IVTURS: a linguistic fuzzy rule-based classification system based on a new Interval-Valued fuzzy reasoning method with TUning and Rule Selection

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***Abstract*—Interval-valued fuzzy sets have been shown to be a useful tool for dealing with the ignorance related to the definition of the linguistic labels. Specifically, they have been successfully applied to solve classification problems, performing simple modifications on the fuzzy reasoning method to work with this representation and making the classification based on a single number.**

In this paper we present IVTURS, a new linguistic fuzzy rule-based classification method based on a new completely interval-valued fuzzy reasoning method. This inference process uses interval-valued restricted equivalence functions to increase the relevance of the rules in which the equivalence of the interval membership degrees of the patterns and the ideal membership degrees is greater, which is a desirable behaviour. Furthermore, their parametrized construction allows the computation of the optimal function for each variable to be performed, which could involve a potential improvement in the system's behaviour. Additionally, we combine this tuning of the equivalence with rule selection in order to decrease the complexity of the system. In this paper we name our method IVTURS-FARC, since we use the FARC-HD method [1] to accomplish the fuzzy rule learning process.

The experimental study is developed in three steps in order to ascertain the quality of our new proposal. First, we determine both the essential role that interval-valued fuzzy sets play in the method and the need for the rule selection process. Next, we show the improvements achieved by IVTURS-FARC with respect to the tuning of the degree of ignorance when it is applied in both an isolated way and when combined with the tuning of the equivalence. Finally, the significance of IVTURS-FARC is further depicted by means of a comparison by which it is proved to outperform the results of FARC-HD and FURIA [2], which are two high performing fuzzy classification algorithms.

***Index Terms*—Linguistic Fuzzy Rule-Based Classification Systems, Interval-Valued Fuzzy Sets, Fuzzy Reasoning Method, Interval-Valued Restricted Equivalence Functions, Tuning, Rule Selection.**

I. Introduction

Fuzzy Rule-Based Classification Systems (FRBCSs) have been widely employed in the field of pattern recognition and classification problems [1]–[8]. Aside from their good performance, FRBCSs are adequate since they also provide a linguistic model interpretable to the users because they are composed of a set of rules composed of linguistic terms [9], [10].

One of the key points in the subsequent success of fuzzy systems (like FRBCSs) is the choice of the membership functions [11]. This is a complex problem due to the uncertainty related to their definition, whose source can be both the intrapersonal and the interpersonal uncertainty associated with the linguistic terms [12], [13].

Interval-Valued Fuzzy Sets (IVFSs) [14] have proven to be an appropriate tool to model the system uncertainties and the ignorance in the definition of the fuzzy terms [15]. An IVFS provides an interval, instead of a single number, as the membership degree of each element to this set. The length of the interval can be seen as a representation of the ignorance related to the assignment of a single number as membership degree [16]. IVFSs have been successfully applied in computing with words [17], mobile robots [18] and image processing [19], [20], among others.

In this paper we present IVTURS, which is short for linguistic FRBCS based on an Interval-Valued fuzzy reasoning method (IV-FRM) with TUning and Rule Selection. The main contribution of IVTURS is a novel IV-FRM in which the ignorance represented by the IVFSs is taken into account *throughout the reasoning process*. To do so, we completely extend the classical fuzzy reasoning method [21] including the computation of the matching degree using Interval-Valued Restricted Equivalence Functions (IV-REFs) [22], [23]. The goal is to show how equivalent are the interval membership degrees of the antecedent of the rules to the ideal interval membership degree ($[1, 1]$). In a nutshell, the higher the equivalence between the example and the antecedent the greater the significance of the rule in the decision process is.

IV-REFs are constructed using parametrized functions. It is therefore easy to construct different functions by modifying the values of their parameters. This fact makes it possible to compute the most suitable set of IV-REFs for each specific problem by defining a genetic tuning to accomplish this optimization problem. Additionally, we combine it with a fuzzy rule selection process as it is a well known synergy in this field to improve both the interpretability and the accuracy of the final fuzzy system [10].

IVTURS is composed of three stages: 1) The generation of an initial Interval-Valued Fuzzy Rule-Based Classification

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System (IV-FRBCS). To do this, we firstly learn the rule base using the recent fuzzy rule learning algorithm known as FARC-HD [1] (Fuzzy Association Rule-based Classification model for High Dimensional problems). Then, we model its linguistic labels with IVFSs and we initialize the IV-REF for each variable of the problem; 2) The application of the new IV-FRM which makes use of IV-REFs and 3) The optimization step using the proposed synergy between the tuning of the equivalence and rule selection. In this paper, our methodology is built-up over the FARC-HD algorithm in order to learn the initial FRBCS, hence denoting the whole model as IVTURS-FARC.

We show the goodness and high potential of the use of IVTURS-FARC firstly by determining the suitability of the synergy produced when combining IVFSs, tuning and rule selection. Next, by studying whether the new IV-FRBCS enhances the results achieved by the tuning of the weak ignorance [24] when it is performed both individually and combined with the new tuning of the equivalence. Furthermore, we analyze the significance of the results obtained with IVTURS-FARC versus the ones achieved by two of the best performing fuzzy methods published in the specialized literature, i.e., the original FARC-HD model [1] and the FURIA algorithm [2]. The performance of the proposals will be evaluated according to the accuracy rate and will be tested over a wide collection of data-sets selected from the KEEL data-set repository[1] [25], [26]. We will use some non-parametric tests [27]–[29] for the purpose of showing the significance of the performance improvements achieved by our proposal.

This paper is arranged as follows: in Section II we recall some preliminary concepts in both FRBCSs and IVFSs theory together with the description of the construction method of IV-REFs used in this paper. Next, we introduce in detail the components of IVTURS-FARC in Section III, which involves the description of the initialization of the IV-FRBCS' components, the definition of the new IV-FRM and the proposal to combine the genetic tuning of the system' parameters with rule selection. The experimental framework and the results obtained by the application of our approaches together with the corresponding analysis are presented in Sections IV and V respectively. We finish the paper with the main concluding remarks in Section VI.

II. PRELIMINARIES

In this section, for the sake of completeness, we first review several preliminary concepts in both FRBCSs and IVFSs. Next, we recall the concept of IV-REFs [22], [23] and the construction method of these functions used in this paper.

A. Fuzzy Rule-Based Classification Systems

There are a lot of techniques used to deal with classification problems in the Data Mining field. Among them, FRBCSs are widely employed as they provide an interpretable model by means of the use of linguistic labels in their rules.

The two main components of FRBCSs are:

- Knowledge Base: it is composed of both the Rule Base (RB) and the Data Base, where the rules and the membership functions are stored respectively.
- Fuzzy reasoning method: it is the mechanism used to classify objects using the information stored in the knowledge base.

In order to generate the knowledge base, a fuzzy rule learning algorithm is applied that uses a set of P labeled patterns $x_p = (x_{p1}, \ldots, x_{pn})$, $p = \{1, 2, \ldots, P\}$ where x_{pi} is the ith attribute value ($i = \{1, 2, \ldots, n\}$). Each of the n attributes is described by a set of linguistic terms together with their corresponding membership functions. In this work, we consider the use of fuzzy rules in the following form:

$$\begin{array}{l}\text{Rule } R_j: \text{ If } x_1 \text{ is } A_{j1} \text{ and } \ldots \text{ and } x_n \text{ is } A_{jn} \\ \text{ then Class} = C_j \text{ with } RW_j\end{array} \tag{1}$$

where R_j is the label of the jth rule, $x = (x_1, \ldots, x_n)$ is an n-dimensional pattern vector, A_{ji} is an antecedent fuzzy set representing a linguistic term, C_j is the class label, and RW_j is the rule weight [30]. Specifically, in this paper we consider the computation of the rule weight using the most common specification, that is, the fuzzy confidence value or certainty factor defined in [31] as:

$$RW_j = CF_j = \frac{\displaystyle\sum_{x_p \in Class C_j} \mu_{A_j}(x_p)}{\displaystyle\sum_{p=1}^{P} \mu_{A_j}(x_p)} \tag{2}$$

where $\mu_{A_j}(x_p)$ is the matching degree of the pattern x_p with the antecedent part of the fuzzy rule R_j.

Let $x_p = (x_{p1}, \ldots, x_{pn})$ be a new pattern to be classified, L denote the number of rules in the RB and M the number of classes of the problem; then, the steps of the fuzzy reasoning method [21] are as follows:

1) *Matching degree*, that is, *the strength of activation of the if-part for all rules in the RB with the pattern* x_p. A conjunction operator (t-norm), *T*, is applied in order to carry out this computation.

$$\mu_{A_j}(x_p) = T(\mu_{A_{j1}}(x_{p1}), \ldots, \mu_{A_{jn}}(x_{pn})), \qquad j = 1, \ldots, L. \tag{3}$$

2) *Association degree*. To compute the *association degree of the pattern* x_p *with the M classes according to each rule in the RB.* To this aim, a combination operator, *h*, is applied to combine the matching degree with the rule weight. When using rules in the form shown in (1) this association degree only refers to the consequent class of the rule (i.e. $k = Class(R_j)$).

$$b_j^k = h(\mu_{A_j}(x_p), RW_j^k), \qquad k = 1, \ldots, M, \quad j = 1, \ldots, L. \tag{4}$$

3) *Pattern classification soundness degree for all classes.* We use an aggregation function, *f*, which combines the positive degrees of association calculated in the previous step.

[1](http://www.keel.es/dataset.php)

$$Y_k = f(b_j^k,\ j = 1,\dots,L \text{ and } b_j^k > 0), \qquad k = 1,\dots,M. \quad (5)$$

4) *Classification*. We apply a decision function F over the soundness degree of the system for the pattern classification for all classes. This function will determine the class label l corresponding to the maximum value.

$$F(Y_1,\dots,Y_M) = \arg\max_{k=1,\dots,M}(Y_k) \quad (6)$$

B. Interval-Valued Fuzzy Sets

In this section we introduce the IVFSs' theoretical concepts which are necessary to understand the paper. First, we recall the definition of IVFSs with our interpretation of their length. Then, we remind both the intersection operation on IVFSs, which will be used to carry out the conjunction among the antecedents of the rules, and the complement operation on IVFS, which is used in the definition of IV-REFs. Next, we present the interval arithmetic that will be used to compute the rule weight as an element of $L([0,1])$. Finally, we introduce the total order relationship for intervals, which will be used in the classification step of the IV-FRM.

Let us denote by $L([0,1])$ the set of all closed subintervals in $[0,1]$, that is,

$$L([0,1]) = \{\mathbf{x} = [\underline{x},\overline{x}] | (\underline{x},\overline{x}) \in [0,1]^2 \text{ and } \underline{x} \leq \overline{x}\}.$$

Definition 1: [32], [33] An interval-valued fuzzy set (or interval type 2 fuzzy set) A on the universe $U \neq \emptyset$ is a mapping $A_{IV} : U \to L([0,1])$, so that

$$A_{IV}(u_i) = [\underline{A}(u_i), \overline{A}(u_i)] \in L([0,1]), \text{ for all } u_i \in U.$$

We denote by $\mathcal{L}$ the length of the interval under consideration, that is

$$\mathcal{L}(A_{IV}(u_i)) = \overline{A}(u_i) - \underline{A}(u_i).$$

The length of the IVFSs can be seen as a representation of the ignorance related to the definition of the membership functions [16]. Independently of the source of the ignorance, it can be quantified by means of weak ignorance functions, as introduced in our previous work [34].

In this paper, we will use two basic operations with IVFSs, namely intersection and complement. On the one hand, we will apply t-norms [35], [36] to model the conjunction among the linguistic variables composing the antecedent of the rules. Therefore, we recall its extension on IVFSs.

Definition 2: [37] A function $\mathbf{T} : (L([0,1]))^2 \to L([0,1])$ is said to be an *interval-valued t-norm* if it is commutative, associative, increasing in both arguments (with respect to the order $\mathbf{x} \leq_L \mathbf{y}$ if and only if $\underline{x} \leq \underline{y}$ and $\overline{x} \leq \overline{y}$), and has the neutral element 1_L.

Definition 3: [37] An interval-valued t-norm is said to be *t-representable* if there are two t-norms T_a and T_b in $[0,1]$, being $T_a \leq T_b$, so that $\mathbf{T}(\mathbf{x},\mathbf{y}) = [T_a(\underline{x},\underline{y}), T_b(\overline{x},\overline{y})]$ for all $\mathbf{x}, \mathbf{y} \in L([0,1])$.

All interval-valued t-norm without zero divisors verify that $\mathbf{T}(\mathbf{x},\mathbf{y}) = 0_L$ if and only if $\mathbf{x} = 0_L$ or $\mathbf{y} = 0_L$.

In this paper, we model the intersection by means of t-representable interval-valued t-norms without zero divisors that will be denoted $\mathbf{T}_{T_a,T_b}$, since they can be represented by T_a and T_b as defined above.

On the other hand, we will use interval-valued fuzzy negations, which are the extension of fuzzy negations on IVFSs, because IV-REFs must fulfill a condition based on them.

In fuzzy set theory a strictly decreasing, continuous function $c : [0,1] \to [0,1]$ so that $c(0) = 1$ and $c(1) = 0$ is called a *strict negation* [38]. If c is also involutive, then it is called a *strong negation*. On this basis, we recall the definition for an interval-valued fuzzy negation.

Definition 4: [37] An interval-valued fuzzy negation is a function $N : L([0,1]) \to L([0,1])$ that is strictly decreasing (with respect to $\leq_L$) so that $N(1_L) = 0_L$ and $N(0_L) = 1_L$. If for all $\mathbf{x} \in L([0,1])$, $N(N(\mathbf{x})) = \mathbf{x}$, N is said to be involutive.

Next, we present the interval arithmetic that we will use to compute the rule weight as an element of $L([0,1])$. This fact allow us to extend the IV-FRM in such a way that the ignorance represented by the IVFSs is taken into account throughout the inference process. A deep study of interval arithmetic can be found in [39].

Let $[\underline{x},\overline{x}]$, $[\underline{y},\overline{y}]$ be two intervals in $\mathbb{R}^+$ so that $\mathbf{x} \leq_L \mathbf{y}$, the rules of interval arithmetic are as follows:

- Addition: $[\underline{x},\overline{x}] + [\underline{y},\overline{y}] = [\underline{x}+\underline{y}, \overline{x}+\overline{y}]$.
- Subtraction: $[\underline{x},\overline{x}] - [\underline{y},\overline{y}] = [|\underline{y}-\overline{x}|, \overline{y}-\underline{x}]$.
- Multiplication: $[\underline{x},\overline{x}] * [\underline{y},\overline{y}] = [\underline{x}*\underline{y}, \overline{x}*\overline{y}]$.
- Division: $[\min(\min(\frac{\underline{x}}{\underline{y}}, \frac{\overline{x}}{\overline{y}}),1), \min(\max(\frac{\underline{x}}{\underline{y}}, \frac{\overline{x}}{\overline{y}}),1)]$.

When we will need to use a total order relationship for intervals, i.e. when the largest interval membership needs to be determined in the last step of the IV-FRM, we will use the one defined by Xu and Yager in [40]: let $[\underline{x},\overline{x}], [\underline{y},\overline{y}] \in L([0,1])$, and let $s([\underline{x},\overline{x}]) = \underline{x} + \overline{x} - 1$ be the score of $[\underline{x},\overline{x}]$ and $h([\underline{x},\overline{x}]) = 1 - (\overline{x} - \underline{x})$ be the accuracy degree of $[\underline{x},\overline{x}]$. Then

- If $s([\underline{x},\overline{x}]) < s([\underline{y},\overline{y}])$, then $[\underline{x},\overline{x}] < [\underline{y},\overline{y}]$;
- If $s([\underline{x},\overline{x}]) = s([\underline{y},\overline{y}])$, then
  a) If $h([\underline{x},\overline{x}]) = h([\underline{y},\overline{y}])$, then $[\underline{x},\overline{x}] = [\underline{y},\overline{y}]$;
  b) if $h([\underline{x},\overline{x}]) < h([\underline{y},\overline{y}])$, then $[\underline{x},\overline{x}] < [\underline{y},\overline{y}]$,

Observe that any two intervals are comparable with this order relation. Moreover, it follows easily that 0_L is the smallest element in $L([0,1])$ and 1_L is the largest. We must remark that in the case of working with intervals in $\mathbb{R}_0^+$, the above described total order relationship works but the domain of the result for both the score and the accuracy degrees changes (from $[-1,1]$ to $[-1,\infty]$ for the score degree and from $[0,1]$ to $[-\infty,1]$ for the accuracy degree).

C. Construction Method of Interval-Valued Restricted Equivalence Functions

This section is aimed at providing an appropriate background about the concept of IV-REFs [22], [23], which is one of the main tools used in our new IV-FRM. IV-REFs are used to quantify the equivalence degree between two intervals. They are the extension on IVFSs of the restricted equivalence functions [41], since they allow to quantify how equivalent two values are.

Definition 5: [41] A function $REF : [0,1]^2 \to [0,1]$ is called a restricted equivalence function associated with a strong negation c, if it satisfies the following conditions:

(R1) $REF(x,y) = REF(y,x)$ for all $x,y \in [0,1]$;
(R2) $REF(x,y) = 1$ if and only if $x = y$;
(R3) $REF(x,y) = 0$ if and only if $x = 1$ and $y = 0$ or $x = 0$ and $y = 1$;
(R4) $REF(x,y) = REF(c(x),c(y))$ for all $x,y \in [0,1]$, c being a strong negation;
(R5) For all $x,y,z \in [0,1]$, if $x \le y \le z$, then $REF(x,y) \ge REF(x,z)$ and $REF(y,z) \ge REF(x,z)$.

We must point out that in this work we use the standard negation, that is, $c(x) = 1 - x$.

Among the methods developed to construct restricted equivalence functions [42] we use the one based on automorphisms, which are defined below.

Definition 6: An automorphism of the unit interval is any continuous and strictly increasing function $\phi : [0,1] \to [0,1]$ so that $\phi(0) = 0$ and $\phi(1) = 1$.

Example 1: The equation $\phi(x) = x^a$, being $a \in (0,\infty)$, generates a family of automorphisms.

- If $a = 1 \to \phi(x) = x$
- If $a = 2 \to \phi(x) = x^2$
- If $a = 0.5 \to \phi(x) = x^{(\frac{1}{2})}$
- If $a = 100 \to \phi(x) = x^{100}$
- If $a = 0.01 \to \phi(x) = x^{(\frac{1}{100})}$

Fig. 1 depicts the behavior of the these five automorphisms.

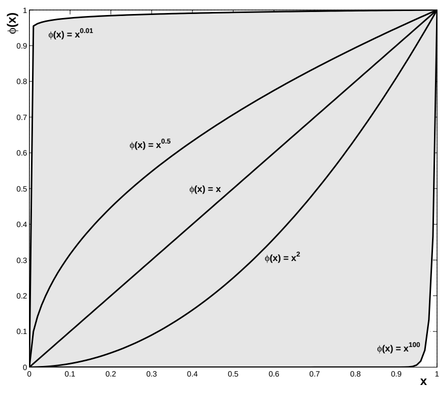

Fig. 1: Example of different automorphisms generated by varying the value of the parameter a as shown in Example 1.

Specifically, the construction method of restricted equivalence functions based on automorphism is the one introduced in Proposition 1.

Proposition 1: [42] If ϕ_1, ϕ_2 are two automorphisms of the unit interval, then

$$REF(x,y) = \phi_1^{-1}(1 - |\phi_2(x) - \phi_2(y)|)$$

with

$$c(x) = \phi_2^{-1}(1 - \phi_2(x))$$

is a restricted equivalence function.

Example 2: Taking $\phi_1(x) = x$, $\phi_2(x) = x$ we obtain the following restricted equivalence function

$$REF(x,y) = 1 - |x - y|,$$

which satisfies conditions (R1)-(R5) with $c(x) = 1 - x$ for all $x \in [0,1]$.

As mentioned previously, IV-REFs are the extension of restricted equivalence functions on IVFSs. Their definition is as follows.

Definition 7: [22], [23] An Interval-Valued Restricted Equivalence Function (IV-REF) associated with a interval-valued negation N is a function

$$IV\text{-}REF : L([0,1])^2 \to L([0,1])$$

so that:

(IR1) $IV\text{-}REF(\mathbf{x},\mathbf{y}) = IV\text{-}REF(\mathbf{y},\mathbf{x})$ for all $\mathbf{x},\mathbf{y} \in L([0,1])$;
(IR2) $IV\text{-}REF(\mathbf{x},\mathbf{y}) = 1_L$ if and only if $\mathbf{x} = \mathbf{y}$;
(IR3) $IV\text{-}REF(\mathbf{x},\mathbf{y}) = 0_L$ if and only if $\mathbf{x} = 1_L$ and $\mathbf{y} = 0_L$ or $\mathbf{x} = 0_L$ and $\mathbf{y} = 1_L$;
(IR4) $IV\text{-}REF(\mathbf{x},\mathbf{y}) = IV\text{-}REF(N(\mathbf{x}),N(\mathbf{y}))$ with N an involutive interval-valued negation;
(IR5) For all $\mathbf{x},\mathbf{y},\mathbf{z} \in L([0,1])$, if $\mathbf{x} \le_L \mathbf{y} \le_L \mathbf{z}$, then $IV\text{-}REF(\mathbf{x},\mathbf{y}) \ge_L IV\text{-}REF(\mathbf{x},\mathbf{z})$ and $IV\text{-}REF(\mathbf{y},\mathbf{z}) \ge_L IV\text{-}REF(\mathbf{x},\mathbf{z})$.

In [22], [23], the authors show several construction methods of IV-REFs. Among them, we make use of the IV-REF construction method given in the following corollary.

Corollary 1: [22], [23] Let REF be a restricted equivalence function and let T and S be any t-norm and any t-conorm in $[0,1]$,

$$\begin{aligned} IV\text{-}REF(\mathbf{x},\mathbf{y}) = \; & [T(REF(\underline{x},\underline{y}), REF(\overline{x},\overline{y})), \\ & S(REF(\underline{x},\underline{y}), REF(\overline{x},\overline{y}))] \end{aligned}$$

is an IV-REF.

Using the construction method of IV-REFs given in Corollary 1 applying the construction method of restricted equivalence functions recalled in Proposition 1 we obtain the IV-REFs construction method used in this paper (Eq.(7)):

$$\begin{gathered} IV\text{-}REF(\mathbf{x},\mathbf{y}) = \\ [T(\phi_1^{-1}(1 - |\phi_2(\underline{x}) - \phi_2(\underline{y})|), \phi_1^{-1}(1 - |\phi_2(\overline{x}) - \phi_2(\overline{y})|)), \\ S(\phi_1^{-1}(1 - |\phi_2(\underline{x}) - \phi_2(\underline{y})|), \phi_1^{-1}(1 - |\phi_2(\overline{x}) - \phi_2(\overline{y})|))] \end{gathered} \tag{7}$$

Example 3: Taking $\phi_1(x) = x$, $\phi_2(x) = x$ we obtain the following IV-REF

$$\begin{aligned} IV\text{-}REF(\mathbf{x},\mathbf{y}) = \; & [T(1 - |\underline{x} - \underline{y}|, 1 - |\overline{x} - \overline{y}|), \\ & S(1 - |\underline{x} - \underline{y}|, 1 - |\overline{x} - \overline{y}|)] \end{aligned}$$

III. A Linguistic Fuzzy Rule-Based Classification System Based on an Interval-Valued Fuzzy Reasoning Method with Tuning and Rule Selection

In our new approach we propose the introduction of the concept of minimum distance classifiers in the IV-FRM of the IV-FRBCSs. To do so, we compute the matching degree between the patterns and the antecedent of the rules using IV-REFs in order to quantify the equivalence degree between the interval membership degree and the ideal interval membership degree ($1_L = [1, 1]$) for each linguistic label composing the antecedent of the rule. The motivation is to strengthen the relevance of the rules with a higher equivalence degree with respect to the new pattern to be classified.

The parametrized construction of the IV-REFs allows an easy generation of many of these functions to be performed. In this manner, we face the problem of choosing a suitable similarity function by applying a genetic tuning, which can lead to an improvement of the behaviour of the system in a general framework by looking for the most appropriate set of IV-REFs to solve each specific problem we deal with.

In the remainder of this section, we first present a general outline of our new method (Section III-A). Then, we describe in detail the initialization of the system' parameters (Section III-B) and the novel IV-FRM making use of IV-REFs (Section III-C). Finally, we introduce the tuning approach used to choose the most appropriate IV-REF for each variable together with the rule selection method (Section III-D).

A. Overviewing IVTURS

This section is aimed at showing a general overview of IVTURS. As depicted in Fig. 2, it is composed of three steps:

1) Initialization of the IV-FRBCS. This step involves the following tasks:
   - The generation of the initial FRBCS by means of the FARC-HD method by Alcalá-Fdez et al. [1].
   - Modelling the linguistic labels of the FRBCS by means of IVFSs.
   - The generation of the initial IV-REF for each variable of the problem.
2) The extension of the fuzzy reasoning method on IVFSs.
3) The application of the optimization approach, which is composed of:
   - The genetic tuning in which we look for the best values of the IV-REFs' parameters.
   - The rule selection process in order to decrease the system's complexity.

As we have mentioned, in this paper our approach combines IVTURS with the FARC-HD method to carry out the fuzzy rule learning process. Therefore, we denote our new proposal as IVTURS-FARC.

In the remainder of this section, we describe in detail each step composing our new method.

B. Initialization of the Interval-Valued Fuzzy Rule-Based Classification System

It is well known that there are two possibilities in the generation of a type-2 model [43]: 1) a partial dependent one, where an initial type-1 fuzzy model is learnt and then used as a smart initialization of the parameters of the type-2 fuzzy model [44]–[46]; 2) a total independent method, where the type-2 fuzzy model is learnt without the help of any base type-1 fuzzy model [47].

In this paper, we use the first option, that is, we generate a base FRBCS using the FARC-HD algorithm, which is based on three stages:

1) Extracting the fuzzy association rules for classification by applying a search tree, whose depth of the branches is limited.
2) Preselecting the most interesting rules using subgroup discovery in order to decrease the computational cost of the system.
3) Optimizing the knowledge base by means of a combination between the well known tuning of the lateral position of the membership functions and a rule selection process.

We make use of the two first stages in order to learn the initial FRBCS, which is the basis of our IV-FRBCS. We must point out that for the learning step we consider triangular membership functions, which are obtained by performing a linear partitioning of the input domain of each variable.

After having the base FRBCS, we model its linguistic labels by means of IVFSs. To this aim we apply the following process:

- We take as the lower bound of each IVFS the initial membership function (the one used in the learning step).
- We generate the upper bound of each linguistic label. For their construction, the amplitude of the support of the upper bounds is determined by the value of the parameter W, which is initially set to 0.25 to achieve an amplitude 50% larger than that of their lower bound counterpart.

An example of these sets is depicted in Fig. 3.

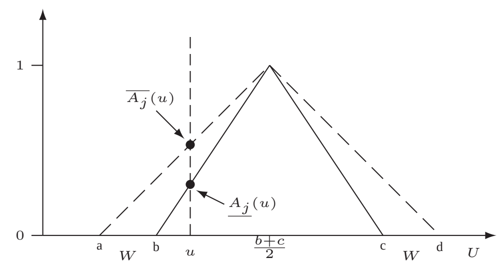

Fig. 3: Example of an initial constructed IVFS as defined in [24], [34]. The solid line is the initial fuzzy set and therefore, it is the lower bound of the IVFS. The dashed line is the upper bound of the IVFS.

Finally, we also have to generate the initial IV-REF associated with each variable of the problem. To this end, we apply the construction method recalled in Section II-C using the identity function as automorphism ($\phi(x) = x$) in every case. In this manner, the initial IV-REF for every variable of the problem is the one that was shown in Example 3.

C. Interval-Valued Fuzzy Reasoning Method

This section is aimed at describing the new IV-FRM. To do so, we modify all the steps of the fuzzy reasoning method [21] recalled in Section II-A. In this way, we develop a method that intrinsically manages the ignorance that the IVFSs represent.

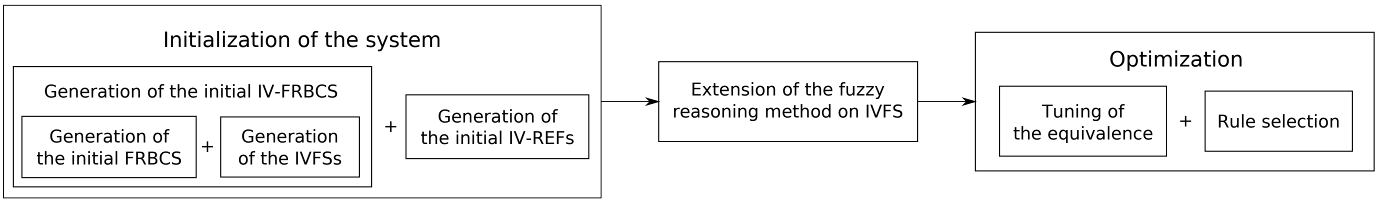

Fig. 2: Flow chart of our new IV-FRBCS.

Let L be the number of rules in the RB and M the number of classes of the problem; If $x_p = (x_{p1}, \ldots, x_{pn})$ is a new pattern to be classified, the steps of the IV-FRM are the following:

1) *Interval matching degree*: we use IV-REFs to compute the similarity between the interval membership degrees (of each variable of the pattern to the corresponding IVFS) and the ideal membership degree, 1_L, and then we apply a t-representable interval-valued t-norm ($\mathbf{T}_{T_a,T_b}$ as introduced in Section II-B) to these results:

$$\begin{gathered}[\underline{A_j}(x_p), \overline{A_j}(x_p)] = \\ \mathbf{T}_{T_a,T_b}(IV\text{-}REF([\underline{A_{j1}}(x_{p1}), \overline{A_{j1}}(x_{p1})], [1,1]), \ldots, \\ IV\text{-}REF([\underline{A_{jn}}(x_{pn}), \overline{A_{jn}}(x_{pn})], [1,1])), \\ j = 1, \ldots, L. \end{gathered} \tag{8}$$

We must point out that the result of the initial IV-REF for each variable is the interval membership degree, since the equation shown in Example 3 is applied. The result of each IV-REF changes according to the values of its parameters. These situations are shown in Example 4, where the result of the initially constructed IV-REF is shown in the first item whereas the two last items show results of IV-REFs when the initial values of their parameters have been modified.

Example 4: Let $[\underline{A}(u), \overline{A}(u)] = [0.6, 0.7]$. The results provided when using IV-REFs (Eq.(7)) constructed from different automorphisms are:

- $\phi_1(x) = x$ and $\phi_2(x) = x$:
$$IV\text{-}REF([0.6, 0.7], [1, 1]) = [0.6, 0.7]$$
- $\phi_1(x) = x^2$ and $\phi_2(x) = x$:
$$IV\text{-}REF([0.6, 0.7], [1, 1]) = [0.77, 0.84]$$
- $\phi_1(x) = x^{0.5}$ and $\phi_2(x) = x$:
$$IV\text{-}REF([0.6, 0.7], [1, 1]) = [0.36, 0.49]$$

2) *Interval association degree*: we apply a combination operator, *h*, to the interval matching degree computed previously and the rule weight:

$$\begin{gathered}[\underline{b_j^k}, \overline{b_j^k}] = h([\underline{A_j}(x_p), \overline{A_j}(x_p)], [\underline{RW_j^k}, \overline{RW_j^k}]) \\ k = 1, \ldots, M, \quad j = 1, \ldots, L. \end{gathered} \tag{9}$$

We must point out that the rule weight is an element of $L([0,1])$. To compute it, we apply the certainty factor (see Eq. (2)) making use of the interval arithmetic introduced in Section II-B. The resulting equation is shown in Eq. (10).

$$[\underline{RW_j}, \overline{RW_j}] = \frac{\sum_{x_p \in ClassC_j} [\underline{A_j}(x_p), \overline{A_j}(x_p)]}{\sum_{p=1}^{P} [\underline{A_j}(x_p), \overline{A_j}(x_p)]} \tag{10}$$

3) *Interval pattern classification soundness degree for all classes*. We aggregate the positive interval association degrees of each class by applying an aggregation function *f*.

$$\begin{gathered}[\underline{Y_k}, \overline{Y_k}] = f([\underline{b_j^k}, \overline{b_j^k}],\ j = 1, \ldots, L \text{ and } [\underline{b_j^k}, \overline{b_j^k}] > 0_L), \\ k = 1, \ldots, M. \end{gathered} \tag{11}$$

4) *Classification*. We apply a decision function F over the interval soundness degree of the system for the pattern classification for all classes:

$$F([\underline{Y_1}, \overline{Y_1}], ..., [\underline{Y_M}, \overline{Y_M}]) = \arg \max_{k=1,\ldots,M}([\underline{Y_k}, \overline{Y_k}]) \tag{12}$$

The last step of the IV-FRM consists of selecting the maximum interval soundness degree. Therefore, in order to to be able to make this decision, we use the total order relationship for intervals [40] presented in Section II-B.

D. Tuning of the Equivalence and Rule Selection

In this proposal, we make use of genetic algorithms with a double aim: 1) to tune the values of the parameters used in the construction of the IV-REFs in order to increase the reasoning capabilities of the IV-FRM and 2) to perform a rule selection process in which we obtain a compact and cooperative fuzzy rule set.

According to Eq. (7), each IV-REF is constructed using the automorphisms ϕ_1 and ϕ_2. In this paper, we take $\phi_1(x) = x^a$ and $\phi_2(x) = x^b$, where $a, b \in (0, \infty)$. Therefore, we can generate a huge number of IV-REFs by taking different values for the parameters a and b. The selection of a suitable IV-REF to measure the equivalence degree in each variable could involve an improvement in the behaviour of the system to solve specific problems.

To face this optimization problem, we propose the tuning of the values of the parameters a and b used to construct the IV-REFs associated with each variable of the problem. In order to cover as much search space as posible, we suggest varying these values in the interval $[0.01, 100]$. This range is due to the fact that when using $\phi(x) = x^{0.01}$ the shape of the function is close to a crisp one and we use the inverse function ($\phi(x) = x^{100}$) in order to provide the same flexibility in both sides of the identity function. The shadow surface of Fig. 1 (Section II-C) depicts the search space covered when using the proposed variation interval, which is almost the whole search space.

On the other hand, regarding the size of the output model, fuzzy rule learning methods usually generate a large number of fuzzy rules so as to achieve a highly accurate system. However, in the fuzzy rule set created we can find irrelevant, redundant, erroneous or conflicting rules, which may perturb the performance of the system [48]. Therefore, a rule reduction process is often applied in order to improve the system's accuracy by removing useless rules and, in turn, easing the readability of the system. There are a lot of methods to deal with the rule reduction process [10], [49]; from among them, we will use a rule selection approach developed using a simple binary codification in order to express whether or not the fuzzy rules belong to the rule set.

In order to accomplish this genetic process we consider the use of the CHC evolutionary model [50] due to both its good properties to deal with complex search spaces [51], [52] and the good results provided in this topic [53], [54]. In the following, we describe the specific features of our evolutionary model:

- **Coding scheme**. Each chromosome is composed of two well differentiated parts implying a double codification scheme: real codification for the tuning of the equivalence (C_E) part and binary coding for the rule selection process (C_R).
  1) *Tuning of the equivalence*. Let n be the number of attributes, the part of the chromosome to carry out the tuning of the IV-REFs is a vector of size $2 \times n$: $C_E = \{a_1, b_1, a_2, b_2, ..., a_n, b_n\}$, where $a_i, b_i \in [0.01, 1.99]$ with $i = 1, 2, ..., n$. Each pair of genes (a_i, b_i) represent the values of the parameters a and b to construct the IV-REF associated with the i^{th} attribute.
  To construct the corresponding IV-REF, we have to adapt the gene values to the interval in which the automorphisms can vary ($[0.01, 100]$). To do so, we adapt the value of each parameter a using the following equation (we consider the same adaptation for parameter b):
  $$a = \begin{cases} a, & \text{if } 0 < a \leq 1 \\ \frac{1}{2-a}, & \text{if } 1 < a < 2 \end{cases} \tag{13}$$
  2) *Rule selection*. Let L be the number of fuzzy rules in the RB, the part of the chromosome to perform the rule selection is a vector of size L, $C_R = \{r_1, ..., r_L\}$ where $r_i \in \{0, 1\}$ with $i = \{1, 2, ..., L\}$, determining the subset of fuzzy rules which compose the final RB as follows: if $r_i = 1$ then $R_i \in$ RB else $R_i \notin$ RB

  Therefore, the whole chromosome scheme is as follows:
  $$C_{E+R} = \{C_E, C_R\}$$
- **Initial Gene Pool**. To include the initial FRBCS in the population, we initialize an individual with all genes with value 1. In this manner, we construct the initial IV-REFs using the identity functions as automorphism for each variable and we include all the fuzzy rules in the RB.
- **Chromosome Evaluation**. We use the most common metric for classification, i.e. the accuracy rate.
- **Crossover Operator**. Due to the double coding scheme of the chromosome, we apply a different crossover operator for each part of the chromosome: the Parent Centrix BLX operator [55] (which is based on the BLX-α) is used for the real coding part and the half uniform crossover scheme [56] is considered for the remainder.
- **Restarting Approach**. To get away from local optima, we consider a restarting approach. To do so, as in the elitist scheme, we include the best global solution found until this moment in the next population and we generate the remaining individuals at random.

For more details about the evolutionary algorithm, please refer to [1].

IV. Experimental Framework

In this section, we first present the real world classification data-sets selected for the experimental study. Next, we introduce the parameter set-up considered throughout this study. Finally, we introduce the statistical tests which are necessary to compare the results achieved throughout the experimental study.

A. Data-sets

We have selected a wide benchmark of twenty-seven real world data-sets selected from the KEEL data-set repository [25], [26], which are publicly available on the corresponding web page [2] including general information about them, partitions for the validation of the experimental results and so on. Table I summarizes the properties of the selected data-sets, showing for each data-set the number of examples (#Ex.), the number of attributes (#Atts.) and the number of classes (#Class.). We must point out that the *magic, page-blocks, penbased, ring, satimage* and *shuttle* data-sets have been stratified sampled at 10% in order to reduce their size for training. In the case of missing values (*crx, dermatology* and *wisconsin*), those instances have been removed from the data-set.

A *5-fold cross-validation model* was considered in order to carry out the different experiments. That is, we split the data-set into 5 random partitions of data, each one with 20% of the patterns, and we employed a combination of 4 of them (80%) to train the system and the remaining one to test it.

B. Methods set-up

This section is aimed at introducing the configurations that have been considered for the different methods used along the experimental study, namely, the FARC-HD method [1], our different versions of this method using IVFSs and the FURIA

[2] http://www.keel.es/dataset.php

TABLE I: Summary Description for the employed data-sets.

Id.	Data-set	#Ex.	#Atts.	#Class.
aus	Australian	690	14	2
bal	Balance	625	4	3
cle	Cleveland	297	13	5
con	Contraceptive	1,473	9	3
crx	Crx	653	15	2
der	Dermatology	358	34	6
eco	Ecoli	336	7	8
ger	German	1,000	20	2
hab	Haberman	306	3	2
hay	Hayes-Roth	160	4	3
hea	Heart	270	13	2
ion	Ionosphere	351	33	10
iri	Iris	150	4	3
mag	Magic	1,902	10	2
new	New-Thyroid	215	5	3
pag	Page-blocks	548	10	5
pen	Penbased	1,992	16	10
pim	Pima	768	8	2
sah	Saheart	462	9	2
spe	Spectfheart	267	44	2
tae	Tae	151	5	3
tit	Titanic	2,201	3	2
two	Twonorm	740	20	2
veh	Vehicle	846	18	4
win	Wine	178	13	3
wiR	Winequality-Red	1,599	11	11
wis	Wisconsin	683	9	2

algorithm [2], which is briefly describe below (please refer to [2] for details).

FURIA [2] builds upon the RIPPER interval rule induction algorithm [57]. The model built by FURIA uses fuzzy rules of the form given in Eq. (1) using fuzzy sets with trapezoidal membership functions. Specifically, FURIA builds the fuzzy rule base by means of these two steps:

1) Learn a rule set for every single class using a one-versus-all decomposition. To this aim, a modified version of RIPPER is applied, which involves a building and an optimization phase.
2) Obtain the fuzzy rules by means of fuzzifying the final rules from the modified RIPPER algorithm in a greedy way.

At classification time, the class predicted by FURIA is the one with maximal support. In case the query is not covered by any rule, a rule stretching method is proposed based on modifying the rules in a local way so as to make them applicable to the query.

Regarding the configurations, for the FARC-HD algorithm we will apply the following one:

- Conjunction operator: product t-norm.
- Combination operator: product t-norm.
- Rule weight: certainty factor.
- Fuzzy reasoning method: additive combination [21].
- Number of linguistic labels per variable: 5 labels.
- $Minsup$: 0.05.
- $Maxconf$: 0.8.
- $Depth_{max}$: 3.
- k_t: 2.

For the IVTURS-FARC, we have considered the following configuration:

- Conjunction operator: product interval-valued t-norm.
- Combination operator: product interval-valued t-norm.
- IVFSs construction:
  - Shape: Triangular membership functions.
  - Upper bound: 50% greater than the lower bound ($W = 0.25$).
- Configuration of the initial IV-REFs:
  - T-norm: minimum.
  - T-conorm: maximum.
  - First automorphism: $\phi_1(x) = x^1$ (a = 1).
  - Second automorphism: $\phi_2(x) = x^1$ (b = 1).

Regarding the genetic tuning with rule selection process, we have used the values suggested in [1], which are:

- Population Size: 50 individuals.
- Number of evaluations: 20,000.
- Bits per gene for the Gray codification (for incest prevention): 30 bits.

Finally, for the parameters of the FURIA algorithm, namely the number of folds and optimizations, we have set their values to 3 and 2 respectively, as recommended by the authors.

C. Statistical Tests for Performance Comparison

In this paper, we use some hypothesis validation techniques in order to give statistical support to the analysis of the results [58], [59]. We will use non-parametric tests because the initial conditions that guarantee the reliability of the parametric tests cannot be fulfilled, which implies that the statistical analysis loses credibility with these parametric tests [27].

Specifically, we use the Friedman aligned ranks test [60] to detect statistical differences among a group of results and the Holm post-hoc test [61] to find the algorithms that reject the equality hypothesis with respect to a selected control method.

The post-hoc procedure allows us to know whether a hypothesis of comparison could be rejected at a specified level of significance α. Furthermore, we compute the adjusted *p*-value (APV) in order to take into account the fact that multiple tests are conducted. In this manner, we can directly compare the APV with respect to the level of significance α in order to be able to reject the null hypothesis.

Furthermore, we consider the method of aligned ranks of the algorithms in order to show graphically how good a method is with respect to its partners. The first step to compute this ranking is to obtain the average performance of the algorithms in each data set. Next, we compute the subtractions between the accuracy of each algorithm minus the average value for each data-set. Then, we rank all these differences in descending order and, finally, we average the rankings obtained by each algorithm. In this manner, the algorithm which achieves the lowest average ranking is the best one.

These tests are suggested in the studies presented in [27]–[29], [58], where its shown that their use in the field of machine learning is highly recommended. A complete description of the test and software for its use can be found on the website: http://sci2s.ugr.es/sicidm/.

V. Analysis of the usefulness of IVTURS-FARC

In this section, we analyse the behaviour of IVTURS-FARC. To do so, we develop an experimental study composed of three steps:

1) We determine the importance of both the rule selection process and, foremost, the IVFSs by comparing IVTURS-FARC versus its fuzzy counterpart with and also without rule selection (Section V-A).
2) We analyse the improvements achieved with respect to our previous proposal in the topic (Section V-B).
3) We study whether IVTURS-FARC improves the results obtained by two state-of-the-art fuzzy classifiers (Section V-C).

The description of the methods used to carry out the two first steps of the experimental study are introduced in Table II. Methods using the prefix FS use the fuzzy system learnt applying the first two stages of the FARC-HD algorithm as defined in [1] and they apply REFs[3] for computing the matching degree. Methods using the suffix T_WI apply our previous proposal for tuning the ignorance degree that each IVFS represents [24].

TABLE II: Description of the methods used in Sections V-A and V-B of the experimental study.

Notation	Linguistic labels	Matching degree	Tuning of the equivalence	Tuning of the ignorance degree	Rule selection
FS_T_E	Fuzzy sets	REFs	Yes	No	No
FS_T_E+R	Fuzzy sets	REFs	Yes	No	Yes
IVFS_T_E	IVFSs	IV-REFs	Yes	No	No
IVFS_T_WI	IVFSs	IV-REFs	No	Yes	No
IVFS_T_E+WI	IVFSs	IV-REFs	Yes	Yes	No
IVTURS_FARC	IVFSs	IV-REFs	Yes	No	Yes

Table III shows the classification accuracy of the different approaches used along the experimental study. Results are grouped in pairs for training and test, where the best global result for each data-set is emphasised in **bold-face**. Vertical lines group the methods involved in each scenario.

In the remainder of this section, we develop the analysis carried out in the three aforementioned scenarios.

A. Determining the suitability of IVTURS-FARC

This section is aimed at showing the goodness of IVTURS-FARC, that is, the combination of IVFSs with the tuning of the equivalence and a rule selection process. To do so, we have to analyse whether the rule selection step strengthens the quality of the results and, most importantly, we must determine whether the use of IVFSs is the main cause of the enhancement achieved. For this reason, we compare IVTURS-FARC with respect to its version without the rule selection process (IVFS_T_E) and also with the fuzzy counterparts (FS_T_E+R and FS_T_E) in order to support the key role that IVFSs play in the system.

Analysing the results achieved by these four approaches, we find that IVTURS-FARC reaches the best performance in twelve out of the twenty-seven data-sets implying a global performance improvement. This fact is confirmed in Fig. 4, where it is clearly shown that our new IV-FRBCS is the best ranking method.

When applying the Friedman aligned ranks test we find a p-value of 5.70E-5, which confirms the existence of statistical

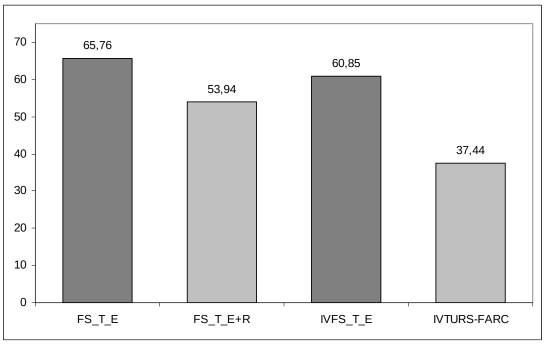

Fig. 4: Rankings of the four versions of IVTURS-FARC.

differences among these four approaches. For this reason, we perform the Holm post-hoc test, the results of which are shown in Table IV, selecting IVTURS-FARC as the control method since it is the best ranking one. These results clearly determine the quality of IVTURS-FARC since: 1) the use of IVFSs leads to the outperforming of the results of the non interval-valued fuzzy versions of IVTURS-FARC with and without the rule selection stage and 2) the rule selection process allows us to enhance the results of the tuning of the equivalence.

TABLE IV: Holm test to compare IVTURS-FARC with respect to its different versions.

i	Algorithm	Hypothesis	APV
1	FS_T_E	Rejected for IVTURS-FARC	0.003
2	IVFS_T_E	Rejected for IVTURS-FARC	0.018
3	FS_T_E+R	Rejected for IVTURS-FARC	0.053

B. Analysing the performance improvement with respect to the tuning of the weak ignorance degree

Once the suitability of the IVTURS-FARC is confirmed, we study whether our new proposal enhances the results of our previous approach to the topic. To do so, we consider the use of the following proposals:

- IVFS_T_WI: the initial IV-FRBCS[4] with a previous tuning approach for modifying the ignorance degree that each IVFS represents [24], that is, we modify the value of the parameter W used in the construction of the IVFSs.
- IVFS_T_E+WI: the initial IV-FRBCS with a tuning approach that simultaneously performs both the tuning of the ignorance degree [24] and the tuning of the equivalence, that is, we modify the values of the parameters W, a and b used in the construction of the IVFSs and the IV-REFs respectively.
- IVTURS-FARC: the proposed method as explained in Section III-A.

From the results in Table III, we can observe that IVTURS-FARC achieves a notable enhancement of the global performance, which is based on the achievement of the best result

[3]These functions allow to quantify the equivalence between two numbers. They are the non interval-valued fuzzy version of the IV-REFs as explained in Section II-C.

[4]The initial IV-FRBCS is the system obtained after the modeling of the linguistic labels of the base fuzzy classifier by means of IVFSs. It uses the new IV-FRM introduced in Section III-C with the identity function as the automorphism for the construction of each IV-REF.

TABLE III: Results in Train (Tr.) and Test (Tst) achieved by the different approaches considered in this paper.

Data Set	FS_T_E $Tr.$	FS_T_E Tst	FS_T_E+R $Tr.$	FS_T_E+R Tst	IVFS_T_E $Tr.$	IVFS_T_E Tst	IVFS_T_WI $Tr.$	IVFS_T_WI Tst	IVFS_T_E+WI $Tr.$	IVFS_T_E+WI Tst	FARC-HD $Tr.$	FARC-HD Tst	FURIA $Tr.$	FURIA Tst	IVTURS-FARC $Tr.$	IVTURS-FARC Tst
aus	88.44	**86.38**	90.65	84.93	88.77	85.07	83.22	83.48	88.84	85.51	90.43	85.51	88.99	86.09	90.04	85.80
bal	84.68	80.96	91.56	82.40	86.92	82.08	88.56	82.40	88.12	82.88	92.12	**87.36**	88.84	83.68	91.84	85.76
cle	83.50	58.92	88.30	54.54	80.81	57.58	80.81	57.25	81.82	59.59	89.56	57.92	62.37	56.57	85.44	**59.60**
con	56.96	53.02	61.80	53.90	57.04	52.68	57.16	52.68	57.37	52.55	62.80	52.68	56.81	**54.17**	59.69	53.36
crx	89.01	85.45	91.58	86.83	89.40	86.53	84.07	82.85	89.47	**87.75**	91.65	86.53	89.70	86.37	91.42	87.14
der	99.44	92.75	100.00	90.49	99.65	**94.69**	98.39	**94.69**	99.65	94.42	100.00	89.94	98.88	93.86	99.86	94.42
eco	88.17	78.60	91.07	**80.07**	86.24	76.20	85.49	77.69	87.43	77.39	92.11	**80.07**	89.66	80.06	89.06	78.58
ger	81.95	71.10	86.98	**73.80**	81.48	71.90	78.25	71.00	81.65	70.70	87.70	71.60	76.90	73.30	85.35	73.10
hab	78.68	**73.84**	82.19	72.19	77.61	72.22	74.43	73.20	78.92	73.51	81.53	71.22	75.57	72.55	80.72	72.85
hay	86.36	76.41	91.28	79.46	86.36	80.23	81.26	77.18	86.36	80.23	91.28	80.20	88.44	**81.00**	91.28	80.23
hea	91.94	**88.15**	94.44	86.67	92.31	86.30	90.09	86.30	92.22	85.19	94.63	84.44	89.72	78.15	93.61	**88.15**
ion	98.29	91.18	98.58	90.60	98.36	91.17	96.15	**92.33**	98.58	90.33	98.50	90.32	97.65	88.91	98.72	89.75
iri	98.17	96.00	98.50	94.67	98.17	**96.67**	97.83	**96.67**	98.33	96.00	98.50	94.00	98.50	94.00	98.17	96.00
mag	82.14	79.13	83.43	80.33	81.32	78.81	78.25	76.18	81.65	78.97	84.46	80.49	84.03	**80.65**	82.28	79.76
new	95.47	92.09	97.91	94.42	96.63	92.09	95.23	93.02	98.14	93.49	98.95	**95.35**	99.07	94.88	98.84	**95.35**
pag	96.44	94.52	96.85	94.89	96.03	94.71	94.66	93.42	96.26	94.52	96.94	94.34	99.54	**95.25**	96.85	95.07
pen	96.09	91.82	97.68	92.00	93.68	91.09	94.45	91.16	94.23	90.18	98.34	**92.64**	98.98	92.45	95.50	92.18
pim	79.56	75.51	82.29	**76.17**	78.71	74.99	75.39	72.00	79.46	74.35	83.66	74.08	79.17	**76.17**	80.57	75.90
sah	79.17	**72.28**	82.74	69.70	78.52	69.28	77.22	70.99	79.06	68.40	83.49	70.77	74.84	70.33	79.55	70.99
spe	91.20	77.51	92.60	77.87	89.42	**80.52**	85.68	77.13	90.08	80.51	92.42	78.64	95.70	77.88	90.45	**80.52**
tae	70.04	52.37	75.50	**54.43**	70.37	53.68	67.55	50.39	72.19	54.34	77.15	48.41	54.63	47.08	73.18	50.34
tit	77.06	77.06	79.07	**78.87**	77.65	77.65	77.65	77.65	77.65	77.65	79.07	**78.87**	78.46	78.51	79.07	**78.87**
two	96.05	89.19	96.76	90.54	95.91	**93.24**	95.71	91.49	96.39	92.97	97.84	89.32	99.46	88.11	96.35	92.30
veh	75.30	64.66	78.69	66.90	70.77	65.26	70.30	62.89	71.57	63.48	80.38	68.44	79.34	**70.21**	72.81	67.38
win	99.44	93.24	99.86	95.49	99.30	96.08	98.88	**97.76**	99.44	94.95	99.86	96.62	99.58	93.78	99.30	97.19
wiR	62.59	59.54	64.93	**59.66**	60.94	58.91	58.30	56.35	61.60	58.97	56.22	53.96	57.12	51.29	62.01	58.28
wis	97.80	96.49	98.61	**96.78**	97.88	96.34	96.12	95.90	98.13	**96.78**	98.76	96.63	98.83	96.63	98.50	96.49
Mean	86.07	79.56	88.66	79.95	85.56	79.85	83.75	79.04	86.10	79.84	88.83	79.64	85.21	79.33	87.42	**80.57**

in more than half of the data-sets. This situation is confirmed in Fig. 5 where it is shown that the best ranking is reached by our new IV-FRBCS.

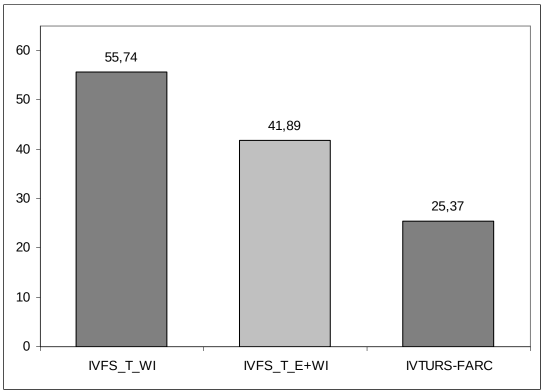

Fig. 5: Rankings of both our new and our previous proposals in the topic.

In order to detect significant differences among the results of the three approaches, we carry out the Friedman aligned rank test. This test obtains a p-value near to zero (3.88E-5), which implies that there are significant differences between the results. For this reason we can apply a post-hoc test to compare our new methodology with the previous ones. Specifically, a Holm test is applied, which is presented in Table V. The statistical analysis reflects that IVTURS-FARC is the best choice among this group of proposals.

TABLE V: Holm test to compare IVTURS-FARC with respect to our previous tuning approaches.

i	Algorithm	Hypothesis	APV
1	IVFS_T_WI	Rejected for IVTURS-FARC	4.21E-6
2	IVFS_T_E+WI	Rejected for IVTURS-FARC	0.019

C. *On the comparison versus state-of-the-art fuzzy classifiers*

In this section, we analyze the performance of IVTURS-FARC against two recognized state-of-the-art classifiers, i.e. the original FARC-HD algorithm by Alcalá-Fdez et al. [1] and the FURIA algorithm by Hühn and Hüllermeier [2].

From the results presented in the last three pairs of columns of Table III, we must highlight the mean performance improvement obtained by our new proposal with respect to the FURIA algorithm (1.22%) and also the notable improvement versus the original FARC-HD method.

In order to compare the results, we have applied the non-parametric tests described in Section IV-C. The p-value obtained by the Aligned Friedman test is 3.20E-5, which implies the existence of significant differences among the three approaches. Fig 6 graphically shows the rankings of the different methods (which have been computed using the Aligned Friedman test).

We now apply Holm's test to compare the best ranking method (IVTURS-FARC) with the remaining ones. Table VI shows that the hypothesis of equality is rejected with the rest of methods with a high level of confidence. Therefore, taking into account the previous findings, we can conclude that IVTURS-FARC is the best performing method among these

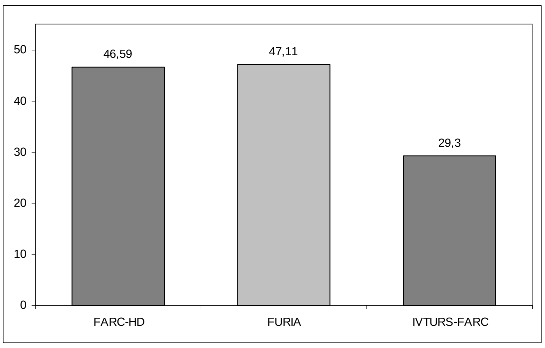

Fig. 6: Rankings of the two state-of-the-art fuzzy classifiers together with IVTURS-FARC.

TABLE VI: Holm test to compare IVTURS-FARC with respect to FARC-HD and FURIA.

i	Algorithm	Hypothesis	APV
1	FURIA	Rejected for IVTURS-FARC	0.011
2	FARC-HD	Rejected for IVTURS-FARC	0.011

three proposals.

VI. Concluding Remarks

In this paper we have introduced a new linguistic fuzzy rule-based classification method called IVTURS. It is based on a new IV-FRM in which all the steps make the computation using intervals, allowing an integral management of the ignorance that the IVFSs represent. The key concept are the IV-REFs, which are applied to compute the matching degree allowing the relevance of the rules with a high interval equivalence degree to be emphasised. Furthermore, the parametrized construction of these functions allows us to compute the most suitable set of IV-REFs so as to solve each specific problem. In this manner, we have defined a genetic process to choose the set of IV-REFs for each problem and also to reduce the complexity of the system by performing a fuzzy rule set reduction process.

For the experimental study we have used an instance of IVTURS using the FARC-HD method, which has been denoted IVTURS-FARC. Throughout the experimental analysis, it has been shown that IVTURS-FARC enhances the results obtained when applying several tuning approaches to the new IV-FRBCS. We must stress that the use of IVFSs allows us to strengthen the quality of the results, since the performance of the fuzzy counterparts of our new IV-FRBCS are highly improved. The highlight of the study is the high accuracy of IVTURS-FARC, since it outperforms the results achieved by two state-of-the-art fuzzy classifiers.

References

[1] J. Alcala-Fdez, R. Alcala, and F. Herrera, "A fuzzy association rule-based classification model for high-dimensional problems with genetic rule selection and lateral tuning," *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 5, pp. 857–872, 2011.

[2] J. Hühn and E. Hüllermeier, "FURIA: an algorithm for unordered fuzzy rule induction," *Data Mining and Knowledge Discovery*, vol. 19, no. 3, pp. 293–319, 2009.

[3] Z. Chi, H. Yan, and T. Pham, *Fuzzy algorithms with applications to image processing and pattern recognition*. World Scientific, 1996.

[4] O. Cordon, M. J. del Jesus, and F. Herrera, "Genetic learning of fuzzy rule-based classification systems cooperating with fuzzy reasoning methods," *International Journal of Intelligent Systems*, vol. 13, no. 10–11, pp. 1025–1053, 1998.

[5] H. Isibuchi, T. Yamamoto, and T. Nakashima, "Hybridization of fuzzy GBML approaches for pattern classification problems," *IEEE Transactions on System, Man and Cybernetics B*, vol. 35, no. 2, pp. 359–365, 2005.

[6] A. González and R. Perez, "Slave: A genetic learning system based on an iterative approach," *IEEE Transactions on Fuzzy Systems*, vol. 7, no. 2, pp. 176–191, 1999.

[7] E. Mansoori, M. Zolghadri, and S. Katebi, "Sgerd: A steady-state genetic algorithm for extracting fuzzy classification rules from data," *IEEE Transactions on Fuzzy Systems*, vol. 16, no. 4, pp. 1061–1071, 2008.

[8] Y.-C. Hu, R.-S. Chen, and G.-H. Tzeng, "Finding fuzzy classification rules using data mining techniques." *Pattern Recognition Letters*, vol. 24, no. 1-3, pp. 509–519, 2003.

[9] E. H. Mamdani, "Applications of fuzzy algorithm for control a simple dynamic plant," *Proceedings of the IEEE*, vol. 121, no. 12, pp. 1585–1588, 1974.

[10] M. J. Gacto, R. Alcalá, and F. Herrera, "Interpretability of linguistic fuzzy rule-based systems: An overview of interpretability measures," *Information Sciences*, vol. 181, no. 20, pp. 4340–4360, 2011.

[11] O. Cordón, F. Herrera, and P. Villar, "Analysis and guidelines to obtain a good uniform fuzzy partition granularity for fuzzy rule–based systems using simulated annealing," *International Journal of Approximate Reasoning*, vol. 25, no. 3, pp. 187–215, 2000.

[12] J. M. Mendel, "Computing with words and its relationships with fuzzistics," *Information Sciences*, vol. 177, no. 4, pp. 988–1006, 2007.

[13] W.-H. Au, K. Chan, and A. Wong, "A fuzzy approach to partitioning continuous attributes for classification," *IEEE Transactions on Knowledge and Data Engineering*, vol. 18, no. 5, pp. 715–719, 2006.

[14] R. Sambuc, "Function ϕ-flous, application a l'aide au diagnostic en pathologie thyroidienne," Ph.D. dissertation, University of Marseille, 1975.

[15] H. Bustince, M. Pagola, E. Barrenechea, J. Fernandez, P. Melo-Pinto, P. Couto, H. R. Tizhoosh, and J. Montero, "Ignorance functions. An application to the calculation of the threshold in prostate ultrasound images," *Fuzzy Sets and Systems*, vol. 161, no. 1, pp. 20–36, 2010.

[16] D. Dubois and H. Prade, "An introduction to bipolar representations of information and preference," *International Journal of Intelligent Systems*, vol. 23, no. 8, pp. 866–877, 2008.

[17] D. Wu and J. M. Mendel, "Computing with words for hierarchical decision making applied to evaluating a weapon system," *IEEE Transactions on Fuzzy Systems*, vol. 18, no. 3, pp. 441–460, 2010.

[18] C. Wagner and H. Hagras, "Toward general type-2 fuzzy logic systems based on zslices," *IEEE Transactions on Fuzzy Systems*, vol. 18, no. 4, pp. 637–660, 2010.

[19] E. Barrenechea, H. Bustince, B. De Baets, and C. Lopez-Molina, "Construction of interval-valued fuzzy relations with application to the generation of fuzzy edge images," *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 5, pp. 819–830, 2011.

[20] P. Couto, A. Jurio, A. Varejao, M. Pagola, H. Bustince, and P. Melo-Pinto, "An IVFS-based image segmentation methodology for rat gait analysis," *Soft Computing - A Fusion of Foundations, Methodologies and Applications*, vol. 15, no. 10, pp. 1937–1944, 2011.

[21] O. Cordón, M. J. del Jesus, and F. Herrera, "A proposal on reasoning methods in fuzzy rule-based classification systems," *International Journal of Approximate Reasoning*, vol. 20, no. 1, pp. 21–45, 1999.

[22] M. Galar, J. Fernandez, G. Beliakov, and H. Bustince, "Interval-valued fuzzy sets applied to stereo matching of color images," *IEEE Transactions on Image Processing*, vol. 20, no. 7, pp. 1949–1961, 2011.

[23] A. Jurio, M. Pagola, D. Paternain, C. Lopez-Molina, and P. Melo-Pinto, "Interval-valued restricted equivalence functions applied on clustering techniques," in *Proceedings of the joint 2009 international fuzzy systems association world congress and 2009 european society of fuzzy logic and technology conference*, J. Carvalho, D. Kaymak, and J. Sousa, Eds., 2009, pp. 831–836, lisbon, Portugal, Jul. 20-24, 2009.

[24] J. Sanz, A. Fernández, H. Bustince, and F. Herrera, "Improving the performance of fuzzy rule-based classification systems with interval-valued fuzzy sets and genetic amplitude tuning," *Information Sciences*, vol. 180, no. 19, pp. 3674–3685, 2010.

[25] J. Alcalá-Fdez, L. Sánchez, S. García, M. J. del Jesus, S. Ventura, J. Garrell, J. Otero, C. Romero, J. Bacardit, V. Rivas, J. C. Fernández, and F. Herrera, "Keel: a software tool to assess evolutionary algorithms

for data mining problems," *Soft Computing*, vol. 13, no. 3, pp. 307–318, 2009.

[26] J. Alcalá-Fdez, A. Fernández, J. Luengo, J. Derrac, S. García, L. Sánchez, and F. Herrera, "KEEL data-mining software tool: Data set repository, integration of algorithms and experimental analysis framework," *Journal of Multiple-Valued Logic and Soft Computing*, vol. 17, no. 2–3, pp. 255–287, 2011.

[27] J. Demšar, "Statistical comparisons of classifiers over multiple data sets," *Journal of Machine Learning Research*, vol. 7, pp. 1–30, 2006.

[28] S. García and F. Herrera, "An extension on "statistical comparisons of classifiers over multiple data sets" for all pairwise comparisons," *Journal of Machine Learning Research*, vol. 9, pp. 2677–2694, 2008.

[29] S. García, A. Fernández, J. Luengo, and F. Herrera, "Advanced nonparametric tests for multiple comparisons in the design of experiments in computational intelligence and data mining: Experimental analysis of power," *Information Sciences*, vol. 180, no. 10, pp. 2044–2064, 2010.

[30] H. Ishibuchi and T. Nakashima, "Effect of rule weights in fuzzy rule-based classification systems," *IEEE Transactions on Fuzzy Systems*, vol. 9, no. 4, pp. 506–515, 2001.

[31] H. Ishibuchi and T. Yamamoto, "Rule weight specification in fuzzy rule-based classification systems," *IEEE Transactions on Fuzzy Systems*, vol. 13, no. 4, pp. 428–435, 2005.

[32] H. Bustince, E. Barrenechea, and M. Pagola, "Generation of interval-valued fuzzy and atanassov's intuitionistic fuzzy connectives from fuzzy connectives and from k_α operators: laws for conjunctions and disjunctions, amplitude," *International Journal of Intelligent Systems*, vol. 23, no. 6, pp. 680–714, 2008.

[33] D. Wu and J. M. Mendel, "Linguistic summarization using if-then rules and interval type-2 fuzzy sets," *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 1, pp. 136–151, 2011.

[34] J. Sanz, A. Fernández, H. Bustince, and F. Herrera, "A genetic tuning to improve the performance of fuzzy rule-based classification systems with interval-valued fuzzy sets: Degree of ignorance and lateral position," *International Journal of Approximate Reasoning*, vol. 52, no. 6, pp. 751–766, 2011.

[35] G. Beliakov, A. Pradera, and T. Calvo, *Aggregation Functions: A Guide for Practitioners. What is an aggregation function*. San Mateo-California: Studies In Fuzziness and Soft Computing, Springer, 2007.

[36] T. Calvo, A. Kolesarova, M. Komornikova, and R. Mesiar, *Aggregation Operators New Trends and Applications: Aggregation operators: properties, classes and construction methods*. Heidelberg: Physica-Verlag, 2002.

[37] G. Deschrijver, C. Cornelis, and E. Kerre, "On the representation of intuitionistic fuzzy t-norms and t-conorms," *IEEE Transactions on Fuzzy Systems*, vol. 12, no. 1, pp. 45–61, 2004.

[38] E. Trillas, "Sobre funciones de negación en la teoría de conjuntos difusos," *Stochastica*, vol. 3, no. 1, pp. 47–59, 1979, reprinted (English version) (1998) in: S. Barro et al. (Eds.), Advances of Fuzzy Logic, tri-Universidad de Santiago de Compostela, pp. 31–43.

[39] G. Deschrijver, "Generalized arithmetic operators and their relationship to t-norms in interval-valued fuzzy set theory," *Fuzzy Sets and Systems*, vol. 160, no. 21, pp. 3080–3102, 2009.

[40] Z. S. Xu and R. R. Yager, "Some geometric aggregation operators based on intuitionistic fuzzy sets," *International Journal of General Systems*, vol. 35, no. 4, pp. 417–433, 2006.

[41] H. Bustince, E. Barrenechea, and M. Pagola, "Restricted equivalence functions," *Fuzzy Sets and Systems*, vol. 157, no. 17, pp. 2333–2346, 2006.

[42] ——, "Image thresholding using restricted equivalence functions and maximizing the measures of similarity," *Fuzzy Sets and Systems*, vol. 158, no. 5, pp. 496—516, 2007.

[43] J. M. Mendel, *Rule-Based Fuzzy Logic Systems: Introduction and New Directions*. Prentice-Hall, Englewood Cliffs, NJ, 2001.

[44] E. A. Jammeh, M. Fleury, C. Wagner, H. Hagras, and M. Ghanbari, "Interval type-2 fuzzy logic congestion control for video streaming across ip networks," *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 5, pp. 1123–1142, 2009.

[45] H. Wu and J. M. Mendel, "Classification of battlefield ground vehicles using acoustic features and fuzzy logic rule-based classifiers," *IEEE Transactions on Fuzzy Systems*, vol. 15, no. 1, pp. 56–72, 2007.

[46] R. Hosseini, S. Qanadli, S. Barman, M. Mazinani, T. Ellis, and J. Dehmeshki, "An automatic approach for learning and tuning gaussian interval type-2 fuzzy membership functions applied to lung cad classification system," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 2, pp. 224 –234, 2012.

[47] D. Wu and W. W. Tan, "Genetic learning and performance evaluation of interval type-2 fuzzy logic controllers," *Engineering Applications of Artificial Intelligence*, vol. 19, no. 8, pp. 829–841, 2006.

[48] R. Alcalá, J. Alcalá-Fdez, M. J. Gacto, and F. Herrera, "Rule base reduction and genetic tuning of fuzzy systems based on the linguistic 3-tuples representation," *Soft Computing*, vol. 11, no. 5, pp. 401–419, 2007.

[49] M. J. Gacto, R. Alcalá, and F. Herrera, "Adaptation and application of multi-objective evolutionary algorithms for rule reduction and parameter tuning of fuzzy rule-based systems," *Soft Computing*, vol. 13, no. 5, pp. 419–436, 2009.

[50] L. Eshelman, *Foundations of Genetic Algorithms*. Morgan Kaufman, 1991, ch. The CHC adaptive search algorithm: How to have safe search when engaging in nontraditional genetic recombination, pp. 265—283.

[51] D. Whitley, S. Rana, J. Dzubera, and K. E. Mathias, "Evaluating evolutionary algorithms," *Artificial Intelligence*, vol. 85, no. 1–2, pp. 245–276, 1996.

[52] A. Fernández, M. J. del Jesus, and F. Herrera, "On the influence of an adaptive inference system in fuzzy rule based classification systems for imbalanced data-sets," *Expert Systems with Applications*, vol. 36, no. 6, pp. 9805–9812, 2009.

[53] R. Alcalá, J. Alcalá-Fdez, and F. Herrera, "A proposal for the genetic lateral tuning of linguistic fuzzy systems and its interaction with rule selection," *IEEE Transactions on Fuzzy Systems*, vol. 15, no. 4, pp. 616–635, 2007.

[54] A. Fernández, M. J. del Jesus, and F. Herrera, "On the 2-tuples based genetic tuning performance for fuzzy rule based classification systems in imbalanced data-sets," *Information Sciences*, vol. 180, no. 8, pp. 1268–1291, 2010.

[55] F. Herrera, M. Lozano, and A. M. Sánchez, "A taxonomy for the crossover operator for real-coded genetic algorithms: An experimental study," *International Journal of Intelligent Systems*, vol. 18, no. 3, pp. 309–338, 2003.

[56] L. J. Eshelman and J. Schafffer, *Foundations of Genetic Algorithms*. Morgan Kaufman, 1993, ch. Real coded genetic algorithms and interval schemata, pp. 187—202.

[57] W. W. Cohen, "Fast effective rule induction," in *Proceedings of the 12th international conference on Machine learning*, 1995, lake Tahoe, CA.

[58] S. García, A. Fernández, J. Luengo, and F. Herrera, "A study of statistical techniques and performance measures for genetics–based machine learning: Accuracy and interpretability," *Soft Computing*, vol. 13, no. 10, pp. 959–977, 2009.

[59] D. Sheskin, *Handbook of parametric and nonparametric statistical procedures*, 2nd ed. Chapman & Hall/CRC, 2006.

[60] J. L. Hodges and E. L. Lehmann, "Ranks methods for combination of independent experiments in analysis of variance," *Annals of Mathematical Statistics*, vol. 33, pp. 482–497, 1962.

[61] S. Holm, "A simple sequentially rejective multiple test procedure," *Scandinavian Journal of Statistics*, vol. 6, pp. 65–70, 1979.

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