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Measurement in Primary School Mathematics and Science Textbooks

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Abstract: The STEM (science, technology, engineering and mathematics) approach to education has acquired considerable prominence among teachers in recent years. Putting forward integrated proposals is nonetheless complex and many educators opt to implement the ones set out in textbooks. We consequently deemed it worthwhile to analyse how content common to mathematics and science is addressed in primary school textbooks with a view to determining whether the approaches adopted complement one another and are compatible with STEM education. More specifically, in light of the importance of measurement in both areas of learning and in everyday life, we describe the meaning of mass and volume found, in two publishers' textbooks. Based on the components of the meaning of measurement and deploying content analysis techniques, we analysed the explanations and tasks set out in these mathematics and science books to identify the similarities and differences in the handling of those magnitudes in the two subjects. Our findings showed the proposals for teaching mass to pursue similar objectives in the earliest grades, addressing matters that could be included in STEM proposals. On the contrary, inconsistencies were detected in the distribution of volume measurement-related content, as well as in the strategies, units and tools used in the two areas.

Keywords: measurement; primary education; textbooks; mathematics education; science education

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1. Introduction

The importance attached to STEM (science, technology, engineering and mathematics) competence has been growing in recent decades. The literature, however, is not unanimous about how such competence should be acquired, with different approaches to STEM education varying in terms of the number of areas addressed and their integration [1]. Mpofu [2], for instance, identified four models. The first entails teaching those disciplines separately and the second in integrating two STEM disciplines, most often mathematics and science. In the third approach, one of the STEM disciplines, normally engineering or technology, is taught as an integral part of the other three, whilst in the fourth model, known as integrated STEM education, all four disciplines are integrated by merging the knowledge and skills characteristic of each. Castro-Rodríguez and Montoro [3] contended that despite the differences in STEM education models, all share three characteristics: reference to a real-world context; interconnection among the various STEM disciplines; and the development of problem-solving skills.

One of the obstacles to sequence STEM proposals in the classroom is the need for cooperation among the teachers involved. In Spanish primary education, the fact that the same general classroom educator also teaches mathematics and the natural sciences to a given class facilitates the introduction of STEM proposals from an interdisciplinary approach that prioritises the integration of those disciplines.

This research starts from an institutional perspective, in which we know that teachers make extensive use of textbooks. Further to the Ministry of Education's Assessment Institute data [4], textbooks are used by virtually all (99.1%) primary school students. The prevalence in classrooms of separate textbooks for natural sciences and mathematics might induce a disconnection between those two subjects. We consequently identified an interest in reviewing two resources indispensable to teaching–learning, the curriculum and natural science and mathematics textbooks, to determine the content included in those two subjects as background information useful for designing integrated STEM proposals. Although it is recognized that the majority of STEM proposals appear on web pages [5], we have relied on the importance of the textbook and its multifunctionality [6] in order to examine its relevance for teaching STEM, in which, according to these authors, there is little research in this regard.

Further to Decree 97/2005 [7] adopted by the Government of Andalusia, a region of Spain, "mathematical thinking contributes to basic competence in science and technology because it affords a fuller understanding and more accurate description of the environment" (p. 317) in respect of three elements: spatial visualisation and orientation, measurement and the use of graphics. It adds that the natural sciences afford an opportunity to "use mathematical tools in significant contexts such as map reading; understanding and defining scales; reading, representing, interpreting and presenting graphics; and using units of measure to contribute to developing competence in mathematics" (p. 62).

The utilitarianism of some mathematical content is attested to by proposals for STEM teaching. In an analysis of STEM proposals drawn from books with a powerful international impact and from highly reputed websites (according to the Portuguese association of mathematics teachers and the Spanish federation of mathematics teachers' societies), Lasa et al. [5] found mathematics to play a basic and utilitarian role in those proposals. They observed the focus to be aimed primarily at measurement, the statistical interpretation of numerical data collected and the use of elementary geometric language.

As a key element in both science and mathematics and frequently present in STEM proposals as well as in everyday life, magnitude measurement is pivotal to the present study. To appreciate to what extent STEM teaching can be carried out with textbooks, we analyse the way mass and volume are handled in primary education mathematics and natural science textbooks, as they are the only two magnitudes listed in the curriculum as content to be addressed in both areas.

1.1. Teaching and Learning Measurement

The concept of measurement sense [8] promotes a functional approach in measurement teaching [9]. In mathematics education, measurement is introduced to students by helping them perceive and compare the magnitude to be measured. For some comparisons, particularly in the absence of a sufficiently large intermediary, a unit of measure appropriate for the object must be chosen and the number of times such a unit 'fits' into the object must be counted, thereby determining its measurement, either in standard or non-standard units. Students must also learn to use measuring tools, understand that measurements are always approximate and develop estimation strategies [10]. In science education, magnitudes are viewed as properties of matter (defined as anything with mass and volume) and classified as general or specific. Density, as a specific property of matter defined as the quotient between mass and volume, is of particular significance in that respect. Science instruction also stresses the use of the International System of Units and of balancing scales and graduated containers [11]. In the final stage of multidimensional magnitude instruction (indirect measurement) formulas are developed or applied to find the area or volume of objects by measuring certain dimensions [10,11]. Maral et al. [12], in turn, contended that a command of measurement-related ideas and procedures is necessary in many scientific experiments. Those authors stressed the importance of establishing measurement consistency and the respective margin of error and analysing possible sources of error in measurements. All the foregoing is discussed in a paper by Passelaigue and Munier [13], who reviewed measurement in the science and mathematics curriculum and the contributions made by research on teaching those two disciplines.

Spanish Royal Decree 126/2014 includes the measurement of object mass, volume and density in the primary education natural sciences curriculum. It also provides that mathematics should cover most of the ideas and procedures mentioned earlier: measuring with standard and non-standard units, choosing the most appropriate unit to express a measurement, identifying the units comprising the International System of Units and estimating and measuring the mass and volume of known objects and spaces. This document likewise defines other skills to be mastered, such as expressing measurements in simple and complex terms, comparing and sorting different quantities of the same magnitude, performing operations with measurements and solving problems that involve measurements [14]. Measurement precision and error are terms not envisaged in the Spanish curriculum until the physics, chemistry and mathematics taught in the latter years of the scientific pathway in secondary education (baccalaureate) [15].

Most research on measurement in mathematics education focuses on length [16,17] and area [18], with very few studies dealing with teaching and learning mass or volume. In one prominent article on mass, Clarke et al. [19] put forward a theoretical structure describing the developmental progression of related skills: perception of and use of language to describe mass; comparing, sorting and matching objects by their mass; precisely quantifying object mass using units and measurement principles; choosing and using of units to estimate and precisely measure mass; and solving problems involving key skills and concepts relating to mass.

The need for tasks geared to perceiving and comparing attributes and using non-standard units prior to introducing standard measurement, along with activities fostering the use of tools, has been stressed both in research and curricular proposals [13]. Differences have nonetheless been identified with respect to the ages deemed most suitable for introducing those ideas. Cheeseman et al. [20], for instance, showed that stage-by-stage progress may be brought forward, proving that with a suitable learning sequence most 6-year-olds are able to measure with non-standard units and half of them and all 7-year-olds with standard units. In a similar vein, pre-school classroom experiences with comparison and measurement of mass using standard units have also been reported [21]. The ability to perceive and compare object mass and volume at very early ages is attested to in educational proposals to introduce floatation [22], although density, which entails understanding and inter-relating two magnitudes (mass and volume), was found to be too complex to be assimilated by such young children. The difficulties faced by students in different years of schooling when comparing and measuring in real-life situations where they cannot perform those tasks directly but must define and deploy strategies have been identified in several studies. Such activities and the subsequent discussion with students of the strategies used have been shown to constitute an opportunity to recognise and correct mistaken conceptions [23,24].

1.2. Textbook Analysis

Textbook analysis and comparison is a line of research widely explored in mathematics [25] and science [26] teaching, due to their high application in the classroom. One particularly prominent study in connection with measurement by Mengual et al. [27] analysed publisher Vicens Vives's primary school mathematics books. The authors found that measurement was the priority activity in the lower 2 years, followed by measurement comparison and sorting, whereas in the upper 4 years the focus was on converting units, geometric measurement and word problems involving measurements. In a study on dimensional estimation, Chang et al. [28] analysed the characteristics of the tasks set out in three textbooks on the grounds of their components. More specifically, they analysed whether tasks defined which object dimension (height, length or width) was to be estimated, whether the starting and end points were clearly established, whether students were allowed to choose the most suitable unit for the task and whether the precision required was stipulated. The final two matters, choice of unit and degree of precision, are applicable not only to estimating dimensions, but to measuring any other magnitude. In another vein, the analysis of secondary school science textbooks conducted by Palacios-Díaz and Criado [29] identified certain shortcomings in the way mass, volume and density are introduced. Those shortcomings include a stress on the application of the mathematical formula to the detriment of a sufficient number of examples of mass or volume of everyday objects and of a qualitative discussion of the relationship among the variables involved.

One article of particular relevance to our study, published in 2013 by Picado et al. [30], analysed the content of a mid-nineteenth century textbook to determine how certain authors responded to the particular needs of contemporary mathematics education, specifically with respect to the introduction of the metric system in an arithmetic textbook. Like those authors, here we used didactic analysis [31] as the theoretical grounds for our study, deeming it a construct apt for in-depth exploration of the various elements of a given mathematical topic [32].

1.3. Meaning of School Mathematics Content

Didactic analysis has proved to be a useful tool for teachers, enabling them to reflect on important items when planning, implementing and assessing their professional endeavour. It requires them to analyse educational content from four perspectives. Teachers must first conduct content analysis to ensure they have a thorough understanding of the mathematics to be taught: definitions, properties, relationships with other conceits; representation systems and situations where they are applicable. They must also acquire an awareness of their students' aptitudes and learning challenges by conducting cognitive analysis. Based on the outcomes of those two analyses, teachers should assign tasks geared to the matter to be learnt, i.e., they should analyse instruction. Lastly, they must assess both students' achievements and the instruction provided [31].

Textbooks may be viewed as teaching materials containing explanations of and tasks relating to the content and skills to be acquired in the various years of schooling. The present study focused on content analysis to explore the meaning of ideas introduced in the classroom, based on the three elements around which the curriculum is organised [33–35]: conceptual structure, representation systems and phenomenology.

Conceptual structure highlights mathematical ideas, procedures and strategies associated with measurement and the relationships among them set out in the textbooks analysed. Establishing relationships between the ideas and procedures associated with measurement calls for examining conceptual assemblages, defined to be 'specific groups of concepts, procedures and relationships that acquire particular importance for they express, organise and summarise coherent content groupings' [32] (p. 86). The primary education curriculum [14], the National Council of Teachers of Mathematics (NCTM) standards [36], training manuals for teachers and the findings of earlier research were reviewed to establish such assemblages. The items comprising conceptual structure can be divided into two knowledge categories, conceptual (facts, ideas and conceptual structures) and procedural (skills, reasoning and strategies). The flow chart in Figure 1 depicts the conceptual structure of measurement in primary education from the standpoint of the meaning of measurement [10]. The initial premise is that, in the absence of sufficiently large intermediaries, magnitudes need to be perceived and compared in many real-life situations. Similarly, commercial transactions depend on the existence of standard units of measure. Such factors should be addressed in the earliest years of schooling, when the comparison and direct measurement of magnitudes should be prioritised [21]. This is usually taught within mathematics classroom during the 1–3 years of primary education. That should be followed in a subsequent stage by instruction in the use of everyday measuring instruments and the introduction of the metric system, which is a goal of both mathematics and science education. In addition, since an estimate of object dimensions often suffices and estimation helps understand measuring processes and can prevent erroneous instrument readings, it should also be taught to even the youngest children [10,13]. Conversely, indirect measurement, relationships between units of measure and operations involving measurements is taught beginning in the fourth year of primary in mathematics classroom, whilst the margin of error associated with measuring instruments is a conceit that should only be addressed in secondary education in both mathematics and science classroom [14,15].

Representation systems show how content and its relationship with other ideas and procedures can be represented. Each system highlights a given characteristic of the mathematical idea. Hence the importance of using different representation systems when teaching mathematical content [37], beginning with the use of manipulatives and working up to symbols [38]. In this study we explored the systems defined by Picado et al. [30]: verbal, the use of written language to refer to ideas, procedures and properties; graphic, the use of pictures or charts; symbolic, the use of numbers and mathematical symbols; tabular, the use of tables; and measuring tools, a specific type of manipulative.

Phenomenology refers to the contexts and situations giving rise or meaning to measurement, i.e., delimiting those situations that attest to its functionality. The PISA 2018 Project [39] proposes four context categories for such situations: personal: where the problems focus on students' or their families' everyday activities, such as food preparation, shopping, travel, sports, family finance or transportation; occupational, covering a wide range of tasks that may be performed at any level of the workforce, from unskilled work to the most demanding levels of professional endeavour, including inventory, costing and ordering material for building, payroll/accounting and quality control; societal, relating to the local or broader (national, worldwide) community, encompassing factors that characterise the community or the domestic economy; scientific, where mathematics are applied to the natural world and scientific and technological issues, as elements of the world of mathematics per se.

As noted above, this study aimed to answer the question: are the existing mathematics and science textbooks suitable for STEM primary school instruction on magnitudes? Consequently, the main objective pursued was to describe the meaning of mass and volume found in primary education mathematics and nature science books.

Inasmuch as the analysis of meanings in classroom mathematics builds on the three elements into which the curriculum is divided, the question guiding that analysis was: what conceptual structures, representation systems and contexts are to be found in those books? Further, are such contexts compatible with the operationality that characterises STEM education? Do they favour skills associated with the meaning of measurement, i.e., useful for everyday life?

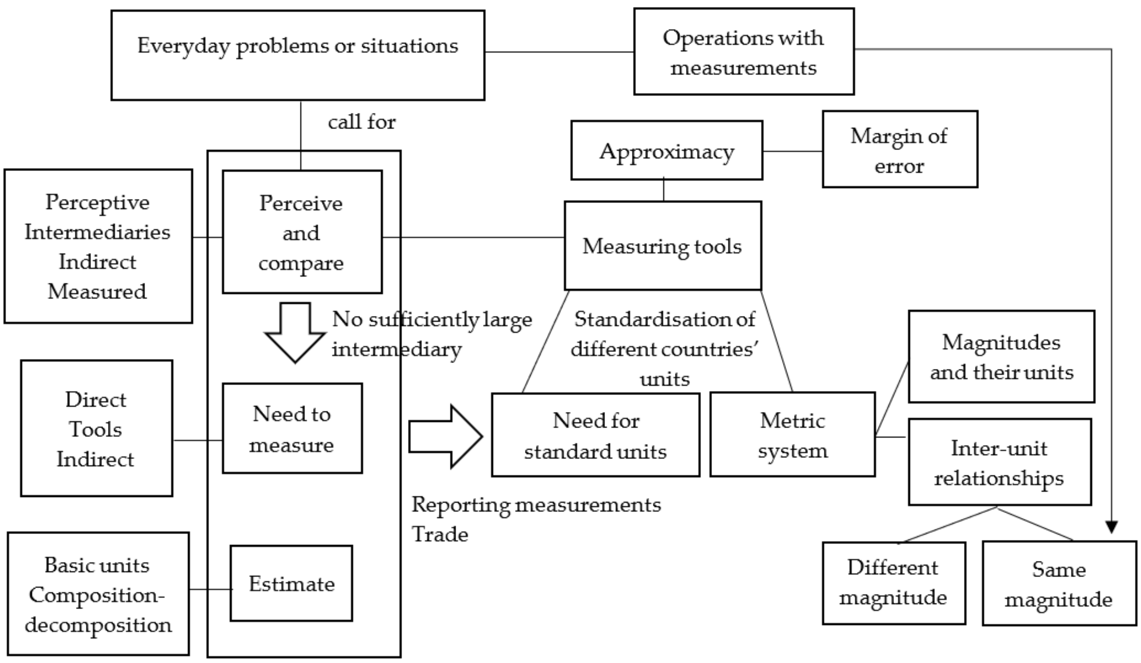


Figure 1. Measurement in primary education: flow chart.

2. Materials and Methods

Qualitative assessment of textbooks is a task performed in many disciplines and is a useful history of science tool for tracking the dissemination of new ideas and analysing teaching practice. Consequently, in light of the objective pursued, the qualitative, descriptive study conducted was based on an analysis of the content [40] of primary education mathematics and nature science (hereafter, science) books in keeping with textual analysis precepts, describing, assessing and characterising the curricular and methodological dimensions of mathematics content [41]. As no general consensus has been reached on a theoretical basis for textual analysis, here we opted to implement content analysis as developed by Rico et al. [31].

In documentary studies, content analysis comprises a number of stages [42]. Once the content to be analysed is defined, the textual unit of analysis must be specified, establishing the fragments from which information is to be extracted and frequency quantified. The categories taken into consideration to code and quantify the units of analysis present in the textbooks analysed are defined and interpreted below.

2.1. Sample

As the objective pursued was to describe and compare the approach adopted in primary education textbooks for teaching two magnitudes, mass and volume, we chose the mathematics and science books for first through sixth year of primary school issued by SM and Anaya, the two top-selling Spanish publishers according to Ministry of Culture and Sport data [43].

Although the initial intention was to analyse the full content of their primary school math and science books, a preliminary analysis advised disregarding the topics where mass or volume appeared as part of the context of an arithmetic problem only, i.e., where they did not form part of the core topic.

After filtering out the irrelevant particulars, we found that both publishers include one topic explicitly dealing with the two magnitudes in their mathematics textbooks for years one through six, whilst SM includes two topics on the matter in its sixth year book. In science, SM includes the topic in the second to fifth year textbooks, whereas Anaya addresses it through sixth year. Nine science and 12 mathematics textbooks were analysed in all.

Most Spanish textbooks, and more specifically those analysed here, are identically structured. Each teaching unit consists primarily in the tasks to be performed by students, explanations of properties or definitions and examples of how to tackle a given task. Bearing that in mind, we selected as units of textual analysis all the fragments requiring a cognitive effort either in the form of tasks or of understanding explanations. The task-related units of analysis established were as delimited in the textbooks themselves. For the explanations, however, the unit of analysis adopted included phrases, paragraphs or graphics describing an inherently meaningful idea, concept or skill. A total of 420 units of analysis were defined, 124 explanation- and 296 task-related. Their distribution by book analysed is given in Table 1.

Table 1. Units of analysis in textbooks.

Year	Mathematics				Science			
	SM		Anaya		SM		Anaya	
	Explan.	Task	Explan.	Task	Explan.	Task	Explan.	Task
1	0	8	1	7	-	-	-	-
2	1	6	1	6	2	1	6	10
3	3	22	5	26	9	7	9	7
4	4	13	2	38	7	5	9	7
5	8	16	2	19	9	12	14	10
6	17	56	7	15	-	-	8	5
Total	33	121	18	111	27	25	46	39

2.2. Categories

The analysis categories used in this study were defined to be specific to each component of the semantic triangle (conceptual structure, representation systems, phenomenology) of classroom mathematics. As most were defined on the grounds of a review of the literature, the procedure deployed was deductive. More specifically, to characterise the representation systems and usage observed in the textbooks, we adopted the categories described in the explanation of those elements in the theoretical background for the study. For instance, only the tasks where students were asked to use a measuring tool were included under the category 'tools', a sub-set of representation systems, deeming mere pictures or drawings of tools to constitute graphic representation. Analogously, in connection with phenomenology, tasks consisting in word problems set in a shop were classified as occupational situations if they involved solving a problem faced by the seller and personal or everyday situations if faced by the buyer. Where no specific context was present, tasks were classified under a fifth class, 'classroom'.

The analysis categories associated with conceptual structure, in turn, were established after reviewing the content and skills characteristic of the meaning of measurement and the findings of earlier studies. Nonetheless, unit of analysis coding revealed a need to segregate or adapt certain classes or create new ones. The categories ultimately defined are listed in Tables 2 and 3 by type of analysis unit.

Table 2. Measurement skills addressed in tasks.

Task	Description
Compare	
Compare	Involves comparing or sorting objects by mass or volume
Use of tools	Involves comparing object mass or volume using tools such as balancing scales or graduated containers
Measure	
Choice of unit	Involves choosing the most suitable unit of measure
Choice of measuring tool	Involves choosing a suitable tool to measure object mass or volume
Non-standard measurement	Involves measurement with non-standard units
Use of tools	Involves measurement with actual tools
Indirect measurement	Involves applying formulas to obtain volume from length or density from mass and volume
Measure (general)	Involves answering open questions to explain the process that would be used to measure or engaging in unrestricted measuring
Approximation or error	Involves discussing possible sources of error in the measurement and the margin of error for the tool and/or delimiting the range of error
Equivalence	Involves converting units, with two sub-categories depending on whether the equivalence involves the same or different magnitudes
Operations	Involves operations calling for switching from one unit to another
Estimate	
Involves specifying object measurements without using measuring tools or iterating the unit of measure, i.e., deploying composition-decomposition strategies	

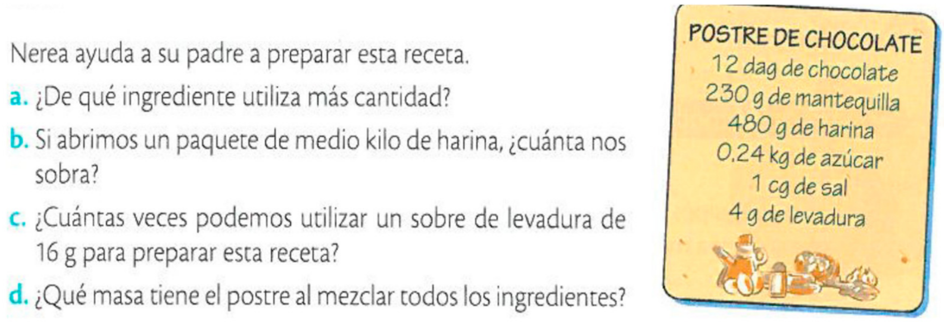
Table 3. Measurement skills addressed in explanations.

Explanation	Description
Perceive and compare	
Definition	Defines object mass or volume
Use of tools	Explains how to use tools to compare object mass or volume
Measure	
Units of measure	Introduces the units of measure for mass or volume
Measuring tools	Specifies the tools to measure mass or volume
Measuring process	Explains what is involved in measurement (unit selection and iteration)
Use of measuring tools	Explains how to use pan or weight balancing scales or graduated containers
Equivalence	Shows the relationship between units of measure or explains how to convert one into the other, with two sub-categories depending on whether the equivalence involves the same or different magnitudes
Indirect measurement	Introduces formulas for finding or explains how to calculate indirect measurements
Approximation or error	Explains the approximate nature of measurement, tool precision or the acceptable margin of error in a given measurement situation
Operations	Explains how to operate with measurements
Estimate	
Explains the utility of estimation in everyday life or furnishes estimation strategies	

2.3. Data Analysis

After establishing the analysis categories, the units of analysis in the third-year mathematics and science textbooks published by SM were coded separately by the first two authors. Their subsequent discussion of the points where disagreement arose resulted in amendments to the description of some of the categories. Using the version described in the preceding sub-section, the one finally adopted, they then proceeded to code all the units of analysis, obtaining a 96.19% agreement between coders (95% in measuring skills, 94.76% in representation systems and 98.8% in context). In the fourth year SM textbook [44], for instance, we found a personal situation word problem that entailed operating with measurements (Figure 2). That unit of analysis (number 180) combined verbal representation to describe the problem with symbolic representation to express the measurements. In other words, each unit of analysis was classified by the idea or procedure involved (conceptual structure), the representation systems used (representation) and the context or situation posed (phenomenology), in addition to information on publisher, subject, year, magnitude (mass or volume) and type of unit of analysis (explanation or task).

After coding the units of analysis, we determined the frequency of appearance of each category as well as inter-category relationships, while interpreting the data.


Nerea ayuda a su padre a preparar esta receta.
a. ¿De qué ingrediente utiliza más cantidad?
b. Si abrimos un paquete de medio kilo de harina, ¿cuánta nos sobra?
c. ¿Cuántas veces podemos utilizar un sobre de levadura de 16 g para preparar esta receta?
d. ¿Qué masa tiene el postre al mezclar todos los ingredientes?

POSTRE DE CHOCOLATE
12 dag de chocolate
230 g de mantequilla
480 g de harina
0,24 kg de azúcar
1 cg de sal
4 g de levadura

Nerea is helping her father make this dessert

a. What ingredient is there most of?
b. If we open a ½ kilo package of flour, how much will be left over?
c. How many times can we make this dessert if we have a 16 g packet of baking powder?

   What is the total mass of the dessert once we mix all the ingredients?

Chocolate Dessert: 12 dag of chocolate, 230g butter, 480g of flour, 0.24 kg of sugar, 1cg of salt, 4 g of baking powder

Figure 2. Sample unit of analysis.

3. Results

This study describes the meanings of mass and volume as defined in primary education mathematics and science books. We consequently arranged the results in terms of conceptual structure, representation systems and contexts or situations involved.

3.1. Conceptual Structure

As discussed in the section on the theoretical grounds for this study and listed in Tables 2 and 3, measurement teaching-learning was divided into three elements: perception and comparison of magnitudes, measurement and estimation. Science books were found to devote 13% of the units of analysis on mass to its definition, from 16% to 23% to comparison and the remainder to measurement. No definition of mass was identified in mathematics books, which were observed to have a lower proportion (11% to 17%) of units of analysis on comparison.

By contrast, volume is defined in the mathematics books in the sixth year, where that magnitude is introduced, accounting for around 4% of the units of analysis. In science books, 18% of the units of analysis on volume deals with its definition. As with mass, in volume, measurement is the element dealt with most profusely, occupying 85% of the units of analysis on this magnitude in both publishers' mathematics and from 65% to 75% of their science books. We found only one unit of analysis on estimating mass in one of each publisher's mathematics books and two (<4%) on estimating volume in one of the publisher's textbooks. Consequently, depending on the publisher and subject, the third element, comparison, is addressed in around 5% to 17% of the units. We deem that frequency to be low and inconsistent with curricular specifications and recommendations on mathematics teaching.

As noted in the methodology, we defined units of analysis on comparison to comprise tasks involving object selection or sorting by mass or volume, distinguishing where they involve the use of tools to perform the comparison. Table 4 shows that the use of tools to compare volume was found in only one of the SM science books, specifically for third year. In contrast, the use of balancing scales as a tool for comparing object mass is frequently included in these books, constituting the object of 40% to 83% of the units of analysis on comparison.

Table 4. Comparison skills by magnitude, subject and publisher.

		Science				Mathematics			
		SM		Anaya		SM		Anaya	
		N	%	N	%	N	%	N	%
Mass	Compare	3	60	1	16.7	6	37.5	5	41.7
	Use of tools	2	40	5	83.3	10	62.5	7	58.3
Volume	Compare	3	75	2	100	2	100	3	100
	Use of tools	1	25	0	0	0	0	0	0

In mathematics, most of the tasks on comparing mass were found in the books for the first three years of primary school. In Anaya, over 60% of the mass comparison tasks were found in the first-year textbook, which was also the sole book that explained how to use a balancing scale to compare the mass of two objects. The tasks involving volume comparison were all found in the sixth-year primary books, inasmuch as that magnitude is not introduced until sixth year. All the foregoing is consistent with methodological guidelines, which specify that comparison is the most suitable approach for introducing magnitudes and their measurements. The distribution of these tasks was observed to follow no established pattern in the science books analysed.

As noted earlier, measurement was the element most frequently found in both subjects. The next question to be answered was, what specific skills were dealt with in each? Figure 3 graphs the percentage of units of analysis dealing with specific skills for measuring mass (left) and volume (right). The data show that the aim primarily pursued in both publishers' science books was to familiarise students with and teach them to use the tools for measuring mass and volume. In mathematics books, on the contrary, no mention was found of tools for measuring volume and balancing scales were observed to be the focus of a mere 10% of the units of analysis dealing with mass. Frequency was highest in second- and third-year Anaya and in third-year SM textbooks. Mathematics books prioritise equivalence between units of measure, both conceptually and procedurally, and operations with units of mass, usually in combination with unit conversion. Those ideas are first introduced in third year, although in Anaya 35% of the units of analysis on measurement also entail equivalence and 45% operations, whilst those percentages are inverted in SM. In tasks involving volume, the use of formulas for indirect measurement is the skill stressed, along with equivalence among units of measure for volume and between those and the units for capacity.

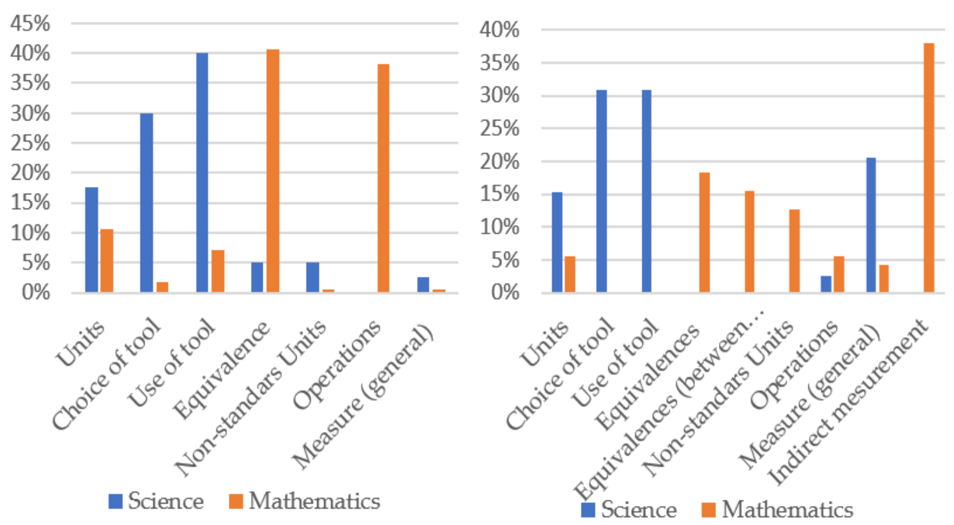
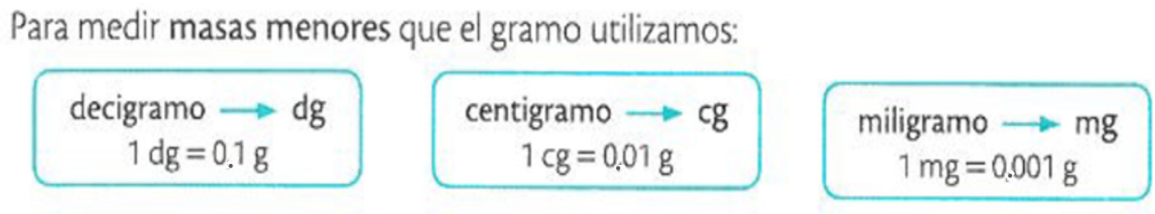


Figure 3. Measurement skills for (**left**) mass and (**right**) volume.

The proportion of tasks requiring volume measurement that fail to specify how that should be done is larger in the science than the mathematics books, whilst the mathematics texts contain more tasks on measurement with non-standard units.

An analysis of the units of measure introduced by year and subject revealed that whereas science books use only g and kg for mass throughout, mathematics books introduce additional units of measure and their equivalents gradually. In second year, the definition reads: 'The kilogramme or kilo is the unit mainly used for mass. It is symbolised as "kg"'. Anaya textbooks, however, add that smaller objects are measured as ½ kg or ¼ kg. The gramme is introduced in third year, when both publishers propose equivalences and operations with measures expressed in natural numbers and the fractions ½ and ¼. Anaya also introduces tonne. All the units of mass in the international system (Figure 4), from milligramme to kilogramme, are introduced beginning in fourth year, although both publishers express equivalences involving units smaller than grammes in decimals beginning in fifth (Figure 5).



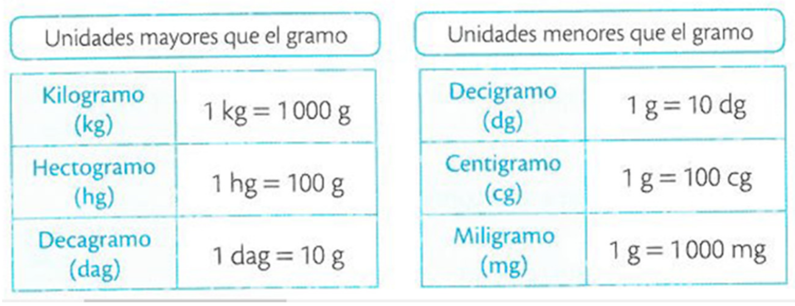
To measure masses smaller than the gramme we use:

decigramme → dg
1dg = 0.1 g

centigramme → cg
1 cg = 0.01 g

milligramme → mg
1 mg = 0.001 g

Figure 4. Equivalence between units of measure (4th year mathematics, SM [44]).



Units greater than the gramme		Units smaller than the gramme	
Kilogramme (Kg)	1 Kg = 1000 g	Decigramme (dg)	1 g = 10 dg
Hectogramme (hg)	1 hg = 100 g	Centigramme (cg)	1 g = 100 cg
Decagramme (dag)	1 dag = 10 g	Milligramme (mg)	1 g = 1000 mg

Figure 5. Equivalence between units of measure (fifth year mathematics, SM [45]).

Switching from one unit to another is illustrated graphically in all cases (Figure 6). In this same vein, the numbers involved in the respective operations increase, successful completion of the tasks depends more on a command of the decimal system (number composition and de-composition) and operations with decimals than understanding the relationship between units of measure. To sum up, the use of mass was found to be essentially consistent in lower year mathematics and science books. The mass of different objects is compared; and the primary units of mass (kg and g), the measuring tools (pan and weight balancing scales) and their correct use are introduced. Nonetheless, the mathematics books also contain tasks involving inter-unit equivalence and word problems calling for operating with measurements. In contrast, a total disconnect was observed in the way the two subjects handle volume, to the extent that they appear to be describing two completely separate ideas.

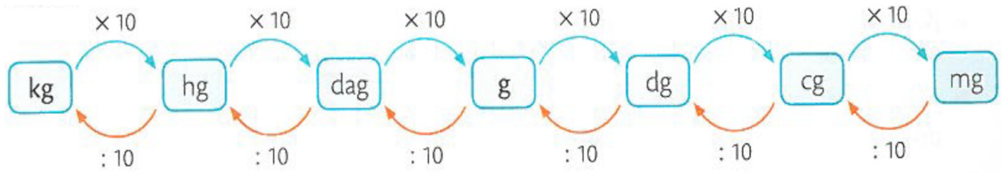
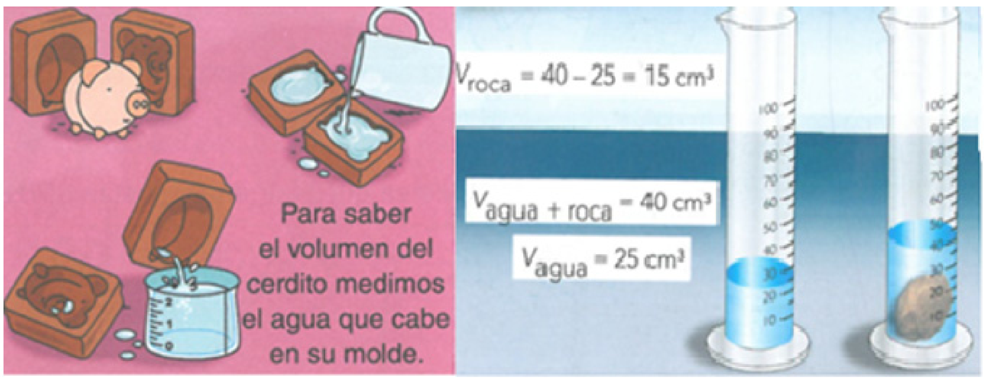


Figure 6. Unit of measure conversion (fourth year mathematics, SM [44]).

In mathematics both publishers use standard (m^3, cm^3) and non-standard (cube of unknown dimensions) cubic units for measuring volume directly, while explaining the equivalence between multiples and sub-multiples of the cubic centimetre. That strategy, in which the number of unit cubes that can be iterated across the object is counted, is only useful for calculating the volume of cube-shaped figures. The science books, on the contrary, use graduated containers to calculate the volume of liquids and any solid object, irrespective of shape. The SM books furnish a full description of how to calculate the volume of liquid displaced in litres or millilitres when an object is completely submerged in a graduated container. Anaya explains that method as well, while also instructing earlier year students to build a mould of the object and measure its capacity to establish the relationship between the two magnitudes (Figure 7). This publisher deploys different units of measure (L, ml, m^3, cm^3) depending on the year of schooling, but fails to establish how they are related.

That relationship is set out in the mathematics books. Nonetheless, whilst the Anaya products contend that "capacity and volume are equivalent magnitudes" SM distinguishes between "volume as the space occupied" and "capacity as the space enclosed by an object". That distinction is not present in the activities included, however, where the volume of a glass container is deemed to be the same as its capacity, for instance.



We can find the piggy bank volume by measuring the amount of water that fits in its mould

$V_{stone} = 40 - 25 = 15 cm^3$
$V_{water + stone} = 40 cm^3$
$V_{water} = 25 cm^3$

Figure 7. Volume measurement in second and sixth year Anaya science books [46,47].

3.2. Systems of Representation

Mathematical ideas can be conveyed using different representation systems. As the number of units of analysis using each system listed in Table 5 shows, the textbooks combine two or more systems to convey the ideas described in the preceding sub-section. Most of the units of analysis in science books use verbal representation, alone or in conjunction with others, graphics in particular. In mathematics, however, those two systems were found with the same frequency as symbols. A few units of analysis were also observed to use tabular representation to compare units of measure, for instance (Figure 4).

Table 5. Representation systems: frequency of use by subject and publisher.

	Science				Mathematics			
	SM		Anaya		SM		Anaya	
	N	%	N	%	N	%	N	%
Verbal	23	44.2	42	49.4	32	20.8	28	21.7
Verbal-graphic	15	38.7	23	27.1	23	14.9	23	17.8
Verbal-symbolic	3	5.8	11	12.9	36	23.4	32	24.8
Verbal-tool	7	13.5	4	4.7	2	1.3	1	0.8
Verbal-tabular	2	3.8			2	1.3	1	0.8
Graphic	1	1.9	2	2.4				
Graphic-symbolic			2	2.4	9	5.8	2	1.6
Graphic-tabular							2	1.6
Symbolic			1	1.2	28	18.2	29	22.5
Symbolic-tabular					2	1.3		
Tabular							1	0.8
Verbal-graphic-symbolic	1	1.9			20	13	6	4.6
Verbal-symbolic-tabular							3	2.3
Verbal-tabular-graphic							1	0.8

That distribution prompts two questions: what did the textbooks use each system for? Did the use of one system or another depend on the measurement skill or idea to be conveyed? Table 6 gives the number of units of analysis for the most frequently used representation systems, broken down by skill. The data show that definitions and choice of units of measure and tools appear primarily in verbal representation only.

Table 6. Relationship between representation system and measurement skill.

	Verbal	Verbal-Graphic	Verbal-Symbolic	Verbal-Tool	Symbolic	Verbal-Graphic-Symbolic
Define	23					
Compare	11	14	5			
Units of measure	28	3	4			
Choose tool	17	9				
Choice of tool	7	44	1	12		2
Equivalence	2	1	20		49	7
Operations	15	7	38		8	2
Measure (gral)	10		2			1
MT	3	2	12		1	14

The textbooks nonetheless combine verbal with graphic representation in explanations or tasks on the use of tools for comparison or measurement (Figure 8). At the same time, verbal representation often appears in conjunction with tasks calling for operations that involve measuring magnitudes. Such tasks are generally contextualised and include units of measure expressed as mathematical symbols. Figure 2 depicts such a case.

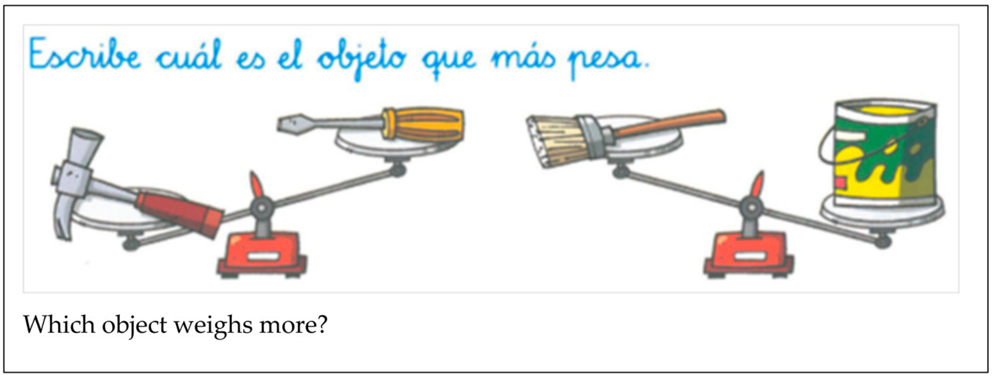


Figure 8. Comparison of mass using tools (first year mathematics, Anaya [48]).

Lastly, high usage rates of symbolic language were found in tasks involving equivalence (Figure 9) or indirect measurement.

0.36 hg = ◆◆◆ dg	6.11cg = ◆◆◆ dag	3002g = ◆◆◆ kg
303 mg = ◆◆◆ g	3.21dag = ◆◆◆ kg	550kg = ◆◆◆ t

Figure 9. Inter-unit of measure equivalence (sixth year mathematics, SM [49]).

3.3. Phenomenology

The meaning of mathematical content includes understanding the contexts and situations giving rise and applicable to the content at issue. As the data in Table 7 show, most of the units of analysis on mass and volume measurement in textbooks were classified under the classroom context category. In mathematics, however, around 12% of the tasks involve personal and in Anaya also occupational contexts (Figure 10).

Table 7. Phenomenology by subject and publisher.

	Science				**Mathematics**			
	SM		**Anaya**		**SM**		**Anaya**	
	N	**%**	**N**	**%**	**N**	**%**	**N**	**%**
Personal	4	7.7		0.0	18	11.7	16	12.4
Occupational	1	1.9	1	1.2	4	2.6	15	11.6
Societal		0.0	1	1.2	2	1.3		0.0
Scientific		0.0		0.0	1	0.6		0.0
Classroom	47	90.4	82	97.6	129	83.8	98	76.0
Total	54		84		154		129	

Most of the word problems in non-classroom contexts were found in mathematics, with most calling for operating with units of measure (Figures 2 and 10), i.e., skills characteristic of the teaching unit on numbers, although they also endow measurement with meaning and utility. In many tasks, such as in Figure 2, the measurements described are scantly realistic, however.

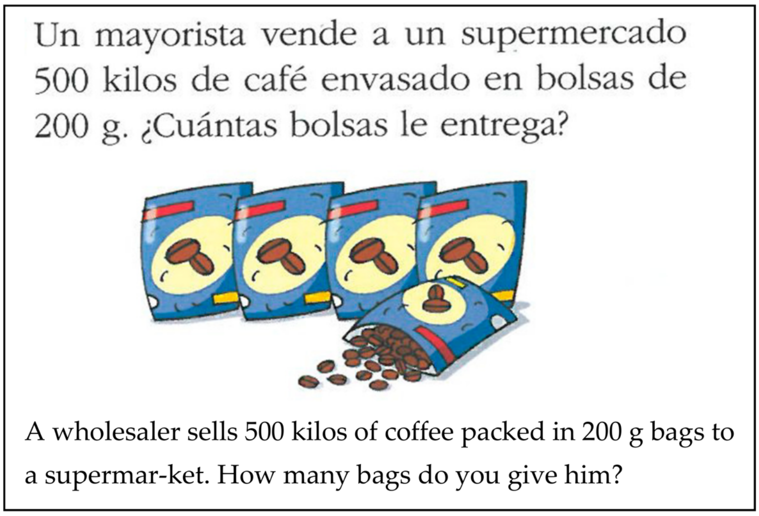


Figure 10. Example of occupational context task (fifth year mathematics, Anaya [50]).

4. Conclusions

This study aimed to explore the applicability of textbooks to STEM education based on their descriptions of the meaning of mass and volume in natural science and mathematics primary school textbooks in an attempt to identify elements common to both subjects and better understand the purpose sought with each. The first step, a review of curricular provisions, was followed by an analysis of the two magnitudes together and then each separately. The elements of the scientific experiments proposed in these books were then examined to detect their merits and short-comings in terms of their applicability to STEM education. A final stage consisted in determining the features of textbook content that would ensure their utility in STEM instruction.

Curricular provisions establish relationships between the two subjects, contending that science is an ideal context for applying mathematical knowledge and mathematics a discipline requisite to the acquisition of scientific competence [7]. Some of the skills associated with measurement appear as objectives shared by the two subjects, and others as mutually complementary content.

In the books analysed, tasks involving the comparison of mass and of volume are put forward as soon as these magnitudes are initially introduced, in keeping with research suggestions about the teaching of measurement [19]. Comparison is present in all science books, at times in connection with the comparison of object density. In mathematics books, in contrast, that skill appears in the earliest years only and in just 10% to 17% of the tasks. That might be attributed to the fact that these matters are introduced in pre-school and, as previous studies showed that when students are provided with suitable instruction, they acquire the ability to measure with standard units in first year primary and even in pre-school [20,21].

The textbooks for the first 3 years of primary education in the two subjects were found to consistently pursue development of the same mass-related skills: to compare objects using pan (as a rule) or weight balancing scales, to understand the main units of mass (g and kg) and to use measuring tools. Verbal and graphic (drawings of the tools) representation systems are used in the explanations and tasks involving the use of tools found in mathematics books. In the science books, in contrast, although many tasks use that same combination, the use of actual measuring tools for such comparisons is recommended more frequently.

In the upper years, the mathematics book approach to teaching mass assumes that students understand the magnitude and focuses rather on procedural considerations such as numerical equivalence among units of measure, simple and complex ways to express measurements, operations with measurements and solving problems involving measurements. In such an approach, students are required to perform calculations and working with numbers that express measurement but not to use specific materials. That gives cause for concern, for as the NCTM [36] notes, "it is unlikely that children can gain a deep understanding of measurement without handling materials, making comparisons physically, and measuring with tools" (p. 44). The skills required are in fact more closely related to understanding and operating with the decimal system than to measurement per se [27]. These elements are cited in Royal Decree 126/2014 [14] under the unit for teaching measurement, while assessment criterion 4 for measurement specifies that students must be able "to use the most common units of measure, convert one unit into others within a given magnitude, express the results in the most suitable unit, explain the procedure followed both orally and in writing and apply it to solve the problem" (p. 19391). That notwithstanding, most of the units of measure appearing in the tasks (Figures 2, 4 and 8) are unlikely to be used in everyday life, among others because contemporary measuring tools use kilogrammes or grammes as the sole units of measure. Furthermore, given the relationships among the units of measure, measurements are understandable using kilogrammes, grammes, milligrammes and tonnes, with no need for the intermediate units. At the same time, whilst mathematics books include some word problems alluding to everyday life (personal context in the PISA definition), their actual utility is questionable inasmuch as they are scantly realistic. In our opinion, such skills would rarely form part of a STEM proposal, for they neither appear in real problems nor constitute the objective of scientific endeavour. In contrast, STEM proposals could be designed to develop mass-related skills in the early years consistent with the approaches adopted in science and mathematics books, thereby serving as a supplementary resource.

Some of the proposals put forward in the two subjects in connection with volume-related skills and strategies were found to be complementary. The skill stressed in science books is volume comparison and measurement with graduated containers, with a few indirect allusions to the relationship between volume and capacity and to the use of liquids to determine the volume of solids. Mathematics books primarily aim to compare and measure the volume of bodies either with unit cubes, or, where these units do not completely fill the geometry at issue, indirectly via formulas based on general decomposition and the application of Cavalieri's principle. Another skill stressed in mathematics books is an understanding of the equivalence between decimal system units. The differences on measuring methods and units of measure used in the textbooks of the two subjects might prevent students from realising that one and the same concept (volume) is involved. That disconnect was highlighted in earlier studies, such as authored by Monterrubio-Pérez et al. [51], who compared physics and mathematics instruction in secondary education. Explicitly addressing the relationship between millilitre and cubic centimetre and analysing the advantages and drawbacks of each method might be a way to enhance students' understanding of these notions and of the relationship between mathematics and science. Similarly, we believe that directly measuring volume with cubes should be introduced before moving on to graduated containers for measuring the volume of a submerged body. Our findings in that regard are similar to the results reported by Palacios-Díaz and Criado [29], who found word problems involving mass and volume in everyday situations to be particularly scant. A similar scarcity of allusions to scientific contexts was observed in this study. The unlike treatment of mass and volume magnitudes observed in the studied textbooks could make it difficult to relate the magnitudes treated in the two subjects, making it complex to understand the concept of density, as indicated [29]. This occurs since in science, mass and volume are related to physical phenomena such as flotation, while in mathematics, mass is studied through comparison with weighing scales as well as the calculation of volume using formulas is emphasized, or, in the best of cases, the counting of unit cubes. As has been stressed in the literature, conducting scientific experiments often calls for an in-depth understanding of the measurement of different magnitudes and their approximate nature [12]. No reference to that issue was found in the textbooks analysed, however. One possible explanation might be that in light of its complexity, that idea is deemed to be more suitable for later years of schooling. The NCTM [36] suggests that students should be familiarised with approximation, precision and accuracy in primary level (specifically in grades 3 to 5 U.S. Educational System) with activities calling for object measurement, comparison of the measurements of several objects and recognition of the inconsistencies. Understanding the need to use a significant number of decimals when calculating measurements is an objective pursued in secondary education. In the Spanish curriculum, one of the items in primary education mathematics content is the "development of strategies to measure figures exactly and approximately" [14], although the study of error types is reserved to secondary education [15]. In another vein, the failure of any of the textbooks analysed to use scientific contexts to stress the need to measure object mass or volume is at least surprising.

Another element highlighted in mathematics education and which has a mere token presence in these textbooks is the estimation of object mass or volume. That ability, in addition to its utility in everyday life (very precise measurement is not always needed nor is a suitable tool always available), may help detect errors in using tools, reading the results or logging the experimental data if the result of the measurement has been previously estimated or predicted with the necessary accuracy. We also believe that addressing approximation and estimation is requisite to a fuller understanding of the nature of mathematics, which while traditionally associated with exactitude also has an approximate dimension, particularly as regards measurement and statistics.

In short, we deem that the teaching approach found in textbooks must be revisited if they are to be used in STEM instruction. In the earliest years, prioritising the use of personal, professional or scientific contexts would further STEM education, characterised by problem solving, everyday situations and inter-disciplinarity [3,52]. Otherwise, such books would be useful only as a supplement or reinforcement of the skills addressed in a given STEM proposal. In later years including measurement in word problems involving scientific experiments or engineering-related issues would afford opportunities to develop an understanding of these ideas and prove the utility of measurement in those pursuits. From that standpoint, textbooks should address particulars such as the approximate nature of measurement, decisions on the level of accuracy required depending on the situation at hand and estimation as a procedure to enhance instrumental readings.

In any case, it is important for teachers to be aware of how the concepts relating to mass and volume are presented in textbooks and how these could enhance learning or possibly lead to misconceptions as well as to promote these important skills through other learning opportunities, including STEM tasks and authentic everyday tasks, in explicit maths or science lessons.

The authors are aware that confining this analysis to only the two best-selling Spanish publishers' books limits the validity of generalising the findings. The sample will be expanded in future research to address that issue.

Designing STEM teaching proposals integrating the four disciplines or highlighting the importance of measurement when seeking answers to scientific questions will call for significant effort on the part of educators. That informs the advisability of focusing future analyses of STEM education proposals on their delivery by pre- and in-service teachers, as well as on those professionals' own proposals in this regard.

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