



# Baryonic content of the pion

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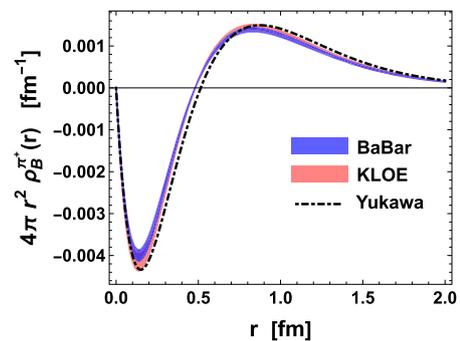
## ABSTRACT

The baryon form factor of charged pions arises since isospin symmetry is broken. We obtain estimates for this basic property in two phenomenological ways: from simple constituent quark models with unequal up and down quark masses, and from fitting to  $e^+e^- \rightarrow \pi^+\pi^-$  data. All our methods yield a positive  $\pi^+$  baryon mean square radius of  $(0.03 - 0.04 \text{ fm})^2$ . Hence, a picture emerges where the outer region has a net baryon, and the inner region a net antibaryon density, both compensating each other such that the total baryon number is zero. For  $\pi^-$  the effect is opposite.

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## 1. Introduction

Since its prediction and subsequent discovery, the pion has been scrupulously investigated as the basic lightest hadron and the pseudo-Goldstone boson of the dynamically broken chiral symmetry. Many of its electroweak and mechanical properties have been studied and determined both experimentally [1] as well as theoretically from a first principles point of view, with notable recent advances from lattice QCD. In this Letter, we draw attention to the *baryonic* structure of the charged pions,  $\pi^+$  and  $\pi^-$ , a remarkable property which, to the best of our knowledge, has not been studied in an explicit manner before. Despite being a zero baryon number state, the composition of the charged pion is not baryonless. We show that simple quark models and data analyses imply a characteristic pattern where the matter and antimatter radial distributions are separated at a distance of  $r \sim 0.5 \text{ fm}$ . For  $\pi^+$ , the inner (outer) region carries a net antibaryon (baryon) density, and opposite for  $\pi^-$  (cf. Fig. 1). The situation is reminiscent of the well-known case of the electric form factor of the neutron which carries no charge, nevertheless possesses a non-zero electric form factor, such that (in the Breit frame) the inner (outer) region has positive (negative) charge density, with the mean squared radius (msr)  $\langle r^2 \rangle_Q^n = -0.1161(22) \text{ fm}^2$ . Similarly, the neutral Kaon  $K^0$  has  $\langle r^2 \rangle_Q^{K^0} = -0.077(10) \text{ fm}^2$  [1] despite its null charge. Even more



**Fig. 1.** The radial baryonic charge distribution in the  $\pi^+$ . The bands correspond to our extraction from the data, and the line to the toy Yukawa model. See the text for details.

striking, the nucleon is known to have a nonvanishing strange form factor despite being strangeless [2,3].

## 2. The baryonic form factor of the pion

### 2.1. Current conservation

To see how the effect arises, let us first recall for completeness some very basic facts. In QCD, one has the conservation laws

$$\partial_\mu [\bar{q}_a(x) \gamma^\mu q_b(x)] = i(m_a - m_b) \bar{q}_a(x) q_b(x), \quad (1)$$

with  $q_j(x)$  denoting the quark field with  $N_c$  colors and flavor  $j = u, d, s, c, b, t$ , which for equal quark masses corresponds to the

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conservation of the vector current and ensures the quark number conservation for any species. For the specific case of the pion, we neglect  $s$  and heavier flavors, as they represent corrections suppressed by the Okubo-Zweig-Iizuka (OZI) rule and are subleading in the large- $N_c$  limit of QCD. In this case the baryon current (isosinglet) and the third isospin component of the isovector current are

$$J_B^\mu = \frac{1}{N_c} (\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d), \quad J_3^\mu = \frac{1}{2} (\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d), \quad (2)$$

where  $N_c \equiv 3$  is assumed in the following. With these definitions, the Gell-Mann–Nishijima formula provides the electromagnetic current  $J_Q^\mu = J_3^\mu + \frac{1}{2}J_B^\mu$ . The baryon, isospin, and charge form factors are defined via the on-shell matrix elements of the corresponding currents, namely

$$\langle \pi^a(p) | J_{B,3,Q}^\mu(0) | \pi^a(p+q) \rangle = (2p^\mu + q^\mu) F_{B,3,Q}^a(q^2), \quad (3)$$

$$\text{with } F_Q^a(q^2) = F_3^a(q^2) + \frac{1}{2}F_B^a(q^2).$$

## 2.2. Charge conjugation and isospin violation

Now come the standard symmetry arguments. Since  $J_B^\mu$  is odd under charge conjugation  $C$ , it implies that for the  $C$ -even neutral pion  $F_B^{\pi^0}(q^2) = 0$  identically, while for the charged pions it provides the relation  $F_B^{\pi^+}(q^2) = -F_B^{\pi^-}(q^2)$ . Similarly, for the case of an exact isospin symmetry, i.e., with  $m_u = m_d$  and neglecting small electromagnetic effects,  $G$ -parity symmetry yields  $F_B^{\pi^\pm}(q^2) = 0$ . However, this is no longer the case in the real world where the isospin is broken.  $G$ -parity ceases to be a good symmetry and  $F_B^{\pi^\pm}(q^2)$  may be – and in fact is – non-zero, with  $F_B^{\pi^+}(q^2) = -F_B^{\pi^-}(q^2) \neq 0$ . Moreover, if we take (say, for  $\pi^+$ )  $Z\bar{d}i\gamma_5 u$  as an interpolating field, then a direct application of the Ward-Takahashi-Green identities for the conserved  $B$  and  $Q$  currents implies (for the canonical pion field)  $F_B^{\pi^\pm}(0) = 0$ ,  $F_Q^{\pi^\pm}(0) = \pm 1$ .

## 2.3. Coordinate space interpretation

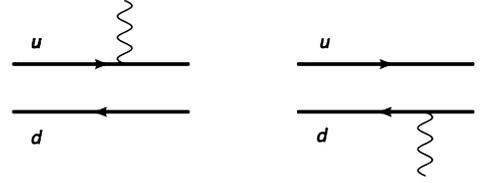
A popular interpretation of form factors is based on choosing the Breit reference frame, where there is no energy transfer. Then the form factor in the space-like region  $q^2 = -\vec{q}^2 \equiv -Q^2 \leq 0$  allows one to construct the (naive) 3-dimensional baryon density as

$$\rho_B(r) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} F_B(-\vec{q}^2). \quad (4)$$

As ambiguities stemming from relativity arise [4,5], it has been argued that a frame-independent interpretation can be formulated in terms of a transverse density in the 2-dimensional impact-parameter  $\vec{b}$  [6], where instead of Eq. (4) one takes the Fourier integral with  $\exp(i\vec{q}_\perp \cdot \vec{b})$  (see, e.g., [7] for a review and Ref. [8]). Here, for our illustrative purpose, we choose to show the  $r$ -space densities, as the  $b$ -space results are simply related and qualitatively the same.

We have no obvious sources coupling the charged pions solely to the baryon current, hence a direct experimental measurement of  $F_B^{\pi^\pm}(q^2)$  is not possible. This is also in common with the neutron electric form factor, where a direct determination is hampered by the absence of free neutron targets and its extraction requires scattering on bound neutrons in the deuteron.<sup>1</sup> The analysis requires

<sup>1</sup> Remarkably, the earliest determinations of the neutron radius were promoted by Fermi and Marshall in 1947 [9] from looking at ultracold neutron - atom scattering transmission experiments. These involve the total but not the differential cross section (see also PDG [1]), hence precluding a form factor determination.



**Fig. 2.** Diagrams for evaluating the form factors of  $\pi^+$  in the impulse approximation. With postulated Yukawa wave functions we get the VMD model in the isovector channel.

an accurate deuteron wave function as well as meson exchange current effects [10], hence the need for additional theoretical input. Returning to the novel case of the so far disregarded pion baryonic form factor, we will content ourselves with rather unsophisticated but complementary and realistic estimates. They are based on dimensional analysis, quark models, and an extraction from  $e^+e^- \rightarrow \pi^+\pi^-$  data analysis. Within uncertainties, our results are consistent.

## 3. Model estimates

### 3.1. Dimensional analysis

A generic order of magnitude estimate of the discussed isospin violating effect can be obtained at the leading order in the pion momenta and the quark mass splitting  $\Delta m \equiv m_d - m_u = 2.8(2)$  MeV (in this work we use  $m_u = 2.01(14)$  MeV and  $m_d = 4.79(16)$  MeV [11]). We expect the two-pion contribution to the baryon current to be of the effective form

$$J_B^\mu = -2i \frac{c\Delta m}{\Lambda^3} \partial_\nu (\partial^\mu \pi^+ \partial^\nu \pi^- - \partial^\nu \pi^+ \partial^\mu \pi^-) + \dots, \quad (5)$$

with  $c$  an undetermined dimensionless number and  $\Lambda$  a typical low energy hadronic scale (say,  $m_\rho \sim 770$  MeV). As it should, this current is odd under  $C$ , is trivially conserved, and its contribution vanishes for  $q^2 = 0$ , providing the form factor  $F_B^{\pi^+}(q^2) = q^2 c \Delta m / \Lambda^3 + \dots$ , with  $\text{msr } \langle r^2 \rangle_B^{\pi^+} = 6c \Delta m / m_\rho^3 \simeq c 0.002 \text{ fm}^2 \simeq c(0.04 \text{ fm})^2$ , a small number compared to the electric charge radius  $\langle r^2 \rangle_Q^{\pi^+} = (0.659(4) \text{ fm})^2 = 0.434(5) \text{ fm}^2$  [1].

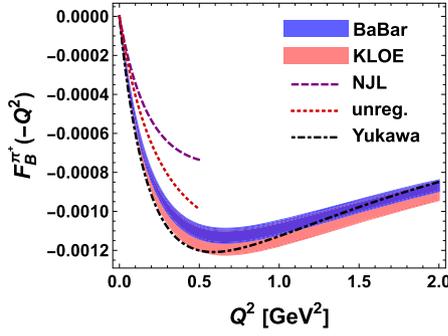
One may seek further guidance in Chiral Perturbation Theory. In particular, the term in Eq. (5) arises starting from the  $\mathcal{O}(p^6)$  chiral Lagrangian [12],

$$\mathcal{L}_{63+65}^{(6)} = \frac{i}{F_2^2} \langle f_{+\mu\nu} (C_{63} \{ \chi_+, u^\mu u^\nu \} + C_{65} u^\mu \chi_+ u^\nu) \rangle, \quad (6)$$

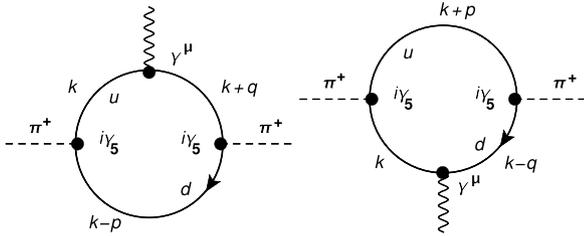
from where we find an explicit relation  $c/\Lambda^3 = \frac{8B_0}{N_c F^4} (2C_{63} - C_{65})$ . It involves two  $C_i$  coefficients, for which currently there are no independent estimates. The naturalness condition yields  $C_i F_\pi^{-4} \sim m_\rho^{-4}$ , hence  $c \sim 1$ , as previously argued.

### 3.2. Yukawa quark model

Next, we come to our quark model estimates for  $\Delta m \neq 0$  effects. To start, we explore the fact that the coordinate representation motivates a toy constituent-quark model based on the familiar *impulse approximation* in nuclear physics (cf. Fig. 2). In this framework  $\rho_B(r) = B_u |\Psi_u(\vec{x})|^2 + B_d |\Psi_d(\vec{x})|^2$ , where the quark baryon numbers are  $B_u = -B_d = \frac{1}{N_c}$ . Motivated by Vector Meson Dominance model (VMD), that provides a reasonable description for the isovector channel,  $F_3^{\pi^+}(q^2) = M_\rho^2 / (M_\rho^2 - q^2)$  (and allows for a simple extension to other pseudoscalar mesons), we take the normalized Yukawa-like probabilities  $|\Psi_i(\vec{x})|^2 = M_i^2 e^{-2M_i r} / (\pi r)$ , with  $M_{u,d} = M \mp \frac{1}{2} \Delta m$  and  $M$  denoting the constituent quark mass. This



**Fig. 3.** The baryon form factor of  $\pi^+$  in the space-like region. The bands correspond to the extraction from the data, and the lines to various models described in the text.



**Fig. 4.** Feynman diagrams for evaluating the baryon form factor of  $\pi^+$  in the NJL model.

ensures that, for  $\Delta m = 0$ , the resulting isovector and charge form factors  $F_{Q,3}^{\pi^+}(-Q^2) = 4M^2/(4M^2 + Q^2)$  reproduce the VMD phenomenology provided we take  $M = \frac{1}{2}m_\rho \simeq 385$  MeV, while for the baryon form factor we find

$$F_B^{\pi^+}(-Q^2) = \frac{1}{N_c} \left[ \frac{4M_u^2}{4M_u^2 + Q^2} - \frac{4M_d^2}{4M_d^2 + Q^2} \right]. \quad (7)$$

Eq. (7) yields  $\langle r^2 \rangle_B^{\pi^+} \simeq 3\Delta m/N_c M^3 \simeq (0.04 \text{ fm})^2$ . Our result for the baryon density is plotted in Fig. 1, where we note a change of sign of the baryon density at  $r_0 = \log(M_d/M_u)/(M_d - M_u) \simeq 1/M \simeq 0.5$  fm. The corresponding form factor is shown in Fig. 3. It takes a minimum  $F_{\min} = -\Delta m/2MN_c \simeq -0.0012$  at  $Q^2 \simeq 4M^2 \simeq 0.5 \text{ GeV}^2$ .

Generalizing to other meson states, such as the Kaons,  $D$ , or  $B$  mesons, with heavy-light constituents, yields a shift of the crossing point to shorter distances. With  $M_a = M + m_a$ , and taking  $m_s = 100$  MeV,  $m_c = 1.27$  GeV,  $m_b = 4.65$  GeV, the toy model gives  $\langle r^2 \rangle_B^{K^{+0}} = (0.22 \text{ fm})^2$ ,  $\langle r^2 \rangle_B^{D^{0-}} = (0.35 \text{ fm})^2$ , and  $\langle r^2 \rangle_B^{B^{+0}} = (0.36 \text{ fm})^2$ , with equal and negative values for the corresponding antiparticles.

### 3.3. Chiral quark model

We now pass to a quark model where the pion is described with a fully relativistic  $\bar{q}q$  dynamics. The Nambu-Jona-Lasinio (NJL) model with constituent quarks (see [13] and Refs. therein) implements the spontaneously broken chiral symmetry, providing the quarks dynamically with a constituent quark mass  $M$ . A leading- $N_c$  diagrammatic representation of the baryon form factor is given in Fig. 4, where we indicate the momenta, and the  $\pi^+ud$  coupling constant of the point-like coupling is  $(M_u + M_d)/\sqrt{2}F_\pi$ , with  $F_\pi$  denoting the pion weak decay constant. The Lorentz, gauge, and chiral symmetries are preserved by implementing the Pauli-Villars regularization [13], imposed in these effective models to suppress the hard momentum contribution from the quark loop.

The result of a standard evaluation has the compact form for the expressions leading in the quark mass splitting  $\Delta m$ :

$$F_Q^{\pi^+}(t) = \frac{4N_c M^2}{F_\pi^2} I_1(t)$$

$$F_B^{\pi^+}(t) = \frac{16M^3 \Delta m}{F_\pi^2} [J(t) - I_2(t)], \quad (8)$$

with the basic one-loop integral evaluated in the Euclidean space as

$$I_p(t) = -i \int \frac{d^4k}{(2\pi)^4} G(k)^p G(k+q),$$

$$J(t) = -i \int \frac{d^4k}{(2\pi)^4} G(k)G(k+q)G(k-p), \quad (9)$$

where the quark propagator is  $G(l) \equiv 1/(l^2 - M^2 + i\epsilon)$ . The subscript ‘reg’ indicates the regularization, imposed in these effective models to suppress the hard momentum contribution from the loop. Explicit calculation in the chiral limit yields the result

$$F_Q^{\pi^+}(t) = 1 + \frac{M^2 N_c \left( 2s - \log \left( \frac{s+1}{1-s} \right) \right)}{4\pi^2 f^2 s} \Bigg|_{\text{reg}}, \quad (10)$$

$$F_B^{\pi^+}(t) = \frac{\Delta m M^3 \left[ \frac{1}{2} \log^2 \left( \frac{s+1}{1-s} \right) + \log \left( \frac{s+1}{1-s} \right) \right]}{\pi^2 F_\pi^2 t} \Bigg|_{\text{reg}},$$

with the short-hand notation  $s = 1/\sqrt{1 - 4M^2/t}$  introduced. The low- $Q^2$  expansion is

$$F_Q^{\pi^+}(-Q^2) = 1 - \frac{N_c Q^2}{24\pi^2 F_\pi^2} + \dots \Bigg|_{\text{reg}}, \quad (11)$$

$$F_B^{\pi^+}(-Q^2) = -\frac{\Delta m Q^2}{24\pi^2 F_\pi^2 M} + \dots \Bigg|_{\text{reg}}.$$

The baryon radius is  $\langle r^2 \rangle_B^{\pi^+} = (0.03 \text{ fm})^2$  with  $M = 0.3 - 0.35$  GeV and the Pauli-Villars cut-off  $\Lambda \simeq 0.7$  GeV, adjusted to give the physical value of  $F_\pi$ . Actually, since the one-loop calculation of Fig. 4 yields a finite result for the electric charge msr [14] and for the baryon msr, without regularization we obtain a numerically very similar (though not identical) value,  $\langle r^2 \rangle_B^{\pi^+} = (\Delta m/N_c M) \langle r^2 \rangle_Q^{\pi^+} \sim (0.03 \text{ fm})^2$ . The baryon form factor in the NJL model with PV regularization (NJL) and in the unregularized case (unreg.) are plotted in Fig. 3 for  $Q^2 \lesssim \Lambda^2 \sim 0.5 \text{ GeV}^2$ . Momenta higher than the cut-off are hard and are outside of the fiducial range of the effective quark models, hence the functions are not plotted there.

### 3.4. Vector meson dominance

Alternatively to quark models, on the basis of the quark-hadron duality we may adopt a purely hadronic description that shall illustrate the significance of the  $\rho - \omega$  mixing in the context of the baryon form factor. In VMD, the physical  $\omega$  and  $\rho^0$  mesons are linear combinations of the isoscalar  $\omega^0$  and isovector  $\rho^3$  states with a mixing angle  $\theta$ . With the current-field identities [15,16] and the matrix elements  $\langle 0 | J_B^\mu | \omega^0 \rangle = \frac{1}{N_c} f_B \epsilon^\mu$  and  $\langle 0 | J_3^\mu | \rho^3 \rangle = \frac{1}{2} f_3 \epsilon^\mu$ , the form factors in the space-like region read

$$F_3(-Q^2) = \frac{f_3}{2} \left[ \frac{\sin \theta g_{\omega\pi\pi}}{Q^2 + m_\omega^2} + \frac{\cos \theta g_{\rho\pi\pi}}{Q^2 + m_\rho^2} \right]$$

$$F_B(-Q^2) = \frac{f_B}{N_c} \left[ \frac{\cos\theta g_{\omega\pi\pi}}{Q^2 + m_\omega^2} - \frac{\sin\theta g_{\rho\pi\pi}}{Q^2 + m_\rho^2} \right] \quad (12)$$

with  $g_{\omega\pi\pi}$  and  $g_{\rho\pi\pi}$  denoting the couplings for  $\omega \rightarrow \pi^+\pi^-$  and  $\rho \rightarrow \pi^+\pi^-$  decays and  $f_{B,3}$  related to  $\rho/\omega \rightarrow \ell^+\ell^-$  decays (see for instance Ref. [17]). The conditions  $F_B(0) = 0$  and  $F_3(0) = 1$  imply  $g_{\rho\pi\pi} = 2m_\rho^2 \cos\theta/f_3$ ,  $g_{\omega\pi\pi} = 2m_\omega^2 \sin\theta/f_3$ , and

$$F_3(-Q^2) = \frac{\cos^2\theta m_\rho^2}{Q^2 + m_\rho^2} + \frac{\sin^2\theta m_\omega^2}{Q^2 + m_\omega^2},$$

$$F_B(-Q^2) = \frac{Q^2 f_B \sin(2\theta)(m_\omega^2 - m_\rho^2)}{f_3 N_c (Q^2 + m_\rho^2)(Q^2 + m_\omega^2)}. \quad (13)$$

The above formula nicely illustrates basic physical features: the association of emergence of  $F_B(-Q^2)$  with the  $\rho - \omega$  mixing, and its vanishing value at  $Q^2 = 0$ .

However, Eqs. (13) hold literally for narrow-width mesons only, which is certainly not the case for the broad  $\rho$  resonance and precludes building a successful phenomenology. For that reason we do not elaborate numerically Eqs. (13), treating them only as a guideline for a more sophisticated analysis of the next section, where the width of resonances is properly incorporated.

#### 4. Data analysis

We now use the available high statistics data in the time-like region to extract the baryon form factor. Actually, the Gell-Mann–Nishijima formula for the form factors would allow for a direct determination if it were not for the fact that, unlike for  $|F_Q^{\pi^\pm}(q^2)|$  accessible from the  $e^+e^- \rightarrow \pi^+\pi^-$  reaction from BaBar [18] and KLOE [19–22]), the  $F_3^{\pi^\pm}(q^2)$  form factor remains unknown. One could in principle consider the flavor-changing current  $J_\mu^+ = \bar{u}\gamma_\mu d$  and the corresponding form factor  $F_+(q^2)$  appearing in the matrix element  $\langle \pi^0 | J_\mu^+ | \pi^+ \rangle$ , determined to a high precision in  $\tau \rightarrow \pi^+\pi^0\nu_\tau$  decays by Belle [23]. In the strict isospin limit ( $\Delta m = 0$ ), the latter is simply related to  $F_3^{\pi^\pm}(q^2)$  via isospin rotation, a relation that has been exploited in the context of the muon ( $g - 2$ ) [24–34], but without paying an effort to extract  $F_B^{\pi^\pm}(q^2)$ . Moreover, the isospin relation no longer holds if  $\Delta m \neq 0$  or the electromagnetic effects are accounted for. Actually, while the isospin version of the Ademollo–Gatto non-renormalization theorem [35] implies  $F_3^{\pi^\pm}(0) = F_+(0) + \mathcal{O}(\Delta m^2)$ , this is no longer true at finite momentum transfer, where  $F_3^{\pi^\pm}(q^2) = F_+(q^2) + \mathcal{O}(\Delta m)$ , comparable itself to the effect we aim to extract,  $F_B^{\pi^\pm}(q^2) = \mathcal{O}(\Delta m)$ . Indeed, our attempts to do so with the aid of dispersion relations provided noisy results. These can be ascribed to the isospin violating corrections, as we detail in the analysis below.

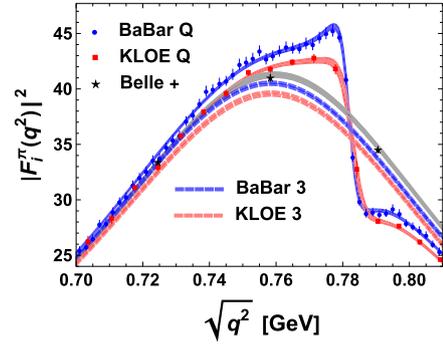
For the above reasons we restrict ourselves to the model decomposition [26]

$$F_Q^{\pi^\pm}(q^2) = F_3(m_{\pi^+}, m_{\pi^0}, m_{\rho^0}, q^2) + \frac{1}{2} F_B^{\pi^\pm}(q^2), \quad (14)$$

to extract  $F_B^{\pi^\pm}(q^2)$ . As a consistency check of the approach, we also fit  $F_+(q^2) = F_3(m_{\pi^+}, m_{\pi^0}, m_{\rho^+}, q^2)$ . We adopt the Gounaris–Sakurai [36] parametrization used in the BaBar analysis [37] for  $F_3(q^2)$ , whereas the  $\rho - \omega$  mixing term is taken in the form

$$\frac{1}{2} F_B^{\pi^\pm}(q^2) = c_{\rho\omega} q^2 D_\rho(q^2) D_\omega(q^2), \quad (15)$$

ensuring that  $F_B^{\pi^\pm}(0) = 0$ . Further,  $c_{\rho\omega}$  is real, not to spoil analyticity (see Eq. (16) below) by a constant non-zero phase. We



**Fig. 5.** Various form factors in the  $\rho - \omega$  region. The points indicate the experimental data from  $e^+e^- \rightarrow \pi^+\pi^-$  (BaBar and KLOE for the  $Q$  form factor), and from  $\tau^- \rightarrow \pi^0\pi^-\nu_\tau$  (Belle for the  $+$  form factor). Our fits are represented with the solid bands following the points, where the widths reflect the statistical errors. We also give the resulting fits to the 3 form factors, indicated with the dashed bands.

**Table 1**

Various estimates for the baryonic radius of  $\pi^+$ .

approach	$\langle r^2 \rangle_B^{\pi^+}$	comment
effective Lagrangian	$c(0.04 \text{ fm})^2$	$c$ - number of order 1
toy Yukawa model	$(0.04 \text{ fm})^2$	
NJL with PV reg.	$(0.03 \text{ fm})^2$	
NJL without reg.	$(0.03 \text{ fm})^2$	
BaBar	$(0.041(1) \text{ fm})^2$	exp. stat. error only
KLOE	$(0.041(1) \text{ fm})^2$	

emphasize these features were not implemented in the BaBar analysis, while the form of  $D_V^{-1}(t) = \tilde{m}_V^2 [m_V^2 - t - im_V \Gamma_V(t)]^{-1}$  is taken as in the BaBar parametrization [37].

The results are shown in Fig. 5. The two highest-reaching bands correspond to our fits to  $F_Q^{\pi^\pm}(q^2)$  from BaBar [18] and KLOE [19–22] with QED effects from vacuum polarization and final state radiation removed. The corresponding  $F_3^{\pi^\pm}(q^2)$  form factors resulting from these fits are shown as the two dashed bands. These can be compared to the fit to the  $F_+(q^2)$  form factor from Belle, shown as a band. The mild difference is clear and can be ascribed to the mentioned isospin-breaking corrections. The obtained value of the mixing parameter from the BaBar/KLOE data is  $c_{\rho\omega} = [36(1)/37(2)] \times 10^{-4} \text{ GeV}^{-2}$ .

To extract the behavior in the spacelike region, we make use of analyticity and the perturbative high-energy behavior at large  $q^2$ ,  $F_B^{\pi^\pm}(q^2) = \mathcal{O}(1/q^2)$  [38], that allows one to write down the unsubtracted (and subtracted) dispersion relations

$$F_B^{\pi^\pm}(q^2) = \frac{1}{\pi} \int_{4m_{\pi^+}^2}^{\infty} ds \frac{\text{Im} F_B^{\pi^\pm}(s)}{s - q^2} = \frac{q^2}{\pi} \int_{4m_{\pi^+}^2}^{\infty} ds \frac{\text{Im} F_B^{\pi^\pm}(s)}{s(s - q^2)}. \quad (16)$$

The results of Fig. 3 show that despite a discrepancy between BaBar and KLOE in the time-like region (cf. Fig. 5), the obtained baryonic form factors in the space-like region are compatible. Likewise, the baryonic radii computed from Eq. (16),  $\langle r^2 \rangle_B^{\pi^+} = (0.0411(7) \text{ fm})^2$  for BaBar and  $(0.0412(12) \text{ fm})^2$  for KLOE, are in a remarkable agreement.

#### 5. Conclusions and outlook

We summarize in Table 1 all the obtained estimates for the baryonic msr of the charged pion, which fall in the range  $\langle r^2 \rangle_B^{\pi^+} = ((0.03 - 0.04) \text{ fm})^2 = (0.001 - 0.002) \text{ fm}^2$ . The agreement with the Yukawa model complies to the natural understanding on the positive sign of the radius for  $\pi^+(ud)$ , where the lighter  $u$  com-

ponent is more extended than the heavier  $\bar{d}$  component. Comparing the estimates of Table 1 to the accuracy of the experimental charge radius,  $\langle r^2 \rangle_Q^\pi = (0.659(4) \text{ fm})^2 = 0.434(5) \text{ fm}^2$  [1], or to the most recent *ab initio* lattice QCD calculations with physical averaged quark masses,  $\langle r^2 \rangle_Q^\pi = (0.648(15) \text{ fm})^2 = 0.42(2) \text{ fm}^2$  [39] and  $\langle r^2 \rangle_Q^\pi = 0.430(5)(12) \text{ fm}^2$  [40], we note that the signal is a factor of 4–10 too small to affect current charge radius determinations. However, the small baryonic msr is also coming from the  $1/N_c$  prefactor preceding the baryon current. Without this factor, one finds  $N_c \langle r^2 \rangle_B^{\pi^+} = \langle r^2 \rangle_u^{\pi^+} - \langle r^2 \rangle_d^{\pi^+} = (0.003 - 0.005) \text{ fm}^2$ , that might be within reach in lattice QCD.

The case of the baryonic content of the Kaon is much more promising. Since for  $\bar{d}$  and  $s$  quarks the baryon number equals minus the charge, in models with structureless constituent quarks (such as our toy Yukawa model)  $\langle r^2 \rangle_B^{K^0} = -\langle r^2 \rangle_Q^{K^0}$ . The Yukawa model yields  $\langle r^2 \rangle_B^{K^0} \simeq (0.22 \text{ fm})^2 \simeq 0.05 \text{ fm}^2$ , whereas PDG [1] quotes  $\langle r^2 \rangle_Q^{K^0} = -(0.28(2) \text{ fm})^2 = -0.077(10) \text{ fm}^2$ , with the uncertainty 5 times smaller than the Yukawa model estimate for the baryon msr. Similarly, VMD predicts  $F_B^{K^0}(q^2) = N_c^{-1}(D_\omega(q^2) - D_\phi(q^2))$ , thus  $\langle r^2 \rangle_B^{K^0} = (0.23 \text{ fm})^2 = 0.052 \text{ fm}^2$ , while a data-driven analysis such as that in Sect. 4 would be more involved. In particular, the  $K\bar{K}$  threshold, well above the  $\rho, \omega$  mesons, would require a more elaborated analysis as outlined in Refs. [41,42].

Hopefully, more accurate experiments and their corresponding analyses of the vector form factors would provide a better understanding of the fundamental issue of the matter-antimatter distribution in pseudoscalar mesons.

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## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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