Economic Research-Ekonomska Istraživanja

ISSN: 1331-677X (Print) 1848-9664 (Online) Journal homepage: https://www.tandfonline.com/loi/rero20

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To cite this article: Xiaomei Mi, Jiamei Li, Huchang Liao, Edmundas Kazimieras Zavadskas, Abdullah Al-Barakati, Ahmed Barnawi, Osman Taylan & Enrique Herrera-Viedma (2019) Hospitality brand management by a score-based q-rung orthopair fuzzy V.I.K.O.R. method integrated with the best worst method, Economic Research-Ekonomska Istraživanja, 32:1, 3266-3295, DOI: 10.1080/1331677X.2019.1658533

To link to this article: https://doi.org/10.1080/1331677X.2019.1658533

ECONOMIC RESEARCH-EKONOMSKA ISTRAŽIVANJA
2019, VOL. 32, NO. 1, 3266–3295
https://doi.org/10.1080/1331677X.2019.1658533

Hospitality brand management by a score-based q-rung orthopair fuzzy V.I.K.O.R. method integrated with the best worst method

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ABSTRACT
Hospitality brand management is a primary concern in the hotel industry and the evaluation of brands can be considered as a decision-making problem with multiple criteria. The evaluation information of brands may be uncertain sometimes. The q-rung orthopair fuzzy set (q-R.O.F.S.), which represents the preference degree of a person from the positive and negative aspects, has turned out to be an efficient tool in depicting uncertainty and vagueness in the decision-making process. This article dedicates to presenting an integrated multiple criteria decision-making method with q-R.O.F.S.. Firstly, a score function of the q-R.O.F.S. is proposed to solve the deficiencies of two existing score functions. Then, a weight-determining method based on the additive consistency of the preference relation is developed. A decision-making method integrating the score function, the best worst method and the VIsekriterijumska optimizacija I KOmpromisno Resenje (V.I.K.O.R.) which means multiple criteria compromise optimisation in English) method is further proposed. Finally, a case study regarding the hospitality brand management is provided to show the applicability and validity of the proposed method.

ARTICLE HISTORY
Received 5 April 2019
Accepted 9 July 2019

KEYWORDS
hospitality brand management; multiple criteria decision-making (M.C.D.M.); q-rung orthopair fuzzy set (q-R.O.F.S.); score function; Best West Method (B.W.M.); VIsekriterijumska optimizacija I KOmpromisno Resenje (V.I.K.O.R.)

JEL CLASSIFICATIONS
C44; C61; D70; D81; L83

1. Introduction

With the development of world economy, the improvement of individuals' living standards, the revolution of transportation means, the deep development of tourism resources and the continuous improvement of tourism service facilities, the world tourism industry has been developed rapidly, and the international tourism market undergoes apparent changes as the blue line in Figure 1 shows. It is undeniable that

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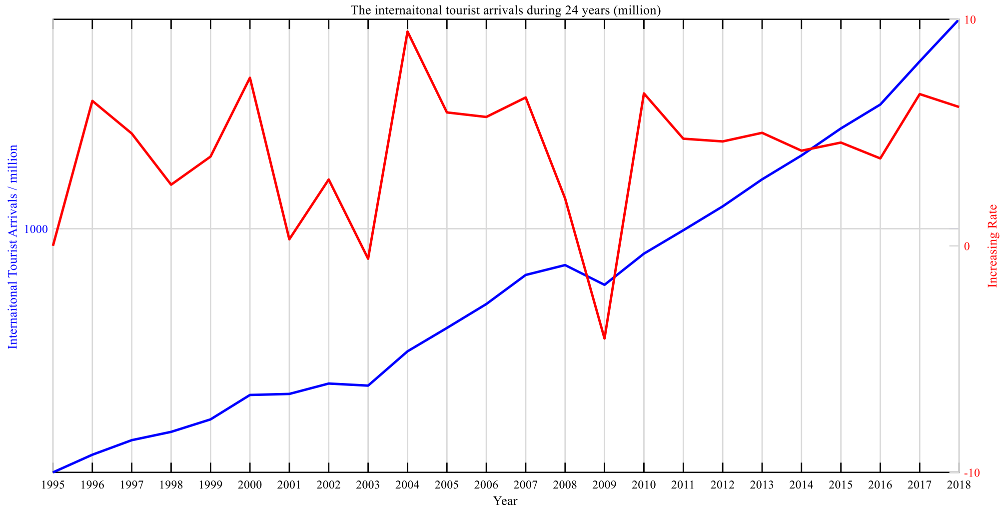


Figure 1. The number and increasing ratio of international tourist arrivals from 1995 to 2018 (million).

the tourism industry is in a vibrant state, having a tendency of development. The report made by the United Nations World Tourism Organization (U.N.W.T.O.) shows that the number of global tourists has increased by 6%, reaching to 1.4 billion (UNWTO, 2018). On the other hand, we can see a significant drop of international tourist arrivals in 2008 from Figure 1. The decline can be ascribed to the economic crisis in 2008 (Alonso-Almeida & Bremser, 2013).

Although the economic instability may have adverse effect on the hotel industry, hotels that focus on high-quality brand image and loyal customer can resist crisis effectively (Alonso-Almeida & Bremser, 2013). Brands can influence consumers' behaviours, and provide the owners with sustainable future business income security to a certain extent. These direct or indirect values generated by brands are often referred to as brand equity. The theory of brand equity was firstly presented by Aaker (1991), who put forward the Customer-Based Brand Equity (C.B.B.E.) framework (including brand loyalty, brand awareness, brand association, and perceived quality components) to investigate the brand equity scientifically. The hospitality brand measured by the C.B.B.E. framework can also be regarded as a Multiple Criteria Decision-Making (M.C.D.M.) problem (Mardani et al., 2015; Roy et al., 2019).

When evaluating the brands of hospitality, the only allowance of crisp values would increase the difficulty of a Decision-Maker (D.M.) in providing information. To reduce the efforts of D.M.s and allow the uncertain information, Atanassov (1986) presented the intuitionistic fuzzy sets by considering the membership degree u and the non-membership degree v and its constraints $u \in [0, 1]$, $v \in [0, 1]$ and $u + v \leq 1$, simultaneously. Sometimes, the preference information provided by a D.M. may be rejected into use, such as (0.7, 0.5) and (0.8, 0.3) which increase the difficulty of human in giving perceptions. To overcome this dilemma, the intuitionistic fuzzy set has been enhanced by Yager (2017) for increasing the flexibility degree in representing preference information. The q-rung orthopair fuzzy set (q-ROFS), a generalised form of intuitionistic fuzzy set, emerges by transforming the membership and non-membership degrees to corresponding power with the power index q whose value ranges from 1 to the infinite number (Yager, 2017), shown as $u^q + v^q \leq 1$. Owing to the advantages of the q-R.O.F.S., it has attracted lots of scholars' attention (Ali, 2018; Liu & Liu, 2018; Peng, Dai, & Garg, 2018; Yager, 2017; Yager, Alajlan, & Bazi, 2018). In these achievements, the score function of q-R.O.F.S. plays a vital role in calculations. Up to now, two score functions of q-R.O.F.S. have been proposed, as listed in Section 2.1, to denote the utility of the assertion in the form of the q-R.O.F.S. (Liu & Liu, 2018; Peng, Dai, & Garg, 2018). For these two score functions, an obvious deficiency is that they failed to distinguish two q-rung orthopair fuzzy numbers (q-R.O.F.N.s) when their membership degrees equal to the non-membership degrees. For example, the score value of the q-R.O.F.N. (0.6, 0.6) equals to the score values of the q-R.O.F.N. (0.7, 0.7). The two score functions take these situations as equal supporting degree, which may lead to misleading results to some extent. This is the first research gap of the current study.

Another primary concern of decision-making problems is to obtain the weights of criteria in the decision-making process (Vetschera, 2017). Among weight-determining methods, the analytic hierarchy process is one of the most famous and applicable methods (William & Xin, 2018). Compared with the analytic hierarchy process, the Best West Method (B.W.M.) (Rezaei, 2015, 2016) is an enhanced version in terms of the specific structure of pairwise comparisons. The comparison structure in B.W.M. is realised by two benchmarks: the best criterion and the worst criterion. Due to the structured comparison process, it correspondingly improves the reliability and consistency of the results calculated by the B.W.M. Lots of research about the B.W.M. focused on the multiplicative consistency property of pairwise comparisons (Mi et al., 2019). Besides, owing to the constraints on limited knowledge, time and experience, the B.W.M. with uncertain information becomes a hot research topic (Mi et al., 2019). This study also talks about the B.W.M. with uncertain information represented by q-R.O.F.N.s and dedicates to overcoming the drawback similar to that of Mou, Xu, and Liao (2017). This is the second research challenge of this study.

Regarding the M.C.D.M. method with q-R.O.F.S. information, till now, as far as we know, only the T.O.D.I.M. method with q-R.O.F.S.s has been studied (Wang and Li, 2018). In the T.O.D.I.M. method, the pairwise outranking values of alternatives are calculated and used for ranking alternatives. In this method, if m alternatives need to be ranked, then, $m(m-1)$ times of computations are conducted over a criterion. Compared with the determined maximal and minimal references MCDM methods, such as the VIsekriterijumska optimizacija I KOmpromisno Resenje (in Serbian; V.I.K.O.R.; multiple criteria compromise optimization in English) (Opricovic & Tzeng, 2004), the T.O.D.I.M. method is time-consuming and complicated. This is the third research challenge of the existing researches.

To fill the above research gaps, this article devotes to presenting an integrated M.C.D.M. method in the q-R.O.F. environment using the B.W.M. to obtain the weights of criteria and the score-based V.I.K.O.R. method to get the ranking list of alternatives. The contributions of this article can be summarised as follows:

1. A score function of the q-R.O.F.S. is introduced and the properties of it are addressed. Comparisons of the proposed score function with other two existing score functions are provided. This fills the first research gap.

2. The B.W.M. in the q-R.O.F. environment is investigated, which broadens the application scope of the B.W.M. and provides an alternative way to determine weights of criteria through the additive consistency property of pairwise comparisons in the q-R.O.F. environment. This defeats the second research challenge.
3. The V.I.K.O.R. integrated with the proposed score function is proposed for solving M.C.D.M. problems with q-R.O.F.S.s. Using the B.W.M. to obtain the weights of criteria, an integrated method named the Q-B.W.M.-S-V.I.K.O.R. method, is proposed. This overcomes the third research challenge.

The rest of this article is organised as follows: Section 2 introduces the preliminaries regarding the q-R.O.F.S. and its existing score functions, the B.W.M. and the V.I.K.O.R. method. Section 3 proposes a novel score function of the q-R.O.F.S. Section 4 introduces the Q-B.W.M.-S-V.I.K.O.R. method. The illustrative case study and comparative analyses are given in Section 5. The article closes with conclusions and future directions in Section 6.

2. Literature review and preliminaries

In this section, we review the knowledge related to the q-R.O.F.S. and its existing score functions, and the characteristics of the B.W.M. and V.I.K.O.R. methods.

2.1. The q-rung orthopair fuzzy set and its score functions

The q-R.O.F.S. was initially proposed by Yager (2017), which is a generalisation of intuitionistic fuzzy set and Pythagorean fuzzy set. The domain of a q-R.O.F.S. is a fixed set X. A q-R.O.F.N. a, an element in q-R.O.F.S., is defined as $a = \{< x, (u_a(x), v_a(x)) > | x \in X\}$, where $u_a(x)$ and $v_a(x)$ denote the membership and non-membership degrees of the element $x \in X$, respectively, and satisfy $0 \leq (u_a(x))^q + (v_a(x))^q \leq 1; q \in [1, +\infty)$. The sum of the power of membership degree, non-membership degree and hesitancy degree should be one, i.e.,$(u_a)^q + (v_a)^q + (\pi_a)^q = 1$. Then, the hesitancy index can be calculated by $\pi_a = [1 - (u_\alpha^q + v_\alpha^q)]^{1/q}$, if we know the values of the membership degree and the non-membership degree. Hence, for simplicity, the tuple (u_a, v_a) is taken into use in this article instead of a complete q-R.O.F.N. form, (u_a, v_a, π_a). For example, for a q-ROFN $a = (0.9,\ 0.5)$, the assertion is supported with 0.9 degree and against with 0.5 degree with the value of q being 3.

Plenty of research has been published since the q-R.O.F.S. appears. There are two feasible ways to solve decision-making problems: aggregation operators and M.C.D.M. methods. Operators of q-R.O.F.S.s have been investigated, such as the weighted averaging operator and the weighted geometric operator (Liu & Liu, 2018), Choquet integral-based operators of q-R.O.F.S.s (Yager et al., 2018) and weighted exponential aggregation operator (Peng et al., 2018). Operations for changing the orbits of q-R.O.F.S.s have been studied (Ali, 2018). Besides, the q-R.O.F.S. has been applied in approximate reasoning (Yager, 2017). To compare the q-R.O.F.N.s, Liu and Liu (2018) proposed the score function of a q-R.O.F.N. $a = (u_a, v_a)$ as:

$$S_L(a) = u_\alpha^q - v_\alpha^q \tag{1}$$

$S_L(a)$ ranges from −1 to 1. The larger the score $S_L(a)$ is, the greater the q-R.O.F.N. a is. However, the score function $S_L(a)$ is not able to distinguish two different q-R.O.F.N.s if they possess the same score values. For example, for two q-R.O.F.N.s $a_1 = (0.6, 0.6)$ and $a_2 = (0.7, 0.7)$, they have the same score value 0 and thus we cannot distinguish them. In this regard, Yager (2017) presented the accuracy function to denote the strength of a commitment, shown as follows:

$$G_Y(\alpha) = u_\alpha^q + v_\alpha^q \tag{2}$$

The value of the accuracy functions belongs to [0,1]. The larger the accuracy degree $G_Y(\alpha)$ is, the greater the q-R.O.F.N. a is.

Besides these functions, Peng et al. (2018) claimed that the hesitancy degree π_α^q has positive and negative effects on the score value when $u>v$ and $u<v$, respectively. Inspired by the common Sigmoid function,[1] they proposed a novel score function of a q-R.O.F.N. as:

$$S_P(\alpha) = u_\alpha^q - \nu_\alpha^q + [\left(e^{u_\alpha^q - \nu_\alpha^q}/(e^{u_\alpha^q - \nu_\alpha^q} + 1)\right) - 1/2] \cdot \pi_\alpha^q \tag{3}$$

where $e^{u_\alpha^q - \nu_\alpha^q}/(e^{u_\alpha^q - \nu_\alpha^q} + 1) - 1/2 > 0$, denoting the net flow of the membership degree being superior to the non-membership degree. The larger the score $S_P(\alpha)$ is, the greater the q-R.O.F.N. a is. However, the score function $S_P(\alpha)$ is also not capable to distinguish the q-R.O.F.N.s whose membership degree equals the non-membership degree.

In conclusion, the two existing score functions lose efficiency when the transformed membership degree equals to the transformed non-membership degree. In this situation, the score functions $S_L(a)$ and $S_P(\alpha)$ obtain the same score value of q-R.O.F.N.s. In this article, we shall propose new score function of q-R.F.O.N.s to tackle this problem.

2.2. The main features of the B.W.M

B.W.M. (Rezaei, 2015, 2016) is a recently developed M.C.D.M. method for decision analysis, especially in acquiring the weights of criteria. The significant characteristic of the B.W.M. is the structured comparison rules to identify the pairwise comparisons between the best criterion c_B and all the other criteria c_j $(j = 1, 2, \ldots, n; j \neq B)$, and the pairwise comparisons between the worst criteria c_W and all the other criteria c_j $(j = 1, 2, \ldots, n; j \neq B; j \neq W)$. In this way, two preference vectors are established regarding the importance of criteria: one is the best-to-others preference vector $BO = (p_{B1}, p_{B2}, \ldots, p_{Bn})$, and the other is the others-to-worst preference vector $OW = (p_{1W}, p_{2W}, \ldots, p_{nW})^T$. In total, if n criteria are evaluated, $2n-3$ times of pairwise comparisons are enough in the B.W.M., while in analytic hierarchic process, the number of pairwise comparisons is $n(n-1)/2$.

In the original B.W.M. (Rezaei, 2015), the weight vector of criteria is calculated through a mathematical model based on the multiplicative consistency of pairwise comparisons. If the pairwise comparisons are perfectly consistent, the relationships between the preference values and the weights of criteria should be $p_{BO} = w_B/w_O$, $p_{OW} = w_O/w_W$ and $p_{BW} = w_B/w_W$. If the consistency property cannot be satisfied, the deviations between them should be as small as possible. Let $\xi = \max\{|p_{Bj} - w_B/w_j|, |p_{jW} - w_j/w_W|, |p_{BW} - w_B/w_W|\}$. Then, a weight-determination model (Model-1) can be established (Rezaei, 2015):

Model-1

$$\begin{aligned} &\min\ \xi \\ &s.t.: |p_{BO} - w_B/w_O| \le \xi; |p_{OW} - w_O/w_W| \le \xi; |p_{BW} - w_B/w_W| \le \xi; w_j \ge 0, \textstyle\sum_{j=1}^{n} w_j = 1 \end{aligned}$$

where $c_O = \{c_j | j = 1, 2, ..., n; j \ne B; j \ne W\}$ are the other criteria excluding the best and worst criteria.

The objective function of Model-1 is to minimise the maximal deviation with respect to all criteria. Solving Model-1, we can obtain the weights of criteria and the minimum deviation ξ^*. The values of $\xi_{\max}$ for Model-1 in the 1–9 scale can be found in Rezaei (2015). If the consistency ratio of $\xi^*/\xi_{\max}$ is not greater than 0.1, the preference vectors are regarded as acceptable consistent and the calculated weights owns the high reliability.

After the B.W.M.'s appearance, it has attracted lots of attention of scholars and fruitful research results regarding it have been published. For more details, please refer to a state-of-the-art survey concerning the B.W.M. (Mi et al., 2019). Mou et al. (2017) concentrated on the uncertain situation where the uncertain information was expressed by intuitionistic fuzzy sets (Atanassov, 1986). Mou et al. (2017) obtained the weights of criteria by separately considering the membership degree and the non-membership degree, which is not appropriate because the membership degree and the non-membership degree are related by the constraint $u + v \le 1$. If the membership degree and the non-membership degree are separately considered, the final weights are not easy to understand from the intuitionistic fuzzy perspective (Mou et al., 2017). However, as we discussed above, only considering the intuitionistic fuzzy information would lose useful preference values offered by D.M.s. This drawback can be remedied by the investigation concerning the B.W.M. with q-R.O.F.S.s. This is the second motivation of this article and we propose the approach specifically in Section 4.2.

2.3. The characteristics of the V.I.K.O.R

V.I.K.O.R. (Opricovic & Tzeng, 2004) is an M.C.D.M. method for solving decision-making problems with conflicting and non-commensurable (different-unit) criteria. Suppose that the evaluations of alternatives $A_1, A_2, ..., A_i, ..., A_m$ over criteria $c_1, c_2, ..., c_j, ..., c_n$ are obtained as f_{ij} $(i = 1, 2, ..., m; j = 1, 2, ..., n)$. The assumption of the V.I.K.O.R. method is that the compromise solutions are acceptable for decisionmakers. To acquire the compromise solution, five steps of the V.I.K.O.R. method are listed as follows:

1. A score function of the q-R.O.F.S. is introduced and the properties of it are addressed. Comparisons of the proposed score function with other two existing score functions are provided. This fills the first research gap.
2. The B.W.M. in the q-R.O.F. environment is investigated, which broadens the application scope of the B.W.M. and provides an alternative way to determine weights of criteria through the additive consistency property of pairwise comparisons in the q-R.O.F. environment. This defeats the second research challenge.
3. Determine the best $f_j^+ = \max_i f_{ij}$ and the worst $f_j^- = \min_i f_{ij}$ values over all benefit criteria $c_j, j = 1, 2, ..., n$ and vice versa.
4. Calculate the superiority value $SV_i = \sum_{j=1}^{n} w_j (f_j^+ - f_{ij})/(f_j^+ - f_j^-)$ and the inferiority value $IV_i = \max_j w_j (f_j^+ - f_{ij})/(f_j^+ - f_j^-)$ of each alternative $A_i, i = 1, 2, ..., n$
5. Compute the overall value OV_i of each alternative by a convex combination with the parameter θ and normalised values SV_i and IV_i. The parameter θ is an index for measuring the relative importance of the superiority value and the inferiority value, If the value of θ is determined as one, the inferiority value can be neglected and vice versa. The value of θ should range from zero to one.
6. Rank the alternatives in ascending orders according to the values of OV_i, SV_i and IV_i, for $i = 1, 2, ..., n$. Three rankings, R_{OV}, R_{SV} and R_{IV}, of the alternatives can be produced.
7. Acceptable analyse from advantages and stabilities aspects based on the ranking results, R_{OV}, R_{SV} and R_{IV}.

   - Advantage acceptable analysis: $OV_{i2} - OV_{i1} \geq 1/(i-1)$, the difference between the alternatives A_{i2} and A_{i1} should be not less than the advantageous threshold $1/(i-1)$, where i is the number of alternatives;
   - Stability acceptable analysis: The best alternative A_i must be ranked at the first position in at least one ranking list, R_{SV} and/or R_{IV}.

If the advantage acceptable condition is not satisfied for all alternatives, the alternatives $A_i, i = 1, 2, ..., n$ would be the compromise solutions. If stability acceptable analysis is not satisfied, the alternatives A_{i2} and A_{i1} should be the compromise solutions.

The V.I.K.O.R. is a useful tool to solve M.C.D.M. problems, particularly in the situation that the evaluations of alternatives over criteria are conflicting. The compromise solutions are feasible and closest to the ideal solution. Owing to this advantage, the V.I.K.O.R. has been investigated from various perspectives (Mardani, Zavadskas, Govindan, Senin, & Jusoh, 2016). Among these researches, the fuzzy extensions of the V.I.K.O.R. play an important role, such as the extensions with the fuzzy information (Sanayei, Mousavi, & Yazdankhah, 2010), intuitionistic fuzzy information (Devi, 2011), hesitant fuzzy information (Liao & Xu, 2013) and linguistic information with hesitation (Liao, Xu, & Zeng, 2015). Up to now, as far as we know, there is no research about the V.I.K.O.R. with the q-R.O.F.S. information. This motivates us to fill this gap by investigating the integrated method of the q-R.O.F. V.I.K.O.R.

3. A new score function for the q-rung orthopair fuzzy set

In this section, we propose a novel score function for the q-R.O.F.S. Properties of the score function and comparisons with two existing score functions are further analysed.

Since the hesitancy degree may influence the score value of a q-R.O.F.S, we define a score function of a q-R.O.F.S. by considering the hesitancy degree, show as:

$$S_M(a) = \frac{2 + u_\alpha^q - v_\alpha^q}{(2 - u_\alpha^q + v_\alpha^q) \times (1 + \pi_\alpha^q)} = \frac{2 + u_\alpha^q - v_\alpha^q}{(2 - u_\alpha^q + v_\alpha^q) \times (2 - u_\alpha^q - v_\alpha^q)} \tag{4}$$

where $S_M(a)$ monotonically increases and decreases regarding u and v, respectively. $(2 + u_\alpha^q - v_\alpha^q)/(2 - (u_\alpha^q - v_\alpha^q))$ is a monotonically increasing function with $u_\alpha^q - v_\alpha^q$. $1 + \pi_\alpha^q$ plays a role in guaranteeing the non-zero property of the denominator.

Figure 2 is drawn to visually demonstrate the score function. It verifies the monotonicity and the value range of the score function regarding the membership degree u and the non-membership degree v. The red colour denotes the close degree to the maximal value, 3, of the score function.

The monotonicity and the value range of the proposed score function are analysed below.

Property 1 (Monotonicity). $S_M(a)$ monotonically increases and decreases regarding u and v, respectively.

Proof. The partial derivatives with respect to u and v of the score function $S_M(a)$ can be calculated as:

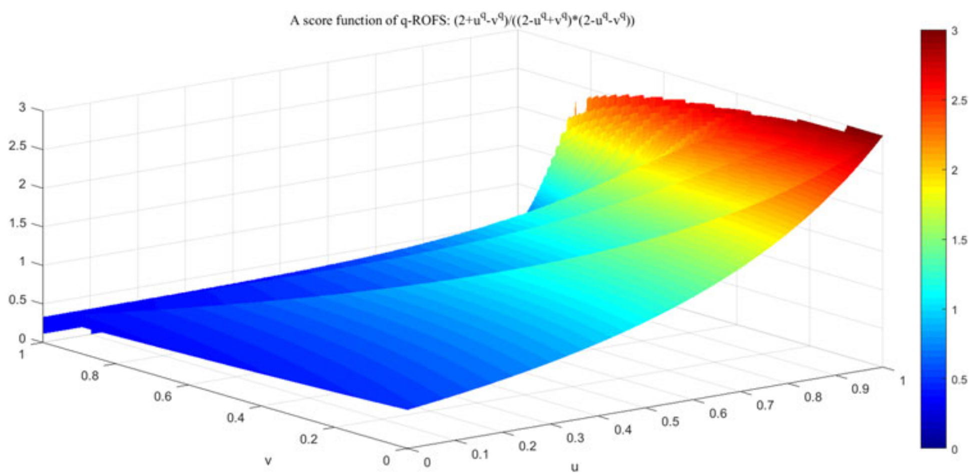


Figure 2. Image of the score function of a q-R.O.F.S.

$$\begin{aligned}\frac{\partial S_M(a)}{\partial u} &= \frac{q\times u^{q-1}\times(u^q-v^q+2)}{(v^q-u^q+2)\times(u^q+v^q-2)^2}-\frac{q\times u^{q-1}}{(v^q-u^q+2)(u^q+v^q-2)}\\ &\quad-\frac{q\times u^{q-1}\times(u^q-v^q+2)}{(v^q-u^q+2)^2\times(u^q+v^q-2)}\\ &=-\frac{q\times u^q(4u^q+4v^q+u^{2q}+v^{2q}-2u^qv^q-12)}{u\times(4u^q-u^{2q}+v^{2q}-4)^2}\\ &=\frac{q\times u^q[8-(u^q-v^q)^2-4(u^q+v^q-1)]}{u\times(4u^q-u^{2q}+v^{2q}-4)^2}\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial S_M(a)}{\partial v} &= \frac{q\times v^{q-1}}{(v^q-u^q+2)\times(u^q+v^q-2)}+\frac{q\times v^{q-1}\times(u^q-v^q+2)}{(v^q-u^q+2)\times(u^q+v^q-2)^2}\\ &\quad+\frac{q\times v^{q-1}\times(u^q-v^q+2)}{(v^q-u^q+2)^2\times(u^q+v^q-2)}\\ &=\frac{q\times v^q\times(4u^q+4v^q-u^{2q}-v^{2q}+2u^qv^q-4)}{v\times(4u^q-u^{2q}+v^{2q}-4)^2}\\ &=\frac{q\times v^q\times\left(4(u^q+v^q-1)-(u^q-v^q)^2\right)}{v\times(4u^q-u^{2q}+v^{2q}-4)^2}\leq 0\end{aligned}$$

Thus, $S_M(a)$ monotonically increases and decreases regarding u and v, respectively.

Property 2 (Boundedness). The value of the score function $S_M(a)$ belongs to $[1/3, 3]$, where $S_M(a)=3$ if and only if $a=(u,v)=(1,0)$, and $S_M(a)=1/3$ if and only if $a=(u,v)=(0,1)$.

Proof. Considering the monotonicity given in Property 1, the maximal and minimal values of $S_M(a)$ should be determined at the endpoints of each variable. In the q-R.O.F.S., we have $u\in[0,1]$, $v\in[0,1]$, $u^q+v^q\in[0,1]$ and $u^q-v^q\in[-1,1]$. Based on these conditions, it is easy to check that the value of $S_{Q-S}(a)$ ranges from 1/3 to 3 and the bounded conditions hold.

Below we distinguish two q-R.O.F.N.s $a_1=(u_1^{q1},v_1^{q1})$ and $a_2=(u_2^{q2},v_2^{q2})$ in two cases. When $u=v$, the two previous score functions obtain the same result, shown as follows:

$$\begin{aligned}S_L(u_1^{q1},v_1^{q1}) &= u_1^{q1}-v_1^{q1}=S_L(u_2^{q2},v_2^{q2})=u_2^{q2}-v_2^{q2}=0\\ S_P(u_1^{q1},v_1^{q1}) &= u_1^{q1}-v_1^{q1}+[(e^{u_1^{q1}-v_1^{q1}}/e^{u_1^{q1}-v_1^{q1}}+1)-1/2]\pi^{q1}\\ &=0+[(e^0/e^0+1)-1/2]\pi^{q1}=0+0\times\pi^{q1}=0\\ S_P(u_2^{q2},v_2^{q2}) &= u_2^{q2}-v_2^{q2}+[(e^{u_2^{q2}-v_2^{q2}}/e^{u_2^{q2}-v_2^{q2}}+1)-1/2]\pi^{q2}\\ &=0+[(e^0/e^0+1)-1/2]\pi^{q2}=0+0\times\pi^{q2}=0\end{aligned}$$

As we can see, $S_P(u,v)$ also takes into account the hesitancy degree; however, it fails to distinguish the q-R.O.F.N.s in such case. For instance, we cannot distinguish (0.6, 0.6) and (0.7, 0.7) by the existing score functions.

We consider a special case where $u_1^{q1} - v_1^{q1} = u_2^{q2} - v_2^{q2}$ with $u_1 \neq v_1$ and $u_2 \neq v_2$. If a q-R.O.F.N. $a_1 = (u_1, v_1)$ is given, there exist lots of solutions for the equation. For example, supposing that $a_1 = (0.8, 0.7)$ is provided, then, a_2 could be $\left(0.6, \sqrt{0.191}\right)$, $\left(0.65, \sqrt{0.2535}\right)$, $\left(0.7, \sqrt{0.321}\right)$ and $\left(0.75, \sqrt{0.3935}\right)$. Besides these four enumerative examples, there still exist other feasible solutions which own the same score values if calculated by the $S_L(u, v)$ and $S_P(u, v)$ score functions.

For these two cases, the accuracy function $G(\alpha) = \mu^q + \nu^q$ was proposed to overcome the weakness of the score function in distinguishing them. When $S(u_1^{q1}, v_1^{q1}) = S(u_2^{q2}, v_2^{q2})$, the accuracy function expresses the opposite of the ignorance.

On the other hand, if we take the score function $S_M(a)$ into use, the preference and indifference relations of q-R.O.F.N.s would be identified by one computation and comparison, i.e., $a_1 > a_2 \Longleftrightarrow S_M(a_1) > S_M(a_2)$ and $a_1 = a_2 \Longleftrightarrow S_M(a_1) = S_M(a_2)$.

In conclusion, the score function S_M is valid and efficient in distinguishing q-R.O.F.N.s. Although the accuracy function could contribute to differentiating q-R.O.F.N.s, the whole comparison process is time-consuming and complex, because it needs additional times to do judgements and calculations. Considering the merits in terms of efficiency and accuracy, we use the score function S_M in developing the Q-B.W.M.-S-V.I.K.O.R. method for M.C.D.M. problems.

4. Integrating a score-based V.I.K.O.R. with the best worst method for M.C.D.M. with q-rung orthopair fuzzy sets

This section introduces the Q-B.W.M.-S-V.I.K.O.R. method, which investigates the V.I.K.O.R. with the B.W.M. in q-R.O.F. environment based on the proposed score function S_M. Section 4.1 introduces the Q-B.W.M. for obtaining the weights of criteria in the q-ROF environment. Section 4.2 concentrates on the Q-S-V.I.K.O.R. method using the proposed score function and the weights acquired in Section 4.1. Section 4.3 gives the algorithm of the Q-B.W.M.-S-V.I.K.O.R. method.

4.1. Obtain the weights of criteria by the Q-B.W.M

Consider a decision-making problem with the potential alternatives $A_1, A_2, \ldots, A_i, \ldots, A_m$, the established criteria $c_1, c_2, \ldots, c_j, \ldots, c_n$ and the performance evaluations $q_{ij}, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$ of alternatives over criteria. In this article, we mainly talk about the q-R.O.F.S. So the evaluations are denoted in the form of q-R.O.F.N.s, shown as follows:

$$\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccccc} c_1 & c_2 & \cdots & c_j & \cdots & c_n \end{array} \\ \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1j} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2j} & \cdots & q_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ q_{i1} & q_{i2} & \cdots & q_{ij} & \cdots & q_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ q_{m1} & q_{m2} & \cdots & q_{mj} & \cdots & q_{mn} \end{pmatrix} \end{array}$$

where q_{ij} is a q-R.O.F.N. in the form of (u_{ij}, v_{ij}). Without loss of generality, the criteria $c_1, c_2, \ldots, c_j, \ldots, c_n$ are assumed to be the benefit criteria. The weights of criteria $c_1, c_2, \ldots, c_j, \ldots, c_n$ could be denoted as $w_1, w_2, \ldots, w_j, \ldots, w_n$. In this article, the weights are of criteria are obtained by the B.W.M. In the following, the specified steps of the Q-B.W.M.-S-V.I.K.O.R. method are given.

We introduce the B.W.M. with the q-R.O.F.S. information based on the additive consistency of q-R.O.F. preference relations in four phases. The first three phases are motivated by the original B.W.M. (Rezaei, 2015) while the last stage focuses on the interval analysis according to the results of first three phases.

The first phase of the Q-B.W.M. is to determine the best criterion c_B and the worst criterion c_W by the decision-maker intuitively, which is the same as that in the original B.W.M. (Rezaei, 2015, 2016).

The second phase of the Q-B.W.M. is to acquire the $2n-3$ preference values in the form of q-ROFNs $p = (u^q, v^q)$. Then, the $q-BO$ vector and the $q-OW$ vector can be obtained as follows:

$$q-BO = (p_{B1}, p_{B2}, \ldots, p_{Bn}) = \Big((u_{B1}, v_{B1}), (u_{B2}, v_{B2}), \ldots, (u_{Bn}, v_{Bn})\Big)$$

$$q-OW = (p_{1W}, p_{2W}, \ldots, p_{nW})^T = \Big((u_{1W}, v_{1W}), (u_{2W}, v_{2W}), \ldots, (u_{nW}, v_{nW})\Big)^T$$

The third phase of the B.W.M. is to construct a model based on the additive consistency of the $q-BO$ and $q-OW$ vectors. The additive consistency of q-R.O.F. preference relations was originally proposed by Zhang, Liao, and Luo (2019) via transforming the q-R.O.F.N.s into intuitionistic fuzzy numbers by:

$$\tilde{u} = u-\nu + (u^q + \nu^q)^{1/q}, \tilde{v} = \nu - u + (u^q + \nu^q)^{1/q} \tag{5}$$

where $\tilde{u}$ and $\tilde{v}$ are the transformed membership and non-membership degrees, respectively, satisfying $\tilde{u} + \tilde{v} \leq 1$. Then, the intuitionistic fuzzy number $(\tilde{u}, \tilde{v})$ can be regarded as an interval number with $\tilde{u}$ being the lower bound and $1-\tilde{v}$ being the upper bound. This section introduces the Q-B.W.M. based on interval analysis. So far, only Rezaei (2016), Ren (2018), and Hafezalkotob et al. (2019) paid attention to the interval analysis regarding the uncertain situation, and they all concentrated on the multiplicative consistency of the preference vectors. This section tries to investigate the Q-B.W.M. based on the additive consistency of preference vectors. In Rezaei (2016), Ren (2018) and Hafezalkotob et al. (2019), the interval analysis of the B.W.M. is based on the midpoints of interval numbers or some specific numbers in intervals. Motivated by Rezaei (2016), Ren (2018) and Hafezalkotob et al. (2019), the midpoint of the interval $[\tilde{u}, 1-\tilde{v}]$ given as Equation (6) is used in the Q-B.W.M.

$$\tilde{\chi} = (\tilde{u} + 1 - \tilde{v})/2 \tag{6}$$

The central idea of the interval analysis in the Q-B.W.M. is based on the additive consistency of the midpoints $\tilde{\chi}_{Bj}$ and $\tilde{\chi}_{jW}$. Using the perfectly additive consistency of fuzzy preference relations (Tanino, 1984), this article identifies the relationships between the weights w_j $(j = 1, 2, ..., n)$ of criteria and the pairwise comparisons as follows:

$$\tilde{\chi}_{Bj} = (w_B - w_j + 1)/2,\ \tilde{\chi}_{jW} = (w_j - w_W + 1)/2,\ j = 1, 2, ..., n \tag{7}$$

where $\tilde{\chi}_{Bj}$ and $\tilde{\chi}_{jW}$ are the transformed values of the $q-BO$ vector and the $q-OW$ vector by Equations (5) and (6), respectively. If the given preference vectors are not totally consistent, the deviation between the both sides of each equation given in Equation (7) should be as small as possible. Motivated by Rezaei (2015), a Q-B.W.M. linear model can be established based on the $2n-3$ values in the preference vectors:

Model-2

$$\begin{aligned} &\min\ \tilde{\xi} \\ &s.t.: |\tilde{\chi}_{Bj} - (w_B - w_j + 1)/2| \le \tilde{\xi}; |\tilde{\chi}_{jW} - (w_j - w_W + 1)/2| \le \tilde{\xi}; w_j \ge 0,\ \sum_{j=1}^{n} w_j = 1 \end{aligned}$$

As we can see, Model-2 is a linear model and thus the global optimal values can be obtained. This is an advantage of the additive consistency over the multiplicative consistency in original B.W.M. Solving Model-2, the weights of criteria can be acquired. Similar to the linear model in Rezaei (2016), the optimal objective function value $\tilde{\xi}^*$ can be directly regarded as the consistency ratio of the pairwise comparison values. If $\tilde{\xi}^* \le 0.1$, then the pairwise comparison values are regarded as acceptably consistent, and thus the weights of criteria are reliable. After the consistency checking process, the first three stages of the Q-B.W.M. are completed.

Based on the results of the first three phases, the interval analysis in the fourth stage to depict the original uncertainty among weights of criteria is implemented. Motivated by the multiplicative interval analysis in analytic hierarchical process (Sugihara, Ishii, & Tanaka, 2004), the interval weights of criteria based on the interval operations are analysed in the following process. The relationships of the weights of criteria and the transformed membership and non-membership degrees by Equation (5) are shown as follows:

$$[\tilde{u}_{Bj}, 1 - \tilde{v}_{Bj}] \subseteq \left[\left((w_B - d_B^1) - (w_j + d_j^2)\right)/2, \left((w_B + d_B^2) - (w_j - d_j^1)\right)/2\right]$$

$$[\tilde{u}_{jW}, 1 - \tilde{v}_{jW}] \subseteq \left[\left((w_j - d_j^1) - (w_W + d_W^2)\right)/2, \left((w_j + d_j^2) - (w_W - d_W^1)\right)/2\right]$$

where d_j^1 and d_j^2 denote the left deviation and the right deviation related to the midpoints w_j of intervals, respectively.

To obtain the interval weights of criteria, the above constraints can be converted into the linear inequalities in Model-3. Model-3 is designed to obtain the interval weights of criteria which depict the original uncertain information in humans' perceptions.

Model-3

$$\begin{aligned}
&\min \lambda \\
&s.t.: \left((w_B - d_B^1) - (w_j + d_j^2)\right)/2 \leq \tilde{u}_{Bj}; \left((w_B + d_B^2) - (w_j - d_j^1)\right)/2 \geq 1 - \tilde{v}_{Bj} \\
&\left((w_j - d_j^1) - (w_W + d_W^2)\right)/2 \leq \tilde{u}_{jW}; \left(w_j + d_j^2 - (w_W - d_W^1)\right)/2 \geq 1 - \tilde{v}_{jW} \\
&\sum_{j=1, j\neq t}^{n} (w_j + d_j^2) + (w_t - d_t^1) \geq 1; \sum_{j=1, j\neq t}^{n} (w_j - d_j^1) + (w_t + d_t^2) \leq 1 \\
&w_j + d_j^2 \geq w_j - d_j^1 \geq 0, \lambda \geq d_j^1 \geq 0, \lambda \geq d_j^2 \geq 0; j = 1, 2, \ldots, n
\end{aligned}$$

where the weights of criteria $w_j (j = 1, 2, \ldots, n)$ are calculated by Model-2 and the objective function value λ is the maximal deviation of interval weights. The constraints $\sum_{j=1, j\neq t}^{n} (w_j + d_j^2) + (w_t - d_t^1) \geq 1$ and $\sum_{j=1, j\neq t}^{n} (w_j - d_j^1) + (w_t + d_t^2) \leq 1$ are normalised restrictions on interval numbers (Sugihara et al., 2004), which plays the same role as $\sum_{j=1}^{n} w_j = 1$ for crisp weights.

Solving Model-3, we obtain the left deviations d_j^1 and the right deviations d_j^2 of the interval numbers, for $j = 1, 2, \ldots, n$. Then, we can get the lower and upper bounds of the interval weight for each criterion as$\bar{w}_j = [w_j - d_j^1, w_j + d_j^2]$.

In conclusion, Model-2 and Model-3 are two linear models owing to the advantage of the additive consistency. Comparing Model-1 (Rezaei, 2016) and Model-1-based interval B.W.M. (Ren, 2018), our linear models (Model-2 and Model-3) are efficient in obtaining the weights of criteria. The B.W.M. under the intuitionistic fuzzy context (Mou et al., 2017) tackled the membership degrees and the non-membership degrees separately, which lose the intrinsic properties of the given preference values. By contrast, the Q-B.W.M. consisting of Model-2 and Model-3 is an appropriate way to obtain the interval weights in the situation that the preference vectors are given in form of q-R.O.F. information.

4.2. A score-based V.I.K.O.R. to rank the alternatives for M.C.D.M. problems

In this section, the main features of the Q-B.W.M.-S-V.I.K.O.R. method are proposed step by step.

The first step is to determine the reference values over all criteria $c_j, j = 1, 2, \ldots, n$. Without loss of generality, the criteria in this article are assumed as benefit criteria. The maximal and minimal reference values of a benefit criterion c_j are expressed as q_j^+ and q_j^-, respectively. With the q-R.O.F.S. information, the maximum and minimum values should be determined by the proposed score function $S_M(a)$ because it considers the membership degree, the non-membership degree and the hesitancy degree simultaneously. That is:

$$q_j^+ = \max_i S_M(q_{ij}), q_j^- = \min_i S_M(q_{ij})$$

The second step is to calculate the q-R.O.F. group utility (Q.G.U.) and the q-R.O.F. individual regret (Q.I.R.) values of each alternative. Linear normalisation is implemented for the given data. After normalisation, for each alternative, the interval Q.G.U. is computed by the weighted interval utilities of all criteria, and the interval Q.I.R. is identified by the maximal value of the interval weighted utility, shown as:

$$QGU_i = \sum_{j=1}^{n} \bar{w}_j (q_j^+ - S_M(q_{ij}))/(q_j^+ - q_j^-) \tag{8}$$

$$QIR_i = \max_j \bar{w}_j (q_j^+ - S_M(q_{ij}))/(q_j^+ - q_j^-) \tag{9}$$

where the interval Q.G.U. and interval Q.I.R. are calculated by the interval weights. As for these two values, the smaller the QGU_i and QIR_i are, the better the corresponding alternative A_i is.

The third step is to compute the overall score of each alternative using QGU_i and QIR_i. The q-R.O.F. overall value (Q.O.V.) of each alternative is calculated by a linear combination with the parameter θ, shown as:

$$QOV_i = \theta \frac{QGU_i - \min_i QGU_i}{\max_i QGU_i - \min_i QGU_i} + (1-\theta) \frac{QIR_i - \min_i QIR_i}{\max_i QIR_i - \min_i QIR_i} \tag{10}$$

where θ denotes the relative power of the group utility and the individual regret values. $\max_i$ and $\min_i$ are designed to denote the maximal and minimal values among the alternatives.

The fourth step is to implement the stable and acceptable advantage analysis over the ranking results for the interval values, QGU_i, QIR_i and QOV_i. The requirement for stable analysis is the same as that in the original V.I.K.O.R. method. That is, the best alternative A_i should occupy the best position at least one times among three rankings. The advantage analysis of intervals is based on the comparison rules of interval numbers (Xu & Da, 2002). Xu and Da (2002) proposed a formula to compare two intervals ψ_1 and ψ_2 by the possibility degree, which is calculated by:

$$P(\psi_1 \geq \psi_2) = \max\{1 - \max\{(\psi_2^+ - \psi_2^-)/(\psi_1^+ - \psi_1^- + \psi_2^+ - \psi_2^-), 0\}, 0\} \tag{11}$$

Motivated by this rule, we modify the advantage analysis into $P(QOV_{i2} - QOV_{i1}) \geq 1/(i-1)$, where ψ_1^+ and ψ_1^- represent the upper and the lower bounds of the interval ψ_1, respectively. In this case, the acceptable advantage exists between two alternatives. If the above advantage and stability acceptable conditions cannot be satisfied, the compromise solutions could be obtained.

4.3. Algorithm of the Q-B.W.M.-S-V.I.K.O.R. method

In this section, we integrate the Q-B.W.M. with the score-based V.I.K.O.R., and propose the algorithm of the Q-B.W.M.-S-V.I.K.O.R. method to rank the alternatives for M.C.D.M. problems with q-rung orthopair fuzzy information. The detailed steps of the algorithm are listed below for applications.

Algorithm (the Q-B.W.M.-S-V.I.K.O.R. method)

Input: The alternatives $A_1, A_2, \ldots, A_i, \ldots, A_m$, the criteria $c_1, c_2, \ldots, c_j, \ldots, c_n$, the best criterion c_B, the worst criterion c_W, the preference vectors $q-BO$, $q-OW$ of criteria and the performance evaluations of alternatives over criteria $q_{ij}, i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n$.

Output: The ranking result of alternatives $A_1, A_2, \ldots, A_i, \ldots, A_m$.

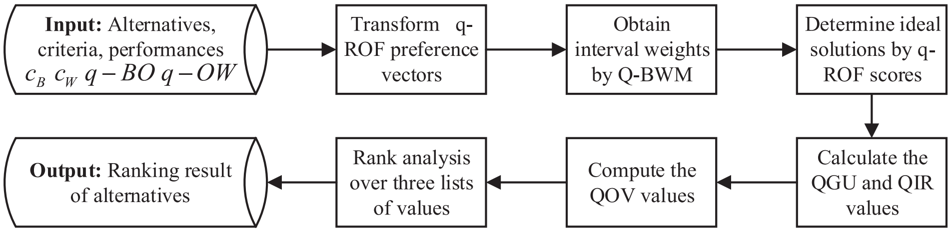


Figure 3. The diagram of the Q-B.W.M.-S-V.I.K.O.R. method.

Step 1: Transform the q-R.O.F. preference vectors, $q-BO$ and $q-OW$, into interpretable interval numbers by Equation (5).
Step 2: Compute Model-2 and Model-3 to acquire the interval weights $\bar{w}_j$ of criteria based on the intervals obtained in Step 1.
Step 3: Determine the positive and negative ideal performances over each criterion by the score function, Equation (4).
Step 4: Calculate the q-R.O.F. group utility QGU_i and q-R.O.F. individual regret QIR_i of each alternative by Equations (8) and (9), respectively.
Step 5: Compute the q-R.O.F. overall value QOV_i of alternatives by Equation (10).
Step 6: Do the stable and acceptable advantage analysis over three ranking lists of QGU_i, QIR_i and QOV_i and obtain the final ranking result of alternatives.

The algorithm of the Q-B.W.M.-S-V.I.K.O.R. method uses the uncertain information in the form of q-R.O.F.N.s. Step 1 implements the transformation process to transform the q-R.O.F.N.s into intervals. In Step 2, Model-2 is used to obtain the potential acceptable weights of criteria. Based on the optimal solutions of Model-2, Model-3 derives the interval weights of criteria. The Q-B.W.M. consists of the input content, Step 1 and Step 2. Steps 3-6 are the prominent content of the Q-S-V.I.K.O.R. method. The proposed score function, Equation (4), is used in the process of determining the ideal solutions of on all criteria. Meanwhile, based on the score function, the q-R.O.F. group utility QGU_i, q-R.O.F. individual regret QIR_i and q-ROF overall value QOV_i can be calculated by Equations (8–10), respectively. The final step is to rank alternatives based on the values of QGU_i, QIR_i and QOV_i. Figure 3 illustrates the procedure intuitively.

5. Hospitality brand management: a case study of the Q-B.W.M.-S-V.I.K.O.R. method

In this section, a case study concerning the hospitality brand management is given. Comparative analyses with existing similar methods are provided to verify the applicability and validation of the Q-B.W.M.-S-V.I.K.O.R. method.

5.1. Case description

In the era of rapid economic globalisation and world economic integration, tourism continues to develop rapidly and steadily, becoming a global strategy, pillar and comprehensive industry (Hashemkhani Zolfani et al., 2015; Meimand et al., 2017; Stanujkic, Zavadskas, & Tamošaitienė, 2015). With the significant increase in the number of hotels, the general improvement of hotel service level and the increasing number of tourists, it is of great significance to manage the hospitality brand scientifically through evaluating and optimising the brands of hotels (Karantzavelou, 2018). There are two understandings of brand from the superficial and commercial aspects as far as the hotel industry is concerned. On the one hand, brand can be a simple concept (Superficially, it can be a name, a symbol, or a trademark), such as the logo of Speed Eight, or the name of home, which means that customers could feel at home when they stay in the hotel. On the other hand, brand can play a commercial role in the process of hotel operation. For instance, if a customer trusts Home Hotel, s/he will choose to stay at home hotel rather than choose other local hotels wherever unfamiliar areas s/he travels in (Prasad & Dev, 2000). In these cases, brands have complex, invisible and intangible value. Brands can be regarded as a visual intangible asset in the hotel industry (Kayaman & Arasli, 2007). It can be understood that brand plays an important role in hotel marketing and customer relationship. Furthermore, hotel brands with high brand equity would make customers have more obvious preferences and purchasing willingness, which indicates that quantifiable brand equity has great significance for the diagnosis and decision-making of hotel brand management.

To know some insights on hospitality brand management, a case study concerning the hospitality brand management is conducted by the proposed C.B.B.E. framework in Aaker (1996). According to Aaker (1996), brand equity has four directly related components: brand loyalty, perceived quality, brand awareness and brand association. In this article, we take these four criteria into consideration in the case study of hospitality brand management. The detailed meanings of four criteria are showed as follows:

- **Brand Loyalty**: Aaker (1991) defined the brand loyalty as 'customer attachment to the brand, which usually translates directly into future sales'. That is to say, if the customer is loyal to the brand, his awareness to this brand is more positive, and will have more purchasing behaviour.
- **Perceived Quality**: Zeithaml (1988) defined the perceived quality as a customer's judgement of the overall superiority or superiority of a product or service, or an objective evaluation of a product by a customer. However, some scholars believed that the high target quality does not always lead to brand equity. Perceived quality can be reflected in many ways, such as shape, appearance and other intrinsic characteristics and external information such as brand name, product information and quality assurance seal (Chaudhuri & Holbrook, 2001);
- **Brand Awareness**: Aaker (1991) described the brand awareness as 'the ability of customers to identify or recall brand as a member of a product category'. It shows the ability of consumers to identify brand in different situations, including brand recognition and brand recall. Brand recognition refers to the ability of consumers to respond and identify the brand when referring to it (Foroudi, 2019). Brand recall refers to the ability of consumers to actively mention the brand when

referring to the product category of the brand. The stronger the brand awareness is, the more consumers will think of the brand when choosing products, and the more likely they will eventually buy the brand;

- **Brand Image**: Aaker (1991) defined the brand image as 'anything that is linked in memory to the brand'. It forms a foundation for brand choice and eventual loyalty (Aaker, 1991). Brand association is the information node associated with brand in consumer brand knowledge system. For a brand, the most direct association of consumers can be a symbol, a product, an enterprise or a person. It can also be the functional, symbolic or experiential benefits of the product. It can be the general attitude and evaluation of the brand as well. Every association of a brand can be measured by three indicators: intensity, identity and uniqueness. The sum of brand association and these three indicators constitute the brand image (Zeithaml, 1988). Brand image is the aggregation of all associations of consumers with brand.

Ten representative hotels, namely Four Seasons Hotel, InterContinental Hotel, Hilton Hotel, Shangri-La Hotel, Peninsula Hotel, Aman Hotel, Mandarin Oriental, Rosewood, Super eight Hotel and Ibis Hotel, are chose for by the D.M. The model evaluates the brand equity of these 10 hotels on the above four criteria: brand loyalty c_1, perceived quality c_2, brand awareness c_3, and brand image c_4. The D.M. identifies brand loyalty (c_1) and brand awareness (c_3) as the best and the worst criteria, respectively. After the D.M. determines the best and worst criteria, we can obtain the preference values between the criteria in the form of q-R.O.F.N.s. The q-B.O. vector, the q-O.W. vector and the performance evaluation matrix of 10 alternatives over four above criteria can be acquired as follows:

$$q\text{-}BO = (p_{11}, p_{12}, p_{13}, p_{14}) = \Big((0.5, 0.5), (0.9, 0.7), (0.9, 0.3), (0.9, 0.4)\Big)$$

$$q\text{-}OW = (p_{13}, p_{23}, p_{33}, p_{43})^T = \Big((0.9, 0.3), (0.9, 0.8), (0.5, 0.5), (0.9, 0.7)\Big)^T$$

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \\ A_{10} \end{array}
\begin{array}{c} \begin{array}{cccc} c_1 & \quad\quad c_2 & \quad\quad c_3 & \quad\quad c_4 \end{array} \\
\begin{pmatrix}
(0.6, 0.5) & (0.6, 0.6) & (0.7, 0.7) & (0.6, 0.5) \\
(0.9, 0.2) & (0.3, 0.4) & (0.1, 0.3) & (0.8, 0.8) \\
(0.2, 0.3) & (0.7, 0.7) & (0.1, 0.8) & (0.3, 0.5) \\
(0.7, 0.7) & (0.4, 0.2) & (0.5, 0.5) & (0.9, 0.5) \\
(0.6, 0.6) & (0.1, 0.6) & (0.7, 0.7) & (0.3, 0.1) \\
(0.6, 0.4) & (0.9, 0.3) & (0.6, 0.6) & (0.3, 0.3) \\
(0.6, 0.4) & (0.9, 0.3) & (0.6, 0.6) & (0.4, 0.4) \\
(0.7, 0.6) & (0.5, 0.5) & (0.1, 0.1) & (0.5, 0.6) \\
(0.9, 0.3) & (0.9, 0.6) & (0.1, 0.9) & (0.9, 0.6) \\
(0.8, 0.2) & (0.4, 0.1) & (0.2, 0.3) & (0.4, 0.4)
\end{pmatrix} \end{array}$$

5.2. Solving the case by the Q-B.W.M.-S-V.I.K.O.R. method

In this section, we implement the case study by the Q-B.W.M.-S-V.I.K.O.R. method. In the following, the detailed computations by the algorithm are given.

Step 1: The q-R.O.F. preference vectors are transformed by Equation (5) into the following interval numbers, shown as follows:

$$q\text{−}BO = ([0.5, 0.5], [0.5865, 0.6135], [0.7743, 0.8257], [0.7424, 0.7576])$$
$$q\text{−}OW = ([0.7743, 0.8257], [0.5415, 0.5585], [0.5, 0.5], [0.5865, 0.6135])^T$$

Step 2: Before using Model-2, we calculated the midpoints of the interval numbers obtained in Step 1, shown as follows:

$$q\text{−}BO' = (\tilde{\chi}_{B1}, \tilde{\chi}_{B2}, \tilde{\chi}_{B3}, \tilde{\chi}_{B4}) = (0.5, 0.6, 0.8, 0.75);$$
$$q - OW' = (\tilde{\chi}_{1W}, \tilde{\chi}_{2W}, \tilde{\chi}_{3W}, \tilde{\chi}_{4W})^T = (0.8, 0.55, 0.5, 0.6)^T$$

where the $q\text{−}BO'$ and the $q\text{−}OW'$ vectors represent the midpoints of intervals.

We then use Model-2 to obtain the weights of criteria, and the model is constructed as follows:

Model-2.1

$$\begin{aligned} &\min \tilde{\xi} \\ &\quad s.t. : |0.6 - (w_1 - w_2 + 1)/2| \le \tilde{\xi}; |0.8 - (w_1 - w_3 + 1)/2| \le \tilde{\xi} \\ &\qquad |0.75 - (w_1 - w_4 + 1)/2| \le \tilde{\xi}; |0.55 - (w_2 - w_3 + 1)/2| \le \tilde{\xi} \\ &\qquad |0.6 - (w_4 - w_3 + 1)/2| \le \tilde{\xi}; w_j \ge 0, \textstyle\sum_{j=1}^{4} w_j = 1 \end{aligned}$$

Solving the above model, we obtain the crisp weights of criteria as $w = (0.55, 0.25, 0.05, 0.15)^T$, and the deviation $\tilde{\xi}^* = 0.05<0.1$, which shows that the given preference vectors are acceptably consistent and thus the crisp weights of criteria are reliable. Then, we do the interval analysis by Model-3 and establish the following model:

Model-3.1

$$\begin{aligned} &\min \lambda \\ &s.t. : \Big((0.55 - d_1^1) - (0.25 + d_2^2)\Big)/2 \le \tilde{u}_{12}; \Big((0.55 + d_1^2) - (0.25 - d_2^1)\Big)/2 \ge 1 - \tilde{v}_{12} \\ &\quad \Big((0.55 - d_1^1) - (0.05 + d_3^2)\Big)/2 \le \tilde{u}_{13}; \Big((0.55 + d_1^2) - (0.05 - d_3^1)\Big)/2 \ge 1 - \tilde{v}_{13} \\ &\quad \Big((0.55 - d_1^1) - (0.15 + d_4^2)\Big)/2 \le \tilde{u}_{14}; \Big((0.55 + d_1^2) - (0.15 - d_4^1)\Big)/2 \ge 1 - \tilde{v}_{14} \\ &\quad \Big((0.25 - d_2^1) - (0.05 + d_3^2)\Big)/2 \le \tilde{u}_{23}; \Big((0.25 + d_2^2) - (0.05 - d_3^1)\Big)/2 \ge 1 - \tilde{v}_{23} \\ &\quad \Big((0.15 - d_4^1) - (0.05 + d_3^2)\Big)/2 \le \tilde{u}_{43}; \Big((0.15 + d_4^2) - (0.05 - d_3^1)\Big)/2 \ge 1 - \tilde{v}_{43} \\ &\quad \textstyle\sum_{j=1, j\ne t}^{n} (w_j + d_j^2) + (w_t - d_t^1) \ge 1; \sum_{j=1, j\ne t}^{n} (w_j - d_j^1) + (w_t + d_t^2) \le 1 \\ &\quad w_j + d_j^2 \ge w_j - d_j^1 \ge 0, \lambda \ge d_j^1 \ge 0, \lambda \ge d_j^2 \ge 0; j = 1, 2, \ldots, n \end{aligned}$$

Solving Model-3.1, the left and right deviations of weights can be derived as $d_j^1 = (0.077, 0.077, 0.05, 0.038)$, $d_j^2 = (0.077, 0.05, 0.04, 0.077)$ and $\lambda = 0.077$. Based on these calculated results, the interval weights of criteria can be obtained as $\bar{w}_j = ([0.4729, 0.6270], [0.1730, 0.3], [0, 0.0899], [0.1119, 0.227])$.

Step 3: After obtaining the score values of performances q_{ij} of alternatives over criteria by Equation (5), we compare these scores and determine the ideal solution for each criterion, shown as follows:

$$q_1^+ = S_M(A_{21}) = 1.9583, q_1^- = S_M(A_{31}) = 0.6032,$$

$$q_2^+ = S_M(A_{62}) = S_M(A_{72}) = 1.9318, q_2^- = S_M(A_{52}) = 0.4615$$

$$q_3^+ = S_M(A_{43}) = 1, q_3^- = S_M(A_{93}) = 0.4286,$$
$$q_4^+ = S_M(A_{44}) = 1.6277, q_4^- = S_M(A_{84}) = 0.6444$$

Step 4: We calculate the weighted q-R.O.F. group utility QGU_i and q-R.O.F. individual regret QIR_i for each alternative in interval forms by Equations (8–10). The detailed results are listed as follows:

$$QGU_i = \begin{pmatrix} 0.6324 & 0.9627 \\ 0.2343 & 0.5093 \\ 0.6926 & 1.1280 \\ 0.4659 & 0.6685 \\ 0.6821 & 1.0472 \\ 0.3609 & 0.5859 \\ 0.4857 & 0.7418 \\ 0.5593 & 0.9349 \\ 0.0510 & 0.1754 \\ 0.2469 & 0.5028 \end{pmatrix}; QIR_i = \begin{pmatrix} 0.4032 & 0.5345 \\ 0.1454 & 0.2522 \\ 0.4730 & 0.6270 \\ 0.3413 & 0.4525 \\ 0.4108 & 0.5446 \\ 0.2569 & 0.3406 \\ 0.3953 & 0.5240 \\ 0.3378 & 0.4478 \\ 0.0388 & 0.0899 \\ 0.1212 & 0.2101 \end{pmatrix}$$

Step 5: We compute the q-R.O.F. overall value QOV_i of each alternative by a linear combination of QGU_i and QIR_i, and the parameter θ is set as 0.5 for considering QGU_i and QIR_i as equal important. The calculated values QOV_i of the 10 alternatives are shown as follows:

$$QOV_i = \begin{pmatrix} 0.5178 & 0.7486 \\ 0.1898 & 0.3807 \\ 0.5828 & 0.8775 \\ 0.4036 & 0.5605 \\ 0.5465 & 0.7959 \\ 0.3089 & 0.4632 \\ 0.4405 & 0.6329 \\ 0.4486 & 0.6914 \\ 0.0449 & 0.1327 \\ 0.1840 & 0.3565 \end{pmatrix}$$

Step 6: We implement the stable analysis and advantage analysis for QOV_i values by the interval comparison rule given as Equation (11), and the final ranking result of the brands is obtained as:

$$A_9 > A_{10} > A_2 > A_6 > A_4 > A_7 > A_8 > A_1 > A_5 > A_3$$

which shows that the alternative A_9 is the best alternative among 10 alternatives.

5.3. Sensitive and comparative analyses

In this section, we conduct the sensitive analysis by changing some conditions in the Q-B.W.M.-S-V.I.K.O.R. method, and compare the method with existing similar methods. The other weight-determination methods are used to calculate the weights of criteria with the q-R.O.F. information. Two other score functions of q-R.O.F.N.s are involved to calculate the group utility and the individual regret values in the V.I.K.O.R. method. The comparison with the Technique for Order Preference by Similarity to an Ideal Solution (T.O.P.S.I.S.) and T.O.D.I.M. methods are provided.

5.3.1. Sensitive analysis regarding the parameter θ

In this section, we do the sensitive analysis concerning the parameter θ. The value range of θ is split into 10 parts and the length of each interval is 0.1. The change of the value of θ would also alter the rank positions of alternatives, shown in Figure 4. From Figure 4, we can find that the alternatives A_1, A_3, A_5, A_6 and A_9 rank the same

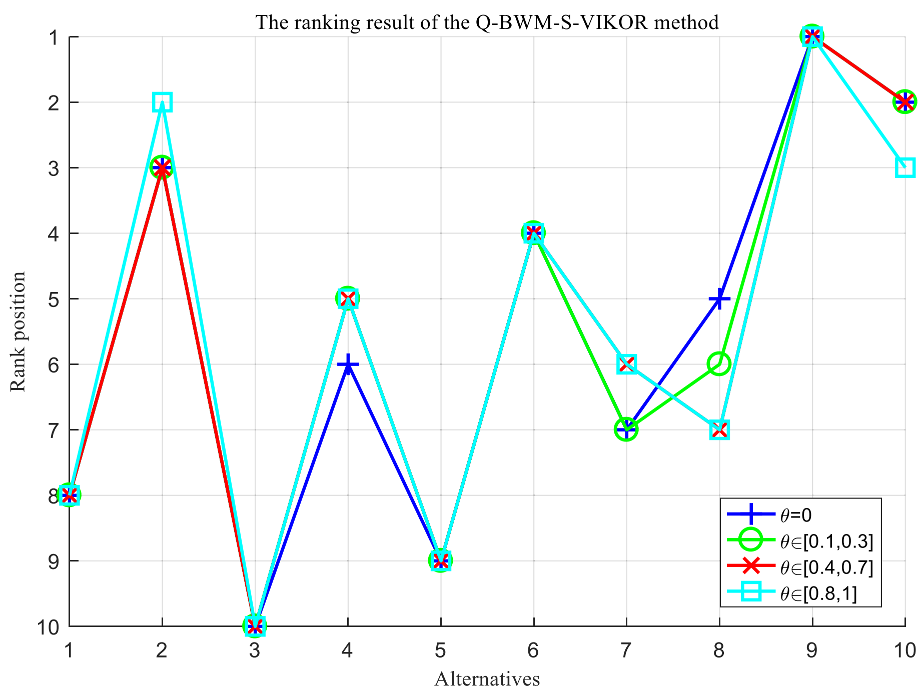


Figure 4. The visual ranking results of the Q-B.W.M.-S-V.I.K.O.R. method with different values of θ.

positions. That is to say, these alternatives obtain the stable positions with changing the values of θ.

When the value of θ belongs to [0.5, 1], the ranking results would focus more on the q-R.O.F. group utility QGU. When the value of θ is less than 0.5, the final ranking result of alternatives would pay more attention to the q-R.O.F. individual regret QIR. The value of QIR plays a role in veto the compensate effect, which means a high average value could cover the disadvantage on some criteria. If the compensate effect is taken into consideration to some extent, the ranking results of alternatives change significantly. Taking the alternatives A_2 and A_{10} with similar performances on criteria for example, if we stress the importance of the q-R.O.F. individual regret QIR, the alternative A_{10} performances better than the alternative A_2. If we think that the q-R.O.F. group utility is much more important than the q-R.O.F. individual regret QIR, the alternative A_2 ranks better than the alternative A_{10}. The Q-B.W.M.-S-V.I.KO.R. method provides optional attitudes in decision-making process.

5.3.2. Solve the case by other weight-determining methods

In this section, the Q-B.W.M. is compared with other methods from two aspects. One is regarding the additive consistency of preference vectors, compared with two related references (Mou et al., 2017) from the theoretical aspect based on Model-2. Another is how to rationally get the interval weights of criteria after obtaining crisp weights, compared with two previous interval analysis references (Ren, 2018; Rezaei, 2016) thorough the calculation steps and values perspective based on Model-3.

The additive consistency of preference vectors has been investigated in by Mou et al. (2017) who paid attention to the uncertain information captured as intuitionistic fuzzy numbers. The additive consistency in Mou et al. (2017) was denoted as $w_B - w_j = a_{Bj}$ and $w_j - w_W = a_{jW}$ where a_{Bj} and a_{jW} represent the preference values in preference vectors. The relations between the weights and preference values are not applicable in reciprocal preference relations whose value scales belong to [0,1]. This article focuses on the reciprocal preference relations and provides an alternative way to implement the B.W.M. in reciprocal preference vectors.

As for the additive consistency in intuitionistic fuzzy environment (Mou et al., 2017), the membership degree and non-membership degree are tackled separately in the B.W.M. model, which lose the intrinsic property of the intuitionistic fuzzy numbers. Furthermore, the deviation objectives based on the additive consistency in Mou et al. (2017) are $w_B/w_j - w_j/w_W - u_{BW}$ and $w_B/w_j - w_j/w_W - v_{BW}$. In the minimal deviation objective function, w_B/w_j and w_j/w_W are established based on the multiplicative consistency, which misunderstood the additive relations between weights and preference values. The Q-B.W.M. proposed in this article transforms the q-R.O.F.N.s into interpretable interval numbers which retain the original property of uncertain information. Besides, the Q-B.W.M. uses the appropriate additive consistency in reciprocal preference relations. In conclusion, from the theoretical perspective, the Q-B.W.M. proposed in this article is able and competent.

Secondly, the operations in the interval analysis for obtaining interval weights of criteria are compared with those in Rezaei (2016) and Ren (2018). After solving Model-2, the interval analysis of weights can be done in two ways in Rezaei (2016) and Ren (2018), respectively. In Rezaei (2016), $2n$ linear models should be established for deriving the maximal and minimal weights of n criteria. Taking the first criterion c_1 for example, based on Model-2, the linear models should be constructed as follows:

$$\begin{array}{ll} \min w_1 & \max w_1 \\ s.t.: |0.6-(w_1-w_2+1)/2| \le \xi_2 & s.t.: |0.6-(w_1-w_2+1)/2| \le \xi_2 \\ \quad |0.8-(w_1-w_3+1)/2| \le \xi_2 & \quad |0.8-(w_1-w_3+1)/2| \le \xi_2 \\ \quad |0.75-(w_1-w_4+1)/2| \le \xi_2 \ ; & \quad |0.75-(w_1-w_4+1)/2| \le \xi_2 \\ \quad |0.55-(w_2-w_3+1)/2| \le \xi_2 & \quad |0.55-(w_2-w_3+1)/2| \le \xi_2 \\ \quad |0.6-(w_4-w_3+1)/2| \le \xi_2 & \quad |0.6-(w_4-w_3+1)/2| \le \xi_2 \\ \quad \xi_2 = 0.05; w_j \ge 0, \sum_{j=1}^4 w_j = 1 & \quad \xi_2 = 0.05; w_j \ge 0, \sum_{j=1}^4 w_j = 1 \end{array}$$

Similarly, for the other criteria c_2, c_3 and c_4, the linear models can be established. After solving 8 linear models for four criteria, the weights of criteria calculated by these eight linear models can be acquired as $w = (0.55, 0.25, 0.05, 0.15)^T$, which is the same as the result of Model-2.1. This shows the acceptable additive consistency of the given preference vectors and the validity of the interval weights. If multiple solutions are desired for capturing the uncertain information, the proposed Q-B.W.M. including the interval analysis would be suitable.

As for the interval analysis in Ren (2018), the left and right deviations comparing the midpoints of the intervals are considered as the equal value for each criterion. Based on this idea, we can establish the interval analysis model as follows:

$$\begin{array}{l} \min \lambda \\ s.t.: \Big((0.55-d_1)-(0.25+d_2)\Big)/2 \le \tilde{u}_{12}; \Big((0.55+d_1)-(0.25-d_2)\Big)/2 \ge 1-\tilde{v}_{12} \\ \quad \Big((0.55-d_1)-(0.05+d_3)\Big)/2 \le \tilde{u}_{13}; \Big((0.55+d_1)-(0.05-d_3)\Big)/2 \ge 1-\tilde{v}_{13} \\ \quad \Big((0.55-d_1)-(0.15+d_4)\Big)/2 \le \tilde{u}_{14}; \Big((0.55+d_1)-(0.15-d_4)\Big)/2 \ge 1-\tilde{v}_{14} \\ \quad \Big((0.25-d_2)-(0.05+d_3)\Big)/2 \le \tilde{u}_{23}; \Big((0.25+d_2)-(0.05-d_3)\Big)/2 \ge 1-\tilde{v}_{23} \\ \quad \Big((0.15-d_4)-(0.05+d_3)\Big)/2 \le \tilde{u}_{43}; \Big((0.15+d_4)-(0.05-d_3)\Big)/2 \ge 1-\tilde{v}_{43} \\ \quad w_j + d_j \ge w_j - d_j \ge 0, \lambda \ge d_j \ge 0; j = 1,2,3,4 \end{array}$$

Solving the above model, the deviations of four criteria are obtained as $d_j = (0.0514, 0.0756, 0.05, 0.077)$. Considering potential deviations into the weights of criteria, the interval weights of criteria can be calculated as: $w' = ([0.4986, 0.6014], [0.1743, 0.3256], [0, 0.1], [0.073, 0.227])^T$. Owing to the same deviation from the midpoints, the ranges of the interval weights in w' are greater than those of the interval weights in $\bar{w}_j$ calculated by the Q-B.W.M. The interval analysis in Ren (2018) lacks the normalisation technique for weights, which may lead that some values of weights could not satisfy the constraint $\sum_{j=1}^4 w_j = 1$. While for the Q-B.W.M., the normalisation technique for intervals is added in the constraints of interval analysis, which guarantees that the picked values in interval weights can meet the constrict of weights $\sum_{j=1}^4 w_j = 1$.

After comparisons, we could draw a conclusion that the Q-B.W.M. proposed in this article is superior to two interval analyses methods to some extent based on the restricted deviations and the normalisation technique for interval weights. The Q-B.W.M. not only studies the additive consistency of the given preference vectors, but also provides an alternative for tackling interval values using the B.W.M.

5.3.3. Solve the case by using other two score functions of q-R.O.F.N.s

In this section, the Q-S-V.I.K.O.R. method based on two existing score functions are analysed using the determined interval weights of criteria in Section 5.2.

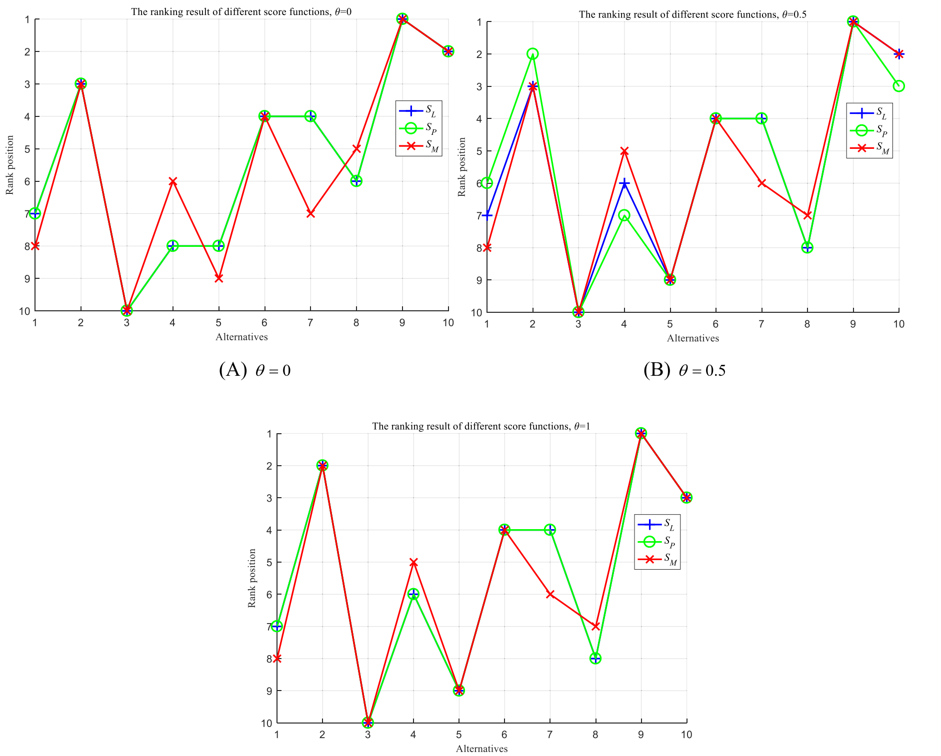
The score values of q-R.O.F.N.s calculated by Equations (1) and (3) are as follows:

$$S_L = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \\ A_{10} \end{matrix} \overset{\begin{matrix} c_1 & \quad c_2 & \quad c_3 & \quad c_4 \end{matrix}}{\begin{pmatrix} 0.110 & 0.000 & 0.000 & 0.110 \\ 0.770 & -0.100 & -0.200 & 0.000 \\ -0.100 & 0.000 & -0.700 & -0.200 \\ 0.000 & 0.200 & 0.000 & 0.604 \\ 0.000 & -0.500 & 0.000 & 0.200 \\ 0.200 & 0.720 & 0.000 & 0.000 \\ 0.200 & 0.720 & 0.000 & 0.000 \\ 0.130 & 0.000 & 0.000 & -0.110 \\ 0.720 & 0.513 & -0.800 & 0.513 \\ 0.600 & 0.300 & -0.100 & 0.000 \end{pmatrix}};$$

$$S_P = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \\ A_{10} \end{matrix} \overset{\begin{matrix} c_1 & \quad c_2 & \quad c_3 & \quad c_4 \end{matrix}}{\begin{pmatrix} 0.121 & 0.000 & 0.000 & 0.121 \\ 0.798 & -0.107 & -0.230 & 0.000 \\ -0.112 & 0.000 & -0.717 & -0.210 \\ 0.000 & 0.220 & 0.000 & 0.625 \\ 0.000 & -0.537 & 0.000 & 0.230 \\ 0.200 & 0.737 & 0.000 & 0.000 \\ 0.200 & 0.737 & 0.000 & 0.000 \\ 0.135 & 0.000 & 0.000 & -0.121 \\ 0.737 & 0.520 & -0.800 & 0.520 \\ 0.600 & 0.337 & -0.112 & 0.000 \end{pmatrix}}$$

Using these two score matrices and the Q-V.I.K.O.R. method, the ranking results of alternatives with different values of θ can be obtained. For $\theta \in [0, 1]$, the ranking results with different values of θ can be divided into ten situations. For analysing the essence of the score functions, the values of θ are determined as 0 and 1. Figure 5 shows the ranking results in these situations.

When the value of θ is set as zero, the q-R.O.F. individual regret QIR value dominates the ranking result. The value of QIR focuses on one specific criterion, which



(A) $\theta = 0$ (B) $\theta = 0.5$

(C) $\theta = 1$

Figure 5. The visual ranking results of different score functions with different values of θ.

shows the disadvantage of alternatives. As shown in Figure 5 (a), the rankings of alternatives based on the score functions S^L and S_P are the same; while the results calculated by the score function S_M show significant difference. The alternatives A_4 and A_5, A_6 and A_7 are ranked at the same positions thorough the score functions S^L and S_P.

When the value of θ equals 0.5, the alternatives A_6 and A_7 are regarded as the same as well. This is mainly caused by the score functions S^L and S_P being incapable of distinguish two q-R.O.F.N.s with the same value of membership degree and non-membership degree. The score function S_M takes the hesitancy degree of q-R.O.F.N.s into consideration reasonably, which shows the advantage of the score function S_M.

If the value of θ is determined as one, the ranking result relies on the value of the q-R.O.F. group utility QGU. As shown in Figure 5 (c), the score functions S^L and S_P deduce the same ranking result. As for the ranking result calculated by the score function S_M, the ranks of alternatives A_2, A_3, A_5, A_6 and A_{10} are different. For the rest alternatives among ten alternatives, these three score functions show the validity of the proposed score function S_M. For example, A_1 and A_8 change their positions and the alternatives A_6 and A_7 are distinguishable. The ties between the alternatives A_1 and A_8, A_6 and A_7 are broken. This also shows the benefit of the score function S_M in breaking ties.

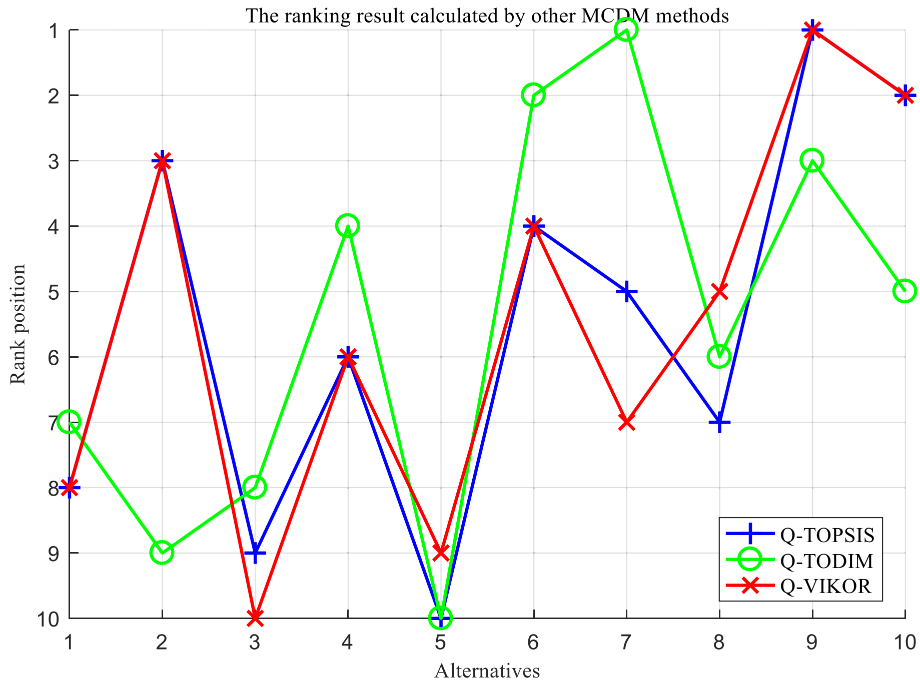


Figure 6. The visual ranking results of different methods.

5.3.4. Solve the case by other M.C.D.M. methods

In this section, the T.O.P.S.I.S. and T.O.D.I.M. methods with q-R.O.F.N.s information are analysed based on the proposed score function S_M.

We use the same data and calculate the ranking results using the T.O.P.S.I.S. method (Opricovic & Tzeng, 2004) and the T.O.D.I.M. method (Wang and Li, 2018). The final results derived by these two methods and that derived by the method proposed in this article are visually displayed in Figure 6.

In Figure 6, the result of the T.O.D.I.M. method (Wang and Li, 2018) shows significantly difference compared with the T.O.P.S.I.S. and V.I.K.O.R. methods. This is mainly caused by the role of weights. The outranking degrees of alternatives in T.O.D.I.M. (Wang and Li, 2018) can be simplified as:

$$\Phi(A_i, A_l) = \begin{cases} \sqrt{w_j\left(S(q_{ij}) - S(q_{lj})\right)} & if \quad q_{ij} \geq q_{lj} \\ -\frac{1}{\sigma}\sqrt{\frac{S(q_{lj})-S(q_{ij})}{w_j}} & if \quad q_{ij}<q_{lj} \end{cases}$$

with the value of σ being determined as one in Wang and Li (2018). Then, the weights of criteria play an important role in retroaction when the performances of alternative A_i on criterion c_j being greater than that of alternative A_l. This is counter-intuitive to some extent. The theoretical analyses of paradoxes in the T.O.D.I.M. method are investigated by Llamazares (2018). The T.O.D.I.M. method is easy to be influenced by the incorrect weights of criteria and the result of final ranking is vulnerable due to inconsistencies existing. During the calculation process, each alternative should be determined as the reference points, and thus the complexity would increase largely. For example, in this article, 10 alternatives over four criteria are necessary to rank. The computation times of the T.O.D.I.M. method is four hundred (10*10*4) because the pairwise comparisons based on all alternatives need to be done. This is not accordant with the intrinsic property, determining the comparison structure based on the best and the worst.

Comparing to the T.O.P.S.I.S. method (Opricovic & Tzeng, 2004), the result calculated by the method proposed in this article achieves consensus with the T.O.P.S.I.S. method on more than half alternatives. The ranks of alternatives A_1, A_2, A_4, A_6, A_9 and A_{10} are stable compared with those deduced by the T.O.P.S.I.S. and V.I.K.O.R. methods. The stable ranks show that these alternatives' disadvantages are not flawed enough to reject the rank positions. Only the alternatives A_3, A_5, A_7 and A_8 exchanges their positions, respectively. This shows that some deficiencies over specific criterion may increase difficulty in ranking better compared with the T.O.P.S.I.S. method. For example, the alternative A_3 performs badly on the most important criterion c_1 with the non-membership degree being greater than the membership degree. The V.I.K.O.R. method proposed in this article reveals the shortcoming of the alternative A_3 and rejects the compensate effect. In conclusion, based on the comparative analyses and discussions, the Q-B.W.M.-S-V.I.K.O.R. method is valid and efficient in the decision-making process.

6. Conclusion

The q-R.O.F.S., a generalisation of intuitionistic fuzzy sets and Pythagorean fuzzy sets, is the focus of this article. To reasonably compare two q-R.O.F.N.s, a score function of the q-R.O.F.S. was proposed by considering the hesitancy degree. The presented score function is able to distinguish the q-R.O.F.N.s with equal membership degrees and non-membership degrees. For decision-making problems, a weight-determination method, B.W.M., was studied in the q-R.O.F. environment based on the additive consistency of reciprocal preference relations. Furthermore, the V.I.K.O.R. method with q-R.O.F.N.s was investigated based on the proposed score function and B.W.M., named as the Q-B.W.M.-S-V.I.K.O.R. method. A case study of the hospitality brand management was solved by the proposed Q-B.W.M.-S-V.I.K.O.R. method. The validity and efficiency of the proposed method were illustrated by the comparative analyses and discussions with some similar methods. The advantages of the proposed Q-B.W.M.-S-V.I.K.O.R. method can be summarised as follows:

1. The score function of the q-R.O.F.S. was proposed by considering the membership degree, non-membership degree and the hesitancy degree at the same time, which is more capable to differentiate q-R.O.F.N.s than two previous score functions.
2. The B.W.M. with the q-R.O.F.N.s information was investigated from the additive consistency perspective, which transforms the nonlinear model in Rezaei (2016)

into a linear one. Then, the global optimal weights can be derived easily. The interval analysis of weights provided potential interval weights of criteria as well.

3. The Q-B.W.M.-S-V.I.K.O.R. method provides an alternative to rank alternatives with the uncertain preferences being given q-R.O.F.N.s. The method ranks alternatives based on the positive and negative ideal references two aspects. The compensate effect of performances can be rejected to some extent which guarantees the reasonability of the ranking result.

In future, we shall address different types of criteria, such as the benefit, the cost and the target, while in this article the criteria are assumed to the benefit criteria. The interactions of criteria may be a good research idea. To depict the interactions, the Sugeno integral (Grabisch, Murofushi, & Sugeno, 2000) in q-R.O.F. environment is an interesting topic. For considering the wisdom of multiple D.M.s, the group decision-making with the proposed method may be a fascinating research point with challenges (Dong, Zhang, & Herrera-Viedma, 2016; Pérez, Cabrerizo, Alonso, & Herrera-Viedma, 2014).

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

The work was supported by the National Natural Science Foundation of China (71771156, 71971145), the Scholarship from China Scholarship Council (No. 201906240161) and the Deanship of Scientific Research (DSR) at King Abdulaziz University, Jeddah (No. RG-10-611-39, No. RG-7-135-38).

Note

1. https://en.wikipedia.org/wiki/Sigmoid_function

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