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Introducing disruption on stagnated Group Decision Making processes using Fuzzy Ontologies

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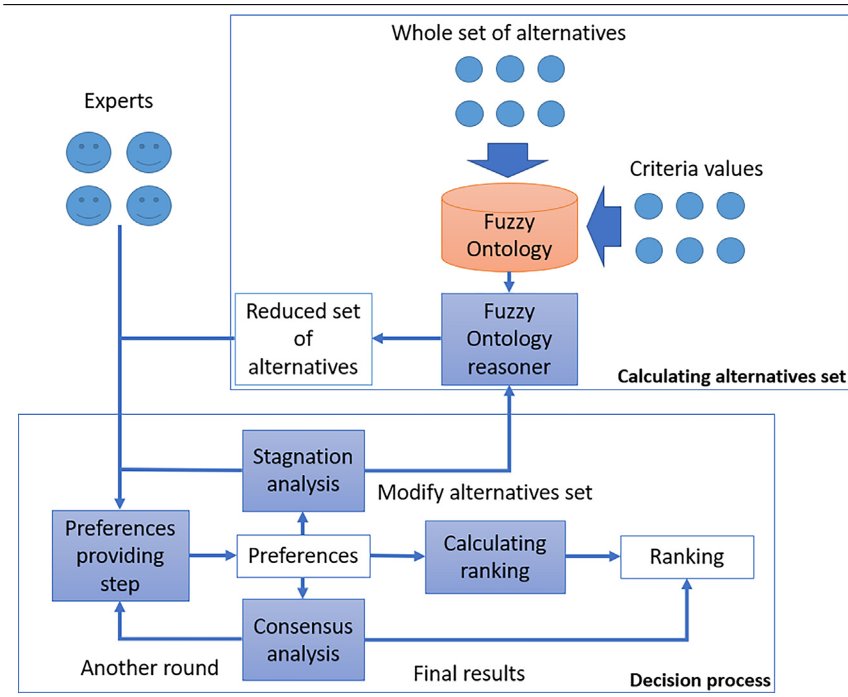
GRAPHICAL ABSTRACT



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ABSTRACT

In Group Decision Making processes, experts debate about how to rank a set of alternatives. It is usual that, at a certain point of the discussion, the debate gets stuck. In this paper, a novel Group Decision Making method for environments with a high number of alternatives is presented. Fuzzy Ontologies are used in order to represent the alternatives and their characteristics. Moreover, a novel stagnation analysis is used in order to determine if the debate gets stuck. If it does, the method modifies the alternatives set in order to introduce new options and remove the least popular ones. This way, the debate can revive since that the new alternatives provide different points of view. The presented

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method helps experts to conduct long and thorough debates in order for them to be able to make effective and reliable decisions.

1. Introduction

Group Decision Making (GDM) is a field that is quite present in the recent literature [1–3]. Traditional GDM environments where experts carry out face-to-face meetings in order to debate about a reduced set of alternatives are now in the past. Nowadays, experts need to make complex GDM processes on social networks [4]. The main challenge is that Internet provides them with plenty of information that they can hardly manage. Decisions are full of alternatives and criteria that experts must take into account if they want to make clever decisions. In this new environment, GDM methods acquire a renewed importance. Their purpose is to assist experts in the process by providing tools and features for helping them to effectively make a clever use of all the available information. Consequently, it is important to develop new methods that allow experts to deal with environments with a high number of alternatives and criteria. It can be considered that a number of alternatives in a GDM problem is high when the experts cannot discuss them at the same time. Generally, it is considered that the number of items that a human can discuss at the same time is 7 [5]. On recent scenarios, it is expected that 1000 or even 10 000 or more alternatives are available. The specific number would totally depend on the discussed topic. For this reason, it is important to develop novel GDM methods that are scalable enough for dealing with any numbers of alternatives.

Another challenge that novel GDM methods must overcome is that the information that the experts deal with is usually coming from users' opinions which makes the information imprecise and difficult to interpret by a computational system. In order to solve this issue, it is possible to use Fuzzy Ontologies (FOs) in order to store and manage the information in an organized way. FOs are a useful tool to use when dealing with imprecise information since it uses fuzzy sets in order to represent the information. Also, they are able to represent high number of items making them able to provide scalability to GDM frameworks that deals with a high number of alternatives and criteria.

When experts carry out their debate, there is usually some point where the discussion gets stuck. This is due to the fact that all the information about the alternatives that the experts are discussing at that moment have been analysed. In these cases, the GDM support system help the experts to effectively continue the debate. For this purpose, the GDM method handling the process would benefit of the inclusion of procedures that help the experts to revive the debate.

Recent GDM papers that deal with a high number of alternatives [6–8] usually tend to create groups of alternatives making the experts to deal with categories instead with concrete solutions. One way of allowing experts to discuss among specific alternatives in this kind of environments, as stated, is by using FOs [9,10]. GDM methods that employ FOs allow the information to be organized and help experts to explore different alternatives. They ask for specific descriptions and retrieve the alternatives that better fulfil them. Afterwards, the discussion is made by using the retrieved small set of alternatives. This oversimplifies the process making the experts not to debate about the full range of possibilities but only a small part of them that are usually similar among them. Therefore, there is a need of new GDM methods for environments with high number of alternatives that use FOs, like the one developed in this paper, which allow experts to debate among alternatives that have different characteristics and not only a small amount of them that are similar among them.

In this paper, a novel GDM process for environments that have a high number of alternatives and criteria is presented. The method uses FOs in order to store the information. The presented paper presents the following advantages and novelties:

- Since FOs are used to store alternatives and criteria information, the proposed method is scalable making it able to deal with any number of alternatives and criteria.
- Although there are a large number of alternatives, experts deal directly with them. Other methods create groups of alternatives to debate about which makes the discussion more superficial.
- A novel stagnation analysis process is designed for reviving the debate in case it gets stuck. The debate is revived by introducing new alternatives for the experts to discuss. The chosen alternatives are selected according to the characteristics desired by the experts. Two approaches are presented. In the first one, alternatives that are different from the most voted ones are presented to the experts. This way, they can debate other points of view. The second approach consists in presenting alternatives that are similar to the most voted ones. This way, experts can find out the best alternative over the ones that fulfil the most popular criteria. These two approaches can be combined in other to discuss different points of view at first, and select the best alternative over a certain set of criteria in the last decision rounds.
- The stagnation analysis allow experts to carry out a thorough debate. Alternatives' set is only modified when the debate is stagnated.
- Since experts rank a subset of alternatives each round, it is possible to analyse the criteria that the most voted alternatives have. Using this information, it is possible to determine which are the criteria values that matter the most to the experts and select new alternatives accordingly.

The presented GDM method requires to know how each of the alternatives fulfil each of the criteria values. Criteria are considered as attributes that are applicable to the alternatives [11]. The information that is unknown to the experts can be inferred from the context by using experiments or by carrying out prior GDM processes.

The paper is organized as follows. In Section 2, basis needed to comprehend the method are presented. In Section 3, the presented method is described in detail. In Section 4, an application example is presented in order to improve the method comprehension. In Section 5, advantages, novelties and drawbacks of the method are discussed. Finally, some conclusions are pointed out.

2. Preliminaries

This section presents the basis needed to comprehend the method. In Section 2.1, GDM methods are presented. In Section 2.2, FOs basis are presented.

2.1. Group decision making

GDM is a quite popular topic nowadays [12,13]. Its main purpose is to define methods that are capable of guiding a set of experts in ranking a set of possible actions or elements called alternatives. Formally, a GDM problem can be defined as follows:

Let $E = \{e_1, \ldots, e_n\}$ be a set of experts and $X = \{x_1, \ldots, x_m\}$ be a set of alternatives. The system's main purpose is to rank the set X by taking into account the preferences provided by E, $P = \{p^1, \ldots, p^n\}$. In the process, it is possible to define the criteria set, $Cr = \{cr_1, \ldots, cr_p\}$. They are the characteristics of the alternatives that have influence over the decision process.

A typical GDM process can be resolved using the following procedure:

- **Discussing the alternatives**: Experts discuss the characteristics of the alternatives and their applicability to the problem at hand. The main purpose of the debate is to expose different points of view and analyse possible solutions. By pointing out the advantages and disadvantages of the alternatives, experts can get an idea of which are the most promising ones.
- **Providing preferences to the system**: After discussing the alternatives, each expert provides his/her preferences to the system. One way of providing this information to the system is by using preference relation matrices [14], P^k, where p_{ij} indicates how expert e_k prefers x_i over x_j. Linguistic label sets can be used in this process. Preference relation matrices allow experts to carry out a pairwise comparison of the alternatives that are involved in the decision. This is easier for them than directly providing a ranking containing all the alternatives at once.
- **Calculating the collective preference matrix**: Once that all the experts' preferences have been provided, they are aggregated into a collective preference matrix that contains the overall opinion of all the experts about the alternatives that are involved in the process. For this purpose, matrices P^k can be aggregated into a single collective preference matrix C by using an aggregation operator [15,16].
- **Calculating consensus**: The ranking calculated in the previous step is built relying only on the preference information provided by the experts. In order to carry out reliable decisions, it is important that the experts intensively discuss and reach an agreement. Consensus measures [17–19] indicate the level of consensus reached among the experts. If the consensus value is low, experts can be asked to carry out more debate before reaching a final agreement. In this case, experts would continue the discussion and the decision process is restarted from the discussing alternatives step. On the contrary, if the consensus value is high, it means that the experts have reached an agreement. Therefore, the calculated ranking can be considered as the final one. Also, it is possible to limit the number of rounds in each GDM process. This way, the decision will not continue endlessly.
- **Ranking the alternatives**: Once that the reached consensus is high enough, the collective preference matrix is used to calculate the ranking of the alternatives.

2.2. Fuzzy Ontologies

FOs are tools that allow the storage and representation of imprecise information. Since decisions usually rely on opinions and imprecise data, FOs can be employed in order to store the information related to the elements that belong to the decision environment. Thanks to FOs, the information is stored in an organized way and queries can be used in order to retrieve any necessary information.

Formally, a FO for storing decision data can be defined as a quintuple $O_F = \{X, CR, R, F, A\}$ [20] where X is the set of alternatives, CR is a set of criteria values, R is a set of relations, F is a set of fuzzy relations and A is a set of axioms. The main purpose of each element is described below [21]:

Table 1
Fuzzy relations between x_1 and x_2 for cr_1.

cr_1	s_1	s_2	s_3	s_4	s_5	s_6	s_7
x_1	0.7	1	0.7	0.2	0	0	0
x_2	0	0	0	0	0.1	0.7	1

Table 2
Fuzzy relations between x_1 and x_2 for cr_2.

cr_2	s_1	s_2	s_3	s_4	s_5	s_6	s_7
x_1	0.8	1	1	0.8	0.1	0	0
x_2	0	0	0	0.1	0.8	1	0.8

- **Alternatives**: They are the components that are described on the FO. In FOs notation, these are the individuals.
- **Criteria**: They are the descriptions that can be assigned to each of the alternatives. In FOs notation, they are called concepts.
- **Relations**: They establish relationships between alternatives and criteria or among alternatives. If an alternative is related to a criterion value, then it means that the criterion is applicable to that alternative.
- **Fuzzy Relations**: Normal relations are binary, that is, only two values, $\{0, 1\}$, are available. This way, alternatives are related or not to the criteria. Fuzzy relations, on the contrary, can establish a fulfilment degree by employing the fuzzy set mathematical environment. Consequently, each alternative is related to each criterion by a certain degree. Thanks to fuzzy relations, it is possible to represent imprecise information on the FO.
- **Axioms**: They establish rules to be fulfilled by the rest of the FO elements.

3. A novel group decision making method for introducing disruption on stagnated processes

In this section, the proposed method is described thoroughly. In Section 3.1, the structure of the FO that stores the information is presented and discussed. In Section 3.2, the process for obtaining the reduced set of alternatives is described. In Section 3.3, the steps used by the novel presented process to manage the GDM process are thoroughly discussed.

3.1. Fuzzy ontology structure

The presented method uses FOs for keeping the information about alternatives organized. Each alternative of the FO is related to each criterion by using a linguistic label set. In order to define this linguistic label set, any number of labels can be used. For exemplary purposes, a granularity value of 7 is chosen since it is a number easy for an expert to tackle and provide sufficient information about how an alternative fulfil a criterion. Therefore, the linguistic label set used is defined as $S^7 = \{s_1, \ldots, s_7\}$. For each label, the membership degree of the relation is defined by employing a value located in the interval [0,1]. The closer the number is to zero, the less the alternative meets the criterion. For instance, let x_1 and x_2 be two alternatives and cr_1 and cr_2 two criteria values. One possible representation of the information related to these elements on the FO would be the one exposed in Tables 1 and 2.

Thanks to FOs, it is possible to represent the alternatives and criteria information in a comfortable way. Moreover, the experts and the computational system can carry out queries in order to retrieve alternatives that fulfil certain criteria. This will facilitate the way in which the system and the experts deal with the large amount of information in the GDM process.

Table 3
Similarity among labels in the set S^7.

	s_1	s_2	s_3	s_4	s_5	s_6	s_7
s_1	1	0.7	0.2	0	0	0	0
s_2	0.7	1	0.7	0.2	0	0	0
s_3	0.2	0.7	1	0.7	0.2	0	0
s_4	0	0.2	0.7	1	0.7	0.2	0
s_5	0	0	0.2	0.7	1	0.7	0.2
s_6	0	0	0	0.2	0.7	1	0.7
s_7	0	0	0	0	0.2	0.7	1

As it can be seen on Tables 1 and 2, it is possible to define fuzzy relations in different ways. The presented representation scheme even allows one alternative to fulfil the concept using several linguistic labels. For instance, labels s_2 and s_3 reach a similarity value of 1 for x_1 and cr_2. This is equal to stating that a hesitant fuzzy relation [10,22] where the set of labels $\{s_2, s_3\}$ is assigned to the relation. s_1 and s_4, since their meaning is close to $\{s_2, s_3\}$ have also a high similarity value. From now on, for exemplary purposes, it is assumed that only one label of S^7 is assigned to each relation. Similarity values of the rest of the labels are calculated using the values on Table 3 [10].

3.2. Fuzzy Ontology reasoning procedure

In this subsection, the process followed by the FO reasoner to obtain the reduced set of alternatives is exposed in detail [23]. In order to retrieve the subset of alternatives that better fulfil the desired criteria, the following steps are used [24]:

- **Providing the query:** A set of criteria that the alternatives must fulfil is indicated to the FO reasoner. It is possible to provide certain weights indicating the importance that fulfilling each criterion has. For instance, in an GDM example that has 12 criteria values, the query below:

  $Q = \{w_1 \cdot cr_1, w_2 \cdot cr_4\}$

  $W = \{0.25, 0.75\}$

  indicates that we are interested in alternatives that better fulfil criteria cr_1 and cr_4 giving more importance to fulfilling criteria cr_4.
- **Calculating the similarity value:** The FO reasoner calculates the similarity value between each alternative and the query. For this purpose, it is possible to use the similarity tables established in the previous section.
- **Retrieving the most fitting alternatives:** The alternatives that have better fulfil the query, that is, the ones that have the highest similarity values, are retrieved. The number of alternatives conforming the reduced set must be determined by the experts depending on how many alternatives do they want to discuss in the debate.

In order to clarify how this process works, the following example is presented.

Example 1. Let define a set of alternatives, $X = \{x_1 \ldots, x_3\}$, and a set of criteria $CR = \{cr_1, \ldots, cr_3\}$. Each criterion can be considered to be low, cr_i^l, or high, cr_i^h. The way that each alternative fulfils each criterion is established in Table 4. The linguistic label set S^7 is used for establishing the relation values and Table 3 is used to establish the similarity among labels. Imagine that, for the FO presented in Table 4, alternatives want to be ranked according to how they fulfil cr_1^l and cr_2^h. Both will be considered of having the same importance. They are only interested in alternatives that have a high fulfilment of the criterion values, s_7. In order to calculate the similarity value, calculations shown in Table 5 are made. For instance, for x_1, the expression $0.5 \cdot 0 + 0.5 \cdot 0$ is obtained. 0.5 value is used as weight since both cr values are considered to have the same importance. Similarity between labels from the query and labels indicating how the alternatives fulfil the criterion is 0 in both cases. In order to obtain those values, Table 3 is used. s_1 and s_3 similarity with s_7 is checked. Once that the similarity values are obtained, alternatives are ranked according to the obtained results. As it can be seen, ranking is as $\{x_3, x_2, x_1\}$, being x_3 the most promising alternative.

Table 4
Relation values for the ontology in Example 1.

	x_1	x_2	x_3
crl_1^l	s_1	s_5	s_7
crl_1^h	s_7	s_5	s_1
crl_2^l	s_3	s_6	s_1
crl_2^h	s_3	s_1	s_6
crl_3^l	s_1	s_6	s_1
crl_3^h	s_6	s_1	s_6

Table 5
Similarity value calculation.

Alternative	Calculations	Similarity value
x_1	$0.5 \cdot 0 + 0.5 \cdot 0$	0
x_2	$0.5 \cdot 0.2 + 0.5 \cdot 0$	0.1
x_3	$0.5 \cdot 1 + 0.5 \cdot 0.7$	0.85

3.3. Group decision making process description

In this paper we develop a GDM process that employs FOs in order to allow experts to carry out discussions over a set of a large number of alternatives. FOs select a reduced set of alternatives and experts carry out a debate among them until the process is stagnated. That is, until experts refuse to change their preferences no matter what is stated on the debate. Once that this occur, the set of alternatives is modified by removing the most undesired alternatives and introducing new ones. Afterwards, the debate continues. The presented GDM method follows the next steps:

1. **Setting the initial GDM round**: FOs are employed in order to calculate the initial reduced set of alternatives. Also, debate parameters are defined.
2. **Providing preferences to the system**: Experts carry out a thorough debate among the reduced set of alternatives and provide their preferences to the system.
3. **Calculating the collective preference value and alternatives ranking**: The information provided by the experts is processed in order to generate the collective matrix and the temporary ranking of results.
4. **Consensus analysis**: Consensus measures are applied in order to determine if the experts agree on the ranking that has been calculated on the previous step. If they do, the decision process ends, and the temporary ranking results become definitive.
5. **Stagnation analysis**: It is possible that the debate stagnates before the experts reach a final consensual decision. This can be detected by analysing the preference matrices of the experts. In this case, it is necessary to update the alternatives set and modify the alternatives that are contained on it. For this purpose, FOs are used in order to retrieve new alternatives that matches the debate necessities. On the contrary, if the debate is not stagnated, another GDM round is performed. It is important that experts carry out a thorough debate among the current set of alternatives before introducing new options.

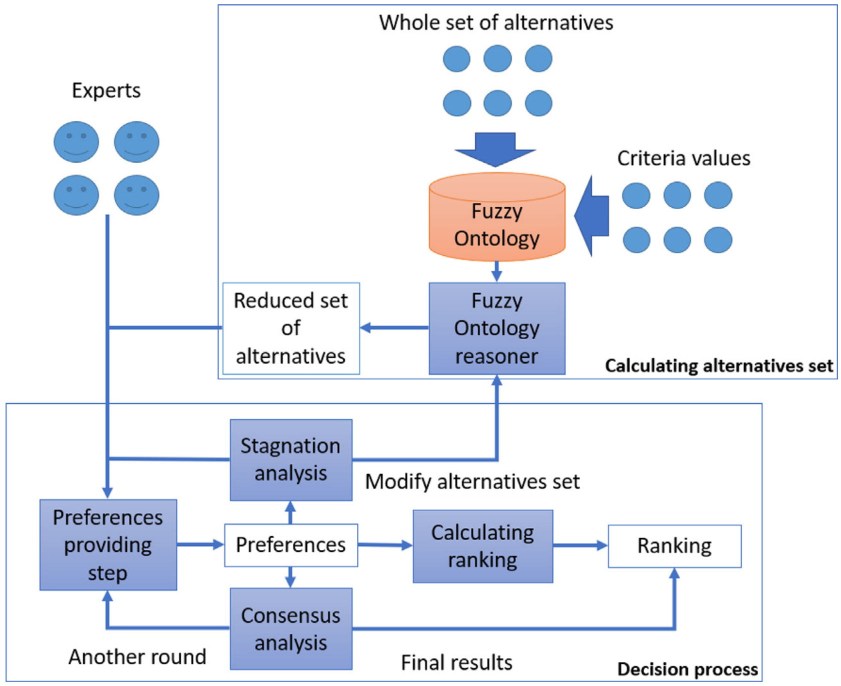

Fig. 1. Overall method scheme.

6. **Determining final results**: Final decision results are presented to the experts.

All these steps are described in detail in the following subsections. A graphical scheme of the presented process can be seen on Fig. 1. The figure has, first, separated into two main courses of action:

- Calculation on the reduced set of alternatives used in the debate round. This process calculates the reduced set of alternatives. For this purpose, a FO query is performed that allows us the retrieve the alternatives that better fulfil it.
- GDM process. Using the reduced set of alternatives calculated, the GDM process is performed. Once that the experts provide the alternatives, its consensus and stagnation are analysed, and the next debate round is set accordingly. More information is provided in the following subsections.

3.3.1. Defining the initial parameters

Before starting the GDM process, there are several parameters to be set [25]:

- **Stagnation threshold,** β: After carrying out the stagnation analysis, a global stagnation value that is located in the interval [0,1] is obtained. The stagnation threshold determines the level of stagnation needed for changing the current set of alternatives.
- **Number of alternatives to replace,** ar: This parameter indicates the number of alternatives that are replaced when the stagnation analysis indicates that the set needs to be updated. The lower the value, the less new alternatives are replaced on the set leading to a low variability in the discussed alternatives. ar is lower than m since it is important to maintain on the reduced alternatives set at least the most popular alternative. As a result, we do not lose the best solution found so far.
- **Consensus threshold,** α: After carrying out the consensus analysis, a global consensus value for the current round of the decision process is calculated. The consensus threshold determines how high that value can be for ending the GDM process.
- **Number of alternatives for discuss in a round,** m: In cases where a large number of alternatives is available, it is unwise to discuss them at the same time. Therefore, experts choose the number of alternatives that they will discuss in the present round. m can be static or can change in each decision round according to experts' necessities. In case there are disagreement about the number of alternatives to select, they can carry out a classic GDM process to decide the number. It is advised to select a relatively large number on the first rounds (9, for instance) to maximize the variety of alternatives available. On the contrary, a shorter one could be selected in the later rounds (4, for instance) in order for experts to focus the discussion on the most preferred ones.
- **Linguistic label set used for the discussion,** S: Experts select the linguistic label set that they want to use on the providing preferences step. If they do not agree on one, each expert can select the linguistic label set that he/she prefers. Information can be uniformed using multi-granular fuzzy linguistic modelling methods [26].
- **Calculating the initial alternatives set,** X_1: The initial set of alternatives that the experts will discuss on the first decision round is calculated. We propose three ways of obtaining this initial set. They all have their advantages and drawbacks. These three options are defined below:
  - **Random selection**: The required alternatives are selected randomly from the whole set of available alternatives. In this option, criteria values are not taken into account. The main advantage of this approach is that it is faster than the others since no FO query is required.
  - **Promote certain characteristics on the initial set of alternatives**: Experts may want the initial set of alternatives to meet a number of characteristics. If this is the case, experts can provide a query to the system and the alternatives that are most similar to the query parameters will fill the reduced alternatives set in the first round. The main advantage of this method is that it allows the experts to focus the debate on the most promising alternatives.

- **Promote variety of views**: According to the optimization problems general scheme, it would be wise to provide the experts with alternatives that are quite different among them. When several rounds have passed and experts have debated about a large number of different alternatives, it is possible to replace the least promising alternatives to alternatives that are similar to the ones that have had the best reception among experts. This way, experts can select the best alternatives among this group. The main advantage of this approach is that experts firstly debate among very different points of view and, as time passes, they focus on the most promising line. This allows them to select, inside the most promising line, the alternative that better fit them. At the end, different points of views and a set of alternatives that fulfil the criteria that had better fit the experts would have been discussed. The main advantage of this approach is that experts do not have to know in advance which is the most promising criteria for them.

- **Maximum number of rounds,** nr: A GDM process can continue endlessly if a high consensus value is not reached. Therefore, there is a need for establishing a maximum number of rounds. It is necessary that the chosen value is high enough to assure that the experts have carried out a thorough discussion. Also, it has to be low enough in order to generate an efficient decision process that do not last long than necessary. There is not an automatic way of establishing the nr value. It totally depends on the time that experts have to make the final decision. If the experts are in a hurry, they can put a low nr value, i.e. 3, 4. On the contrary, they could establish a higher value like 10 or 11.

Once all these parameters have been set, the GDM process can start.

3.3.2. Providing preferences to the system

Once the initial parameters have been defined, experts can start the debate. In this step, experts highlight the advantages and disadvantages of the alternatives included in the reduced set of alternatives for everyone to know their suitability. Once that the debate is made, experts provide their preferences to the system using preference relation matrices [27]. For this purpose, they carry out a pairwise comparison among the alternatives included on the reduced set of alternatives and they provide their opinions using preference relation matrices.

Formally, in the round o, e_k provide a preference relation P^{ko} whose values, p_{ij}^{ko}, indicate how much e_k prefers alternative x_i over x_j. In order to ease the way that experts use to express themselves, they use linguistic label sets for providing their preferences. Therefore, each p_{ij}^{ko} value is a label from a specific linguistic label set.

3.3.3. Calculating the collective preference values and alternatives ranking

Once that the experts have provided their preferences for the round o, the collective preference matrix, C^o and the temporary alternatives ranking of the round, R^o are calculated. In order to calculate the collective preference matrix, the mean operator over the preference values indexes is applied [28]:

$$C^o = \frac{\sum_{k=1}^{n} index(p_{ij}^{ko})}{n} \tag{1}$$

where *index* function indicates the index of a linguistic label set and p_{ij}^{ko} indicates how much e_k prefers x_i over x_j on round o.

To calculate the ranking of alternatives, it is possible to use the guided dominance degree (GDD) operator [10] over the values of the collective preference matrix, $C^o = \{c_{ij}\}, i,j \in [1,n]$. Its expression is shown below:

$$GDD_i^o = \phi(c_{i1}^o, c_{i2}^o, \ldots, c_{i(i-1)}^o, c_{i(i+1)}^o, \ldots, c_{in}^o) \tag{2}$$

where ϕ is the mean operator The ranking of alternatives for the current round, R^o, is calculated by sorting the alternatives according to their associated *GDD* value. Once that the temporary ranking of alternatives has been calculated, stagnation and consensus analysis of the process are performed.

3.3.4. Consensus analysis

Consensus measures [25,29] are used in order to determine if the experts have reached an agreement or, on the contrary, more debate is carried out.

First of all, it is necessary to establish a similarity measure between two different preferences matrices [30]. For that purpose, the following expression can be applied:

$$sim(P^{io}, P^{jo}) = 1 - |P^{io} - P^{jo}| \tag{3}$$

where P^{io} is the preference matrix that expert e_i has provided on round o.

By adding all the similarity matrices calculated using *sim* over the preference matrices of the experts, a collective consensus matrix is obtained. The following expression can be applied for this purpose:

$$CM^o = \phi(sim(P^{io}, P^{jo}))\ i,j = 1,\ldots,n;\ \ i<j \tag{4}$$

where ϕ is the mean operator. By using the obtained CM^o values, it is possible to calculate several consensus values according to the alternatives, experts and the decision round. These measures provide information about the consensus reached on round o by taking into account different aspects of the decision. Below, we list all the consensus values that can be calculated along with their corresponding expressions:

- **Consensus between two experts**: It is possible to calculate the consensus between experts e_i and e_j in round o using the following expression:

$$sim(P^{io}, P^{jo}) = 1 - |P^{io} - P^{jo}| \tag{5}$$

- **Distance of one expert to the mean**: It is interesting to know how far is one expert to the overall opinion of all the experts. It is possible to calculate the distance that expert e_i has to the overall opinion for the decision round o using the following expression:

$$sim(P^{io}, C^o) = 1 - |P^{io} - C^o| \tag{6}$$

- **Consensus at pair of alternatives level**: Each position of the CM^o matrix calculated on expression (4) indicates the consensus of one alternative over the others. For instance, cm_{ij}^o indicates the consensus that experts have on the value indicating how much x_i is preferred over x_j.
- **Consensus for each alternative**: It is possible to calculate the consensus for alternative x_l in round o, ca_l^o by applying the following expression:

$$ca_l^o = \frac{\sum_{k=1,k\neq l}^{m}(cm_{lk} + cm_{kl})}{2(m-1)} \tag{7}$$

where m is the number of alternatives. and cm the consensus at pair of alternatives level.

- **Global consensus**: Finally, it is possible to calculate an overall consensus value for decision round o if all the ca_l^o values are aggregated as follows:

$$gc^o = \frac{\sum_{l=1}^{n} ca_l^o}{m} \quad (8)$$

This value can be used in order to measure the overall consensus reached on the round. If the consensus is over α, experts agree on the decision and the process can end. Otherwise, it is recommended that another decision round is performed.

3.3.5. Stagnation analysis

It is quite common that after a few decision rounds, experts have already discussed all the advantages and drawbacks of the initially proposed set of alternatives. One of the main novelties of the presented method is that we have designed a novel stagnation analysis procedure that allows the debate to revive. The presented method analyses the criteria associated with the alternatives and they provide the experts new information to discuss. First of all, it is necessary to determine if the debate has stagnated. For this purpose, the preferences provided by the experts in the last three rounds are analysed. If there are changes, it means that experts are still effectively discussing the current set of alternatives. In other words, they are still doubting and changing their opinions about the alternatives. On the contrary, if the information provided does not change, experts need new information to discuss.

First of all, preferences from the three previous GDM rounds are used in order to calculate the global stagnation degree. The stagnation value reached by one expert in the round o can be calculated using the following expression:

$$Stag_{e_i} = \phi(sim(P^{i(o-2)}, P^{i(o-1)}), sim(P^{i(o-1)}, P^{i(o)})) \quad (9)$$

As it can be seen, the similarity value between the preference from one round to the previous is used. As a result, it is possible to measure the variability between rounds. For calculating the global stagnation degree, the following expression can be used.

$$GStag = \phi(Stag_{e_i}), i = 1\ldots, n \quad (10)$$

The obtained $GStag$ value can be used in order to determine if the debate is stagnated or, on the contrary, the discussion can continue with the same set of alternatives. Once that the alternative set has changed, three decision rounds have to pass before being able to apply the stagnation analysis to the decision process again.

In case that the stagnation level is high and there is a need of changing the alternatives set, the following steps are followed:

1. **Selecting the *ar* worst alternatives**: It is important to maintain on the reduced alternatives set the ones that are more popular among the experts. Therefore, only the worst classified alternatives are replaced. Parameter ar establishes the number of alternatives to be replaced. The higher ar is, the more alternatives are included in the set. It is important to always maintain the most popular alternatives on the reduced alternatives set in order to always keep the best solutions found so far.
2. **Replacing the alternatives**: There are several schemes that can be used in order to replace the alternatives. All of them are based on the criteria values associated to the discussed alternatives. Criteria values provide us with descriptions about the alternatives and can help us to select a set of alternatives from the FO that will replace the worst ones in the reduced alternatives set. From now on, the criteria values of one alternative will be represented using the following tuple:

$$Cri(x_j) = (cr_1(x_j), \ldots, cr_i(x_j) \ldots, cr_p(x_j)) \quad (11)$$

where $cr_i(x_j)$ is the index of the label that defines the relation between x_j and the criteria value cr_i. According to this representation, it is possible to carry out the replacement process using three different approaches. Each of them has a different purpose and they can be combined in a hybrid approach. We define them below:

- **Exploitation approach**: This approach tries to fill the reduced alternatives set with elements that are similar to the ones that occupy the first places of the ranking. Its main purpose is to allow the experts to find the best alternative among the ones that fulfil the preferred criteria. The idea of this approach matches to the exploitation procedure that is quite present in optimization procedures. The query that will be performed over the FO to obtain the required alternatives for the new set is as follows:

$$Q(round(\phi(Cri(R_1^o) + Cri(R_2^o)))) \quad (12)$$

where R_1^o and R_2^o are the two best alternatives on the ranking and *round* is the rounding operator. As a result of this approach, the obtained new alternatives will inherit the good characteristics that the best alternatives on the ranking have. It is possible to choose more than two alternatives for applying this approach. Nevertheless, taking into account the usual number of alternatives that are tackled by the experts in decision processes, we consider two a good choice. This way, only the best criteria configuration is taken into account on the query.
- **Exploration approach**: This approach has the opposite intention of the previous one. Its main purpose is to promote variety among the alternatives and allow experts to explore alternatives that are different from the already discussed ones. Its idea matches to the idea of exploration that is present in optimization algorithms [31–33]. The query that will be used to select the ar alternatives that will replace the worst alternatives on the ranking is built as follows:

$$Q(round(\phi(Cri(R_1^o) + Cri(R_2^o)))^C) \quad (13)$$

where C defines the complementary operation over the labels that represent the indexes. This approach determines the most promising criteria values and fill the reduced alternatives set with alternatives that fulfil the opposite. As a result, experts are able to discuss alternatives that are totally different from the ones that have already been discussed. Therefore, the main purpose of this approach is to introduce new solutions and points of view to the debate.

Example. Let x_3 be an alternative that has 7 criteria values associated. The S^7 linguistic label set is used in order to represent the information on the FO. Also, $cri(x_3) = (7, 4, 3, 1, 7, 7, 1)$. It is possible to apply the complementary operator, ^{C} to the tuple as follows:

$$(7, 4, 3, 1, 7, 7, 1)^C = (1, 4, 5, 7, 1, 1, 7)$$

- **Hybrid approach**: Most of the optimization algorithms available in the literature employs the following scheme. First, exploratory processes are applied in order to find the most promising area inside the search zone. Next, exploitation processes are employed in order to determine the best value on the selected promising area. By applying the exploration approach in the first decision rounds and the exploitation in the last ones, a similar scheme can be built. Therefore, the hybrid approach defines a new parameter, *extime* that indicates how many times the exploration approach is performed. Once that the exploration approach has been applied *extime* times, the exploitation approach is applied during the rest of the decision process instead. As a result, exploration of new alternatives is performed over the first part of the decision. Once that experts have carried out to the top of the list the alternatives that fulfil certain criteria, it is possible to search for alternatives that have that desirable characteristics in order to select the best-related alternative.

The selection of the approach depends on the problem and the experts. There is not an approach that works better in every situation. If the experts are very confident on how the alternatives should fulfil the criteria, it is probably better to go for the exploitation approach since it provides alternatives that fulfil criteria in a similar way. On the contrary, if experts are unsure, they can choose the exploration approach. This way, they can discuss about very different alternatives. Finally, the hybrid approach combines both exploration and exploitation approach buy it requires a long debate since it requires a set of rounds for exploration and another for the exploitation part.

Once that the *ar* new alternatives have been selected from the FO, they replace the alternatives that occupy the worst positions on the ranking.

3. **Restarting the debate**: Once that the reduced alternatives set has been recalculated, a new decision round is started. It is important to notice that the stagnation analysis will not be executed until the first three decision rounds are performed.

3.3.6. Determining final results

The described GDM process goes under several rounds until one of the following situations occur:

- **The reached consensus is higher than** α: If all the experts agree on a specific alternative ranking, it means that the solution satisfies all of them. Therefore, the alternative that occupies the top of the ranking is chosen as the final result. In order to avoid fast convergence without analysing enough alternatives, it is possible to establish a minimum number of stagnation points in the decision process. This way, it is assured that experts debate about a significant number of alternatives before generating a final decision result.
- **The maximum number of rounds,** nr**, is reached:** When the number of maximum rounds, nr, is reached, it means that the decision has been extended long enough. Therefore, the alternative that occupies the top of the ranking is selected as the decision result.

4. Example

This section presents a use case example whose main purpose is to enhance the comprehension of the reader about the novel proposed method. A company formed by four directives, $E = \{e_1, e_2, e_3, e_4\}$, needs to decide which smartphone should they buy to their employees. For this purpose, they will decide about 150 different mobile phones from different brands, $X = \{x_1, \ldots, x_{150}\}$. Experts inferred 12 different criteria, $CR = \{cr_1, \ldots, cr_{12}\}$, that they can use to describe the features of each mobile phone. Each of these features are specified in Table 6. The consensus threshold is set on 0.75 and the stagnation one on 2 in the interval [0,6].

Table 6
Criteria description.

Criteria	Description	Criteria	Description
cr_1	CPU speed	cr_7	OS support
cr_2	Size	cr_8	Storage
cr_3	Weight	cr_9	Security
cr_4	Connectivity	cr_{10}	Screen refresh
cr_5	Memory	cr_{11}	Price
cr_6	Camera	cr_{12}	Battery life

Furthermore, experts decide to debate about 5 different alternatives in each round. 3 alternatives will be replaced if the stagnation analysis process decides that new alternatives are included on the reduced alternatives set. This setting will allow new alternatives to be discuss while, at the same time, maintains the two best options. At the same time, 5 alternatives are a good number for the experts since it ensures that they do not get lost by having to discuss too many options at the same time.

First of all, the initial reduced set of alternatives is calculated. In this example, experts do not have a set of favourite criteria. Therefore, a set of 5 alternatives that fulfil different criteria values will be included on the initial set. As a result, alternatives that are quite different among them are included in the initial set. This will enhance exploration. This A set of 5 different queries, each one looking for candidates that fulfil different criteria are performed. These queries are listed below:

$$Q_1 = \{7, 7, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$$
$$Q_2 = \{0, 0, 0, 0, 0, 0, 0, 0, 0, 7, 7, 0\}$$
$$Q_3 = \{0, 0, 7, 0, 0, 0, 0, 7, 0, 0, 0, 7\}$$
$$Q_4 = \{0, 0, 0, 7, 7, 0, 0, 0, 0, 0, 0, 0\}$$
$$Q_5 = \{0, 0, 0, 0, 0, 7, 7, 0, 0, 0, 0, 0\}$$

Positions labelled as 0 are the ones that will not be taken into account in the search. Moreover, values in the set $\{1, \ldots, 7\}$ indicate the index of the searched label. Since only 5 alternatives are allowed, it is not possible to select one alternative per each criterion. Since this is not possible, the defined queries try to cover the maximum number of possible options. In posterior rounds, alternatives that fulfil criteria values in other ways will be discussed. It is assumed in this example that all the criteria values are equally important.

After taking the best result of each query, the initial reduced set of alternatives, RX, is as:

$$RedX = \{x_{50}, x_{77}, x_{25}, x_{17}, x_{57}\}$$

Once that this set is calculated, experts start the discussion. Afterwards, they provide their preferences to the system using the linguistic label set S^7:

$$P^{11} = \begin{pmatrix} - & s_4 & s_4 & s_2 & s_3 \\ s_6 & - & s_4 & s_4 & s_5 \\ s_1 & s_3 & - & s_2 & s_1 \\ s_1 & s_2 & s_4 & - & s_2 \\ s_2 & s_2 & s_4 & s_3 & - \end{pmatrix} P^{21} = \begin{pmatrix} - & s_2 & s_3 & s_3 & s_4 \\ s_3 & - & s_2 & s_1 & s_2 \\ s_2 & s_2 & - & s_1 & s_3 \\ s_5 & s_4 & s_3 & - & s_7 \\ s_2 & s_1 & s_2 & s_4 & - \end{pmatrix}$$

$$P^{31} = \begin{pmatrix} - & s_1 & s_2 & s_3 & s_4 \\ s_6 & - & s_5 & s_4 & s_6 \\ s_2 & s_1 & - & s_1 & s_3 \\ s_3 & s_2 & s_5 & - & s_3 \\ s_2 & s_1 & s_4 & s_1 & - \end{pmatrix} P^{41} = \begin{pmatrix} - & s_1 & s_2 & s_1 & s_3 \\ s_6 & - & s_7 & s_5 & s_6 \\ s_1 & s_2 & - & s_1 & s_2 \\ s_3 & s_1 & s_2 & - & s_4 \\ s_2 & s_2 & s_1 & s_3 & - \end{pmatrix}$$

The preference relation matrices provided by the experts are aggregated into the single collective matrix C^1. The resulting matrix is shown below:

$$C^1 = \begin{pmatrix} - & 2 & 2.75 & 2.25 & 3.5 \\ 5.25 & - & 4.5 & 3.5 & 4.75 \\ 1.5 & 2 & - & 1.25 & 2.25 \\ 3 & 2.25 & 3.5 & - & 4 \\ 2 & 1.5 & 2.75 & 2.75 & - \end{pmatrix}$$

By applying expression (2), the temporary alternatives ranking is calculated.

The obtained *GDD* values are shown below:

$$GDD = \{0.27, 0.58, 0.125, 0.364, 0.21\}$$

For instance, the value 0.27 is calculated as follows:

$$\left(\frac{2+2.75+2.25+3.5}{4} - 1\right)/6 = 0.27$$

This generates the following temporary ranking: $R^1 = \{x_{77}, x_{17}, x_{50}, x_{57}, x_{25}\}$. Temporary results are shown to the experts. Based on that information they continue the debate and start round two. After three rounds have passed, it is possible to apply the stagnation analysis in order to determine if the debate needs to be revived with new alternatives. In the three rounds that have passed, e_1 has provided the following information to the system:

$$P^{11} = \begin{pmatrix} - & s_4 & s_4 & s_2 & s_3 \\ s_6 & - & s_4 & s_4 & s_5 \\ s_1 & s_3 & - & s_2 & s_1 \\ s_1 & s_2 & s_4 & - & s_2 \\ s_2 & s_2 & s_4 & s_3 & - \end{pmatrix} P^{12} = \begin{pmatrix} - & s_6 & s_4 & s_2 & s_3 \\ s_6 & - & s_4 & s_4 & s_5 \\ s_2 & s_2 & - & s_3 & s_1 \\ s_1 & s_2 & s_3 & - & s_2 \\ s_2 & s_2 & s_4 & s_3 & - \end{pmatrix}$$

$$P^{13} = \begin{pmatrix} - & s_5 & s_4 & s_2 & s_3 \\ s_6 & - & s_5 & s_4 & s_5 \\ s_2 & s_3 & - & s_3 & s_1 \\ s_1 & s_2 & s_3 & - & s_2 \\ s_1 & s_2 & s_4 & s_3 & - \end{pmatrix}$$

In order to calculate e_1 associated stagnation value, the following calculation is performed:

$$Stag_{e_1} = \phi(sim(P^{12}, P^{11}), sim(P^{13}, P^{12}))$$

whose result is 0.2. The same calculation is performed for experts e_2 and e_3. Resulting values are 0.3 and 0.34 respectively. By applying the mean operator, the global stagnation value is 0.28. This value is expressed in the range [0,6] since there are 7 labels on the linguistic label set used. Since the obtained value is less than the established threshold value, 2, it can be considered that the debate is stagnated. Therefore, the actual reduced set of alternatives is modified. This is the first time that the replacement process is applied, therefore, the exploration approach will be used. Alternatives x_{50}, x_{57} and x_{25} are replaced because they occupy the last positions on the ranking. For this purpose, a query is built using the most voted alternatives, that is, x_{77} and x_{17}. Labels associated with the relations for both alternatives and the calculation of the query are shown in Table 7.

Once that the query is performed, the three alternatives located on the top of the FO resulting ranking are selected to replace the three least voted alternatives of the reduced set. In this case, the selected alternatives are $\{x_{35}, x_2, x_{120}\}$. Therefore, the new set of alternatives is as:

$$RedX = \{x_{77}, x_{17}, x_{35}, x_2, x_{120}\}$$

Once that the set of alternatives are presented to the experts, a new decision round is made. Preferences provided by the experts

Table 7
Labels associated to the relations of x_{77} and x_{17}.

	x_{77}	x_{17}	Mean	Query
cr_1	s_3	s_2	2.5	5
cr_2	s_1	s_6	3.5	4
cr_3	s_1	s_7	3	5
cr_4	s_2	s_7	4.5	3
cr_5	s_5	s_7	6	1
cr_6	s_7	s_3	5	3
cr_7	s_7	s_7	7	1
cr_8	s_7	s_3	5	3
cr_9	s_6	s_2	4	4
cr_{10}	s_7	s_1	4	4
cr_{11}	s_7	s_2	4.5	3
cr_{12}	s_6	s_6	6	1

in round 4 are specified below:

$$P^{14} = \begin{pmatrix} - & s_3 & s_5 & s_3 & s_2 \\ s_6 & - & s_2 & s_1 & s_4 \\ s_2 & s_3 & - & s_4 & s_2 \\ s_3 & s_2 & s_3 & - & s_2 \\ s_6 & s_6 & s_5 & s_7 & - \end{pmatrix} P^{24} = \begin{pmatrix} - & s_4 & s_3 & s_2 & s_4 \\ s_3 & - & s_2 & s_2 & s_1 \\ s_2 & s_3 & - & s_3 & s_5 \\ s_2 & s_3 & s_1 & - & s_5 \\ s_6 & s_7 & s_6 & s_6 & - \end{pmatrix}$$

$$P^{34} = \begin{pmatrix} - & s_3 & s_1 & s_2 & s_2 \\ s_2 & - & s_3 & s_5 & s_5 \\ s_1 & s_1 & - & s_3 & s_4 \\ s_2 & s_1 & s_4 & - & s_2 \\ s_7 & s_7 & s_7 & s_6 & - \end{pmatrix} P^{44} = \begin{pmatrix} - & s_1 & s_1 & s_1 & s_2 \\ s_1 & - & s_2 & s_1 & s_3 \\ s_2 & s_3 & - & s_2 & s_1 \\ s_1 & s_2 & s_3 & - & s_2 \\ s_6 & s_6 & s_5 & s_6 & - \end{pmatrix}$$

After aggregating the preference information, the collective preference matrix, C^4, is calculated:

$$C^4 = \begin{pmatrix} - & 2.75 & 2.5 & 2 & 2.5 \\ 3 & - & 2.25 & 2.25 & 3.25 \\ 1.75 & 2.5 & - & 3 & 3 \\ 2 & 2 & 2.75 & - & 2.75 \\ 6.25 & 6.5 & 5.75 & 6.25 & - \end{pmatrix}$$

Finally, the GDD vector in round 4 is as:

$$GDD = \{0.24, 0.281, 0.26, 0.23, 0.864\}$$

Therefore, the ranking of this round is as:

$$R^4 = \{x_{120}, x_{17}, x_{35}, x_{77}, x_2\}$$

By using the preferences provided by the experts to the system, it is possible to calculate the consensus measures exposed on Section 3.3.4. By applying expression (4), it is possible to calculate the global consensus matrix as follows:

$$CM^4 = \begin{pmatrix} - & 0.75 & 0.611 & 0.833 & 0.833 \\ 0.556 & - & 0.917 & 0.639 & 0.638 \\ 0.917 & 0.833 & - & 0.833 & 0.611 \\ 0.833 & 0.833 & 0.75 & - & 0.75 \\ 0.917 & 0.889 & 0.806 & 0.917 & - \end{pmatrix}$$

By aggregating the columns, it is possible to calculate the consensus reached for each alternative. The obtained values are shown below:

$$CA = \{0.806, 0.827, 0.771, 0.806, 0.709\}$$

Finally, by aggregating *CA* values, it is possible to calculate the global consensus value for round 4:

$$GC^4 = 0.784$$

Since the consensus value is above 0.75, the decision process can end. To allow the experts to make a thorough debate, it is possible to establish a minimum number of alternatives replacements before reaching the final decision result. This way, the process assures that a significant number of alternatives is discussed.

Finally, in Table 8, consensus values reached for each pair of experts is shown.

Table 8
Consensus among experts in round 4.

	e_2	e_3	e_4
e_1	0.18	0.193	0.1333
e_2	–	0.187	0.18
e_3	–	–	0.167

5. Discussion

In this paper, a novel GDM method that works over environments that have a large number of alternatives is presented. Information about the alternatives is stored in a FO. The process includes a stagnation analysis procedure whose purpose is twofold:

- **Enrich the debate by avoiding stagnation**: It is usual that the debate gets stagnated after several decision rounds. Therefore, there is a need for techniques that are capable of introducing new information that revives the debate. It is important to maintain the balance between a thorough debate among the elements of the set alternatives and the inclusion of new information. If new information is introduced before the experts have not fully discussed the previous one, some important facts could be lost and the final decision result is hindered. On the contrary, it is important to introduce some new information from time to time in order for the experts to explore a wide range of alternatives. If the information that the experts discuss does not change, they do not consider new possibilities.
- **Find the most suitable alternatives for the experts to discuss:** The novel developed stagnation analysis procedure allows the experts to discuss in environments that have a large number of alternatives. Since it is impossible to discuss all of them at the same time due to the high amount of information, the designed system creates reduced sets of alternatives that the experts can discuss. By using the criteria values associated with the alternatives, the novel proposed method selects the alternatives that have the most promising characteristics. Experts indirectly lead this process by ranking the reduced alternatives set or, also, they can do it directly by indicating which characteristics should the alternatives fulfil. This information is processed by the FO reasoner which provides the most adequate alternatives to discuss. Moreover, experts can explore different points of view by choosing alternatives that have different criteria.

In the recent literature, there already exist some GDM methods that deal with a large number of alternatives. For instance, in [34,35], authors classify the alternatives in clusters and experts provide the preference matrices according to these clusters. The main disadvantage that these methods have is that experts have to deal with clusters of alternatives instead of alternatives themselves. Because of this, experts lose sight of the features of each of the alternatives that conform the group making the debate ends in a too general discussion of groups of solutions. Furthermore, criteria are only taken into account in order to conform the groups. On the contrary, our method allows experts to modify the importance given to the criteria during the debate.

According to the use of FOs, there are already some methods that use them in order to deal with a large number of alternatives. For instance, [10,36,37]. These methods use FOs in order to represent alternatives and employ queries in order to retrieve a reduced set of alternatives for the experts to discuss about. Their main disadvantage is that the experts' discussion gets reduced to a certain set of alternatives. The rest of the alternatives that are available on the FO are never discussed or even known by the experts.

Table 9
Comparison table between the presented method and other ones on the literature. C refers to clustering approaches while F refers to FO existing approaches.

Feature	F [10,36,37]	C [34,35]	P
Alternatives are discussed individually.	Yes	No	Yes
Alternatives are selected according to the information that arise on the debate.	No	No	Yes
All alternatives can appear on the debate.	No	Yes	Yes
The importance given to the different criteria of the alternatives can be modified during the debate.	No	No	Yes
The method establishes measures to modify the alternatives while at the same time, promoting a thorough debate.	No	No	Yes
Employ learning algorithms to deal with the large number of alternatives.	No	Yes	Yes
Uses guided exploration–exploitation approach to find the best alternative.	No	No	Yes
Takes into account criteria to select the best alternatives	Yes	No	Yes

The method proposed on this article overcomes the disadvantages of both approaches:

- Alternatives are discussed individually, that is, experts can focus on the specific advantages and drawbacks that each of the alternatives have.
- Since the alternatives set changes over time, experts are not limited to only discuss a certain fixed set of alternatives. Any alternative can be discussed on the debate.
- Another disadvantage that recent methods have is that they do not take into account how the debate is going on when deciding which alternatives should be discussed. Our method dynamically updates the alternatives set from time to time based on the results achieved on the debate. The exploration–exploitation technique allows experts to discuss alternatives that are different among them while, near the end of the process, they can discuss the alternatives that have the desirable features that the experts have decide that they should have along the debate.
- The presented method is capable of guiding the experts in their decision assuring a good balance between a thorough debate of the alternatives and the inclusion of new information to discuss. The rest of the methods group the alternatives or select the reduced alternatives set without taking into account the debate carried out by the experts.

Finally, a summary table comparing the presented method with other ones on the literature is presented on Table 9.

6. Conclusions

In this paper, a novel GDM process for environments with a large number of alternatives is presented. It is impossible for experts to discuss all the alternatives at the same time. Therefore, our method allows them to discuss a reduced set of alternatives in each round. The set of alternatives to be discussed are selected in a way that experts discuss the alternatives that better fulfil certain criteria. This will ensure that alternatives that have different outcomes and are different among them will be discussed by the experts. This will lead to a better decision result since a large number of possibilities will be analysed. In order to make a fair and objective selection of the alternatives to discuss, FOs has been employed. FOs store the information about alternatives and how they fulfil certain criteria. Thanks to this, it is possible to objectively select alternatives that better fulfil certain criteria. They will create reduced set of alternatives for the experts to discuss about. This way, the experts to deal with and discuss a lot of different options and points of views. It is recommended that, at first, alternatives with different criteria values are discussed. This will help experts to decide which type of alternatives do they prefer. Afterwards, FOs can provide alternatives that are similar among them in order to select the alternative that better improves the outcome.

The proposed method includes measures that indicate when the debate has been stagnated. This way, alternatives set changes only when it is assured that the experts have finished discussing about the current ones. This is an important point since experts need to thoroughly discuss them before changing the subject.

Thanks to FOs, it is possible to focus the discussion on alternatives that have certain characteristics. This can be done explicitly asking the experts or obtaining the information from the ranking of alternatives. FOs provide tools to perform all these tasks. Therefore, our method can adapt to any possible need that the experts have about the debate.

For future research, we will believe that the method would benefit by improving the way that experts provide their preferences to the system. For instance, sentiment analysis could be used in order for the experts to express themselves using natural language.

CRediT authorship contribution statement

J.A. Morente-Molinera: Conceptualization, Methodology, Formal analysis, Software, Validation, Writing – original draft, Investigation, Data curation, Visualization. **A. Morfeq:** Visualization, Funding acquisition, Supervision. **R. Al-Hmouz:** Funding acquisition, Writing – review & editing, Supervision. **E.B. Ashary:** Funding acquisition, Writing – review & editing. **J.F. Su:** Validation, Methodology, Visualization. **E. Herrera-Viedma:** Writing – review & editing, Supervision, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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