

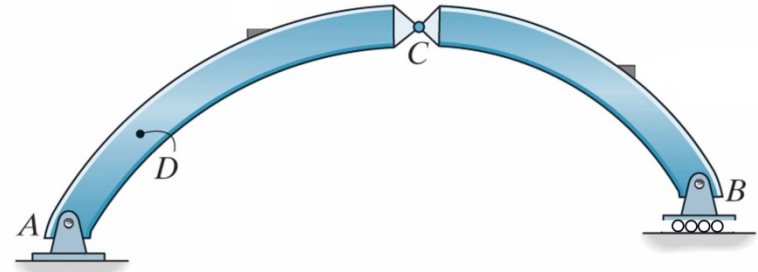
**SISTEMAS HIPERESTÁTICOS
DE NUDOS RÍGIDOS**

ÍNDICE

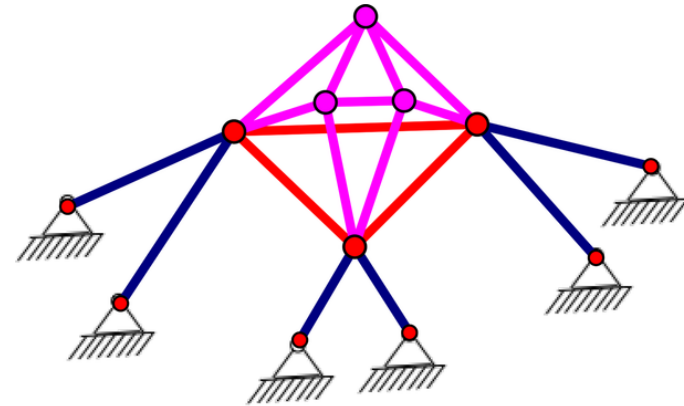
1. Hiperestatismo
2. Concepto de rigidez
3. Métodos de análisis
 - Pendiente-deformación
 - Cross
 - Rigideces

HIPERESTATISMO

Hipostático



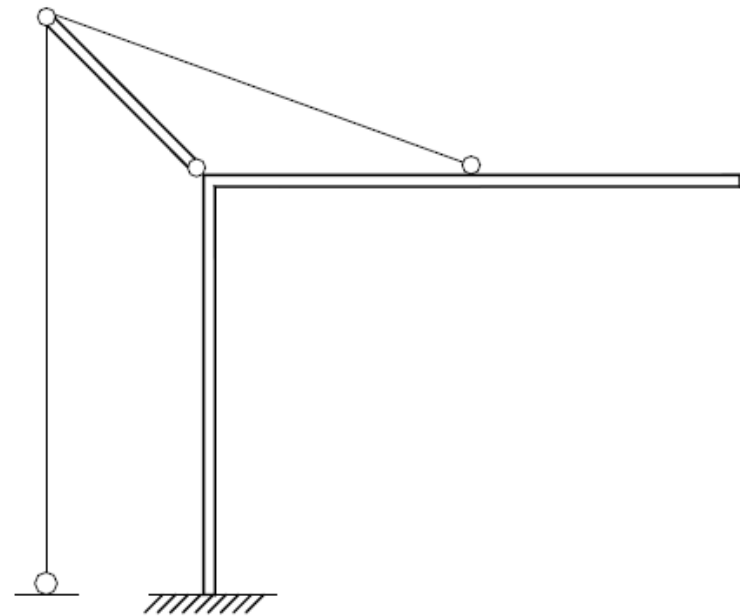
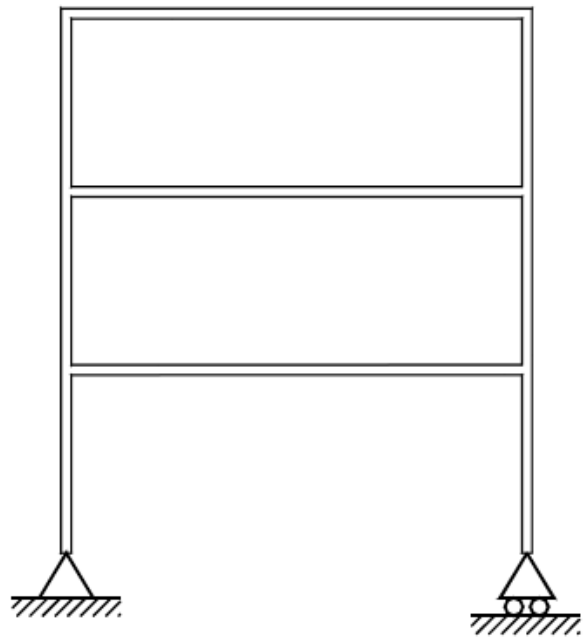
Isostático



Hiperestático



HIPERESTATISMO

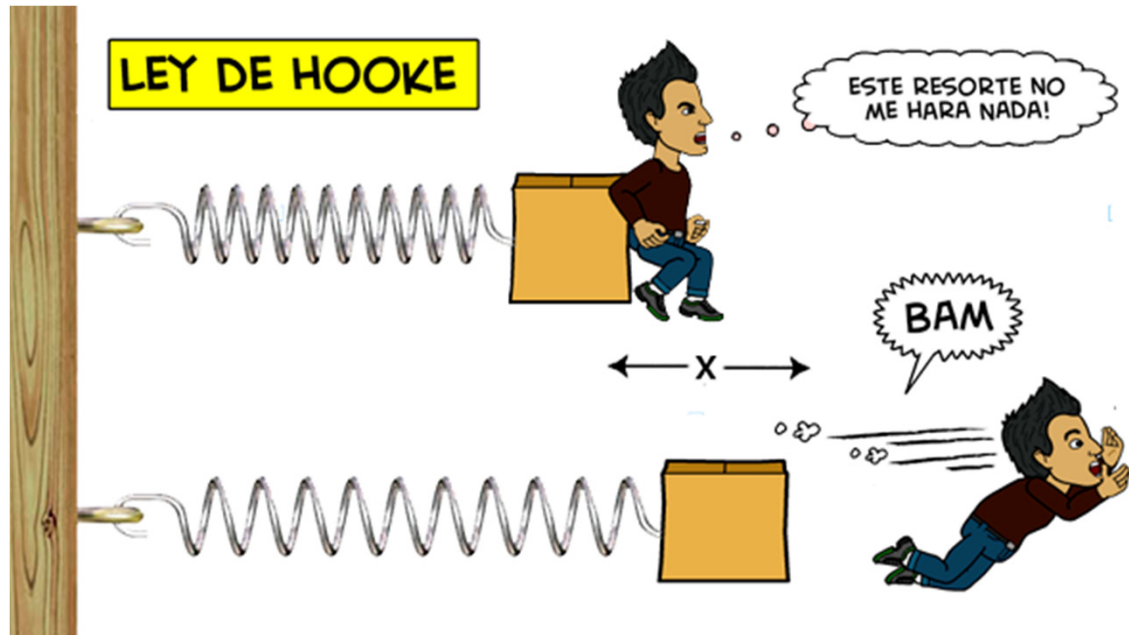




**NO PODEMOS CALCULAR HIPERESTÁTICAS
CON LAS 3 ECUACIONES DE EQUILIBRIO**

**NECESITAMOS AÑADIR
ECUACIONES DE DEFORMACIÓN**

RIGIDEZ

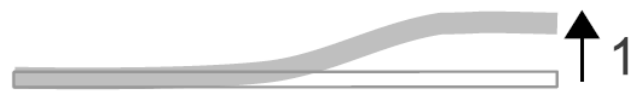


RIGIDECES BARRA

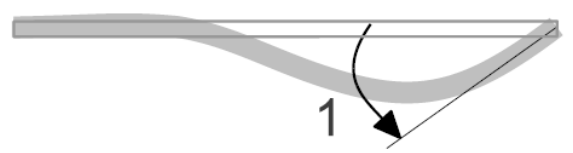
Rigidez axial



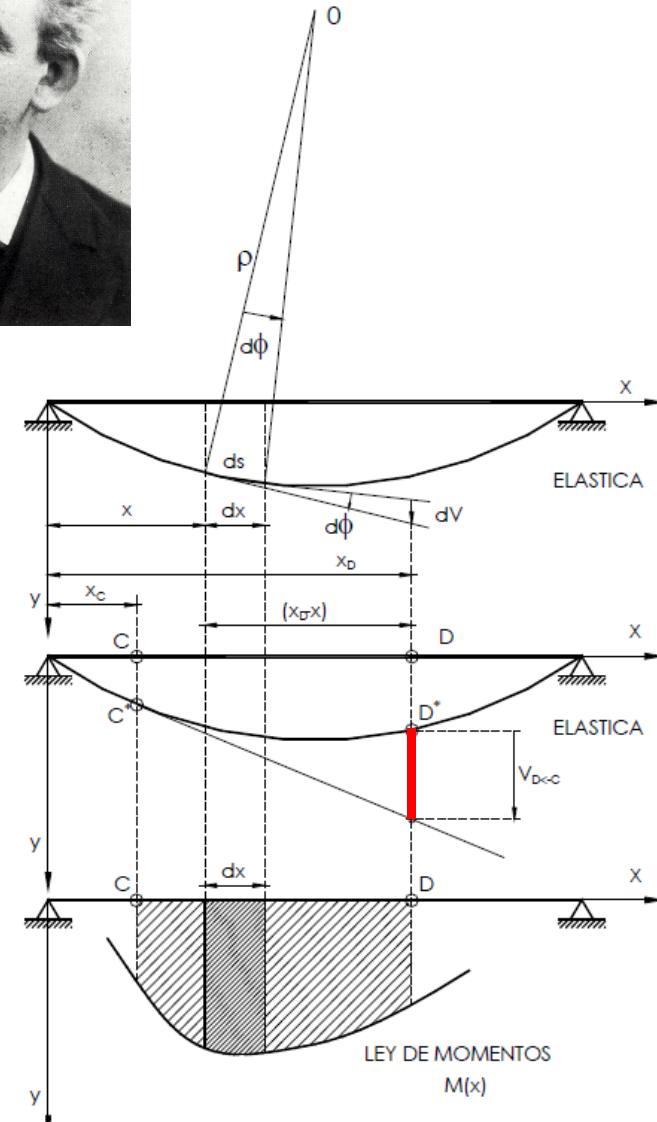
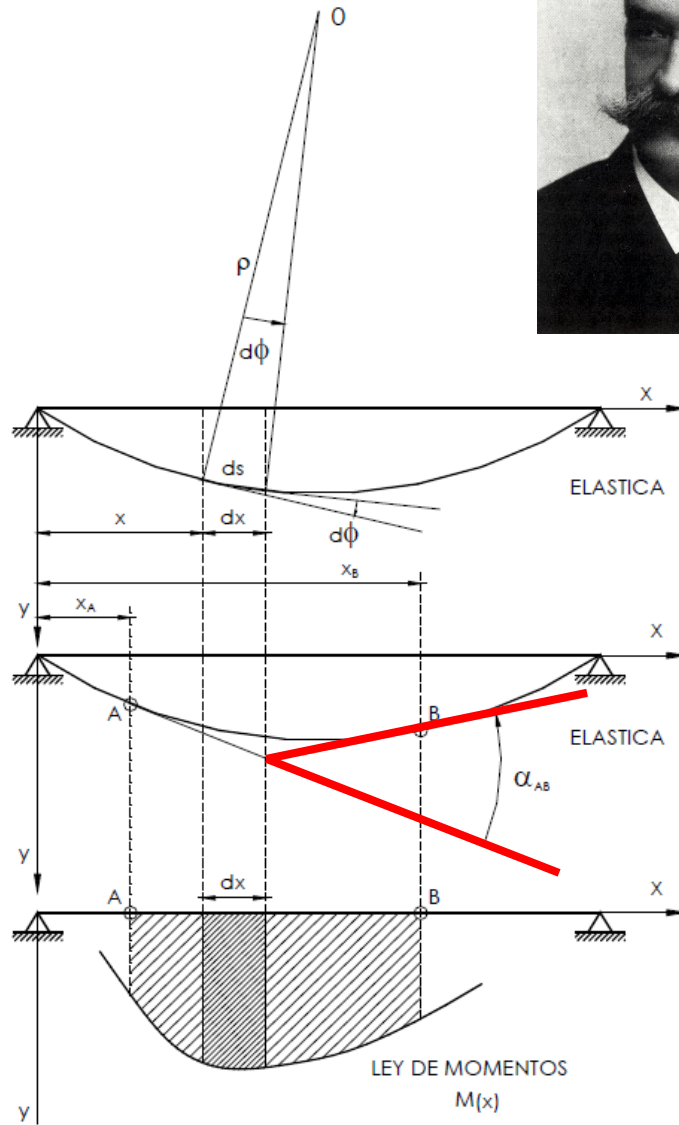
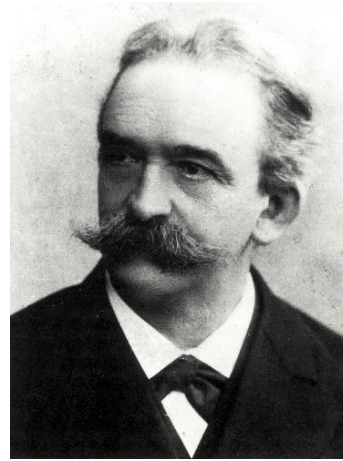
Rigidez a cortante



Rigidez a flexión



(teoremas de Mohr)



MÉTODO DE CROSS

Requisitos

- Nudos rígidos
- Estructura intraslacional
- Cargas en barras

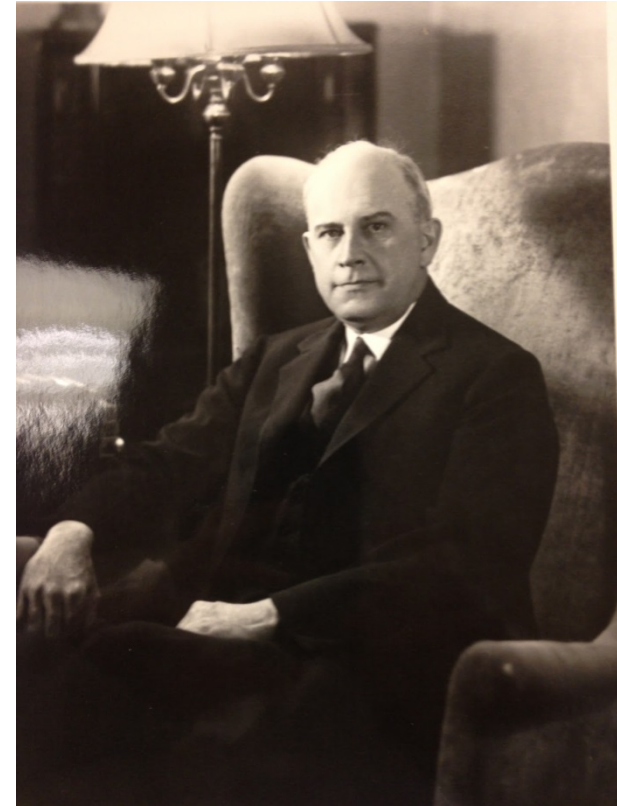
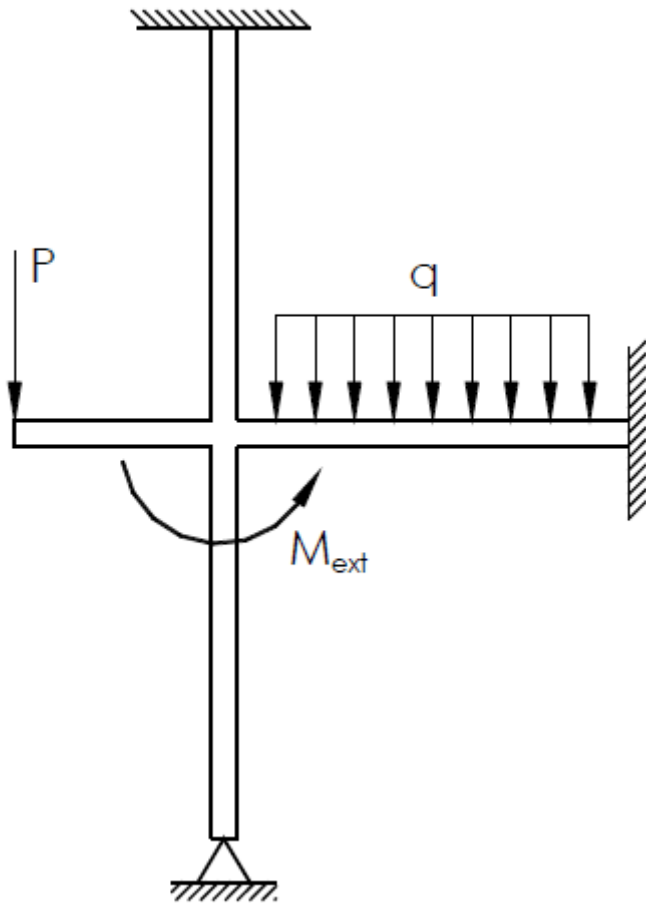
PASOS

- 1) Identificación del modelo, verificación de requisitos
- 2) Rigideces barras
- 3) Coeficientes de reparto
- 4) Coeficientes de transmisión
- 5) Momentos de empotramiento perfecto
- 6) Iteraciones de liberación → **Equilibrio**
- 7) Reacciones de extremos de barra
- 8) Reacciones externas
- 9) Solicitaciones



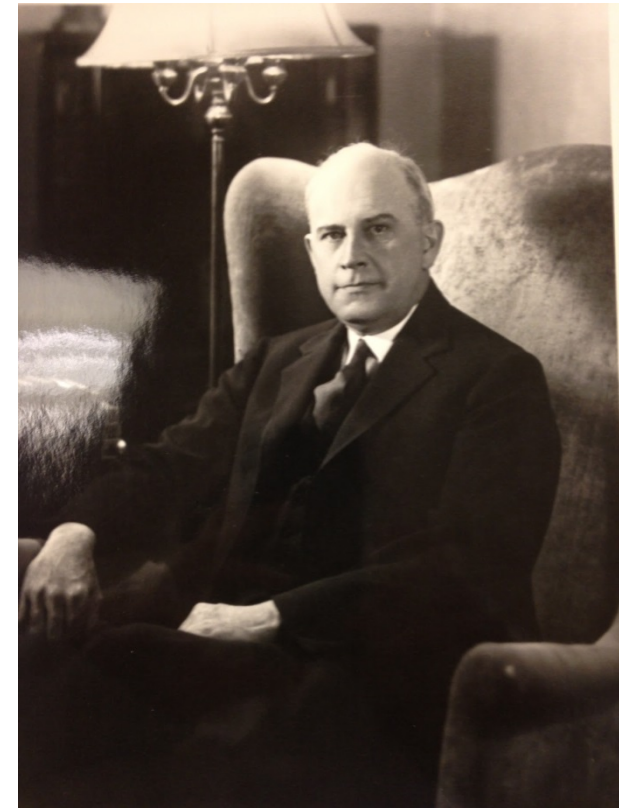
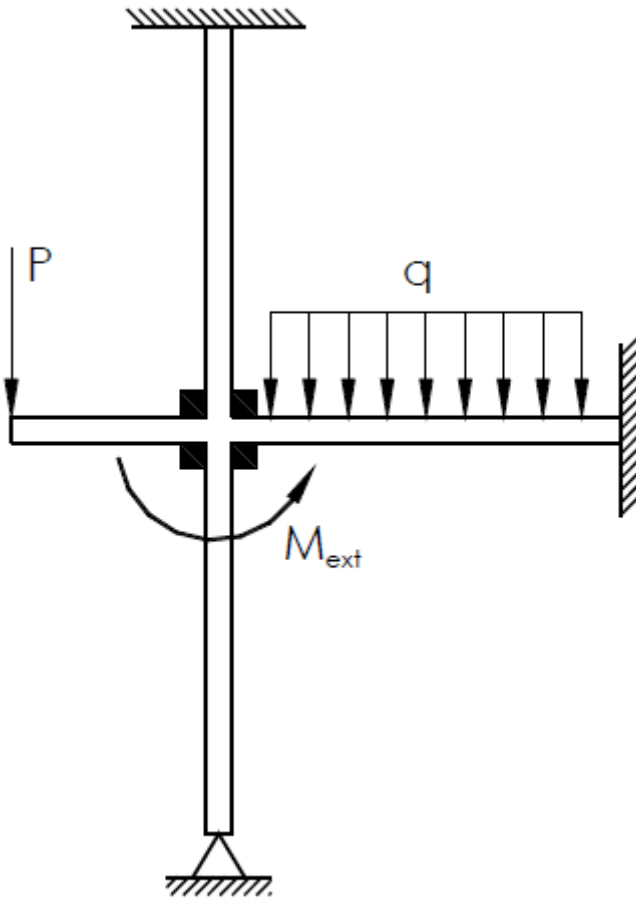
MÉTODO DE CROSS

1) Identificación del modelo, verificación de requisitos



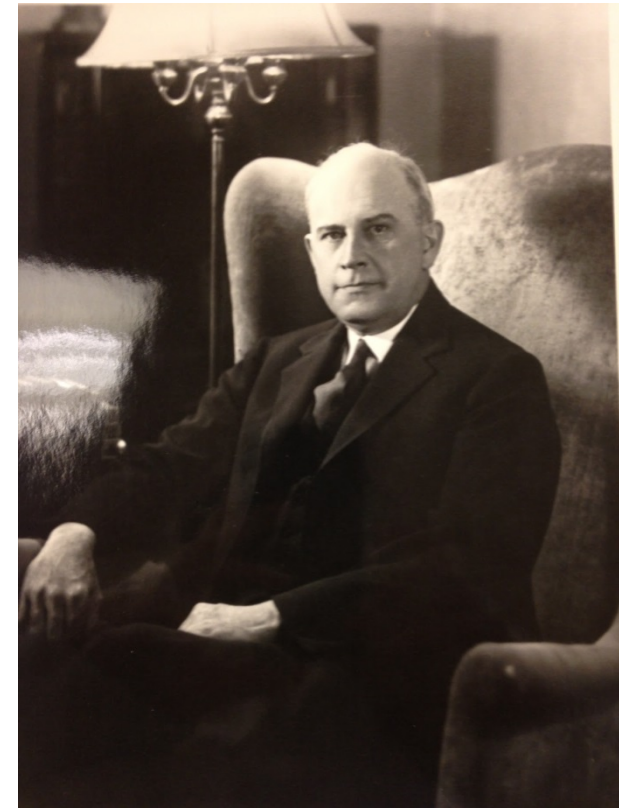
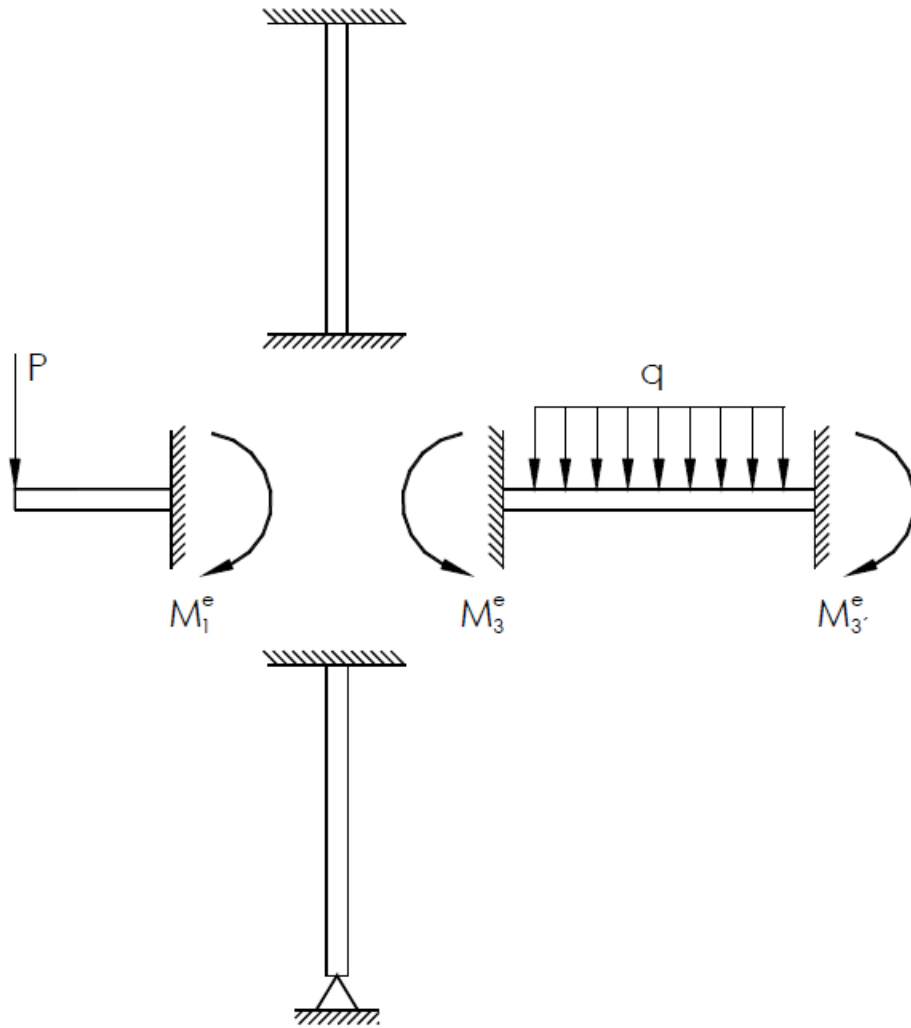
MÉTODO DE CROSS

- 2) Rigideces barras
- 3) Coeficientes de reparto
- 4) Coeficientes de transmisión



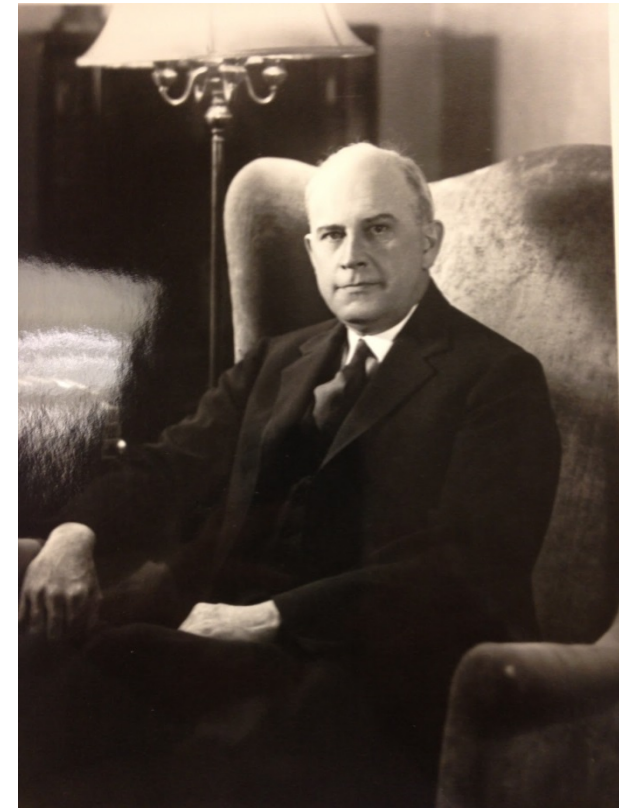
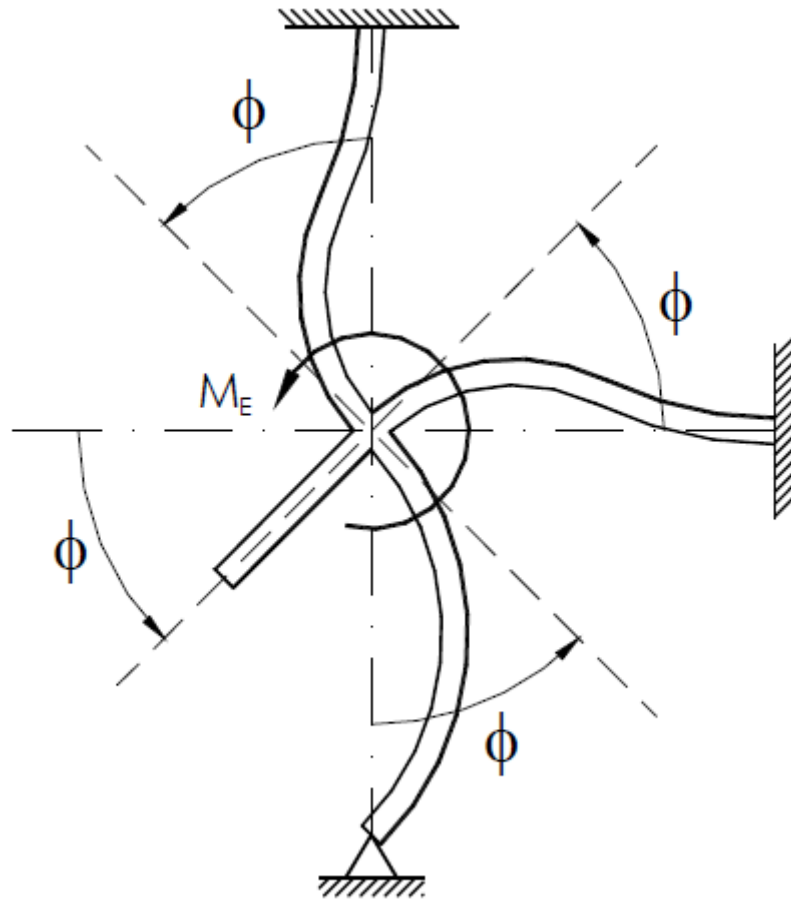
MÉTODO DE CROSS

5) Momentos de empotramiento perfecto



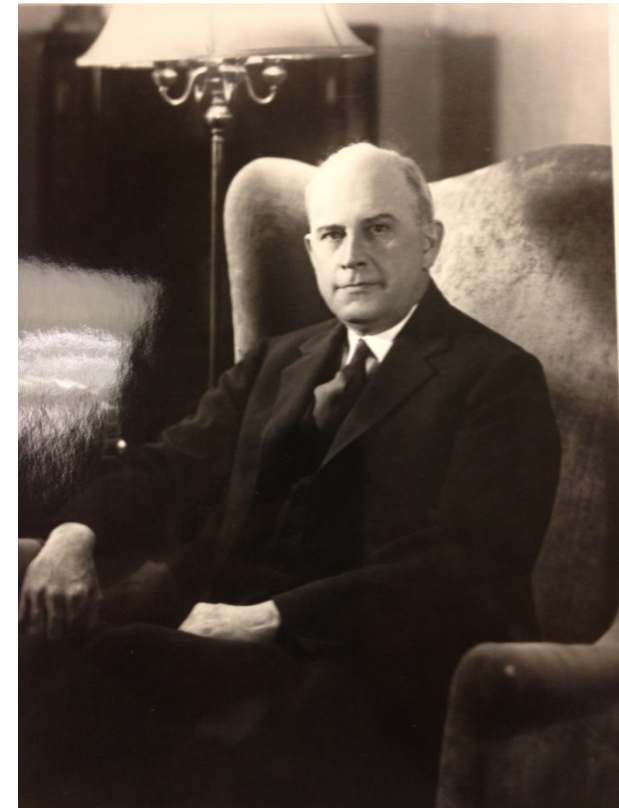
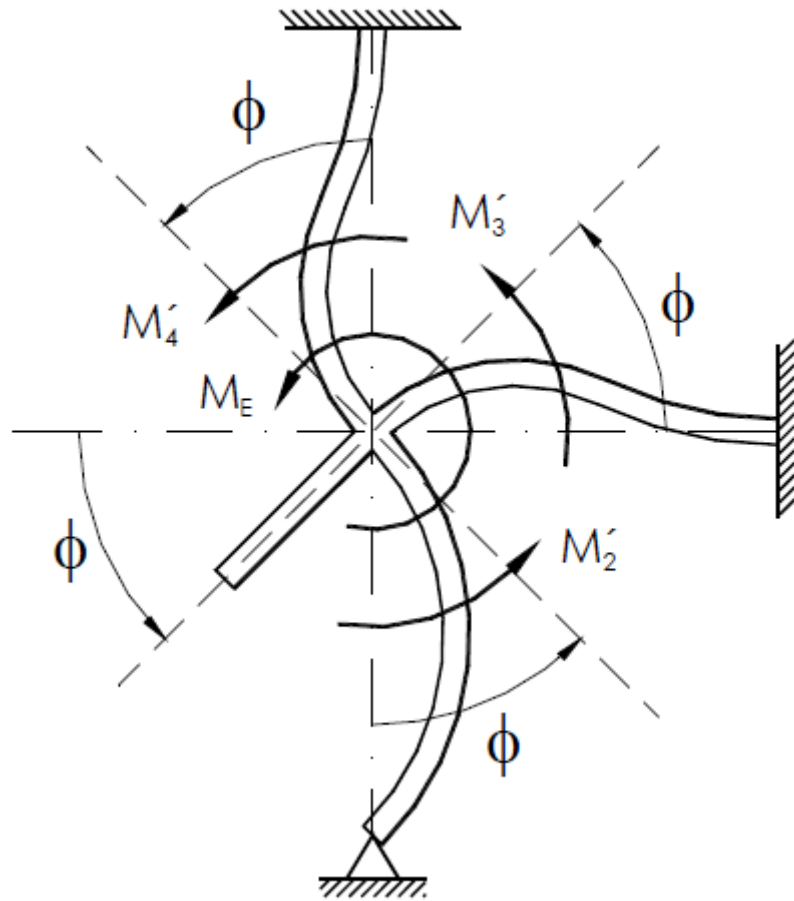
MÉTODO DE CROSS

6) Iteraciones:
liberación



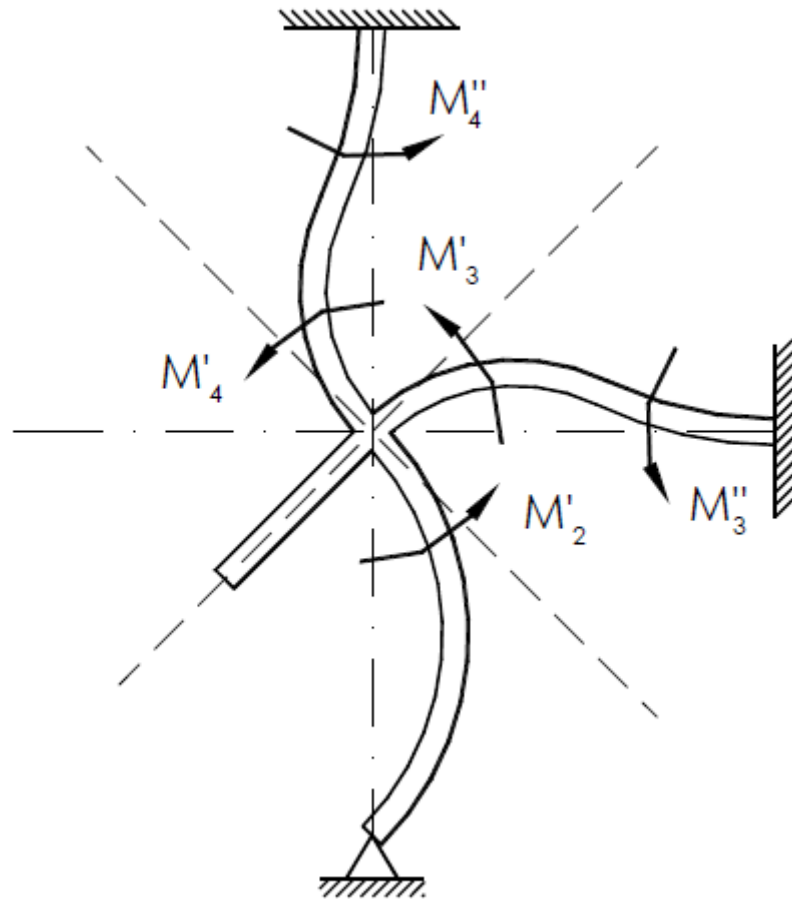
MÉTODO DE CROSS

6) Iteraciones:
reparto

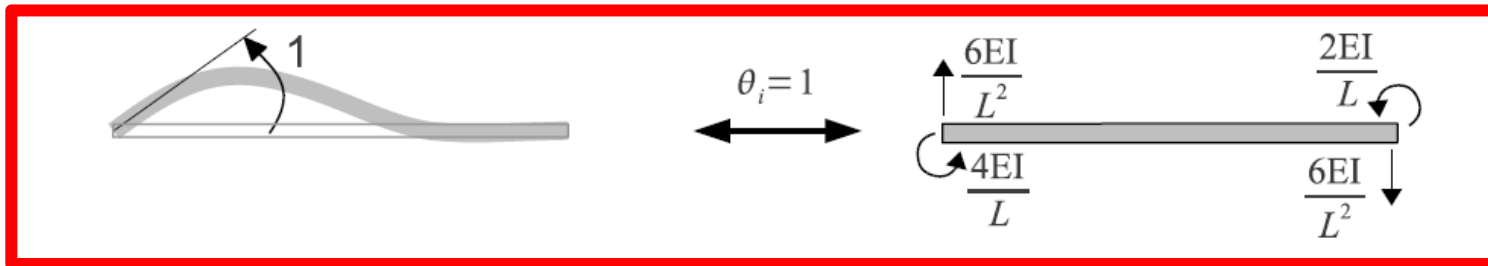
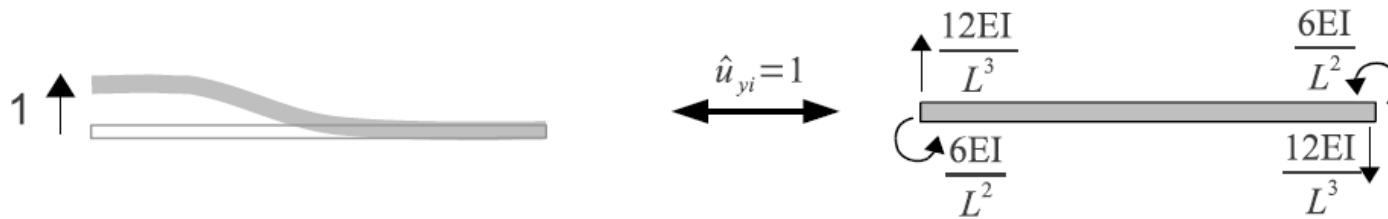
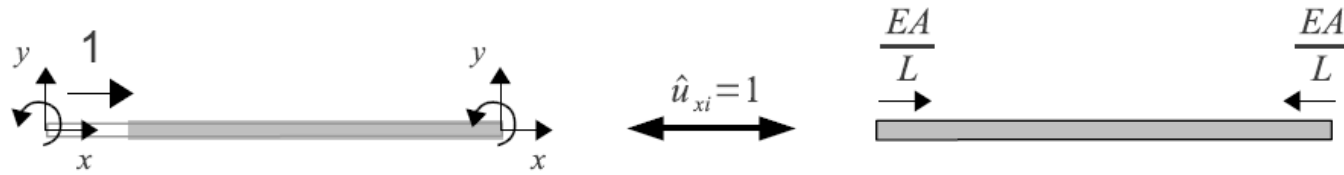


MÉTODO DE CROSS

6) Iteraciones:
transmisión



RIGIDES BARRA



RIGIDECES BARRA, COEFICIENTES DE REPARTO Y DE TRANSMISIÓN

Barras biempotradas (Fig. 10-6):

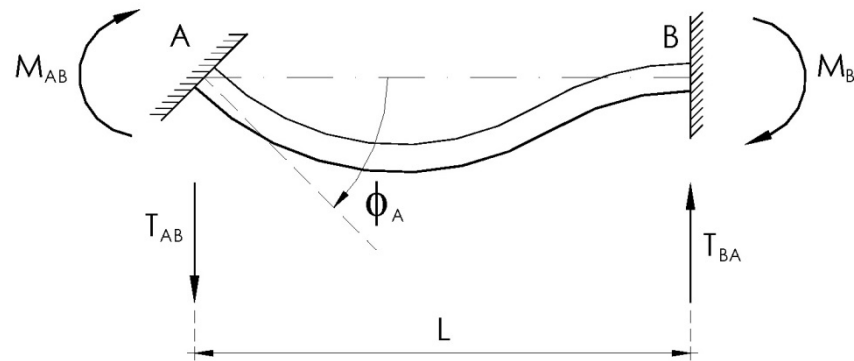


Fig. 10-6 Barra biempotrada

Para barras con plano central de simetría y sección constante:

$$K = \frac{4 \cdot E \cdot I_z}{L} \quad t = \frac{1}{2}$$

RIGIDECES BARRA, COEFICIENTES DE REPARTO Y DE TRANSMISIÓN

Barras empotradas-articuladas (Fig. 10-7) :

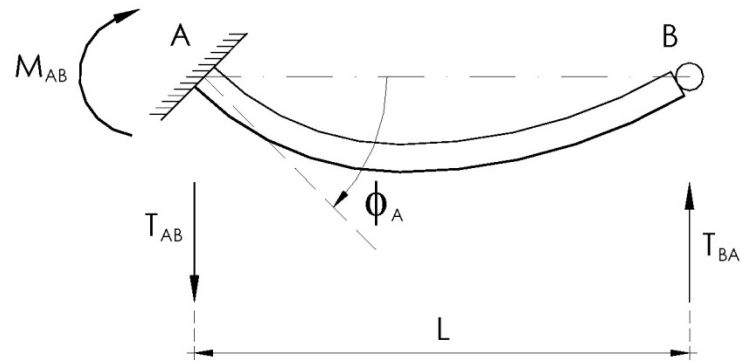


Fig. 10-7 Barra empotrada-articulada

Para barras con plano central de simetría y sección constante:

$$K' = \frac{3 \cdot E \cdot I_z}{L} \quad t' = 0$$

RIGIDECES BARRA, COEFICIENTES DE REPARTO Y DE TRANSMISIÓN

Voladizos (Fig. 10-8):

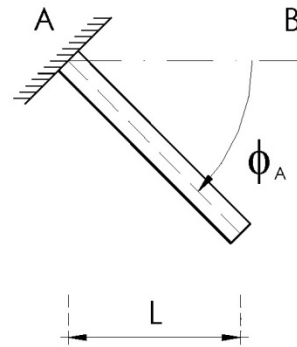


Fig. 10-8 Voladizo

Para barras con plano central de simetría y sección constante:

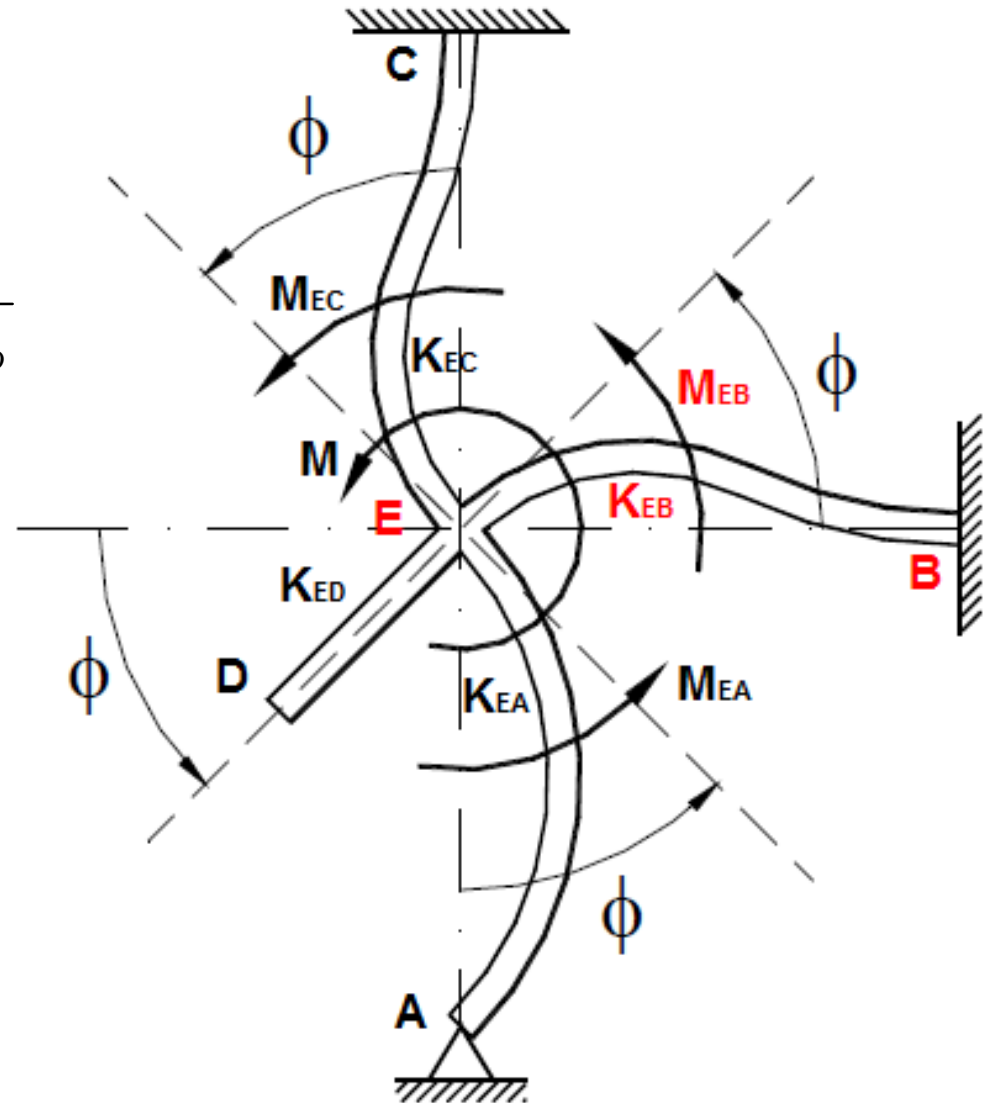
$$K'' = 0$$

$$t'' = 0$$

RIGIDECES BARRA, COEFICIENTES DE REPARTO Y DE TRANSMISIÓN

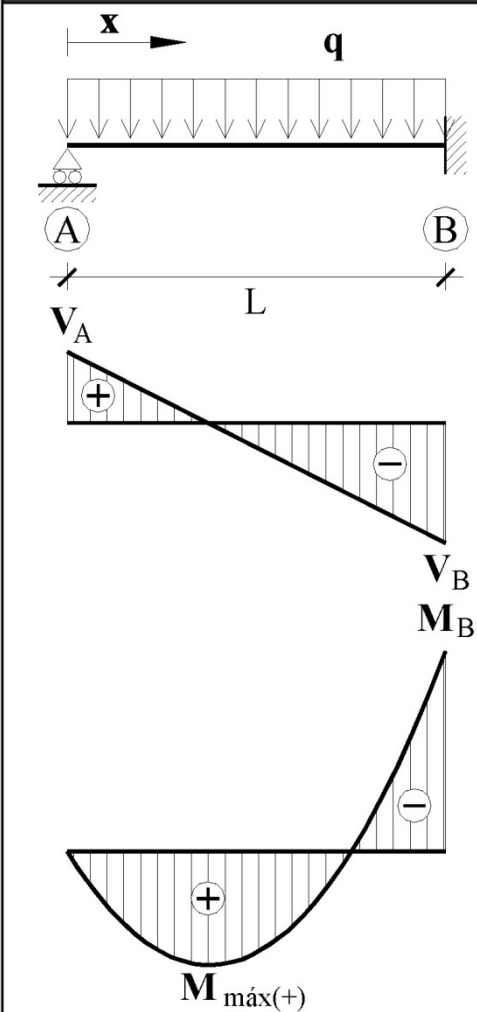
Coeficiente de reparto
en el nudo **E** hacia la
barra **EB**:

$$c_{EB} = \frac{K_{EB}}{\sum_i K_{Ei}} = \frac{K_{EB}}{K_{EA} + K_{EB} + K_{EC} + K_{ED}}$$



MOMENTOS DE EMPOTRAMIENTO PERFECTO

VIGA SIMPLE APOYADA-EMPOTRADA: carga uniforme q en todo el vano.



Reacciones y solicitaciones

Reacciones: $R_A = \frac{3}{8}qL$

$R_B = \frac{5}{8}qL$

Cortantes: $V_{AB} = qL \left(\frac{3}{8} - \frac{x}{L} \right)$

$V_A = \frac{3}{8}qL$

$V_B = -\frac{5}{8}qL$

Flectores: $M_{AB} = \frac{qx}{8}(3L - 4x)$

$M_B = -\frac{qL^2}{8}$

$M_{\text{máx}(+)} = \frac{9}{128}qL^2$ para $x = \frac{3}{8}L$

$M_x = 0$ para $x = \frac{3}{4}L$

Deformaciones

Giros: $\varphi_A = -\frac{qL^3}{48EI}$

Elástica: $y_{AB} = \frac{qx}{48EI}(L + 2x)(L - x)^2$

Flecha máxima: $y_{\text{máx}} = \frac{qL^4}{185EI}$ para $x = \frac{1 + \sqrt{33}}{16}L$

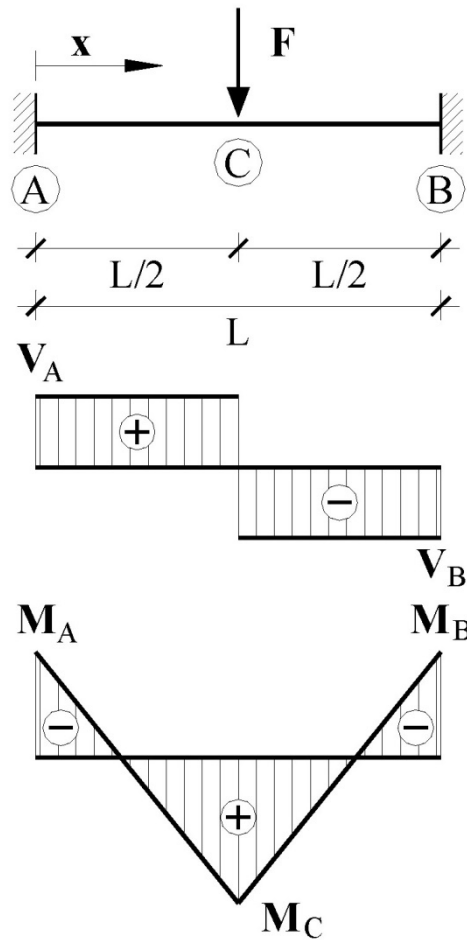
MOMENTOS DE EMPOTRAMIENTO PERFECTO

VIGA SIMPLE APOYADA-EMPOTRADA: carga puntual F centrada.

	<p align="center">Reacciones y solicitaciones</p>
<p>Reacciones: $R_A = \frac{5}{16}F$ $R_B = \frac{11}{16}F$</p> <p>Cortantes: $V_{AC} = \frac{5}{16}F$ $V_{CB} = -\frac{11}{16}F$</p> <p>Flectores: $M_{AC} = \frac{5}{16}Fx$ $M_{CB} = \frac{FL}{16} \left(11 \frac{L-x}{L} - 3 \right)$</p> <div style="border: 2px solid red; padding: 5px; display: inline-block;"> <p>$M_B = -\frac{3}{16}FL$ $M_C = \frac{5}{32}FL$</p> </div>	
<p align="center">Deformaciones</p>	
<p>Giros: $\varphi_A = -\frac{Fa^2}{32EI}$ $\varphi_C = \frac{FL^2}{128EI}$</p> <p>Elástica:</p> <p>$y_{AC} = \frac{FL^2}{96EI} x \left(3 - 5 \frac{x^2}{L^2} \right)$ $y_{CB} = \frac{FL}{96EI} (L-x)^2 \left(9 - 11 \frac{L-x}{L} \right)$</p> <p>Flechas:</p> <p>$y_C = \frac{7FL^3}{768EI}$ $y_{\text{máx}} = \frac{FL^3}{48\sqrt{5}EI}$ para $x = \frac{L}{\sqrt{5}}$</p>	

MOMENTOS DE EMPOTRAMIENTO PERFECTO

VIGA SIMPLE EMPOTRADA: carga puntual F centrada.



Reacciones y solicitaciones

Reacciones: $R_A = R_B = \frac{F}{2}$

Cortantes: $V_{AC} = \frac{F}{2}$ $V_{CB} = -\frac{F}{2}$

Flectores: $M_{AC} = \frac{FL}{8} \left(4 \frac{x}{L} - 1 \right)$ $M_{CB} = \frac{FL}{8} \left(3 - 4 \frac{x}{L} \right)$

$M_A = M_B = -\frac{FL}{8}$ $M_{\text{máx}} = M_C = \frac{FL}{8}$ para $x = \frac{L}{2}$

Deformaciones

Elástica:

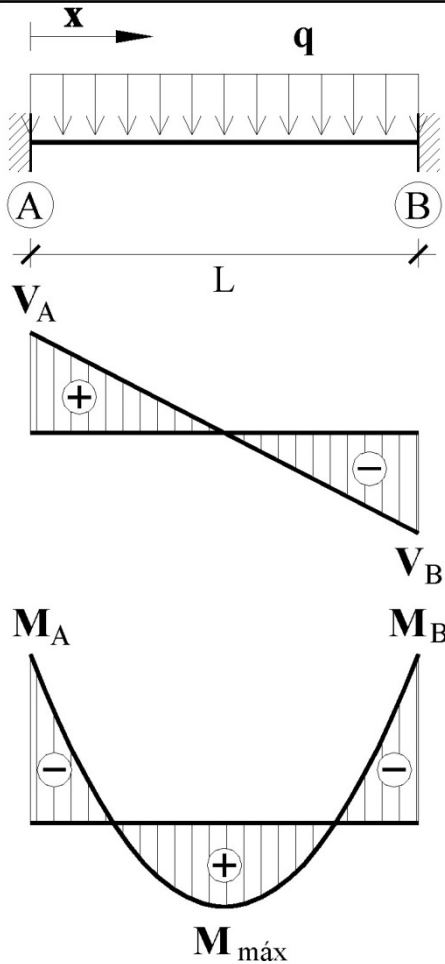
$y_{AC} = \frac{FLx^2}{48EI} \left(3 - 4 \frac{x}{L} \right)$ $y_{CB} = \frac{F(L-x)^2}{48EI} (4x - L)$

Flecha máxima:

$y_{\text{máx}} = y_C = \frac{FL^3}{192EI}$ para $x = \frac{L}{2}$

MOMENTOS DE EMPOTRAMIENTO PERFECTO

VIGA SIMPLE EMPOTRADA: carga uniforme q en todo el vano.



Reacciones y solicitaciones

Reacciones: $R_A = R_B = \frac{qL}{2}$

Cortantes: $V_{AB} = q \left(\frac{L}{2} - x \right)$ $V_A = -V_B = \frac{qL}{2}$

Flectores: $M_{AB} = -\frac{q}{12} (L^2 - 6Lx + 6x^2)$ $M_A = M_B = -\frac{qL^2}{12}$

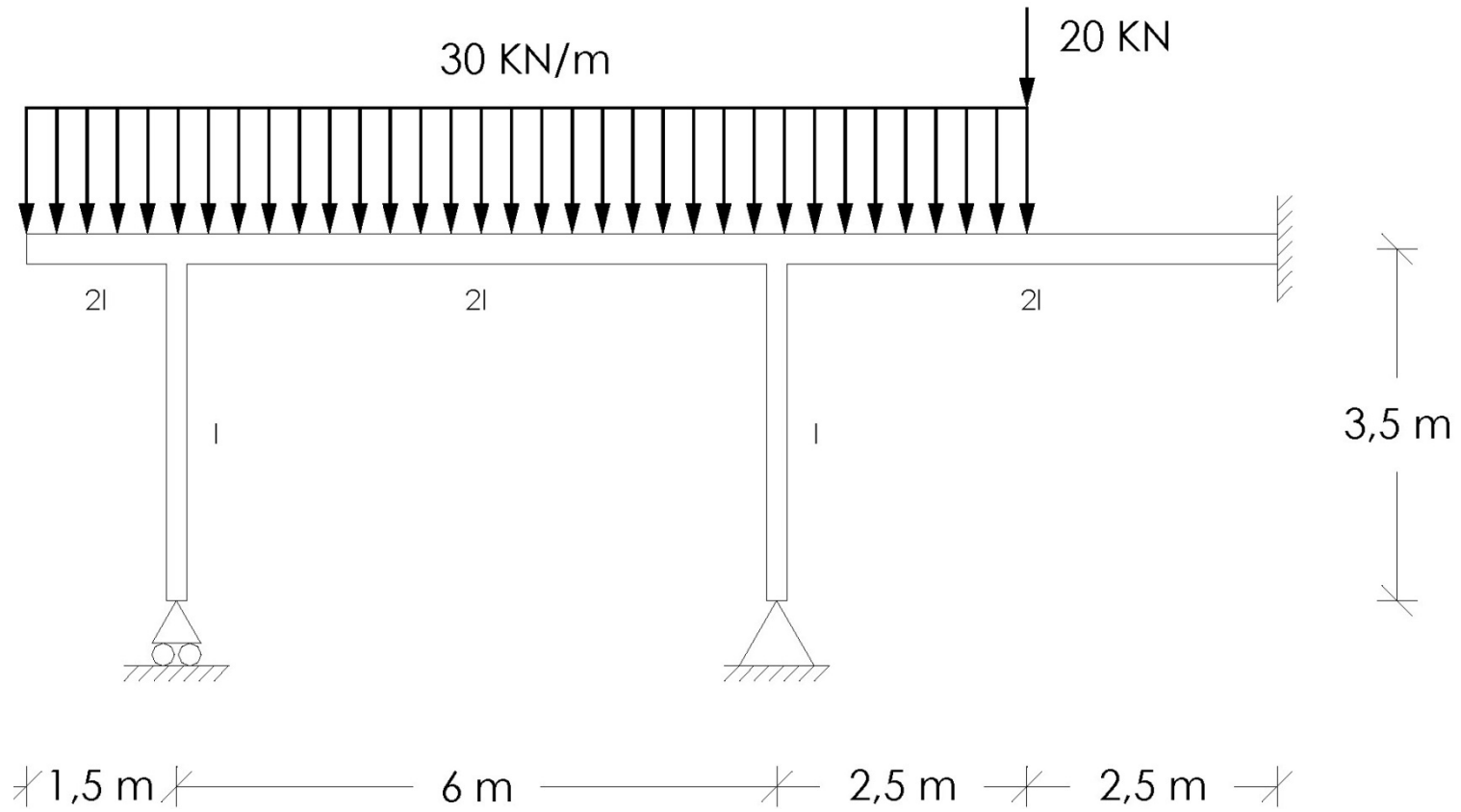
$M_{\text{máx}} = \frac{qL^2}{24}$ para $x = \frac{L}{2}$ $M_x = 0$ para $x = 0,2113L$

Deformaciones

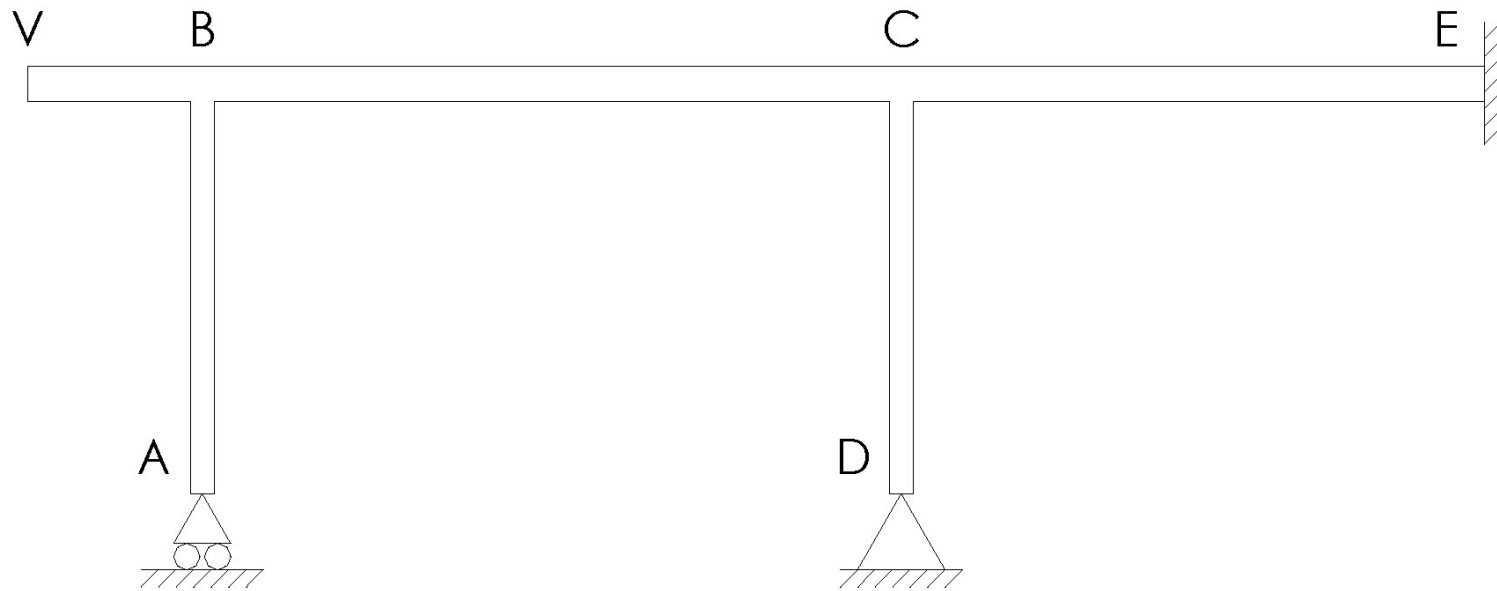
Elástica: $y_{AB} = \frac{qL^4}{24EI} \left(\frac{x}{L} - \frac{x^2}{L^2} \right)^2$

Flecha máxima: $y_{\text{máx}} = \frac{qL^4}{384EI}$ para $x = \frac{L}{2}$

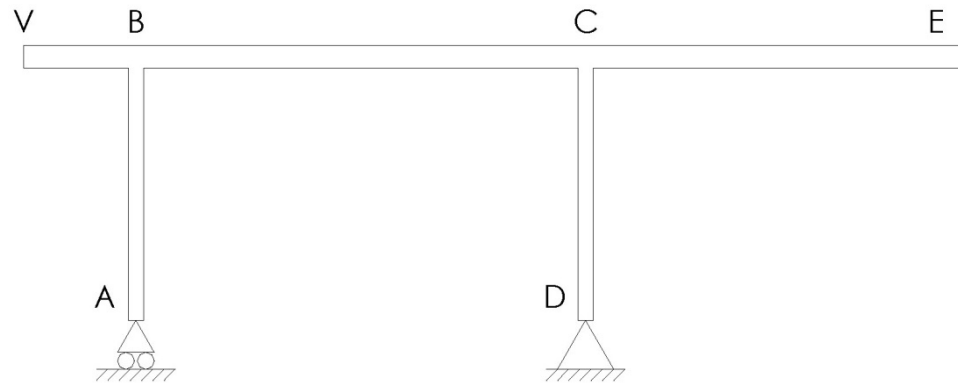
MÉTODO DE CROSS



MÉTODO DE CROSS

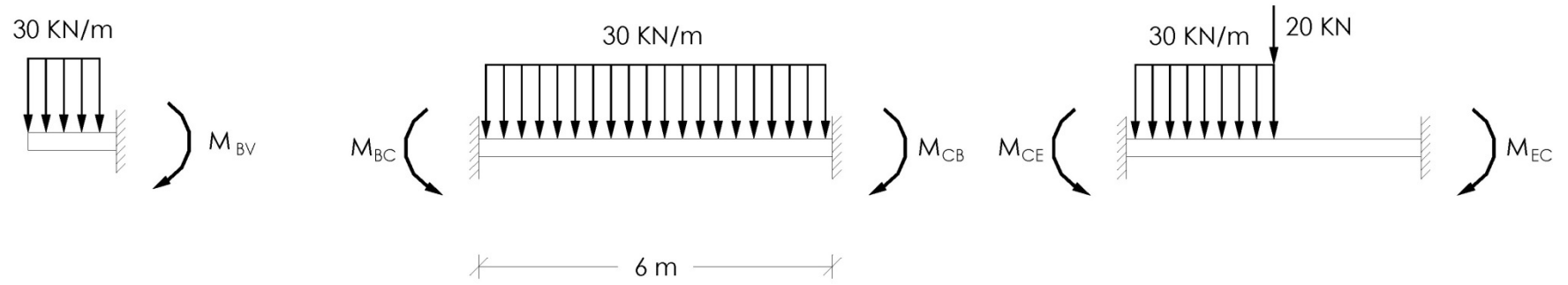


MÉTODO DE CROSS

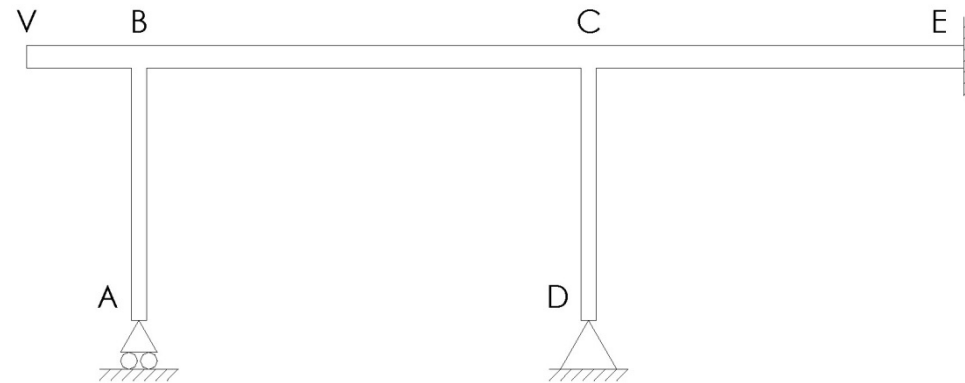


$$\begin{aligned}
 K_{BA} &= 0 & c_{BC} &= \frac{K_{BC}}{K_{BC}} = 1 \\
 K_{BV} &= 0 & c_{CB} &= \frac{K_{CB}}{K_{CB} + K_{CD} + K_{CE}} = 0,29 & t_{BC} &= 0,5 \\
 K_{BC} &= \frac{4 \cdot E \cdot I}{L} & c_{CD} &= \frac{K_{CD}}{K_{CB} + K_{CD} + K_{CE}} = 0,25 & t_{CB} &= 0 \\
 K_{CB} &= \frac{3 \cdot E \cdot I}{L} & c_{CE} &= \frac{K_{CE}}{K_{CB} + K_{CD} + K_{CE}} = 0,46 & t_{CD} &= 0 \\
 K_{CD} &= \frac{3 \cdot E \cdot I}{L} & & & t_{CE} &= 0,50 \\
 K_{CE} &= \frac{4 \cdot E \cdot I}{L} & c_{CB} + c_{CD} + c_{CE} &= 0,29 + 0,25 + 0,46 = 1
 \end{aligned}$$

MÉTODO DE CROSS

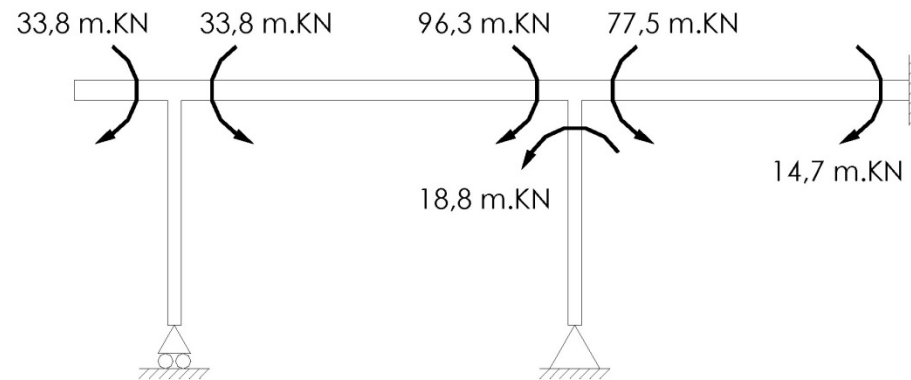


MÉTODO DE CROSS



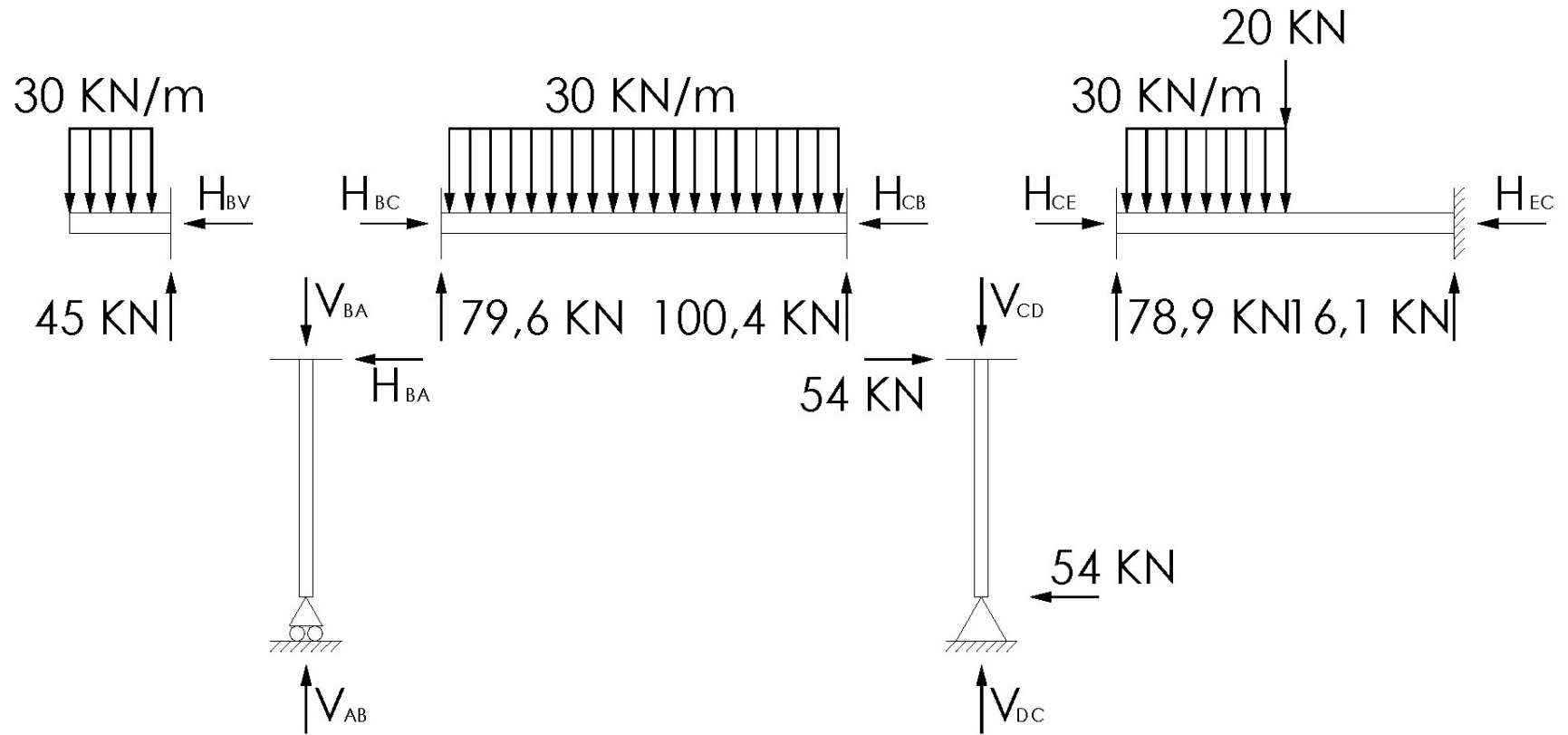
NUDO	B			C			E
EXTRE.	BV	BA	BC	CB	CD	CE	EC
Coe. rep.	0	0	1	0,29	0,25	0,46	---
Coe. tra.	0	0	0,5	0	0	0,5	---
M.E.P.	- 33,8	0	+ 90	- 90	0	+ 43	- 32
B: - 5,62	0	0	- 56,2	- 28,1			
C: + 7,51				+ 21,8	+ 18,8	+ 34,5	+ 17,2
M. Final.	- 33,8	0	+ 33,8	- 96,3	+ 18,8	+ 77,5	- 14,7

MÉTODO DE CROSS

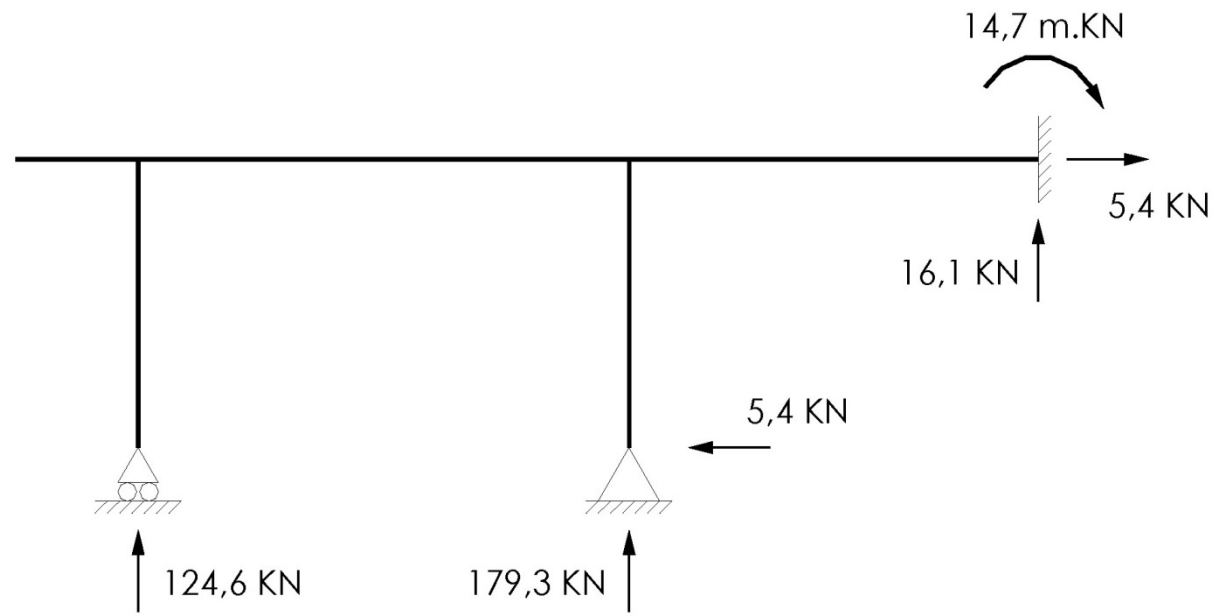


NUDO	B			C			E
EXTRE.	BV	BA	BC	CB	CD	CE	EC
Coe. rep.	0	0	1	0,29	0,25	0,46	---
Coe. tra.	0	0	0,5	0	0	0,5	---
M.E.P.	- 33,8	0	+ 90	- 90	0	+ 43	- 32
B: - 5,62	0	0	- 56,2	- 28,1			
C: + 7,51				+ 21,8	+ 18,8	+ 34,5	+ 17,2
M. Final.	- 33,8	0	+ 33,8	- 96,3	+ 18,8	+ 77,5	- 14,7

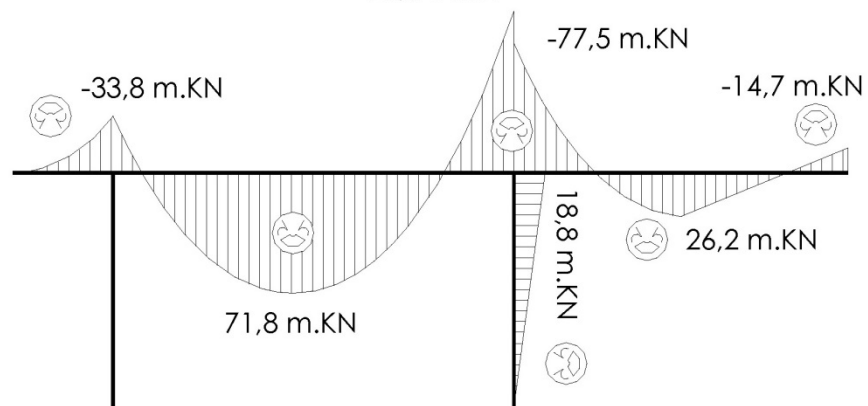
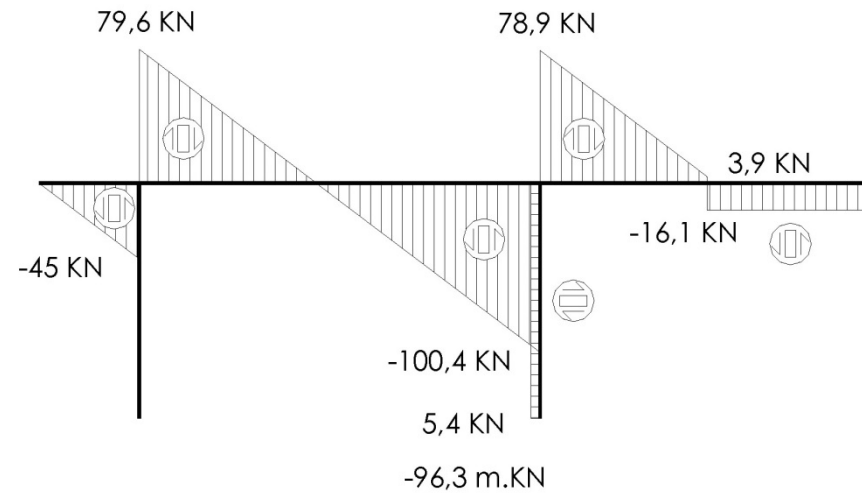
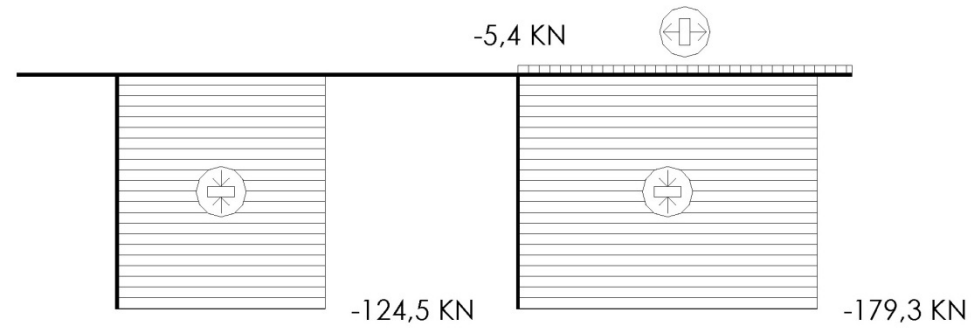
MÉTODO DE CROSS



MÉTODO DE CROSS

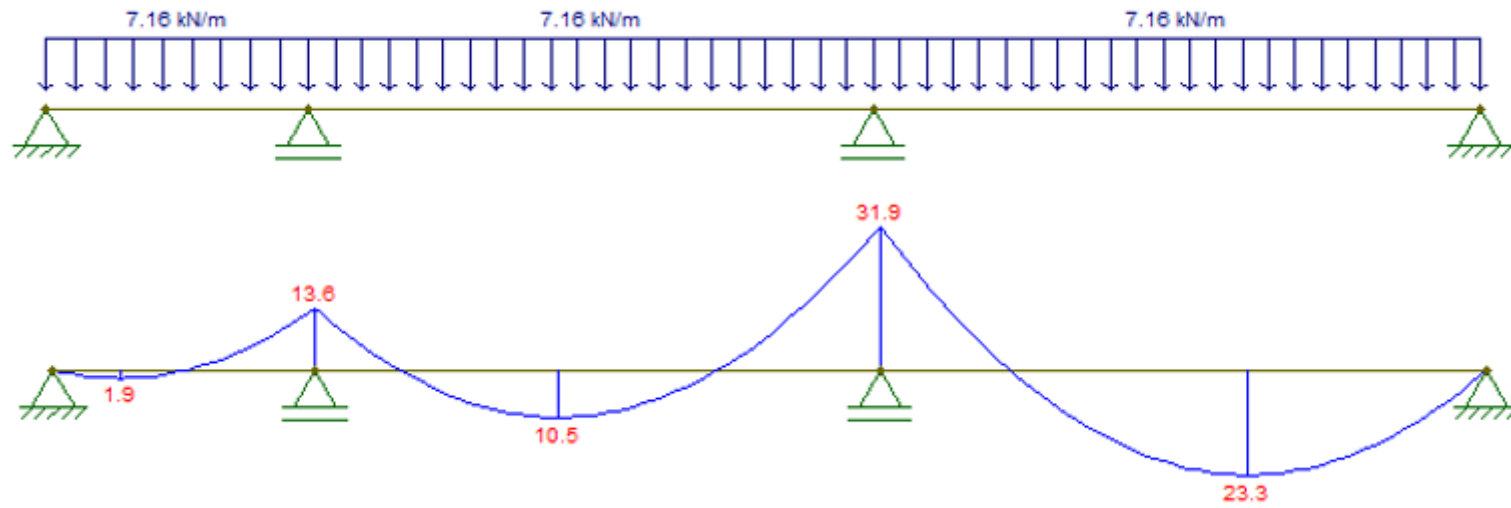


MÉTODO DE CROSS



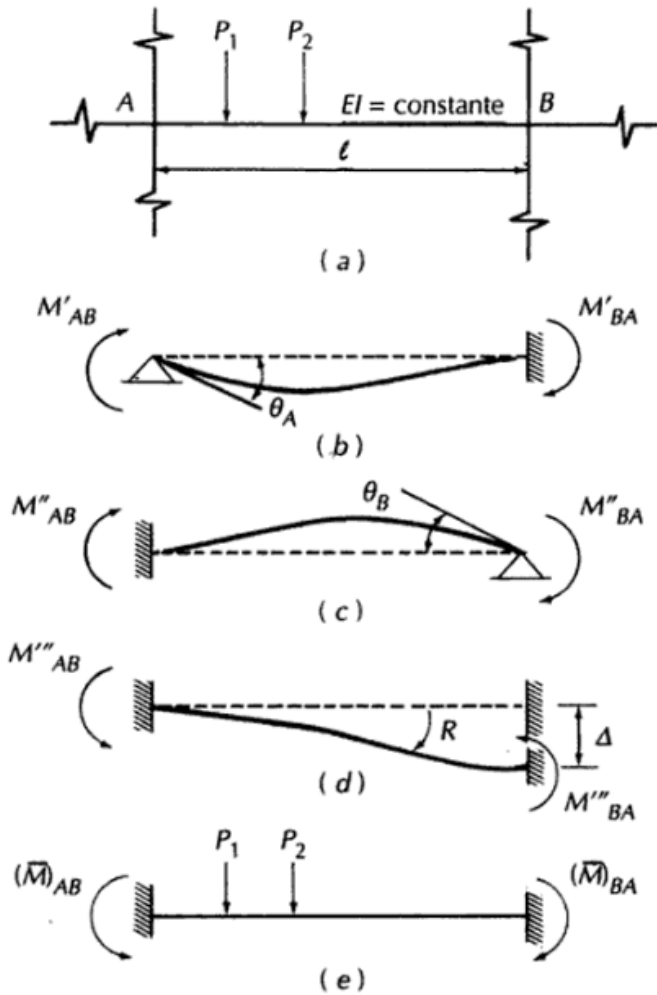
MÉTODO PENDIENTE-DEFORMACIÓN

Sólo para vigas continuas



MÉTODO PENDIENTE-DEFORMACIÓN

Sólo para vigas continuas



$$M_{AB} = \frac{4EI\theta_A}{l} + \frac{2EI\theta_B}{l} - \frac{6EI\Delta}{l^2} \pm \bar{M}_{AB}$$