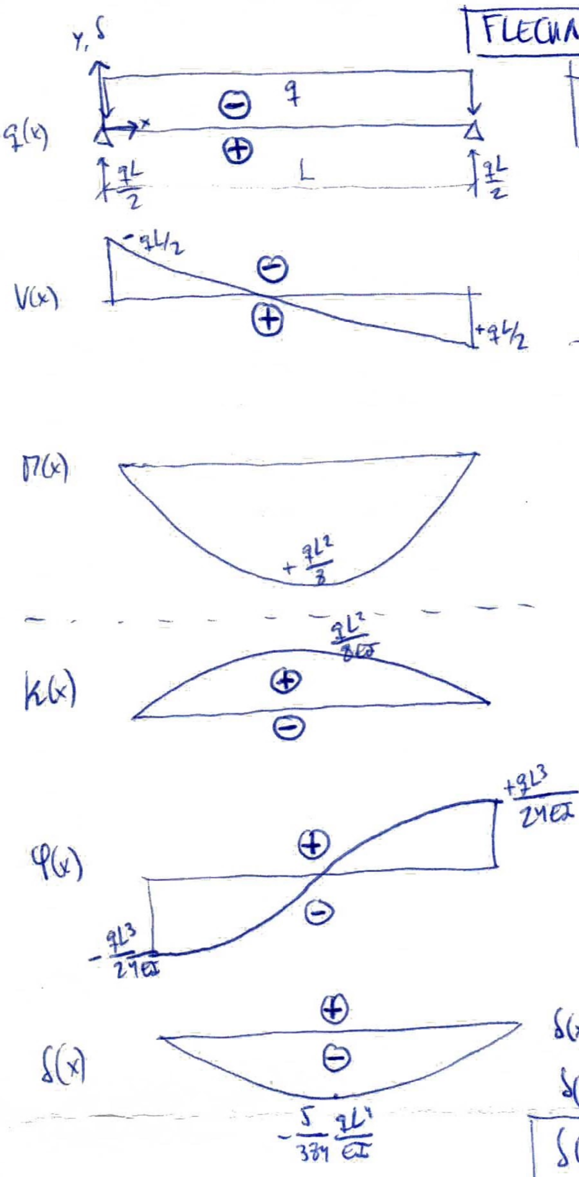
FLECHAS: MÉTODO DE LA ELÁSTICA



$\boxed{q(x) = -q} = -V'(x)$ (cte)

$\boxed{V(x) = -\frac{qL}{2} + qx} = -M'(x)$ $\begin{cases} V(0) = -\frac{qL}{2} \\ V(\frac{L}{2}) = 0 \\ V(L) = \frac{qL}{2} \end{cases}$

$M(x) = \frac{qL}{2}x - \frac{qx^2}{2} = \boxed{-\frac{q}{2}x^2 + \frac{qL}{2}x}$ $\begin{cases} M(0) = 0 \\ M(\frac{L}{2}) = \frac{qL^2}{8} \\ M(L) = 0 \end{cases}$

$K(x) = \frac{M(x)}{EI} = \frac{1}{EI}\left(-\frac{q}{2}x^2 + \frac{qL}{2}x\right) = \boxed{\frac{q}{EI}\left(-\frac{1}{2}x^2 + \frac{L}{2}x\right)}$ $\begin{cases} K(0) = 0 \\ K(\frac{L}{2}) = \frac{qL^2}{8EI} \\ K(L) = 0 \end{cases}$

$\varphi(x) = \int K(x)dx + C_1 = \frac{q}{EI}\left(-\frac{1}{6}x^3 + \frac{L}{4}x^2\right) + C_1$

$\varphi(\frac{L}{2}) = 0 \Rightarrow \frac{q}{EI}\left(-\frac{L^3}{6 \cdot 8} + \frac{L^3}{4 \cdot 4}\right) + C_1 = 0 \Rightarrow C_1 = \frac{qL^3}{EI}\left(\frac{1}{48} - \frac{1}{16}\right) =$

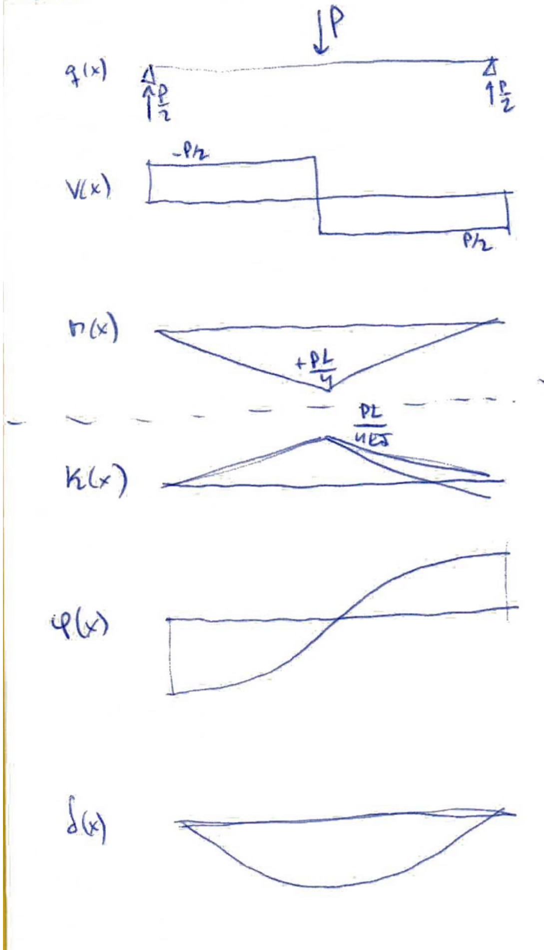
$= \frac{qL^3}{EI}\left(\frac{1-3}{48}\right) = -\frac{qL^3}{24EI} = \varphi(0)$

$\boxed{\varphi(x) = \frac{q}{EI}\left(-\frac{1}{6}x^3 + \frac{L}{4}x^2 - \frac{L^3}{24}\right)}$ $\begin{cases} \varphi(0) = -\frac{qL^3}{24EI} \\ \varphi(\frac{L}{2}) = 0 \\ \varphi(L) = \frac{qL^3}{24EI} \end{cases}$

$\delta(x) = \int \varphi(x)dx + C_2 = \frac{q}{EI}\left(-\frac{1}{24}x^4 + \frac{L}{12}x^3 - \frac{L^3}{24}x\right) + C_2$

$\delta(0) = 0 \Rightarrow C_2 = 0$

$\boxed{\delta(x) = \frac{q}{EI}\left(-\frac{1}{24}x^4 + \frac{L}{12}x^3 - \frac{L^3}{24}x\right)}$ $\begin{cases} \delta(0) = 0 \\ \delta(\frac{L}{2}) = \frac{5}{384}\frac{qL^4}{EI} \\ \delta(L) = 0 \end{cases}$



$\boxed{q(x) = 0} = V'(x)$

$\boxed{V(x) = \begin{cases} -\frac{P}{2}, & x \le \frac{L}{2} \\ \frac{P}{2} & x > \frac{L}{2} \end{cases}}$

$M(x) = \begin{cases} \boxed{\frac{P}{2}x} & x \le \frac{L}{2} \\ \frac{P}{2}x - P(x - \frac{L}{2}) = \boxed{-\frac{P}{2}x + \frac{PL}{2}} & x \ge \frac{L}{2} \end{cases}$

$\boxed{K(x) = \begin{cases} \frac{P}{2EI}x & , x \le \frac{L}{2} \\ \frac{P}{2EI}(-x + L) & , x \ge \frac{L}{2} \end{cases}}$

$\varphi(x) = \int K(x)dx + C_1 = \begin{cases} \frac{P}{4EI}x^2 + C_1 \xrightarrow{\varphi(\frac{L}{2}) = 0} C_1 = -\frac{PL^2}{16EI} \\ \frac{P}{4EI}(-x^2 + 2Lx) + C_1' \rightarrow C_1' = -\frac{P}{4EI}\left(-\frac{L^2}{4} + 2\frac{L^2}{2}\right) = -\frac{3PL^2}{16EI} \end{cases}$

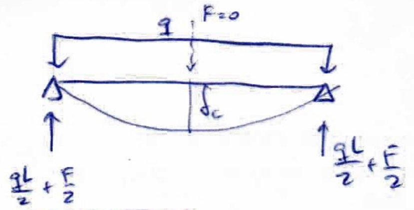
$\boxed{\varphi(x) = \begin{cases} \frac{P}{4EI}\left(x^2 - \frac{L^2}{4}\right) & , x \le \frac{L}{2} \\ \frac{P}{4EI}\left(-x^2 + 2Lx - \frac{3L^2}{4}\right) & , x \ge \frac{L}{2} \end{cases}}$

$\delta(x) = \int \varphi(x)dx + C_2 = \begin{cases} \frac{P}{4EI}\left(\frac{x^3}{3} - \frac{L^2}{4}x\right) + C_2 \\ \frac{P}{4EI}\left(-\frac{x^3}{3} + Lx^2 - \frac{3L^2}{4}x\right) + C_2' \end{cases}$

$C_2 = 0$; $C_2' = \frac{P}{4EI}\left(\frac{L^3}{3} - L^3 + \frac{3}{4}L^3\right) = \frac{PL^3}{4EI}\left(\frac{4 - 12 + 9}{12}\right) = \frac{PL^3}{48EI}$

$\delta(x) = \begin{cases} \frac{P}{4EI}\left(\frac{1}{3}x^3 - \frac{L^2}{4}x\right) & \delta(0) = 0 ✓ ; \ \delta(\frac{L}{2}) = \frac{P}{4EI}\left(\frac{L^3}{3 \cdot 8} - \frac{L^3}{8}\right) = \frac{P}{4EI}\left(\frac{L^3 - 3L^3}{24}\right) = -\frac{PL^3}{48EI} ✓ \\ \frac{P}{4EI}\left(-\frac{1}{3}x^3 + Lx^2 - \frac{3L^2}{4}x + \frac{L^3}{12}\right) & \delta(\frac{L}{2}) = \frac{P}{4EI}\left(-\frac{L^3}{3 \cdot 8} + \frac{L^3}{4} - \frac{3L^3}{8} + \frac{L^3}{12}\right) = \frac{PL^3}{4EI}\left(\frac{-1 + 6 - 9 + 2}{24}\right) \end{cases}$ ✓

FLECHAS: OTROS MÉTODOS



$$M(x)=\begin{cases}\frac{qL}{2}x+\frac{F}{2}x-\frac{qx^2}{2} & ,\ 0\le x\le \frac{L}{2}\\ \frac{qL}{2}x+\frac{F}{2}x-\frac{qx^2}{2}-F\left(x-\frac{L}{2}\right)=\frac{qL}{2}x+\frac{F}{2}x-\frac{qx^2}{2}-Fx+\frac{FL}{2} & ,\ \frac{L}{2}\le x\le L\end{cases}$$

Castigliano

$$\delta_c=\frac{\partial U}{\partial F}=\frac{\partial}{\partial F}\int_0^L\frac{M^2}{2EI}dx=\frac{1}{2EI}\int_0^L\frac{\partial M^2}{\partial F}dx=\frac{1}{2EI}\int 2M\frac{\partial M}{\partial F}dx=\frac{1}{EI}\int_0^L M\frac{\partial M}{\partial F}dx=\frac{1}{EI}\left(\int_0^{L/2}M\frac{\partial M}{\partial F}dx+\int_{L/2}^{L}M\frac{\partial M}{\partial F}dx\right)$$

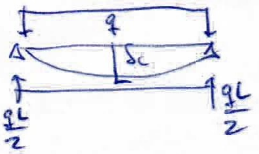
$$=\frac{1}{EI}\left(\int_0^{L/2}\left(\frac{qL}{2}x+\cancel{\frac{F}{2}x}-\frac{qx^2}{2}\right)\frac{x}{2}dx+\int_{L/2}^{L}\left(\frac{qL}{2}x+\cancel{\frac{F}{2}x}-\frac{qx^2}{2}-\cancel{Fx}+\cancel{\frac{FL}{2}}\right)\left(\frac{x}{2}-x+\frac{L}{2}\right)dx\right)=$$

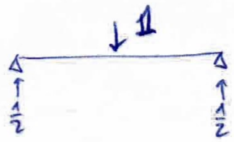
$$=\frac{1}{EI}\left(\int_0^{L/2}\left(\frac{qLx}{2}-\frac{qx^2}{2}\right)\frac{x}{2}dx+\int_{L/2}^{L}\left(\frac{qLx}{2}-\frac{qx^2}{2}\right)\left(\frac{L}{2}-\frac{x}{2}\right)dx\right)=\frac{q}{4EI}\left(\int_0^{L/2}(Lx^2-x^3)dx+\int_{L/2}^{L}(L^2x\underbrace{-Lx^2-Lx^2}_{-2Lx^2}+x^3)dx\right)$$

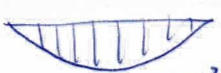
$$=\frac{q}{4EI}\left(\left[\frac{Lx^3}{3}-\frac{x^4}{4}\right]_0^{L/2}+\left[\frac{L^2x^2}{2}-\frac{2Lx^3}{3}+\frac{x^4}{4}\right]_{L/2}^{L}\right)=\frac{q}{4EI}\left(\frac{L^4}{24}-\frac{L^4}{64}+\frac{L^4}{2}-\frac{2L^4}{3}+\frac{L^4}{4}-\frac{L^4}{8}+\frac{2L^4}{24}-\frac{L^4}{64}\right)=$$

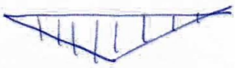
$$=\frac{qL^4}{4EI}\ \frac{96-128+48-24+16-3}{192}=\boxed{\frac{5}{384}\frac{qL^4}{EI}}$$

Carga unidad





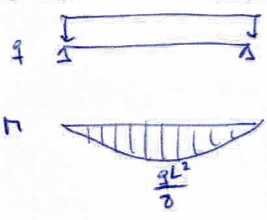


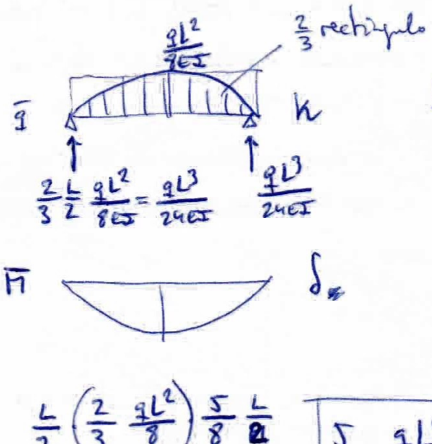


$$M_R(x)=\frac{qL}{2}x-\frac{qx^2}{2}\qquad M_I(x)=\begin{cases}\frac{1}{2}x & ,\ 0\le x\le\frac{L}{2}\\ \frac{1}{2}x-1\left(x-\frac{L}{2}\right)=\frac{x}{2}-x+\frac{L}{2}=\frac{L}{2}-\frac{x}{2} & ,\ \frac{L}{2}\le x\le L\end{cases}$$

$$\delta_c=\int_0^L\frac{M_I(x)\cdot M_R(x)}{EI}dx=\frac{1}{EI}\int_0^{L/2}\left(\frac{qL}{2}x-\frac{qx^2}{2}\right)\frac{1}{2}x\,dx+\frac{1}{EI}\int_{L/2}^{L}\left(\frac{qL}{2}x-\frac{qx^2}{2}\right)\left(\frac{L}{2}-\frac{x}{2}\right)dx=\cdots=\boxed{\frac{5}{384}\frac{qL^4}{EI}}$$

Viga conjugada





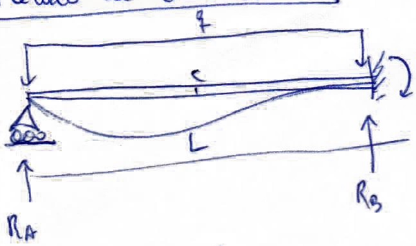
$$\delta_c=\frac{qL^3}{24EI}\frac{L}{2}-\frac{qL^3}{24EI}\cdot\frac{3}{8}\frac{L}{2}=\frac{qL^3}{24EI}\frac{L}{2}\left(1-\frac{3}{8}\right)=\boxed{\frac{5}{384}\frac{qL^4}{EI}}$$

Mohr

$$\delta_{C\to B}=\frac{S_{M,C\to B}}{EI}=\frac{\frac{L}{2}\left(\frac{2}{3}\frac{qL^2}{8}\right)\frac{5}{8}\frac{L}{2}}{EI}=\boxed{\frac{5}{384}\frac{qL^4}{EI}}$$

Método de la elástica

$$\boxed{\delta_c = \frac{qL^4}{192EI}}$$



3 tipos condiciones
- Contorno / frontera
- Continuidad
- Simetría

$$M(x) = R_A x - \frac{qx^2}{2}$$

$$K(x) = \frac{M(x)}{EI} = \frac{1}{EI}\left(Rx - \frac{qx^2}{2}\right)$$

$$\varphi(x) = \int K(x)\,dx + C_1 = \frac{1}{EI}\int\left(Rx - \frac{qx^2}{2}\right)dx + C_1 = \frac{1}{EI}\left(\frac{Rx^2}{2} - \frac{qx^3}{6}\right) + C_1$$

$$\varphi(L) = 0 \Rightarrow \frac{1}{EI}\left(\frac{RL^2}{2} - \frac{qL^3}{6}\right) + C_1 = 0 \Rightarrow C_1 = \frac{1}{3EI}\left(\frac{qL^3}{3} - RL^2\right)$$

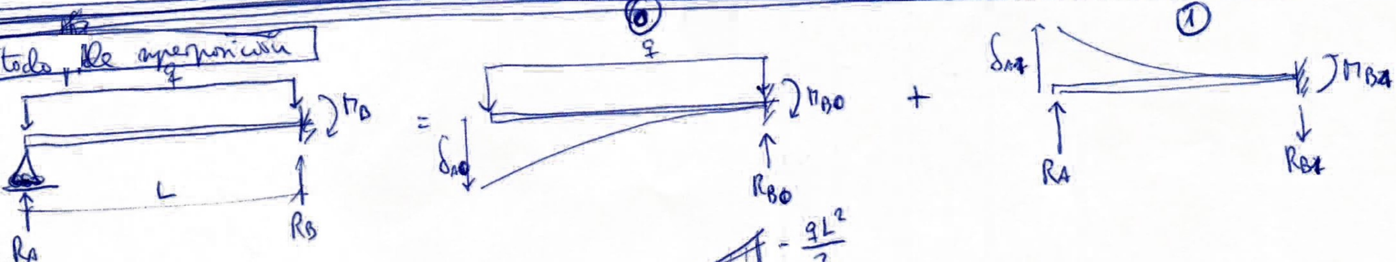
$$\varphi(x) = \frac{1}{EI}\left(\frac{Rx^2}{2} - \frac{qx^3}{6} - \frac{RL^2}{2} + \frac{qL^3}{6}\right) = \frac{1}{EI}\left[\frac{q}{6}(L^3 - x^3) + \frac{R}{2}(x^2 - L^2)\right] = \frac{1}{2EI}\left[\frac{q}{3}(L^3 - x^3) + R(x^2 - L^2)\right]$$

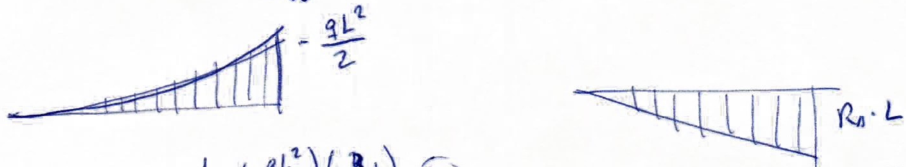
$$\delta(x) = \int \varphi(x)\,dx + C_2 = \frac{1}{2EI}\left[\frac{q}{3}\left(L^3x - \frac{x^4}{4}\right) + R\left(\frac{x^3}{3} - L^2x\right)\right] + C_2$$

$$\delta(0) = 0 \Rightarrow C_2 = 0$$

$$\delta(L) = 0 \Rightarrow \frac{1}{2EI}\left[\frac{q}{3}\left(L^4 - \frac{L^4}{4}\right) + R\left(\frac{L^3}{3} - L^3\right)\right] = \frac{1}{2EI}\left(\frac{qL^4}{4} - \frac{2}{3}RL^3\right) = 0 \Rightarrow \boxed{R = \frac{3}{8}qL}$$

Método de superposición



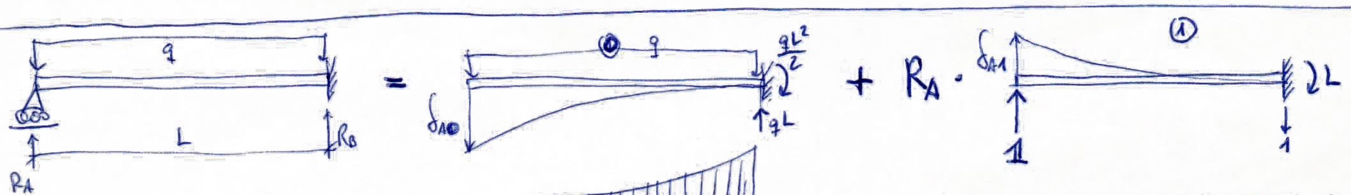


(Calculando desplazamientos con Mohr)

$$\delta_{A0} = \delta_{B\to A} = \frac{S_{M,B\to A}}{EI} = \frac{\frac{1}{3}L\left(-\frac{qL^2}{2}\right)\left(-\frac{3}{4}L\right)}{EI} = \frac{qL^4}{8EI}$$

$$\delta_{A1} = \delta_{B\to A} = \frac{\frac{1}{2}L\,R_A L\cdot\frac{2}{3}L}{EI} = \frac{R_A L^3}{3EI} \rightarrow$$

$$\Rightarrow \boxed{R_A = \frac{3}{8}qL}$$



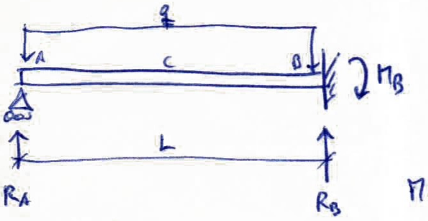
$$M_0(x) = -\frac{qx^2}{2}$$

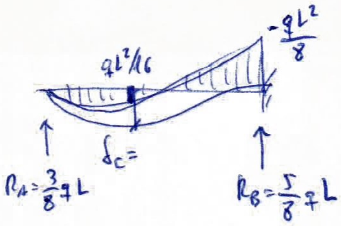
$$\delta_{A0} = \int_0^L \frac{M_0(x)\,M_1(x)}{EI}dx = \frac{1}{EI}\int_0^L\left(-\frac{qx^2}{2}\right)x\,dx = \frac{-q}{2EI}\int_0^L x^3dx = \frac{-q}{2EI}\left[\frac{x^4}{4}\right]_0^L = \frac{-qL^4}{8EI}$$

(Calculando desplazamientos con carga unidad)

$$\delta_{A1} = \int_0^L \frac{M_1^2(x)}{EI} = \frac{1}{EI}\int_0^L x^2dx = \frac{1}{EI}\left[\frac{x^3}{3}\right]_0^L = \frac{L^3}{3EI}$$

$$\delta_A = \delta_{A0} + R_A\cdot\delta_{A1} \Rightarrow 0 = -\frac{qL^4}{8EI} + R_A\frac{L^3}{3EI} \Rightarrow \boxed{R_A = \frac{3}{8}qL}$$





$$M_C = \frac{q L/2}{8}\left(\underbrace{3L - 4\frac{L}{2}}_{L}\right) = \frac{qL^2}{16}$$

$$\delta_C = \frac{q\frac{L}{2}}{48EI}\left(L + 2\frac{L}{2}\right)\left(L - \frac{L}{2}\right)^2 = \frac{q\frac{L}{2}}{48EI} 2L \frac{L^2}{4} =$$

$$= \frac{qL^4}{192EI}$$

$$M(x) = R_A x - \frac{qx^2}{2}$$

$\boxed{\text{Castigliano}}$

$$U = \int_0^L \frac{M^2(x)}{2EI} dx = \frac{1}{2EI}\int_0^L \left(R_A x - \frac{qx^2}{2}\right)^2 dx = \frac{1}{2EI}\int_0^L \left(R_A^2 x^2 + \frac{q^2x^4}{4} - 2R_A x \frac{qx^2}{2}\right) dx = \frac{1}{2EI}\int_0^L \left(R_A^2 x^2 + \frac{q^2 x^4}{4} - R_A q x^3\right) dx$$

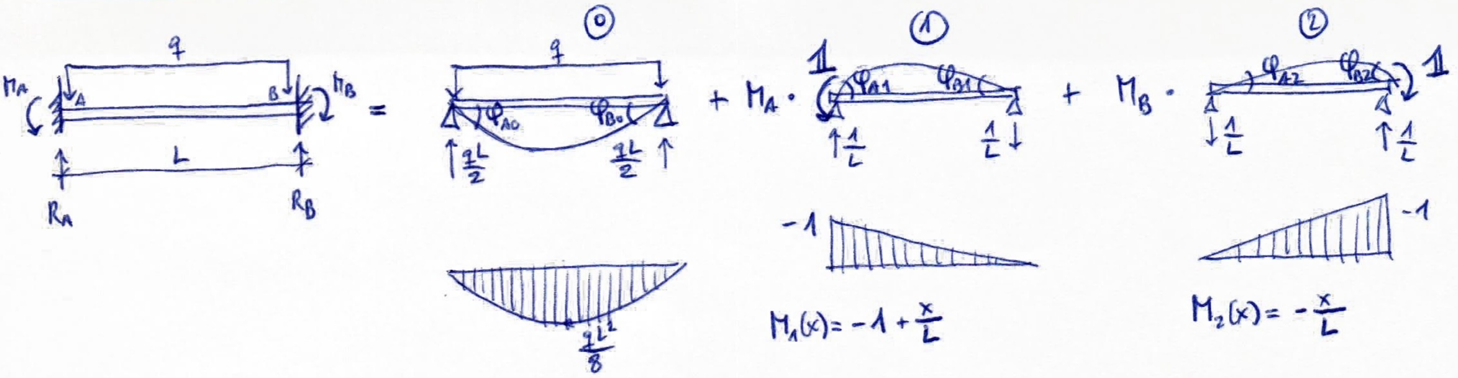
$$= \frac{1}{2EI}\left[R_A^2 \frac{x^3}{3} + \frac{q^2 x^5}{20} - R_A q \frac{x^4}{4}\right]_0^L = \frac{1}{2EI}\left(R_A^2 \frac{L^3}{3} + \frac{q^2 L^5}{20} - \frac{R_A q L^4}{4}\right)$$

$$\delta_A = \frac{\partial U}{\partial R_A} = \frac{1}{2EI}\left(2R_A \frac{L^3}{3} - \frac{qL^4}{4}\right) = 0 \Rightarrow R_A = \frac{\frac{qL^4}{4}}{\frac{2L^3}{3}} = \boxed{\frac{3}{8} qL}$$

Más sencillo: (no integrar)

$$U = \int_0^L \frac{M^2(x)}{2EI}\ ; \quad \delta_A = \frac{\partial}{\partial R_A}\int_0^L \frac{M^2}{2EI} dx = \frac{1}{2EI}\int_0^L \frac{\partial M^2}{\partial R_A} dx = \frac{1}{2EI}\int_0^L 2M \frac{\partial M}{\partial R_A} dx = \frac{1}{EI}\int_0^L \left(R_A x - \frac{qx^2}{2}\right) x \, dx =$$

$$= \frac{1}{EI}\int_0^L \left(R_A x^2 - \frac{qx^3}{2}\right) dx = \frac{1}{EI}\left[R_A \frac{x^3}{3} - \frac{qx^4}{8}\right]_0^L = \frac{1}{EI}\left(\frac{R_A L^3}{3} - \frac{qL^4}{8}\right) = 0 \Rightarrow \boxed{R_A = \frac{3}{8} qL}$$



$M_1(x) = -1 + \frac{x}{L}$ $\qquad$ $M_2(x) = -\frac{x}{L}$

$M_0(x) = \frac{qL}{2}x - \frac{qx^2}{2}$

$$\varphi_{A0} = \int_0^L \frac{M_0 M_1}{EI}dx = \frac{1}{EI}\int_0^L \left(\frac{qLx}{2} - \frac{qx^2}{2}\right)\left(\frac{x}{L} - 1\right)dx = \frac{q}{2EI}\int_0^L (Lx - x^2)\left(\frac{x}{L} - 1\right)dx = \frac{q}{2EI}\int_0^L \left(x^2 - \frac{x^3}{L} - Lx + x^2\right)dx =$$

$$= \frac{q}{2EI}\int_0^L \left(-\frac{x^3}{L} + 2x^2 - Lx\right)dx = \frac{q}{2EI}\left[-\frac{x^4}{4L} + \frac{2x^3}{3} - \frac{Lx^2}{2}\right]_0^L = \frac{q}{2EI}\left(-\frac{L^4}{4L} + \frac{2L^3}{3} - \frac{L^3}{2}\right) = \frac{qL^3}{2EI}\frac{-3+8-6}{12} = -\frac{qL^3}{24EI}$$

(Por 1er Teorema Mohr: $\varphi_{A0} = -\frac{2}{3}\frac{qL^2}{8}\frac{L}{2}\frac{1}{EI} = -\frac{qL^3}{24EI}$; $\varphi_{B0} = -\frac{qL^3}{24EI} = \varphi_{A0}$

$$\varphi_{A1} = \int_0^L \frac{M_1^2}{EI}dx = \frac{1}{EI}\int_0^L \left(-1 + \frac{x}{L}\right)^2 dx = \frac{1}{EI}\int_0^L \left(1 - \frac{2x}{L} + \frac{x^2}{L^2}\right)dx = \frac{1}{EI}\left[x - \frac{x^2}{L} + \frac{x^3}{3L^2}\right]_0^L = \frac{1}{EI}\left(L - L + \frac{L}{3}\right) = \frac{L}{3EI}$$

$$\varphi_{B1} = \int_0^L \frac{M_1 M_2}{EI}dx = \frac{1}{EI}\int_0^L \left(-1 + \frac{x}{L}\right)\left(-\frac{x}{L}\right)dx = \frac{1}{EI}\int_0^L \left(\frac{x}{L} - \frac{x^2}{L^2}\right)dx = \frac{1}{EI}\left[\frac{x^2}{2L} - \frac{x^3}{3L^2}\right]_0^L = \frac{1}{EI}\left(\frac{L}{2} - \frac{L}{3}\right) = \frac{L}{6EI} = \frac{\varphi_{A1}}{2}$$

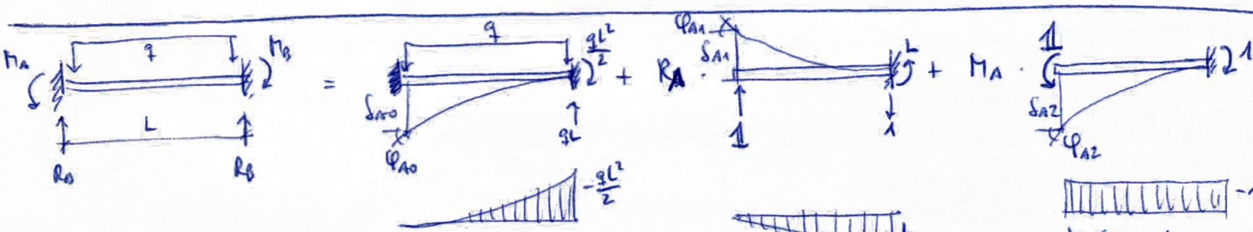
$$\varphi_{A2} = \varphi_{B1} = \frac{L}{6EI}$$

$$\varphi_{B2} = \varphi_{A1} = \frac{L}{3EI}$$

$$\begin{cases} \varphi_A = \varphi_{A0} + M_A \cdot \varphi_{A1} + M_B \cdot \varphi_{A2} \Rightarrow 0 = -\frac{qL^3}{24EI} + \frac{L}{3EI}M_A + \frac{L}{6EI}M_B \longrightarrow \underbrace{\left(\frac{L}{3EI} + \frac{L}{6EI}\right)}_{\frac{L}{2EI}} M_A = \frac{qL^3}{24EI} \to \\ \varphi_B = \varphi_{B0} + M_A \cdot \varphi_{B1} + M_B \cdot \varphi_{B2} \Rightarrow 0 = -\frac{qL^3}{24EI} + \frac{L}{6EI}M_A + \frac{L}{3EI}M_B \end{cases}$$

$$\Rightarrow \boxed{M_A = \frac{qL^2}{12} = M_B}$$

$$\frac{L}{6EI}M_A - \frac{L}{6EI}M_B = 0 \Rightarrow M_A = M_B$$



$M_0(x) = -\frac{qx^2}{2}$ $\qquad$ $M_1(x) = x$ $\qquad$ $M_2(x) = -1$

$$\delta_{A0} = \int_0^L \frac{M_0 M_1}{EI}dx = \frac{1}{EI}\int_0^L -\frac{qx^3}{2}dx = \frac{1}{EI}\left[-\frac{qx^4}{8}\right]_0^L = -\frac{qL^4}{8EI} \; ; \; \varphi_{A0} = \frac{1}{EI}\int_0^L \frac{qx^2}{2}dx = \frac{1}{EI}\left[\frac{qx^3}{6}\right]_0^L = \frac{qL^3}{6EI}$$

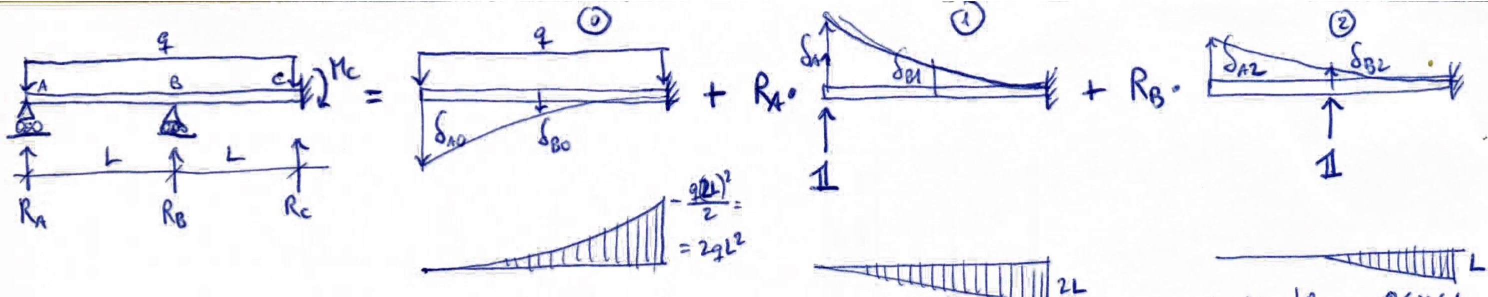
$$M_B = -\frac{qL^2}{4} + \frac{2}{3}L\frac{qL}{2} = qL^2\left(\frac{1}{3} - \frac{1}{4}\right) \boxed{= \frac{qL^2}{12}}$$

$$\delta_{A1} = \frac{1}{EI}\int_0^L x^2 dx = \frac{L^3}{3EI} \; ; \; \varphi_{A1} = \frac{1}{EI}\int_0^L -x\,dx = \frac{1}{EI}\left[\frac{-x^2}{2}\right]_0^L = \frac{-L^2}{2EI} \; ; \; \delta_{A2} = \frac{-L^2}{2EI} \; ; \; \varphi_{A2} = \frac{1}{EI}\int_0^L dx = \frac{L}{EI}$$

$$\begin{cases} \delta_A = \delta_{A0} + R_A \cdot \delta_{A1} + M_A \cdot \delta_{A2} \Rightarrow 0 = -\frac{qL^4}{8EI} + \frac{L^3}{3EI}R_A - \frac{L^2}{2EI}M_A \Rightarrow \quad M_A = \frac{\left(\frac{qL^4}{8} - \frac{L^3}{3}R_A\right)2}{-L^2} = -\frac{qL^2}{4} + \frac{2L}{3}R_A \\ \varphi_A = \varphi_{A0} + R_A\,\varphi_{A1} + M_A \cdot \varphi_{A2} \Rightarrow 0 = \frac{qL^3}{6EI} + \frac{L^2}{2EI}R_A + \frac{L}{EI}M_A \longrightarrow -\frac{qL^3}{6EI} = \frac{-L^2}{2EI}R_A + \frac{L}{EI}\left(-\frac{qL^2}{4} + \frac{2L}{3}R_A\right) \Rightarrow \end{cases}$$

$$\Rightarrow \quad -\frac{qL^3}{6EI} + \frac{qL^3}{4EI} = R_A\left(-\frac{L^2}{2EI} + \frac{2L^2}{3EI}\right) \Rightarrow qL^3\left(-\frac{1}{6} + \frac{1}{4}\right) = R_A L^2$$

$$qL^3\left(\frac{1}{12}\right) = R_A L^2 \frac{1}{6} \Rightarrow \boxed{R_A = \frac{qL}{2}}$$



$M_0(x) = -\frac{qx^2}{2}$ $\qquad M_1(x) = x$ $\qquad M_2(x) = \begin{cases} 0 & , 0 \le x \le L \\ x - L & L < x \le 2L \end{cases}$

$$\delta_{A0} = \int_0^{2L} \frac{M_0 \cdot M_1}{EI} dx = \frac{1}{EI}\int_0^{2L}\left(-\frac{qx^2}{2}\right) x\, dx = -\frac{q}{2EI}\left[\frac{x^4}{4}\right]_0^{2L} = -\frac{q\cdot 16L^4}{8EI} = -\frac{2qL^4}{EI}$$

$$\delta_{B0} = \int_0^{2L} \frac{M_0 M_2}{EI} dx = \frac{1}{EI}\left(\int_0^{L}\left(-\frac{qx^2}{2}\right)\cdot 0\, dx + \int_L^{2L}\left(-\frac{qx^2}{2}\right)(x-L)\, dx\right) = \frac{1}{EI}\int_L^{2L}\left(-\frac{qx^3}{2} + \frac{qx^2L}{2}\right)dx = \frac{q}{2EI}\int_L^{2L}\left(-x^3 + Lx^2\right)dx =$$

$$= \frac{q}{2EI}\left[-\frac{x^4}{4} + \frac{Lx^3}{3}\right]_L^{2L} = \frac{q}{2EI}\left(-\frac{16L^4}{4} + \frac{L(8L^3)}{3} + \frac{L^4}{4} - \frac{L^4}{3}\right) = \frac{qL^4}{2EI}\left(-4 + \frac{8}{3} + \frac{1}{4} - \frac{1}{3}\right) = \frac{qL^4}{2EI}\cdot\frac{-48+32+3-4}{12} =$$

$$= -\frac{17}{24}\frac{qL^4}{EI}$$

$$\delta_{A1} = \int_0^{2L} \frac{M_1^2}{EI} dx = \frac{1}{EI}\int_0^{2L} x^2 dx = \frac{1}{EI}\left[\frac{x^3}{3}\right]_0^{2L} = \frac{8}{3}\frac{L^3}{EI}$$

$$\delta_{B1} = \int_0^{2L} \frac{M_1 M_2}{EI} dx = \frac{1}{EI}\int_L^{2L} x(x-L)\, dx = \frac{1}{EI}\int_L^{2L}\left(x^2 - xL\right)dx = \frac{1}{EI}\left[\frac{x^3}{3} - \frac{Lx^2}{2}\right]_L^{2L} = \frac{1}{EI}\left(\frac{8L^3}{3} - \frac{L^3}{3} - \frac{4L^3}{2} + \frac{L^3}{2}\right) = \frac{L^3}{EI}\,\frac{14-9}{6} =$$

$$= \frac{5}{6}\frac{L^3}{EI}$$

$$\delta_{A2} = \int_0^{2L} \frac{M_1 M_2}{EI} dx = \delta_{B1} = \frac{5}{6}\frac{L^3}{EI}$$

$$\delta_{B2} = \int_0^{2L} \frac{M_2^2}{EI} dx = \frac{1}{EI}\int_L^{2L}(x-L)^2 dx = \frac{1}{EI}\int_L^{2L}\left(x^2 - 2Lx + L^2\right)dx = \frac{1}{EI}\left[\frac{x^3}{3} - Lx^2 + L^2x\right]_L^{2L} = \frac{1}{EI}\left(\frac{8L^3}{3} - \frac{L^3}{3} - 4L^3 + L^3 + 2L^3 - L^3\right) =$$

$$= \frac{1}{EI}\left(\frac{7}{3}L^3 - 2L^3\right) = \frac{L^3}{EI}\,\frac{7-6}{3} = \frac{L^3}{3EI}$$

$$\begin{cases} \delta_A = \delta_{A0} + R_A\cdot\delta_{A1} + R_B\cdot\delta_{A2} \Rightarrow \\ \delta_B = \delta_{B0} + R_A\cdot\delta_{B1} + R_B\cdot\delta_{B2} \Rightarrow \end{cases}$$

$$\Rightarrow \begin{cases} 0 = -\frac{2qL^4}{EI} + \frac{8}{3}\frac{L^3}{EI}R_A + \frac{5L^3}{6EI}R_B \Rightarrow \frac{8}{3}R_A + \frac{5}{6}R_B = 2qL \Rightarrow R_B = \frac{6}{5}\left(2qL - \frac{8}{3}R_A\right) = \frac{12qL}{5} - \frac{16}{5}R_A \\ 0 = -\frac{17}{24}\frac{qL^4}{EI} + \frac{5L^3}{6EI}R_A + \frac{L^3}{3EI}R_B \Rightarrow \frac{5}{6}R_A + \frac{1}{3}R_B = \frac{17}{24}qL \rightarrow \frac{5}{6}R_A + \frac{1}{3}\left(\frac{12qL}{5} - \frac{16}{5}R_A\right) = \frac{17}{24}qL \Rightarrow \end{cases}$$

$$\Rightarrow \frac{5}{6}R_A + \frac{4qL}{5} - \frac{16}{15}R_A = \frac{17}{24}qL \Rightarrow R_A\left(\frac{5}{6} - \frac{16}{15}\right) = qL\left(\frac{17}{24} - \frac{4}{5}\right) \Rightarrow \boxed{R_A = 0'39\,qL}$$

$$\boxed{R_B = 1'15\,qL}$$

$$\boxed{(R_C = 0'46\,qL)}$$