psi_R^q = \begin{pmatrix} -i\sigma^2 (\widetilde{q}_-^c)_R \\ i\,(\chi_+^q)_R \\ -i\sigma^2 q_{HR} \end{pmatrix}, \quad \widehat{\Psi}_R^q = \begin{pmatrix} -i\sigma^2 (\widetilde{q}_+^c)_R \\ i\,(\chi_-^q)_R \\ 0_2 \end{pmatrix} \tag{C.3}$$

with transformation properties

$$\Psi_R^q \xrightarrow{G_g} U_g \Psi_R^q, \quad \widehat{\Psi}_R^q \xrightarrow{G_g} \widehat{U}_g \widehat{\Psi}_R^q, \quad \Psi_R^q \xrightarrow{\mathrm{T}} \Omega \Psi_R^q, \quad \widehat{\Psi}_R^q \xrightarrow{\mathrm{T}} -\Omega \widehat{\Psi}_R^q \tag{C.4}$$

through the Yukawa Lagrangians

$$\mathcal{L}_{Y_H}^q = -\kappa_q f \left(\overline{\Psi}_2^q \xi + \overline{\Psi}_1^q \Sigma_0 \xi^\dagger \right) \Psi_R^q + \mathrm{h.c.}, \tag{C.5}$$

$$\mathcal{L}_{\widehat{Y}_H}^q = -\widehat{\kappa}_q f \left(\overline{\Psi}_2^q \widehat{\xi} - \overline{\Psi}_1^q \Sigma_0 \widehat{\xi}^\dagger \right) \widehat{\Psi}_R^q + \mathrm{h.c.}. \tag{C.6}$$

On the other hand, for the top quark one implements the collective symmetry breaking mechanism in order to avoid quadratically divergent contributions to the Higgs mass parameter proportional to the top Yukawa coupling. For that purpose, inspired by the LHT, for the third family of quarks one introduces the incomplete SU(5) multiplets

$$Q_1^t = \begin{pmatrix} -i\sigma^2 \mathcal{T}_{1L} \\ iT_{1L} \\ 0_2 \end{pmatrix}, \quad Q_2^t = \begin{pmatrix} 0_2 \\ iT_{2L} \\ -i\sigma^2 \mathcal{T}_{2L} \end{pmatrix}, \tag{C.7}$$

where T_{1L} and T_{2L} are the left-handed components of the top quark partners that are responsible for the cancellation of the top quark quadratic divergence to the Higgs mass. Their transformation properties under the gauge group and T-parity read

$$Q_1^t \xrightarrow{G_g} V_g^* Q_1^t, \quad Q_2^t \xrightarrow{G_g} V_g Q_2^t, \quad Q_2^t \xrightarrow{\mathrm{T}} \Omega\Sigma_0 Q_1^t. \tag{C.8}$$

Adding the right-handed field t_R and the right-handed counterparts of the top partners T_{1R}, T_{2R} one builds the Yukawa Lagrangian that provides the top couplings to the Higgs and masses for the top quark partners

$$\begin{aligned}\mathcal{L}_Y^t = &- i\frac{\lambda_1 f}{4}\epsilon_{ijk}\epsilon_{xy}\left[\left(\overline{Q}_1^t\right)_i \Sigma_{jx}\Sigma_{ky} + \left(\overline{Q}_2^t\Sigma_0\Omega\right)_i \widetilde{\Sigma}_{jx}\widetilde{\Sigma}_{ky}\right] t_R \\ &- \frac{\lambda_2 f}{\sqrt{2}}\left(\overline{T}_{1L}\widehat{X}T_{1R} + \overline{T}_{2L}\widehat{X}^* T_{2R}\right) + \text{h.c.},\end{aligned} \tag{C.9}$$

where $\{i,j,k\} = 1,2,3$ and $\{x,y\} = 4,5$.

However, one may notice that the left-handed multiplets Ψ_1^q, Q_1^t and Ψ_2^q, Q_2^t have the same transformation properties under the gauge group and T-parity. In particular, we might identify the top partners $T_{1L} \to \chi_{1L}$ and $T_{2L} \to \chi_{2L}$. Hence, we can eliminate $Q_{1,2}^t$ and plug the third family of Ψ_1^q and Ψ_2^q in the Yukawa Lagrangian (C.9). Since the T-even combination of χ_{1L} and χ_{2L} couples to $(\chi_+)_R$ through (C.5) and gets a mass proportional to κ_q while the T-odd combination couples to $(\chi_-)_R$ through (C.6) acquiring a mass proportional to $\widehat{\kappa}_q$ (see § 4.1.4), the Yukawa coupling λ_2 responsible for providing masses to the top partners can be set to zero. One may think that in this case, the mirror partner quarks would also couple to t_R. However, this is not the case because the indices of Ψ_1^q, $\Sigma_0\Psi_2$ are contracted with ϵ_{ijk} with $\{i,j,k\} = 1,2,3$ and thus the mirror partner quarks, living in the last two components of Ψ_1^q, $\Sigma_0\Psi_2$, do not receive new couplings from (C.9).

C.2. Top and top partner masses

This simplification in the quark sector affects to the mass of the top quark and its corresponding partners. For the sake of simplicity let us consider a diagonal κ_q coupling in flavour space. After defining the T-even and T-odd eigenstates (see § 4.1.4), the T-even fields mass matrix reads

$$-\mathcal{L}_{t,\kappa_q,\widehat{\kappa}_q} \supset \left(\bar{t}_L, (\overline{\chi}_+^q)_L\right)\begin{pmatrix}\frac{\lambda_1 v}{\sqrt{2}} & 0 \\ \frac{\lambda_1 f}{\sqrt{2}} & \sqrt{2}\,(\kappa_q)_{33} f\end{pmatrix}\begin{pmatrix} t_R \\ (\chi_+^q)_R\end{pmatrix} + \text{h.c.}. \tag{C.10}$$

At leading order this matrix can be diagonalized by a redefinition of the right-handed fields that can be read from eq. (3.86) by the substitution $\lambda_2 \to 2(\kappa_q)_{33}$

$$\begin{pmatrix} t_R \\ (\chi_+^q)_R\end{pmatrix} \to \begin{pmatrix} c_R & s_R \\ -s_R & c_R\end{pmatrix}\begin{pmatrix} t_R \\ (\chi_+^q)_R\end{pmatrix}, \quad c_R = \frac{2\,(\kappa_q)_{33}}{\sqrt{\lambda_1^2 + 4\,(\kappa_q)_{33}^2}}, \quad s_R = \frac{\lambda_1}{\sqrt{\lambda_1^2 + 4\,(\kappa_q)_{33}^2}}. \tag{C.11}$$

Thus the physical mass of the SM top quark and its T-even top partner is given by

$$m_t = \frac{1}{\sqrt{2}}\frac{\lambda_1\,(\kappa_q)_{33}}{\sqrt{\lambda_1^2 + 4\,(\kappa_q)_{33}^2}}, \tag{C.12}$$

$$M_{\chi_+^q} = \sqrt{\frac{\lambda_1^2 + 4\left(\kappa_q\right)_{33}^2}{2}} f. \tag{C.13}$$

As a consequence, the relation between the couplings λ_1 and $\left(\kappa_q\right)_{33}$ is

$$\frac{1}{\lambda_1^2} + \frac{1}{4\left(\kappa_q\right)_{33}^2} = \left(\frac{v}{\sqrt{2}m_t}\right)^2. \tag{C.14}$$

On the other hand, the T-odd top partner does not mix with t_R due to T-parity and its mass comes exclusively from $\mathcal{L}^q_{\widehat{Y}_H}$. At leading order

$$M_{\chi_-^q} = \sqrt{2}\widehat{\kappa} f. \tag{C.15}$$

C.3. Physical scalar masses and Higgs potential

At this point, one may wonder whether this simplification in terms of fermion fields has also effects on the scalar field masses, in particular on the Higgs mass. For that purpose let us apply once again the BFM to these Yukawa Lagrangians. Performing the substitutions

$$\Psi_1^q = \mathcal{A}\widetilde{\Psi}_1^q, \quad \Psi_2^q = \mathcal{A}\widetilde{\Psi}_2^q, \quad \Psi_R^q = \mathcal{A}\widetilde{\Psi}_R^q, \quad \widehat{\Psi}_R^q = \mathcal{B}\widetilde{\widehat{\Psi}}_R^q, \tag{C.16}$$

where

$$\mathcal{A} = \begin{pmatrix} -i\sigma^2 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & -i\sigma^2 \end{pmatrix}, \quad \mathcal{B} = \begin{pmatrix} -i\sigma^2 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0_{2\times 2} \end{pmatrix}. \tag{C.17}$$

Collecting the left-handed and right-handed quantum fields in vectors

$$\begin{aligned} v_L^T &= \left(\psi_2^{qT}, \psi_1^{qT}, \bullet, \bullet, \bullet\right), \\ v_R^T &= \left(\bullet, \bullet, \psi_R^{qT}, \widehat{\psi}_R^{qT}, \mathrm{t}_R^T\right), \end{aligned} \tag{C.18}$$

we can compute the matrix r_q in eq. (4.104) needed to obtain the quadratic and logarithmic contributions to the scalar potential in eqs. (4.147) and (4.148),

$$\mathcal{L}^{(2,q)}_{Y_H,\widehat{Y}_H,t} = -\overline{v}_L r_q v_R + \text{h.c.} \tag{C.19}$$

with

$$r_q = \begin{pmatrix} 0 & 0 & (r_{2R})_{mn\beta}{}^{\alpha} & (r_{2\widehat{R}})_{mn\beta}{}^{\alpha} & (r_{2t})_{m\beta}{}^{\alpha} \\ 0 & 0 & (r_{1R})_{mn\beta}{}^{\alpha} & (r_{1\widehat{R}})_{mn\beta}{}^{\alpha} & (r_{1t})_{m\beta}{}^{\alpha} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{C.20}$$

where the greek indices are the SU(3) color indices and $l_q = r_q^\dagger$. The components of the matrix r_q are defined as

$$(r_{2R})_{mn\beta}{}^{\alpha} = \kappa_q f \left(\mathcal{A}^\dagger \xi \mathcal{A}\right)_{mn} \delta^\alpha_\beta, \tag{C.21}$$

$$\left(r_{2\widehat{R}}\right)_{mn\beta}{}^{\alpha} = \widehat{\kappa}_q f \left(\mathcal{A}^\dagger \widehat{\xi} \mathcal{B}\right)_{mn} \delta^\alpha_\beta, \tag{C.22}$$

$$\left(r_{2t}\right)_{m\beta}{}^{\alpha} = \frac{i}{4}\lambda_1 f \left(\mathcal{A}^\dagger \Sigma_0 \Omega\right)_{mi} \epsilon_{ijk}\epsilon_{xy}\widetilde{\Sigma}_{jx}\widetilde{\Sigma}_{ky}\delta^\alpha_\beta, \tag{C.23}$$

$$\left(r_{1R}\right)_{mn\beta}{}^{\alpha} = \kappa_q f \left(\mathcal{A}^\dagger \Sigma_0 \xi^\dagger \mathcal{A}\right)_{mn} \delta^\alpha_\beta, \tag{C.24}$$

$$\left(r_{1\widehat{R}}\right)_{mn\beta}{}^{\alpha} = -\widehat{\kappa}_q f \left(\mathcal{A}^\dagger \Sigma_0 \widehat{\xi}^\dagger \mathcal{B}\right)_{mn} \delta^\alpha_\beta, \tag{C.25}$$

$$\left(r_{1t}\right)_{m\beta}{}^{\alpha} = \frac{i}{4}\lambda_1 f \mathcal{A}^\dagger_{mi}\epsilon_{ijk}\epsilon_{xy}\Sigma_{jx}\Sigma_{ky}\delta^\alpha_\beta. \tag{C.26}$$

Thus, using the expression for the quadratic divergences in eq. (4.147) one obtains

$$V^{\Lambda^2}_{L=1} = -\frac{3\Lambda^2}{128\pi^2}\lambda_1^2 f^2 \epsilon_{ijk}\epsilon_{ij'k'}\epsilon_{xy}\epsilon_{x'y'}\left(\Sigma_{jx}\Sigma_{ky}\Sigma^\dagger_{j'x'}\Sigma^\dagger_{k'y'} + \widetilde{\Sigma}_{jx}\widetilde{\Sigma}_{ky}\widetilde{\Sigma}^\dagger_{j'x'}\widetilde{\Sigma}^\dagger_{k'y'}\right), \tag{C.27}$$

containing the usual contribution to the triplet Φ mass and to the Higgs quartic coupling in eqs. (4.193) and (4.194), respectively,

$$V^{\Lambda^2}_{L=1} \supset \frac{3\lambda_1^2}{4\pi^2}\Lambda^2 \mathrm{tr}\left(\Phi^\dagger\Phi\right) + \frac{3\lambda_1^2}{16\pi^2}\frac{\Lambda^2}{f^2}\left(H^\dagger H\right)^2, \tag{C.28}$$

obtained in Chapter 4.

On the other hand, from eq. (4.148) the logarithmically divergent contributions read

$$\begin{aligned}
\mathcal{V}^{\log\Lambda^2}_{L=1,q} = \frac{3}{16\pi^2}\log\Lambda^2\Big[& \frac{1}{16^2}\lambda_1^4 f^4\left(\epsilon_{ijk}\epsilon_{ij'k'}\epsilon_{xy}\epsilon_{x'y'}\Sigma^\dagger_{jx}\Sigma^\dagger_{ky}\Sigma_{j'x'}\Sigma_{k'y'} + \epsilon_{ijk}\epsilon_{ij'k'}\epsilon_{xy}\epsilon_{x'y'}\widetilde{\Sigma}^\dagger_{jx}\widetilde{\Sigma}^\dagger_{ky}\widetilde{\Sigma}_{j'x'}\widetilde{\Sigma}_{k'y'}\right)^2 \\
&+ \frac{1}{8}\left(\kappa_q\kappa_q^\dagger\right)_{33}\lambda_1^2 f^4 \epsilon_{ijk}\epsilon_{ij'k'}\epsilon_{xy}\epsilon_{x'y'}\left(\widetilde{\Sigma}_{jx}\widetilde{\Sigma}_{ky}\widetilde{\Sigma}^\dagger_{j'x'}\widetilde{\Sigma}^\dagger_{k'y'} + \Sigma_{jx}\Sigma_{ky}\Sigma^\dagger_{j'x'}\Sigma^\dagger_{k'y'}\right) \\
&+ \frac{1}{8}\left(\kappa_q\kappa_q^\dagger\right)_{33}\lambda_1^2 f^4 \epsilon_{ijk}\epsilon_{i'j'k'}\epsilon_{xy}\epsilon_{x'y'}\Big(\left(\Omega\Sigma_0\Sigma\right)_{ii'}\Sigma_{jx}\Sigma_{ky}\widetilde{\Sigma}^\dagger_{j'x'}\widetilde{\Sigma}^\dagger_{k'y'} + \mathrm{h.c.}\Big) \\
&+ \frac{1}{8}\left(\widehat{\kappa}_q\widehat{\kappa}_q^\dagger\right)_{33}\lambda_1^2 f^4 \epsilon_{ijk}\epsilon_{i'j'k'}\epsilon_{xy}\epsilon_{x'y'} \\
&\quad\times\Big((\widehat{\xi}\Sigma_0)_{bi}\widetilde{\Sigma}_{jx}\widetilde{\Sigma}_{ky}\widetilde{\Sigma}^\dagger_{j'x'}\widetilde{\Sigma}^\dagger_{k'y'}(\Sigma_0\widehat{\xi}^\dagger)_{i'b} + (\widehat{\xi}\Sigma_0\Omega)_{ir}\widetilde{\Sigma}_{jx}\widetilde{\Sigma}_{ky}\widetilde{\Sigma}^\dagger_{j'x'}\widetilde{\Sigma}^\dagger_{k'y'}(\Omega\Sigma_0\widehat{\xi})_{i'b}\Big) \\
&- \frac{1}{8}\left(\widehat{\kappa}_q\widehat{\kappa}_q^\dagger\right)_{33}\lambda_1^2 f^4 \epsilon_{ijk}\epsilon_{i'j'k'}\epsilon_{xy}\epsilon_{x'y'}\Big((\widehat{\xi}\Sigma_0)_{bi}\Sigma_{jx}\Sigma_{ky}\widetilde{\Sigma}^\dagger_{j'x'}\widetilde{\Sigma}^\dagger_{k'y'}(\Omega\Sigma_0\widehat{\xi})_{i'b} + \mathrm{h.c.}\Big) \\
&- 2\mathrm{tr}\left(\kappa_q\kappa_q^\dagger\widehat{\kappa}_q\widehat{\kappa}_q^\dagger f^4\right)\sum_{a=1}^{3}\sum_{b=1}^{5}\left(\widehat{\Sigma}^\dagger_{ab}\Sigma_{ba} + \Sigma^\dagger_{ab}\widehat{\Sigma}_{ba}\right)\Big],
\end{aligned} \tag{C.29}$$

where we find

$$\begin{aligned}
\mathcal{V}^{\log\Lambda^2}_{L=1,q} \supset & -\frac{3f^2}{8\pi^2}\log\Lambda^2\lambda_1^2(\kappa\kappa^\dagger)_{33}(H^\dagger H) + \frac{3f^2}{5\pi^2}\log\Lambda^2\lambda_1^2(\widehat{\kappa}_q\widehat{\kappa}_q^\dagger)_{33}\widehat{\eta}^2 \\
& + \frac{3f^2}{8\pi^2}\log\Lambda^2\mathrm{tr}(\kappa_q\kappa_q^\dagger\widehat{\kappa}_q\widehat{\kappa}_q^\dagger)\times\left(6H^\dagger H + \frac{36\widehat{\eta}^2}{5} + 8\mathrm{tr}(\widehat{\omega}^\dagger\widehat{\omega})\right).
\end{aligned} \tag{C.30}$$

The first term gives a negative contribution to the Higgs mass squared parameter μ^2 that substitutes that proportional to $\lambda_1^2\lambda_2^2$ in eqs. (4.198) and (B.1). The second term is a new contribution to the mass of the singlet $\widehat{\eta}$, while the third term is the usual contribution coming from the κ Yukawa couplings. Thus, after adding the contribution coming from leptons and gauge bosons evaluated in Chapter 4, the parameters of the Higgs potential read

$$\mu^2 = \frac{f^2}{16\pi^2} \log \Lambda^2 \left(-6\lambda_1^2 (\kappa\kappa^\dagger)_{33} + 6g^4 + \frac{2}{5} g'^4 + 12T_\kappa \right), \tag{C.31}$$

$$\lambda = \frac{1}{16\pi^2} \frac{\Lambda^2}{f^2} \left(g^2 + g'^2 + 3\lambda_1^2 \right). \tag{C.32}$$

Minimizing the Higgs potential provides the expression for the Higgs mass and *vev* in terms of μ^2 and λ, $M_h^2 = -2\mu^2$ and $v = \sqrt{-\mu^2/\lambda}$. In terms of the Higgs mass and *vev* the rest of the scalar field masses are given by

$$M_\Phi^2 = 2\frac{f^2}{v^2} M_h^2 \tag{C.33}$$

$$M_{\widehat{\omega}}^2 = 8M_h^2 \frac{g^4 + T_\kappa}{6\lambda_1^2(\kappa\kappa^\dagger)_{33} - 6g^4 - \frac{2}{5}g'^4 - 12T_\kappa}, \tag{C.34}$$

$$M_{\widehat{\eta}}^2 = \frac{72}{10} M_h^2 \frac{2T_\kappa + 9\lambda_1^2(\kappa\kappa^\dagger)_{33}}{6\lambda_1^2(\kappa\kappa^\dagger)_{33} - 6g^4 - \frac{2}{5}g'^4 - 12T_\kappa}. \tag{C.35}$$

The relation between the masses of the real triplet $\widehat{\omega}$ and singlet $\widehat{\eta}$ is

$$M_{\widehat{\eta}}^2 = \frac{9}{10} \frac{2T_\kappa + 9\lambda_1^2(\kappa\kappa^\dagger)_{33}}{g^4 + T_\kappa} M_{\widehat{\omega}}^2. \tag{C.36}$$

Appendix D

NLHT Feynrules/FeynArts/FormCalc model file

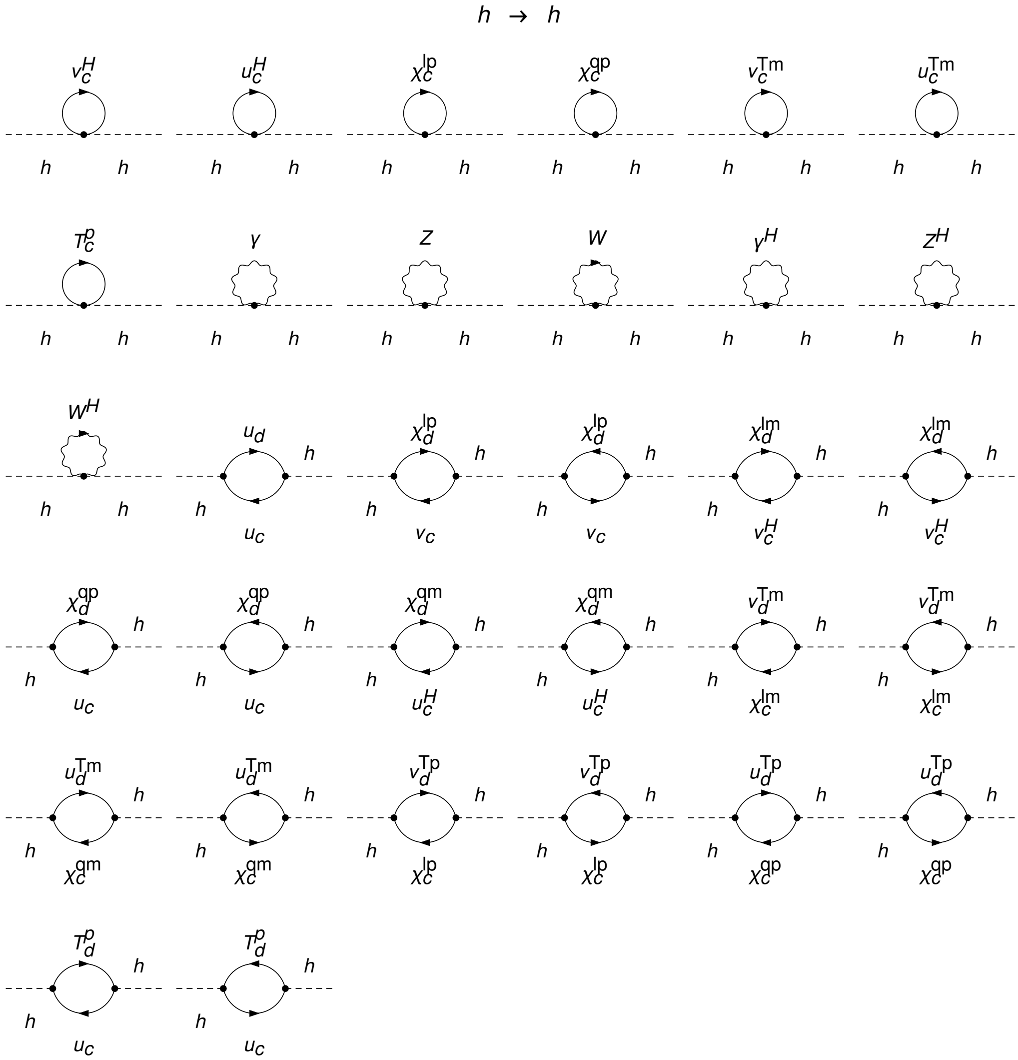
The NLHT presented in Chapter 4 has been implemented in FeynRules [160] in the SM symmetric phase. However, the diagonalization of the mass matrix of the neutral leptons and the up-type quarks in table 4.3 at order v^2/f^2 is still a work in progress. Nevertheless, to crosscheck some of the results we showed in Chapter 4 one can neglect mixing terms between fermions.

Once the Feynman rules are generated, we used FeynArts [164] and FormCalc [165] to compute at one loop the masses of the light pseudo-Goldstone bosons that arise from the logarithmic part of the Coleman-Weinberg potential: the Higgs h, the real triplet $\widehat{\omega}$ and the singlet $\widehat{\eta}$. The mass of the complex triplet Φ cannot be evaluated directly using FormCalc. This is because FormCalc works with the dimensional regularization scheme that allows to parametrize only the logarithmic divergences through the identification $\log \Lambda^2 \leftrightarrow 2/\epsilon$ with $\epsilon = 4 - d$ while the mass of the complex triplet comes from the quadratically divergent part of the Coleman-Weinberg potential.

The Feynman diagrams that contribute to the scalar masses and the corresponding results are depicted in figs. D.1, D.2, D.3 and D.4. To compare with our previous results in Chapter 4 one has to take into account the notation by FeynRules and FormCalc for couplings and energy scales. These are shown in table D.1.

NLHT	FeynRules & FormCalc
g	g2
g'	g1
λ_1	c1
λ_2	c2
κ_l	kappal
κ_q	kappaq
$\widehat{\kappa}_l$	kappalhat
$\widehat{\kappa}_q$	kappaqhat
f	fH
v	vev

TABLE D.1: Comparison between the notation used in this Thesis and the FeynRules/FormCalc notation.



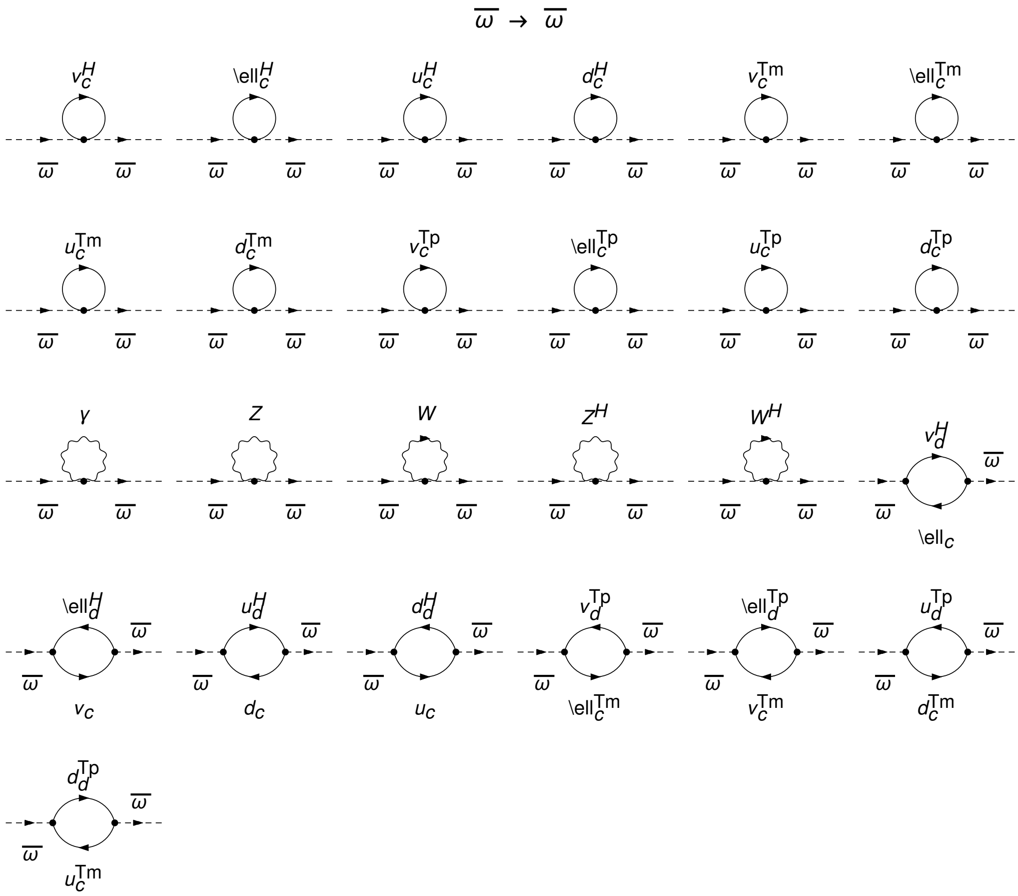
(a)

In[∘]:= $\mu 2$

Out[∘]= $$\frac{3\,\mathrm{Trace}[\kappa l.\kappa ldag.\kappa lhat.\kappa lhatdag]}{4\,fH^2\,\pi^2}+\frac{9\,\mathrm{Trace}[\kappa q.\kappa qdag.\kappa qhat.\kappa qhatdag]}{4\,fH^2\,\pi^2}-\frac{3\,c1^2\,c2^2\,fH^2}{16\,\pi^2}+\frac{fH^2\,g1^4}{40\,\pi^2}+\frac{3\,fH^2\,g2^4}{8\,\pi^2}$$

(b)

FIGURE D.1: One-loop Feynman diagrams that contribute to the Higgs mass parameter μ^2 and result.



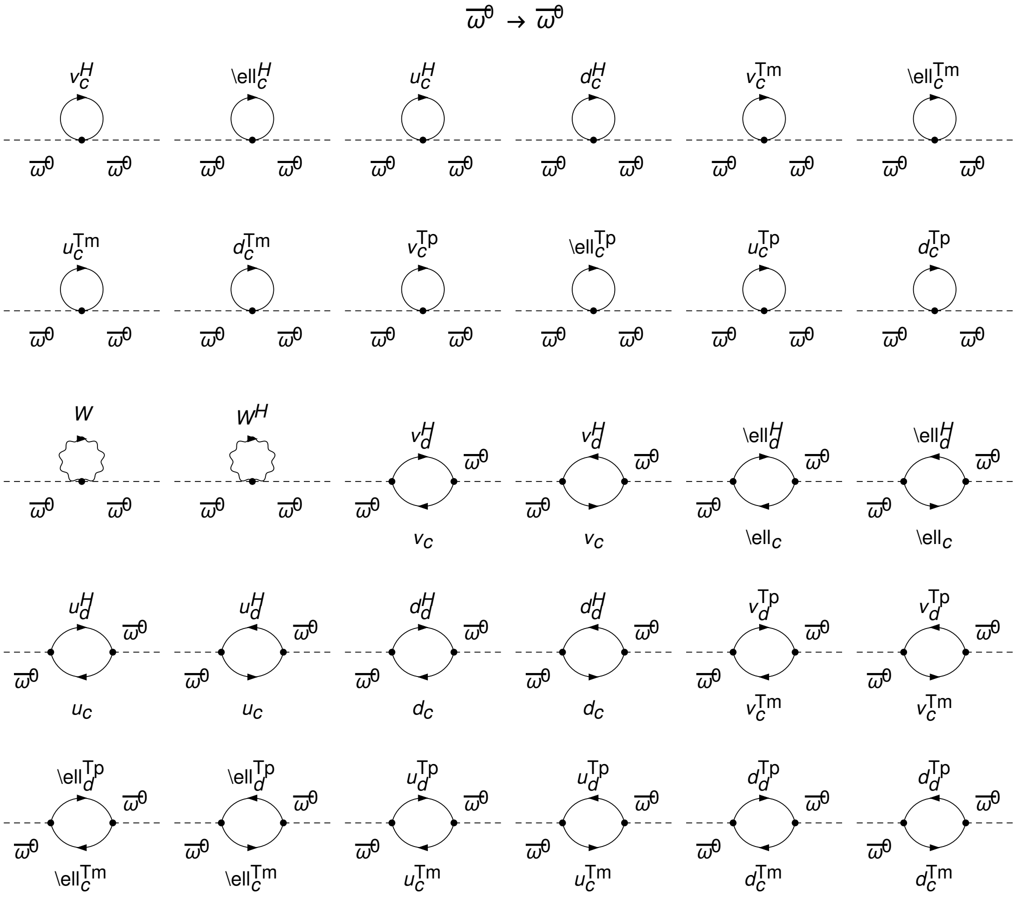
(a)

In[∘]:= **Mω2**

$$\text{Out}[\circ]= \frac{\text{Trace}[\kappa\text{l}.\kappa\text{ldag}.\kappa\text{lhat}.\kappa\text{lhatdag}]}{\text{fH}^2\,\pi^2} + \frac{3\,\text{Trace}[\kappa\text{q}.\kappa\text{qdag}.\kappa\text{qhat}.\kappa\text{qhatdag}]}{\text{fH}^2\,\pi^2} + \frac{\text{fH}^2\,\text{g2}^4}{\pi^2}$$

(b)

FIGURE D.2: One-loop Feynman diagrams that contribute to the charged components of the real triplet $\widehat{\omega}$ mass and result.



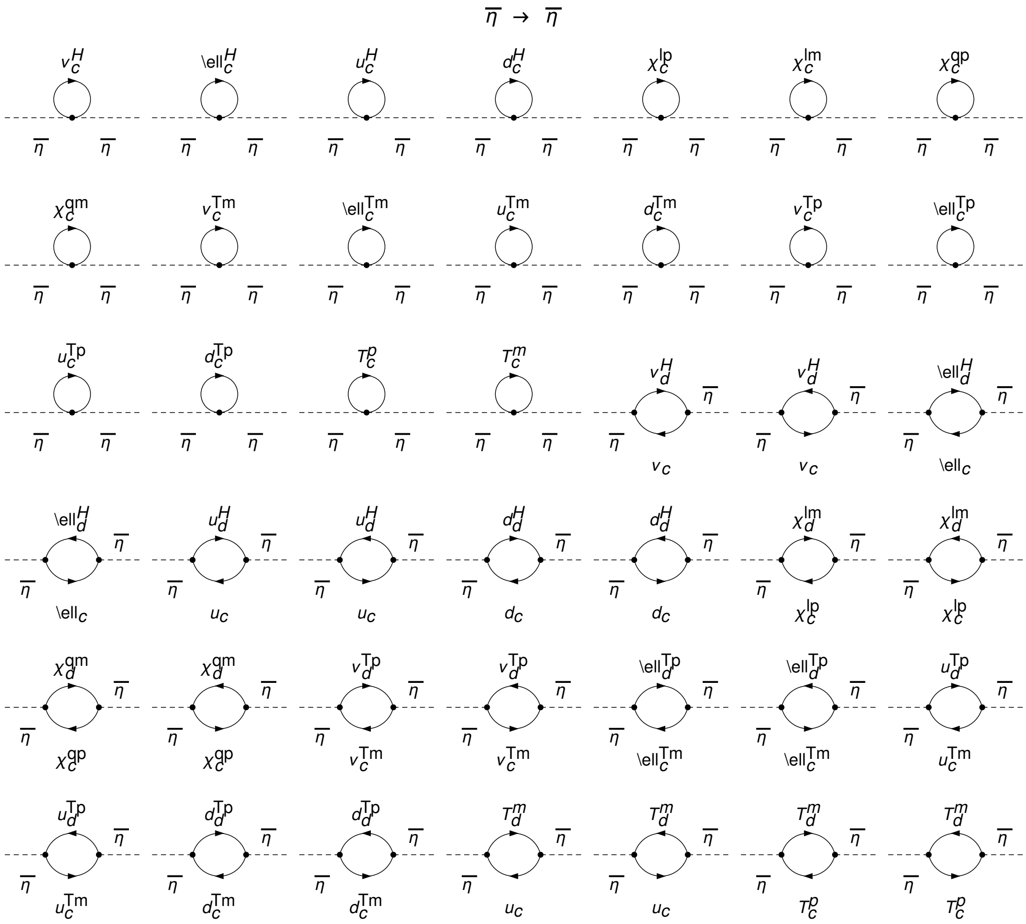
(a)

In[∘]:= **Mω02**

Out[∘]= $\frac{\text{Trace}[\kappa\text{l}.\kappa\text{ldag}.\kappa\text{lhat}.\kappa\text{lhatdag}]}{\text{fH}^2\,\pi^2} + \frac{3\,\text{Trace}[\kappa\text{q}.\kappa\text{qdag}.\kappa\text{qhat}.\kappa\text{qhatdag}]}{\text{fH}^2\,\pi^2} + \frac{\text{fH}^2\,\text{g2}^4}{\pi^2}$

(b)

FIGURE D.3: One-loop Feynman diagrams that contribute to the neutral component of the real triplet $\widehat{\omega}$ mass and result.



(a)

In[∘]:= **Mη2**

Out[∘]= $$-\frac{9\,\text{Trace}[\kappa\text{l}.\kappa\text{ldag}.\kappa\text{lhat}.\kappa\text{lhatdag}]}{5\,\text{fH}^2\,\pi^2}-\frac{27\,\text{Trace}[\kappa\text{q}.\kappa\text{qdag}.\kappa\text{qhat}.\kappa\text{qhatdag}]}{5\,\text{fH}^2\,\pi^2}$$

(b)

FIGURE D.4: One-loop Feynman diagrams that contribute to the singlet $\widehat{\eta}$ mass and result.

List of Figures

1.1. Most relevant one-loop quadratically divergent contributions to the Higgs mass squared in the SM in the gauge eigenbasis. 6
1.2. Fine tuning required to obtain the experimental value of the Higgs mass in the SM for $\Lambda_{\rm SM} \approx 10$ TeV. Figure taken from [49]. 6

2.1. Illustration of the vacuum misalignment mechanism for the case of $G = \mathrm{SO}(3)$ and $H = \mathrm{SO}(2)$, where $\vec{F} = \Sigma_0$ and $\theta = \Pi$. (Figure extracted from [80]). 19
2.2. One-loop contribution to the Higgs mass squared from gauge bosons in the gauge eigenbasis. The big blobs represent off-diagonal mass insertions proportional to $g_1 g_2$ and $g_1' g_2'$. 25
2.3. One-loop contribution to the Higgs mass squared from the top quark and top partners in the gauge eigenbasis. The big blobs are mass insertions proportional to the λ_1, λ_2 Yukawa couplings in $\mathcal{L}_t^1$ and $\mathcal{L}_t^2$ in eqs. (2.74) and (2.75), respectively. Diagrams with only t_L and t_R are absent because they are $\mathrm{SU}(3)_1$ symmetric. . . 26
2.4. One-loop contribution to the Higgs mass squared from the top quark and top partners in the gauge eigenbasis in the toy model with T-parity. The big blobs are mass insertions proportional to the Yukawa couplings λ_1 and λ_2. 29
2.5. One-loop contribution to the Higgs mass squared from $\mathcal{L}_H$ in the gauge eigenbasis. The big blobs represent off-diagonal mass insertions proportional to κ and mass insertions proportional to M_x. 31

3.1. Topologies contributing to $h \to \bar{\ell}\ell'$. 56
3.2. In blue and orange respectively, real and imaginary parts of the one loop function in $c_L^{\rm phys}$. The singlet χ_+ decouples. 64
3.3. Feynman diagrams that generate the divergent operators in eq. (3.158) in LFV Higgs decays in the T-odd χ scenario. 66
3.4. Diagrammatic expansion of the tree-level integration out of χ_+ in eq. (3.165). . . 68
3.5. Topologies contributing to $\gamma \to \bar{\ell}\ell'$. 72
3.6. Allowed mass region for the mirror leptons mass m_{ℓ_H} versus the quasi-Dirac neutrino mass m_χ for different values of the NP scale f. Solid lines are fixed by the upper bound of 0.03 on the mixing between the SM and heavy neutrinos for $f = 2$ TeV and 2.5 TeV. The dashed lines correspond to a fixed value of $\kappa_l = 0.2$ according to [131]. 73
3.7. Feynman diagrams that lead to neutrino mass generation in the radiative inverse seesaw scenario with a T-odd singlet (χ_-). 74

4.1. One-loop Feynman diagrams that would lead to unacceptable quadratic divergences in the Higgs mass if the mixed Lagrangian of eq. (4.13) did not vanish. . . 84
4.2. One-loop Feynman diagrams contributing to the quadratic divergences of the Higgs self-energy from the fermion sector in the LHT extended with $\widehat{\Psi}_L$ (4.15). Diagrams (a) and (b) from $\mathcal{L}_{Y_H}$ cancel each other, whereas (c) and (d) from $\mathcal{L}_Y^{\hat{\xi}\xi}$ (4.18) are proportional to κ'^2 and do *not* cancel. 86

4.3. One-loop Feynman diagrams contributing to the quadratic divergences of the Higgs self-energy from the fermion sector in the NLHT. Diagrams (a) and (b) arise from $\mathcal{L}_{Y_H}$ and cancel each other, as in fig. 4.2. The rest stem from $\mathcal{L}_{\widehat{Y}_H}$ in eq. (4.25) and also cancel among themselves. 88
4.4. The interval of $\lambda_1 \in [1.05, 1.71]$ yielding a top quark-partner mass M_{T_+} below the scale Λ on the left-hand-side plot determines the range of possible values of $\lambda_2 \in [1.22, 3.10]$ through eq. (3.87) on the plot of the right-hand side. These intervals are independent of f but the mass scales are proportional to f. 115
4.5. Left: masses of the Higgs and the new T-odd scalars as a function of T_κ for a chosen value of $\lambda_1 = 1.5$. They are naturally light except for T_κ close to its minimum ($T^{\max}(\lambda_1 = 1.5) \approx 0.90$). Right: mass of the new T-odd singlet scalar field $\widehat{\eta}$ as a function of the allowed values of $\lambda_1 \in [1.05, 1.71]$ when $M_{\widehat{\eta}} = M_{\widehat{\omega}}$. . . 115
4.6. Region in the $M_{A_H} - m_{\ell_H}$ plane compatible with A_H as dark matter candidate (white). The A_H constitutes 100% (50%) of the dark matter abundance for masses within the orange (green) bands. The grey region is excluded because it would yield dark matter overabundance and the blue region is excluded because A_H would not be the LTP. 117
4.7. Values of f compatible with $M_{T_-} > 2$ TeV as a function of λ_1 (light blue). The region is further constrained (darker blue) by the conditions $T^{\max}_{\kappa,\widehat{\eta}}(\lambda_1) > T^{\min}_{\kappa,\widehat{\omega}}(\lambda_1, f)$ (upper bound) and $T^{\max}_{\kappa,\widehat{\eta}}(\lambda_1) > T^{\min}_{\kappa,q_H}(f)$ (lower bound) as explained in the text. 118
4.8. Correlation between the heavy quarks Yukawa coupling κ_q and the top quark Yukawa coupling λ_1 for different values of the scale f. 118
4.9. Ranges of allowed masses of the new T-odd scalars for different values of the scale f as a function of λ_1 119
4.10. Typical NLHT spectrum for $f \approx 2.7$ TeV and $\lambda_1 = 1.2$. 119
4.11. Decay widths for the triplet and singlet scalar fields normalized to the Higgs width for $\lambda_1 = 1.1$ and $\lambda_1 = 1.5$ (lighter color). The upper and lower bounds correspond to the maximum and minimum values of the scalar masses. 120

B.1. One-loop Feynman diagrams contributing to the quadratic and logarithmic divergences of the Higgs self-energy from the fermion sector in the LHT with a T-odd SU(2) singlet. 144
B.2. One-loop Feynman diagrams contributing to the quadratic and logarithmic divergences of the Higgs self-energy from the fermion sector in the LHT with a T-even SU(2) singlet. 145

D.1. One-loop Feynman diagrams that contribute to the Higgs mass parameter μ^2 and result. 154
D.2. One-loop Feynman diagrams that contribute to the charged components of the real triplet $\widehat{\omega}$ mass and result. 155
D.3. One-loop Feynman diagrams that contribute to the neutral component of the real triplet $\widehat{\omega}$ mass and result. 156
D.4. One-loop Feynman diagrams that contribute to the singlet $\widehat{\eta}$ mass and result. . . 157

List of Tables

1.1. Quantum numbers of the different SM constituents in the weak eigenbasis. . . . 3

2.1. Quantum numbers for quarks. The gauged hypercharge $\mathrm{U}(1)_i$ is the sum of that in $\mathrm{SU}(3)_i$ and the one in the extra $\mathrm{U}(1)'_i$. 26
2.2. Quantum numbers for quarks transforming in a linear representation of the global group. The gauged hypercharge $\mathrm{U}(1)_i$ is the sum of that inside $\mathrm{SU}(3)_i$ and the one in the extra $\mathrm{U}(1)'_i$. 29
2.3. Quantum numbers for quarks transforming in a non linear representation of the global group. The gauged hypercharge is the sum of that inside $\mathrm{SU}(3)_V$ and the one in the extra $\mathrm{U}(1)'_{1+2}$. 30

3.1. Hypercharges under $\mathrm{U}(1)'_1 \times \mathrm{U}(1)'_2 \subset SU(5)$ of the Σ components in eqs. (3.46) and (3.47). 45
3.2. Hypercharge assignments under $\mathrm{U}(1)'_1 \times \mathrm{U}(1)'_2 \subset \mathrm{SU}(5)$ and $\mathrm{U}(1)''_1 \times \mathrm{U}(1)''_2$ for fermions transforming in a linear representation of the gauge group. The hypercharges under the gauge group $\mathrm{U}(1)_1 \times \mathrm{U}(1)_2$ come from $Y_j = Y'_j + Y''_j$, $j = 1,2$. 46
3.3. Hypercharge assignments under the diagonal $\mathrm{U}(1)' \subset \mathrm{SO}(5)$ and $\mathrm{U}(1)''$ for fermions transforming in a non linear representation of the gauge group. 46
3.4. Order of the mixing between the top quark and the rest of T-even up-type quarks once the diagonalization in eq. (3.86) is performed. A dot means that they are connected by the mass term and a dash indicates that no mixing term is generated to order v^2. 53
3.5. As in table 3.4 but for the neutral lepton fields, SM up-type quarks of the first two generations and rest of the new LHT up-type quarks. 54
3.6. Divergent contributions proportional to $\frac{1}{\epsilon}$, with $\epsilon = 4 - d$ the extra dimensions in dimensional regularization, of each particle set running in the loop and topology in fig. 3.1 contributing at $\mathcal{O}(1)$. A dash means that the field set does not run in the diagram, whereas a dot indicates that infinite and finite parts vanish. 57
3.7. As in table 3.6 but to $\mathcal{O}(v^2/f^2)$. 57
3.8. Scalar-Fermion-Fermion couplings at $\mathcal{O}(v^2/f^2)$. 61
3.9. Scalar-Scalar-Fermion-Fermion couplings at $\mathcal{O}(v^2/f^2)$. 61
3.10. Scalar-Vector-Vector and Vector-Fermion-Fermion couplings at $\mathcal{O}(v^2/f^2)$. The right-handed Vector-Fermion-Fermion couplings g_R vanish. 61
3.11. Scalar-Scalar-Vector and Scalar-Scalar-Scalar couplings at $\mathcal{O}(v^2/f^2)$. 61
3.12. Order 1 singlet and neutrino divergent contributions proportional to $\frac{1}{\epsilon}$, with $\epsilon = 4 - d$ the extra dimensions in dimensional regularization. A dash means that the field set does not run in the diagram, whereas a 0 indicates that only the infinite part vanishes. 63
3.13. As in table 3.12 but to order v^2/f^2 singlet and neutrino divergent contributions proportional to $\frac{1}{\epsilon}$, with $\epsilon = 4 - d$ the extra dimensions in dimensional regularization. A dash means that the field set does not run in the diagram, whereas a 0 indicates that only the infinite part vanishes. 63

3.14. Scalar-Fermion-Fermion couplings at $\mathcal{O}(v^2/f^2)$ involving ν_H and a T-odd singlet. 65
3.15. Scalar-Scalar-Fermion-Fermion couplings at $\mathcal{O}(v^2/f^2)$ involving a T-odd singlet. 65
3.16. Order $\frac{v^2}{f^2}$ T-odd singlet contributions to the UV divergent part of $h \to \bar{\ell}_i \ell_j$. 66
3.17. As in table 3.2 for the extra χ_{1L} and χ_{2L} left-handed fields and the new scalar fields x, $\widetilde{x}$ required for the left-handed couplings of χ_- to ℓ. 67
3.18. Limits on the mixing between the SM and the heavy quasi-Dirac neutrinos from electroweak precision data (top) and from lepton flavour violating processes (bottom). The sum on the repeated index $j = 1, 2, 3$ is understood. 71
3.19. Vector-Vector-Vector, Scalar-Scalar-Vector couplings and Scalar-Vector-Vector couplings for $\gamma \to \bar{\ell}\ell'$. 71

4.1. Quantum numbers for fermions transforming in a linear representation. These fields are singlets under $[\mathrm{SU}(2)'' \times \mathrm{U}(1)'']^2$. 89
4.2. Quantum numbers for right-handed fermions transforming in a non linear representation. 90
4.3. Order of the mixing between neutral fields and up-type quarks in the NLHT. A dot means that they are connected by the mass term and a dash indicates that no mixing term is generated to order v^2. 95

B.1. Comparison between the LHT and NLHT heavy fermion sector. A dash means that the corresponding fields are absent in one model with respect to the other. In gray color we indicate the fields that are optional in the LHT. **5**, **5*** are the fundamental and antifundamental SU(5) representations while $\underline{\mathbf{5}}$ is the fundamental SO(5) representation. 142
B.2. Feynman rules needed to compute the contribution of the heavy leptons to the Higgs mass parameter in the LHT with a T-even SU(2) singlet. 143
B.3. Feynman rules needed to compute the contribution of the heavy leptons to the Higgs mass parameter in the LHT with a T-even SU(2) singlet. 145

D.1. Comparison between the notation used in this Thesis and the FeynRules/FormCalc notation. 153

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