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Computing with Words in Decision support Systems: An overview on Models and Applications

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Abstract

Decision making is inherent to mankind, as human beings daily face situations in which they should choose among different alternatives by means of reasoning and mental processes. Many of these decision problems are under uncertain environments with vague and imprecise information. This type of information is usually modelled by linguistic information because of the common use of language by the experts involved in the given decision situations, originating *linguistic decision making*. The use of linguistic information in decision making demands processes of Computing with Words to solve the related decision problems. Different methodologies and approaches have been proposed to accomplish such processes in an accurate and interpretable way. The good performance of linguistic computing dealing with uncertainty has caused a spread use of it in different types of decision based applications. This paper overviews the more significant and extended linguistic computing models due to its key role in linguistic decision making and a wide range of the most recent applications of linguistic decision support models.

Keywords: Decision support systems, linguistic decision making, computing with words

1. Introduction

Human activities are very diverse and it is fairly common the necessity in many of them of decision making processes. Decision making can be seen as a process composed of different phases such as information gathering, analysis and selection based on different mental and reasoning processes that led to choose a *suitable* alternative among a set of possible alternatives in a given activity [24,54].

Remarkably, decision making is a core area in a wide range of disciplines such as engineering, psychology, operations research, artificial intelligence, etc. Because of this variety of disciplines, decision problems have been classified in decision theory attending to their framework and elements [23]. Sometimes the solving process of a decision making problem is straightforward by using an algorithmic approach, these situations are so-called well-structured problems. However many decision problems cannot be solved in this way because decisions might be related to changing environments, the existence of vagueness and uncertainty in the decision framework, and so on. The latter problems, so-called ill-structured problems [114], are quite common in real problems of the aforementioned disciplines.

In this paper we focus on ill-structured decision problems dealing with vague and imprecise information, i.e., decision making under uncertainty. Classical decision theory provides probabilistic models to manage uncertainty in decision problems but in many of them it is easy to observe that a lot of aspects of these uncertainties have a non-probabilistic character since they are related to imprecision and vagueness of meanings [64]. Linguistic descriptors are often used by experts in such a type of problems. Therefore, taking into account that linguistic terms are fuzzy judgments rather than probabilistic values among the appropriate tools to overcome these difficulties of managing and modelling this type of uncertainties, fuzzy logic and fuzzy set theory [45,107] arise to facilitate the managing of uncertainty in decision processes [9,54] and the fuzzy linguistic approach [108,109,110] provides a direct way to represent the linguistic information by means of linguistic variables. The use of linguistic information thus enhances the reliability and flexibility of classical decision models [66].

The use of linguistic information plays a key role not only in linguistic decision making [33,35,63] but also in other fields [2,43,44,75,85] that need to operate with linguistic information. Computing with words (CW) has recently become an important research topic in which different methodologies and approaches have been proposed. Since CW deals with words or sentences defined in a natural or artificial language instead of numbers, it emulates human cognitive processes to improve solving processes of problems dealing with uncertainty. Consequently, CW has been applied as computational basis to linguistic decision making [35], because it provides tools close to human beings reasoning processes related to decision making, which improve the resolution of decision making under uncertainty as linguistic decision making.

This paper overviews the most wide-spread methodologies of CW used in linguistic decision making [16,35,37,89,97], including a short list of those [5,47,84,87,88] that are interesting for specific decision situations but they have not been intensively used yet. It further presents in depth the most recent decision applications based on CW over the last years regarding real world applications.

The paper is structured as follows, Section 2 overviews CW and its use in decision making. Section 3 reviews both linguistic modelling and computing, specially the computing models most wide-used in linguistic decision making. Section 4 lists recent applications based on linguistic decision making. And Section 5 concludes the paper.

2. Computing with Words in Decision Making

In many real decision situations is straightforward the use of linguistic information due to the nature of different aspects of the decision problem. In such situations one common approach to model the linguistic information is the fuzzy linguistic approach [108,109,110] that uses the fuzzy set theory [107] to manage the uncertainty and model the information.

Zadeh [108] introduced the concept of linguistic variable as *"a variable whose values are not numbers but words or sentences in a natural or artificial language"*. A linguistic value is less precise than a number but it is closer to human cognitive processes used to solve successfully problems dealing with uncertainty. Formally a linguistic variable is defined as follows.

Definition 1 [108]: *A linguistic variable is characterized by a quintuple (H,T(H),U,G,M) in which H is the name of the variable; T(H) (or simply T) denotes the term set of H, i.e., the set of names of linguistic values of H, with each value being a fuzzy variable denoted generically by X and ranging across a universe of discourse U which is associated with the base variable u; G is a syntactic rule (which usu-*

ally takes the form of a grammar) for generating the names of values of H; and M is a semantic rule for associating its meaning with each H, $M(X)$, which is a fuzzy subset of U.

Fig. 1 shows a linguistic term set with the syntax and semantics of their terms.

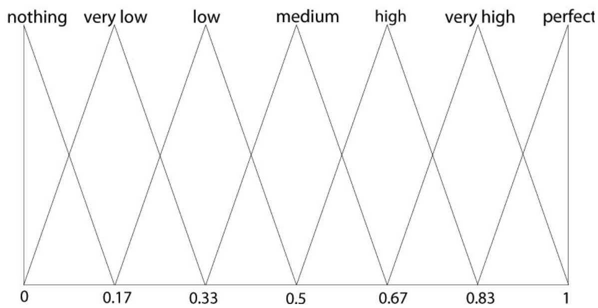

Fig. 1. A seven-term set with its semantics

One crucial aspect to determine the validity of a CW approach is the selection of the membership functions for the linguistic term set. There exist different approaches to choose the linguistic descriptors and different ways to define their semantics [101,109,110].

It is necessary to analyze the phases of a linguistic decision scheme as long as the linguistic information is formally modelled. A common decision resolution scheme consists of two main phases [76]:

1. An *aggregation phase* that aggregates the values provided by the experts to obtain a collective assessment for the alternatives.

2. An *exploitation phase* of the collective assessments to rank, sort or choose the best one/s among the alternatives.

Herrera and Herrera-Viedma [35] analyzed how should the previous scheme change in linguistic decision making? They pointed out the necessity of introducing two new steps previously to the application of both the aggregation and exploitation phases by the following resolution scheme:

1. *The choice of the linguistic term set with its semantics.* It establishes the linguistic expression domain in which experts provide their linguistic assessments about alternatives according to their knowledge.

2. *The choice of the aggregation operator of linguistic information.* An appropriate aggregation operator of linguistic information is chosen for aggregating the linguistic assessments. The appropriateness of the operator depends on each single decision problem.

3. *The choice of the best alternatives.* The best alternative/s are chosen according to the linguistic assessments provided by the experts. It is carried out by the two phases of the common resolution scheme:

   (a) *Aggregation phase of linguistic information:* It obtains a linguistic collective assessment for each alternative by aggregating the experts linguistic assessments under the chosen linguistic aggregation operator.

   (b) *Exploitation phase:* It ranks the alternatives by using the collective linguistic assessment obtained in the previous phase in order to choose the best alternative/s.

Looking at this linguistic solution scheme, it is clear the necessity of linguistic computational models that allow *computations* with linguistic information in order to obtain accurate results and provide a *representation* that facilitates the interpretability of them.

3. Linguistic Computational Models

Due to the relevance of linguistic decision making in real problems and the necessity of methodologies for CW, there exist different linguistic computational models. We shall pay more attention to those that have been wide-used in linguistic decision making. We consider the analysis not only of the computational model but also of its linguistic representation utilized to represent the results.

3.1. Linguistic computational model based on membership functions

A) Representation

This computational model is based on the fuzzy linguistic approach and represents the linguistic information according to Definition 1 (See Fig. 1).

B) Computation

This computational model makes the computations directly on the membership functions of the linguistic terms by using the Extension Principle [45]. The fuzzy arithmetic provides as result of a computation, $\widetilde{F}$, regarding a set of n linguistic labels in the term set, $T(H)$, a fuzzy number, $F(\mathscr{R})$, that usually does not match any linguistic label in $T(H)$. From these results we have:

i) In those problems that accuracy outweighs interpretability (ranking purposes). The results are expressed by the fuzzy numbers themselves using fuzzy ranking procedures to obtain a final order of the alternatives [1,27].

ii) If an interpretable and linguistic result is demanded then an approximation function $app_1(\cdot)$ is applied to associate the fuzzy result $F(\mathscr{R})$ to a label in $T(H)$ [16,58,102]:

$$T(L)^n \xrightarrow{\widetilde{F}} F(\mathscr{R}) \xrightarrow{app_1(\cdot)} T(L)$$

The approximation process implies a loss of information and lack of accuracy of the results.

3.2. Linguistic computational model based on type-2 fuzzy sets

A) Representation

This computational model makes use of type-2 fuzzy sets (see Fig. 2) to model the linguistic assessments [67,86,87].

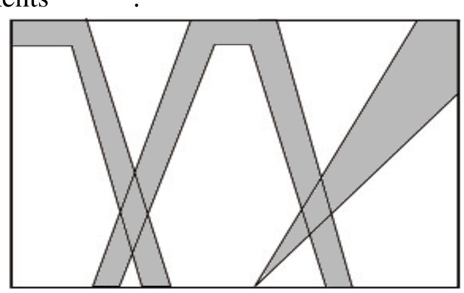

Fig. 2. Linguistic terms represented by Type-2 fuzzy sets

The use of type-2 fuzzy sets has been justified in different ways:

- In [86]: *"Type-1 representation is a 'reductionist' approach for it discards the spread of membership values by averaging or curve fitting techniques and hence, camouflages the 'uncertainty' embedded in the spread of membership values."*
- In [68] *"Words mean different things to different people and so are uncertain. We therefore need a fuzzy set model for a word that has the potential to capture its uncertainties, and an interval type-2 fuzzy set should be used as a fuzzy set model of a word."*

B) Computation

The majority of the contributions in the field use interval type-2 fuzzy sets (a particular kind of type-2 fuzzy sets) which maintain the uncertainty modelling properties of general type-2 fuzzy sets but reducing the computational efforts that are needed to operate with them. In [20,113] the Linguistic Weighted Average and the Linguistic OWA operators based on the type-2 representation are presented. They can be seen as respective extensions of the Fuzzy Weighted Aggregation and OWA operators where both weights and attributes are words modelled by interval type-2 fuzzy sets.

As the previous linguistic model revised, this type-2 fuzzy sets based model needs to approximate the resulting type-2 fuzzy set from a linguistic operation by mapping the result into a linguistic assessment producing a loss of information.

3.3. Linguistic symbolic computational models based on ordinal scales

Symbolic models have been widely used in CW because of their simple computational processes and high interpretability. The initial proposal for a symbolic model [99] uses max-min operators, and new symbolic proposals [17,97] introduce aggregation based symbolic models. We shall review different linguistic symbolic computational models based on ordinal scales.

3.3.1. Linguistic symbolic computational model based on ordinal scales and max-min operators

A) Representation

This model [99] represents the information according to the fuzzy linguistic approach (See Fig. 1) but imposes a linear ordering to the linguistic term set $S = \{s_1, s_2, \ldots, s_g\}$ such that $s_i \geqslant s_j \Leftrightarrow i \geqslant j$.

B) Computation

It uses the ordered structure of the linguistic term set to accomplish symbolic computations in such ordered linguistic scales that the classical operators Max, Min and Neg are proposed:

- $Max(s_i, s_j) = S_i$ if $s_i \geqslant s_j$,
- $Min(s_i, s_j) = S_i$ if $s_i \leqslant s_j$ and
- $Neg(s_i) = s_{g-i+1}$ where g is the cardinality of S.

More operators have been proposed for this model, Yager [100,103] studied several aggregation operators for ordinal information such as weighted norm operators, uninorm operators and ordinal mean type operators. Buckley [3] proposed different variations of the median, max and min operators to aggregate linguistic opinions and criteria.

3.3.2. Linguistic symbolic computational model based on convex combinations

A) Representation

This model [17] is an extension of the previous one that is based on the same representation basis.

B) Computation

It provides a wider range of aggregation operators by using a convex combination of linguistic labels [17], which directly acts over the label indexes, $\{0, .., g\}$, of the linguistic term set, $S = \{s_0, \ldots, s_g\}$, in a recursive way producing a real value on the granularity interval, $[0, g]$, of the linguistic term set S. Note that this model usually assumes that the cardinality of the linguistic term set is odd and that linguistic labels are symmetrically placed around a middle term. As a result of a convex combination aggregation does not match usually with a term of the label set S, it is also necessary to introduce an approximation function $app_2(\cdot)$ to obtain a solution in the term set S:

$$S^n \xrightarrow{C} [0, g] \xrightarrow{app_2(\cdot)} \{0, ..., g\} \rightarrow S$$

Similarly to the model presented in Section 3.1, the approximation process produces a loss of information in the final results.

Aggregation operators based on this linguistic model are the Linguistic Ordered Weighted Averaging (LOWA) operator [36] (based on the OWA operator and the convex combination of linguistic labels), the Linguistic Weighted Disjunction (LWD), Linguistic Weighted Conjunction (LWC), the Linguistic Weighted Averaging (LWA) [34], the Linguistic Aggregation of Majority Additive (LAMA) operator [73] and the Majority Guided Induced Linguistic Aggregation Operators [41].

3.3.3. Linguistic symbolic computational model based on virtual linguistic terms

A) Representation

Xu [97] introduced this model to increase the accuracy and operators in processes of CW. To do so, the discrete term set $S = \{s_{-\frac{g}{2}}, \ldots, s_0, \ldots, s_{\frac{g}{2}}\}$, with $g+1$ being the cardinality of S, is extended into a continuous term set $\overline{S} = \{s_\alpha | \alpha \in [-t, t]\}$, where t ($t >> g/2$) is a sufficiently large positive integer. If $s_\alpha \in S$, then s_α is called an *original linguistic term*, otherwise, s_α is called a *virtual linguistic term*. Fig. 3 shows a discrete term set $S = \{s_{-3}, \ldots, s_3\}$ (original linguistic terms) that is extended to a continuous term set in which virtual linguistic terms as $s_{-0.3} \in [-3, 3]$ can be obtained and manipulated to avoid loss of information.

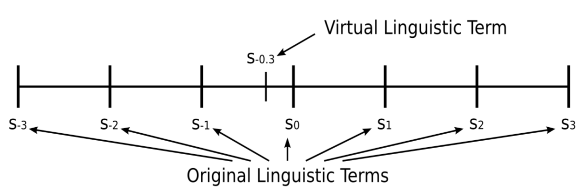

Fig. 3. Example of the linguistic model proposed by Xu [97]

According to Xu this extension allows to preserve all given information in the problem (avoiding the loss of information presents in classical linguistic symbolic computational models). Xu stated that, *"in general, the decision maker uses the original linguistic terms to evaluate alternatives, and the virtual linguistic terms only appear in operation"*.

B) Computation

Let $s_\alpha, s_\beta \in \overline{S}$, be any two linguistic terms and $\mu, \mu_1, \mu_2 \in [0,1]$. Then to accomplish processes of CW with this representation model, Xu introduced the following operational laws [96,98]:

1. $(s_\alpha)^\mu = s_{\alpha^\mu}$
2. $(s_\alpha)^{\mu_1} \otimes (s_\alpha)^{\mu_2} = (s_\alpha)^{\mu_1+\mu_2}$
3. $(s_\alpha \otimes s_\beta)^\mu = (s_\alpha)^\mu \otimes (s_\beta)^\mu$
4. $s_\alpha \otimes s_\beta = s_\beta \otimes s_\alpha = s_{\alpha\beta}$
5. $s_\alpha \oplus s_\beta = s_{\alpha+\beta}$
6. $s_\alpha \oplus s_\beta = s_\beta \oplus s_\alpha$
7. $\mu s_\alpha = s_{\mu\alpha}$
8. $(\mu_1 + \mu_2) s_\alpha = \mu_1 s_\alpha \oplus \mu_2 s_\alpha$
9. $\mu(s_\alpha \oplus s_\beta) = \mu s_\alpha \oplus \mu s_\beta$

Importantly, this symbolic computational model uses a term set that changes during the computations as new virtual terms are created in the computing processes. The appearance of virtual terms without syntax either semantics limits the interpretability of the results of this computational model. Therefore, this model also needs an approximation process, implying lack of accuracy, if the results of the operations are virtual linguistic terms (and they will usually be virtual ones) and the problem looks for interpretable final results in the original linguistic term set. Otherwise they can be used for ranking purposes.

3.4. A 2-tuple Linguistic computational model: A symbolic model extending the use of indexes

The 2-tuple linguistic model [37] is a symbolic computational one introduced by Herrera and Martínez in order to improve the accuracy and facilitate the processes of CW by treating the linguistic domain as continuous but keeping the linguistic basis (syntax and semantics). To do so, this model extends the fuzzy linguistic representation adding a new parameter.

A) Representation

The modelling of the linguistic information is based on the concept of *symbolic translation* and uses it for representing the linguistic information by means of a pair of values, so-called *linguistic 2-tuple*, (s_i, α) where s_i is a linguistic term and α is a numerical value representing the *Symbolic Translation*.

Definition 2 [37]: *Let β be the result of symbolic aggregation over a set of labels $\{s_k \in S, k = \{1,...,n\}\}$ assessed in the linguistic term set $S = \{s_0, \ldots, s_g\}$, hence $\beta \in [0,g]$. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values such that i represents a term index in the interval of granularity $\{0,1,...,g\}$ and $\alpha \in [-0.5, 0.5)$ is the "difference of information" between β and the index of the closest linguistic term s_i in S. α is then so-called a Symbolic Translation.*

This representation model defines a set of transformation functions between numeric values and linguistic 2-tuples to facilitate linguistic computational processes [37].

Definition 3 [37]: *Let $S = \{s_0, \ldots, s_g\}$ and $\beta \in [0,g]$ be a set of linguistic terms and the result of a symbolic aggregation operation respectively. The 2-tuple associated with β is then obtained by the function $\Delta : [0,g] \longrightarrow S \times [-0.5, 0.5)$ defined as:*

$$\Delta(\beta) = (s_i, \alpha), \text{ with } \begin{cases} i = \text{round}(\beta), \\ \alpha = \beta - i, \end{cases} \quad (1)$$

where round *assigns to β the integer number, $i \in \{0,1,\ldots,g\}$, closest to β.*

We note that Δ is a one-to-one mapping [37,38] and $\Delta^{-1} : S \times [-0.5, 0.5) \longrightarrow [0,g]$ is defined by $\Delta^{-1}(s_i, \alpha) = i + \alpha$. In this way, the 2-tuple of $S \times [-0.5, 0.5)$ will be identified with the numerical values in the interval $[0,g]$.

Obviously the conversion between a linguistic term into a linguistic 2-tuple consists of adding a value 0 as symbolic translation: $s_i \in S \Longrightarrow (s_i, 0)$.

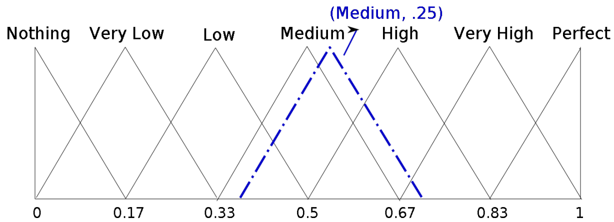

Fig. 4. A linguistic 2-tuple representation

Figure 4 shows an example of a 2-tuple linguistic label that expresses the equivalent information of the result of a symbolic aggregation operation. Let us suppose that $\beta = 3.25$ is a value representing the result of a symbolic aggregation operation on the set of labels, $S = \{s_0 : Nothing, \ldots, s_6 : Perfect\}$, then the 2-tuple that expresses the equivalent information to β is $(Medium, .25)$.

B) Computation

Together the representation model, a linguistic computational approach based on the functions Δ and Δ^{-1} was also defined in [37], where some classical aggregation operators as the Arithmetic Mean, the Weighted Average Operator, the Ordered Weighted Aggregation (OWA) operator, the LOWA operator were extended for the linguistic 2-tuple. Other aggregation operators for the linguistic 2-tuple were later defined as the Lattice-based Linguistic-Valued Weighted Aggregation (LVWA) [51] and the LAMA operator [73].

3.4.1. Proportional 2-tuple linguistic computational model: An extension of the linguistic 2-tuple model

Even though the 2-tuple is a quite recent model, it has attracted attention in the specialized literature and some extensions to the 2-tuple linguistic model have been developed. The proportional 2-tuple introduced by Wang and Hao [89] develops a new way to represent the linguistic information that is a generalization and extension of 2-tuple linguistic representation model [37].

A) Representation

This model represents the linguistic information by means of proportional 2-tuple, such as $(0.2A, 0.8B)$ for the case when someone's grades in the answer scripts of a whole course are distributed as $20\%A$ and $80\%B$. The authors pointed out that if B were used as the approximative grade then some performance information would be lost. This proportional 2-tuple model is based on the concept of *symbolic proportion* [89].

Definition 4. *Let $S = \{s_0, s_1, \ldots, s_g\}$ be an ordinal term set, $I = [0,1]$ and*

$$IS \equiv I \times S = \{(\alpha, s_i) : \alpha \in [0,1] \text{ and } i = 0, 1, \ldots, g\} \tag{2}$$

where S is the ordered set of $g+1$ ordinal terms $\{s_0, \ldots, s_g\}$. Given a pair (s_i, s_{i+1}) of two successive ordinal terms of S, any two elements (α, s_i), (β, s_{i+1}) of IS is so-called a symbolic proportion pair and α, β are a pair of symbolic proportions of the pair (s_i, s_{i+1}) if $\alpha + \beta = 1$. A symbolic proportion pair (α, s_i), $(1-\alpha, s_{i+1})$ is denoted by $(\alpha s_i, (1-\alpha)s_{i+1})$ and the set of all the symbolic proportion pairs is denoted by $\overline{S}$, i.e., $\overline{S} = \{(\alpha s_i, (1-\alpha)s_{i+1}) : \alpha \in [0,1] \text{ and } i = 0, 1, \ldots, g-1\}$.

$\overline{S}$ is called the *ordinal proportional 2-tuple set* generated by S and the members of $\overline{S}$, *ordinal proportional 2-tuple*, which are used to represent the ordinal information for CW.

In a similar way to the symbolic 2-tuple [37], Wang and Hao introduced functions to facilitate the computations with this type of representation.

Definition 5. *Let $S = \{s_0, s_1, \ldots, s_g\}$ be an ordinal term set and $\overline{S}$ be the ordinal proportional 2-tuple set generated by S. The function $\pi : \overline{S} \to [0, g]$ is defined by*

$$\pi((\alpha s_i, (1-\alpha)s_{i+1})) = i + (1-\alpha), \tag{3}$$

where $i = \{0, 1, \ldots, g-1\}, \alpha \in [0,1]$ and π is called the position index function of ordinal 2-tuples

Note that, under the identification convention by Eq. (2), the position index function π becomes a one-to-one mapping from $\overline{S}$ to $[0, g]$ and its inverse $\pi^{-1} : [0, g] \to \overline{S}$ is defined by

$$\pi^{-1}(x) = ((1-\beta)s_i, \beta s_{i+1}) \tag{4}$$

where $i = E(x)$, E is the integer part function, $\beta = x - i$.

B) Computation

To operate with linguistic information under proportional 2-tuple contexts, Wang and Hao expanded the computational techniques for symbolic information to proportional 2-tuple and underlying definitions of linguistic labels and linguistic variables are taken into account in the process of aggregating linguistic information by assigning canonical characteristic values of the corresponding linguistic labels [89,90].

3.4.2. *Others 2-tuple based linguistic computational models*

Quite recently two new linguistic computational models based on extensions and/or hybridizing with the 2-tuple linguistic representation model have been presented in [19,49].

- An extended 2-tuple fuzzy linguistic representation that fuses the virtual linguistic terms [97] (see Section 3.3.3) and the linguistic 2-tuple model [37] is presented by Deng-Feng [49] that transforms virtual terms into *original* linguistic values by using a representation based on the 2-tuple so-called extended 2-tuple. This representation and the computational model based on virtual linguistic terms are used to introduce a Multi-attribute Group Decision Making method based on the generalized induced OWA operators.
- Dong et al. [19] introduced the concept of numerical scale, which extends the linguistic 2-tuple [37] and the proportional 2-tuple models [89], together with the concepts of transitive calibration matrix its consistent index and an optimization model to compute the numerical scale of the linguistic term set from the previous matrix. With the aim to complete the 2-tuple based models for CW and make the information of the decision maker more consistent in different decision situations.

3.5. Others linguistic computational models

As it was aforementioned, because of the high attention that CW has received in the last years additionally to the previous wide-used models in linguistic decision making, other new approaches and methodologies for CW have been introduced in the specialized literature:

- Lawry presents both an alternative approach to CW based on mass assignment theory and a new framework for linguistic modelling that avoid some of the complexity problems that arise by the use of the extension principle in Zadeh's CW methodology [46,47,48].
- Rubin defines CW as a symbolic generalization of fuzzy logic [77].
- Ying et al. propose a new formal model for CW based on fuzzy automatas whose inputs are strings of fuzzy subsets of the input alphabet [5,106].
- Wang et al. extend Ying's work considering CW via a different computational model, in particular, Turing machines [88].
- Tang et al. present a new linguistic modelling that can be applied in CW which does not directly rely on fuzzy sets to model the meaning of natural language terms but uses some fuzzy relations between the linguistic labels to model their semantics [84].
- Türkşen proposes the use of meta-linguistic axioms as a foundation for CW as an extension of fuzzy sets and logic theory [87].

Finally, we remark that the management of perceptions is also highly related to linguistic information and to human cognitive processes, a historical review of computing with perceptions was introduced by Mendel [69].

4. Recent Applications of CW in Decision Making

Once we have reviewed the preponderant position that the linguistic information plays in decision making under uncertainty and the different computing models proposed in the literature to manage such information. In this section we review recent decision applications (published in the specialized literature in 2007-2010) based on linguistic models.

Despite the wide range of applications in which linguistic decision based models have been applied, we have organized the application papers according to the following areas:

- **Industrial Applications**: Different key strategic selection industrial processes that are complex to solve due to their uncertain environments have been considered under linguistic decision models.

Table 1. Industrial applications

Applications	*Papers*	*Year*
Supplier selection & evaluation	Chang et al. [7]	2007
	Li et al. [50]	2007
	Onut et al. [70]	2009
	Zhang et al. [112]	2009
	Sanayei et al. [78]	2010
Location selection	Chou et al. [11]	2008
	Önüt et al. [71]	2008
	Anagnostopoulos et al.[1]	2008
	Demirel et al. [18]	2010
Material, stock and systems selection	Lin et al. [52]	2007
	Gharehgozli et al. [30]	2008
	Lu et al. [57]	2009
Manufacturing flexibility evaluation	Chuu [12]	2007
	Chuu [13]	2009

- **Internet based services**: The viral growth of Internet has provoked the necessity of solving different problems related to its services, such as, to retrieve customized products or information from huge data bases or to manage social networks issues in the web 2.0. For all these problems different linguistic decision based solutions have been proposed.

Table 2. Internet based linguistic applications

Applications	*Papers*	*Year*
Information retrieval	Herrera-Viedma et al. [39,40]	2007
	Liu [53]	2009
	Pei et al. [72]	2009
Recommender systems	Martínez et al. [65]	2007
	Martínez et al. [60]	2008
	Porcel et al. [74]	2009
	Castellano and Martínez [6]	2009
	Wang [93]	2009
Web quality	Herrera-Viedma et al. [42]	2007
Social Networks	Yager [104]	2010

- **Resource management**: The management of resources is a really complex task. Moreover, if the imprecision and subjectivity of the related information of such problems are taken into consideration. Different management applications based on linguistic decision making have been proposed.

Table 3. Resource management linguistic based applications

Applications	*Papers*	*Year*
Sustainable energy management	Doukas et al. [22]	2007
	Doukas and Psarras [21]	2009
Water resources management	Fu [27]	2008
	Sen and Altunkaynak [80]	2009
	Zarghami & Szidarovszky [111]	2009
Human resources management	Yang et al. [105]	2007
	Genevois et al. [29]	2008
	Sun et al. [82]	2008
	Tai et al. [83]	2009
	de Andrés et al. [15,14]	2010
Knowledge management	Wang et al. [91]	2007
	Fan et al. [26]	2009
Situation awareness	Lu et al. [56]	2008

- **Evaluation**: Decision analysis has been widely used in evaluation processes. The existence of real evaluation problems dealing with uncertain, vague and imprecise information that fits pretty well linguistic decision analysis has derived in many linguistic evaluation proposals.

Table 4. Linguistic Evaluation Processes

Applications	*Papers*	*Year*
Projects evaluation & selection	G. Büyüközkan et al. [4]	2008
	Halouani et al. [32]	2009
	Sánchez et al. [79]	2009
Engineering evaluation	Martínez et al. [63]	2007
Sensory evaluation	Zou et al. [115]	2008
	Martínez [59]	2007
	Martínez et al. [62]	2008
	Chen et al. [10]	2009
	Martínez et al. [61]	2009
Investments evaluation	Shevchenko et al.[81]	2008
New Product Development	Fan et al. [25]	2009
	Wang [92]	2009

- **Other applications**: Additionally to the previous ones, linguistic decision models have been applied to other applications.

Table 5. Other Applications

Applications	*Papers*	*Year*
Situation Assessment	Lu et al. [55]	2008
Investment improvement	Güngör et al. [31]	2007
Voting systems	García and Martínez [28]	2009
Risk assessment	Wang et al. [94]	2007
	Chang and Wang [8]	2010
	Xu et al. [95]	2010

5. Conclusions

The frequency that human beings face decision making problems defined under uncertain situations, in which the use of linguistic information to describe such uncertainty has produced that linguistic decision making became a common process in real world applications. The modelling and treatment of linguistic information for necessary computing with words processes are crucial. Therefore in this paper we have reviewed different linguistic computing models with their respective linguistic representations paying more attention to those ones that have been widely used in linguistic decision making. We have not described the decision models based on CW, which can be found in the review presented by Herrera et al.[33]. Eventually to show the usability and advantages that the linguistic information produces in decision making, we have presented a not exhaustive but rather a wide and recent list of applications.

An associated website at `http://sci2s.ugr.es/CWDM/` includes a more exhaustive list of most publications in the specialized literature about the topic.

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