Available online at www.sciencedirect.com

ScienceDirect

Procedia Computer Science 55 (2015) 593 – 602

Procedia Computer Science

Information Technology and Quantitative Management (ITQM 2015)

On multi-granular fuzzy linguistic modelling in decision making

J. A. Morente-Molinera[a], I. J. Pérez[b,*], R. Ureña[a], E. Herrera-Viedma[a]

[a]Dept. of Computer Science and Artificial Intelligence, University of Granada, jamoren@decsai.ugr.es, viedma@decsai.ugr.es, raquel@decsai.ugr.es, Granada, Spain

[b]Dept. of Computer Sciences and Engineering, University of Cadiz, ignaciojavier.perez@uca.es, Cadiz, Spain

Abstract

Nowadays, the human-computer interaction is being a hot topic. In such a way, several methods have been proposed to deal with multi-granularity when people with different knowledge level express their preferences on the same concept using linguistic notation, that is, words instead of numbers. This is a critical problem in group decision making scenarios, but all the existing approaches have their own advantages and drawbacks. Therefore, some work better in certain environments than others. In such a way, choosing the best method in each situation is critical for obtaining good quality results. In this contribution, an analysis on the different fuzzy linguistic multi-granular modelling approaches is presented in order to provide the reader some advice of what method should be chosen depending on the problem and the quality of results that the user expects to obtain.

© 2015 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

Peer-review under responsibility of the Organizing Committee of ITQM 2015

Keywords: Computing with words; Group decision making; Fuzzy linguistic modelling; Multi-granular linguistic information.

1. Introduction

Group decision making (GDM) is used to obtain the best solution(s) for a problem according to the information provided by some decision makers. Usually, each decision maker (expert) may approach the decision process from a different angle, but they have a common interest in reaching an agreement on taking the best decision. Concretely, in a GDM problem we have a set of different alternatives to solve the problem and a set of experts which are usually required to provide their preferences about the alternatives by means of a particular preference format.

In an ideal GDM situation, all the experts could express their preferences in a precise way by using numerical values. Unfortunately, in many cases, due to the experts background or the kind of information, experts can not represent their preferences precisely in a quantitative way. In these cases, it seems to be more adequate the use of qualitative concepts instead of numerical values. Several authors have provided interesting results on GDM with the help of fuzzy set theory [1]. Even, some of them have proposed the necessity of a linguistic approach to model that situations [2]. A linguistic approach is an approximate technique which represents qualitative aspects as linguistic values by means of linguistic variables, that is, variables whose values are not numbers but

*Corresponding author

1877-0509 © 2015 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).
Peer-review under responsibility of the Organizing Committee of ITQM 2015
doi:10.1016/j.procs.2015.07.049

words or sentences in a natural or artificial language. In the current literature, it is possible to find two kinds of fuzzy linguistic approaches in order to represent linguistic information [3]: traditional fuzzy linguistic approach and ordinal fuzzy linguistic approach. The former is more classical and is based on the membership functions associated to each label [1], while the latter is based on the symbolic ordinal representation of the labels [2]. The symbolic approximation approach has awakened high interest among the scientific community because of its simplicity and application possibilities.

An important parameter to determine a linguistic approach is the number of linguistic variables, that is, the cardinality of the term set. There are cases where experts are non homogeneous in the sense of they have different background and levels of knowledge about the alternatives, and as consequence, they might use linguistic term sets with different granularity to express their preferences. In such cases, we say that GDM problem is defined in a multigranular fuzzy linguistic context [4].

The multi-granular fuzzy linguistic modelling (FLM) is appropriate in cases where several information providers need different criteria to express their preferences. For example, this could happen when they have different knowledge levels and need different expression linguistic domains with a different granularity and/or semantics. Multi-granular FLM has been applied successfully in areas such as information retrieval, recommender systems, consensus, web quality and decision making.

The aim of this paper is to show a comprehensive presentation of the state of the art of all known multi-granular FLM approaches, with an in-depth analysis of the respective problems and solutions. Methods selected after carrying out a systematic review process have been classified into six different categories:

1. Traditional multi-granular FLM based on fuzzy membership functions.
2. Ordinal multi-granular FLM based on a basic Linguistic Term Set.
3. Ordinal multi-granular FLM based on 2-tuple FLM.
4. Ordinal multi-granular FLM based on hierarchical trees.
5. Multi-granular FLM based on qualitative description spaces.
6. Ordinal multi-granular FLM based on discrete fuzzy numbers.

This paper is organized as follows. Section 2 presents some background information about GDM problems and multi-granular FLM. In Section 3, different multi-granular fuzzy linguistic approaches are described. Finally, some conclusions are pointed out.

2. GDM problems and multi-granular FLM background

A GDM problem is classically defined as a decision situation where a set of experts, $E = \{e_1, e_2, \ldots, e_m\}$ ($m \geq 2$), express their preferences about a set of feasible alternatives, $X = \{x_1, x_2, \ldots, x_n\}$ ($n \geq 2$), and they work to achieve a consensual solution. In many decision situations it is assumed that each expert e_i provides his/her preferences by means of a fuzzy preference relation, $P_{e_i} = [p_i^{lk}],\ l, k \in \{1, \ldots, n\}$ with $p_i^{lk} = \mu_{P_{e_i}}(x_l, x_k)$ assessed in the unit interval $[0, 1]$ and being interpreted as the preference degree of the alternative x_l over x_k according to the expert e_i. Another possibility is that experts use linguistic preference relations to represent their preferences, i.e., with $p_i^{lk} = \mu_{P_{e_i}}(x_l, x_k)$ assessed in a linguistic term set (LTS) S. The ideal situation for GDM problems defined in linguistic contexts would be that all the experts use the same LTS S to express their preferences about the alternatives. However, in some cases, experts may belong, e.g., to distinct research areas and, therefore, could have different background and levels of knowledge. A consequence of this is that they need to express their preferences by using LTSs with different granularity, $S_i = \{s_0^i, \ldots, s_{g_i}^i\},\ i \in \{1, 2, \ldots, m\}$. In these cases, we say that the GDM problem is defined in a multi-granular fuzzy linguistic context [4].

Multi-granular FLM was first introduced by Herrera et al. [4]. They designed a GDM method where each expert can use a different ordinal LTS in order to provide his/her preferences. In such a way, they defined a new fuzzy linguistic framework to make decisions that allowed experts to express their preferences using the concept of linguistic variable introduced by Zadeh [1], but in a more flexible way, i.e., using different LTS to express the different assessments of the linguistic variable. This multi-granular fuzzy linguistic approach was introduced assuming that the qualitative information in the GDM problem was modeled using an ordinal fuzzy linguistic approach.

The ordinal fuzzy linguistic approach is defined by considering a finite and totally ordered label set $S = \{s_i\}, i \in \{0, \ldots, \mathcal{T}\}$ in the usual sense, i.e., $s_i \geq s_j$ if $i \geq j$, and with odd cardinality (typically 7 or 9 labels). The mid term represents an assessment of *approximately 0.5*, and the rest of the terms are placed symmetrically around it. The semantics of the LTS is established from the ordered structure of the term set by considering that each linguistic term in the pair $(s_i, s_{\mathcal{T}-i})$ is equally informative. For example, we can use the following set of seven labels to provide the expert preferences: S ={ *N = None,VL = Very_Low, L = Low, M = Medium, H = High, VH = Very_High, T = Total* }. An important issue to analyze is the "granularity of uncertainty", i.e, the cardinality of the LTS. The granularity of S should be small enough so as not to impose useless precision levels on the users but large enough to allow a discrimination of the assessments in a limited number of degrees.

Sometimes, the semantics of S can be completed by associating to the labels any fuzzy numbers defined on the unit interval $[0, 1]$. One way to characterize a fuzzy number is by using a representation based on parameters of its membership function.

3. Analysis of multi-granular FLM methods

In this section, the main primary studies about multi-granular linguistic approaches are described. As mentioned in the introduction, the multi-granular linguistic approaches are organized into six different methodologies.

3.1. Traditional multi-granular FLM based on fuzzy membership functions

This methodology follows a traditional multi-granular FLM based on membership functions approach [5, 6]. The next scheme is used in order to deal with multi-granular information:

1. All the labels belonging to different LTSs present an associated semantics represented by membership functions.
2. Computations are carried out on the membership functions of the labels.
3. Unless some kind of transformation is performed, computation results are expressed using the membership functions instead of particular linguistic labels.

Generally, Trapezoidal Fuzzy Numbers (TFNs) are used in order to represent the information and carry out the required computations. They have a strong mathematical environment that let us to work with a wide range of operations. The disadvantage is that it is a troublesome task to express them linguistically making the results interpretation and data providing become difficult tasks for common people. Experts can provide their preferences using an ordinal LTS. In such a way, the experts provide labels that are translated into TFNs in order to carry out the GDM computations. This solves the data providing problem but not the results interpretation, because we have to translate a membership function into a particular label of the original LTS. Because labels have their own semantics, results that do not match to any of the label semantics are obtained after performing computations. No solutions are presented in the analysed papers to this issue. The authors probably did not considered this to be a problem because they were only calculating an alternative ranking. Nevertheless, if consensus approaches want to be applied to these methods, results cannot be given to the experts using TFNs because they lack interpretation. One way of solving this issue, although it could imply a loss of information, consists in assigning to each obtained TFN that label whose semantics (also a TFN) is the closest one. Using TFNs, it is also possible to avoid the labels unification process. For instance, in [7], authors present a novel way of aggregating LTSs using TFNs and the T1OWA operator. The main novelty of this method is that it is not necessary to carry out an unification process, labels from different LTSs can be aggregated directly using the T1OWAoperator.

Regular LTSs are not the only way for experts to provide their preferences. For example, uncertain linguistic terms (ULTS) can be employed [6]. ULTSs allow users to provide their preferences using a label interval instead of a unique label. This way, preference constructions like *I prefer x_1 to x_2 with a between high and very high degree* can be used. ULTSs also help in the task of classifying linguistic terms that do not belong to any LTS. So, they can be part of the linguistic term interval that better suits them.

Once that all the preferences are expressed in terms of an unique TFN, calculation of the collective preference information piece is carried out. For this process, goal programming model is a preferred method. Then, the TFN whose distance to all the provided TFNs is minimum is selected. Finally, ranking is made through the selection process. Closeness coefficients using the TOPSIS principle [6] or the calculation of dominance degree [5] are two possibilities. In Figure 1, the explained process can be seen schematically.

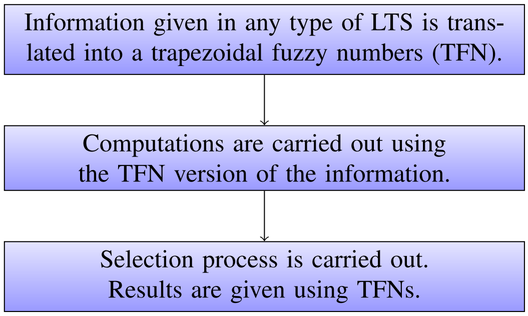

Fig. 1. Scheme of traditional multi-granular FLM based on fuzzy membership functions approaches.

3.2. Ordinal multi-granular FLM based on a Basic LTS

Multi-granular GDM methods classified in this category follow the next steps (see Figure 2):

1. **Providing preferences**: Experts provide their preferences using the LTS that better fits them.
2. **Making the information uniform**: All the provided information is expressed using a unique LTS that is called BLTS. In such a way, the same LTS is employed for all the preference values and any operation can be carried out.
3. **Computing collective values**: All the provided information is aggregated into a collective piece of information.
4. **Selection phase**: Using the collective preference values and any selection criteria, ranking among the alternatives is made.

Different ways of representation of preferences could be used:

- **Balanced ordinal LTSs**: [4, 8]: Balanced ordinal LTSs are LTSs whose number of linguistic terms is odd and they are equally distributed in an ordinal scale. Labels belonging to other LTSs are translated to the LTS that has the highest granularity in order to carry out computations. This could become a disadvantage if operations with a high number of labels are inadequate or not desired. The method presented in [8] established this requirement and allowed the use of any balanced LTS as the BLTS. In such a way, small granularity LTSs can become the BLTS easing the subsequent operations. It should be pointed out that the smaller the BLTS, the less the representation capability is and, consequently, the more loss of information is produced. The main problem of using balanced LTSs is that they have several restrictions that reduce flexibility. If balanced ordinal LTSs are used, information can be aggregated using OWA operator and selection process can be carried out using the non-dominance degree.
- **Unbalanced LTSs** [9]: In order to introduce flexibility in the way that experts express their preferences, unbalanced LTSs can also be used in this methodology. The main problem is that not every unbalanced LTS is allowed, only the ones having most of the labels concentrated near the LTS medium term having the same number of labels before and after it. Although this is a clear representational advantage, the allowed unbalanced LTSs are not the most ideal ones. In general, experts are interested in unbalanced LTSs that have labels concentrated in the right side or the left side of the medium label. For example, they can require more options when trying to give a positive answer than a negative one. They can be more interested and need more specification when trying to rate how positive an alternative is than how negative it is. In the recent literature, this issue is not yet resolved. Another disadvantage is that linguistic terms are indexed using fractions instead of natural numbers. This reduces readability and introduces complexity to the model.
- **Uncertain LTSs** [9] : If uncertain LTSs are used, information can be aggregated using the ULWA operator and selection process can be carried out using, for example, the degree of possibility.

3.3. Ordinal multi-granular FLM based on 2-tuple representation model

Ordinal multi-granular FLM based on 2-tuple and Linguistic Hierarchies [10, 11, 12] uses the 2-tuple representation model in order to deal with multi-granular information. 2-tuple linguistic modelling provides an easy

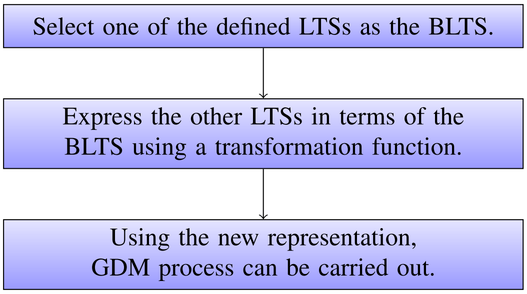

Fig. 2. Ordinal multi-granular FLM based on Basic LTS methodology scheme.

way of dealing with LTSs and operating with them without any loss of information. Furthermore, they are able to represent elements that do not belong to the initial LTS. These properties can be used in order to develop methods that deal with multi-granular linguistic information.

2-tuple linguistic modelling is based on the concept of symbolic translation. Let β be the result of an aggregation of the indexes of a set of terms assessed in a LTS S whose cardinality is $g+1$. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values such that $i \in [0, g]$ and $\alpha \in [-0.5, 0.5)$ then α is called a symbolic translation where $round()$ is the usual round operation.

Let $S = \{s_i | i = 0, 1, 2, \ldots, g\}$ be a LTS and $\beta \in [0, g]$ a value representing the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following expression:

$$\Delta : [0, g] \rightarrow S \times [0.5, 0.5) \tag{1}$$

$$\Delta(\beta) = (s_i, \alpha), \text{ with } \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-0.5, 0.5) \end{cases} \tag{2}$$

where s_i has the closest index label to β and α is the value of the symbolic translation.

Let $S = \{s_i | i = 0, 1, 2, \ldots, g\}$ be a LTS and (s_i, α) be a 2-tuple. There exists a function Δ^{-1} such that from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g] \subset \mathfrak{R}$:

$$\Delta^{-1} : S \times [0.5, 0.5) \rightarrow [0, g] \tag{3}$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta \tag{4}$$

A conversion of any linguistic label into a linguistic 2-tuple can be performed directly adding a zero value as a symbolic translation

$$\Delta(s_i) = (s_i, 0), i = 0, 1, 2, \ldots, g \tag{5}$$

Two different approaches that use 2-tuple representation method can be followed:

- **Use of generalized linguistic 2-tuple variable** [12]: All the 2-tuple LTSs are expressed using the generalized linguistic 2-tuple variable [13]. Posterior GDM operations are carried out using it. The main advantage of this approach is the simplicity. Its main drawback is that all the GDM processes have to be carried out using an specific unchanging representation. The scheme used by this approach follows the next steps:
  1. Experts express their preferences using their preferred LTSs.
  2. LTSs are translated into 2-tuple linguistic information using equation (5).
  3. 2-tuple linguistic information provided by the users are translated into the generalized linguistic 2-tuple variable.
  4. Information can be then aggregated and selection processes can be applied in order to obtain the alternatives ranking. IVTWA operator [12] can be used for the aggregation and selection process can be carried out comparing the obtained collective values for each alternative.

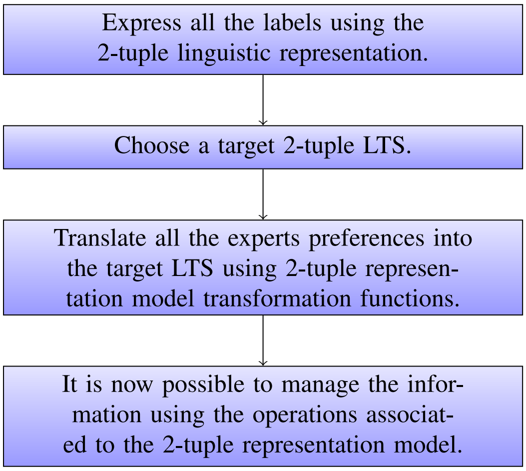

Fig. 3. Ordinal multi-granular FLM based on 2-tuple scheme.

- **Linguistic hierarchy building** [10, 11]: A hierarchy is built using the LTSs. Although this approach is more complex, the hierarchy allows translations among the different LTSs that conforms it. In such a way, any LTS of the hierarchy can be used for carry out computations. The scheme used by this method follows the following steps:
  1. A hierarchy is built using the LTSs that experts use to express themselves.
  2. One of the LTSs that conforms the hierarchy is used as the target LTS for carrying out the computations.
  3. Preferences provided by the users using different LTSs are translated into the target LTS. Thanks to 2-tuple representation model and Linguistic Hierarchies (LHs), it is possible to carry out this process without loss of information.
  4. Information is aggregated and selection process is carried out in order to calculate the alternatives ranking.

  In general, each level of a LH represents a unique LTS. This way, expressions that let us express labels from one level into another one within the hierarchy are defined. These methods tend to be efficient and are able to provide results in a linguistic manner without needing to use, in most of the cases, a defuzzification process. They also avoid loss of precision in the fusion of multi-granular linguistic information. The main drawback of this method is that only the LTSs that define the hierarchy can be used by the experts in the decision process. It is also not possible to use LTSs that are unbalanced [14] or have atypical characteristics. Another problem is that all the LTSs that conform the LH have to keep their formal modal points from one level to the next. This problem is partially solved in [11]. Authors allow the creation of hierarchies that do not have to keep all the former modal points of the previous levels. Nevertheless, it requires the creation of a new level that usually has an enormous granularity value. For this reason, several ways of minimizing the granularity level are provided. Nonetheless, the model can become extremely complex and difficult to manage. The granularity increases with the number of levels of the hierarchy making the hierarchies with a high number of levels become unmanageable. A scheme of the process followed in this methodology can be seen in Figure 3.

3.4. Ordinal multi-granular FLM based on hierarchical trees

Hierarchical trees [15] are a special hierarchical construction that is built directly over the labels, without taking into account any semantics associated to them. Each level of the tree represents a different LTS. The closer the LTS is to the tree root, the less granularity it has. Comparing to LHs, Hierarchical trees are more flexible

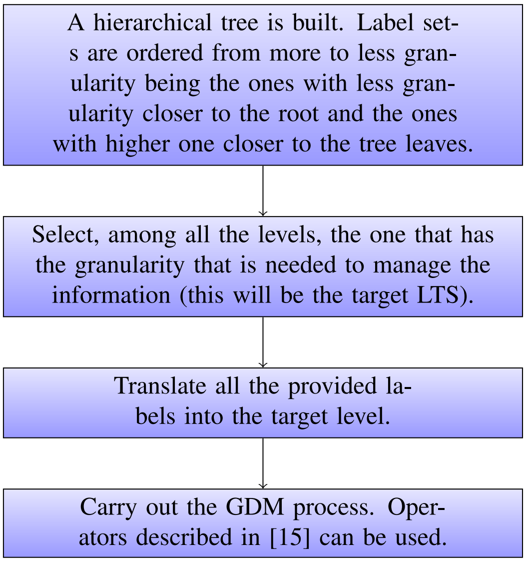

Fig. 4. Ordinal multi-granular FLM based on hierarchical trees scheme.

because any structure is valid as long as each label is connected to one label from the previous level and at least another one of the next. Its main disadvantage is that translations from labels belonging to lower granularity LTSs to labels belonging to high granularity ones can lead into the creation of a set of transformation rules.

Aggregation and selection phase can be done using a choice function based on the satisfactory principle [15]. Satisfactory principle claims that *it is perfectly satisfactory to select an alternative as the best if its performance is as at least good as all the others under the same evaluation scheme*. The process carried out by this method can be seen in Figure 4.

3.5. Multi-granular FLM based on qualitative description spaces

This method uses description spaces in order to represent the information [16]. A description space is an ordered triple $(\Lambda, \mathbb{S}_n, \mu)$ where $\Lambda = \{a_t | t \in I, I \subset \mathbb{R}\}$ is a set of features, $\mathbb{S}_n$ is an order-of-magnitude space with granularity n and μ is a normalized measure defined in $\mathbb{S}_n$, such that all features in Λ are assessed by $\mathbb{S}_n$ labels [16]. In GDM problems, Λ is used to represent the alternatives or, in the case of multi-criteria GDM, the different criteria used. $\mathbb{S}_n$ is the label set used by the experts to express their preferences and μ can be used to give weights to the different labels of the LTS. In such a way, unbalanced LTSs with more granularity in one of the extreme sides of the interval can be defined. Consequently, experts could have more specification possibilities and precision when giving positive evaluations than negative, if necessary.

A qualitative assessment Q can be associated to a description space. Given a description space $(\Lambda, \mathbb{S}_n, \mu)$, a qualitative assessment Q is a mapping $Q : \Lambda \to \mathbb{S}_n$. It can be seen straightforward from this definition that qualitative assessments will be used in the GDM problems in order to represent the preferences provided by the experts. It can be seen that description spaces provide a mathematical representation that fits perfectly a GDM problem representation.

In order to introduce multi-granularity in the described environment, the generalized description space concept is introduced. A generalized description space is an ordered triple such that

$$\left(\Lambda, \bigcup \mathbb{S}_{n_i}, \{\mu\}\right) = \left(\biguplus_{i=1}^{r} \Lambda_i, \bigcup_{i=1}^{r} \mathbb{S}_{n_i}, \{\mu_1, \dots, \mu_r\}\right) \tag{6}$$

where $r > 1$. A generalized description space consists in the disjoint union, $\biguplus$, of different sets of features, Λ_i,

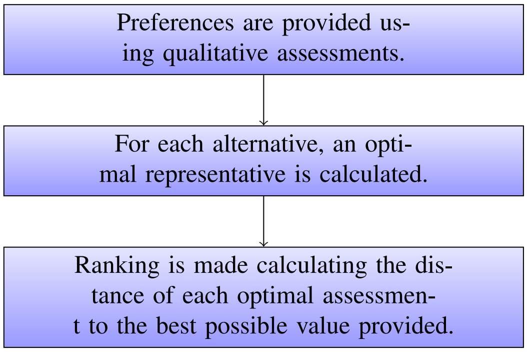

Fig. 5. Ordinal multi-granular FLM based on qualitative description spaces methodology scheme.

where each of them admits qualitative descriptions in $\mathbb{S}_{n_i}$ and normalized measures μ_i defined in $\mathbb{S}_{n_i}$. In such a way, in order to carry out a GDM process, this methodology follows the next steps:

1. Experts give their qualitative assessments using the LTS that they prefer.
2. Find the optimal representative for each of the alternatives. This is done selecting the label that has the less distance to all the provided labels for the same alternative.
3. Distance from the best possible ranking value for each alternative in each description space to the provided one is calculated.
4. Alternatives are ordered by the distances computed to the best possible ranking value.

We should point out that this methodology does not use semantics associated to the labels and, in addition, it does not require any label translation. Therefore, no loss of information is produced in conversions. LTSs with a even number of labels are supported. Its main drawback consists in that multi-granularity is associated to alternatives, that is, the same LTS must be chosen to describe all the preferences provided for an specific alternative. Thus, experts must use the LTS associated with the alternative in order to describe it, not the one that they could prefer. A scheme of the followed process can be seen in Figure 5.

3.6. Ordinal multi-granular FLM based on discrete fuzzy numbers

In [17], the concept of Subjective Linguistic Hierarchy (SLH) is introduced. A SLH is a LH that is built using LTSs whose linguistic terms are represented by Discrete Fuzzy Numbers [17]. Therefore, experts do not provide a single label, they provide a list of all the labels with an associated number in the interval [0,1] that represents the level of agreement that the expert has with that label in the corresponding description. It should be pointed out that value 1 must be assigned to at least one of the labels and monotonicity properties must be fulfilled. For example, taking into account the LTS $S = \{s_1, \ldots, s_7\}$, a preference value of $\{0.3/s_5, 1/s_6, 0/s_7\}$ indicates that the preference provided by the expert matches perfectly the label s_6, it has something to do with label s_5 and s_7 should not be considered as a provided label for that description. This type of description is really flexible because it allows the addition of degrees to the labels. Nevertheless, this flexibility reduces the methodology simplicity. Thus, it is more complicated for experts to provide their preferences because they have to think on the numerical degree that they want to give to the labels. Of course, this can be solved if experts are provided with a set of labels that are lately translated into discrete fuzzy numbers. As most of the methodologies explained in previous subsections, this methodology follows the next scheme:

1. **Providing preferences:** Experts express their preferences by means of DFNs. It is also possible to provide single labels due to the fact that a label s_i can be expressed as the DFN $\{1/s_i\}$. This reduces complexity allowing experts to express themselves using labels instead of complex DFNs.
2. **Uniforming preferences:** An LTS is chosen as the target for computations. Sometimes, it can be necessary to carry out translations from LTSs with small granularity to LTSs with bigger granularities. To do so, completions are employed [17]. That is, some labels of the origin LTS are translated directly into labels from the target LTS and others are inferred using the surrounding valuations of the labels belonging to the

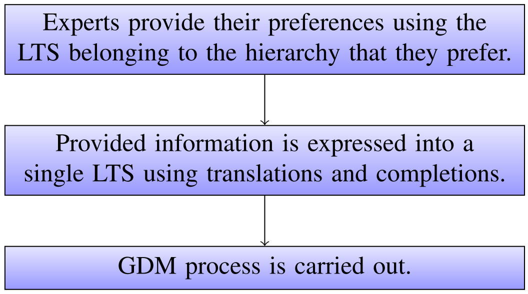

Fig. 6. Ordinal multi-granular FLM based on discrete fuzzy numbers methodology scheme.

target LTS. This way of solving this problem is based in the assumption that labels that are close have similar valuations. For example, if an expert has provided a high valuation value to a high position linguistic term it is not probable that they provide a high intensity value to a term located in a low position. Thanks to completions, it is possible to carry out translations from one level of the hierarchy to another one, allowing us to manage linguistic terms belonging to different LTSs from the hierarchy.

3. **Aggregation phase:** Aggregation functions are used to add the discrete fuzzy numbers into a collective value. Several aggregation functions for discrete fuzzy numbers are exposed in [18].
4. **Selection phase:** Method described in [19] can be used. According to it, left and right dominance concept is applied over the collective matrix calculated in the previous step in order to obtain the ranking of alternatives.

A scheme of the followed process can be seen in Figure 6.

4. Conclusions

The multi-granular fuzzy linguistic modelling allows the use of several linguistic term sets in fuzzy linguistic modelling. This is quite useful when the problem involves several people with different knowledge levels since they could describe each item with different precision and they could need more than one linguistic term set. Multi-granular fuzzy linguistic modelling has been frequently used in group decision making field due to its capability of allowing each expert to express his/her preferences using his/her own linguistic term set. In this contribution, we provide some insights about the evolution of multi-granular fuzzy linguistic modelling approaches during the last years and discuss their drawbacks, advantages and the type of problems and environments where they are more suitable.

The main features of each method are exposed in Table 1. In such a way, researchers can use this information to analyse and select the method that better fits the environment where they want to apply the multi-granular FLM method. As it was aforementioned, it is important to point out that there not exists a method that is better than others for all the possible applications.

Acknowledgements

This paper has been developed with the financing of FEDER funds in FUZZYLING-II Project TIN2010-17876, TIN2013-40658-P and Andalusian Excellence Projects TIC-05299 and TIC-5991.

Table 1. Comparative about techniques used for dealing with multi-granular information.

Technique	Refs	Loss of data	Repr. type	Complexity	Set restrictions	Results in input sets
MFLM based on fuzzy membership functions	[5] [6] [7]	No	Semantic	Medium	Medium	No
FLM based on a Basic LTS	[4] [8] [9]	No	Semantic	Medium	Medium	No
MFLM based on 2-tuple	[10] [11] [12]	No	Symbolic	Low	High	Yes
MFLM based on Hierarchical trees	[15]	Yes	Symbolic	Low	Low	Yes
MFLM based on description spaces	[16]	No	Symbolic	High	Low	Yes
MFLM based on discrete fuzzy numbers	[17]	No	Symbolic	Low	Low	Yes

References

[1] Zadeh LA. The concept of a linguistic variable and its application to approximate reasoning-I. Information sciences 1975;8:199-249.
[2] Alonso S, Pérez IJ, Cabrerizo FJ, Herrera-Viedma E. A linguistic consensus model for web 2.0 communities. Applied Soft Computing 2012;13:149-157.
[3] Herrera F, Alonso S, Chiclana F, Herrera-Viedma E. Computing with words in decision making: foundations, trends and prospects. Fuzzy Optimization and Decision Making 2009;8:337-364.
[4] Herrera F, Herrera-Viedma E, Martínez L. A fusion approach for managing multi-granularity linguistic term sets in decision making. Fuzzy Sets and Systems 2000;114:43-58.
[5] Jiang YP, Fan ZP, Ma J. A method for group decision making with multi-granularity linguistic assessment information. Information Sciences 2008;178:1098-1109.
[6] Zhang Z, Guo C. A method for multi-granularity uncertain linguistic group decision making with incomplete weight information. Knowledge-Based Systems 2012;26:111-119.
[7] Mata F, Gonzaga L, Shang-Ming Z. Type-1 owa methodology to consensus reaching processes in multi-granular linguistic contexts. Knowledge-Based Systems 2014;58:11-22.
[8] Chen Z, Ben-Arieh D. On the fusion of multi-granularity linguistic label sets in group decision making. Computers and Industrial Engineering 2006;51:526-541.
[9] Xu Z. An interactive approach to multiple attribute group decision making with multigranular uncertain linguistic information. Group Decision and Negotiation 2009;18:11-145.
[10] Herrera F, Martínez L. A model based on linguistic 2-tuples for dealing with multigranular hierarchical linguistic contexts in multiexpert decision-making. Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on 2001;31:227-234.
[11] Espinilla M, Liu J, Martínez L. An extended hierarchical linguistic model for decision-making problems. Computational Intelligence 2011;27:489-512.
[12] Zhang H. The multiattribute group decision making method based on aggregation operators with interval-valued 2-tuple linguistic information. Mathematical and Computer Modelling 2012;56:27-35.
[13] Chen CT, Tai WS. Measuring the intellectual capital performance based on 2-tuple fuzzy linguistic information, in: The 10th Annual Meeting of APDSI, Asia Pacific Region of Decision Sciences Institute, 2005;20.
[14] Herrera-Viedma E, López-Herrera A G. A model of an information retrieval system with unbalanced fuzzy linguistic information. International Journal of Intelligent Systems 2007;22:1197-1214.
[15] Huynh VN, Nakamori Y. A satisfactory-oriented approach to multiexpert decision-making with linguistic assessments. Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on 2005;35:184-196.
[16] Roselló L, Sánchez M, Agell N, Prats F, Mazaira FA. Using consensus and distances between generalized multi-attribute linguistic assessments for group decision-making. Information Fusion 2011;32:65-75.
[17] Massanet S, Riera JV, Torrens J, Herrera-Viedma E. A new linguistic computational model based on discrete fuzzy numbers for computing with words. Information Sciences 2014;258:277-290.
[18] Mas M, Monserrat M, Torrens J. Kernel aggregation functions on finite scales. constructions from their marginals. Fuzzy Sets and Systems 2014;241:27-40.
[19] Chen LH, Lu HW. An approximate approach for ranking fuzzy numbers based on left and right dominance. Computers and Mathematics with Applications 2001;41:1589-1602.