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Every commutative JB*-triple satisfies the complex Mazur–Ulam property

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Abstract

We prove that every commutative JB*-triple, represented as a space of continuous functions $C_0^{\mathbb{T}}(L)$, satisfies the complex Mazur–Ulam property, that is, every surjective isometry from the unit sphere of $C_0^{\mathbb{T}}(L)$ onto the unit sphere of any complex Banach space admits an extension to a surjective real linear isometry between the spaces.

Keywords Isometry · Tingley's problem · Mazur–Ulam property · Abelian JB*-triples

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1 Introduction

New recent advances continue improving our understanding of Tingley's problem by enlarging the list of positive solutions, and the range of spaces satisfying the Mazur–Ulam property. As introduced in [8], a Banach space X satisfies the Mazur–Ulam property if every surjective isometry from its unit sphere onto the unit sphere of any other Banach space admits an extension to a surjective real linear isometry between the spaces. It is worth to note that this property was previously considered by Ding in [10] without an explicit name (see also [23, page 730]). A remarkable outstanding discovering has been obtained by Banakh in [1], who has proved that every 2-dimensional Banach space X satisfies the *Mazur–Ulam property*. This is, in fact, the culminating point of deep technical advances (see [2, 6]).

The abundance of unitary elements in unital C*-algebras, real von Neumann algebras and JBW*-algebras is a key property to prove that these spaces together with all JBW*-triples satisfy the Mazur–Ulam property (cf. [3, 13, 16]). A prototypical example of non-unital C*-algebra is given by the C*-algebra $K(H)$, of all compact operators on an infinite dimensional complex Hilbert space H, or more generally, by a compact C*-algebra (i.e., a c_0-sum of $K(H)$-spaces). Compact C*-algebras and weakly compact JBW*-triples are in the list of Banach spaces satisfying the Mazur–Ulam property (see [18]).

Tingley's problem is also studied in the case of certain function algebras and spaces. The first positive solution to Tingley's problem for a Banach space consisting of analytic functions, apart from Hilbert spaces, was obtained by Hatori et al. in [12], where a proof is given for any surjective isometry between the unit spheres of two uniform algebras (i.e., closed subalgebras of $C(K)$ containing the constants and separating the points of K). Hatori has gone further by showing that every uniform algebra satisfies the *complex Mazur–Ulam property*, i.e., every surjective isometry from its unit sphere onto the unit sphere of any complex Banach spaces admits an extension to a real linear mapping between the spaces [11, Theorem 4.5].

The non-unital analogue of uniform algebras is materialized in the notion of uniformly closed function algebra on a locally compact Hausdorff space L. We recently showed that each surjective isometry between the unit spheres of two uniformly closed function algebras on locally compact Hausdorff spaces admits an extension to a surjective real linear isometry between these algebras (see [9]). In the just quoted reference we also proved that Tingley's problem admits a positive solution for any surjective isometry between the unit spheres of two commutative JB*-triples, which are not, in general, subalgebras of the algebra $C_0(L)$ of all complex-valued continuous functions on L vanishing at infinity (see Sect. 3 for details). In this note we shall employ a recent tool developed by Hatori in [11] to infer that a stronger conclusion holds, namely, every commutative JB*-triple satisfies the complex Mazur–Ulam property. Among the consequences we derive that every commutative C*-algebra enjoys the complex Mazur–Ulam property.

2 Preliminaries

We shall briefly recall some basic terminology to understand the sufficient condition in [11, Proposition 4.4] to guarantee that a Banach space satisfies the complex Mazur–Ulam property. Let X be a real or complex Banach space, and let X^*, $S(X)$ and $\mathcal{B}_X$ denote the dual space, the unit sphere and the closed unit ball of X, respectively. It is known, thanks to Hahn–Banach theorem or Eidelheit's separation theorem, that maximal convex subsets of $S(X)$ and maximal proper norm closed faces of $\mathcal{B}_X$ define the same subsets (cf. [21, Lemma 3.3] or [22, Lemma 3.2]). The set of all maximal convex subsets of $S(X)$, equivalently, all maximal proper norm closed faces of $\mathcal{B}_X$, will be denoted by $\mathfrak{F}_X$. For each $F \in \mathfrak{F}_X$ there exists an extreme point φ of the closed unit ball $\mathcal{B}_{X^*}$ such that $F = \varphi^{-1}\{1\} \cap S(X)$ (cf. [21, Lemma 3.3]). The set of all extreme points φ of $\mathcal{B}_{X^*}$ for which $\varphi^{-1}\{1\} \cap S(X)$ is a maximal convex subset of $S(X)$ will be denoted by $\mathcal{Q}_X$. On the latter set we consider the equivalence relation defined by

$$\varphi \sim \phi \Leftrightarrow \exists \gamma \in \mathbb{T} = S(\mathbb{K}) \text{ with } \varphi^{-1}\{1\} \cap S(X) = (\gamma\phi)^{-1}\{1\} \cap S(X),$$

where $\mathbb{K} = \mathbb{R}$ if X is a real Banach space and $\mathbb{K} = \mathbb{C}$ if X is a complex Banach space. A *set of representatives* for the quotient set $\mathcal{Q}_X/\sim$ (or for $\mathfrak{F}_X$) will consist in a subset $\mathbf{P}_X$ of $\mathcal{Q}_X$ which is formed by precisely one, and only one, element in each equivalence class of $\mathcal{Q}_X/\sim$. According to this notation, for each $F \in \mathfrak{F}_X$ there exists a unique $\varphi \in \mathbf{P}_X$ and $\gamma \in \mathbb{T}$ such that $F = F_{\varphi,\gamma} := \{x \in S(X) : \varphi(x) = \gamma\}$ (cf. [11, Lemma 2.5]), that is, the elements in $\mathfrak{F}_X$ are bijectively labelled by the set $\mathbf{P}_X \times \mathbb{T}$, and we can define a bijection $\mathcal{I}_X : \mathfrak{F}_X \to \mathbf{P}_X \times \mathbb{T}$ labelling the set $\mathfrak{F}_X$.

For example, by the classical description of the extreme points of the closed unit ball of the dual of a $C(K)$ space as those functionals of the form $\lambda\delta_t(f) = \lambda f(t)$ ($f \in C(K)$) with $t \in K$, $\lambda \in \mathbb{T}$, the set $\mathbf{P}_{C(K)} = \{\delta_t : t \in K\}$ is a set of representatives for $\mathfrak{F}_{C(K)}$. It is shown in [11, Example 2.4] that for a uniform algebra A over a compact Hausdorff space K, the set $\{\delta_t : t \in \mathrm{Ch}(A)\}$ is a set of representatives for A, where $\mathrm{Ch}(A)$ denotes the Choquet boundary of A.

Let A, B be non-empty closed subsets of a metric space (E, d). The usual Hausdorff distance between A and B is defined by

$$d_H(A,B) = \max\left\{ \sup_{a\in A} d(a,B), \sup_{b\in B} d(b,A) \right\}.$$

We shall employ this Hausdorff distance to measure distances between elements in $\mathfrak{F}_X$.

According to [11], a Banach space X satisfies the *condition of the Hausdorff distance* if the elements in $\mathfrak{F}_X$ satisfy the following rules:

$$d_H(F_{\varphi,\lambda}, F_{\varphi',\lambda'}) = \begin{cases} |\lambda - \gamma\lambda'|, & \text{if } \varphi^{-1}\{1\} \cap S(X) = (\gamma\varphi')^{-1}\{1\} \cap S(X), \\ 2, & \text{if } \varphi \nsim \varphi'. \end{cases} \tag{1}$$

for $\varphi, \varphi' \in \mathcal{Q}_X$ and $\lambda, \lambda' \in \mathbb{T}$. Let $\mathbf{P}_X \subset \mathcal{Q}_X$ be a set of representatives for $\mathfrak{F}_X$.

Under the light of [11, Lemma 3.1] to conclude that a complex Banach space X together with a set of representatives $\mathbf{P}_X$ satisfies the condition of the Hausdorff distance, it suffices to prove that

$$F_{\varphi,\lambda} \cap F_{\varphi',\lambda'} \neq \emptyset \quad \text{for any } \varphi \neq \varphi' \text{ in } \mathbf{P}_X, \lambda, \lambda' \text{ in } \mathbb{T}. \tag{2}$$

Let us go back to the set $\mathcal{Q}_X$ determining the set $\mathfrak{F}_X$ of all maximal proper norm closed faces of $\mathcal{B}_X$. For $\varphi \in \mathcal{Q}_X$ and $\alpha \in \overline{\mathbb{D}} = \mathcal{B}_{\mathbb{K}}$ ($\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$), we set

$$M_{\varphi,\alpha} = \left\{ x \in S(X) : d\left(x, F_{\varphi, \frac{\alpha}{|\alpha|}}\right) \leq 1 - |\alpha|, \ d\left(x, F_{\varphi, -\frac{\alpha}{|\alpha|}}\right) \leq 1 + |\alpha| \right\},$$

where $\frac{\alpha}{|\alpha|} = 1$ if $\alpha = 0$. It is known that for each φ in a set of representatives $\mathbf{P}_X$, the inclusion

$$M_{\varphi,\alpha} \subseteq \{x \in S(X) : \varphi(x) = \alpha\}$$

holds for all $\alpha \in \overline{\mathbb{D}}$ (cf. [11, Lemma 4.3]). O. Hatori has recently established that a complex Banach space X, together with a set of representatives $\mathbf{P}_X$ for $\mathfrak{F}_X$, satisfying the condition of the Hausdorff distance and the equality:

$$M_{\varphi,\alpha} = \{x \in S(X) : \varphi(x) = \alpha\},$$

for each φ in $\mathbf{P}_X$ and $\alpha \in \overline{\mathbb{D}}$, satisfies the complex Mazur–Ulam property (cf. [11, Proposition 4.4]).

3 The complex Mazur–Ulam property for commutative JB*-triples

We shall avoid the axiomatic definition of commutative JB*-triples and we shall simply recall their representation as function spaces. By the Gelfand theory for JB* -triples (see [14, Corollary 1.11]), each abelian JB*-triple can be identified with the norm closed subspace of $C_0(L)$ defined by

$$C_0^{\mathbb{T}}(L) := \{a \in C_0(L) : a(\lambda t) = \lambda a(t) \text{ for every } (\lambda, t) \in \mathbb{T} \times L\},$$

where L is a principal $\mathbb{T}$-bundle, that is, a subset of a Hausdorff locally convex complex space such that $0 \notin L$, $L \cup \{0\}$ is compact, and $\mathbb{T}L = L$ (see also [7, §4.2.1] or [5, 9]).

We can state next the main result of the paper.

Theorem 3.1 *Let L be a principal $\mathbb{T}$-bundle. Then, $C_0^{\mathbb{T}}(L)$ satisfies the complex Mazur–Ulam property, that is, for each complex Banach space X, every surjective isometry $\Delta : S(C_0^{\mathbb{T}}(L)) \to S(X)$ admits an extension to a surjective real linear isometry $T : C_0^{\mathbb{T}}(L) \to X$.*

The proof will be obtained after a series of technical results via [11, Proposition 4.4].

Although for each locally compact space $\tilde{L}$, the Banach space $C_0(\tilde{L})$ is isometrically isomorphic to a $C_0^{\mathbb{T}}(L)$ space (cf. [17, Proposition 10]), there exist principal $\mathbb{T}$-bundles L for which the space $C_0^{\mathbb{T}}(L)$ is not isometrically isomorphic to a $C_0(L)$ space (cf. [14, Corollary 1.13 and subsequent comments]). Therefore, there exist abelian JB*-triples which are not isometrically isomorphic to commutative C*-algebras. The next corollary is a weaker consequence of our previous theorem.

Corollary 3.2 *Every abelian C*-algebra (that is, every $C_0(\tilde{L})$ space) satisfies the complex Mazur–Ulam property.*

Compared with previous results, we observe that as a consequence of the result proved by Hatori for uniform algebras in [11, Theorem 4.5] every unital abelian C* -algebra satisfies the complex Mazur–Ulam property. Actually, all unital C*-algebras enjoy the Mazur–Ulam property [16]. In the case of real-valued continuous functions, Liu proved that for each compact Hausdorff space K, $C(K,\mathbb{R})$ satisfies the Mazur–Ulam property (see [15, Corollary 6]).

Let $\tilde{L}$ be a locally compact Hausdorff space. A closed subspace E of $C_0(\tilde{L},\mathbb{K})$, separates the points of $\tilde{L}$ if for any $t_1 \neq t_2$ in $\tilde{L}$ there exists a function $a \in E$ such that $a(t_1) \neq a(t_2)$. Following [11], we shall say that E satisfies the condition (r) if for any t in the Choquet boundary of E, each neighborhood V of t, and $\varepsilon > 0$ there exists $u \in E$ such that $0 \leq u \leq 1 = u(t)$ on $\tilde{L}$ and $0 \leq u \leq \varepsilon$ on $\tilde{L}\backslash V$. The proof of Corollary 5.4 in the preprint version of [11] (see arXiv:2017.01515) affirms that each closed subspace E of $C_0(\tilde{L},\mathbb{R})$ separating the points of $\tilde{L}$ and satisfying a stronger assumption than condition (r) has the Mazur–Ulam property. After some private communications with O. Hatori we actually learned that property (r) is enough to conclude that any such closed subspace E satisfies the Mazur–Ulam property. Actually the desired conclusion can be derived from [4, Theorem 2.4] by just observing that condition (r) implies that the isometric identification of E in $C_0\left(\overline{Ch(E)}\right)$ is C-rich, and hence a lush space. Corollary 3.9 in [20] implies that E has the Mazur–Ulam property.

We focus now on the main goal of this section. Henceforth, let L be a principal $\mathbb{T}$ -bundle and $L_0 \subset L$ a maximal non-overlapping set, that is, L_0 is maximal satisfying that for each $t \in L_0$ we have $L_0 \cap \mathbb{T}t = \{t\}$ (its existence is guaranteed by Zorn's lemma).

Assume that a Banach space Y satisfies the following property: for every extreme point $\varphi \in \partial_e(\mathcal{B}_{Y^*})$, the set $\{\varphi\}$ is a weak*-semi-exposed face of $\mathcal{B}_{Y^*}$. It is clear that each extreme point $\varphi \in \partial_e(\mathcal{B}_{Y^*})$ is determined by the set $\{\varphi\}_, = \varphi^{-1}(1) \cap S(Y)$. Hence, the equivalence relation $\sim$ defined in Sect. 2 (cf. [11, Definition 2.1]) can be characterized in the following terms: for $\varphi, \psi \in \partial_e(\mathcal{B}_{Y^*})$, we have $\varphi \sim \psi \Leftrightarrow \varphi = \gamma\psi$ for some $\gamma \in \mathbb{T}$. Since $Y = C_0^{\mathbb{T}}(L)$ satisfies the mentioned property, the set $\{\delta_t : t \in L_0\}$ is a set of representatives for the relation $\sim$. We know that each maximal proper face F of the closed unit ball of $C_0^{\mathbb{T}}(L)$ is of the form:

$$F = F_{\delta_{t_0},\lambda} = F_{t_0,\lambda} := \{a \in S(C_0^{\mathbb{T}}(L)) : \delta_{t_0}(a) = a(t_0) = \lambda\}$$

for some $(t_0, \lambda) \in L_0 \times \mathbb{T}$ (cf. [9, Lemma 3.5]).

We can now begin with the technical details for our arguments.

Lemma 3.3 *If $t_1 \neq t_2$ in L_0, then there exist open $\mathbb{T}$-symmetric subsets $V_1, V_2 \subset L$ satisfying: $\mathbb{T}t_j \subset V_j$, $\overline{V_j}$ is compact for $j = 1, 2$, and $V_1 \cap V_2 = \overline{V_1} \cap \overline{V_2} = \emptyset$.*

Proof Since L_0 is non-overlapping, we know that $\mathbb{T}t_1$ and $\mathbb{T}t_2$ are disjoint compact subsets of L. Hence $\mathbb{T}t_1 \subset L\backslash\mathbb{T}t_2$, where $L\backslash\mathbb{T}t_2$ is $\mathbb{T}$-symmetric and open. By a basic topological argument (cf. [19, Theorem 2.7] whose $\mathbb{T}$-symmetric version remains true), there exists a $\mathbb{T}$-symmetric open set $V_1 \subset L$ with $\mathbb{T}$-symmetric compact closure satisfying $\mathbb{T}t_1 \subset V_1 \subset \overline{V_1} \subset L\backslash\mathbb{T}t_2$. Now, having in mind that $\overline{V_1}$ is $\mathbb{T}$-symmetric and compact with $\overline{V_1} \cap \mathbb{T}t_2 = \emptyset$, we deduce that $L\backslash\overline{V_1}$ is an open $\mathbb{T}$-symmetric set containing $\mathbb{T}t_2$. We can find another open $\mathbb{T}$-symmetric subset V_2 with $\mathbb{T}$-symmetric compact closure such that $\mathbb{T}t_2 \subset V_2 \subset \overline{V_2} \subset L\backslash\overline{V_1}$. $\square$

Corollary 3.4 *Let $t_1 \neq t_2$ in L_0 and $\lambda_1, \lambda_2 \in \mathbb{T}$. Then, there exist a function $a \in S(C_0^{\mathbb{T}}(L))$ such that $a(t_j) = \lambda_j$ for $j = 1, 2$.*

Proof Lemma 3.3 assures the existence of disjoint open $\mathbb{T}$-symmetric neighbourhoods with compact closure W_1 and W_2 of t_1 and t_2, respectively. By [9, Remark 3.4], there exist functions $a_1, a_2 \in S(C_0^{\mathbb{T}}(L))$ such that $a_j(t_j) = 1$ and $a_j|_{L\backslash W_j} \equiv 0$ for $j = 1, 2$. The function $a := \lambda_1 a_1 + \lambda_2 a_2$ satisfies the desired properties by the disjointness of W_1 and W_2. $\square$

***Remark* 3.5** The previous corollary shows that $F_{t_1,\lambda} \cap F_{t_2,\lambda'} \neq \emptyset$ for any $t_1 \neq t_2$ in L_0 and $\lambda, \lambda' \in \mathbb{T}$. Therefore, the space $C_0^{\mathbb{T}}(L)$ satisfies (2) and hence the condition of the Hausdorff distance (cf. (1) and (2) or [11, Lemma 3.1]).

We shall next show that $C_0^{\mathbb{T}}(L)$ satisfies the second hypothesis in [11, Proposition 4.4].

Proposition 3.6 *The identity $M_{t_0,\alpha} = M_{\delta_{t_0},\alpha} = \{a \in S(C_0^{\mathbb{T}}(L)) : a(t_0) = \alpha\}$ holds for every $t_0 \in L_0$ and $\alpha \in \overline{\mathbb{D}}$.*

Proof We only need to show the inclusion $\supseteq$, because, as we commented above, the reciprocal content always holds. Take any $a \in S(C_0^{\mathbb{T}}(L))$ such that $a(t_0) = \alpha$. We shall discuss first the case where $|\alpha| = 1$. In such a case we have $a \in F_{t_0,\frac{\alpha}{|\alpha|}} = F_{t_0,\alpha}$ and $-a \in F_{t_0,-\frac{\alpha}{|\alpha|}} = F_{t_0,-\alpha}$. Thus, $d(a, F_{t_0,\frac{\alpha}{|\alpha|}}) \leq d(a, a) = 0 = 1 - |\alpha|$ and $d(a, F_{t_0,-\frac{\alpha}{|\alpha|}}) \leq d(a, -a) = 2 = 1 + |\alpha|$.

We assume next that $\alpha = 0$. For each $\varepsilon > 0$ we can find an open neighbourhood U_ε of t_0 and an element $a_\varepsilon \in S(C_0^{\mathbb{T}}(L))$ such that $\|a - a_\varepsilon\| < \varepsilon$ and $a_\varepsilon|_{U_\varepsilon} \equiv 0$. By applying a basic topological argument, we may assume that U_ε is $\mathbb{T}$-symmetric and has compact $\mathbb{T}$-symmetric closure. Then, by [9, Remark 3.4], there exists a function $b_\varepsilon \in F_{t_0,1}$ (and thus $-b_\varepsilon \in F_{t_0,-1}$) with $b_{\varepsilon}|_{L\setminus U_\varepsilon} \equiv 0$. By combining these conclusions we have

$$d(a, F_{t_0,\pm 1}) \leq \|a \mp b_\varepsilon\| \leq \|a - a_\varepsilon\| + \|a_\varepsilon \mp b_\varepsilon\| < \varepsilon + \max\{\|b_\varepsilon\|, \|a_\varepsilon\|\} \leq 1 + \varepsilon.$$

Therefore, $d(a, F_{t_0,\pm 1}) \leq 1 = 1 \mp |\alpha|$.

We finally assume that $0 < |\alpha| = |a(t_0)| < 1$. Take two continuous functions $h_1, h_2 : [0,1] \to [-1,1]$ such that $h_1(0) = h_2(0) = 0$, $h_1(1) = h_2(1) = 1$, $h_1(|\alpha|) = 1$, $h_2(|\alpha|) = -1$, and both are affine on the intervals $[0, |\alpha|]$ and $[|\alpha|, 1]$. Consider the two functions $a_j : L \to \mathbb{C}$, $j = 1, 2$, defined by

$$a_j(s) = \begin{cases} 0, & \text{if } a(s) = 0, \\ \frac{a(s)}{|a(s)|}\, h_j\big(|a(s)|\big), & \text{if } a(s) \neq 0. \end{cases}$$

It is not hard to check that $a_j \in S(C_0^{\mathbb{T}}(L))$; furthermore, $a_1 \in F_{t_0, \frac{\alpha}{|\alpha|}}$ and $a_2 \in F_{t_0, -\frac{\alpha}{|\alpha|}}$. Take now any $s \in L$ such that $a(s) \neq 0$. Then we have

$$\begin{aligned} |a(s) - a_j(s)| &= \left| a(s) - \frac{a(s)}{|a(s)|}\, h_j\big(|a(s)|\big) \right| = |a(s)| \left| 1 - \frac{h_j\big(|a(s)|\big)}{|a(s)|} \right| \\ &= \Big| |a(s)| - h_j\big(|a(s)|\big) \Big| \leq 1 + (-1)^j |\alpha| \quad (j = 1, 2), \end{aligned}$$

and clearly the equality $|a(s) - a_j(s)| = \Big| |a(s)| - h_j\big(|a(s)|\big) \Big|$ also holds when $a(s) = 0$. We, therefore, have $d(a, a_j) \leq 1 + (-1)^j |\alpha|$ $(j = 1, 2)$, which implies that $a \in M_{t_0,\alpha}$. □

***Proof of Theorem* 3.1** Remark 3.5 and Proposition 3.6 guarantee that $C_0^{\mathbb{T}}(L)$ satisfies the hypotheses in [11, Proposition 4.4] for the set of representatives given by L_0, and the just quoted proposition gives the desired conclusion. □

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