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Training Teachers to Teach Probability: A Promising Research Area

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Abstract In this commentary, I reflect on the relevant role that the education of teachers imply to assure a successful teaching of probability at school levels. Unfortunately, research on this issue is still scarce. In this sense, the four papers included in this section are welcome because they approach this research problem from different perspectives. I analyse these perspectives using the Didactic-Mathematics Knowledge and Competences Model (CCDM) developed at the University of Granada, Spain.

Résumé Dans ce commentaire, je réfléchis sur les implications du rôle pertinent joué par la formation des enseignants afin de garantir un enseignement réussi des probabilités aux divers niveaux scolaires. Malheureusement, les recherches existantes sur cette question sont encore rares. Dans ce sens, les quatre articles inclus dans cette section sont les bienvenus, car ils abordent ce problème de recherche à partir de différentes perspectives. J'analyse celles-ci en utilisant le modèle de connaissances et de compétences didactiques et mathématiques (CCDM) développé à l'Université de Grenade, en Espagne.

Keywords Training teachers · Probability · Analysis of research

Training Teachers to Teach Probability

Today, there is ample agreement that probability is a part of mathematics with many applications in daily life and an instrumental role in many disciplines. Moreover, many professions require basic probabilistic literacy (Gal, 2002; 2005) and reasoning (Borovcnik, 2016) to make informed decisions under uncertainty. Due to all these reasons, probability is included in the secondary and high school curricula around the world and since the first grade of primary school in countries like Spain (MEFP, 2022).

An essential issue to ensure the success of teaching mathematics is the adequate training of the teachers responsible for this content, because what students' learn is related to the teacher' knowledge, competence, beliefs, and teaching practices (Eichler, 2011). Consequently, the number and variety of

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research exploring the teachers' knowledge, competence, attitudes, beliefs, and teaching practices has increased enormously. However, little of this research is focused on the education of teachers to teach probability.

In Batanero et al. (2004a, b), we analysed why teaching probability is difficult for mathematics teachers, and part of these reasons still holds. There is no specific preparation of teachers, but only (at best) general training in mathematics education. Due to the little time allowable and to the many topics teachers need to study during their initial education, mathematics teachers frequently lack specific preparation in probability education. This preparation should include a deep learning of the mathematical characteristics of probability in its different meanings and the related pedagogical knowledge. It is also important to consider the specific character of probability, which is contrary to the deterministic tradition in other mathematics areas.

To orient mathematics teacher training, our research group suggests the Didactic-Mathematics Knowledge and Competences Model (CCDM) (Godino et al., 2017). The teacher needs *common mathematical knowledge* relating to a particular level of education where he teaches and *advanced knowledge of the mathematical content* that allows him or her to articulate this knowledge with the teaching of the topic in higher educational levels.

Furthermore, the teacher needs *didactic-mathematical knowledge* of the different facets that affect the instructional process: *epistemic* (specialized mathematical knowledge), *ecological* (relation of the subject with other themes in the curriculum and with society), *cognitive* (students' learning, difficulties, and reasoning), *affective* (students' attitudes, beliefs, and emotions), *mediational* (technological and temporal resources), and *interactional* (management of the discourse in the classroom). In addition, the teacher must be competent to:

(a) Recognise the various meanings of the content concerned and their interconnectedness,
(b) Identify the diversity of objects and processes involved in the tasks he or she proposes to the students,
(c) Plan and manage the teaching of a theme,
(d) Identify the rules governing the topic and the norms implied in teaching, and
(e) Assess the didactic suitability of instructional processes.

Examples of Research Included in this Monograph

I was asked to comment four of the papers related to the education of teachers included in this monograph. This research is not trivial since the teachers' initial preparation and teaching experience is varied.

Moreover, when we speak of probability, we have to consider its multiple meanings (Batanero et al., 2005; Gillies, 2000). Several approaches (intuitive, classical, frequentist, subjective, or axiomatic) are introduced at different teaching levels. Moreover, some teachers prefer or are more acquainted with one of several of these views. We need to describe these preferences and knowledge to organise the mathematics teacher's education.

To contribute new knowledge in this regard, Machuca studies the mathematical work of a small group of pre-service and in-service teachers when solving a mathematical task in Chile. Using the Mathematical Working Space (MWS) (Kuzniak, 2011) theoretical framework, she analyses the mathematical objects involved in the task, under semiotic, instrumental, and discursive dimensions, identifying privileged vertical planes, the paradigm applied, and the different strategies used by teachers. The framework articulates epistemological and cognitive aspects through the semiotic, instrumental, and discursive dimensions to better understand, from a didactic point of view, what mathematical work is involved in an educational context.

The task admits two different assumptions about the related random experiment and two correct possible answers, which might have confused some of the pre-service teachers. Machuca remarks that the probability for an event is not unique and depends on the person's interpretation of the random experiment. This assertion also coincides with Devlin's (2014) conjecture about the generalized misconception that events have a unique probability. In Machuca' research, the sample space of the experiment was not always explicitly described by the teachers, and there was scarce use of tree diagrams. The author performs a detailed prior analysis of possible solutions and strategies to the problem and a posterior study of the solutions provided by the two groups of teachers. The author also describes some errors in the teachers' production, for example, not considering the order of events in the solution or misunderstanding the task conditions.

Even if the sample of teachers is small and the author only analyse a task in the classical view of probability, the paper highlights some relevant points to reinforce the training of teachers. In our opinion, the task would benefit if the teachers were also asked to describe the difficulties that students could find in the solution of the problem proposed and were requested to explicitly list the mathematical objects and process that the students need to acquire to solve the problem.

Following a previous survey performed on 626 teachers in Québec (Martin et al., 2018), in this new paper, Vincent Martin, Mathieu Thibault, and Marianne Homier analyse interviews with eight teachers who took part in the previous survey and whose self-reported teaching practices were exemplary. Using content analysis of the interviews performed on this small sample, the authors explore issues related to these teachers' teaching practices that can be generalised to other teachers in Québec.

The authors remark those points in which there was agreement in the interviews because the participants' discourse was relatively homogenous. All the teachers agreed on the social usefulness of probability and the interest of including this theme in their students' education. Participants also coincided in giving relevance to the theoretical approach to probability in their teaching, as well as in the main probabilistic concepts to teach and in the use of traditional probabilistic tasks with their students. They also were conscious of the relevance of increasing their professional development to improve their teaching of probability.

Five other themes produced more diversified and sometimes diverging answers in the interviews, as there was not full agreement in the teachers' responses. All the teachers reported that they introduced the frequentist approach of probability to their students. However, how this introduction was performed and the tasks proposed to the students were different for each teacher. There were also distinct ways to connect the different approaches to probability in the classroom. The subjective approach to probability only occupied a marginal place in the curriculum and the teaching. All the participants used a diversity of unusual tasks and different manipulatives and technology in their teaching to exploit the frequentist approach. Regarding technology, there was ample agreement on the added value of technological tools for exploration and amplification of ideas about randomness and probability; however, the teachers reported a diversity of resources and different uses of the same. In summary, this paper provides insights about points in which the teachers need further preparation and also revealed the variety of approaches possible to teach probability in the same educational levels.

Haneet Ganhdi was also interested in the different meanings of probability and randomness and explored which of these different meanings are implemented in the Indian curriculum, textbooks, and by the teachers. In the Indian school curriculum, the concepts of probability are introduced in middle grades within the chapter on Statistics, mostly in the classical and frequentist views of probability that dominate the textbooks. However, the author indicates that in some textbooks, examples of empirical introduction to probability can be found with a tendency towards the propensity interpretation of probability (Gillies, 2000). Beyond this, all the examples and problems are mostly dealt with classical probability or require conducting experiments with common manipulative random generators, such as a fair coin or a dice.

Gandhi was interested in understanding the view of probability and randomness that sustain the Indian teachers' strategies to introduce probability to their students. She performed some interviews to 42 in-service teachers having more than 10 years of experience of teaching mathematics to Indian middle-grade students. She provided these teachers with equilateral tetrahedrons and asked them whether they would use these devices to introduce probability in their classroom. She used a tool not common in the Indian classrooms because she believed that resources with which people may not be familiar while constructing probabilistic concepts help to know the teachers' thoughts and their basis for making probabilistic and didactic decision. Gandhi also asked to the 31 teachers who agreed to use this device or were undecided about its use to explain how they will use the tetrahedron in the teaching of probability and whether they prefer it to the cube or not.

In the analysis of the responses, the author focused on knowing how these teachers make transitions between the classical, empirical, and propensity interpretations of probability. Following and interpretative paradigm, she investigated the teachers' prevalent epistemic considerations as they worked with a resource that has characteristics similar to a commonly used object to teach randomness, the cube. The teachers' description of their pedagogic strategies related to an unfamiliar object (in teaching probability), helped the author to uncover the teachers' dispositions to the perceptual properties, structural aspects of the random generator, and to other externally generating conditions as determinants for their probabilistic reasonings. The author concluded that, although the classical and frequentist views of probability dominate the curriculum and textbooks, the propensity view (Gillies, 2000) tends to serve as a prelude to the other views of probability. The author discusses the difficulty of considering the propensity view in the probability curriculum when trying to interpret randomness. In summary, this paper reflects on the need to consider and relate the different views of probability and how the teachers need to be conscious of the relationships and complementarity of these different views and the advantages and difficulties involved in each of them.

The main focus of the paper by Salinas-Herrera and Salinas Hernández is the teaching of the normal distribution, a complex topic which has a fundamental role in probability and inference (Batanero et al., 2004a, b). Salinas-Herrera and Salinas-Hernández describe an experience with 18 high school students who were introduced to the convergence of the binomial distribution to the normal distribution using the Fathom simulation capabilities. In previous courses, the students had acquired formal knowledge of the normal and binomial distributions. The authors explain that the students were weak in mathematics and elected statistics and probability because they assumed this topic to be easy.

The authors report how six mathematics teachers worked together to design and analyse the teaching experience for the students, following the Lesson Study method (Murata, 2011). The investigation used the Documentational Approach to Didactics (DAD) (Trouche et al., 2020) theoretical framework. The perspective of this model is to study how teachers interact (design, use, adapt, and modify the lesson) with curriculum and other resources during their practice. The analysis also applied the concept of a scheme (Vergnaud, 2011). This concept organizes observable behaviour and the underlying thinking activity.

The authors present the analysis of the first teaching session. A conclusion is that the students engaged best with a situation that was familiar to them, both experientially and conceptually. The simulation with the software also facilitated the students' learning. Most students correctly interpreted the simulation results and used them to solve a probability problem. However, the relation between the problem posed and the simulation model (constructed by the teacher) was not evident for some students. Not all the students accepted the convergence of the binomial towards the normal distribution when the number of trials increases. The law of large numbers did not appear spontaneously, and the teacher explicitly should introduce this theorem after the simulation.

The teacher studied with his colleagues the development of each session. Before teaching, some teachers were unfamiliar with the software and sceptical about its role in the specific theme. This view

changed in part of the teachers after observing and analysing the lessons. Consequently, the use of the Lesson Study method provided a space for discussion and enrichment for these teachers.

Final Reflections

The four papers analysed in the previous section reflect the variety of approaches with which research in probability or statistics education is carried out nowadays by different researchers. As a synthesis of the reading of these four papers, a first reflection is that they represent the diversity and richness of the education of teachers to teach probability as a research topic. We observe part of the numerous theoretical frameworks that sustain research in mathematics education and a variety of methods, such as Lesson Study, interviews with a small sample of teachers selected after a questionnaire applied to a wide sample, analysis of solutions to mathematical problems, or interviews about how to use a random generator.

Some of the papers focused on the teachers' knowledge and their educational needs in specific topics, such as randomness, solving probability problems, the normal distribution, and how this topic is facilitated by manipulatives or technological support. The Canadian researchers were interested in a more general view of the teaching practices of teachers and the beliefs that supported these practices.

The papers deal with different components of the CCDM (Godino et al., 2017): common and advanced mathematical knowledge of probability and randomness (Gandhi; Machuca), mediational facet of the teachers' didactic knowledge (Gandhi; Martin et al.; Salinas & Salinas), and ecological facet (perception of the role of statistics in society; Martin et al.). Some competences in the model are also investigated. For example, Salinas and Salinas analyse the teachers' competence to plan, manage, and perform a retrospective analysis of the teaching of the normal distribution; Martin et al. investigate the competence to recognise and connect the different meanings of probability; and Gandhi related to the use of non-usual random generators to introduce probability in the classroom.

In summary, this small sample of papers is some way representative of the research on teachers' education to teach probability. Of course, there is still much work to do to investigate the teachers' knowledge and competence to teach probability and about their attitudes towards probability and its teaching (Estrada & Batanero, 2019). For example, more research is needed on the cognitive and affective facets of teacher's didactic knowledge, that is, the teachers' knowledge of the students' learning, reasoning, and strategies when working with probability and on the teacher's knowledge of students' attitudes and beliefs about probability. More research is also needed on the competence to identify the diversity of objects and processes involved in the tasks that the teachers propose to their students, and on the competence to assess the didactic suitability of instructional processes. Hopefully, the papers analysed encourage other researchers to pursue this important problem area.

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Declarations

Conflict of Interest The author declares no competing interests.

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