Mathematics Teacher's Specialised Knowledge of Prospective Primary Teachers: An Explorative Study

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The importance of good mastery of specialised mathematics knowledge for mathematics teachers and prospective teachers is now recognised internationally. This paper presents an exploratory study aimed at investigating how mathematics education courses affect the Mathematics Teachers' Specialised Knowledge (MTSK) of prospective primary teachers. A questionnaire was administered to a sample of 118 future teachers, and an analysis of their answers was carried out with the MTSK model. The research highlights positive feedback on the effectiveness of the mathematics education laboratory that was staged, and provides an outline of the beliefs and the knowledge of the prospective teachers.

Keywords: MTSK model; Primary education; Prospective primary teachers

El conocimiento especializado del profesor de matemáticas de futuros maestros de primaria: un estudio exploratorio

La importancia de un buen dominio del conocimiento matemático especializado para los futuros profesores de matemáticas está aceptada internacionalmente. Este artículo presenta un estudio exploratorio destinado a investigar cómo los cursos de educación matemática afectan el Conocimiento Especializado de Profesor de Matemáticas (MTSK) de futuros profesores de educación primaria. Se administró un cuestionario a una muestra de 118 futuros profesores y se analizaron sus respuestas con el modelo MTSK. La investigación pone de relieve los comentarios positivos sobre la eficacia del laboratorio de enseñanza de las matemáticas que se puso en marcha, y permite realizar un esbozo de las creencias y los conocimientos de los futuros profesores.

Palabras clave: Educación Primaria; Modelo MTSK; Profesores de primaria en formación

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The degree courses in the Faculty of Education represent a convergence and practical implementation of various fields of education, covering both disciplinary and general didactics. The relationship between different types of knowledge has been the subject of intense debate in pedagogy (Depaepe, Verschaffel, & Kelchtermans, 2013; Gardner, 1999). However, the necessity to implement coherent courses for future primary school teachers has directed research in searching for common points of convergence; the need to access a coherent and non-fragmented set of knowledge is growing stronger (Ferretti & Maffia, 2019). Besides, it seems legitimate to affirm that the combination of the various didactics can actually contribute to an effective development of the teachers' professionalism (Tamir, 1988). The not-always positive relationship of students with the discipline (e.g., Di Martino, Coppola, Mollo, Pacelli, & Sabena, 2013), leads to a continuous tension in the focus on teaching methodologies, often resulting in gaps in mathematical knowledge. This contribution presents a research study conducted with pre-service teachers of the Faculty of Education aimed at investigating the disciplinary knowledge together with pedagogical knowledge that prospective teachers should possess and their beliefs about them. In particular, we outline the results of a questionnaire designed to investigate whether, and how, mathematics education courses affect the necessary specialised mathematics knowledge of prospective teachers; analysis of the responses is carried out using the Carrillo-Yáñez and colleagues' (2018) framework of Mathematics Teachers' Specialised Knowledge (MTSK). In our research, we assume that disciplinary knowledge is not sufficient for prospective teachers, content knowledge of the discipline is essential to the process of teaching/learning of mathematics, but, as pointed out by Shulman (1986), this content knowledge should be intertwined with pedagogical knowledge and this is fundamental in the design, development, and assessment phase of classroom activities. In this paper, we investigate the prospective teachers' specialised knowledge at the end of a laboratory of mathematics education and, in order to have a detailed characterisation of the situation, we framed their knowledge and their beliefs in the sub-domain of the MTSK model (Carrillo-Yáñez et al., 2018).

THEORETICAL FRAMEWORK

The importance of the disciplinary knowledge of teachers who deal with a specific discipline is internationally recognised; already in the mid-80s, Shulman (1986; 1987) focused attention on Subject Knowledge for Teaching and proposed a model aimed at outlining the areas of knowledge that teachers should possess, in terms of Pedagogical Content Knowledge (PCK). His innovation was the denotation of a new knowledge of the content, specific to teaching. Inspired by these initial studies, over the last few years several more studies have tackled aspects concerning both content knowledge and how it is taught (e.g., Depaepe et al., 2013). To investigate the knowledge of teachers, the studies did not start from the list of contents in the school curriculum but instead focused on empirical approaches to understanding the mathematical content needed for teaching by investigating its basis, role, and relevance. These studies (of which one of the most relevant is Ball, Thames, and Phelps, 2008) not only contributed to the improvement of PCK in identifying subdomains, but also provided a framework for conceptualization of the knowledge and mathematical skills necessary for teaching by identifying the Specialized Content Knowledge (SCK). As pointed out by Carrillo-Yáñez and colleagues (2018), the object of analysis in the MKT model is assessment of the mathematical knowledge used by teachers to carry out their work, as opposed to their own, entire knowledge; so, Ball et al. (2008) focused their attention on classroom performance. For analysis of the protocols of our research, we turned to the subdomains of the MTSK model ideated by Carrillo-Yáñez and colleagues (2018), which allowed us to conduct a fine-grained analysis of prospective teachers' specialised mathematics knowledge. Drawing on Shulman (1986), in the MTSK model there are two extensive areas of knowledge: Mathematical Knowledge (MK)—the knowledge possessed by a mathematics teacher in terms of a scientific discipline within an educational context—and Pedagogical Content Knowledge (PCK)—the knowledge relating to mathematical content in terms of teaching-learning. In line with the literature on affective domain (e.g., Leder & Forgasz, 2006) the MTSK model also considers the beliefs about mathematics and its teaching and learning; these lie at the *centre* of the model to "underline the reciprocity between beliefs and knowledge domains" (Carrillo-Yáñez et al., 2018, p.240). In the model, MK and PCK are each divided into three sub-domains. MK is composed of Knowledge of Topics (KoT), mathematics content itself, Knowledge of the Structure of Mathematics (KSM), the interlinking systems which bind the subject, and Knowledge of Practices in Mathematics (KPM), how one proceeds in mathematics. In the MTSK model, PCK "is a specific type of knowledge of pedagogy which derives chiefly from mathematics" (Carrillo-Yáñez et al., 2018, p. 246). The three subdomains of PCK are Knowledge of Mathematics Teaching (KMT), Knowledge of Features of Learning Mathematics (KFLM), knowledge associated with features inherent to the learning and the teaching of mathematics, intrinsically bound up with mathematical content, and Knowledge of Mathematics Learning Standards (KMLS), where the term Learning Standards means any tool designed to measure students' level of ability to understand, construct, and use mathematics, which can be applied at any specific school level. In order to make the analysis more comprehensive, the deepening of the subdomains will be outlined in the analysis section.

We framed our research with the MTSK model as set out by Carrillo-Yáñez and colleagues (2018):

> *The MTSK model takes an analytical focus with the aim of gaining insight into the teacher's knowledge, specifically the elements which go to make this knowledge up and the interactions between them. It is, then, pre-eminently directed towards studying the knowledge, which the teacher puts into use. To this effect, we bring to bear domains and sub-domains under the hypothesis that the knowledge in question can be mapped onto these. When we say that a teacher needs knowledge pertaining to a particular sub-domain, we are not referring to a predetermined list of contents; rather, we mean that the teacher must necessarily have knowledge which can be located in this sub-domain. In this regard, teacher trainers could make use of the MTSK model for organising the perceived training needs of their trainees.* (p. 239)

The aim of our research was to develop tasks that could investigate the Mathematics Teachers' Specialised Knowledge developed by prospective mathematics teachers. In detail: What are students' self-perceptions regarding their mobilised knowledge concerning the teaching-learning processes of mathematics after attending a mathematics teaching laboratory? Is it possible to detect their declared knowledge within the framework of MTSK?

The participating prospective teachers are part of a five-year degree course, at the end of which they will qualify as teachers for pre-primary and primary school level (after these five-year students obtain Level 7 of the European Qualification Framework). We administered the questionnaire in the context of a Mathematics Education class, a one-semester course with a final exam; MTSK was not presented to prospective teachers during this course.

METHODOLOGY

For this research, an exploratory study was conducted with 118 students attending the Faculty of Education at the Free University of Bozen-Bolzano. Students attended the second year of the course of study; very few (less than 10%) had already had experience as an in-service teacher. The majority of the sample is female; in fact, there are seven males. As part of their studies, the prospective teachers attend three courses of Mathematics Education; the focus of this research is a laboratory activity afferent to the course of Foundations and Didactic of Mathematics. The laboratory is part of the "Eduspace–Multilab research Project" of the Faculty of Education of the Free University of Bozen-Bolzano and the activities and materials used during the laboratory are planned and developed as part of this project. The Eduspace–Multilab research project is a research environment aimed at the development of educational activities within a combined theory and practice framework. Within the mathematics area of this project, the laboratory materials are planned and created; some of these are generated during the activities with children and others inside the laboratory with prospective teachers (also in the laboratory involved in this research). The laboratory lasted 12 hours and focused on activities about numbering systems of the ancient Egyptians, Mayans, and Sumerians. After a brief historical introduction, the numerical systems of the ancient civilizations, with the support of the artefacts constructed ad hoc, were analysed. In line with Furinghetti (2007), our approach of using history of mathematics in teacher education can be effective and create opportunities and appropriate situations for learning. As stated by Lara-Alecio, Irby, and Morales-Aldana (1998),

> *Using such examples from a cultural and historical perspective, teachers can offer models of study for abstract concepts of numerals and number or amount. Not only does this approach have the advantage of modelling an abstract concept, it also has implicit ethnomathematical, cultural links that could be used by the elementary school teacher to advance multicultural attitudes in the classroom and assist students of Mayan descent to feel pride in their own culture.* (p. 4)

In addition to a geographical and historical contextualization of the Mayan civilization, an analysis of the development of numbering systems was also carried out during the laboratory. As we will see in the next paragraph, this contextualization was impactful in the learning process of prospective teachers. From the point of view of mathematical contents, non-positional or non-10-based numbering systems were presented and analysed, with the aim of strengthening knowledge of the position numerical system and decimal numerical system. As far as the contents are concerned, the laboratory focused on a crucial issue regarding the future teachers' professional development; literature highlights that prospective teachers show many gaps and weaknesses about arithmetic knowledge and, in particular, regarding the positional numeric system (De Castro, Castro, & Segovia, 2004; Montes et al., 2015).

In line with a socio-constructivist prospective, the laboratory was conducted in interactive modality and prospective teachers participated actively. At the end of the laboratory, in order to answer our research questions, we built and administered an open-ended questionnaire. The questionnaire was validated with a smaller group of students the previous year. During the 2017/18 academic year, the first version of the questionnaire was administered to a group of students (31) who followed the same laboratory on numbering systems. Given the incompleteness of the answers to the first question, the question "if so, what?" has been added in the final version of the questionnaire, which is referred to in the research presented in this contribution. From a substantial qualitative point of view, the results of the first trial do not differ from those obtained in the research.

In details, the questions, designed in order to assess different aspect of MTSK, are as follows:

- ♦ Has the laboratory enriched your knowledge from the point of view of mathematical contents? Have you acquired new mathematical knowledge? If so, what?
- ♦ What information about the learning/teaching of one or more of the contents discussed in the laboratory have you acquired?

The questions were asked in a generic way (and not in reference to the individual subdomains of the MTSK model) in order to investigate the presence or absence of changes/beliefs of future teachers both in terms of Mathematical Content Knowledge and in terms of the Pedagogical Content Knowledge. As we will see in the next paragraph, the analysis of the responses has been done by making a categorization based on the individual sub-domains of the MTSK model (Carrillo-Yáñez et al., 2018).

In the questionnaires, a large answer space was allotted to each question (approximately 30 empty lines) to fill in; the questionnaire was administered (anonymously) at the end of the laboratory. Just to avoid contractual responses (Brousseau, 1997), during the design phase it was chosen to administer the questionnaire anonymously and upon administering it, besides underlining the fact of anonymity, it was communicated to the students that their answers would be used to evaluate the laboratory and, therefore, to try to improve it. The students were therefore asked to respond freely and seriously and without condescension. The students were given half an hour to complete the questionnaire and all of them answered the questions in depth. Being considered an internal laboratory activity, the response rate was 100%. Obviously, we do not intend to generalize, the study is local as it involves a specific sample of students and refers to certain contents (although they were intentionally selected). In detail, a qualitative analysis through an inductive content analysis was carried out (Patton, 2002). The answers refer to the subdomains of the MTSK model; we carried out a classification of answers investigating the presence of MTSK in the sense of Carrillo-Yáñez and colleagues (2018) and categorising the answers with the sub-domains of model.

Analysis of the Questionnaires

Both for MK and for PCK, declared evidence, with different frequency and facets, were found in each of the sub-domains. We will analyse the answers provided by the students in reference to each sub-domain.

Knowledge of Topics (KoT)

In almost all the analysed questionnaire responses, prospective teachers declared an increase in their KoT. The majority of prospective teachers said that the laboratory was helpful in strengthening their knowledge regarding basic concepts and the positional numerical system. 73 of them claimed that before the laboratory they had no mastery of the concept of the positioning numerical system and 91 said they realised that they did not know what a base 10 system meant.

Some prospective teachers stated that:

PT_51: Yes, the laboratory has enriched my knowledge of positional and decimal system, which I thought I knew but didn't.

PT_73: Analysing the numerical systems of the Egyptians and the Sumerians, which are not positional, strengthened my idea of a positional system... but analysing the numbering system Maya totally enlightened me! I realized that until then I had no idea what it meant that our system was based on 10!!

PT_109: I never really noticed what it meant that ours is a decimal system.

Other students (39) talked about their increased knowledge in terms of knowledge of other numerical systems, with phrases such as:

PT_81: I learned the existence of other numerical systems.

PT_05: I learned how the ancients counted.

In line with Carrillo-Yáñez and colleagues (2018), we found evidence of the type of problems the content can be applied to; we found this evidence for example in phrases such as:

PT_16: I learned what problems to propose on the decimal system.

Other prospective teachers refer to the importance of *representations of numbers* and the fact that they became:

PT_06: Aware that numbers can also be represented in other ways.

In general, in most of the responses in which we found an increase in students' perception in terms of KoT, reference is made to the part of the laboratory regarding the Mayan numbering system. As we can read in Anderson (1971), working with a different numbering system requires a different level of abstraction and allows us to highlight the advantages/disadvantages of each numbering system typology and, from the students' answers, it emerges that these facts seem to be perceived by most of them. The reasoning about these issues allowed students to strengthen their knowledge on the decimal positional system, knowledge that was previously poor (as most of them declared).

Lastly, 7 participants stated that the laboratory did not increase their knowledge from a mathematical point of view.

Knowledge of the Structure of Mathematics (KSM)

The KSM identifies the teacher's knowledge of connections among different mathematical items. Connections are referring to temporal consideration, related to mathematics and the question of sequencing, as regards to complexity or simplification. In analysed responses, we found evidence in both directions. For instance, one prospective teacher declared that:

PT_83: I thought a lot about zero and its birth.

PT_20: In this laboratory the thing that struck me the most were the reflections on zero and its necessity.

In 27 responses, reference is made to the concept of zero –to its birth from a historical point of view and to its necessity in the positional numbering systems and its absence in the non-positional numbering systems. During the laboratory, it was mentioned that some scholars, such as Salyers (1954), argue that the Mayans discovered the zero independently of the Hindus. Others declare that the Mayans presented "man's first positional arithmetical system, one involving the concept of zero; this is among the most brilliant intellectual achievements of all time" (Morley, 1915, p. 454). As can be seen from the answers, these mathematical historical notes have remained very impressive to the students and spurred them to reason about its meaning within numbering systems.

In 13 responses, the prospective teachers declared that they reasoned and reflected on the connections and differences between the basic concept and the concept of position. One participant explicitly declared the following:

PT_48: I have reflected on the basic and positions concepts; they are often considered together although they are very different. The thing that made me think more is that the way of saying the decimal-positional system intrinsically hides two concepts closely connected but also very distinct. I find that the basic concept is very complex and reasoning on different numbering systems has simplified my understanding.

With regard to MK, this type of knowledge is the least detected in the questionnaire answers analysed.

Knowledge of Practices in Mathematics (KPM)

The analysis was carried out according to the definition of Franke, Kazemi, and Battey (2007) as adopted in the MSKT model, in which practice is understood as actions occurring in teaching and learning processes that involve both teacher and students individually and in interaction. In the MSKT model, KPM focused on "the working of mathematics rather than the process of teaching it" (Carrillo-Yáñez et al., 2018, p. 244). The KPM is linked to meta-knowledge of mathematics (in the sense of Robert & Robinet, 1996) and focuses specifically on means of production and mathematical functioning. In this sense, most of the participants in our research study conducted a meta-cognitive reflection on positional and decimal system concepts, identifying these factors in the exploratory activities during the laboratory. Most prospective teachers (more than 90%) who participated, at the time of the research had not yet had any experience in the classroom as an in-service teacher, so the statements made refer only to beliefs about their future practices. One prospective teacher declared that:

PT_20: Understanding the Mayan numbering system based on 20 and making the jump between the powers of 20 helps to understand the meaning of tens and units.

And another participant recognised the difficulties of producing the argumentation about the process analysed:

PT_12: It is not easy to explain how to represent numbers in different numbering systems. I found it difficult to explain how I had done it.

In general, from most of the students' declarations, it emerged that they believed that the approach used and meta-analysed during the laboratory could have positive implications within the teaching and learning process of numerical systems, therefore they declared that it is their intention to propose it in their future classes. And this is line with literature, such as with Lara-Alecio, Irby, and Morales-Aldana (1998), that highlight that mathematics teachers can use cultural and historical perspectives from different civilizations, such as the Mayan, to offer models of studying the numbers and this could have advantageous aspects from the point of view of learning outreach.

Knowledge of Features of Learning Mathematics (KFLM)

In the MTSK model, the germinal idea of Shulman (1986) is followed, in which PCK is considered the knowledge of mathematical content in terms of teaching and learning. In MTSK, the relation with pedagogy is explained:

> *In our view, the specific focus of PCK is related to mathematics itself. More than being about the intersection between mathematical and general pedagogical knowledge, it is a specific type of knowledge of pedagogy, which derives chiefly from mathematics. Hence, we do not include in this sub-domain general pedagogical knowledge applied to mathematical contexts, but rather only that knowledge in which the mathematical content determines the teaching and learning which takes place. It is in this domain that the research literature in mathematics education plays a major role as a source of knowledge for teachers*. (Carrillo-Yáñez et al., 2018, p. 246)

In particular, the KFLM is associated with features inherent to learning mathematics and focuses on mathematical content as the object of learning. Many protocols refer to this subdomain:

PT_34: Living it first-hand, I learned the difficulties that my students will have when learning the basic concept.

PT_56: The laboratory well illustrated the difficulties in understanding the positional and decimal system.

PT_57: Facing numbering systems with a different basis helped me to understand the fact that our numbering system is in base 10.

PT_101: I believe that, like us, our students will find it difficult to understand the numbering system.

PT_116: To understand the basic concept, I think it is very useful to work with numbering systems that have different bases from ours; to understand the concept of positioning it is useful to work with additional systems.

PT_119: I learned from the lab that although it seems that children have acquired the basic and positional concept, in reality this can only be a superficial impression which needs further exploration

In details, as we can be seen here to, in most cases (in particular, among the examples shown, PT_34 and PT_101) prospective teachers refer to beliefs about their perception of what the student's learning process will be. Noteworthy is the fact that the prospective teachers' sentences show that the laboratory has raised prospective teachers' awareness of how much the concept of place value is a widespread difficulty at primary school level. These thoughts are in line with what has already been shared by the international literature for which the place value is one of the most arduous concepts to be acquired in arithmetic learning (Baroody, 1990).

In almost all the analysed responses, there was declared evidence of KFLM (mainly as regards the basic concept); only in 3 responses was there no trace.

Knowledge of Mathematics Teaching (KMT)

This subdomain aims to frame the "knowledge intrinsically bound up with content to the exclusion of aspects of general pedagogical knowledge" (Carrillo-Yáñez et al., 2018, p. 247). This knowledge can involve awareness of the potential of activities, strategies, and techniques for teaching specific mathematical content. The analysis of responses highlighted many references to their beliefs about KMT, both regarding their planning of classroom activities and of strategies and tools to use in their future teaching practices. Many prospective teachers referred to their future designs of learning opportunities, for instance:

PT_06: I will re-propose the activities seen during the laboratory, to introduce the decimal positioning system.

PT_24: I found the activities seen very effective for the process of teaching and learning of basic and positional concepts. I'll definitely take that into account when I teach.

PT_39: I will certainly introduce the numbering systems of Egyptians, Sumerians and Mayans when I have to reinforce the concepts of our positional system.

Some participants referred to assessment practices, for example:

PT_17: This seems to me a great way to test the knowledge of our positional/decimal system.

Many teachers (58), referred to the materials used during the laboratory. For example:

PT_66: I have already done some research to reconstruct the materials used, I find them really effective and suitable.

PT_91: I really liked the Sumerian stones, I think I'll make them for my students.

PT_92: I think the use of magnets for the Mayans is really useful in order not to get lost while you count.

Most of the prospective teachers (89), stated that they appreciated the approach used.

PT_06: I believe that the way in which the activities are presented is also crucial.

PT_20: I liked the fact that I enjoyed it; it's rare while doing math. I want to entertain my students. Yes, I think that's the most important thing I took away from this; we had fun while understanding how the Mayans counted. We finally figured it out.

Overall, the fact that the majority of the prospective teachers appreciated the laboratory modality and that they are convinced to employ it within their future classes is a very favourable fact on account that it was observed that the use of mathematics laboratory enhanced achievement in mathematics (Okigbo & Osuafor, 2008).

Although this knowledge might be based on theories of mathematics education tackled during the course to which the laboratory is afferent, there are very few theoretical references in the responses of future teachers (theoretical references with explicit references were found only in seven answers).

Knowledge of Mathematics Learning Standards (KMLS)

This knowledge involves the understanding which mathematics might be taught at any particular level; this type of knowledge can be found both in official documents, like National Guidelines, and in non-official documents, for example those from NCTM (2000).

During the course, the Italian Ministry regulation, the National Guidelines (NG) for the first cycle of education and Kindergarten schools (MIUR, 2012), were analysed in depth. Specifically, the learning objectives to be attained at the end of class 3 and at the end of class 5 of Primary School were explored; the goals for the development of the competencies to reach at the end of the Primary School were also analysed. In particular, during the laboratory it was highlighted how the proposed activities were in accordance with the NG.

Specifically, in the Italian NG there is a learning objective which underlines the importance of "knowing systems of notation of numbers that are or have been in use in places, times, and cultures different from ours" (p. 50, MIUR, 2012). It is precisely to this learning objective that the 17 references to the NG are addressed, among which, for example:

PT_12: Thanks to the laboratory I found that the NG explicitly requires us to look at mathematical approaches other than our own.

Four future teachers made an explicit reference to the goal for the development of the competences "Students [..] develop a positive attitude towards mathematics" (p. 49, MIUR, 2012), e.g.,

PT_45: I believe that activities like these are the activities in mathematics that help to develop the positive attitude towards mathematics, also mentioned in the NG but little taken into account in the usual practices.

DISCUSSION AND CONCLUSION

The research carried out is aimed at investigating whether and how the activities of a laboratory of mathematics education, conducted in a Faculty of Education, affect the perceptions of the mathematics and its teaching knowledge of future teachers of mathematics. The research focus is the specialised mathematical knowledge for teaching; an initial exploratory study was carried out and participants' answers were read within the Carrillo-Yáñez and colleagues (2018) MTSK framework. In analysing the statements of the prospective teachers, all the areas of the model emerged. In almost all the statements analysed (118), both MK and PCK aspects were highlighted and, in most cases knowledge and beliefs are strictly intertwined. In this paper, examples of evidence for each of the sub-domains of MK and PCK inside prospective teachers statements were given. Concerning the MK, the sub-domain that emerged least in the prospective teachers' declarations is the KPM; this may be due to the fact that prospective teachers have little actual school experience. As far as this subdomain is concerned, it is not surprising that only beliefs have been detected regarding matters concerning future practices This may also be why, as regarding the PCK, in the KMLS sub-domain there are no explicit references to the sequencing of arguments and their time sub-sequentially. The references found in the statements refer more to transversal knowledge and skills, such as the importance of knowing numbering systems other than our own and ensuring that pupils develop positive attitudes to mathematics.

MTSK is a construct created to frame the knowledge of mathematics teachers; in this research it has been shown that it is also suitable for framing the beliefs and the mobilised knowledge of prospective teachers. Not being able to investigate their practices, it has been analysed what they think and declare in reference to them, the limit of the research is that of being able to investigate only what they declare. The research offers insights and shows that framing the knowledge of the prospective teachers with the MTSK model provides timely information about their beliefs and knowledge.

In general, a lot of prospective teachers stated that the activities carried out in the laboratory showed a different mathematics from what they were used to or what their beliefs in mathematics were and they explicitly refer to a change in their beliefs and attitudes towards mathematics. Much research has studied the attitude of prospective teachers towards arithmetic (e.g., Dutton, 1951; Smith, 1964) and how the teacher's attitude can influence the student's attitude and achievement (Phillips, 1973). Other studies, that underlie the effects of a preparatory mathematics program to change the prospective teachers' attitudes and beliefs towards mathematics (Philippou & Christou, 1998; Swars et al., 2009). Raymond and Santos (1995) emphasize how innovation in the preparation of mathematics teachers challenges mathematical beliefs. Furthermore, explicit declaration of change of beliefs towards mathematics reveals in 51 protocols, and this is considered a particularly positive outcome, as there was no explicit request to do so in the questionnaire.

It is clear that, by working appropriately on disciplinary didactics and general didactics, Faculty of Education students can appreciate the consistency of the knowledge acquired within the degree course and therefore devote such knowledge to the interpretation of behaviour of primary school students. We are aware that any attempt to seek for regularities presupposes a simplification of reality and diversity, but our intention has been to look for the most frequent and common evidence of prospective teachers' knowledge and beliefs, in order to obtain efficient information on their learning of mathematics education. We believe that the results presented in this paper can offer food for thought as regards to the design and implementation of activities aimed at trainee primary school mathematics teachers and their impact on acquired knowledge.

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